2nd EDITION

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a complete manual

ENGINEERING ECONOMICS

BACHELOR OF ENGINEERING

POKHARA UNIVERSITY

SALIENT FEATURES

Precise and to the point explanations
Comprehensive Coverage of the Syllabus
Solved Out 2011 to 2020 Board Exam's Questions
Solved Out 40 Plus Relevant Numerical Examples
Systematic Subject Matter with Solved Additional Problems

Er. Sanjaya Chauwal

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CHAPTER 1

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1.1 DEFINITION OF ECONOMICS

Economics is the science that deals with the production, distribution and consumption of goods and services or the material welfare of human kind. Economics can be broken down into two main disciplines: macro economics and microeconomics. Macroeconomics deals with the behaviour of economics on a large scale, usually the economics of countries or regions. Microeconomics, on the other hand, usually addresses individual agent.

Lion Robbins in 1932 defined "Economics is the science which studies human behaviour as a relationship between ends and scare means which have alternative uses."

Marshall in 1980 defined "Economics is on the one side, a study of wealth and on the other more important side is a part of the study of man." So, according to him, economics is concerned with human welfare that means man is primary importance and wealth is only secondary importance. *i.e.*, wealth for man.

The following are the economic goals:

i) Efficiency
ii) Price stability
iii) Growth
iv) A high level of employment
v) An equitable distribution of income

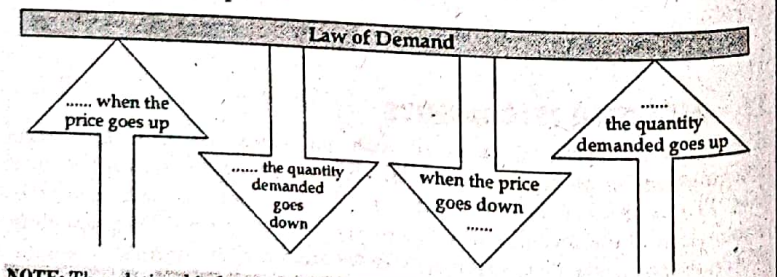
Some of the above goals are interdependent. The economic goals are not always complementary; in many cases they are in conflict. *For example;* any move to have a significant reduction in unemployment will lead to an increase in inflation.

1.2 DEMAND

Demand in economics is the consumer's desire and ability to purchase a goods or service. Demand in economics is how many goods and services are bought at various prices during a certain period of time. "Every demand or want supported by the willingness and ability to buy" constitutes demand for a particular product or services.

1.2.1 Law of Demand

Law of demand states the relationship between quantity demanded and price of the commodity. It explains the relationship between price and quantity demanded. It states: "All other things being equal, demand varies inversely with price".



NOTE: The relationship between price and quantity is inverse.

Factors Influencing Demand

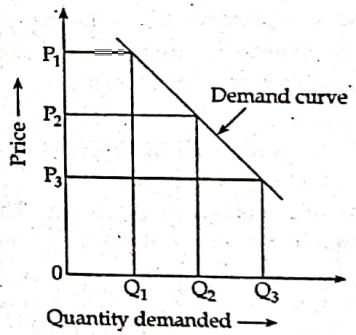
The shape of the demand curve is influenced by the following factors;

i) Income of the people
ii) Taste of consumers
iii) Prices of related goods
iv) Custom and fashions
v) Weather
vi) Future expectations
vii) Price of commodity, etc.

Description

The law of demand states that other factors being constant, price and quantity demanded of any good and service are inversely related to each other. When the price of a product increases, the demand for the same product will fall. Law of demand explains consumer's choice behavior when the price changes. In the market, assuming other factors affecting demand being constant, when the price of a good rises, it leads to a fall in the demand of that good. This is the natural consumer choice behavior. This happens because a consumer hesitates to spend more for the good with the fear of going out of cash.

Figure below shows the demand curve which is downward sloping. Clearly, when the price of the commodity increases from price P_3 to P_2, then its quantity demand comes down from Q_3 to Q_2 and then to Q_1 and vice-versa.



The higher the price of good, lower will be demand and vice-versa.

1.3 LAW OF DIMINISHING MARGINAL UTILITY

The law of diminishing marginal utility states that all else equal as consumption increases the marginal utility derived from each additional unit declines **or,** The law of diminishing marginal utility states that the additional utility gained from an increase in consumption decreases with each subsequent increase in the level of consumption. Marginal utility is the change in utility due to a one-unit change in the level of consumption.

The law of diminishing marginal utility states the marginal utility gradually decreases with the level of consumption utility being defined as benefit or satisfaction.

The law of diminishing marginal utility is similar to the law of diminishing returns which states that as the amount of one factor of production increases as all other factors of production are held the same, the marginal return (extra output gained by adding an extra unit) decreases. As the rate of commodity acquisition increases, marginal utility decreases. It commodity consumption continue to rise, marginal utility at some point may fall to zero, reaching maximum total utility. Further increase in the consumption of units of commoditities causes the marginal utility to become negative; this signifies dissatisfaction.

For examples;

i) Beyond some point, further doses of antibiotics would kill no pathogens at all and might even become harmful to the body.

ii) To satiate thirst, a person drinks water but beyond a point, consumption of more water might take the person vomit, hence leading to negative marginal and this diminished total utility.

The law is based on certain assumptions:

- Utility is measurable. The measurement unit is util.
- Continuous consumption.
- Price of the substitute do not change.
- Commodity is measurable.
- Consumer taste and preferences unchanged.
- Various units of commodity are homogenous in characteristics.
- Suitable and similar units of commodity.

Limitations:

- This law does not hold well in the rare collections. *For example;* collection of ancient coins, stamps etc.
- This law is not fully applicable to money. The marginal utility of money declines with richness but never fails to zero.
- It does not apply to the art, innovation and knowledge.
- The utility increases due to demonstration. It is a natural element.
- The law is not applicable for precious goods.
- Historical things are also included in exceptions to this law.
- Law does not operate if consumer behaves in irrational manner.

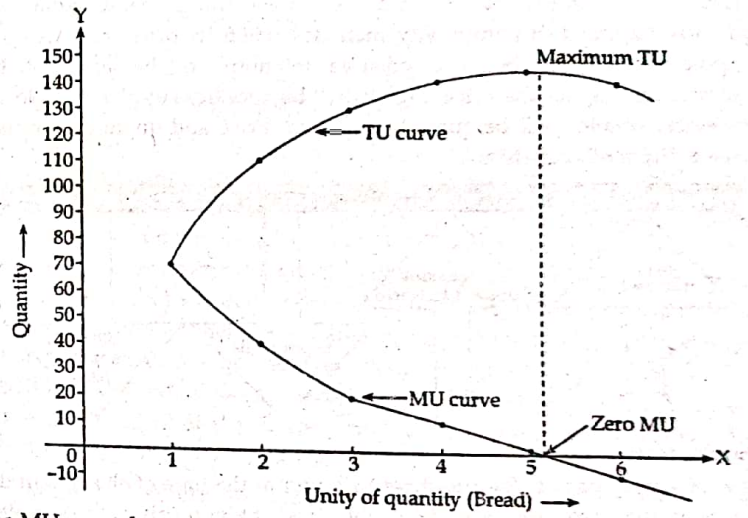
1.3.1 Explanation with Schedule and Diagram

Suppose a person eats bread and 1st unit of bread gives him maximum satisfaction. When he will eat 2nd bread, his total satisfaction would increase. But the utility added by 2nd unit of bread will be less than the 1st bread and this marginal utility will continue to decrease as he will keep

on eating additional piece of bread and once his stomach is full, marginal utility will become negative, decreasing the total utility.

Slices of bread	Total Utility (TU)	Marginal Utility (MU)
0	0	-
1	70	70
2	110	40
3	130	20
4	140	10
5	145	3
6	140	-10

In the graph below, the total utility (TU) curve is increasing at decreasing rate and it becomes maximum when marginal utility is zero. When MU is negative, TU declines plotting the above data on a graph gives,



The MU curve has negative slope and cross the OX-axis on 6th unit. He does not want to consume another *i.e.*, 6th unit gives him negative utility (-10) instead of positive satisfaction which leads to decrease his total utility by 140.

1.3.2 Utility and Marginal Utility

1.3.2.1 Utility

Utility is the level of satisfaction to a consumer by consuming goods or services *For example;* Bread satisfies hunger, TV satisfies the want for entertainment, etc.

1.3.2.2 Marginal Utility (MU)

Marginal utility refers to the extra utility a consumer gets from one additional unit of a specific product. It is the utility derived by single unit of consumption.

$$MU = \frac{\text{Change in total utility}}{\text{Change in quantity consumption}}$$

$$= TU_{N+1} - TU_N$$

Here, TU_N = Total utility up to n^{th} unit of outcome consume

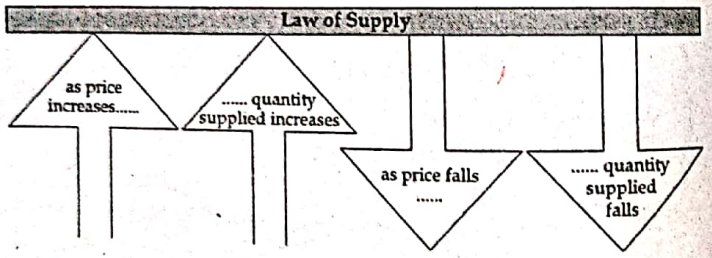
TU_{N+1} = Total utility up to $(n + 1)^{th}$ unit of outcome consume

1.4 SUPPLY

Supply is defined as the desire with ability to sell and willingness to sell. Supply is a fundamental economic concept that describes the total amount of a specific goods or service that is available to consumers

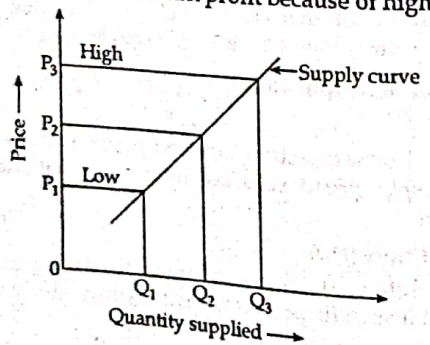
1.4.1 Law of Supply

Law of supply states the relationship between quantity supplied and price of the commodity. It states "All other things being equal, the quantity supplied of commodity increases when its price increases and vice-versa". Supply bears a positive relations to the price of the commodity. Higher the price, higher will be quantity supplied and lower the price, smaller will be quantity supplied. Price and quantity supplied have a direct relationship.



Description

Law of supply depicts the producer behavior at the time of changes in the prices of goods and services. When the price of a good rises, the supplier increases the supply in order to earn profit because of higher prices.



The above diagram shows the supply curve that is upward sloping (positive relation between the price and quantity supplied). When the price of the good was at P_3, supplies were supplying Q_3 quantity. As the price starts rising, the quantity supplied also starts rising.

Factors affecting supply

- New inventions
- Price of related goods
- Taxes and subsides
- Development of infrastructure state of natural resources
- Change in money income
- Price of commodity
- Production technology
- State of natural resources
- Weather

1.5 ELASTICITY OF DEMAND (E_d)

Inelasticity and elasticity of demand refer to the degree to which demand responds to a change in another economic factor, such as price, income level or substitute availability. Elasticity measures how demand shifts when other economic factors change. When fluctuating demand is unrelated to an economic factor, it is called inelasticity. The elasticity of demand or demand elasticity refers to how much sensitive demand for a good is compared to changes in other economic factors, such as price or income. The elasticity of demand helps companies to income. The elasticity of demand helps companies to predict changes in demand based on a number of different factors, including changes in price and the market entry of competitive goods.

The change in quantity demanded due to change in price, income etc is called elasticity of demand. It is a technical term used by the economists to describe the degree of responsiveness of the demand for the commodity to change in price and so on. The elasticity of demand is a measure of change in the quantity demanded in response to the change in the price of the commodity.

Mathematically,

$$E_d = \frac{\text{Proportionate change in quantity demanded}}{\text{Proportionate change in any one quantitative determinant of demand}}$$

The elasticity of demand is classified as:

i) Price elasticity of demand
ii) Income elasticity of demand
iii) Cross elasticity of demand

1.5.1 Price Elasticity of Demand (E_p)

The elasticity of demand is commonly referred to as price elasticity of demand because the price of a good or service is the most common economic factor used to measure it.

Price elasticity of demand (E_p) is defined as the degree of responsiveness of quantity demanded to a change in its price.

$$E_p = \frac{\text{Proportionate change in quantity demanded of the commodity}}{\text{Proportionate change the price of the commodity}}$$

$$= \frac{\left(\frac{\Delta Q}{Q}\right)}{\left(\frac{\Delta P}{P}\right)}$$

$$\therefore \quad E_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

where, ΔQ = Change in quantity demanded

ΔP = Change in price

P = Initial price

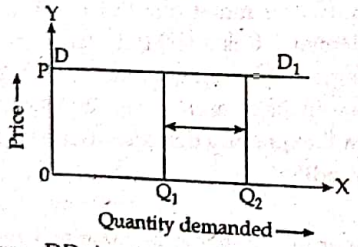
Q = Initial quantity demanded

For example, a change in the price of a luxury car can cause a change in the quantity demanded. If a luxury car producer has a surplus of cars, they may reduce their price in an attempt to increase demand. The extent of the price change will determine whether or not the demand for the good changes and if so, by how much.

Types or degree of price elasticity of demand

1. Perfectly elastic demand ($E_p = \infty$)

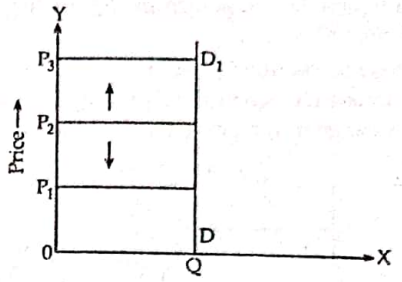
The demand is said to be perfectly elastic if the quantity demanded increases infinitely or by unlimited quantity with a small fall in price or quantity demanded falls to zero with a small rise in price.



In the figure, the curve DD_1 is a horizontal straight line parallel to the X-axis. It shows that negligible change in price causes infinite fall or rise in quantity demanded.

2. Perfectly inelastic demand ($E_p = 0$)

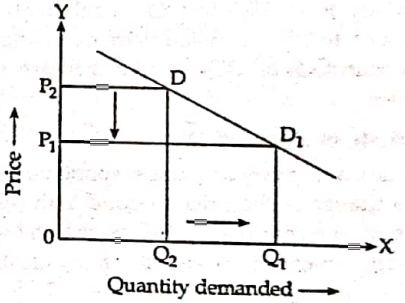
When there is no change in the demand for a product due to the change in the price, then the demand is said to be perfectly inelastic.



In the figure, the curve DD_1 is a straight vertical line which shows that the demand remains unchanged irrespective of change in the price. *i.e.*, quantity OQ remains unchanged at different prices P_1, P_2, P_3, ...

3. Relatively elastic demand ($E_p > 1$ to ∞)

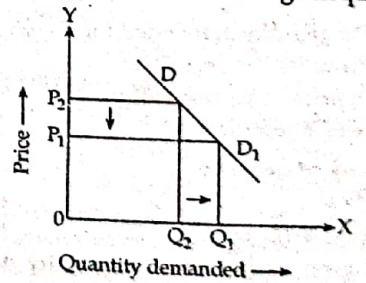
The demand is relatively elastic demand when the proportionate change in the demand for a commodity is greater than the proportionate change in its price.

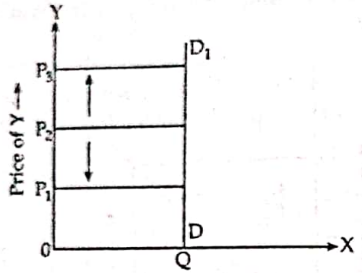


In the figure, the demand curve is gradually sloping which shows that a proportionate change in quantity from OQ_2 to OQ_1 is greater than the proportionate change in the price from OP_1 to OP_2.

4. Unitary elastic demand ($E_p = 1$)

The demand is unitary elastic demand when the proportionate change in the price of a product results in the same change in quantity demanded.





Quantity demanded for X⟶

In figure, vertical straight line DD_1 shows zero cross elasticity of demand. Here, increase in price of commodity Y (car) does not effect to demand for commodity X (cloth) because these goods are non related with each other.

1.5.3 Income Elasticity of Demand (E_Y)

The income elasticity of demand is also known as the income effect. The income level of a given population can influence the demand elasticity of goods and services.

Income elasticity of demand refers to the change in quantity demanded due to the result of change in income, other things remaining constant.

$$E_Y = \frac{\text{Proportionate change in quantity demanded}}{\text{Proportionate change in income}}$$

$$= \frac{\left(\frac{\Delta Q}{Q}\right)}{\left(\frac{\Delta Y}{Y}\right)} = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

where, ΔQ = Change in quantity demanded

ΔY = Change in income

Y = Initial income

Q = Initial demand of quantity

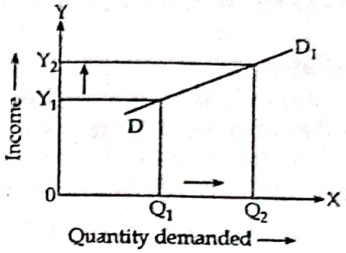
For example; Suppose than an economic event leads to many workers being laid off. During this time period, people may decide to save their money rather than upgrading their smart phones or buying designer purses. This would lead to luxury items becoming more elastic. In other words, a slight change in income level would lead to a significant change in the consumption of luxury goods.

Types/degrees of income elasticity of demand

It is of five types. They are;

1. **Greater than unity income elasticity ($E_y > 1$)**

If the percentage change in quantity demanded for a commodity is greater than percentage change in income of the consumer, it is said to be greater than unity income elasticity.

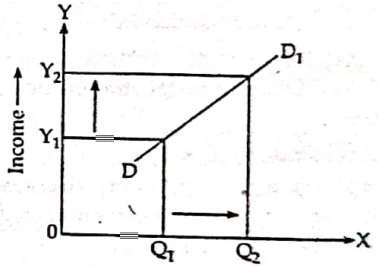


Quantity demanded ⟶

In the figure, flatter demand curve DD_1 shows income elasticity is greater than unity. When income increases from OY_1 to OY_2, demand increases from OQ_1 to OQ_2. *i.e.*, $Q_1Q_2 > Y_1Y_2$.

2. **Equal to unity income elasticity ($E_y = 1$)**

If the percentage change in quantity demanded for a commodity is equal to percentage change in income of the consumer, then such elasticity is called equal to unity.

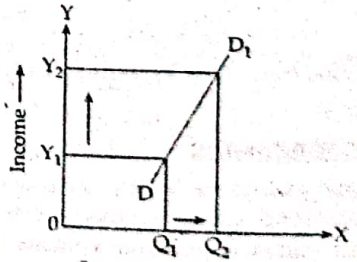


Quantity demanded ⟶

In the figure, $Y_1Y_2 = Q_1Q_2$, the increase in income from OY_1 to OY_2 leads to increase in demand from OQ_1 to OQ_2 *i.e.*, change in demand and change in income are same.

3. **Less than unity income elasticity ($E_y < 1$)**

If the percentage change in quantity demanded for a commodity is less than percentage change in income of the consumer, then such elasticity is said to be less than unity income elasticity.

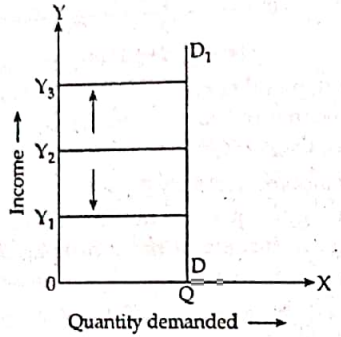


Quantity demanded ⟶

Here, the change in quantity demanded $Q_1 Q_2$ is less than the change in income $Y_1 Y_2$.

4. Zero income elasticity ($E_y = 0$)

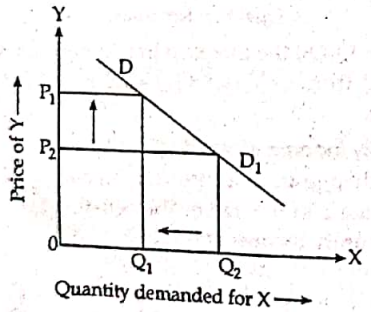
If the quantity demanded for a commodity remains constant with any rise or fall in income of the consumer, then such elasticity is known as zero income elasticity.



Here, demand of quantity remains constant whatever changes in the income of the consumer. DD_1, a vertical straight demand curve indicates zero income elasticity.

5. Negative income elasticity ($E_y < 0$)

If the quantity demanded for a commodity decreases with the rise in income of the consumer and vice-versa, then such type of elasticity is called negative income elasticity of demand.



Here, increase in income from Y_1 to Y_2 leads to the decrease in quantity demanded from Q_1 to Q_2.

1.6 ENGINEERING ECONOMICS

Science is a field of study where the basic principles of different physical systems are formulated and tested. Engineering is the application of science. It establishes varied application systems based on different scientific principles. Engineering economics deals with the methods that

enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

Engineering economics is the application of economic technique to the evaluation of design and engineering alternatives. Engineering economics is the study of how engineers choose to optimize their designs and construction methods to produce objects and systems that will optimize their efficiency and hence the satisfaction of their clients.

Engineering economics, previously known as engineering economy, is a subset of economics concerned with the use and application of economic principles in the analysis of engineering decisions.

Special characteristics of engineering economics are:

i) Engineering economics is closely related or aligned with conventional micro economics.

ii) Engineering economics is devoted to the problem solving and decision making at the operation level.

iii) Engineering economics mainly uses the body of economic concepts and principles.

iv) Engineering economics integrates economic theory with engineering practice.

v) Engineering economics removes complicated abstract issues of economical theory.

vi) Engineering economics is useful to identify alternative uses of limited resources and to select the preferred course of action.

1.7 PRINCIPLE OF ENGINEERING ECONOMICS

Engineering economics principle focus on the process used to make an economics based decision, not on the decision itself. These principles are crucial to the decision making and accomplishment of good engineering economy studies. The seven principle of engineering economics are:

i) Develop the alternatives
ii) Focus on differences
iii) Use a consistent view point
iv) Use of common unit of measure
v) Consider all relevant criteria (social and environmental aspect)
vi) Make uncertainty explicit
vii) Revisit your decision (self evaluation)

I) Develop the alternatives

The alternatives need to be identified and then defined for subsequent analysis. Since the decision is among alternatives, developing and defining comprehensive list of alternatives for detailed evaluation is important creativity and innovation are essential.

ii) Focus on differences

Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision. Outcomes that are common to all alternatives can be disregarded in the process of compassion and decision.

iii) Use a consistent viewpoint

The prospective outcomes of the alternatives, selection of the criteria and other, should be consistently developed from a defined viewpoint. For *example*; the perspective of employees is used for the problem of designing the employee benefit package.

iv) Use a common unit of measure

Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will simplify the analysis of the alternatives. For measuring economic consequences, a monetary unit such as dollars is the common measure. Using more than one monetary unit for economic analysis will complicate the overall analysis of a project.

v) Consider all relevant criteria

Selection of a preferred alternative (decision making) requires the use of a criteria (or several criteria). *For example*; long term interest.

vi) Make uncertainty explicit

Risk and uncertainty are inherent in estimating the future outcomes of the alternatives and should be recognized in the analysis and comparison.

vii) Revisit your decisions

Improved decision making results from an adaptive process. The initial projected outcomes of the selected alternative should be subsequently compared with the actual results achieved.

The first two principles setup the thought process. The next three principles focus on evaluation criteria and last two principle focus on analysis.

1.8 APPLICATION OF ENGINEERING ECONOMY

Decisions are made routinely to choose one alternative over another by individuals in everyday life; by engineers on the job; by managers who supervise the activities of others; by corporate presidents who operate a business; and by government officials who work for the public good. Most decisions involve money called capital or capital funds, which is usually limited in amount. The decision of where and how to invest this limited capital is motivated by a primary goal of adding value as future, anticipated results of the selected alternative are realized. Engineers play a vital role in capital investment decisions based upon their ability and experience to design analyze and synthesis. The factors upon which a

decision is based are commonly a combination of economic and non-economic elements. Engineering economy involves formulating, estimating and evaluating the expected economic outcomes of alternatives designed to accomplish a defined purpose. Mathematical techniques simplify the economic evaluation of alternatives. Because the formulas and techniques used in engineering matters, they are equally useful in business and government, as well as for individuals.

An engineering economy study involves many elements: Problem identification, definition of the objective, cash flow estimation, financial analysis and decision making. Implementing a structured procedure is the best approach to select the best solution to the problem.

The steps in an engineering economy study are as follows:

a) Indentify and understand the problem; identify the objective of the project.
b) Collect relevant, available data and define viable solution alternatives.
c) Make realistic cash flow estimates.
d) Identify an economic measure of worth criterion for decision making.
e) Evaluate each alternative; consider non-economic factors; use sensitive analysis as needed.
f) Select the best alternative.
g) Implement the solution and monitor the results.

The application of engineering economy are diverse and found in most areas of an organization. Engineering economy enables Engineers to consider all aspects of investment from both the technical and financial view points. Engineering economy furnishes several pattern of analysis to determine rate of return, annual costs and payout periods which all serve as bases for decision.

They are typically classified as:

i) Equipment replacement
ii) Equipment and process selection
iii) Cost reduction
iv) Capital budgets allocation
v) Service improvement
vi) New product and production expansion, etc.

8. What are the factors affecting demand of a commodity? [2012/Spring, 2018/Fall]

Answer:

The factors affecting demand of a commodity are:

i) Price of commodity
ii) Size of population
iii) Future expectation
iv) Weather
v) Custom and fashion
vi) Income of consumer
vii) Price of related goods
viii) Quality of goods, etc.

9. Define supply. [2014/Fall, 2016/Fall]

Answer: See the topic 1.4.

10. Explain law of supply. [2012/Spring, 2013/Spring]

Answer: See the topic 1.4.1.

11. Define utility/ marginal utility. [2012/Spring, 2010/Fall, 2014/Fall, 2016/Fall]

Answer: See the topic 1.3.2.

12. Explain law of diminishing utility with suitable example and figure. [2010/Fall, 2011/Spring, 2012/Fall, 2012/Spring, 2018/Fall]

Answer: See the topic 1.3.

13. Define elasticity of demand. [2011/Fall, 2011/Spring, 2013/Fall, 2010/Fall, 2014/Spring]

Answer: See the topic 1.5.

14. Describe three kinds of elasticity of demand. [2011/Fall, 2013/Spring, 2010/Fall, 2016/Fall]

Answer: See the topic 1.5.

15. Describe all types of price elasticity of demand with suitable figures. [2014/Spring]

Answer: See the topic 1.5.1.

16. What do you mean by demand, elasticity of demand and types of elasticity of demand? [2019/Fall]

Answer: See the topics 1.2, 1.5, 1.5.1, 1.5.2 and 1.5.3

17. How you justify the statement. "Demand creates supply"? [2020/Fall]

Answer: See the additional question number 1.

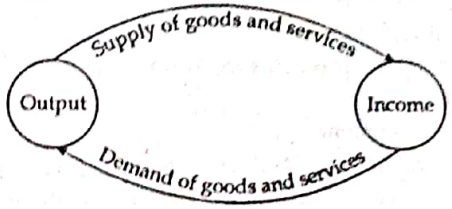
ADDITIONAL PROBLEMS

1. "Supply creates its own demand". Justify this statement

Answer:

J.B. say, a French economist of 19th century, arrested that, "supply creates its own demand". This appears to be simple proposition but has had many different meanings, and many set of reasoning. According to this concept.

Whenever any product is produced, the demand of that product is also simultaneously generated on account of or through the process of payment of remuneration to the factors of production. In other words, every output produced, results in an equivalent demand being generated which leads to its sale so that there is no surplus output or over production.



Say's law is explained with the help of simplified circular flow figure. Say's law means that supply creates its own demand for goods and services.

Jean Baptiste Say, as a French Economist is most commonly identified with Say's Law which states that "Supply creates its own demand". Goods and services produced can be absorbed by demand until Market Equilibrium. Classical Economist said that lack of aggregate demand in an economy won't happen. Goods produced by an economy will be bought by the society. The Substantiality of Say's Law is to reinforce that market can be an instrument of efficient resource allocation through exchange of economics. Based on that statement we also can say that, when supply is increased, the price decreases. A decrease in price leads to an increase in demand. So technically supply creates its own demand.

Say's Law also has various interpretations. The long-run version is that there cannot be overproduction of goods in general for a very long time because those who produce the goods, by their act of producing, produce the purchasing power to buy other goods. So, certainly in the long-run version is correct. Given enough time, supply will create its own demand.

In another side, there are also economists that argued with Say's Law. They are "Deman-Siders" also known as Keynessians. They said that "Demand creates Supply". They insist that short-run economic fluctuations are caused by shocks to the economy that leave aggregate demand below full capacity. In their view since less money is spent on goods and services, businesses must lay people off, which further reduces

spending, resulting in more layoffs, causing even less spending, and so the cycle goes. The policy argument is that government must step in and spend in order to prop up total demand. So, it means that through demand, supply will be created.

2. "Engineering economics is all about decision making." Explain.

Answer:

Economic analysis and decision making are important engineering considerations because Engineers are capable of evaluating decisions both technically and from a business perspective. The term engineering economic decision refers to all investment decisions relating to engineering projects. The five main types of engineering economic decisions are:

- Equipment and process selection
- Equipment replacement
- New product and product expansion
- Cost reduction
- Service improvement

Engineering economics helps to deal with the identification of economic choices, and is concerned with the decision making of engineering problems of economic nature. The factors of time and uncertainty are the defining aspects of any engineering economic decisions. A decision is simply the selection of best one from two or more options in course of action whether it takes place in construction or production, service or manufacturing industries, private or public agencies.

The techniques and methodology of engineering economy can assist in decision making process from the following grounds,

- Understand the problem
- Define objective
- Collect relevant information
- Develop possible alternatives
- Identify the criteria for decision making
- Select the best alternative
- Evaluate each alternative
- Effective implementation

The principles of engineering economics are mandatory to the decision making. Hence, "Engineering economics is all about decision making".

CHAPTER 2

COST CONCEPT AND FUNDAMENTALS OF COST ACCOUNTING

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2.1 COST

Cost refers to the amount of expenditure incurred in acquiring something. In general terms, cost refers to an amount to be paid or given up for acquiring any resources or services. The organization decision of maximizing profit depends on the behavior of its cost and revenues.

In economics, cost can be defined as monetary valuation of efforts, materials, resources, time and utilities consumed, risk incurred and opportunity forgone in production of a good or service.

Cost can be broadly classified into variable and overhead cost. Variable cost varies with the volume of production while overhead cost is fixed, irrespective of the production volume.

Variable cost can be further classified into direct material cost, direct labour cost and direct expenses. The overhead cost can be classified into factory overhead, administration overhead, selling overhead and distribution overhead. Direct material costs are those costs of materials that are used to produce the product. Direct labour cost is the amount of wages paid to the direct labour involved in the production activities. Direct expenses are those expenses that vary in relation to the production volume, other than the direct material costs and direct labour costs.

Overhead cost is the aggregate of indirect material costs, indirect labour costs and indirect expenses. Administration overhead includes all the costs that are incurred in administering the business. Selling overhead is the total expense that is incurred in the promotional activities and the expenses relating to sales force. Distribution overhead is the total cost of shipping the items from the factory site to the customer sites.

The selling price of a product is derived as shown below;

i) Direct material costs + Direct labour costs + Direct expenses = Prime cost.

ii) Prime cost + Factory overhead = Factory cost.

iii) Factory cost + Office and administrative overhead = Costs of production.

iv) Cost of production + Opening finished stock = Closing finished stock = Cost of goods sold.

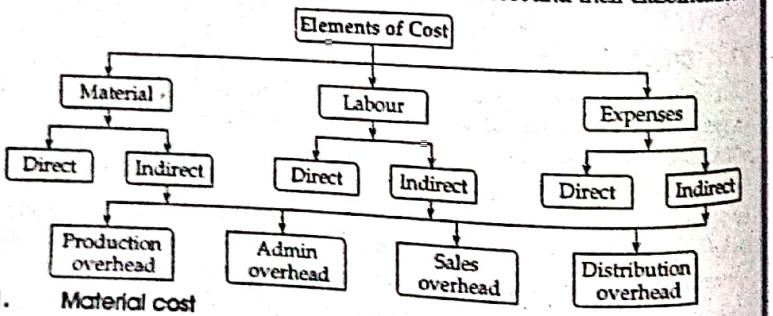
v) Cost of goods sold + Selling and distribution overhead = Cost of sales.

vi) Cost of sales + Profit = Sales.

vii) Sales/Quantity sold= Selling price per unit.

In the above calculations, if the opening finished stock is equal to the closing finished stock, then the cost of production is equal to the cost of goods sold.

The following chart shows the various elements of cost and their classification:



1. Material cost

Material implies a substance from which a product is made. This include the cost of commodities and materials used by the organization.

i) Direct material cost

Refers to a material that is directly related to specific product, job or process. *For example;* golds for making jewellery, etc.

ii) Indirect material cost

Materials which cannot be identified with the individual cost centre, assist the manufacturing process and doesn't become an integral part of finished goods. *For example;* oils and lubricants, etc.

2. Labour cost

Labour cost is the main element of cost. An organization requires labour to convert raw materials into finished goods.

i) Direct labour cost

Refers to labour that takes an active part in manufacturing a product. The costs related to direct labour are called labour costs. These costs very directly with change in the level of output thus it is referred as a variable express.

ii) Indirect labour cost

The indirect labour costs may or may not vary with the change in the volume of output.

3. Expenses

Refers to the costs that are incurred in production of finished goods other than material costs and labour cost.

i) Direct expenses

Imply the expenses that are directly or easily allocated to a particular cost centre or cost units. These expenses are called chargeable expenses. *For example;* cost of patents, royalties, etc.

ii) Indirect expenses

Refers to expenses that cannot be allocated to specific cost center or cost units. *For example;* rent, depreciation, insurance, etc.

4. Overhead cost

Overhead is defined as the aggregate of the cost of indirect materials, indirect labour and such other expenses including services as cannot conveniently be charged direct to specific cost of units. Thus, overhead are all expenses other than direct expenses.

i) Manufacturing overheads

It is the indirect expenses of operating manufacturing divisions of a concern and covers all indirect expenditure incurred. *For example;* repairs and maintenance of taxed assets, wages of indirect labour, etc.

ii) Administrative overhead

It is the indirect expenditure incurred in formulating the policy, directing the organization, controlling and managing the operations of an undertaking which is not related directly to a research, development or production. *For example;* office rent, light, heat, credits approval, etc.

(iii) Sales overhead

It refers to those indirect costs which are associated with marketing and selling (excluding distribution) activities. *For example;* advertisement charges, samples and free gifts, etc.

(iv) Distribution overhead

It comprises all expenditure incurred from the time the product is completed in the works until it reaches its destination. *For example,* expenses on delivery van trucks, ware house staff salaries, etc.

2.1.1 Manufacturing Cost

Manufacturing cost is the sum of costs of all resources consumed in the process of making a product. The manufacturing cost is defined as the costs that are incurred during the production of a product. These costs include direct material costs, direct labour costs and manufacturing overhead costs. Manufacturing cost = cost of (labour + overhead + materials).

Direct material	Milk for curd, cloth in pant, plastic for toys, wood for table, etc.
Direct labour	Salary for labour, salary for security guards, salary for supervisors, etc.
Direct expenses	Tax, depreciation, insurance, cost of patents, design cost, etc.

2.1.2 Non-Manufacturing Cost

Non-manufacturing costs are the cost for the company operations that are not directly related to manufacturing. It includes marketing costs and administrative costs. Marketing costs include all costs necessary to secure customer orders and get the finished product or service into hands of and get the finished product or service into hands of the customer.

For example; marketing costs-secure customer order, shipping, commissions, advertisement, etc.

For example; Administrative costs-public relations cost, management salaries, office supplies, etc.

2.2 COST FOR BUSINESS DECISION

2.2.1 Differential Cost

Differential cost is the difference between cost of two alternative decisions or of a change in output levels. A differential cost can be a variable cost, a fixed cost or a mix of the two-there is no differentiation between these types of costs. *For example;* Decision regarding purchase of the machinery out of two. The differential cost is the difference in the prices paid plus the cost of operating the machines.

2.2.2 Differential Revenue

Differential revenue is the difference in sales that will be generated by two different courses of action. The concept is commonly used when evaluating which of two (or more) investments to make in a businesses.

For example; You have a job paying Rs. 30,000 per month in your hometown. You have a job offer in neighbouring city that pays Rs. 35,000 per month. The commuting cost to the city is Rs. 2,000.

Hence, differential cost = Rs. 2,000

Differential revenue = Rs. 35,000 – Rs. 30,000 = Rs. 5,000

2.2.3 Opportunity Cost

Opportunity cost is defined as the potential benefit that is given up as you seek an alternative course of action. In fact, virtually every alternative has some opportunity cost associated with it.

In an economic sense, opportunity cost could mean the contribution to income that is forgone by not using a limited resource in the best way possible. Or we may view opportunity costs as cash flows that could be generated from an asset the firm already owns, provided that such flows are not used for the alternative in question. In general, accountants do not post opportunity cost in the accounting records of an organization. However, this cost must be explicitly considered in every decision.

For example; The opportunity cost of starting your own business in the wages you give up by working for another company.

For example; The opportunity cost of watching TV on a weeknight is the benefit you could have gotten from studying.

2.2.4 Sunk Cost

Sunk cost is a cost that has already been incurred and cannot be recovered. Sunk costs are not relevant to decisions, because they cannot be changed regardless of what decision is made now or in the future.

For Example;

- You bought a bike that cost Rs. 2 lakh two years ago. The 2 lakh cost is sunk because whether you drive it, park it, trade it, you cannot change that 2 lakh cost.
- Cost of marketing
- Feasibility study

2.2.5 Marginal Cost

Marginal cost is defined as the added cost that would result from increasing the rate of output by a single unit. The marginal cost of production is the change in total costs that come from making one additional item. The marginal cost formula can be used in financial modeling to optimize the generation of cash flow.

The revenue that can be obtained from selling one more unit of product is

called marginal revenue. The cost involved in producing one more unit of product is called marginal cost. The difference between the unit sales price and the unit variable cost is the producers marginal contribution, also known as marginal income. This means that each unit sold contributes towards absorbing the company's fixed costs.

$$\text{Marginal cost} = \frac{\text{Change in costs}}{\text{Change in quantity}}$$

$$\therefore \quad MC = \frac{\Delta TC}{\Delta Q}$$

For example; if a firm produces 'X' unit at a cost of Rs. 300, 'X + 1' units at a cost of Rs. 320, then the cost of an additional unit will be Rs. (320–300) = Rs. 20 which is marginal cost.

2.2.6 Fixed Cost

Fixed costs are the cost that the firm has to pay independently of whether it is operating or not. *For example;* rent of a building

Fixed costs are expenses that have to be paid by a company, independent of any business activity.

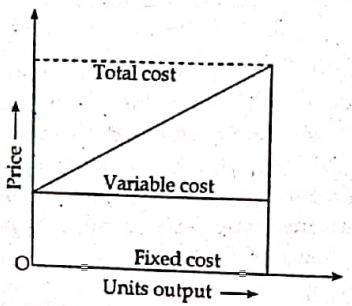
2.2.7 Variable Cost

Variable costs are the cost that varies with the level of output.

For example; Wages paid to labour, cost of raw materials, etc.

2.2.8 Total Cost

Total cost is the summation of fixed cost and the variable cost



2.2.9 Prime Costs

The sum total of direct material, direct labour and direct expenses. *For example;* cost of raw materials.

2.2.10 Overhead Cost

Overhead cost refer to an ongoing expense of operating a business. There are essentially two types of business overheads-administrative overheads and manufacturing overheads.

For example; telephone bills, legal fees, supplies, repairs, etc.

BOARD EXAM SOLVED PROBLEMS

1. Following are the data for the production of 100 badminton rackets:

Labour rate	**Rs. 40/hr**
Leather	**50 m at Rs. 200/meter**
Gut	**300 m at Rs. 50/meter**
Graphite	**100 kg at Rs. 200/kg**

Total annual factory overhead = Rs. 5,00,000

Total annual direct labour hour = 25,000 hours

Labour hours needed = 200 hours

Show the cost breakdown and calculate the total cost per racket.

[2015/Fall]

Solution:

Cost	Amount (Rs)
Manufacturing cost:	
Direct material cost:	
Leather (50 m × Rs. 200)	10,000
Gut (300 m × Rs. 50)	15,000
Graphite (100 kg × Rs. 200)	20,000
Direct labour cost (200 hrs × Rs. 40)	8,000
Direct overhead cost	5,00,000
Non-manufacturing cost:	
Sales and marketing	-
Administrative	-
Indirect over head cost	-
Total cost for 100 rackets	5,53,000
∴ Total cost per racket	5,530

2. Write short notes on: Type of cost **[2016/Fall]**

Answer: Types of cost are:

i) Fixed and variable costs
ii) Direct and indirect costs
iii) Actual and opportunity costs
iv) Real and prime costs
v) Implicit and explicit costs
vi) Total costs, marginal cost and average costs
vii) Sunk costs and incremental costs
viii) Recurring costs and non-recurring costs
ix) Historical costs and replacement costs
x) Short run costs and long-run costs

See the topic 2.2.

3. Explain different types of cost involved in manufacturing products with suitable example. [2015/Spring]

Answer: See the topic 2.1.1

4. Define

i) Manufacturing cost: [2014/Spring, 2016/Spring, 2018/Spring]

Answer: See the topic 2.1.1

ii) Non-manufacturing cost: [2014/Spring, 2016/Spring, 2018/Spring]

Answer: See the topic 2.1.2

iii) Opportunity cost: [2013/Fall, 2016/Spring, 2017/Fall, 2018/Fall, 2018/Spring]

Answer: See the topic 2.2.3

iv) Sunk cost: [2013/Fall, 2017/Fall, 2018/Fall, 2018/Spring]

Answer: See the topic 2.2.4

v) Marginal cost: [2017/Fall, 2018/Fall]

Answer: See the topic 2.2.5.

vi) Fixed cost: [2013/Fall, 2018/Fall]

Answer: See the topic 2.2.6

vii) Variable cost: [2013/Fall, 2018/Fall]

Answer: See the topic 2.2.7

viii) Differential cost: [2018/Fall]

Answer: See the topic 2.2.1

ix) Prime costs: [2018/Fall]

Answer: See the topic 2.2.9

x) Overhead costs: [2018/Fall]

Answer: See the topic 2.2.10

5. What do you mean by cost? Explain the elements of cost. [2012/Spring, 2012/Fall, 2014/Fall]

Answer: See the topic 2.1.

6. Explain manufacturing cost, non-manufacturing cost, opportunity cost and marginal cost with suitable examples. [2019/Fall], [2020/Fall]

Answer: See the topics 2.1.1, 2.2.2, 2.2.3 and 2.2.5.

CHAPTER 3

TIME VALUE OF MONEY

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3.1 INTEREST

Interest is the money paid regularly at a particular rate for the use of money lent or for delaying the repayment of a debt.

In other word, interest rate is the rental value of money. It represents the growth of capital per unit period. The period may be a month, a quarter, semi-annual or a year. Money can be lent and unpaid in many ways and equally, usually, however, at the end of each interest period, the interest earned on the end of each interest period, the interest earthed on the principal amount is calculated according to a specified interest rate. The

two computational schemes for calculating this earned interest are said to yield either simple interest or compound interest. Engineering economic analysis uses the compound-interest scheme almost exclusively.

3.1.1 Simple Interest

Simple interest is the interest earned on only the principal amount during each interest period. In other word, the interest earned during each interest period does not earn additional interest in the remaining period even though you do not withdraw it.

In general, for a deposit of P rupees at a simple interest rate of i for N periods, the total earned interest would be,

$$I = (iP)\,N$$

The total amount available at the end of N periods thus would be,

$$F = P + I = P + iP\,N = P\,(1 + iN)$$

Simple interest is commonly used with add-on loans or bonds.

where, P = A sum of money at a time chosen as time zero for purpose of analysis; sometimes referred to as the present value or present worth.

F = A future sum of money at the end of the analysis period. This sum may be specified as E_N.

i = Interest rate per interest period.

N = Total number of interest periods.

3.1.2 Compound Interest

Compound interest is the addition of interest to the principal sum of a loan or deposits, or in other words, interest on interests.

It is the result of reinvesting interest, rather than paying it out, so that interest in the next period is then earned on the principal sum plus previously, accumulated interest. Compound interest is standard in finance and economics. When calculating compound interest, the number of compounding periods make a significance difference. The basic rule is that the higher the number of compounding periods, the greater the amount of compound interest.

This interest earning process repeats and after N periods, the total accumulated value, F will grow to $F = P\,(1 + i)^N$

where, i = Effective interest rate per interest period

3.1.3 Effective Interest Rate

The actual rate of interest earned during one year is called effective interest rate.

The effective annual interest rate is the real return on savings account or any interest-paying investment when the effects of compounding over time are taken into account. It also reveals the real percentage rate owned in interest on a loan, a credit rate or any other debt.

Assuming that the nominal interest rate is r, and M compounding periods occur during the year, we can calculate the effective annual interest rate.

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1 \qquad (1)$$

when m = 1, we have the special case of annual compounding when m = 1 equation (1) reduces it to $i_a = r$. That is when compounding takes place once annually, the effective interest is equal to the nominal interest.

3.1.4 Nominal Interest Rate

An interest is called nominal if the frequency of compounding period is not identical to the time unit (usually a year).

The nominal interest rate, also known as an annualized percentage rate or APR is the periodic interest rate multiplied by the number of periods per year. *For example*; a nominal annual interest rate of 12% based on monthly compounding means a 1% interest rate per month (compounded). A nominal interest rate for compounding periods less than a year is always lower than the equivalent rate with annual compounding.

It is denoted by r.

$$r = N\left(\frac{T}{P}\right)^{\frac{1}{Ny}} - N$$

Relationship between I and r

Let 'r' be the nominal annual interest rate, 'i' be the effective annual interest rate, 'm' be the number of compounding period per year, 'n' be the invested years and 'P' be the principal amount.

The equivalence between the two rates suggests that if a principal 'P' is invested for 'n' years, the two compound amounts, would be same, *i.e.*,

$$\text{or,} \quad P\,(1 + i)^N = P\left(1 + \frac{r}{m}\right)^{Nm}$$

Dividing both sides by P,

$$\text{or,} \quad (1 + i)^N = \left(1 + \frac{r}{m}\right)^{Nm}$$

Taking the n^{th} root of both sides result in,

$$\text{or,} \quad (1 + i) = \left(1 + \frac{r}{m}\right)^m$$

$$\therefore \quad i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

When m = 1, $i_{eff} = r$ and

$$\text{or,} \quad \left(1 + \frac{r}{m}\right)^m = 1 + i_{eff}$$

$$\text{or,} \quad \left(1 + \frac{r}{m}\right) = (1 + i_{eff})^{1/m}$$

or, $\frac{r}{m} = (1 + i_{eff})^{1/m} - 1$

$\therefore \quad i_{monthly} = (1 + i_{eff})^{1/m} - 1$

3.2 ECONOMIC EQUIVALENCE

Calculations for determining the economic effects of one or more cash flows are based on the concept of the economic equivalence. Economic equivalence exists between cash flows that have the same economic effect and therefore could be traded for one another in the financial market place, which we assure to exist.

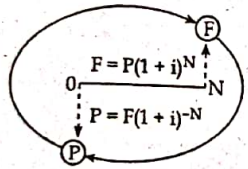
Economic equivalence refers to the fact that a cash flow whether a single payment or a series of payments can be converted to an equivalent cash flow at any point in time.

For example; we could find the equivalent future value F of a present amount P at interest rate i at period n; or we could determine the equivalent present value P of N equal payments A.

The preceding strict concept of equivalence which limits us to converting a cash flow into another equivalent cash flow, may be extended to include the comparison of alternatives. For example, we could compare the value of two proposals by finding the equivalent value of each at any common point in time. If financial proposals that appear to be quite different turn out to have the same monetary value, then we can be economically indifferent to choosing between them. In terms of economic effect, one would be an even exchange for the other, so no reason exists to prefer one over the other in terms of their economic value.

Using compound interest to establish economic equivalence

- If you deposit P dollars today for N periods at i, you will have F dollars at the end of period N.
- F dollars at the end of period N is equal to a single sum of P dollars now if your earning power is measured in terms of the interest rate i.



3.2.1 Present Worth (PW)

In this method, all the cash inflows and outflows are discounted to the beginning points at an interest rate for the economic study.

Present value is the concept that states an amount of money today is worth more than that same amount in the future. In other words, money received in the future is not worth as much as an equal amount received today.

For example, if an investor receives Rs. 10,000 today and can earn a rate of return 5% per year, the Rs. 10,000 today is certainly worth more than receiving Rs. 10,000 five years from now. If an investor waited five years for Rs. 10,000 there would be opportunity cost or the investor would lose out on the rate of return for the five years.

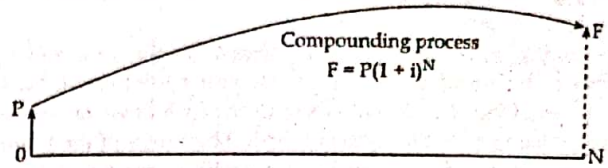


Figure: Finding an equivalent future value of a current cash payment

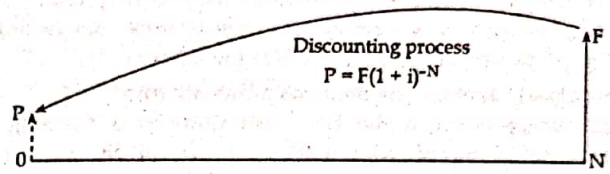
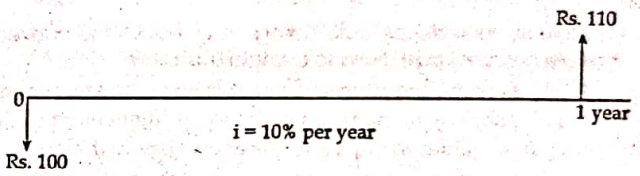


Figure: Finding an equivalent present value of a future cash payment

For example; Different sum of money at different times may be equal in economic value at a given rate.



Here, Rs. 100 now is economically equivalent to Rs. 110 one year from now, if the Rs. 100 is invested at a rate of 10% per year.

3.2.2 Annual Worth (AW)

It provide basis for measuring investment worth in a series of equal payments at the each period.

The annual worth is commonly used for comparing alternatives. Annual worth means that all incomes and disbursements (irregular and uniform) are converted into an equivalent uniform annual (end-of-period) amount, which is the same each period.

3.2.3 Future Worth (FW)

The future worth of money relates how much a current investment will be worth in the future.

The future value or future worth is important to investors and financial planners as they use it to estimate how much an investment made today will be worth in the future. Knowing the future value enables investors to

make sound investment decisions based on their anticipated needs. However, external economic factors, such as inflation can adversely affect the future value of the asset by eroding its value.

3.2.4 Equivalence Calculations: General Principles

I) Principle I: Equivalence calculations made to compare alternatives require a common time basis

When selecting a point in time at which to compare the value of alternative cash flows, we commonly use either the present time, which yields what is called the present worth of the cash flows or some point in the future, which yields their future worth. The choice of the point in time often depends on the circumstances surrounding a particular decisions, or it may be chosen for convenience. For instance, if the present worth is known for the first two of three alternatives, all three may be compared simply by calculating the present worth of the third.

II) Principle II: Equivalence depends on interest rate

The equivalence between the two cash flows is a function of the magnitude and timing of individual cash flows and the interest rate or rates that operate on those flows. This principle is easy to grasp in relation to our simple example; Rs. 1,000 received now is equivalent to Rs. 1,762.34 receive five years from now only at a 12% interest rate. Any change in the interest rate will destroy the equivalence between these two sums.

III) Principle III: Equivalence calculations may require the conversion of multiple payment cash flows to a single cash flow

Part of the task of comparing alternative cash flow series involves moving each individual cash flow in the series to the same single point in the time and summing these values to yield a single equivalent cash flow.

IV) Principle IV: Equivalence is maintained regardless of point of view

As long as we use the same interest rate in equivalence calculations, equivalence can be maintained regardless of point of view.

3.3 CASH FLOW DIAGRAM

The sums of money recorded as receipts or disbursements in a projects financial records are called cash flows. Examples of cash flows are deposits to a bank, dividend interest payments, loan payment, operating and maintenance costs and trade-in salvage on equipment whether the cash flow is considered to be a receipt or disbursement, depends on the project under consideration. *For example*, interest paid on a sum in a bank account will be considered a disbursement to the bank and a receipt to the holder of the account.

The graphical representation of the cash flows *i.e.*, both cash out flows and cash inflows with respect to a time scale is generally referred as cash flow diagram. The cash outflows (*i.e.*, expenses) are generally represented by vertical downward arrows whereas the cash inflows (*i.e.*, revenue or

income) are represented by vertically upward arrows. The number of interest period is shown on the time scale.

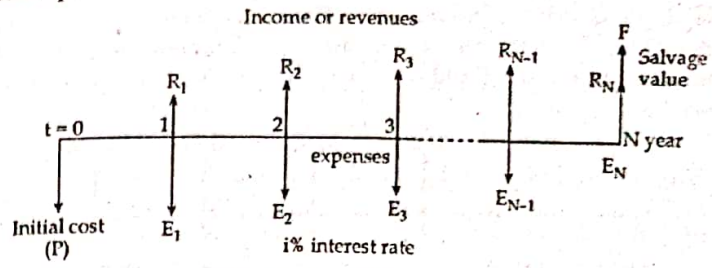


Figure: Cash flow diagram

The following conventions are used to standardize cash flow diagrams;

i) The horizontal (time) axis is marked off in equal increments one per period, up to the duration of the project.

ii) Receipts are represented by arrow directed upward. Disbursements are represented by arrows directed downward. The arrow length is approximately proportional to the magnitude of the cash flow.

iii) Two or more transfers in the same period are place end to end and these may be combined.

iv) Expenses occurred before t = 0 are called sunk costs. Sunk costs are not relevant to the problem unless they have tax consequences in an after tax analysis.

3.3.1 Types of Cash Flows

We can classify class flow transactions into five category, they are;

i) Single cash flow.
ii) Uniform series cash flow.
iii) Linear gradient series cash flow.
iv) Geometric gradient series cash flow.
v) Irregular or mixed series cash flow.

I) Single cash flow

The simplest case involves the equivalence of a single present amount and its future worth. Thus, the single cash flow formulas deal with only two mounts; a single present amount P and its future worth F.

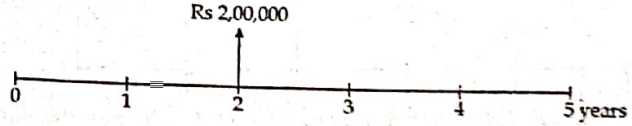


Figure: Single cash flow

II) Equal or uniform series

The most familiar category includes transactions arranged as a series of equal cash flows at regular intervals, known as an equal payment series

or uniform series. *For Example*, this category describes the cash flows of the common installment loan contract which arranges the repayment of a loan in equal periodic installments. The repayment of loan in equal periodic installments. The equal-cash-flow formulas deal with the equivalence relations P, F and A (the constant amount of the cash flows in the series).

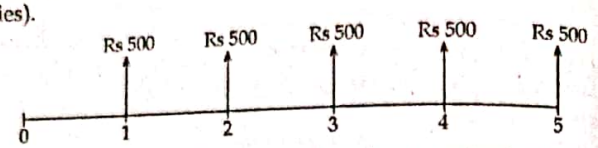


Figure: Equal or uniform payment series at regular intervals.

III) Linear gradient series

While many transactions involve series of cash flows, the amounts are not always uniform, they may, however, vary in some regular way. One common pattern of variation occurs when each cash flow in a series increases or decreases by a fixed amount (figure).

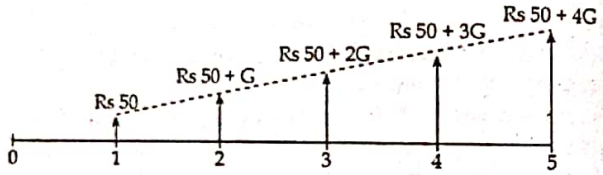


Figure: Linear gradient series where each cash flow in the series increases or decreases by a fixed amount G.

In addition to using P, F and A, the formulas employed in such problem involve a constant amount G of the change in each cash flow.

IV) Geometric gradient series

Another kind of gradient series is formed when the series in a cash flow is determined not by some fixed amount like Rs. 500 but by some fixed rate, expressed as a percentage. In the formulas dealing with such series the rate of change is represented by a lowercase g.

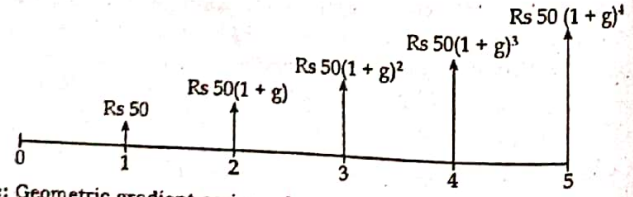


Figure: Geometric gradient series, where each cash flow in the series increases or decreases by a fixed rate (percentage) g.

V) Irregular or mixed series

A series of cash flows may be irregular, in that it does not exhibit a regular overall pattern. Even in such a series, however, one or more of the

patterns already identified may appear over segments of time in the total length of the series. The cash flows may be equal. *For example*, for 5 consecutive periods in a 10-period series. When such patterns appear, their results included in calculating an equivalent value for the entire series.

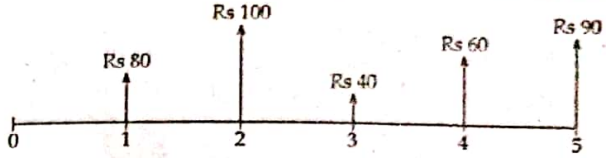


Figure: Irregular payment series which exhibits no regular overall pattern.

3.3.2 Interest formulas

The notations which are used in various interest formulae are as follows;

P = Principal amount

n = Number of interest periods

i = Interest rate

F = Future amount at the end of year n.

A = Equal amount deposited at the end of every interest period.

G = Uniform amount which will be added/subtracted period after period to/from the amount of deposit A_1 at the end of period.

I) Single payment compound amount

Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram of this situation is shown in figure.

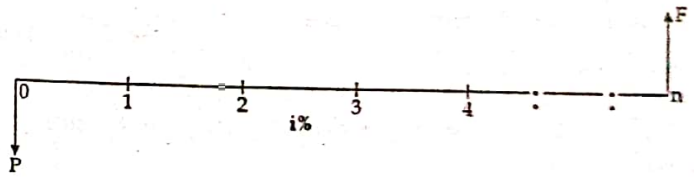


Figure: Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is,

$$F = P(1 + i)^n$$
$$= P(F/P, i, n)$$

where, (F/P, i, n) is called as single-payment compound amount factor.

Example

A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution:

Here;

P = Rs. 20,000

i = 18% compounded annually

n = 10 years

$F = P(1 + i)^n$

$= P(F/P, i, n)$

$= 20,000 (F/P, 18\%, 10)$

$= 20,000 \times 5.234$

= Rs. 1,04,680

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

ii) Single-payment present worth amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period. The corresponding cash flow diagram is shown in figure.

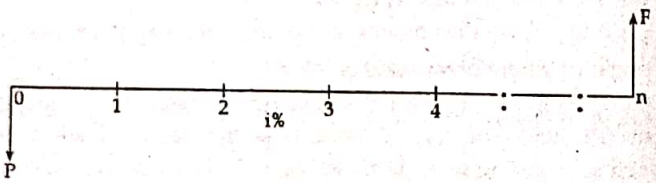


Figure: Cash flow diagram of single- payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1 + i)^n} = F(P/F, i, n)$$

where, (P/F, i, n) is termed as single-payment present worth factor.

Example

A person wishes to have a future sum of Rs. 1,00,000 for his son; education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution:

Here;

F = Rs. 1,00,000

i = 15% compounded annually

n = 10 years

$$P = \frac{F}{P(1 + i)^n} = F(P/F, i, n)$$

$= 1,00,000 (P/F, 15\%\ 10)$

$= 1,00,00 \times 0.2472$

= Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

iii) Equal-payment series compound amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the n^{th} interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in figure.

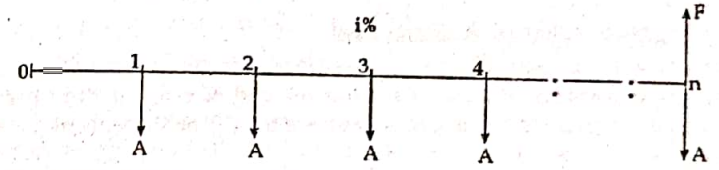


Figure: Cash flow diagram of equal-payment series compound amount.

Here; A = Equal amount deposited at the end of each interest period

n = Number of interest periods

i = Rate of interest

F = Single future amount

The formula to get F is,

$$F = A\frac{(1 + i)^n - 1}{i} = A(F/A, i\%, n)$$

where, (F/A, i, n) is termed as equal-payment series compound amount factor.

Example

A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution:

Here;

A = Rs. 10,000

n = 25 years

i = 20%

F = ?

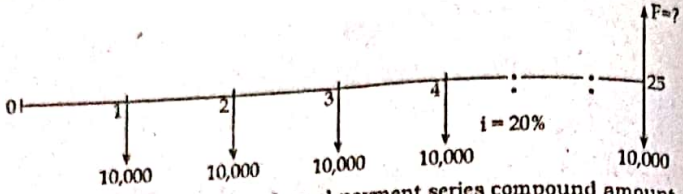


Figure: Cash flow diagram of equal-payment series compound amount.

$$F = \frac{A(1+i)^n - 1}{i} = A\,(F/A, i, n)$$

$= 10,000\,(F/A, 20\%, 25)$

$= 10,000 \times 471.981$

$=$ Rs. 47,19,810

The future sum of the annual equal payment after 25 years is equal to Rs. 47,19,810.

iv) Equal-payment series sinking fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the n^{th} interest period at an interest rate of i.

The corresponding cash flow diagram is shown in figure.

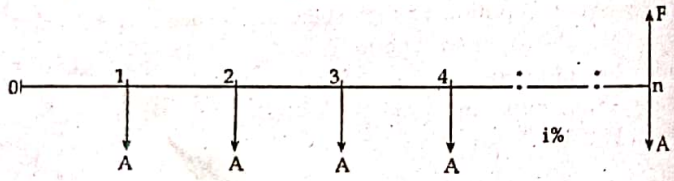


Figure: Cash flow diagram of equal-payment series sinking fund.

The formula to get F is,

$$A = F\frac{i}{(1+i)^n - 1} = F\,(A/F, i, n)$$

where, (A/F, i, n) is called as equal-payment series sinking fund factor.

Example

A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

Solution:

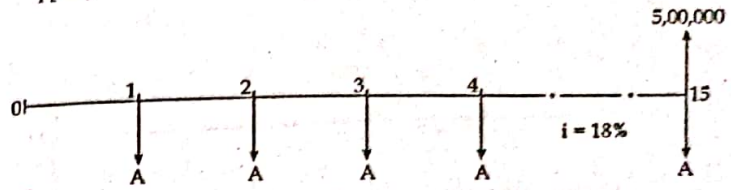
Here;

$F =$ Rs. 5,00,000

$n = 15$ years

$i = 18\%$

$A = ?$



Here; $A = F\dfrac{i}{(1+i)^n - 1}$

$= F\,(A/F, i, n)$

$= 5,00,000\,(A/F, 18\%, 15)$

$= 5,00,000 \times 0.0164$

$=$ Rs. 8,200

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

v) Equal-payment series present worth method.

The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in figure.

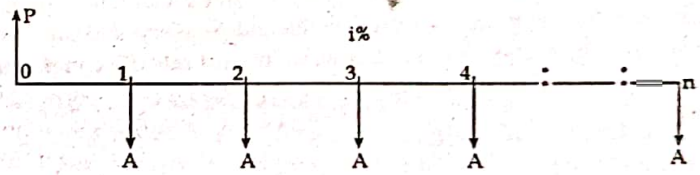


Figure: Cash flow diagram of equal-payment series present worth amount.

The formula to compute P is,

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A\,(P/A, i, n)$$

where, (P/A, i, n) is called equal- payment series present worth factor.

Example

A company wants to set up a reserve which will help the company to have on annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employers welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

Solution:

Given that;

$A =$ Rs. 10,00,000

i = 15%
n = 20 years
P = ?

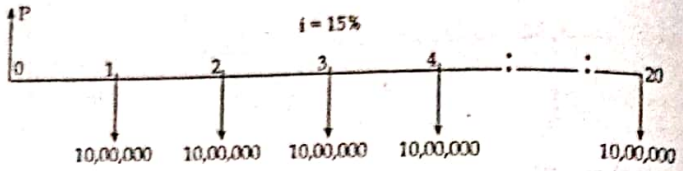


Figure: Cash flow diagram of equal-payment series present worth amount

Here;

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$= A\,(P/A, i, n)$$

$$= 10,00,000 \times (P/A, 15\%, 20)$$

$$= 10,00,000 \times 6.2593$$

$$= Rs.\ 62,59,300$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

vi) Equal payment series capital recovery amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period.

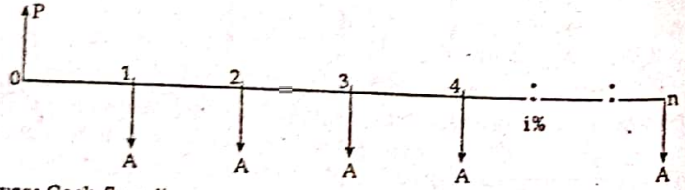


Figure: Cash flow diagram of equal-payment series capital recovery amount

The formula to compute P is as follows,

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P\,(A/P, i, n)$$

where, (A/P, i, n) is called equal-payment series capital recovery factor.

Example

A man gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution:
Here;

P = Rs. 10,00,000
i = 18%
n = 15 years
A = ?

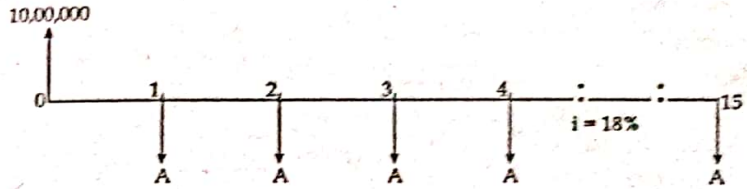


Figure: Cash flow diagram of equal-payment series capital recovery amount

We have,

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= P(A/P, i, n)$$

$$= 10,00,000 \times (A/P, 18\%, 15)$$

$$= 10,00,000 \times 0.1964$$

$$= Rs.\ 1,96,400$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

vii) Uniform gradient series annual equivalent amount

The objective of this mode is to find the annual equivalent amount of a series with an amount A_1 at the end of the first year and with an equal increment (G) at the end if each of the following n-1 years with an interest rate i compounded annually. The corresponding cash flow diagram is shown in figure.

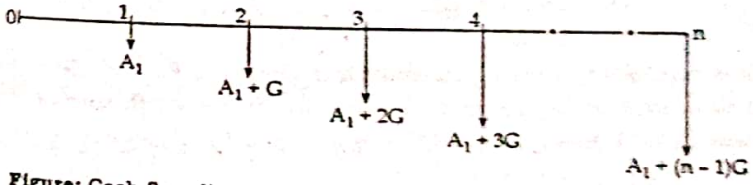


Figure: Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute A under this situation is,

$$A = A_1 + \frac{G(1+i)^n - in - 1}{i(1+i)^n - 1} = A_1 + G\,(A/G, i, n)$$

where, (A/G, i, n) is called uniform gradient series factor.

Example

A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000 at the end of the first year and therefore he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution:

Here;

A_1 = Rs. 4,000

G = Rs. 500

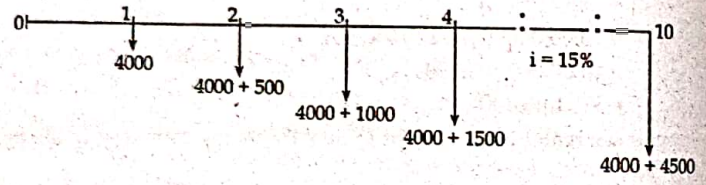
i = 15%

n = 10 years

A =?

F =?

The cash flow diagram is shown in figure.



Now,

$$A = A_1 + \frac{G(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A_1 + G\,(A/G, i, n)$$

$$= 4{,}000 + 500\,(A/G, 15\%, 10)$$

$$= 4{,}000 + 500 \times 3.3832$$

$$= \text{Rs. } 5{,}691.50$$

This is equivalent to paying an equivalent amount of Rs. 5,691.50 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th years is obtained as follows;

$$F = A\,(F/A, i, n)$$

$$= A\,(F/A, 15\%, 10)$$

$$= 5{,}691.50 \times (20.304)$$

$$= \text{Rs. } 1{,}15{,}562.25$$

At the end of the 10th year, the compounded amount of all his payments will be Rs. 1,15,562.25.

3.3.3 Interest Factor and Symbols of Single Cash Flows

To find	Given	Factor	Factor name	Functional symbol
Single cash flow				
F	P	$(1+i)^N$	Single payment compound amount	(F/P, i%, N)
P	F	$(1+i)^{-N}$	Single payment present worth	(P/F, i%, N)
Uniform series annuities				
F	A	$\frac{(1+i)^N - 1}{i}$	Uniform series compound amount	(F/A, i%, N)
A	F	$\frac{i}{(1+i)^N - 1}$	Sinking fund	(A/F, i%, N)
P	A	$\frac{(1+i)^N - 1}{i(1+i)^N}$	Uniform series present worth	(P/A, i%, N)
A	P	$\frac{i(1+i)^N}{(1+i)^N - 1}$	Capital recovery	(A/P, i%, N)

NOTE

To find F, when P, i and N are given,

$$F = P\,(1+i)^N$$

Here, $(1+i)^N$ is called single payment compound amount factor. It is denoted by (F/P, i%, N) and it is read as 'finding F given P at i% interest for N periods.'

Interest Calculation for Uniform Gradient

Let, G be uniform gradient amount, then

i) $$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

i.e., $$F = \frac{G}{i}(F/A, i\%, N) - \frac{NG}{i}$$

ii) $$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

iii) $$P = G\left[\frac{(1+i)^N - 1 - Ni}{i^2(1+i)^N}\right]$$

3.3.4 Geometric Gradient Series

Many engineering economic problems-particularly those relating to construction costs-involve cash flows that increase or decrease over time, not by a constant amount (as with a linear gradient), but rather by a constant percentage (a geometric gradient). This kind of cash flow is called compound growth. Price changes caused by inflation are a good example of a geometric gradient series. If we use g to designate the percentage change in a payment from one period to the next, the

magnitude of the n^{th} payment, A_n is related to the first payment A_1 by the formula,

$$A_n = A_1 (1 + g)^{n-1}, n = 1, 2 \ldots\ldots, N$$

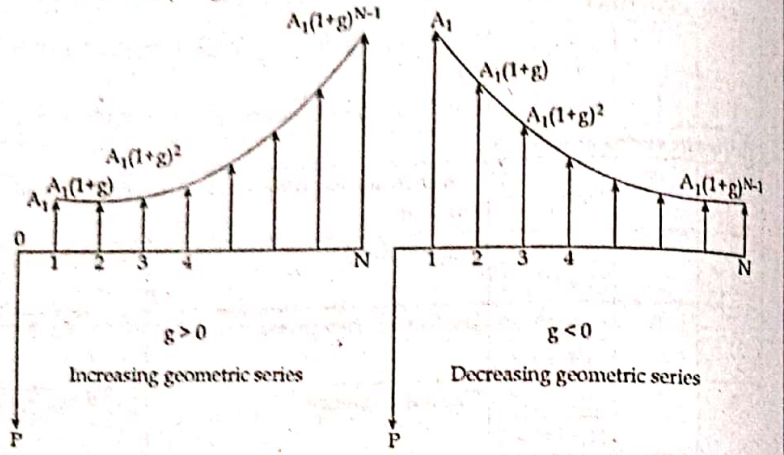


Figure: A geometrically increasing or decreasing gradient series at a constant rate g.

The variable g can take either a positive or negative sign, depending on the type of cash flow. It g > 0, the series will increase and if g < 0, the series will decrease.

Present worth factor: Find P, given A_1, g, i and N.

The present worth of any cash flow F_n at interest rate i is,

$$P_n = A_n (1 + i)^{-n} = A_1 (1 + g)^{n-1} (1 + i)^{-n}$$

Applying single-payment present worth factor to each term of the series,

$$P = \sum_{n=1}^{N} A_1 (1 + g)^{n-1} (1 + i)^{-n} \quad (1)$$

Bringing the constant term $A_1 (1 + g)^{-1}$ outside the summation yields,

$$P = \frac{A_1}{(1 + g)} \sum_{n=1}^{N} \left[\frac{1 + g}{1 + i}\right]^n \quad (2)$$

Let, $a = \frac{A_1}{1 + g}$ and $x = \frac{1 + g}{1 + i}$

Rewriting equation (2) as,

$$P = a (x + x^2 + x^3 + \ldots\ldots\ldots + x^n) \quad (3)$$

Since the summation in equation (3) represents the first N terms of a geometric series, we may obtain the closed- form expression as follows; Multiplying equation (3) by x to get,

$$xP = a (x^2 + x^3 + x^4 + \ldots\ldots\ldots + x^{N+1}) \quad (4)$$

Subtracting equation (4) from (3); we get,

$$P - xP = a (x - x^{N+1})$$

or, $$P(1 - x) = a(x - x^{N+1})$$

or, $$P = a \frac{(x - x^{N+1})}{1 - x} \quad (x \neq 1)$$

If we replace the original values for a and x, we get,

$$P = \begin{cases} A_1 \left[1 - \frac{(1 + g)^N (1 + i)^{-N}}{i - g}\right] & \text{if } i \neq g \\ \frac{NA_1}{(1 + i)} & \text{if } i = g \end{cases}$$

or, $$P = A_1 (P/A_1, g, i, N)$$

The factor within brackets is called the geometric gradient-series present worth factor and is designated $(P/A_1, g, i, N)$.

In the special case where, i = g,

$$P = \left[\frac{A_1}{(1 + i)}\right] N.$$

3.3.5 Linear Gradient Series

Engineers frequently encounter situations involving periodic payments that increase or decrease by a constant amount (G) from period to period. These situations occur often enough to warrant the use of special equivalence factors that relate the arithmetic gradient to other cash flows. Figure illustrates a strict gradient series, $A_n = (n - 1)$ G. Note that the series is at the end of the first period with a zero value. The gradient G can be either positive or negative. If G > 0, the series is referred to as an increasing gradient series. If G < 0, it is a decreasing gradient series.

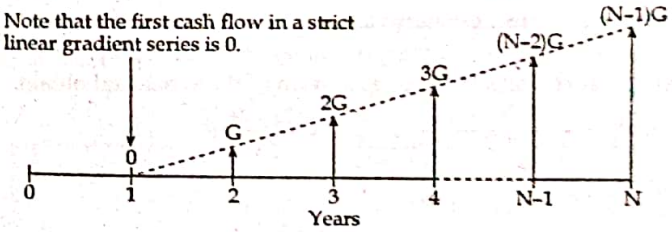
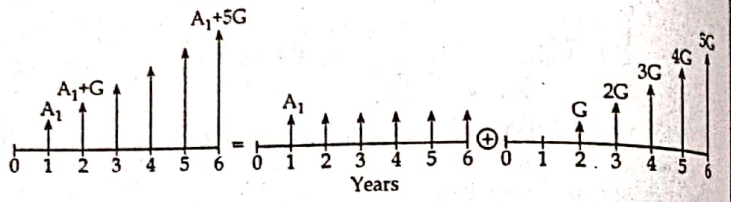


Figure: A cash flow diagram for a strict gradient series.

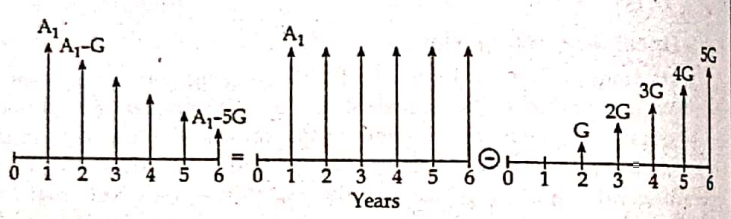
Unfortunately, the strict form of the increasing or decreasing gradient series does not correspond with the form that most engineering economic problem take. A typical problem involving a linear gradient includes an initial payment during period 1 that increases G during some number of interest periods, a situation illustrated in figure. This contrasts with the strict form illustrated in figure, in which no payment is made during period 1 and the gradient is added to the previous payment beginning in period 2.

Gradient series as composite series

In order to utilize the strict gradient series to solve typical problems, we must view cash flows as shown in figure as a composite series or a set of two cash flows, each corresponding to a form that we can recognize and easily solve; a uniform series of N payments of amount A_1 and a gradient series of increments of constant amount G. The need to view cash flows that involve linear gradient series as composites of two series is very important in solving problems, as we shall now see.



(a) Increasing gradient series.



(b) Decreasing gradient series

Figure: Twp types of linear gradient series as composite of a uniform series of N payments of A_1 and the gradient series of increments of constant amount G.

Present worth factor: Linear gradient: Find P, given G, N and i

To find an expression for the present amount P, we apply the single payment present worth factor to each term of the series and obtain.

$$P = 0 + \frac{G}{(1+i)^2} + \frac{2G}{(1+i)^3} + \ldots\ldots + \frac{(N-1)G}{(1+i)^N}$$

$$P = \sum_{n=1}^{N} (n-1)\,G\,(1+i)^{-n} \qquad (1)$$

Let, $G = a$ and, $\frac{1}{(1+i)} = x$, then,

$$P = 0 + ax^2 + 2ax^3 + \ldots\ldots + (N-1)\,ax^N$$

$$= ax\,[0 + x + 2x^2 + \ldots\ldots + (N-1)x^{N-1}] \qquad (2)$$

Since an arithmetic geometric series $\{0, x, 2x^2, \ldots\ldots, (N-1)x^{N-1}\}$ has the finite sum

$$0 + x + 2x^2 + \ldots\ldots + (N-1)\,x^{N-1} = \left[\frac{1 - Nx^{N-1} + (N-1)\,x^n}{(1-x)^2}\right]$$

Rewriting equation (2) as,

$$P = ax^2\left[\frac{1 - Nx^{N-1} + (N-1)\,x^N}{(1-x)^2}\right] \qquad (3)$$

Replacing the original values for A and x, we get,

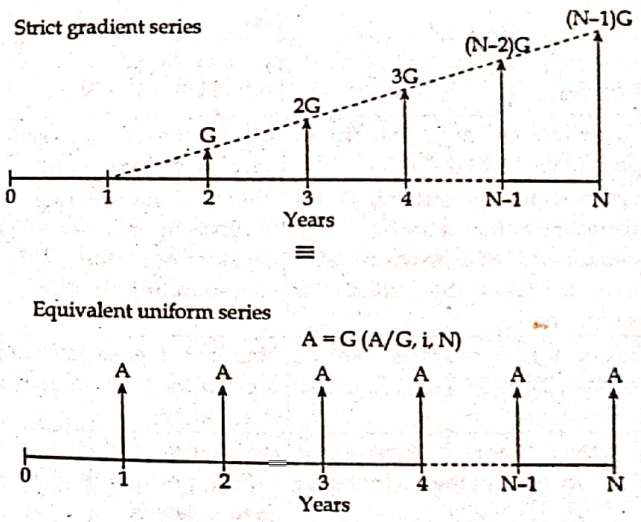
$$P = G\left[\frac{(1+i)^N - iN - 1}{i^2\,(1+i)^N}\right] = G\,(P/G, i, N) \qquad (4)$$

The resulting factor in brackets is called the gradient series present worth factor, which we denote as (P/G, i, N).

Gradient to equal payment series conversion factor: find A, given G, I and N

$$A = G\left[\frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]}\right] = G\,(A/G, i, N)$$

where, the resulting factor in brackets is referred to as the gradient to equal payment series conversion factor and is designated (A/G, i, N).



BOARD EXAM SOLVED PROBLEMS

1. Define interest. [2018/Fall, 2019/Spring]

Answer: See the topic 3.1.

2. Define effective interest rate and nominal interest rate. [2015/Fall, 2018/Spring, 2018/Fall]

Answer: See the topic 3.1.4 and 3.1.5

3. Differentiate between effective and nominal interest rate. [2012/Fall, 2013/Fall, 2016/Spring, 2018/Spring]

S.N.	Effective interest rate	Nominal interest rate
i.	Represents interest that affects the real value of money after one corresponding period.	Represents the interest rate that is convertible more than once in a corresponding period.
ii.	It is greater than nominal interest rate.	It is less than effective interest rate.
iii.	$i = \left(\frac{T}{P}\right)^{\frac{1}{Y}} - 1$	$R = N\left(\frac{T}{P}\right)^{\frac{1}{NY}} - N$
iv.	Often referred to as the annual percentage yield (APY)	Often referred to as the annual percentage rate (APR)
v.	Effective interest rate, i, is the actual rate that applies for a stated period of time which takes into account the effect of compounding.	Nominal interest rate, r, is an annual interest rate without considering the effect of compounding.
vi.	Effective interest rates can be used in the time value formulas or equations.	Nominal interest rate cannot be used in the time value formulas or equations.
vii.	*For example;* for an interest rate of 5.6% per month, the real interest rate is $\left(1 + \frac{0.0140}{4}\right)^4 - 1$ = 1.41% per quarter	*For example;* for an interest rate of 5.6% per month, the nominal interest rate is $= \frac{5.6}{4}$% per quarter = 1.40% per quarter

4. Explain about time value of money. [2013/Fall, 2015/Fall, 2016/Spring, 2018/Fall, 2019/Spring]

Answer:

Time value of money is the idea that money that is available at the present time is worth more than the same amount in the future, due to its potential earning capacity. It is funded on time preference. The time value of money is an important concept to investors because a dollar on hand today is worth more than a dollar promised in the future. The dollar on hand today can be used to invest and earn interest or capital gains.

When we deal with large amount of money, long periods of time or high interest rates, the change in the value of a sum of money over time becomes extremely significant. *For example;* at a current annual interest rate of 10%, Rs. 1 crore will earn Rs. 10,00,000 in interest in a year, thus, to wait a year to receive Rs. 1 crore clearly involves a significant sacrifice. When deciding among alternative proposals, we must take into account the operation of interest and the time value of money in order to make valid comparisons of different amounts at various times. The way interest operates reflects the fact that money has a time value.

Option A: Rs. 10,000 → Rs. 10,000 + interest

Option B: Rs. 10,000 – interest → Rs. 10,000

By receiving Rs. 10,000 today (option A), you are poised to increase the future value of your money by investing and gaining interest over a period of time. If you receive the money three years down the line (option B), you don't have time on your side, and the payment received in three years would be your future value.

5. How does compound interest different from simple interest? Why does bank use concept of compound interest instead of simple interest? [2018/Fall, 2019/Spring]

Answer:

S.N.	Simple interest	Compound interest
i)	This is calculated only on original principal.	This is calculated on interest earned and the principal.
ii)	Interest earned is not reinvested. Hence, it is not used in interest calculations for the following periods.	Interest earned during the previous period is added to the principal. The total amount will become new principal and all the money will earn interest.
iii)	Principal remains constant.	Principal goes on changing during entire borrowing period.
iv)	Interest charged on principal only.	Interest charged on principal plus accumulated interest.
v)	Simple interest is linear in nature.	Compound interest is exponential in nature.

vi)	Simple interest is calculated using the following formula: Simple interest = $P \times r \times n$ where, P = Principal amount r = Annual interest rate n = Term of loan, in years	Compound interest is calculated using the following formula: Compound interest $= P(1 + r)^t - P$ where, t = Number of years interest is applied.
vii)	Simple interest is smaller than compound interest.	Compound interest is larger than simple interest.
viii)	Returns earned on the investment are lower.	Returns earned on the investment are higher.

Interest can compound either frequently (daily or monthly) or infrequently (quarterly, semi-annually or yearly). The more often your interest compounds, the more interest you will earn on your investment in compound interest rate. When it comes to investing, compound interest is better since it allows funds to grow at a faster rate than they would in an account with a simple interest rate. So, bank use concept of compound interest instead of simple interest.

6. Describe about cash flow diagram. **[2017/Spring]**

Answer: See the topic 3.3

7. What is the effective interest rate if the nominal rate is 9% per year, a 365 day is used and the compounding period is (i) yearly (ii) quarterly (iii) daily (iv) hourly (v) continuously: **[2015/Fall]**

Solution:

Nominal interest rate (r) = 9%

We know,

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

i) $i_{eff}\text{ - yearly} = \left(1 + \frac{0.09}{1}\right)^1 - 1 = 9\%$

ii) $i_{eff}\text{ - quarterly} = \left(1 + \frac{0.09}{4}\right)^4 - 1 = 9.3083\%$

iii) $i_{eff}\text{ - daily} = \left(1 + \frac{0.09}{365}\right)^{365} - 1 = 9.4162\%$

iv) $i_{eff}\text{ - hourly} = \left(1 + \frac{0.09}{8760}\right)^{8760} - 1 = 9.4174\%$

v) $i_{eff}\text{ - continuous} = e^r - 1 = e^{0.09} - 1 = 9.4174\%$

8. Sarita wants to deposit Rs. 15,000 in every year into a bank at an interest rate of 12% per year, compounded semi-annually. What will be the maturity amount after 5 years? **[2013/Fall]**

Solution:

Here; Number of years (N) = 5

Interest rate (r) = 12% compounded semiannually

Amount deposited every year (A) = Rs. 15,000

Future amount after 5 year (F) = ?

We know,

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.12}{2}\right)^2 - 1 \qquad [\because m = 2; \text{compounded semi annually}]$$

$\therefore\ i_{eff} = 12.36\%$

Then,

$\therefore\ F = A\,(F/A, i\%, N)$

or, $F = A \times \left[\frac{(1+i)^N - 1}{i}\right]$

or, $F = \text{Rs. } 15{,}000 \times (F/A, 12.36\%, 5)$

or, $F = \text{Rs. } 15{,}000 \times \left[\frac{(1 + 0.1236)^5 - 1}{0.1236}\right]$

or, $F = \text{Rs. } 95{,}976.67$

Hence, maturity amount after 5 years is Rs. 95976.67

9. How many deposits of Rs. 5,000 each should make per month so that the final accumulation amount will be Rs. 1,00,000 if the bank interest rate is 12% per year? **[2014/Fall]**

Solution:

Here; Monthly deposit (A) = Rs. 5,000

Future amount (F) = Rs. 1,00,000

Interest rate (i_y) = 12% per year

$i_{monthly} = (1 + i_y)^{1/m} - 1$

or, $i_m = (1 + 0.12)^{1/12} - 1$

or, $i_m = 0.00948$

$\therefore\ i_m = 0.948\%$

We have,

$F = A\,(F/A, i\%, N)$

or, $F = A\left[\frac{(1+i)^N - 1}{i}\right]$

or, $1{,}00{,}000 = 5{,}000\left[\frac{(1 + 0.00948)^N - 1}{0.00948}\right]$

or, $0.1896 = (1.00948)^N - 1$

or, $1.1896 = (1.00948)^N$

Taking $\log_e$ on both sides,

or, $\log_e(1.1896) = N \times \log_e(1.00948)$

or, $0.0754009 = N \times 0.0040977$

∴ $N = 18.40$ deposits

Hence, it needs 19 deposits.

10. A father deposits a sum of Rs. 1,00,000 in a bank for his son's education who will be admitted to a professional course after 5 years. The bank pays 12% interest rate per year, compounded monthly. Find the future amount of the deposited money at the time of admitting his son in the professional course. **[2014/Fall, 2012/Spring]**

Solution:

Here; Principal amount (P) = Rs. 1,00,000

N = 5 years

r = 12% compounded monthly

so, $i = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.01268$

∴ $i = 12.68\%$

We know,

$F = P(F/P, i\%, N)$

or, $F = P(1+i)^N = 1{,}00{,}000\ (F/P, 12.68\%, 5)$

$= 1{,}00{,}000 \times (1 + 0.1268)^5 = 1{,}00{,}000 \times 1.8165$

∴ F = Rs. 1,81,650

Hence, future amount is Rs. 1,81,650

11. Sabina deposits a sum of Rs. 10,00,000 in a bank at an interest rate of 12% per year. What will be the future amount after 5 years? If compounded:

i) Weekly

ii) Quarterly

iii) Annually

Solution: **[2016/Fall]**

Here; P = Rs. 10,00,000

N = 5 years

r = 12%

i) $i_{weekly} = \left(1 + \frac{0.12}{52}\right)^{52} - 1 = 12.734\%$

Future amount after 5 years compound weekly is,

or, $F = P(1+i)^N = 10{,}00{,}000 \times (1 + 0.12734)^5 = \text{Rs. } 18{,}20{,}851.734$

ii) $i_{quarterly} = \left(1 + \frac{0.12}{4}\right)^4 - 1 = 12.55\%$

or, $F = P(1+i)^N = 10{,}00{,}000\ (1 + 0.1255)^5 = \text{Rs. } 18{,}06{,}040.548$

iii) $i_{annual} = \left(1 + \frac{0.12}{1}\right)^1 - 1 = 12\%$

or, $F = P(1+i)^N = 10{,}00{,}000\ (1 + 0.12)^5 = \text{Rs. } 17{,}62{,}341.683$

12. You wish to study your son in medical college after 20 years Recently government has fixed total 35 lakh to complete MBBS studies. How much do you need to deposit on each year to meet your desire if bank provides 10% interest rate per year for your fixed account? **[2017/Spring]**

Solution:

Here;

N = 20 years

r = 10% per year = i

F = Rs. 35,00,000

A = ?

We know,

$A = F(A/F, i\%, N)$

$= 35{,}00{,}000\ (A/F, 10\%, 20)$

$= 35{,}00{,}000 \times \left[\frac{10\%}{(1 + 10\%)^{20} - 1}\right]$

$= 35{,}00{,}000 \times \frac{0.01}{0.22019}$

$= 35{,}00{,}000 \times 0.045415$

or, A = 1,58,952.50

∴ A = Rs. 1,58,952.50

Hence, I must deposit Rs. 1,58,952.50 each year for 20 years.

13. Ram invested at high yield account aimed to get the double of his investment at the end of 10 years. Compute the effective interest year he received on the account. **[2017/Spring]**

Solution:

N = 10 years

i_{eff} = ?

Let, Present investment (P) = P

Future sum (F) = 2P

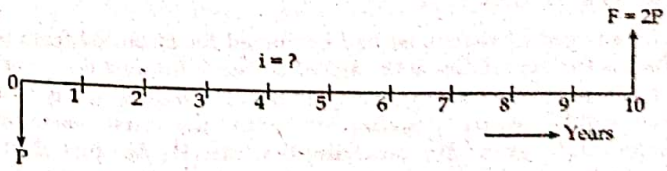


Figure: Cash flow diagram

We know,

$F = P(1 + i)^N$

or, $2P = P(1 + i)^N$

or, $2 = (1 + i)^{10}$

or, $2^{1/10} = (1 + i)$

or, $1.0718 = 1 + i$

or, $i = 0.0718$

$\therefore i = 7.18\%$

Hence, effective interest rate is 7.18% per year.

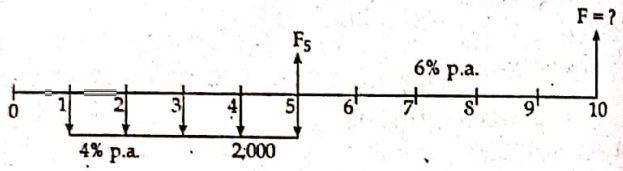
14. An individual makes five annual deposits of Rs. 2,000 in a savings account that pays interest at a rate of 4% interest rate change to 6% per year. Five years after last deposit, the accumulated money is withdrawn from account. How much is withdrawn? [2013/Spring]

Solution:

A = Rs. 2,000 annually

i = 4% per year

N = 5 years + 5 years



$F_5 = A(F/A, i\%, N)$

$= 2{,}000 \times (F/A, 4\%, 5)$

$= 2{,}000 \times \left[\frac{(1 + 0.04)^5 - 1}{0.04}\right]$

$= 2{,}000 \times 5.4163$

$= \text{Rs. } 10{,}832.60$

Now,

$F_{10} = F_5 \times (F/P, i\%, N)$

$= 10{,}832.60 \times (F/P, 6\%, 5)$

$= \text{Rs. } 14{,}496.18$

Hence, Rs. 14,496 is withdrawn.

15. A man aged 40 years now had borrowed Rs. 50,00,000 from bank for his further studies at the age of 20 years. Interest was changed at 11% per year compounded quarterly. He wishes to pay loan in semiannual equal installments with the first installment beginning 5 years after receiving the loan. He has just clear his loan now what amount did he pay in each installments? [2012/Fall, 2016/Spring]

Solution:

P = Rs. 50,00,000

i = 11% compounded quarterly

$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.11}{4}\right)^4 - 1 = 11.46\%$

$i_{semi\text{-}annually} = (1 + i_{eff})^{1/m} - 1 = (1 + 0.1146)^{1/2} - 1 = 5.58\%$

Now,

$F = P(F/P, i\%, N)$

$= 50{,}00{,}000\ (F/P, 11.46\%, 20)$

$= 50{,}00{,}000 \times (1 + 0.1146)^{20}$

$= 50{,}00{,}000 \times 8.7575$

$= \text{Rs. } 4{,}37{,}87{,}500$

Using uniform series compound factor,

$F = A(F/A, i\%, N)$

or, $4{,}37{,}87{,}500 = A(F/A, 5.58\%, 30)$

or, $4{,}37{,}87{,}500 = A\left[\frac{(1 + 0.0558)^{30} - 1}{0.0558}\right]$

or, $4{,}37{,}87{,}500 = A \times 73.4514$

$\therefore A = \text{Rs. } 5{,}96{,}142.483$

Hence, he paid Rs. 5,96,142.483 on each installment. [Hint: He paid each installment semi-annually after 5 years from receiving the loan. So, N = 2 × 15 times = 30 times]

16. Calculate the future worth of the following cash flows deposited at 8% compounded continuously for 5 years. [2017/Fall]

i) Rs. 50,000 at the beginning of each year

ii) Rs. 50,000 at the end of each year

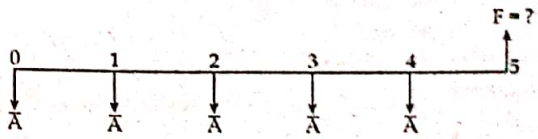
Solution:

N = 5 years

r = 8% compounded continuously

$\bar{A}$ = Rs. 50,000

i) Rs. 50,000 at the beginning of each year:

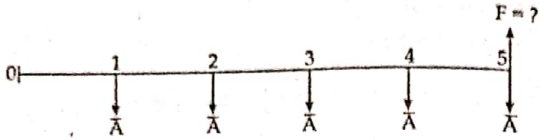


$F = \bar{A}(F/\bar{A}, r\%, N) \times e^r$

$= 50{,}000\left[\frac{e^{rN} - 1}{e^r - 1}\right] \times e^r = 50{,}000\left[\frac{e^{0.08 \times 5} - 1}{e^{0.08} - 1}\right] \times e^{0.08}$

$\therefore F = \text{Rs. } 3{,}19{,}850$

ii) Rs. 50,000 at the end of each year:



$$F = \bar{A}\,(F/\bar{A}, r\%, N) = \bar{A}\left(\frac{e^{rN}-1}{e^{r}-1}\right) = 50{,}000\left(\frac{e^{0.08\times5}-1}{e^{0.08}-1}\right)$$

$\therefore$ F = Rs. 2,95,258

17. A 30 years old person is planning for his retired life. He plans to invest an equal sum of Rs. 25,000 at the end of every year. The bank gives 6% interest rate compounded semi-annually. How much money can be withdrawn from his account when he is 60 years old? [2012/Spring]

Solution:

A = Rs. 25,000 every year

r = 6% compounded semi-annually

F = ?

N = 30 years

$$\therefore \quad i = \left(1+\frac{r}{m}\right)^{m} - 1 = \left(1+\frac{0.06}{2}\right)^{2} - 1 = 6.09\%$$

Now, using uniform series compound amount factor,

$$F = A\,(F/A, i\%, N) = 25{,}000\,(F/A, 6.09\%, 30)$$

$$= 25{,}000 \times \left[\frac{(1+0.0609)^{30}-1}{0.0609}\right]$$

$\therefore$ F = Rs. 20,08,048

Hence, he can draw Rs. 20,08,048 from his account when he is 60 years old.

18. Rabindra deposits Rs. 1,00,000 at a bank account that provides 12% interest for 5 years. How much balance will Rabindra have at the end of 5th year, if bank calculates interest on simple interest basis? How much will he have if interest is compounded annually? [2012/Spring]

Solution:

Principal (P) = Rs. 1,00,000

Interest = 12% per year

Number of year, N = 5 year

Future amount, F = ?

We have,

i) Simple interest,

$$F = P\,(1 + i \times N) = 1{,}00{,}000 \times (1 + 0.12 \times 5) = \text{Rs. } 1{,}60{,}000$$

ii) Compound interest,

$$F = P\,(1+i)^{N} = 1{,}00{,}000 \times (1+0.12)^{5} = \text{Rs. } 1{,}76{,}235$$

The future value under compound interest system is always higher than that of simple interest system because the compound interest is calculated on the interest earned and the principal while simple interest rate is calculated only on the original principal amount.

19. If you have Rs. 10,00,000 loan now from a bank, how much Rs. should you pay as installment per two month for 5 years if bank interest rate is 12% per year? [2018/Fall]

Solution:

P = Rs. 10,00,000

N = 5 years

A = ?

i = 12% per year

$i_{monthly} = (1+i)^{1/12} - 1$

But, installment is paid per two months in a year,

$$i_{mon} = (1+i)^{1/6} - 1 = (1+0.12)^{1/6} - 1 = 1.907\%$$

Now,

$$A = P\,(A/P, i\%, N)$$

$$= 10{,}00{,}000\,(A/P, 1.907\%, 30)$$

$$= 10{,}00{,}000\left[\frac{i(1+i)^{N}}{(1+i)^{N}-1}\right] = 10{,}00{,}000\left[\frac{0.01907\,(1+0.01907)^{30}}{(1+0.01907)^{30}-1}\right]$$

$$= 10{,}00{,}000 \times \frac{0.03361}{0.76246}$$

$$= 10{,}00{,}000 \times 0.044081$$

$\therefore$ A = Rs. 44,081

Hint:

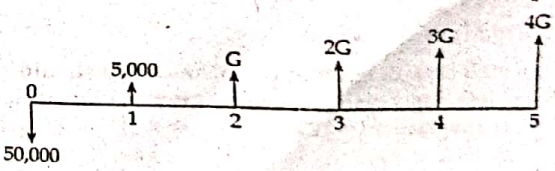
Total number of months = 5 × 12 = 60 months

But installment is paid per 2 months so

$$N = \frac{60}{2} = 30$$

Hence, Rs. 44,081 should be paid as installment per two months for 5 years for the loan of 10,00,000 if interest rate is 12% per year.

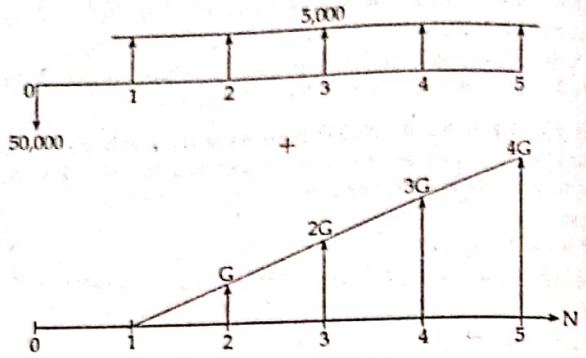
20. Find the value of G if i = 12%. [2014/Spring]



Solution:

i = 12%

G = ?



Applying uniform series PW factor and gradient series PW factor,

$50,000 = 5,000\ (P/A, 12\%, 5) + G\ (P/G, 12\%, 5)$

or, $50,000 = 5,000 \times \left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] + G\left[\frac{(1+i)^N - 1 - Ni}{i^2(1+i)^N}\right]$

or, $50,000 = 5,000 \times 3.6048 + G \times 6.3971$

or, $G = \frac{31,976}{6.3971}$

∴ $G = Rs.\ 5,000$

Hence, value of G is Rs. 5,000

21. What is the future equivalent of Rs. 40,00,000 per year that flows continuously for 11 years if nominal interest is 12% compounded continuously? [2011/Fall]

Solution:

$\bar{A} = Rs.\ 40,00,000$

$N = 11$ year

$r = 12\%$ per year

$F = ?$

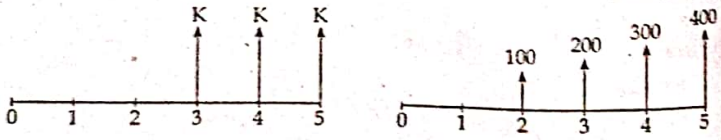
By using continuous compounding,

$$F = \bar{A}\ (F/\bar{A}, r\%, N) = \bar{A}\left(\frac{e^{rN} - 1}{r}\right) = 40,00,000\left(\frac{e^{0.12 \times 11} - 1}{0.12}\right)$$

∴ $F = Rs.\ 9,14,47,380$

Future equivalent is Rs. 9,14,47,380

22. For the cash flow diagram given below, what should be the value of K on left hand cash flow diagram to be equal to right hand cash flow diagram if i = 12%? [2015/Spring]



Solution:

$i = 12\%$

$K = ?$

We have,

$$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

or, $F = \frac{100}{0.12}\left[\frac{(1+0.12)^5 - 1}{0.12}\right] - \frac{5 \times 100}{0.12} = 5294.039 - 4166.67$

or, $F = Rs.\ 1,127.369$

Now,

$A = F\ (A/F, i\%, N)$ [∵ For 1st figure]

$= F(A/F, 12\%, 3) = 1,127.369 \times 0.2963 = Rs.\ 334$

Hence, value of K is Rs. 334

23. Ramesh, an engineer is planning to place 20% of his salary, which is Rs. 2,50,000 per year at present, each year in mutual fund. He expects 7% of his salary increase each year for next 15 years. If the mutual fund will average 10% annual return, what will be the sum amount at the end of 15 years? If salary increases by Rs. 25,000 per year. What will be the amount? [2018/Spring]

Solution:

Here; Number of year, N = 15 years

Increment rate of salary = 7% per year = g

Present salary = Rs. 2,50,000

% of salary for deposit = 20%

Year	Total salary increase @ 7% per year	20% deposit
0	2,50,000	50,000
1	2,67,500	53,500
2	2,86,225	57,245
3	3,06,260	61,252
4	3,17,699	65,540
5	3,50,638	70,127
6	3,75,182	75,036
7	4,01,445	80,289
8	4,29,546	85,909
9	4,49,615	91,923
10	4,91,788	98,357
11	5,26,213	1,05,242
12	5,63,048	1,12,609
13	6,02,461	1,20,492
14	6,44,633	1,28,926
15	6,89,758	1,37,951

Here;

P = Rs. 50,000

A' = Rs. 53,500

g = 7%

i = 10% = Annual return rate

N = 15 year

We know,

$$F = P(F/P, i\%\ N) + A'(F/A', g, i, N)(F/P, i, N)$$

$$= 50,000(F/P, 10\%, 15) + 53,500(F/A', 7\%, 10\%, 15) \times (F/P, 10\%, 15)$$

$$= 2,08,860 + \frac{18163.77}{0.03} \times 4.1772$$

$\therefore$ F = Rs. 27,37,984.

If salary increases by Rs. 25,000 per year,

Year	Salary increased by Rs. 25,000 per year	20% deposit
0	2,50,000	50,000
1	2,75,000	55,000
2	3,00,000	60,000
3	3,25,000	65,000
4	3,50,000	70,000
5	3,75,000	75,000
6	4,00,000	80,000
7	4,25,000	85,000
8	4,50,000	90,000
9	4,75,000	95,000
10	5,00,000	1,00,000
11	5,25,000	1,05,000
12	5,50,000	1,10,000
13	5,75,000	1,15,000
14	6,00,000	1,20,000
15	6,25,000	1,25,000

Now,

P = Rs. 50,000

A = Rs. 55,000

G = Rs. 5,000

We know,

$$F = P(F/P, i\%, N) + A(F/A, i\%, N) + G(F/G, i\%, N)$$

$$= 50,000(F/P, 10\%\ 15) + 55,000(F/A, 10\%, 15) + 5,000 \times (F/G, 10\%, 15)$$

$$= 50,000 \times 4.1772 + 55,000 \times 31.7725 + \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$= 2,08,860 + 17,487.50 + \frac{5000}{0.10}\left[\frac{(1+0.10)^{15} - 1}{0.10}\right] - \frac{15 \times 5000}{0.10}$$

$$= 2,08,860 + 17,47,487.50 + 15,88,624 - 7,50,000$$

$\therefore$ F = Rs. 27, 94, 971.50

24. How many rupees Ms. Reshma should deposit now so that she can withdraw Rs. 2,00,000 every five years if the bank interest rate is 10% per year?

Solution:

i = 10% per

Interest rate for the end of each 5 year

$$i_5 = (1+i)^N - 1 = (1+0.10)^5 - 1$$

$\therefore$ i_5 = 61.051% for each period

As, Ms. Reshma can withdraw Rs. 2,00,000 every 5 years for infinite period, applying CW method,

$$CW = \frac{AW}{i_5} = \frac{2,00,000}{0.6105} = \text{Rs. } 3,27,595$$

Hence, Ms. Reshma should deposit Rs. 3,27,595 now for getting Rs. 2,00,000 every 5 year.

25. Find the compound amount, if the investment is done Rs. 5,000 with the interest rate 12% per year and compounded weekly for 2 years. [2010/Fall]

Solution:

P = Rs. 5,000

N = 2 years

r = 12% per year compounded weekly

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.12}{52}\right)^{52} - 1$$

$\therefore$ i_{eff} = 12 .74%

so, $F = P(1+i)^N = 5,000 \times (1+0.1274)^2$

$\therefore$ F = Rs. 6,355.15

Hence, compounded amount is Rs. 6,355.15.

26. Ram invested Rs. 15,000 in a high yield account. At the end of 30 years, he closed the account and received Rs. 5,39,250. Compute the effective interest rate he received on the account. [2010/Fall]

Solution:

P = Rs. 15,000

N = 30 years

F = Rs. 5, 39, 250

i = ?

We have,

$P = F(1+i)^{-N}$

or, $15{,}000 = 5{,}39{,}250\,(1+i)^{-30}$

or, $0.027816 = (1+i)^{-30}$

or, $(1+i) = \sqrt[30]{0.027816}$

or, $1 + i = 1.12682$

or, $i = 0.12682$

$\therefore$ $i = 12.68\%$

Hence, effective interest rate is 12.68% compounded annually.

27. What is the interest rate if your amount will be double in 5 years? **[2019/Spring]**

Solution:

Here;

$N = 5$ years

Present sum (P) = P

Future sum (F) = 2P

We know,

$F = P(1+i)^N$

or $2P = P(1+i)^5$

or $2 = (1+i)^5$

or $2^{1/5} = (1+i)$

or, $1.1487 = 1 + i$

or $i = 0.1487$

$\therefore$ $i = 14.87\%$

The interest rate should be 14.87% per year to double the amount in 5 years.

28. If you deposit Rs. 5,000 per month for two years, what will be the amount at the end of five years if bank interest rate is 5% in every six month? **[2020/Fall]**

Solution:

Here;

A = Rs. 5,000 per month for two years

Interest rate, r = 5% per six months

Effective interest rate,

$$i_{eff} = \left(1+\frac{r}{2}\right)^2 - 1 = \left(1+\frac{0.05}{2}\right)^2 - 1 = 0.050625 = 5.0625\% \text{ per year}$$

and, Monthly interest rate,

$$i_{mon} = (1+i_{eff})^{1/m} - 1 = (1+0.050625)^{1/12} - 1 = 0.00412 = 0.412\%$$

Amount at the end of 2 years,

$F = A(F/A, i\%, N)$

$= 5000\,(F/A, 0.412\%, 24)$

$$= 5000 \times \left[\frac{(1+0.00412)^{24}-1}{0.00412}\right]$$

= Rs. 125831.84

Now, this amount is kept at bank for 3 years. Number of compounding period in 3 years,

$N = 3 \times 2 = 6$

$i_{semi\text{-}annually} = (1+0.0506)^{1/2} - 1 = 0.025 = 2.5\%$

Hence, $F = P(F/P, 2.5\%, 6) = 125831.84 \times (1+0.025)^6$ = Rs. 145926.36

Hence, amount at end of 5 years is Rs. 1,45,926.36.

29. Suppose a farmer want to save money semi-annually in a financial company for the engineering education of his daughter of 2 years old. How much money does he need to save per period if she will need 20,00,000 when her age will be 18 years old? The company compounded the money semi-annually and interest rate is 12%? **[2019/Fall]**

Solution:

Here,

Interest rate, r = 12%, compounded semi-annually

Amount required at the age of 18, F = Rs. 20,00,000

Time period, n = 18 – 2 = 16 years

Now, Effective interest rate,

$$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.12}{2}\right)^2 - 1 = 0.1236 = 12.36\%$$

Since time period for each period is semi-annual,

$i_{semi\text{-}annual} = (1+i_{eff})^{1/m} - 1 = (1+0.1236)^{1/2} - 1 = 0.06 = 6\%$

Number of periods, $N = 2 \times n = 2 \times 16 = 32$

Hence,

$F = A(F/A, i\%, N)$

or $20{,}00{,}000 = A \times (F/A, 6\%, 32)$

or $$20{,}00{,}000 = A \times \left[\frac{(1+0.06)^{32}-1}{0.06}\right]$$

or $20{,}00{,}000 = A \times 90.8898$

$\therefore$ $$A = \frac{20{,}00{,}000}{90.8898} = \text{Rs. } 22{,}004.669$$

Hence, that farmer needs to save Rs. 22,004.669 per period.

ADDITIONAL PROBLEMS

1. Find out the effective and nominal interest rate per year if the monthly interest rate is 1.85%

Solution:

$i_{monthly} = 1.85\%$

$i_{eff} = (1 + i_{mon})^{12} - 1$

$= (1 + 0.0185)^{12} - 1$

$= 24.60\%$

Hence, effective interest rate is 24.60% per year, and

$$i_{eff} = \left(1 + \frac{r}{m}\right)^{m} - 1$$

or, $0.2460 = \left(1 + \frac{r}{12}\right)^{12} - 1$

or, $0.2460 + 1 = \left(1 + \frac{r}{12}\right)^{12}$

or, $1.246 = \left(1 + \frac{r}{12}\right)^{12}$

or, $(1.246)^{1/12} = 1 + \frac{r}{12}$

or, $1.0185 - 1 = \frac{r}{12}$

or, $r = 12 \times 0.0185$

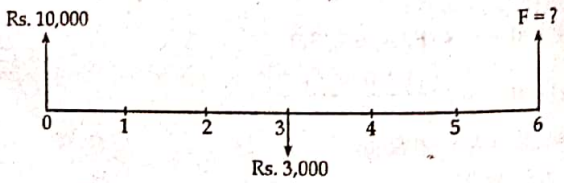
or, $r = 0.22196$

$\therefore$ $r = 22.20\%$

Hence, nominal interest rate is 22.20% per year

2. Pragya deposits Rs. 10,000 now in a bank which gives 7% interest rate per year. She draws Rs. 3,000 at the end of 3rd year. What will be the remaining amount at the end of 6th year?

Solution:



$i = 7\%$ per year

At the end of 3rd year, accumulated amount is,

$F = P(1 + i)^N = 10,000(1 + 0.07)^3 = \text{Rs. } 12,250.43$

And after drawing Rs. 3000, the remaining deposit amount at the end of 3rd year will be,

$= \text{Rs. } 12,250.43 - \text{Rs. } 3,000$

Hence, at the end of 6th year,

$F = P_3(1 + i)^{6-3} = 9250.43 \times (1 + 0.07)^3$

$\therefore$ $F_6 = \text{Rs. } 11,332.18$

3. If Prashansa wish to draw Rs. 22,000 per month for 2 years. How much should she deposit at present for that when rate of interest is 8% per year?

Solution:

$A = \text{Rs. } 22,000$ per month

$i = 8\%$ per year

$N = 2$ years $= 2 \times 12$ months $= 24$ months

We have,

$i_{monthly} = (1 + i_{eff})^{1/12} - 1 = (1 + 0.08)^{1/12} - 1$

$\therefore$ $i_{monthly} = 0.00643$

Then,

$P = A(P/A, i\%, N)$

$= \text{Rs. } 22,000\ [P/A, 0.00643, 2 \times 12]$

$$= \text{Rs. } 22,000 \times \left[\frac{(1 + 0.00643)^{24} - 1}{0.00643 \times (1 + 0.00643)^{24}}\right]$$

$= \text{Rs. } 22,000 \times 22.1739$

$\therefore$ $P = \text{Rs. } 4,87,828$

Hence, Prashansa must deposit Rs. 4,87,828 at present.

4. Bank 'A' offers 6% per year but compounded daily. Bank 'B' offers 6.5% per year but compounded monthly, which one do you prefer? Why?

Solution:

Effective interest rate for Bank 'A'

$$i_A = \left(1 + \frac{r}{m}\right)^{m} - 1 = \left(1 + \frac{0.06}{365}\right)^{365} - 1$$

$\therefore$ $i_A = 6.183\%$

and, effective interest rate for bank 'B'

$$\therefore \quad i_B = \left(1 + \frac{0.065}{12}\right)^{12} - 1 = 6.70\%$$

Hence, effective interest of bank B > Bank A. So, I would prefer Bank 'B'.

5. If a saving bank pays 2% interest every three months, what are the nominal and effective interest rate per year?

Solution:

$i_{quarterly} = 2\% = 0.02$

$\therefore \quad i_{eff} = (1 + i_{quarterly})^4 - 1 = (1 + 0.02)^4 - 1 = 8.24\%$

We know,

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

or, $0.0824 = \left(1 + \frac{r}{4}\right)^4 - 1$

or, $0.0824 + 1 = \left(1 + \frac{r}{4}\right)^4$

or, $1.0824 = \left(1 + \frac{r}{4}\right)^4$

or, $\frac{r}{4} = (1.0824)^{1/4} - 1$

or, $r = 4 \times 0.01999$

or, $r = 0.07996$

$\therefore \quad r = 8\%$ per year

Hence, nominal interest rate = 8% per year

Effective interest rate = 8.24% per year

6. Suppose, Sagar make a series of annual deposit into a bank account that pays 10% interest. The initial deposit at the end of the first year is Rs. 1,200. The deposit amount decline by Rs. 200 in each of the next four years. How much would he have immediately after the 5th deposit?

Solution:

N = 5 years

i = 10% per year

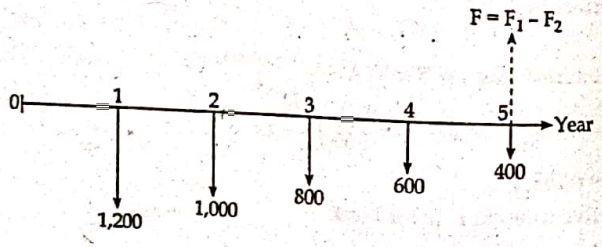
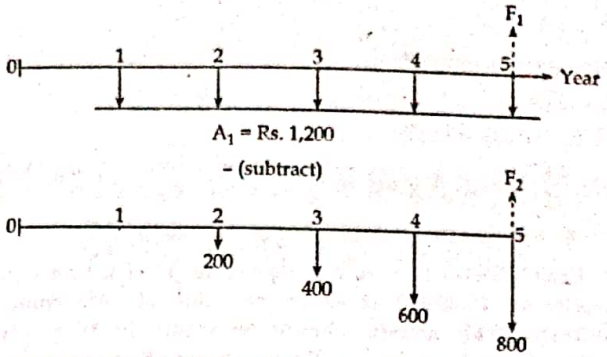


Figure: Original cash flow

The cash flow includes a decreasing gradient series. For a decreasing gradient series, the solution is obtained by separating the flow into two components: a uniform series and an increasing gradient that is subtracted from the uniform series.



Hence, $F = F_1 - F_2 = A_1 (F/A, i\%, 5) - G (P/G, i\%, N) (F/P, i\%, N)$

$= 1,200 \times 6.105 - 200 \times 6.862 \times 1.611 = \text{Rs. } 5115$

7. You have just purchased 100 shares of general electrical stock at Rs. 600 per share. You will sell the stock when its market price has doubled. If you expect the stock price to increase 20% per year, how long do you anticipate waiting before selling the stock?

Solution:

P = Rs. 600 × 100 = Rs. 60,000

F = Rs. 1,200 × 100 = Rs. 1,20,000

i = 20% per year

N = ?

We know, using the single payment compounded amount factor,

$F = P(1 + i)^N$

or, $1,20,000 = 60,000 \times (1 + 0.20)^N$

or, $2 = (1.2)^N$

Taking in on both sides,

or, $\ln(2) = N \times \ln(1.2)$

or, $0.693 = N \times 0.1823$

$\therefore \quad N = 3.80$ years

Hence, it will take 3.80 years

8. How long does it take for an investment to triple if the interest rate is 8% compounded weekly?

Solution:

Let, P = P

F = 3P

r = 8% compounded weekly

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.08}{52}\right)^{52} - 1 = 8.322\%$$

N = ?

We have,

$$F = P(1+i)^N$$

or, $3P = P(1+0.08322)^N$

or, $3 = (1.0832)^N$

or, $N \ln(1.0832) = \ln(3)$

or, $N = \frac{1.0986}{0.0799} = 13.75$ years

Hence, it takes 13.75 years to triple the investment.

9. A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution:

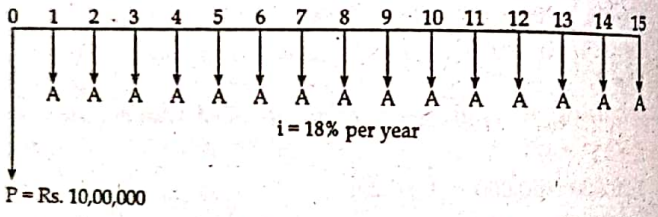
P = Rs. 10,00,000 i = 18% per year

N = 15 years A = ?

We know,

$A = P(A/P, i\%, N) = 10,00,000\ (A/P, 18\%, 15) = 10,00,000 \times 0.1964$

$\therefore$ A = Rs. 1,96,400



Hence, the company has to pay Rs. 1,96,400 per year up to 15 years and total amount to be paid to bank is

= Rs. 1,96,400 × 15 = Rs. 2,946,000

10. What will be the maturity amount of Rs. 100,000 after 5 years for nominal interest rate of 7% when compounded quarterly?

Solution:

P = Rs. 1,00,000

N = 5 years

r = 7% compounded quarterly

$$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.07}{4}\right)^4 - 1 = 7.19\% \text{ per year}$$

Now,

$\therefore$ $F = P(1+i)^N = Rs.\ 1,00,000\ (1+0.0719)^5$

$= Rs.\ 1,00,000 \times 1.4148 = Rs.\ 1,41,505$

CHAPTER 4

BASIC METHOD OF ENGINEERING ECONOMIC STUDIES

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4.1 MINIMUM ATTRACTIVE RATE OF RETURN (MARR)

MARR is the minimum interest rate that encourages the investor to invest in financial projects. MAAR is the interest rate at which a firm can always earn or borrow money under a normal operating environment. It is the rate at which NPW analysis should be conducted. Also, termed as hurdle rate for an investment usually, selection of the MARR is a policy decision made by top management. MARR shows how much an investor is interested in investment. MARR is determined from the opportunity cost view point.

4.2 PAYBACK PERIOD METHOD

The payback period is the length of time required to recover the cost of an investment. It is one of the simplest investment appraisal techniques.

4.2.1 Simple Payback Period (θ)

Simple payback period denotes the time period required to breakeven on an investment without considering time value of money.

$$SPP\ (\theta) = \frac{\text{Initial investment}}{\text{Net cash inflows per period}}$$

or, $$\theta = \sum_{C=1}^{\theta} (R_C - E_C) - I \geq 0$$

where, R_C = Revenues for the C^{th} year

E_C = Expenses for the C^{th} year

when $\theta = N$, the salvage value is included in determination of payback period.

4.2.2 Discounted Payback Period (θ')

Discounted payback period denotes the number of years required to recover the investment from discounting cash flows *i.e.*, considering time value of money.

$$\theta' = \sum_{C=1}^{\theta'} (R_C - E_C)\,(P/F, i\%, C) - I = 0$$

where, I = Initial investment

A discounted payback period gives the number of years it takes to break even from undertaking the initial expenditure by discounting future cash flows and recognizing the time value of money.

4.3 EQUIVALENT WORTH METHODS

This method converts all cash flows into equivalent worth at present or future time by using an interest rate equal to MARR.

4.3.1 Present Worth (PW) Method

It is based on the concept of equivalent worth of all cash flows to the base point in time called the present. In this method, all the cash inflows and outflows are discounted to the beginning point of time at an interest rate for the economic study. It is also known as net present value (NPV).

The PW, present worth of a project, can be calculated by the equivalence of all cash inflows (such as receipts) and all cash outflows (such as expenses) to the current time at an interest rate that is a MARR. When the calculate PW is positive, the project is acceptable for further pursuits.

The factors considered during PW analysis are as follows.

i) Purchase price of equipment needed.
ii) Estimated useful life.
iii) Revenue accrued.
iv) Interest rate.
v) Salvage value.
vi) Energy and other utility costs.
vii) Operational and maintenance costs.

The effect of inflation and deflation on raw materials and energy costs can be taken into account as well, using inflation factors. The inflation factor gives the rate at which the price go up every year.

When N becomes infinity, PW of such projects can also be called CW, the capitalized worth of the projects. The CW method is popular when endowments are established and public projects with indefinite lives are pursued.

It is calculated as:

$$PW\ (i\%) = \sum_{C=0}^{N} F_C\,(1+i)^{-C}$$

where, i = Effective interest rate or MARR

C = Index for each compounding period

F_C = Future cash flow at the end of period C.

N = Number of compounding periods.

Decision criteria

If PW (i%) > 0, accept the project
If PW (i%) < 0, reject the project
If PW (i%) = 0, remain indifferent

4.3.2 Future Worth (FW) Method

Future worth method is used particularly in an investment situation where we need to compute the equivalent worth of the project at the end of its investment period. It is the equivalent worth of all cash flows at the end of study period.

The FW (future worth) of a project can be calculated by the equivalence of all cash inflows such as receipts and all cash outflows such as expenses to a future time at an interest rate that is a MARR. When the calculated PW is positive, the project is acceptable for further pursuits. The period of time, N, can be obtained suitably from the cash flow diagram.

It is calculated as:

$$FW\ (i\%) = \sum_{C=0}^{N} F_C\,(1+i)^{N-C}$$

Decision criteria

If FW (i%) > 0, accept the project
If FW (i%) = 0, remain indifferent
If FW (i%) < 0, reject the project

4.3.3 Annual Worth Method

It provides basis for measuring investment worth into a series of equal payments at the end of each period. The AW of a project is annual equivalent revenue or savings minus annual equivalent expenses, less its annual capital recovery (CR) amount.

It is calculated as:

AW (i%) = R – E – CR(i%)

where, R = Annual equivalent revenue

E = Annual equivalent expenses

CR = Annual equivalent capital recovery amount

Decision criteria

If AW (i%) > 0, accept the project

If AW (i%) < 0, reject the project

If AW (i%) = 0, remain indifferent

Capital asset includes initial investment amount (I) and salvage value (S) of any business operation, so, CR cost can be calculated as,

CR (i%) = I (A/P, i%, N) – S (A/F, i%, B)

or CR (i%) = (I – S) (A/F, i%, N) + I (i%)

or, CR (i%) = (I – S) (A/P, i%, N) + S (i%)

4.4 RATE OF RETURN METHOD

This method is especially relevant to evaluate mutually exclusive alternative projects. It is a relative percentage method which measures the yield as a percentage of investment over the life of a project.

4.4.1 Internal Rate of Return (IRR)

This is the most widely used method of evaluation of a capital project is the internal rate of return method or IRR. Other phrases used for the same method are investor's method, discounted cash flow method, profitability interest, etc. An interest rate is calculated, at which point the PW of the project goes from negative territory to positive territory. This interest rate is called to IRR of the project. In addition, the IRR rate has to be greater than the MARR rate in order for the project to be considered acceptable. It can be viewed as another break-even point.

The IRR can be calculated using graph paper. The PW of the project can be evaluated at various interest rates such as 2%, 4%, 6%, 8%, 10%, 12%, 14%, ..., etc. The PW is plotted as a function of interest rate. The x-intercept or the interest rate at which the PW emerges into the positive territory from negative territory is called the IRR. Software is available to perform the trial and error calculations. For example, the MS excel spreadsheet has an IRR (range, guess) function call for obtaining the IRR for a set of cash flows.

Although a popular method, some pitfalls of an IRR include the occurrence of multiple values for the IRR. The variation of PW as a function of interest rate, i, is a function of several factors and may vary from project to project. When multiple values of an IRR result, one of the other methods of evaluation of the project such as FW, AW, payback period, etc. can be used instead of the IRR method. An IRR method also assumes that the revenues generated are reinvested back into the business.

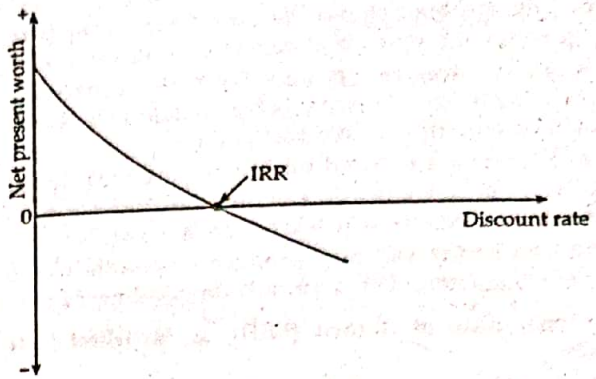


Figure: IRR

IRR is the interest rate at which the net present value of all cash flows both positive and negative from a project or investment equals zero.

i.e., $PW_{Inflow} = PW_{outflow}$

or, $PW_{Inflow} - PW_{outflow} = 0$

Based on PW formulation

$$PW\ (i\%) = \sum_{C=0}^{N} R_C\ (P/F, i'\%\ C) - \sum_{C=0}^{N} E_C\ (P/F, i'\%\ C) = 0$$

where, R_C = Revenues for the C^{th} year

E_C = Expenses for the C^{th} year

N = Project life

Based on FW formulation

$$FW\ (i\%) = \sum_{C=0}^{N} R_C\ (F/P, i'\%, N - C) - \sum_{C=0}^{N} E_C\ (F/P, i'\%, N - C) = 0$$

Based on AW formulation

$$AW\ (i\%) = \sum_{C=0}^{N} R_C\ (F/P, i'\%, N - C)\ (A/F, i'\%, N) - \sum_{C=0}^{N} E_C$$

$$(F/P, i'\%\ N - C)\ (A/\ F, i'\%, N) = 0$$

Steps for calculating IRR

Step 1: Develop an equation for equivalent worth of any point of time indicating rate of interest i'% whose value is to be found out.

Step 2: Equate the developed equation to zero

Step 3: Solve it to get i'% which is required IRR.

Decision criteria

If IRR > MARR, accept the project

If IRR < MARR, reject the project

If IRR = MARR, remain indifferent

Drawbacks of IRR method

i) When the algebraic sign of the cash flows changes more than o... in the series, it is possible to obtain multiple rates of return.

ii) IRR method does not consider the scale of investment. It can ... misleading when choosing between mutually exclusive proje... that have substantially different outlays.

iii) The IRR method is based on the assumptions that the recove... funds, if not consumed in each time period are reinvested at i... rather than at MARR which is not always practical.

iv) Time consuming and may produce inaccurate results due to ... use of linear interpolation formula for non-linear function.

4.4.2 External Rate of Return (ERR) Or Modified Rate of Retur... (MIRR)

ERR method directly takes into account the interest rate (∈) external to project at which net cash flows generated or required by the project ov... its life can be reinvested or borrowed. ERR method eliminated th... drawback of reinvestment assumption to some extent.

ERR is i'% at which

$$\sum_{C=0}^{N} E_C\,(P/F, \in\%, C)\,(F/P, i'\%, N) = \sum_{C=0}^{N} R_C\,(F/P, \in\%, N-C)$$

where, ∈ = External reinvestment rate per period.

Decision rule

If ERR > MARR, accept the project
If ERR < MARR, reject the project
If ERR = MARR, remain indifferent

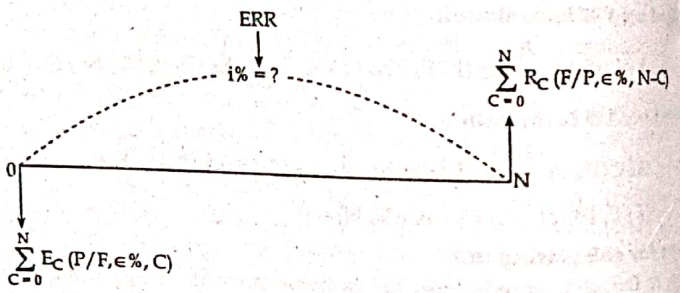


Figure: ERR

Steps for calculating ERR

Step 1: All net cash outflows are discounted to time zero at ∈% per compounding period.

Step 2: All net cash inflows are compounded to period N at ∈%.

Step 3: Calculate ERR by making equivalence between the above two quantities.

4.5 BENEFIT COST RATIO METHOD (BCR)

BCR method is defined as the ratio of the equivalent worth of benefits to the equivalent worth of costs. It is an indicator used in cost benefit analysis, that attempts to summarize the overall value for money of a project. It is also known as the saving investment ratio.

Two commonly used formulation of B/C ratio are as follows:

1. Conventional B/C ratio

It is the ratio of gross benefits to costs.

i) With PW formulation

$$\text{B/C ratio} = \frac{PW\,(B)}{PW\,(I) - PW\,(S) + PW\,(O \text{ and } M)}$$

ii) With FW formulation:

$$\text{B/C ratio} = \frac{FW\,(B)}{FW\,(I) - FW(S) + FW\,(O \text{ and } M)}$$

iii) With AW formulation:

$$\text{B/C ratio} = \frac{AW\,(B)}{AW\,(I) - AW\,(S) + AW\,(O \text{ and } M)}$$

2. Modified B/C ratio

It is the ratio of net benefits to costs

i) With PW formulation:

$$\text{B/C ratio} = \frac{PW\,(B) - PW\,(O \text{ and } M)}{PW\,(I) - PW\,(S)}$$

ii) With AW formulation:

$$\text{B/C ratio} = \frac{AW\,(B) - AW\,(O \text{ and } M)}{AW\,(I) - AW\,(S)} = \frac{AW\,(B) - AW\,(O \text{ and } M)}{CR}$$

iii) With FW formulation:

$$\text{B/C ratio} = \frac{FW\,(B) - FW\,(O \text{ and } M)}{FW\,(I) - FW(S)}$$

where, B = Benefits of the proposed project
S = Salvage value of the proposed project
CR = Capital recovery amount
O and M = Operation and maintenance cost of proposed project
I = Initial investment of the proposed project

Decision criteria

If B/C ratio > 1, accept the project
If B/C ratio < 1, reject the project
If B/C ratio = 1, remain indifferent

Relationship among MARR, IRR and ERR

- If IRR < MARR, then IRR < ERR < MARR
- If IRR > MARR, then IRR > ERR > MARR
- If IRR = MARR, then IRR = ERR = MARR

BOARD EXAM SOLVED PROBLEMS

1. Difference between IRR and ERR. [2018/Fall]

Answer:

S.N.	IRR	ERR
i)	It is the discount rate that makes the net present value of all cash flows from a particular project equals to zero.	ERR method directly takes into account the interest rate (ϵ) external to a project at which net cash flows generated by the project over its life can be reinvested or borrowed.
ii)	IRR sometime gives multiple rate of return.	ERR never gives multiple rate of return.
iii)	It is time consuming method.	It is not time consuming method.

2. Define payback period, simple payback period and discounted payback period. [2012/Fall, 2018/Fall, 2017/Fall]

Answer: See the topic 4.2, 4.2.1, 4.2.2

3. Define cost benefit ratio. [2012/Spring, 2014/Fall]

Answer: See the topic 4.5

4. Define life cycle cost and explain it categories. [2010/Fall, 2016/Fall, 2017/Spring]

Answer:

Life cycle costing is a system that tracks and accumulates the actual costs and revenues attributable to cost object from its invention to its abandonment. Life cycle cost is defined as the sum of all expenditures associated with the item or project during its entire life service.

Life cycle cost consists following three categories:

i) Initial cost
ii) Operating and maintenance costs
iii) Disposal costs

i) Initial costs

Initial costs is the total investment required to get the item or project ready for service and such costs are non-recurring nature over the life of the item or project. The initial or first cost may consists following elements:

- Machine equipment costs
- Land and building costs
- Manufacturing costs
- Installation costs
- Development and engineering design costs etc

ii) Operating and maintenance costs

Operating and maintenance costs are recurring costs that are necessary to operate and maintain an item or project during its useful life. This costs consists of following elements:

- Raw material cost
- Transportation cost
- Labour wages and salaries cost
- Cost for spares, corrective maintenance
- Overhead items cost (tax, insurance premium, overtime expenses, etc.)

iii) Disposal costs

Disposal cost is the cost or gain of getting rid of assets after use. It may include the net remaining worth, as well as the cost of transferring or destroying the assets. It includes following elements:

- Labour cost for removal of the item
- Material costs (tool, fuel, packing material, etc.)

The salvage value is determined by deducting the cost of disposal from the market value of the asset at the time of disposal. Salvage value may be positive or negative.

∴ Life cycle costs = First costs + Lifetime operating costs
+ Lifetime maintenance costs
+ Disposal costs
– Residual value

5. Find the present equivalent from the cash flow given if interest rate is 11% per year using uniform gradient method.

End of year	Cash flow (Rs)
1	400
2	600
3	800
4	1000
5	1200
6	1400

[2014/Fall]

Solution:

i = 11% per year
G = Rs. 200

PW (i%) = $A_0(P/A, i\%, N) + G(P/G, i\%, N)$
= $400 \times (P/A, 11\%, 6) + 200 (P/G, 11\%, 6)$
= $400 \times 4.2305 + 200 \times 9.2972$
= 1692.20 + 1859.44

∴ PW(11%) = Rs. 3,551.64

6. For the project given below: [2015/Spring]

Initial investment	**Rs. 90,000**
Project life	**6 years**
Annual revenue	**Rs. 27,000**
Salvage value	**Rs. 10,000**
Annual expenses	**Rs. 8,000**

i) What is the IRR for the project?

ii) If MARR = 15%, is the project feasible?

Solution:

I = Rs. 90,000

N = 6 years

R = Rs. 27,000

S = Rs. 10,000

E = Rs. 8,000

MARR = 15%

We know,

Setting the given information into PW formulation

$$PW = -90,000 + (27,000 - 8,000)(P/A, i\%, 6) + 10,000 \times (P/F, i\%, 6)$$

IRR is that rate which makes PW = 0,

$$PW = -90,000 + 19,000\,(P/A, i'\%, 6) + 10,000\,(P/F, i'\%, 6) = 0$$

If i' = 10%,

$$PW(10\%) = -90,000 + 19,000 \times 4.3553 + 10,000 \times 0.5645$$
$$= -\text{Rs. } 1604.3 \text{ (-ve)}$$

If i' = 8%

$$PW(8\%) = -90,000 + 19,000 \times 4.6229 + 10,000 \times 0.6302$$
$$= \text{Rs. } 4,137 \text{ (+ve)}$$

Now,

$$IRR = i_1' + \frac{PW_1}{PW_1 - PW_2} \times (i_2' - i_1') = 8 + \frac{4,137}{4,137 + 1,604.30} \times (10 - 8)$$

$\therefore$ IRR = 9.441%

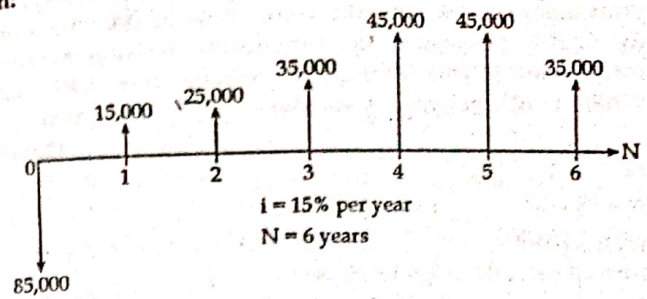
Since, IRR (9.441%) < MARR (15%), the project is unfeasible.

7. Calculate simple and discounted payback period using MARR = 15%.

Period	Cash flow (Rs.)
0	-85,000
1	15,000
2	25,000
3	35,000
4	45,000
5	45,000
6	35,000

[2015/Spring]

Solution:



Simple payback period:

EOY	Net cash flow	Cumulative cash flow
0	-85,000	- 85,000
1	15,000	-70,000
2	25,000	-45,000
3	35,000	-10,000
4	45,000	35,000
5	45,000	8,000
6	35,000	1,15,000

Payback period lies between 3 and 4 years.

Hence, simple payback period, $\theta = 3 + \frac{10,000}{45,000} = 3.23$ years

and, **Discounted payback period:**

EOY	Net cash flow, F	Discounted cash flow into present @ 15%, P	Cumulative cash flow (Rs)
0	-85,000	-85,000	-85,000
1	15,000	13,043.48	-71,956.52
2	25,000	18,903.59	-53,052.93
3	35,000	23,013.06	-30,039.87
4	45,000	25,728.89	-4,310.98
5	45,000	22,373	18,062.02
6	35,000	15,131.46	33,193.50

$P = F(1 + i)^{-N}$

Here, cumulative cash flow turns positive in period 5. Hence, discounted payback period lies between 4 and 5.

By interpolating,

$$\theta' = 4 + \frac{4,310.98}{22,373} = 4 + 0.1926 = 4.20 \text{ years}$$

Hence, discounted payback period is 4.20 years.

8. **A new machine costing Rs. 250,000 is estimated to have life of 1 years and expected annual revenue is Rs. 50,000 with annual co Rs. 12,500. Determine the investment decision based on PV formulation to this machine, if salvage value is Rs. 70,000 an MARR is 10% per year. Also, Make cash flow diagram.** [2014/Spring]

Solution:

N = 10 years

I = Rs. 250,000

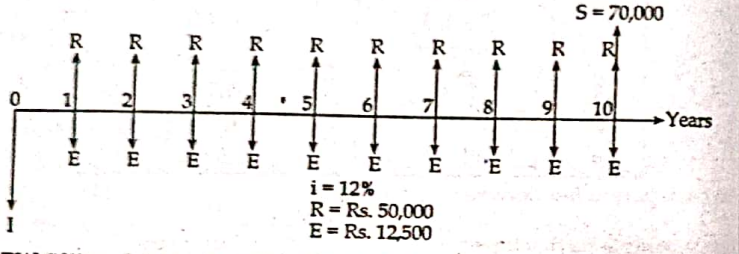
Annual revenue, AR = Rs. 50,000

Annual cost AE = Rs. 12,500

Salvage value, S = Rs. 70,000

MARR = 10% per year

Cash flow diagram:



$$PW(i\%) = -I + (A - E)(P/A, i\%, N) + S(P/F, i\%, N)$$

$$= -250,000 + (50,000 - 12,500) \times (P/A, 10\%, 10) + 70,000 \times (P/F, 10\%, 10)$$

$$= -2,50,000 + 37,500 \times 6.1446 + 70,000 \times 0.3855$$

$\therefore$ PW = Rs. 7,407.50

Since, PW (10%) > 0, the investment is acceptable.

9. **Find present equivalent from the cash flow given if interest rate is 11% per year using uniform gradient method.** [2011/Spring]

End of year	Cash flow
1	–40,000
2	–50,000
3	–60,000
4	–70,000
5	–80,000
6	–1,00,000

Solution:

i = 11% per year

$F = P(F/A, i\%, N) + G(F/G, i\%, N)$

$$F_5 = -40,000(F/A, 11\%, 5) + \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$= -40,000 \times 6.2278 + \frac{(-10,000)}{0.11}\left[\frac{(1+0.11)^5 - 1}{0.11}\right] - 5 \times \frac{(-10,000)}{0.11}$$

$$= -2,49,112 - 5,66,164 + 4,54,546$$

$\therefore$ $F_5 = -6,09,842$

and, $F_6 = F_5 \times (F/P, 11\%, 1) = -6,09,842 \times 1.110 = -$ Rs. 676,926

Thus,

$\therefore$ $PW(11\%) = F_6(P/F_6, 11\%, 6) = -6,76,926 \times 0.5346 = -$ Rs. 3,61,884

10. **From the following cash flow information, calculate PW, AW and FW by assuming rate of interest is 7% per year that compounds semi annually.**

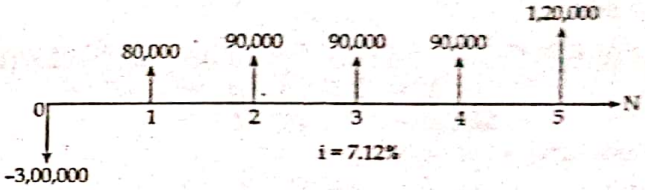
EOY	0	1	2	3	4	5
Cash flow	–3,00,000	80,000	90,000	90,000	90,000	1,20,000

[2015/Fall]

Solution:

r = 7% per year

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.07}{2}\right)^2 - 1 = 0.0712 = 7.12\%$$



$$PW = -3,00,000 + 80,000 \times (1.0712)^{-1} + 90,000(1.0712)^{-2} + 90,000(1.0712)^{-3} + 90,000(1.0712)^{-4} + 1,20,000 \times (1.0712)^{-5}$$

$$= -3,00,000 + 74,682.6 + 78,433.5 + 73,220 + 68,353.5 + 85,080$$

$\therefore$ PW (7.12%) = Rs. 79,769.5

Now,

$$AW(7.12\%) = PW(A/P, i\%, N)$$

$$= 79,769.5 \times (A/P, 7.12\%, 5)$$

$$= 79,769.5 \times \left[0.0712 \times \frac{(1+0.0712)^5}{(1+0.0712)^5 - 1}\right]$$

$$= 79,769.5 \times 0.2447$$

$\therefore$ AW (7.12%) = Rs. 19,519.60

and, $FW(7.12\%) = PW(F/P, i\%, N)$

$$= PW(F/P, 7.12\%, 5)$$

$$= 79,769.5 \times (1 + 0.0712)^5$$

$\therefore$ FW (7.12%) = Rs. 1,12,509.62

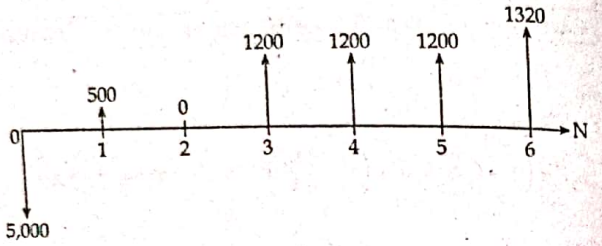
11. Calculate present worth, future worth and annual worth from the following net cash flows. MARR is 12% per year.

End of year	Net cash flow
0	-5,000
1	-500
2	0
3	1,200
4	1,200
5	1,220
6	1,320

[2013/Fall]

Solution:

MARR = 12% per year



$$PW(12\%) = -5{,}000 - 500(1.12)^{-1} + 0 + 1{,}200(1.12)^{-3} + 1{,}200(1.12)^{-4} + 1{,}220(1.12)^{-5} + 1{,}320(1.12)^{-6}$$

$$= -2{,}468.66$$

$$AW(12\%) = PW(A/P, i\%, N)$$

$$= PW \times \left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = -2{,}468.66 \times \left[\frac{0.12 \times (1+0.12)^6}{(1+0.12)^6 - 1}\right]$$

$$= -2{,}468.66 \times 0.24322$$

$$= -\text{Rs. } 600$$

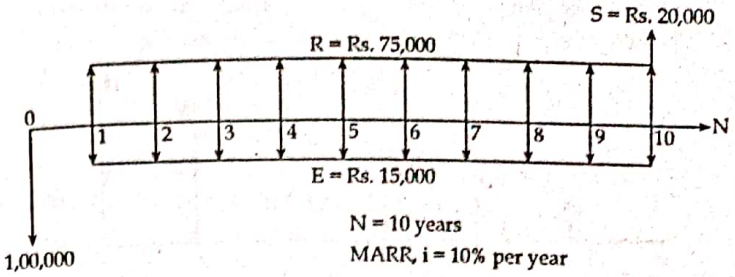
$$FW(12\%) = PW(F/P, 12\%, 6) = -2{,}468.66 \times 1.9376 = -\text{Rs. } 4{,}783.74$$

12. Find both types of B/C ratio using PW formulation for a project having first investment cost Rs. 1,00,000, project life 10 years, salvage value Rs. 20,000, annual benefit Rs. 75,000, annual O and M cost Rs. 15,000 and MARR = 10%. [2015/Fall, 2011/Spring]

Solution:

Here; Investment (I) = Rs. 1,00,000
Salvage value (S) = Rs. 20,000
Annual benefit (B) = Rs. 75,000
Annual expenses (O and M) = Rs. 15,000
MARR = 10% per year
Project life (N) = 10 years

We know,



N = 10 years
MARR, i = 10% per year

$$PW(B) = B(P/A, i\%, N)$$

$$= 75{,}000 \times (P/A, 10\%, 10)$$

$$= 75{,}000 \times 6.1446$$

$$= \text{Rs. } 4{,}60{,}845$$

$$PW(\text{O and M}) = 15{,}000 \times (P/A, 10\%, 10)$$

$$= 15{,}000 \times 6.1446$$

$$= \text{Rs. } 92{,}169$$

$$PW(S) = 20{,}000 \times (P/F, 10\%, 10)$$

$$= 20{,}000 \times 0.3855$$

$$= \text{Rs. } 7{,}710$$

$$\text{Now, Conventional BCR} = \frac{PW(B)}{I - PW(S) + PW(\text{O and M})}$$

$$= \frac{4{,}60{,}845}{1{,}00{,}000 - 7{,}710 + 92{,}169}$$

$$= 2.498 > 1$$

$$\text{Modified BCR} = \frac{PW(B) - PW(\text{O and M})}{I - PW(S)}$$

$$= \frac{4{,}60{,}845 - 92{,}169}{1{,}00{,}000 - 7{,}710}$$

$$= 3.995 > 1$$

Since, BCR > 1, project is feasible to invest.

13. Find PW from the cash flow given if interest rate is 11% per year using uniform gradient method.

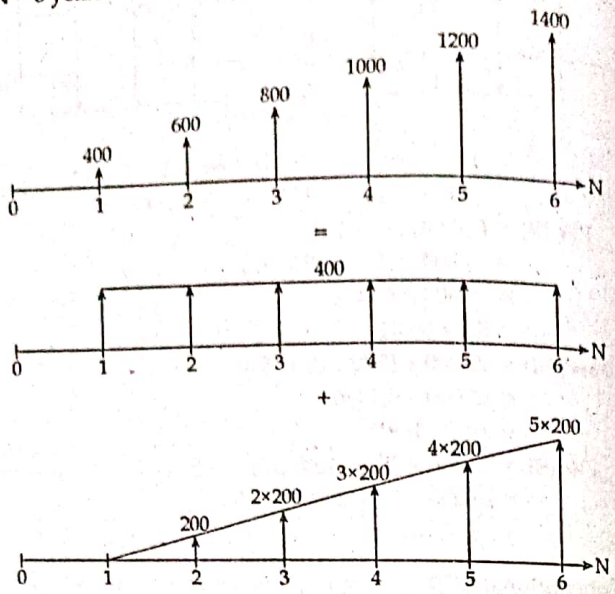
EOY	Cash flow
1	400
2	600
3	800
4	1,000
5	1,200
6	1,400

[2014/Fall]

Solution:

i = 11% per year

N = 6 years



∴ PW (11%) = 400 (P/A, 11%, 6) + 200 (P/G, 11%, 6)

= 400 × 4.2305 + 200 × 9.2972

= Rs. 3,551.64

14. Find out the both types of B/C ratio using PW and AW method.

Initial investment = Rs. 4,00,000

Annual benefit = Rs. 1,50,000

Annual cost = Rs. 30,000

MARR = 12% per year

Useful life, = 8 years

Salvage value = Rs. 50,000

[2013/Fall, 2014/Fall, 2016/Fall, 2016/Spring, 2012/Spring, 2010/Fall]

Solution:

Here;

I = Rs. 4,00,000

N = 8 years

MARR = 12% per year

AB = Rs. 1,50,000

S = Rs. 50,000

O and M = Rs. 30,000

i) With PW formulation

PW (I) = Rs. 4,00,000

PW (B) = 1,50,000 (P/A, 12%, 8)

= 1,50,000 × 4.9676

= Rs. 7,45,140

PW (S) = 50,000 (P/F, 12%, 8) = 50,000 × 0.4039 = Rs. 20,195

PW (O and M) = 30,000 (P/A, 12%, 8)

= 30,000 × 4.9676

= Rs. 1,49,028

Thus,

$$\therefore \text{Conventional B/C ratio} = \frac{\text{PW (B)}}{\text{PW (I)} - \text{PW(S)} + \text{PW (O and M)}}$$

$$= \frac{7,45,140}{4,00,000 - 20,195 + 1,49,028}$$

$$= 1.409 > 1$$

$$\therefore \text{Modified B/C ratio} = \frac{\text{PW (B)} - \text{PW (O and M)}}{\text{PW (I)} - \text{PW(S)}}$$

$$= \frac{7,45,140 - 1,49,028}{4,00,000 - 20,195}$$

$$= 1.659 > 1$$

ii) Using AW formulation:

AW (I) = 4,00,000 (A/P, 12%, 8)

= 4,00,000 × 0.2013

= Rs. 80,520

AW (B) = Rs. 1,50,000

AW(O and M) = Rs. 30,000

AW (S) = 50,000 (A/F, 12%, 8)

= 50,000 × 0.0813

= Rs. 4,065

$$\therefore \text{Conventional B/C ratio} = \frac{\text{AW (B)}}{\text{AW (I)} - \text{AW (S)} + \text{AW (O and M)}}$$

$$= \frac{1,50,000}{80,520 - 4,065 + 30000}$$

$$= 1.409 > 1$$

and,

$$\therefore \text{Modified B/C ratio} = \frac{\text{AW (B)} - \text{AW (O and M)}}{\text{AW (I)} - \text{AW (S)}}$$

$$= \frac{1,50,000 - 30,000}{80,520 - 4,065}$$

$$= 1.569 > 1$$

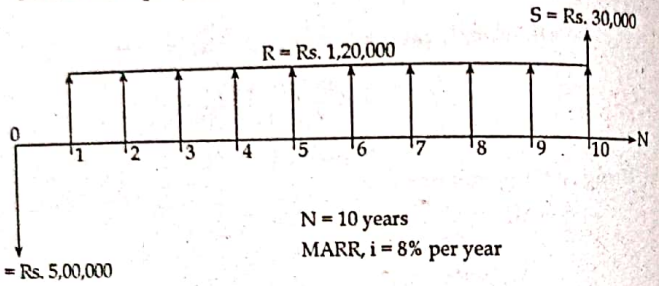
Since, B/C ratio > 1, investment is feasible.

15. Evaluate the IRR for the following project and decide whether the project is acceptable or not? Also draw the unrecovered investment balance diagram.
Initial investment = Rs. 5,00,000
Annual revenue = Rs. 1,20,000
Salvage value = Rs. 30,000
Useful life year = 10 year
MARR = 8% **[2014/Fall, 2011/Fall, 2010/Fall]**

Solution:

Here;

I = Rs. 5,00,000
AR = Rs. 1,20,000
S = Rs. 30,000
N = 10 years
MARR = 8% per year



Using PW formulation,

$$PW(i\%) = -I + R(P/A, i\%, N) + S(P/F, i\%, N)$$
$$PW(i\%) = -5,00,000 + 120,000\,(P/A, i\%, 10) + 30,000 \times (P/F, i\%, 10)$$

IRR is that rate at which PW = 0,

Let, i' = 20%

$$PW(20\%) = -5,00,000 + 120,000 \times 4.1925 + 30,000 \times 0.1615$$
$$= 7,945\ (+ve)$$

Let, i' = 21%,

$$PW(21\%) = -5,00,000 + 120,000 \times 4.054 + 30,000 \times 0.1486$$
$$= -\text{Rs. } 9,062\ (-ve)$$

Now,

$$IRR = i_1'\% + \frac{PW_1}{PW_1 - PW_2} \times (i_2' - i_1')\%$$

$$= 20\% + \frac{7,945}{7,945 + 9,062} \times (21 - 20\%) = 20\% + 0.467\%$$

$$\therefore \quad IRR = 20.467\%$$

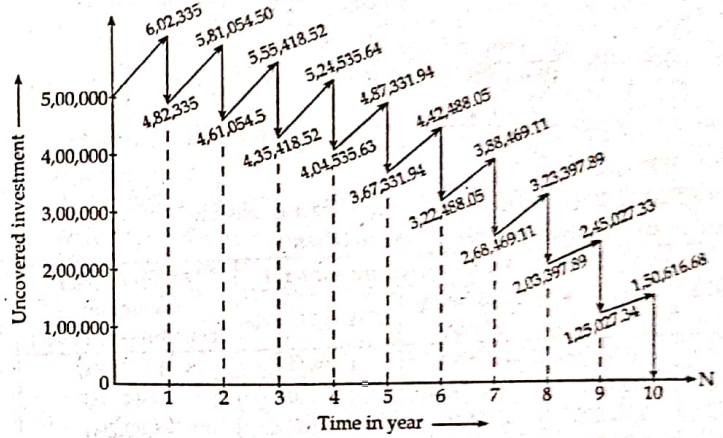
Since, IRR (20.467%) > MARR (8%), the project is feasible.

Investment Balance diagram:

Unrecovered investment calculation in tabular form:

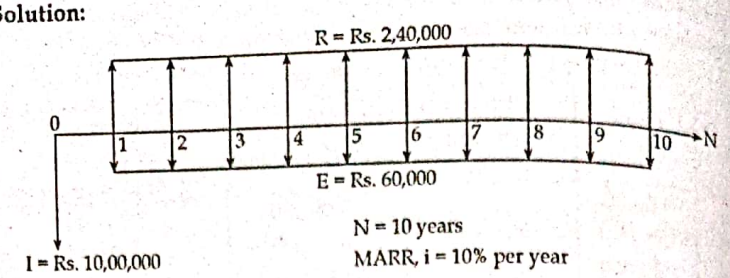
$\therefore$ EOY value = (1 + i) × Beginning of year value

EOY	Cash flow	Unrecovered investment		Unrecovered investment (Revenue-expenses)
		Beginning of year	EOY @ 20.467%	
0	-5,00,000	-	-5,00,000	-5,00,000
1	1,20,000	-5,00,000	-6,02,335	-4,82,335
2	1,20,000	-4,82,335	-5,81,054.50	-4,61,054.5
3	1,20,000	-4,61,054.5	-5,55,418.52	-4,35,418.52
4	1,20,000	-4,35,418.52	-5,24,535.64	-4,04,535.63
5	1,20,000	-4,04,535.63	-4,87,331.94	-3,67,331.94
6	1,20,000	-3,67,331.94	-4,42,488.05	-3,22,488.05
7	1,20,000	-3,22,488.05	-3,88,469.11	-2,68,469.11
8	1,20,000	-2,68,469.11	-3,23,397.89	-2,03,397.89
9	1,20,000	-2,03,397.89	-2,45,027.33	-1,25,027.34
10	1,50,000	-1,25,027.34	-1,50,616.68	-616.68



16. Evaluate the IRR of the following project, identify whether the project is feasible or not. Also, draw investment balance diagram.
Initial investment = Rs. 10,00,000
Annual revenues = Rs. 2,40,000
Annual cost = Rs. 60,000
Useful life year = 10 years
MARR = 10% **[2014/Spring, 2013/Spring, 2010/Fall]**

Solution:



N = 10 years
MARR, i = 10% per year

Using PW formulation,

$$PW(i\%) = -I + (R - E)(P/A, i\%, N)$$
$$= -10,00,000 + (240,000 - 60,000) \times (P/A, i\%, 10)$$
$$= -10,00,000 + 1,80,000\ (P/A, i\%, 10)$$

IRR is that rate which makes PW = 0

$$PW = -10,00,000 + 1,80,000\ (P/A, i'\%, 10) = 0$$

If i'% = 10%

$$PW(10\%) = -10,00,000 + 1,80,000 \times 6.1446 = \text{Rs. } 1,06,028\ (+ve)$$

If i'% = 13%

$$PW(13\%) = -10,00,000 + 1,80,000 \times 5.4262 = -\text{ Rs. } 23,284\ (-ve)$$

Now,

$$IRR = i_1'\% + \frac{PW_1}{PW_1 - PW_2} \times (i_2' - i_1')\%$$

$$= 10\% + \frac{1,06,028}{1,06,028 + 23,284} \times (13 - 10)\% = 10\% + 2.46\%$$

$\therefore$ IRR = 12.46%

Since IRR (12.46%) > MARR (10%), the project is feasible.

Unrecovered investment calculation in table:

EOY	Cash flow	Unrecovered investment		Unrecovered investment (revenue-expenses)
		Beginning of year	EOY @ 12.46%	
0	-10,00,000	-10,00,000	-10,00,000	-10,00,000
1	1,80,000	-10,00,000	-1124600	-9,44,600
2	1,80,000	-9,44,600	-10,62,298	-8,82,298
3	1,80,000	-8,82,298	-9,92,232	-8,12,232
4	1,80,000	-8,12,232	-9,13,436	-7,33,436
5	1,80,000	-7,33,436	-8,24,822	-6,44,822
6	1,80,000	-6,44,822	-7,25,167	-5,45,167
7	1,80,000	-5,45,167	-6,13,095	-4,33,095
8	1,80,000	-4,33,095	-4,87,058	-3,07,058
9	1,80,000	-3,07,058	-3,45,318	-1,65,318
10	1,80,000	-1,65,318	-1,85,916	-5,916

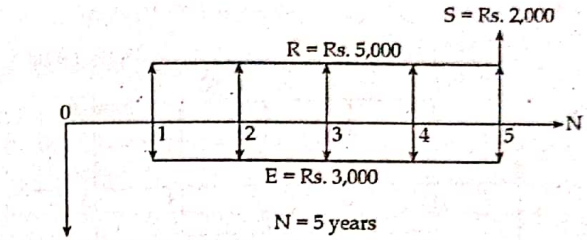
Hence, unrecovered investment is Rs. 5916. (Draw investment balance diagram as previous question).

17. Determine the IRR of the following project. Also present unrecovered investment balance in graph and table.
Initial investment = Rs. 1,50,000
Life span = 5 yrs.
Annual revenue = Rs. 5,000
Annual cost = Rs. 3,000
Salvage value = Rs. 2,000
MARR = 8% **[2015/Fall]**

Solution:

Here;

N = 5 years
S = Rs. 2,000
AR = Rs. 5,000
I = Rs. 1,50,000
MARR = 8% per year
AE = Rs. 3,000



N = 5 years
MARR, i = 8% per year

Using PW formulation,

$$PW = -1,50,000 + (5,000 - 3,000)(P/A, i\%, 5) + 2,000 \times (P/F, i\%, 5)$$
$$PW(10\%) = -1,50,000 + 2,000\ (P/A, i\%, 5) + 2,000 \times (P/F, i\%, 5)$$

IRR is that rate which makes PW = 0

$$PW = -1,50,000 + 2,000\ (P/A, i\%, 5) + 2,000 \times (P/F, i\%, 5) = 0$$

Let i' = 10%

$$PW(10\%) = -150,000 + 2,000 \times 3.7908 + 2,000 \times 0.6209$$
$$= -\text{ Rs. } 1,41,176\ (-ve)$$

Let i' = 7%

$$PW(7\%) = -1,50,000 + 2,000 \times 4.1002 + 2,000 \times 0.7130$$
$$= -\text{ Rs. } 1,40,373.6\ (-ve)$$

Let, i' = 1%

$$PW(1\%) = -1,50,000 + 2,000 \times 4.8534 + 2,000 \times 0.9515$$
$$= -\text{ Rs. } 1,38,390\ (-ve)$$

Since, the investment is very high comparative to the annual revenue and salvage value. So, PW(i%) is always negative. *i.e.*, question should be corrected to get IRR.

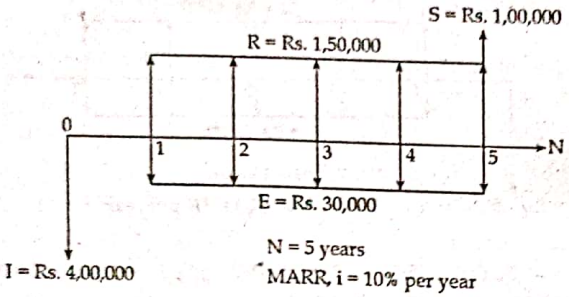
18. Find simple and discounted payback period and justify investment with the given cash flow information.

Initial investment = Rs. 4,00,000
Annual revenue = Rs. 1,50,000
Annual cost = Rs. 30,000
Annual life = 5 year
Annual MARR = 10%
Salvage value = Rs. 1,00,000 **[2015/Fall, 2017/Fall]**

Solution:

Here;

I = Rs. 4,00,000
AR = Rs. 1,50,000
AE = Rs. 30,000
S = Rs. 1,00,000
MARR = 10%
N = 10%



i) Simple payback method:

Year	Net cash flow (Rs.)	Cumulative cash flow (Rs.)
0	–4,00,000	–4,00,000
1	1,20,000	–2,80,000
2	1,20,000	–1,60,000
3	1,20,000	–40,000
4	1,20,000	80,000
5	2,20,200	3,00,000

Net cash flow = R – E = 1,50,000 – 30,000 = 1,20,000

Hence, cumulative cash flow turns to be positive in period 4. Thus, payback period lies between period 3 and 4. By interpolating,

$$\theta = 3 + \frac{40,000}{1,20,000}$$

$$\therefore \quad \theta = 3.33 \text{ years}$$

ii) Discounted payback period:

Year	Net cash flow, F	Discounted cash flow into present @ 10%; P	Cumulative cash flow
0	–4,00,000	–4,00,000	–4,00,000
1	120,000	1,09,091	–2,90,909
2	1,20,000	99,174	–1,91,735
3	1,20,000	90,158	–1,01,577
4	1,20,000	81,961	–19,616
5	2,20,000	136,603	1,16,987

$PW = F(1 + i)^{-N}$

$PW_1 = 1,20,000\ (1 + 0.10)^{-1} = \text{Rs. } 1,09,091$

$PW_2 = 1,20,000\ (1 + 0.10)^{-2} = \text{Rs. } 99,174$

$PW_3 = 1,20,000\ (1 + 0.10)^{-3} = \text{Rs. } 90,158$

$PW_4 = 1,20,000\ (1 + 0.10)^{-4} = \text{Rs. } 81,961$

$PW_5 = 2,20,000\ (1 + 0.10)^{-5} = \text{Rs. } 1,36,603$

Hence, discounted payback period $= 4 + \frac{19,616}{1,36,603}$

$\theta' = 4.144$ years

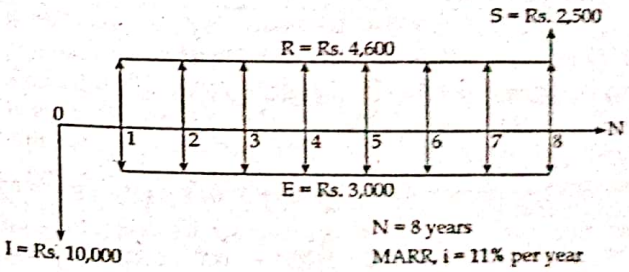
Since the investment is recovered with the given life span, the project is feasible.

19. Determine conventional and modified B/C ratio for the given projects if interest rate is 11%.

Investment = Rs. 10,000
Life of project = 8 year
Annual benefits = Rs. 4,600
Annual cost = Rs. 3,000
Salvage value = Rs. 2,500 **[2011/Fall, 2013/Spring, 2017/Spring]**

Solution:

Here; I = Rs. 10,000 N = 8 year
AB = Rs. 4,600 AE = Rs. 3,000
S = Rs. 2,500



Using annual worth method,

AW (I) = 10,000 × (A/P, 11%, 8) = 10,000 × 0.1943 = Rs. 1,943

AW (B) = Rs. 4,600

AW (O and M) = Rs. 3,000

AW (S) = Rs. 2,500 × (A/F, 11%, 8)

= 2,500 × 0.0843

= Rs. 210.75

$$\text{Conventional BCR} = \frac{\text{AW (B)}}{\text{AW (I)} - \text{AW (S)} + \text{AW (O and M)}}$$

$$= \frac{4,600}{1,943 - 210.75 + 3,000}$$

$$= 0.972 < 1$$

$$\text{Modified BCR} = \frac{\text{AW (B)} - \text{AW (O and M)}}{\text{AW (I)} - \text{AW(S)}}$$

$$= \frac{4,600 - 3,000}{1,943 - 210.75}$$

$$= 0.924 < 1$$

Since, B/C ratio < 1, reject the project

20. Evaluate IRR of the following project, identify whether the project is feasible or not.

Initial investment = Rs. 25,000

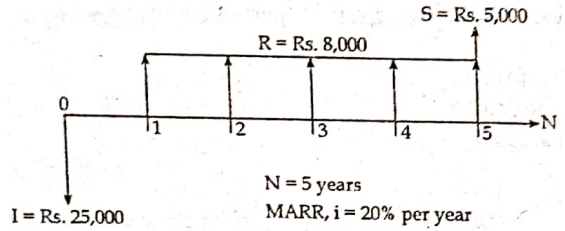
Annual revenue = Rs. 8,000

Salvage value = Rs. 5,000

Useful life = 5 years

MARR = 20% **[2013/Fall, 2012/Fall]**

Solution:



PW = – 25,000 + 8,000 (P/A, i'%, 5) + 5,000 (P/F, i'%, 5)

IRR is that rate which makes PW =0

Let, i' = 20%

PW(20%) = –25,000 + 8,000 × 2.9906 + 5,000 × 0.4019 = Rs. 934 (+ ve)

Let, i' = 21%

PW(21%) = –25,000 + 8,000 × 2.9259 + 5,000 × 0.3855 = Rs. 334 (+ve)

Let, i' = 22%

PW(22%) = –25,000 + 8,000 × 2.8634 + 5,000 × 0.3669

= – Rs. 243.30(–ve)

so, $\text{IRR} = 21\% + \frac{334}{334 + 243.30} \times (22 - 21)\% = 21\% + 0.5786\%$

∴ IRR = 21.5786%

Since IRR (21.578%) > MARR (20%) so, the project is feasible.

21. Find both types of B/C ratio using AW formulation of the following projects and find whether the project is feasible or not.

Initial investment = Rs. 5,00,000

Annual revenue = Rs. 80,000

Salvage value = Rs. 10,000

Life = 20 years

MARR = 10%

Annual O and M = Rs. 15,000 **[2014/Spring, 2012/Fall]**

Solution:

AW (I) = 5,00,000 (A/P, 10%, 20)

= 5,00,000 × 0.1175

= Rs. 58,750

AW (B) = Rs. 80,000

AW (S) = 10,000 (A/F, 10%, 20)

= 10,000 × 0.0175

= Rs. 175

AW (O and M) = Rs. 15,000

$$\text{Conventional BCR} = \frac{80,000}{58,750 - 175 + 15,000} = 1.087 > 1$$

and, $\text{Modified BCR} = \frac{80,000 - 15,000}{58,740 - 175} = 1.1096 > 1$

Since, BCR is greater than 1, the project is feasible.

22. A multipurpose hydroelectric project under consideration of the government, whose estimated benefits and costs expected to be derived from the project are listed as;

End of year	Annual cash flow (Rs.)
Initial cost	18,00,00,000
Annual power sales	1,20,00,000
Annual flood control saving	50,00,000
Annual irrigation benefits	80,00,000
Annual recreation benefits	40,00,000
Annual O and M costs	50,00,000

Suggest, based on B/C ratio, the government about implementing the project for 40 years. MARR = 15% **[2018/Spring]**

Solution:

Here;

I = Rs. 18,00,00,000

AR = 1,20,00,000 + 50,00,000 + 80,00,000 + 40,00,000 = Rs. 2,90,00,000

AE = 50,00,000

N = 40 years

MARR = 15%

PW (B) = 2,90,00,000 × (P/A, 15%, 40)

= 2,90,00,000 × 6.6418

= Rs. 19,26,12,200

PW (O and M) = 50,00,000 (P/A, 15%, 40)

= 50,00,000 × 6.6418

= Rs. 3,32,09,000

Now, Conventional BCR = $\frac{19,26,12,200}{18,00,00,00 + 3,32,09,000 - 0} = 0.904 < 1$

and, Modified BCR = $\frac{19,26,12,200 - 3,32,09,000}{18,00,00,000 - 0} = 0.886 < 1$

Since, B/C ratio smaller than 1, it is not feasible to implement this project for 40 years.

23. An investment of Rs. 100,000 can be made in a project that will produce uniform annual revenue of Rs. 62,100 for 5 years and then have a market salvage value of Rs. 20,000. Annual expenses will be Rs. 30,000 each year. Company accepts project that earns 10% or more, evaluate IRR of this project and suggest whether the project is feasible or not? Also draw an investment balance diagram. [2018/Spring]

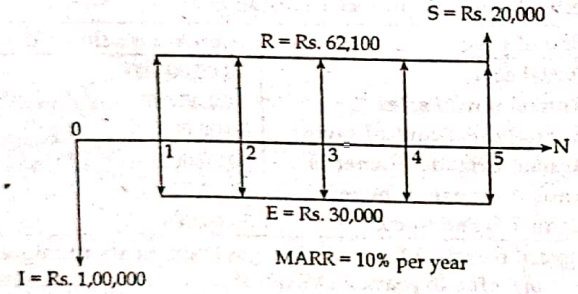
Solution:

Here;

I = Rs. 1,00,000 AR = Rs. 62,100 per year

N = 5 year S = Rs. 20,000

AE = Rs. 30,000 per year MARR = 10%



We know,

Using PW formulation,

PW = -1,00,000 + (62,100 - 30,000)(P/A, i%, 5) + 20,000 × (P/F, i'%, 5)

IRR is that rate which makes PW = 0

PW = -1,00,000 + 32,100 (P/A, i%, 5) + 20,000 × (P/F, i'%, 5)

= 0

If i' = 10%,

PW(10%) = -1,00,000 + 32,100 × 3.7908 + 20,000 × 0.6209

= Rs. 34,102.68

If i' = 15%

PW(15%) = -1,00,000 + 32,100 × 3.3522 + 20,000 × 0.4972

= Rs. 17,549.62

If i' = 20%

PW(20%) = -1,00,000 + 32,100 × 2.9906 + 20,000 × 0.4019

= Rs. 4,036

If i' = 21%

PW(21%) = -1,00,000 + 32,100 × 2.9259 + 20,000 × 0.3855

= Rs. 1631.39 (+ve)

If i' = 22%

PW(22%) = -1,00,000 + 32,100 × 2.8634 + 20,000 × 0.3699

= - Rs. 686.86 (-ve)

$\therefore$ IRR = $i_1' + \frac{PW_1}{PW_1 - PW_2} \times (i_2' - i_1')$

= $21 + \frac{1,631.39}{1,631.39 + 686.86} \times (22 - 21)$

= 21 + 0.7037

= 21.703%

Since, MARR (10%) < IRR (21.703%), the project is feasible.

Unrecovered investment calculation in tabular form:

EOY	Cash flow	Unrecovered investment		Unrecovered investment
		Beginning of year	EOY @ 21.70%	(Revenue - Expenses)
0	1,00,000	-	-1,00,000	-1,00,000
1	32,100	-1,00,000	-1,21,700	-89,600
2	32,100	-89,600	-1,09,043	-76,943.20
3	32,100	-76,943.20	-93,640	-61,540
4	32,100	-61,540	-74,894.18	-42,794.18
5	52,100	-42,794.18	-52,080	-80

Unrecovered balance diagram at IRR = 21.703%

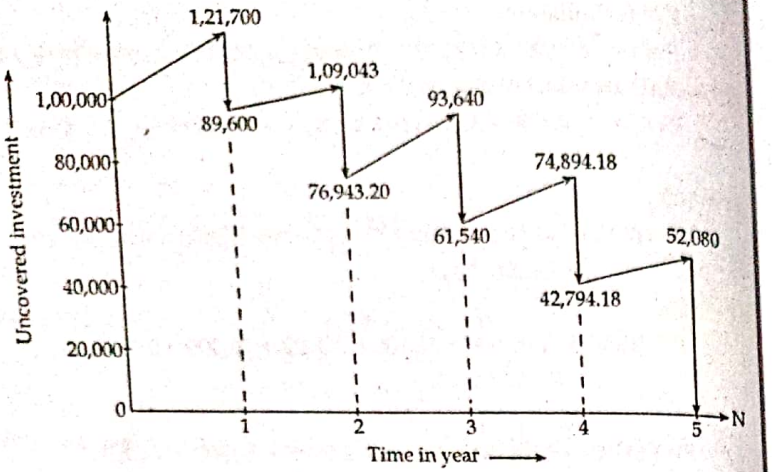


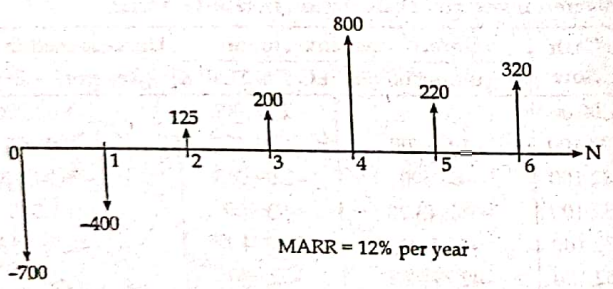
Figure: Investment Balance diagram.

24. **Evaluate the following project by the simple payback period, present worth and future worth method. The cash flow of the project are as follows: If the MARR is 12% per year.**

End of year	Net cash flow (Rs.)
0	-700
1	-400
2	125
3	200
4	800
5	220
6	320

[2016/Fall, 2017/Fall]

Solution:



Simple payback method:

EOY	Net cash flow	Cumulative cash flow
0	-700	-700
1	-400	-1,100
2	125	-975
3	200	-775
4	800	25
5	220	245
6	320	565

Simple payback period

$$\theta = 3 + \frac{775}{800} = 3.968 \text{ years}$$

Since, payback period is within the useful life, this project is feasible.

Present worth method:

$$\therefore \quad PW(12\%) = -700 - 400\,(1.12)^{-1} + 125(1.12)^{-2} + 200\,(1.12)^{-3} + 800\,(1.12)^{-4} + 220\,(1.12)^{-5} + 320\,(1.12)^{-6}$$

$$= -\text{Rs. } 19.767$$

Future worth method:

$$\therefore \quad FW(12\%) = PW\,(F/P, 12\%, 6) = -19.767 \times 1.9738 = -\text{Rs. } 39$$

25. **Evaluate IRR of the following and identify whether the project is feasible or not.**

Initial investment = Rs. 6,00,000

Annual revenue = Rs. 2,50,000

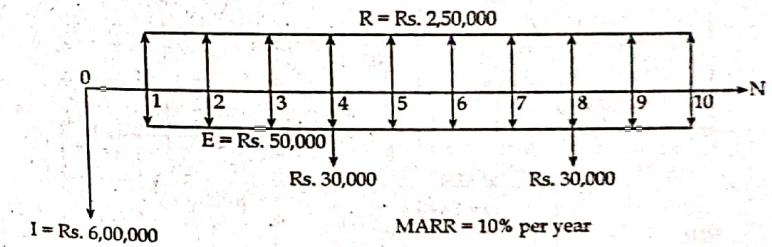
Annual cost = Rs. 50,000

Useful life = 10 years

MARR = 10% per year

Repair and maintenance at 4th and 8th year = Rs. 30,000 [2016/Fall]

Solution:



Using PW formulation,

$$PW = -6,00,000 + (2,50,000 - 50,000)\,(P/A, i\%, 10) - 30,000 \times (P/F, i\%, 8) - 30,000\,(P/F, i\%, 4)$$

IRR is that rate which makes PW = 0

$$PW = -6,00,000 + 2,00,000\,(P/A, i'\%, 10) - 30,000\,(P/F, i\%, 8) - (P/F, i\%, 4)$$
$$= 0$$

If i' = 10%

$$PW(10\%) = -6,00,000 + 2,00,000 \times 6.1446 - 30,000 \times 0.4665 - 30,000 \times 0.6830$$
$$= \text{Rs. } 5,94,435$$

If i' = 30%

$$PW(30\%) = -6,00,000 + 2,00,000 \times 3.0915 - 30,000 \times 0.1226 - 30,000 \times 0.3501$$
$$= \text{Rs. } 4,119 \text{ (+ve)}$$

If i' = 35%

$$PW(35\%) = -6,00,000 + 2,00,000 \times 2.7150 - 30,000 \times 0.0906 - 30,000 \times 0.3011$$
$$= -\text{Rs. } 68,751 \text{ (-ve)}$$

Thus, $IRR = i_1' + \frac{PW_1}{PW_1 - PW_2} \times (i_2' - i_1') = 30 + \frac{4,119}{4,119 + 68,751} \times (35 - 30)$

$$\approx 30 + 0.283$$
$$= 30.283\%$$

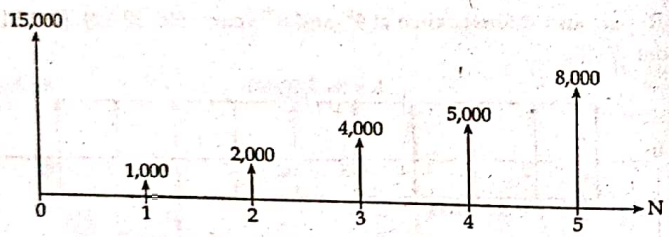
Since, IRR (30.283%) > MARR (10%), the project is feasible.

26. Find IRR, MIRR, discounted payback period of the following project (assume MARR = 10%)

Year	0	1	2	3	4	5
Amt (Rs)	-15,000	1,000	2,000	4,000	5,000	8,000

[2016/Spring]

Solution:



MARR = 10% per year
N = 5 years

i) IRR

Using PW formulation:

$$PW = -15,000 + 1,000\,(P/F, i\%, 1) + 2,000\,(P/F, i\%, 2) + 4,000 \times (P/F, i\%, 3) + 5,000\,(P/F, i\%, 4) + 8,000\,(P/F, i\%, S)$$

IRR → PW (i%) = 0

or, $-15,000 + 1,000\,(P/F, i'\%, 1) + 2,000\,(P/F, i'\%, 2) + 4,000\,(P/F, i'\%, 3) + 5,000\,(P/F, i'\%, 4) + 8,000(P/F, i'\%, 5) = 0$

If i' = 10%

$$PW(10\%) = -15,000 + 1,000 \times 0.9091 + 2,000 \times 0.8264 + 4,000 \times 0.7513 + 5,000 \times 0.6830 + 8,000 \times 0.6209$$
$$= -\text{Rs. } 1,050.50$$

If i' = 8%

PW (8%) = -Rs. 64.25 (-ve)

If i' = 7%

PW (7%) = Rs. 465.01 (+ve)

Hence, $IRR = 7\% + \frac{465.01}{465.01 + 64.25} \times (8 - 7)\% = 7\% + 0.8786\%$

∴ IRR = 7.8786%

ii) MIRR/ERR

Discounting all the cash outflows to present time at ∈ = 15% (assume E_C = 15%)

= 15,000

ΣE_C = Rs. 15,000

Compounding all the cash inflows to future time at ∈ = 15%

$$= 1,000\,(F/P, 15\%, 4) + 2,000\,(F/P, 15\%, 3) + 4,000\,(F/P, 15\%, 2) + 5,000\,(F/P, 15\%, 1) + 8,000$$
$$= 1,000 \times 1.7490 + 2,000 \times 1.5209 + 4,000 \times 1.3225 + 5,000 \times 1.1500 + 8,000$$

∴ ΣR_C = Rs. 23,830.80

We have,

$\Sigma E_C\,(F/P, i'\%\ 5) = 23830.80$

or, $15,000 \times (F/P, i'\%\ 5) = 23,830.80$

or, $15,000 \times (1 + i'\%)^5 = 23,830.80$

or, $(1 + i')^5 = 1.5887$

or, $1 + i' = 1.0970$

∴ i' = 0.09700

∴ ERR → i'% = 9.70%

iii) Discounted payback period:

EOY	Net cash flow, F	Discounted cash flow into present @ 10%, PW	Cumulative cash flow
0	-15,000	-15,000	-15,000
1	1,000	909.09	-14,090.90
2	2,000	1,652.89	-12,438.00
3	4,000	3,005.25	-9,433
4	5,000	3415	-6,018
5	8,000	4967	-1,051

Here, $PW = F(1+i)^{-N}$

Since, the investment amount is not recovered within the given time, discounted payback period cannot be calculated.

27. Pokhara photocopy center is considered to purchase a new photocopy machine costing Rs. 1,00,000 and expected salvage value Rs. 30,000 at the end of 10th year. The machine will save Rs. 20,000 by consuming electricity of Rs. 6,000 per year. Find IRR and interpret your result when MARR is 8% per year. [2017/Fall]

Solution:

Here;

Investment (I) = Rs. 1,00,000
Salvage value (S) = Rs. 30,000
Number of year (N) = 10 years
Annual revenue (R) = Rs. 20,000
Annual expenses (E) = Rs. 6,000
MARR = 8% per year

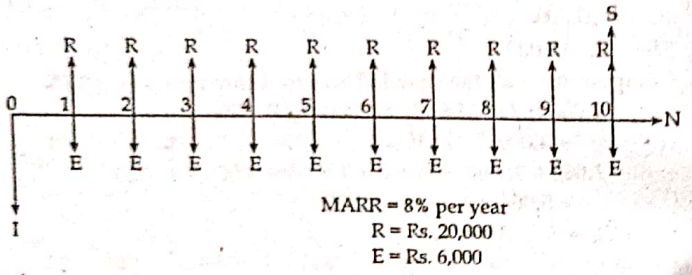


Figure: Cash flow diagram.

We know,

Setting the given information into PW formulation,

$$PW = -1{,}00{,}000 + (20{,}000 - 6{,}000)(P/A, i\%, N) + 30{,}000 \times (P/F, i\%, N)$$

IRR is that rate which makes PW = 0,

$$PW = -1{,}00{,}000 + 14{,}000\,(P/A, i'\%, 10) + 30{,}000 \times (P/F, i'\%, 10)$$

If i' = 10%

$$PW = -1{,}00{,}000 + 14{,}000 \times 6.1446 + 30{,}000 \times 0.3855$$
$$= -\text{ Rs. } 2{,}410.6$$

If i' = 9%

$$PW(9\%) = -1{,}00{,}000 + 14{,}000 \times 6.4177 + 30{,}000 \times 0.4224$$
$$= \text{Rs. } 2{,}519.80$$

Now,

$$IRR = 9\% + \frac{2{,}519.80}{2{,}519.80 + 2{,}410.6} \times (10 - 9)\%$$
$$= 9\% + 0.5110\% = 9.5110\%$$

Since, IRR (9.5110%) > MARR 8%), hence the purchase of photocopy machine is feasible.

28. A company is investing the purchase of new equipment. Interest rate is 9% per year. The cash flow for the equipment is as follows: Initial investment is Rs. 50,000, annual operating cost Rs. 2,000 annual income Rs. 9,000 and salvage value Rs. 10,000, life 10 years

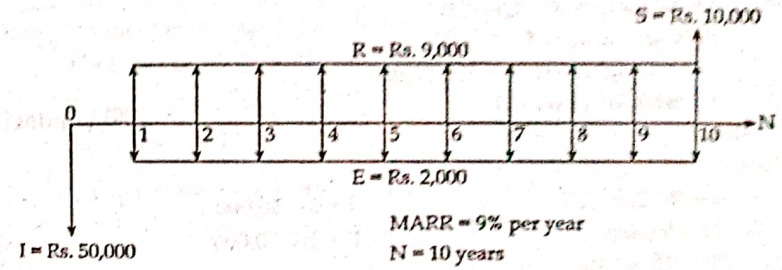
i) Is the investment worth under taking?

ii) What should be the minimum annual benefit for making if a worthy of investment at 9% rate of return? [2017/Fall]

Solution:

Here;

Interest rate = 9% per year
I = Rs. 50,000
AR = Rs. 9,000
AE = Rs. 2,000
S = Rs. 10,000
N = 10 years



Using PW formulation,

$$PW(i\%) = -I + (R - E)(P/A, i\%, N) + S(P/F, i\%, N)$$
$$PW(i\%) = -50{,}000 + (9{,}000 - 2000)(P/A, 9\%, 10) + 10{,}000 \times (P/F, 9\%, 10)$$
$$= -\text{ Rs. } 852.10$$

i) Since, PW (9%) < 0, this investment is not worth undertaking

Or,

For making the project worthy investment,

i.e., PW (i%) ≥ 0

According to question, given annual benefit is

Rs. (9,000 – 2000) = Rs. 7,000

IRR is that rate which makes PW = 0

$$PW = -50{,}000 + 7{,}000\,(P/A, i'\%, 10) + 10{,}000\,(P/F, i'\%, 10) = 0$$

If i' = 10%

$$PW(10\%) = -50{,}000 + 7{,}000 \times 6.1446 + 10{,}000 \times 0.3855$$
$$= -\text{ Rs. } 3{,}132.80$$

If i' = 9%

PW (9%) = -50,000 + 7,000 × 6.4177 + 10,000 × 0.4224

= - Rs. 852.10 (-ve)

If i' = 8%

PW (8%) = 50,000 + 7,000 × 6.7101 + 10,000 × 0.4632

= Rs. 1,602.7 (+ve)

so, $IRR = 8\% + \frac{1,602.7}{1,602.7 + 852.10} \times (9 - 8)\% = 8\% + 0.653\%$

∴ IRR = 8.653%

Since, IRR (8.653) < MARR (9%), project is not feasible

ii) For minimum benefit annually (B)

PW (9%) = -50,000 + B (P/A, 9%, 10) + 10,000 (P/F, 9%, 10)

or, 0 = -50,000 + B × 6.4177 + 10,000 × 0.4224

or, 45,776 = B × 6.4177

∴ B = Rs. 7,132.7735

Hence, minimum annual benefit should be Rs. 7,132.7735.

29. **Find IRR and unrecovered value.**
Investment = Rs. 2,50,000
Salvage value = Rs. 50,000
Net annual revenue = Rs. 70,000
Number of years = 5 **[2017/Spring]**

Solution:

Here;

I = Rs. 2,50,000 S = Rs. 50,000

N = 5 years R = Rs. 70,000

Let, MARR = 10%

We have,

Using PW setting

PW = -250,000 + 70,000 (P/A, i%, 5) + 50,000 (P/F, i%, 5)

IRR is that rate which makes PW = 0,

or, PW = -250,000 + 70,000 (P/A, i'%, 5) + 50,000 (P/F, i'%, 5) = 0

If i' = 10%

PW(10%) = -2,50,000 + 70,000 × 3.7908 + 50,000 × 0.6209

= Rs. 46,401 (+ve)

If i' = 13%

PW(13%) = -2,50,000 + 70,000 × 3.5172 + 50,000 × 0.5428

= Rs. 23,344 (+ve)

If i' = 16%

PW(16%) = -2,50,000 + 70,000 × 3.2743 + 50,000 × 0.4761

= Rs. 3,006 (+ve)

If i' = 18%

PW(18%) = -2,50,000 + 70,000 × 3.1272 + 50,000 × 0.4371

= - Rs. 9,241 (-ve)

so, $IRR = 16\% + \frac{3,006}{3,006 + 9,241} \times (18 - 16)\%$

= 16% + 0.4908%

∴ IRR = 16.50%

Unrecovered value:

EOY	Cash flow	Unrecovered investment		Unrecovered investment (revenue - expenses)
		Beginning of year, P	EOY @ 16.50% F	
0	2,50,000	-	-250,000	-2,50,000
1	70,000	-2,50,000	-2,91,250	-2,21,250
2	70,000	-2,11,250	-2,57,756	-1,87,756
3	70,000	-1,87,756	-2,18,736	-1,48,735
4	70,000	-1,48,735	-1,73,276	-1,03,276
5	1,20,000	-1,03,276	-1,20,317	-316

Hence, unrecovered value is Rs. 316

∴ $F = P(1 + i)^N$

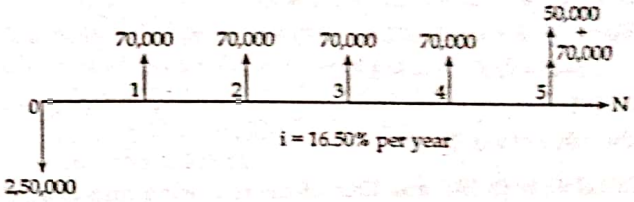


Figure: Cash flow diagram

30. **A project has the following cash flow:**

Years	Cash flows (Rs.)
0	-500
1	+ 202
2	-X
3	196
4	350
5	451

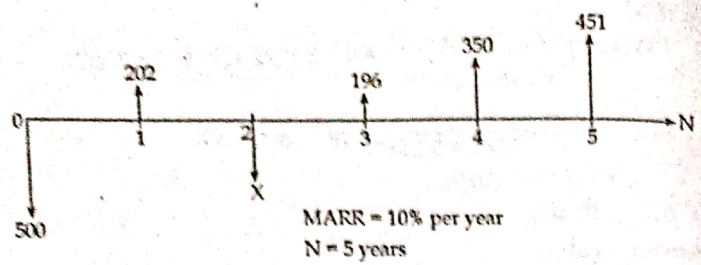
MARR = 10%, ERR = 14.14%, find the value of X. **[2017/Spring]**

Solution:

ERR = 14.14% = i'%

MARR = 10%

Assume, ∈ = 12%



Discounting all cash flows to year zero at $\in = 12\%$

$= 500 + x\,(P/F, 12\%, 2)$

$= 500 + x\,(1 + 0.12)^{-2}$

$\Sigma E_C = 500 + x \times 0.7972$

Compounding all cash inflows to year 5 at 12%

$= 202\,(1.12)^{+4} + 196\,(1.12)^{+2} + 350\,(1.12)^{1} + 451$

$\Sigma R_C = 1{,}406.70$

Establishing the economic equivalence of the two equation at i%,

or, $\Sigma E_C\,(F/P, i'\%, N) = \Sigma R_C$

or, $(500 + x \times 0.7972) \times (F/P, i'\%\ 5) = 1{,}406.70$

or, $(500 + x \times 0.7972) \times 1.9373 = 1{,}406.70$

or, $500 \times 1.9373 + x \times 0.7972 \times 1.9373 = 1{,}406.70$

or, $x \times 1.5444 = 1{,}406.70 - 968.633$

$\therefore\ x = 284$

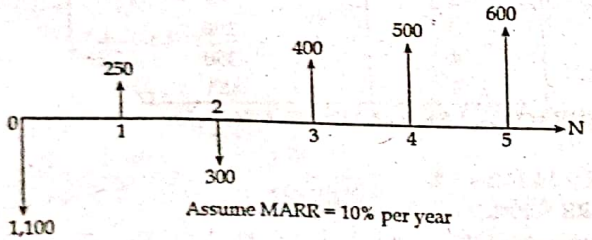
Hence, the value of x is Rs. 284.

31. Calculate both IRR and ERR of the following cash flow. Explain why these values are different.

Year	0	1	2	3	4	5
Cash flow	-1,100	250	-300	400	500	600

[2018/Fall]

Solution:



i) **IRR:**

Using PW formulation,

$PW = -1100 + 250\,(P/F, i'\%, 1) - 300\,(P/F, i'\%, 2) + 400 \times (P/F, i'\%, 3) + 500\,(P/F, i'\%, 4) + 600\,(P/F, i'\%, 5)$

IRR is that rate which makes PW = 0

If i = 10%

$PW(10\%) = -1100 + 250 \times 0.9090 - 300 \times 0.8264 + 400 \times 0.7513 + 500 \times 0.6830 + 0.6209 \times 600$

$= -106.11$ (+ve)

If i = 8%

PW (8%) = -32.32 (-ve)

If i = 7%

PW (7%) = 7.372 (+ve)

Hence, IRR lies between 7% and 8%

$$IRR = 7\% + \frac{7.372}{7.372 + 32.32} \times (8 - 7)\% = 7\% + 0.186\% = 7.186\%$$

ii) **ERR:**

Discounting all cash inflows to year zero at $\in = 12\%$

$= 1{,}100 + 300\,(P/F, i\%, N)$

$= 1{,}100 + 300\,(P/F, 12, 2)$

$= 1{,}100 + 300 \times 0.7872$

ΣE_C = Rs. 1,339.16

Compounding all cash outflow to year 5 at $\in = 12\%$

$\Sigma R_C = 250\,(F/P, 12\%, 4) + 400\,(F/P, 12\%, 2) + 500 \times (F/P, 12\%, 1) + 600$

ΣR_C = Rs. 2,055.14

We have,

ERR $\Rightarrow \Sigma E_C\,(F/P, i'\%, N) = \Sigma R_C$

or, $1{,}339.16 \times (F/P, i'\%, 5) = 2{,}055.14$

or, $(1 + i')^5 = 1.5347$

or, $(1 + i') = \sqrt[5]{1.5347}$

or, $i' = 1.0894 - 1$

$\therefore\ i'\% = 8.944\%$

Hence, ERR = 8.944%

When the algebraic sign of the cash flow changes more than once in the series, it is possible to obtain multiple rates of return and in IRR the reinvestment assumption is taken. So, the value of IRR and ERR are different.

32. Find both types of BCR using FW formulation where initial investment is Rs. 5,00,000, annual income is Rs. 100,000 and decrease by Rs. 10,000 per year; annual cost is Rs. 20,000 and

increases by Rs. 2,000 per year; useful life = 10 years and salvage value is Rs. 1,50,000 and MARR = 11%. [2018/Fall]

Solution:

Here; MARR = 11% per year

I = Rs. 5,00,000

AR = Rs. 1,00,000

AE = Rs. 20,000

Annual income decreases by Rs. 10,000 per year.

Annual cost increases by Rs. 2,000 per year.

S = Rs. 1,50,000

We have,

$$\begin{aligned}PW(B) &= 1,00,000\,(1.11)^{-1} + 90,000\,(1.11)^{-2} + 80,000\,(1.11)^{-3} \\ &\quad + 70,000\,(1.11)^{-4} + 60,000\,(1.11)^{-5} + 50,000\,(1.11)^{-6} \\ &\quad + 40,000 \times (1.11)^{-7} + 30,000\,(1.11)^{-8} \\ &\quad + 20,000\,(1.11)^{-9} + 10,000\,(1.11)^{-10} \\ &= 90,090.09 + 73,046.02 + 58,495.31 + 46,111.16 + 35,607.08 \\ &\quad + 26,732.04 + 19,266.3 + 13,017.18 + 7,818.50 + 3,521.8 \\ &= \text{Rs. } 3,73,703\end{aligned}$$

Or, Using gradient formula,

$$\begin{aligned}PW(B) &= 1,00,000\,(P/A, 11\%, 10) - 10,000\,(P/G, 11\%, 10) \\ &= 1,00,000 \times 5.88992 - 10,000 \times 21.5217 \\ &= \text{Rs. } 3,73,703\end{aligned}$$

$$PW(I) = \text{Rs. } 5,00,000$$

$$\begin{aligned}PW(S) &= 1,50,000 \times (P/F, 11\%, 10) = 1,50,000 \times 0.3552 \\ &= \text{Rs. } 53,280\end{aligned}$$

$$\begin{aligned}PW(O \text{ and } M) &= 20,000\,(P/A, 11\%, 10) + 2,000\,(P/G, 11\%, 10) \\ &= 20,000 \times 5.8892 + 2,000 \times 21.5217 = \text{Rs. } 1,60,827.40\end{aligned}$$

Now,

$$\therefore \text{Conventional BCR} = \frac{PW(B)}{PW(I) - PW(S) + PW(O \text{ and } M)}$$

$$= \frac{3,73,703}{5,00,000 - 53,280 + 1,60,827.40}$$

$$= \frac{3,73,703}{5,00,000 - 53,280 + 1,60,827.40}$$

$$= 0.615$$

$$\therefore \text{Modified BCR} = \frac{PW(B) - PW(O \text{ and } M)}{PW(I) - PW(S)}$$

$$= \frac{3,73,703 - 160827.40}{5,00,000 - 53,280}$$

$$= 0.476$$

Using FW:

$$FW(S) = \text{Rs. } 1,50,000$$

$$FW(I) = 5,00,000 \times (F/P, 11\%, 10) = 5,00,000 \times 2.8394 = 14,19,700$$

$$\begin{aligned}FW(B) &= 1,00,000\,(F/A, 11\%, 10) - 10,000\,(F/G, 11\%, 10) \\ &= 1,00,000 \times 16.7220 - \left[\frac{10,000}{i}(F/A, 11\%, 10) - \frac{NG}{i}\right] \\ &= 16,72,200 - \frac{10,000}{0.11} \times 16.7220 + \frac{10 \times 10,000}{0.11}\end{aligned}$$

$$FW(B) = \text{Rs. } 10,61,110$$

$$\begin{aligned}FW(O \text{ and } M) &= 20,000\,(F/A, 11\%, 10) + 2,000\,(F/G, 11\%, 10) \\ &= 20,000 \times 16.7220 + \left[\frac{2,000}{0.11} \times 16.7220 - \frac{10 \times 2,000}{0.11}\right] \\ &= 3,34,440 + 3,04,036 - 1,81,818 \\ &= \text{Rs. } 4,56,658\end{aligned}$$

Now,

$$\text{Conventional BCR} = \frac{10,61,110}{14,19,700 - 1,50,000 + 4,56,658} = 0.615$$

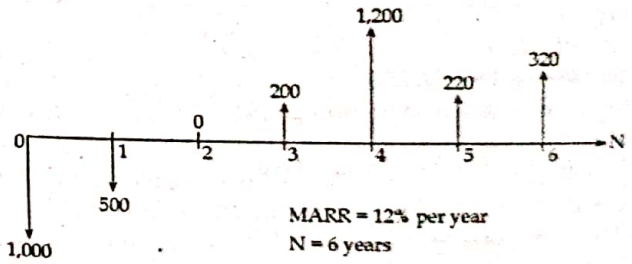
and, $$\text{Modified BCR} = \frac{10,61,110 - 456,658}{14,19,700 - 1,50,000} = 0.476.$$

33. Calculate the simple payback period and discounted payback period from the following cash flows, if MARR = 12%.

EOY	Net cash flow
0	-1000
1	-500
2	0
3	200
4	1,200
5	220
6	320

[2012/Spring]

Solution:



Simple payback period:

EOY	Net cash flow	Cumulative cash flow
0	-1,000	-1,000
1	-500	-1,500
2	0	-1,500
3	200	-1,300
4	1,200	-100
5	220	120
6	320	440

Here, cumulative cash flow turns to positive in period 5. Hence, payback period lies between 4 and 5.

$$\theta = 4 + \frac{100}{220} = 4.46 \text{ years.}$$

Discounted payback period:

EOY	Net cash flow, F	Discounted cash flow into present @ 12%, P	Cumulative cash flow
0	-1,000	-1,000	-1,000
1	-500	-446.43	-1,446.43
2	0	0	-1,446.43
3	200	142.36	-1,304.07
4	1,200	762.63	-541.44
5	220	124.84	-416.60
6	320	162.12	-254.48

Cumulative cash flow does not turn to positive value in given time period. Hence, discounted payback period cannot be obtained.

34. Calculate discount payback of following given cash flow of the engineering project, when MARR is 20%.

EOY	0	1	2	3	4	5
Net cash flows	-25,000	+8,000	+8,000	+8,000	+8,000	+13,000

[2020/Fall]

Solution:

Here; MARR = 20%

Investment, I = – Rs. 25,000

We know, discounted payback period,

$$\theta' = \sum_{C=1}^{\theta'} (R_C - E_C)(P/F, i\%, C) - I = 0$$

$C = 1$

where, R_C = Revenue in year C

E_C = Expenses in year C

From the given values, for end of year 1,

$$= 8{,}000\,(P/E.\ 20\%, 1) - 25{,}000$$

$$= 8{,}000 \times \frac{1}{(1+0.2)^1} - 25{,}000$$

$$= -18{,}333.33 < 0$$

Similarly, for other values given, we do similar calculation which is tabulated as,

EOY	Net cash flow, F (Rs.)	Discounted cash flow into present @ 20%, P (Rs.)	Cumulative cash flow (Rs.)
0	-25,000	-25,000	-25,000
1	8,000	6,666.67	-18,333.34
2	8,000	5,555.56	-12,777.78
3	8,000	4,629.63	-8,148.15
4	8,000	3,858.024	-4,290.15
5	13,000	5,224.41	934.29

Here, cumulative cash flow turns positive in period 5. Hence, payback period lies between 4 and 5.

By interpolating,

$$\theta' = \text{Minimum year} + \frac{\text{Amount to be recovered in minimum year}}{\text{Next year PW}}$$

$$= 4 + \frac{4{,}290.12}{5{,}224.41}$$

$$= 4 + 0.8211$$

$$= 4.8211 \text{ periods}$$

35. Select the best project by using IRR method when MARR is 8%

	Project A	Project B
Initial investment	3,00,000	5,00,00
Annual revenue	90,000	1,75,000
Life year	6	6
Salvage value	10,000	1,00,000

[2020/Fall]

Solution:

Here;

MARR = 8%

For project A:

Initial investment, I = Rs. 3,00,000

Annual revenue, R = Rs. 90,000

Life year, N = 6 years

Salvage value, S = Rs. 10,000

Using net present worth formula,

$$PW(i\%) = -I + R(P/A, i\%, N) + S(P/F, i\%, N)$$
$$= -3,00,000 + 90,000(P/A, i\%, 6) + 10,000(P/F, i\%, 6)$$

When i = 20%,

$$PW(20\%) = -3,00,000 + 90,000(P/A, 20\%, 6) + 10,000(P/F, 20\%, 6)$$
$$= Rs.\ 2,644.9$$

When i = 24%,

$$PW(24\%) = -3,00,000 + 90,000(P/A, 24\%, 6) + 10,000(P/F, 24\%, 6)$$
$$= -Rs.\ 25,406.71$$

so, IRR for project A $= 20 + \frac{2,644.9}{2,644.9 - (-25,406.71)} \times (24 - 20)$

$= 20 + 0.38$

$= 20.38\%$

For project B:

Initial investment, I = Rs. 5,00,000

Annual revenue, R = Rs. 1,75,000

Life year, N = 6 years

Salvage value, S = Rs. 1,00,000

We know,

$$PW(i\%) = -5,00,000 + 1,75,000(P/A, i\%, 6) + 1,00,000(P/F, i\%, 6)$$

When i = 16%

$$PW(16\%) = -5,00,000 + 1,75,000(P/A, 16\%, 6) + 1,00,000(P/F, 16\%, 6)$$
$$= Rs.\ 1,85,873$$

When i = 28%,

$$PW(28\%) = -5,00,000 + 1,75,000(P/A, 28\%, 6) + 1,00,000(P/F, 28\%, 6)$$
$$= Rs.\ 5,628.82$$

When i = 32%,

$$PW(32\%) = 5,00,000 + 1,75,000(P/A, 32\%, 6) + 1,00,000(P/F, 32\%, 6)$$
$$= -Rs.\ 37,602.78$$

When i = 30%

$$PW(30\%) = -5,00,000 + 1,75,000(P/A, 30\%, 6) + 1,00,000(P/F, 30\%, 6)$$
$$= -5,00,000 + 1,75,000 \times 2.643 + 1,00,000 \times 0.2071$$
$$= -Rs.\ 16,775$$

so, IRR of project B $= 28 + \frac{5,628.82}{5,628.82 + 16,775} \times (30 - 28)$

$= 28 + 0.5025$

$= 28.5025\%$

Here, IRR of project B is greater than that of project A and both IRR are greater than MARR. Hence, project B should be selected.

36. Find out the both types of B/C ratios using present worth and annual worth method.
Initial investment = Rs. 5,00,000
Annual benefit = Rs. 1,50,000
Annual cost = Rs. 30,000
Salvage value = Rs. 40,000
MARR = 12% per year
Useful life = 6 years **[2019/Spring]**

Solution:

Here;

Initial investment (I) = Rs. 5,00,000

Annual benefit (B) = Rs. 1,50,000

Annual cost (O and M) = Rs. 30,000

Salvage value (s) = Rs. 40,000

MARR (i) = 12% per year

Useful life (N) = 6 years

a) Present worth method:

$$PW(B) = 1,50,000 \times (P/A, 12\%, 6$$
$$= 1,50,000 \times \left[\frac{(1 + 0.12)^6 - 1}{0.12 \times (1 + 0.12)^6}\right]$$
$$= Rs.\ 6,16,711.1$$

$$PW(S) = 40,000(P/F, 12\%, 6)$$
$$= 40,000 \times \frac{1}{(1 + 0.12)^5}$$
$$= Rs.\ 20,265.245$$

$$PW(O \text{ and } M) = 30,000(P/A, 12\%, 6) = Rs.\ 1,23,342.22$$

Now,

Conventional B/C ratio;

$$B/C = \frac{PW(B)}{PW(I) - PW(S) + PW(O \text{ and } M)}$$
$$= \frac{6,16,711.1}{5,00,000 - 20,265.245 + 1,23,342.22}$$
$$= 1.023$$

Modified B/C ratio:

$$B/C = \frac{PW(B) - PW(o \text{ and } m)}{PW(I) - PW(S)}$$
$$= \frac{6,16,711.1 - 1,23,342.22}{5,00,000 - 20,265.245}$$
$$= 1.028$$

b) Annual worth method

$$AW(I) = 5,00,000(A/P, 12\%, 6)$$

$$= 5{,}00{,}000 \times \left[\frac{0.12 \times (1+0.12)^6}{(1+0.12)^6 - 1}\right]$$

$$= \text{Rs. } 1{,}21{,}612.86$$

$$AW\,(S) = 40{,}000\,(A/F, 12\%, 6)$$

$$= 40{,}000 \times \frac{0.12}{(1.12^6 - 1)}$$

$$= \text{Rs. } 4{,}929.029$$

Now,

Conventional B/C ratio:

$$B/C = \frac{AW\,(B)}{AW\,(I) - AW\,(S) + AW\,(O \text{ and } M)}$$

$$= \frac{1{,}50{,}000}{1{,}21{,}612.86 - 4{,}929.029 + 30{,}000}$$

$$= 1.023$$

Modified B/C ratio:

$$B/C = \frac{AW\,(B) - AW\,(O \text{ and } M)}{AW\,(I) - AW\,(S)}$$

$$= \frac{1{,}50{,}000 - 30{,}000}{1{,}21{,}612.86 - 4{,}929.029}$$

$$= 1.028$$

37. Define cost-benefit analysis. **[2019/Spring]**

Answer:

A cost-benefit analysis is a systematic process that businesses use to analyze which decisions to make and which to forgo. A cost-benefit analysis is the process used to measure the benefits of a decision or taking action minus the costs associated with taking that action. It involves measurable financial matrices such as revenue earned or costs saved as a result of the decisions to purse a project. It also include intangible benefits and costs or effects from a decision such as employees morale and customer satisfaction.

38. Consider an investment project with the following cash flow.

End of year (EOY)	Net cash flow
0	-2,30,000
1	-70,000
2	0
3	80,000
4	1,20,000
5	1,70,000

Compute the IRR of the project and determine its acceptability at MARR = 10% **[2019/Spring]**

Solution:

Here; MARR = 10%

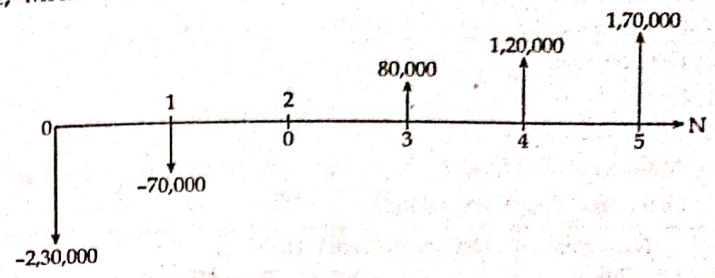


Figure: Cash flow diagram

Now,

$$PW\,(i\%) = -2{,}30{,}000 - 70{,}000\,(P/F, i\%, 1) + 0 + 80{,}000\,(P/F, i\%, 3) + 1{,}20{,}000\,(P/F, i\%, 4) + 1{,}70{,}000\,(P/F, i\%, 5)$$

$$= -2{,}30{,}000 - \frac{70{,}000}{(1+i)^1} + \frac{80{,}000}{(1+i)^3} + \frac{1{,}20{,}000}{(1+i)^4} + \frac{1{,}70{,}000}{(1+i)^5}$$

When i = 10%,

$$PW(10\%) = -2{,}30{,}000 - \frac{70{,}000}{(1+0.1)^1} + \frac{80{,}000}{(1+0.1)^3} + \frac{1{,}20{,}000}{(1+0.1)^4} + \frac{1{,}70{,}000}{(1+0.1)^5}$$

$$= -\text{Rs. } 46{,}012.94$$

When i = 5%,

$$PW(5\%) = -2{,}30{,}000 - \frac{70{,}000}{(1+0.05)^1} + \frac{80{,}000}{(1+0.05)^3} + \frac{1{,}20{,}000}{(1+0.05)^4} + \frac{1{,}70{,}000}{(1+0.05)^5}$$

$$= \text{Rs. } 4{,}364.08$$

Now,

$$IRR = 5 + \frac{4{,}364.08}{4{,}364.08 - (-46{,}012.94)} \times (10 - 5) = 5.43\ \%$$

Since, IRR (5.43%) < MARR (10%), the project is not acceptable for investment.

39. Write short notes on: Benefit cost ratio. **[2019/Spring]**

Answer: See the topic 4.5.

40. Compute discounted payback period and modified B/C ratio from the following data.

Initial investment (I) = Rs. 10,00,000
Annual revenue (R) = Rs. 1,80,000
Annual cost (C) = Rs. 60,000
Useful life, (N) = 10 years
MARR (i) = 5%
Salvage value (S) = Rs. 1,50,000 **[2019/Fall]**

Solution:

Given that;

I = Rs. 10,00,000

C = Rs. 60,000

R = Rs. 180000

N = 10 years

i = 5%

S = Rs. 1,50,000

Annual net cash flow, F = R – C = Rs. 1,80,000 – 60,000 = Rs. 1,20,000

a) Discounted payback period

EOY	Net cash flows F (Rs.)	Discounted cash flow into present @ 5%, P (Rs.)	Cumulative cash flow
0	-10,00,000	-10,00,000	-10,00,000
1	1,20,000	1,14,285.71	-8,85,714.29
2	1,20,000	1,08,843.54	-7,76,870.75
3	1,20,000	1,03,660.51	-6,73,210.24
4	1,20,000	98,724.3	-5,74,485.94
5	1,20,000	94,023.14	-48,0462.8
6	1,20,000	89,545.85	-3,90,916.95
7	1,20,000	85,281.76	-3,05,635.19
8	1,20,000	81,220.72	-2,24,414.47
9	1,20,000	77,353.07	-1,47,061.4
10	2,70,000	1,65,756.58	18,695.18

Hence, cumulative cash flow turns to positive in year 10. Hence, discounted payback period is between 9 and 10 years.

By interpolating, we get,

$$\theta' = 9 + \frac{1,47,061.4}{16,576.58} \times (10 - 9) = 9.89 \text{ years}$$

b) Modified B/C ratio with PW formulation

PW (I) = Rs. 10,00,000

PW (R) = Rs. 1,80,000 (P/A, 5%, 10) = Rs. 13,89,912.29

PW (C) = 60,000 (P/A, 5%, 10) = Rs. 4,63,304.1

PW (S) = 1,50,000 (P/F, 5%, 10) = Rs. 92,086.99

Now,

$$\text{B/C ratio} = \frac{PW(R) - PW(C)}{PW(I) - PW(S)} = \frac{13,89,912.29 - 4,63,304.1}{10,00,000 - 92,086.99} = 1.02$$

41. Select the best project by using IRR method when MARR is 8%. Use incremental analysis if necessary.

	Project A	Project B
Initial investment	3,00,000	5,00,000
Annual revenue	1,50,000	1,75,000
Life year	6	6
Salvage value	70,000	1,00,000

[2019/Fall]

Solution:

MARR, i = 8%

For project A:

Initial investment, I = Rs. 3,00,000

Annual revenue, R = Rs. 1,50,000

Life year revenue, N = 6

Salvage value, S = Rs. 70,000

Now,

$$PW(i\%) = -I + R(P/A, i\%, N) + S(P/F, i\%, N)$$
$$= -3,00,000 + 1,50,000(P/A, i\%, 6) + 70,000(P/F, i\%, 6)$$
$$= -3,00,000 + 1,50,000 \times \frac{(1+i)^6 - 1}{i \times (1+i)^6} + 70,000 \times \frac{1}{(1+i)^6}$$

When i = 10%,

$$PW(10\%) = -3,00,000 + 1,50,000(P/A, 10\%, 6) + 70,000(P/F, 10\%, 6)$$
$$= -3,00,000 + 1,50,000 \times \left[\frac{(1+0.1)^6 - 1}{0.1 \times (1+0.1)^6}\right] + 70,000 \times \frac{1}{(1+0.1)^6}$$
$$= \text{Rs. } 3,92,802.28$$

When i = 25%

$$PW(25\%) = -3,00,000 + 1,50,000(P/A, 25\%, 6) + 70,000(P/F, 25\%, 6)$$
$$= \text{Rs. } 1,61,063.68$$

When i = 50%

$$PW(50\%) = -3,00,000 + 1,50,000(P/A, 50\%, 6) + 70,000(P/F, 50\%, 6)$$
$$= -\text{ Rs. } 20,192.04$$

When i = 45%,

$$PW(45\%) = -3,00,000 + 1,50,000(P/A, 45\%, 6) + 70,000(P/F, 45\%, 6)$$
$$= \text{Rs. } 4,999.99$$

Now,

$$IRR_A = 45 + \frac{4,999.99}{4,999.99 - (-20,192.04)} \times (50 - 45) = 45.99\%$$

For project B:

Initial investment, I = Rs. 5,00,000

Annual revenue, R = Rs. 1,75,000

Life year, N = 6 years

Salvage value, S = 1,00,000

We know,

$PW(i\%) = -5,00,000 + 1,75,000\ (P/A, i\%, 6) + 1,00,000\ (P/F, i\%, 6)$

When i = 20%,

$PW(20\%) = -5,00,000 + 1,75,000\ (P/A, 20\%\ 6) + 1,00,000\ (P/F, 10\%, 6)$

$= Rs.\ 1,15,454.07$

When i = 25%,

$PW(25\%) = -5,00,000 + 1,75,000\ (P/A, 25\%, 6) + 1,00,000\ (P/E, 25\%, 6)$

$= Rs.\ 42,713.6$

When i = 30%,

$PW(30\%) = -5,00,000 + 1,75,000\ (P/A, 30\%, 6) + 1,00,000\ (P/F, 30\%, 6)$

$= -Rs.\ 16,801.84$

Now,

$$IRR_B = 25 + \frac{42,713.6}{42,713.6 - (-16,801.84)} \times (30 - 25) = 28.59\%$$

Since IRR of project A is greater than that of project B which are greater than MARR, project A is the best choice:

Incremental analysis:

Using project A as base alternative,

	Project A	Project B	Project B-project A
Initial investment	3,00,000	5,00,000	2,00,000
Annual revenue	1,50,000	1,75,000	25,000
Salvage value	7,000	1,00,000	30,000

We know,

$PW(i\%) = -2,00,000 + 25,000\ (P/A, i\%, 6) + 30,000\ (P/F, i\%, 6)$

When i = 2%

$PW(2\%) = -2,00,000 + 25,000\ (P/A, 2\%, 6) + 30,000\ (P/F, 2\%, 6)$

$= -Rs.\ 33,325.09$

When i = 8%

$PW(8\%) = -2,00,000 + 25,000\ (P/A, 8\%, 6) + 30,000\ (P/F, 8\%, 6)$

$= -Rs.\ 65,522.92$

From this, we can see that the IRR_{B-A} will be less than 8%. So, the higher investment of project B is not justified. Thus, best alternative is project A.

ADDITIONAL PROBLEMS

1. Explain the factors affecting the determination of MARR.

Answer:

Factors affecting the determination of MARR are:

i) The amount of money available for investment and the source and the cost of these funds.

ii) The number of good projects available for investment. *i.e.*, greater number of projects leads greater alternatives.

iii) The conscious risks which is associated with investment opportunities. *i.e.*, risk level.

iv) The purpose of investment alternatives and their financial attractiveness. *i.e.*, is project essential or elective?

v) The types of organization involved (*i.e.*, government, private or public). In general, government organizations provides less MARR than the private organizations.

2. What is the difference between financial and economic analysis?

Answer:

S.N.	Financial analysis	Economic analysis
i)	Includes only project participants.	Includes all member of society.
ii)	Uses financial prices	Uses economic prices.
iii)	The main purpose of the financial analysis is to use the project cash flow forecast to calculate suitable net return indicators, particularly from market point of view of the investor.	The economic analysis appraises the project's contribution to the economic welfare of the region or country. It is made on behalf of the whole society instead of just the owners of the infrastructure, as in the financial analysis.
iv)	It provides financing plan for the proposed investment.	It provides information for the decision rule.
v)	Taxes, fees are included on cost side because these are financial cost.	Taxes, fees are excluded from cost and benefit side because these are transfer of resources for the economic growth.
vi)	Cost measurement is monetary cost.	Cost measurement is opportunity cost.

3. For the cash flow given at 10% interest, compute the value of 'C' for net amount equal to be zero at the end of 8 years

Year	1	2	3	4	5	6	7	8
Cash flow	0	-100	-200	-300	0	c	c	c

Solution:

i = 10% per year

N = 8 year

Using FW formulation,

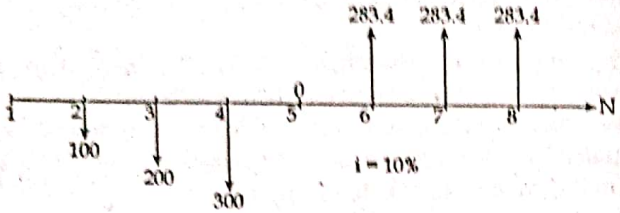
or, $0 - 100 \times (1.1)^6 - 200\,(1.1)^5 - 300\,(1.1)^4 + 0 + c(1.1)^2 + c \times (1.1)^1 + c = 0$

or, $0 - 177 - 322 - 439 + 0 + 1.21c + 1.1c + c = 0$

or, $3.31c = 938$

∴ $c = 283.4$

Hence the value of c is Rs. 283.4



4. Determine both types of B/C ratio from the following cash flow

Initial investment = Rs. 3,00,000

Annual revenue = Rs. 85,000

Annual costs = Rs. 15,000

Salvage value = 20% of initial investment

Useful life = 6 years

MARR = 10% per year

Solution:

Here;

I = Rs. 30,000

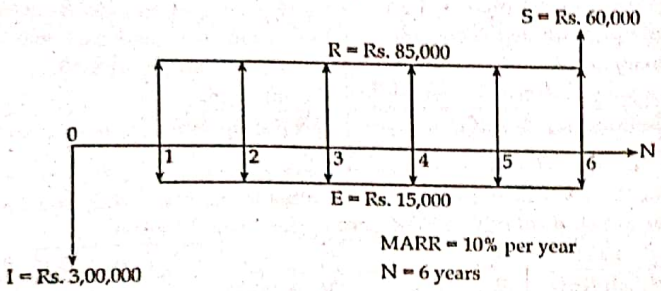
R = Rs. 85,000

O and M = Rs. 15,000

MARR = 10%

N = 6 years

S = 20% of Rs. 3,00,000 = Rs. 60,000



PW formulation:

PW (I) = Rs. 3,00,000

PW (B) = 85,000 (P/A, 1%, 6) = 85,000 (P/A, 10%, 6)
= 85,000 × 4.3553 = Rs. 3,70,200.50

PW(S) = 60,000 (P/F, 10%, 6) = 60,000 × 0.5645
= Rs. 33870

PW (O and M) = 15,000 (P/A, 10%, 6)
= 15,000 × 4.3553
= Rs. 65,329.50

Now,

$$\text{Conventional BCR} = \frac{\text{PW (B)}}{\text{PW (I)} - \text{PW (S)} + \text{PW (O and M)}}$$

$$= \frac{3,70,200.50}{3,00,000 - 33,870 + 65,329.50} = 1.11 > 1$$

and, $$\text{Modified B/C ratio} = \frac{\text{PW (B)} - \text{PW (O and M)}}{\text{PW (I)} - \text{PW (S)}}$$

$$= \frac{3,70,200.50 - 65,329.50}{3,00,000 - 33,870} = 1.14 > 1$$

AW formulation:

AW (I) = 3,00,000 (A/P, 10%, 6)
= 3,00,000 × 0.2296
= Rs. 68,880

AW (S) = S (A/F, i%, N)
= 60,000 × (A/F, 10%, 6)
= 60,000 × 0.1296
= Rs. 7,776

AW (B) = Rs. 86,000

AW (O and M) = Rs. 15,000

Now,

$$\text{Conventional BCR} = \frac{\text{AW (B)}}{\text{AW (I)} - \text{AW (S)} + \text{AW (O and M)}}$$

$$= \frac{85,000}{68,880 - 7,776 + 15,000}$$

$$= 1.11 > 1$$

$$\text{Modified BCR} = \frac{\text{AW (B)} - \text{AW (O and M)}}{\text{AW (I)} - \text{AW (S)}}$$

$$= \frac{85,000 - 15,000}{68,880 - 7,776} = 1.14 > 1$$

FW formulation:

FW (I) = 3,00,000 (F/P, 10%, 6) = 3,00,000 × 1.7716
= Rs. 531,480

FW (B) = 85,000 × (F/A, 10%, 6) = 85,000 × 7.7156
= Rs. 6,55,826

FW (S) = Rs. 60,000

FW (O and M) = 15,000 (F/A, 10%, 6) = 15,000 × 7.1156
= Rs. 1,15,734

$$\text{Conventional BCR} = \frac{\text{FW (R)}}{\text{FW (I)} - \text{FW (S)} + \text{FW (O and M)}}$$

$$= \frac{6,55,826}{5,31,480 - 60,000 + 1,15,734} = 1.11 > 1$$

$$\text{Modified BCR} = \frac{\text{FW (B)} - \text{FW (O and M)}}{\text{FW (I)} - \text{FW (S)}}$$

$$= \frac{6,55,826 - 1,15,734}{5,31,480 - 60,000} = 1.14 > 1$$

Since, BCR > 1, the investment is fruitful.

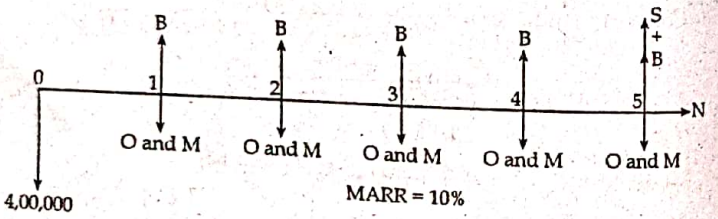
5. NEC is considering to purchase a new machine costing Rs. 4,00,000 having salvage value Rs. 1,00,000 at the end of 5th year. The use of machine will increase revenue by Rs. 1,50,000 that needs fuel cost of Rs. 30,000 per year. Find the following values when MARR = 10%

i) PW, FW and AW
ii) IRR
iii) BCR by PW
iv) Simple and discounted payback period

Solution:

Here;

I = Rs. 4,00,000
B = Rs. 1,50,000 per year
MARR = 10% per year
N = 5 year
O and M = Rs. 30,000 per year
S = Rs. 1,00,000



i) PW(10%) = –4,00,000 + (1,50,000 – 30,000) (P/A, 10%, 5)
+ 1,00,000 (P/F, 10%, 5)
= –4,00,000 + 120,000 × 3.7908 + 1,00,000 × 0.6209
= Rs. 1,16,986

AW(10%) = –400,000 (A/P, 10% 5) + (1,50,000 – 30,000)
+ 1,00,000 (A/F, 10%, 5)
= –40,000 × 0.2638 + 1,20,000 + 1,00,000 × 0.1638
= Rs. 30,860

FW(10%) = –4,00,000 (F/P, 10%, 5) + 1,20,000 (F/A, 10%, 5) + 100,000
= –4,00,000 × 1.6105 + 1,20,000 × 6.1051 + 100,000
= Rs. 1,88,412

ii) Setting equation in PW formation

PW = –4,00,000 + 1,20,000 (P/A, i'%, 5) + 1,00,000 (P/F, i%, 5) = 0

Let, i% = 10%

PW(10%) = Rs. 1,16,986 (+ve)

Let, i% = 20%

PW(20%) = –4,00,000 + 1,20,000 × 2.9906 + 1,00,000 × 0.4019 = 0
= – Rs. 938 (–ve)

Let, i% = 19%

PW(20%) = –4,00,000 + 1,20,000 + 1,00,000 × 0.4019
= Rs. 8,812 (+ve)

Then,

$$\therefore \quad \text{IRR} = 19\% + \frac{8812}{8812 + 938} \times (20 - 21)\% = 19.90\%$$

iii) $$\text{Conventional BCR} = \frac{\text{PW (B)}}{\text{PW (I)} - \text{PW (S)} + \text{PW (O and M)}}$$

$$= \frac{5,68,620}{4,00,000 - 62090 + 113724}$$

$$= 1.25 > 1$$

$$\text{Modified BCR} = \frac{\text{PW (B)} - \text{PW (O and M)}}{\text{PW (I)} - \text{PW (S)}}$$

$$= \frac{5,68,620 - 1,13,724}{4,00,000 - 62,090}$$

$$= 1.35 > 1$$

iv) Simple payback period

EOY	Net cash flow	Cumulative cash flow
0	–4,00,000	–4,00,000
1	1,20,000	–2,80,000
2	1,20,000	–1,60,000
3	1,20,00	–40,000
4	1,20,000	80,000
5	2,20,000	3,00,000

Hence,

$$\therefore \quad \text{Simple payback period, } \theta = 3 + \frac{40,000}{1,20,000} = 3.34 \text{ years}$$

Now, Discounted cash flow:

EOY	Net cash flow	Discounted cash flow into present	cumulative cash flow
0	-4,00,000	-4,00,000	-4,00,000
1	120,000	1,09,090.9	-2,90,909.09
2	120,000	99,173.55	-1,91,735.5
3	120,000	90,157.77	-1,01,577.8
4	120,000	81,961.61	-19,316.08
5	2,20,000	1,36,602.70	1,16,986

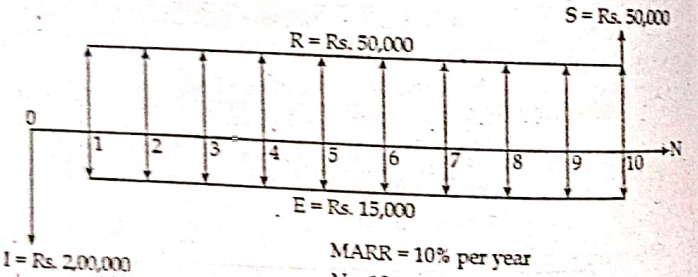
Then,

$\therefore$ Discounted payback period (θ') $= 4 + \frac{19,616.08}{1,36,602.70} = 4.15$ years

6. **Check the feasibility of the following investment by using PW, AW and FW methods. What is the capital recovery amount of this project?**

I = Rs. 2,00,000
S = Rs. 50,000
N = 10 years
R = Rs. 50,000
E = Rs. 15,000
MARR = 10% per year

Solution:



$PW(i\%) = -I + (R - E)(P/A, i\%, N) + S(P/F, i\%, N)$

or, $PW(10\%) = -2,00,000 + (50,000 - 15,000) \times (P/A, 10\%, 10) + 50,000 (P/F, 10\%, 10)$
$= -2,00,000 + 35,000 \times 6.1446 + 50,000 \times 0.3855$
$= Rs. 34,336$

$\therefore$ $AW(i\%) = PW(i\%) \times (A/P, i\%, N)$
$AW(10\%) = Rs. 34,336 \times (A/P, 10\%, 10)$
$= Rs. 34,336 \times 0.1627$
$= Rs. 5,586.47$

and, $FW(i\%) = PW(i\%) \times (F/P, i\%, N)$
$\therefore$ $FW(10\%) = Rs. 34,336 \times (F/P, 10\%, 10)$
$= Rs. 34,336 \times 2.5937$
$= Rs. 89,057.28$

$\therefore$ Capital recovery $= (I - S)(A/P, i\%, N) + S(i\%)$
$CR(10\%) = R - E - AW$
$= Rs. (50,000 - 15,000 - 5,586.47)$
$= Rs. 29,413.53$

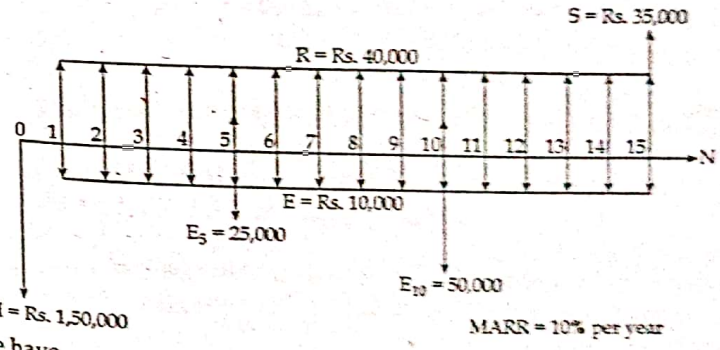
Hence, project is feasible as PW, AW, FW and CR > 0

7. **Evaluate the project on the basis of the PW method when the MARR is 10%**

Investment	Rs. 1,50,000
Salvage value	Rs. 35,000
Project life	15 years
Annual expenses	Rs. 10,000
Annual revenue	Rs. 40,000
Overhead cost at end of 5th year	Rs. 25,000
Overhead cost at end of 10th year	Rs. 50,000

Solution:

Here; I = Rs. 1,50,000
N = 15 year
S = Rs. 35,000
E = Rs. 10,000
R = Rs. 40,000
E_5 = Rs. 25,000
E_{10} = Rs. 50,000



We have,

$PW(i\%) = -I + (R - E)(P/A, i\%, N) + S(P/F, i\%, N) - E_5(P/F, i\% 5) - E_{10}(P/F, i\%, 10)$

$= -1,50,000 + (40,000 - 10,000)\,(P/A, 10\%, 15) + 35,000 \times (P/F, 10\%, 15) - 25,000\,(P/F, 10\%, 5) - 50,000 \times (P/F, 10\%, 10)$

$= -1,50,000 + 30,000 \times 7.6061 + 35,000 \times 0.2394 - 25,000 \times 0.6209 - 50,000 \times 0.3855$

$= -1,50,000 + 2,28,183 + 8,379 - 15,522.5 - 19,275$

$\therefore$ PW(10%) = Rs. 51,764.50

and, FW (10%) = PW (10%) × (F/P, 10%, 15)

= Rs. 51,764.50 × 4.1772

= Rs. 2,16,231

8. Find both types of B/C ratio using FW formulation.

Initial investment = Rs. 2,00,000

Project life = 5 years

Salvage value = Rs. 50,000

Annual O and M costs = Rs. 20,000

MARR = 17%

Annual benefits = Rs. 50,000 at the end of year 1 and increasing by Rs. 25,000 each year for remaining years.

Solution:

FW (I) = 2,00,000 (F/P, 17%, 5)

FW (I) = 2,00,000 × 2.1930 = Rs. 4,38,489.60

FW (S) = Rs. 50,000

FW (O and M) = 20,000 (F/A, 17%, 5)

= 20,000 × 7.0144

= Rs. 1,40,288

FW (B) = Rs. 50,000 (F/A, 17%, 5) + 25,000 (F/G, 17%, 5)

= Rs. 50,000 × 7.0144 + 25,000 × 11.8494

= Rs. 3,50,720 + Rs. 2,96,235

= Rs. 6,46,955

Now,

$$\text{Conventional B/C ratio} = \frac{\text{FW (B)}}{\text{FW (I)} - \text{FW (S)} + \text{FW (O and M)}}$$

$$= \frac{6,46,955}{4,38,489.6 - 50,000 + 1,40,288}$$

= 1.23 > 1, project is accepted.

$$\text{Modified B/C ratio} = \frac{\text{FW (B)} - \text{FW (O and M)}}{\text{FW (I)} - \text{FW (S)}}$$

$$= \frac{6,46,955 - 1,40,288}{4,38,489.60 - 50,000}$$

= 1.31 > 1, project is accepted.

CHAPTER 5

COMPARATIVE ANALYSIS OF ALTERNATIVES

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5.1 COMPARING MUTUALLY EXCLUSIVE ALTERNATIVES HAVING SAME USEFUL LIFE BY PAYBACK PERIOD METHOD; EQUIVALENT WORTH METHOD; RATE OF RETURN METHOD AND BENEFIT COST RATIO METHOD

When the useful life of an alternative is equal to be selected study period, adjustments to the cash flows are not required. The most straight forward technique for comparing the mutually exclusive alternatives when all useful life's are equal to the study period is to determine the equivalent worth of each alternative based on total investment at i = MARR. Then, for investment alternatives, the one with the greatest positive equivalent

worth is selected. And in the case of cost alternatives, the one with the least negative equivalent worth is selected.

The economic ranking of mutually exclusive alternatives will be the same when using the three methods of equivalent worth method. Consider the general case of two alternatives A and B. If PW $(i\%)_A$ < PW $(i\%)_B$, then the AW and FW analysis will result in the same preference for alternative B. Do not compare the IRRS of mutually exclusive alternatives (or IRRs. of the difference between mutually exclusive alternative) against those of other alternatives.

When you compare exclusive alternatives with the same revenues, they are compared on a cost-only basis. In this situation (because you are minimizing costs rather than maximizing profits), you should accept the project that results in the smallest or least negative NPW. Similarly, use same decision criteria for AW, FW, PW and BCR method while comparing and selecting between multiple alternatives.

5.2 COMPARING MUTUALLY EXCLUSIVE ALTERNATIVES HAVING DIFFERENT USEFUL LIFE

When the useful likes of mutually exclusive alternatives are unequal, the repeatability assumption or co-terminated assumption or capitalized worth method can be used.

5.2.1 Repeatability Assumption

It assumes that the study period is either indefinitely long or equal to common multiple of the lives of the alternatives. The cash flow associated with an alternative's initial life span are representative of what will happen in succeeding life spans.

This can be used when the useful lives of equipment in alternative A and alternative B are different from each other. The study period is selected as the lowest common multiple of the two useful lives.

For example, say the decision analysis is to select the best photocopier for the department. Copier A is imported from America and has a useful life of nine years. Copier B is from a domestic supplier with a useful life of six years. Then the study period is set at 18 years; 18 is the LCM of (6, 9). Two A copies are allowed to function in series in time, thus lasting $2 \times 9 = 18$ years. In a similar manner, three copies of B are allowed to function one after the next, thus lasting $3 \times 6 = 18$ years. The study period of 18 years is now the same for both copiers A and B. The purchase price, annual supplies cost, maintenance cost, utility cost, labor cost and salvage value obtained at the end of use of the machine can all be accounted for two copies of A and three copies of B. In some cases, the quantity may have to be multiplied by the numbers of copies-it depends on the given application.

Sometimes, the LCM may have to be the multiple of two useful lives. For example, should copier A have a life of five years and copier B have a life of three years, the study period can be selected as LCM (3, 5) = 15 years. In this case, five copiers A have to be considered as a basis and three copiers of B have to be considered as a basis. In such cases, projections into the future may cause errors such as changes in interest rates, inflation rates, etc. This method can also be used when the study period tends to infinity or is indefinitely long.

Example

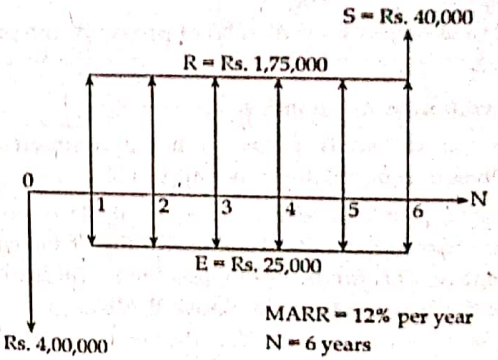
Recommend the best project from the following two projects assuming repeatability.

Project	A	B
Initial investment, Rs.	4,00,000	7,00,000
Annual Revenue, Rs.	1,75,000	2,50,000
Annual cost, Rs.	25,000	25,000
Salvage value, Rs.	40,000	70,000
Useful life	6	8
MARR	12% per year	

Solution:

Study period: L.C.M. of 6 and 8 years = 24 years

Project A: It requires 4 times repetition.

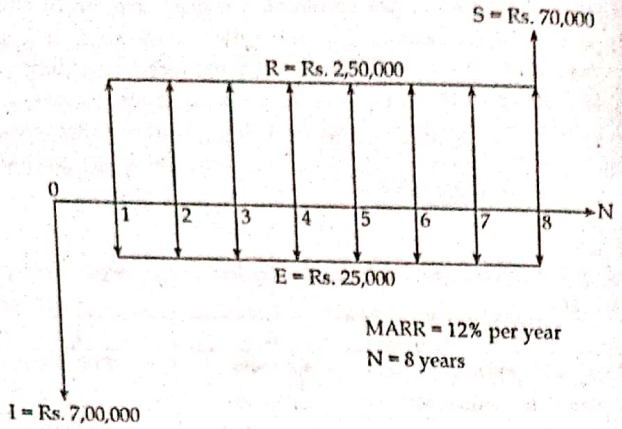


PW(12%) = –4,00,000 – 4,00,000 (P/F, 12%, 6) – 4,00,000(P/F, 12%, 12)
– 4,00,000 (P/F, 12%, 18) + 150,000(P/A, 12%, 24)
+ 40,000 (P/F, 12%, 6) 40,000 (P/F, 12%, 12)
+ 40,000 (P/F, 12% 18) + 40,000 (P/F, 12%, 24)

= –4,00,000 – 400,000 × 0.50669 – 400,000 × 0.2567
+ 400,000 × 0.130 + 150,000 × 7.7843 + 40,000 × 0.5066
+ 40,000 × 0.2567 + 40,000 × 0.130 + 40,000 × 0.659

= Rs. 4,48,693

Project B: It requires 3 times repetition.



$$PW(12\%) = -7,00,000 - 7,00,000\,(P/F, 12\%, 8) - 7,00,000(P/F, 12\%, 16) + (2,50,000 - 25,000)\,(P/A, 12\%, 24) + 70,000(P/F, 12\%, 8) + 70,000\,(P/F, 12\%, 16) + 70,000\,(P/F, 12\%, 24)$$

$$= -7,00,000 - 7,00,000 \times 0.4039 - 7,00,000 \times 0.1631 + 2,25,000 \times 7.7843 + 70,000 \times 0.4039 + 70,000 \times 0.1631 + 70,000 \times 0.659$$

Since PW(12%) of project B > PW(12%) of project A, the project B is highly recommended.

5.2.2 Co-Terminated Assumption

It is preferred if the study period is not a common multiple of the alternatives lives or repeatability is not applicable.

The study period is truncated from the useful life of equipment in one alternative in order to have equal the useful life of the equipment in the second alternative. This can be used when the useful life of alternative A, NA is greater than useful life of alternative B, NB *i.e.*, NA > NB. The study period can be selected as NB. All the revenues and expenses for alternative A are then revaluated for a co-terminated period of NB. This is feasible because NB < NA.

When Study Period > Useful Life

Calculate the FW at the end of useful life and move this to the end of the study period using the MARR.

When Study Period < Useful Life

Truncate the alternative at the end of the study period using an estimated or imputed market value.

Example

Using co-terminated assumption recommend the best project.

Project	A	B
I	1,50,000	2,00,000
AR	90,000	1,00,000
AE	20,000	22,000
S	5,000	1,00,000
N	2 years	4 years
MARR	10% per year	

Solution:

For project A:

$$PW(10\%) = -150,000 + 70,000\,(P/A, 10\%, 2) + 50,000\,(P/F, 10\%, 2)$$

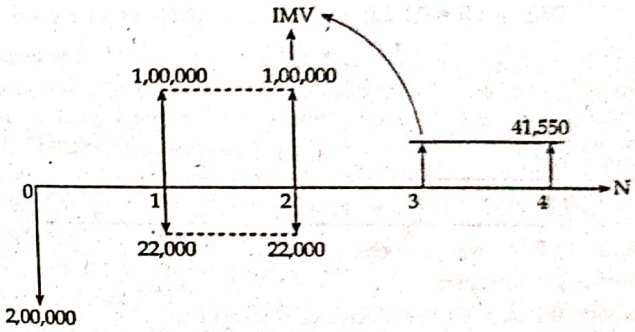
$$= -150,000 + 70,000 \times 1.7355 + 50,000 \times 0.8264 = \text{Rs. } 12,805$$

For project B:

It should terminate at the end of second year *i.e.*, useful life of project A.

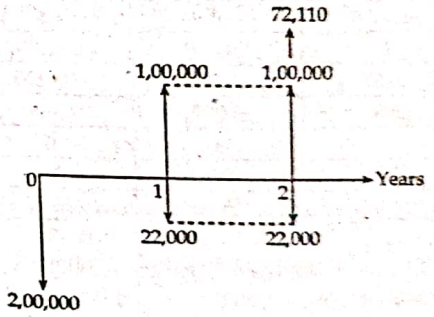
$$CR(10\%) = 2,00,000\,(A/P, 10\%, 4) - 1,00,000\,(A/F, 10\%, 4)$$

$$= 2,00,000 \times 3.155 - 1,00,000 \times 0.2155 = \text{Rs. } 41,550$$



PW of CR at the end of second year:

$$PW_{CR}(10\%) = 41,550\,(P/A, 10\%, 2) = 72,110$$



PW of project B terminating at the end of second year,

$\therefore$ PW(10%) = 2,00,000 + 78,000 (P/A, 10%, 2) + 72,110 (P/F, 10%, 2%)
= 2,00,000 + 78,000 × 1,7355 + 72,110 × 0.8264
= – Rs. 5,039

$\therefore$ PW of A > PW of B, so select project A.

5.2.3 Capitalized Worth (CW) Method

Capitalized worth is the present worth of all revenues or expenses over an infinite life of time. It is a special variation of present worth. If only expenses are considered, we call it capitalized cost method. We know,

$$PW(i\%) = A\,(P/A, i\%, N)$$

When $N \to \infty$, PW is called as capitalized worth.

Now,

$$CW(i\%) = PW_{N\to\infty}(i\%)$$

$$= A\left[\lim_{N\to\infty} \frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$= A\left[\lim_{N\to\infty} \left(\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{1}{i(1+i)^N}\right)\right]$$

$$= A\left[\lim_{N\to\infty} \left(\frac{1}{i} - \frac{1}{i(1+i)^N}\right)\right]$$

$$= A\left(\frac{1}{i} - 0\right)$$

$\therefore$ $$CW(i\%) = \left(\frac{A}{i}\right)$$

Hence to find CW of any project:

→ Find AW of project.

→ Divide this AW by the MARR or interest rate.

Example

Solve by capitalized worth method and select best project.

Project	A	B
I	50,000	1,20,000
AE	10,000	10,000
AR	9,000	6,000
S	0	0
N	20	30
MARR	15%	15%

Solution:

AW_A = –50,000 (A/P, 15%, 20) + (9,000 – 10,000)
= –50,000 × 0.1598 – 1,000
= – Rs. 8,990

AW_B = –1,20,000 (A/P, 15%, 30) + (6,000 – 10,000)
= –1,20,000 × 0.1523 – 4,000
= – Rs. 22,276

Now,

$$CW_A = \frac{AW_A}{i} = \frac{-8,990}{0.15} = -59,933.33$$

$$CW_B = \frac{AW_B}{i} = \frac{-22,276}{0.15} = -1,48,506.67$$

Both have negative value. But least negative value is of project A compares to project B. So, select project A.

5.3 COMPARING MUTUALLY EXCLUSIVE, CONTINGENT AND INDEPENDENT PROJECTS IN COMBINATION

In the real world of engineering practice, however, it is typical for us to have two or more choices of projects that are not independent of one another in seeking to accomplish a business objective. Often, various projects or investment under consideration do not have the same duration or do not match the desired study period. Adjustments must be made to account for the differences.

5.3.1 Independent Project

An independent project is one where the decision is to accept or reject the project has no effect on any other project being considered by the company. The cash flows of an independent project have no effect on the cash flow of other projects or divisions of the business. For two independent projects, we can have four investment alternatives:

- To accept the both project
- To reject the both project
- To accept only the first project
- To accept only the second project

For example; The decision to replace a company's computer system would be considered independent of a decision to build a new factory.

5.3.2 Dependent Project

The projects related to one another in such a way that the acceptance or rejection of one project influences the acceptance of other projects are called dependent projects. Two such types of dependencies are mutually exclusive projects contingent projects.

5.3.2.1 Mutually Exclusive

Mutually exclusive projects are those when one project is chosen all other are excluded in the group. Such cases come into decision making process mostly because of limitation of capital to invest.

For example; Buying Vs leasing an automobile for business use, when one alternative is accepted the other is excluded.

5.3.2.2 Contingent Project

Two or more projects are said to be contingent if the acceptance of one project requires the acceptance of another project.

For example; The purchase of a computer printer is dependent upon the purchase of a computer, but the computer may be purchased without purchasing the printer.

Formulation of mutually exclusive combination of projects:

i) Independent projects case

Suppose there are two independent projects A and B. Then,

Mutually exclusive combination	Project		Explanation
	A	B	
1	0	0	Accept none
2	1	0	Accept A
3	0	1	Accept B
4	1	1	Accept A and B

ii) Mutually exclusive project case

Suppose there are three mutually exclusive projects A, B and C. Then,

Mutually exclusive combination	Project			Explanation
	A	B	C	
1	0	0	0	Accept none
2	1	0	0	Accept A
3	0	1	0	Accept B
4	0	0	1	Accept C

iii) Contingent project case

Let, there are three projects A, B and C where A is contingent on the acceptance of both B and C, and C is contingent on the acceptance of B. then,

Mutually exclusive combination	Project			Explanation
	A	B	C	
1	0	0	0	Accept none
2	0	1	0	Accept B
3	0	1	1	Accept B and C
4	1	1	1	Accept A, B and C

BOARD EXAM SOLVED PROBLEMS

1. **What do you mean by dependent, independent and mutually exclusive project? Explain with examples.**
[2011/Spring, 2016/Fall, 2016/Spring]

Answer: See the topic 5.3.1, 5.3.2, 5.3.2.1.

2. **Write short notes on mutually exclusive projects and its combination.**
[2017/Spring]

Answer: See the topic 5.3.2.1

A company is considering two independent sets of mutually exclusive projects. That is, project A_1 and A_2 are mutually exclusive as are B_1 and B_2. However, the selection of any project from the set of projects A_1 and A_2 is independent of the selection of any project from the set of project B_1 and B_2.

Mutually exclusive combination for two independent sets of mutually exclusive projects.

Mutually exclusive combination	Project				Explanation
	A_1	A_2	B_1	B_2	
1	0	0	0	0	Accept none
2	1	0	0	0	Accept A_1
3	0	1	0	0	Accept A_2
4	0	0	1	0	Accept B_1
5	0	0	0	1	Accept B_2
6	1	0	1	0	Accept A_1 and B_1
7	1	0	0	1	Accept A_1 and B_2
8	0	1	1	0	Accept A_2 and B_1
9	0	1	0	1	Accept A_2 and B_2

3. **Two mutually exclusive projects are shown below select the best project by using PW method.**

Project	M	N
Capital Investment, Rs.	5,00,000	7,00,000
Net annual revenue, Rs.	1,10,000	1,10,000
Salvage value	40,000	40,000
Useful life (years)	3	4
MARR	10% per years	

[2013/Spring]

Solution:

Study period: LCM of 3 and 4 = 12 years.

Project A:

It requires 4 times repetition.

Using PW formulation:

PW(10%) = -5,00,000 - 5,00,000 (P/F, 10%, 3) - 5,00,000 (P/F, 10% 6)

$= -5,00,000 (P/F, 10\%, 9) + 1,10,000(P/A, 10\%, 12) + 40,000 (P/F, 10\%, 3) + 40,000 (P/F, 10\%, 6) + 40,000 (P/F, 10\%, 9) + 40,000 (P/F, 10\%, 12)$

$= -5,00,000 - 5,00,000 \times 0.7513 - 5,00,000 \times 0.5645 - 5,00,000 \times 0.4241 + 1,10,000 \times 6.8137 + 40,000 \times 0.7513 + 40,000 \times 0.5645 + 40,000 \times 0.4241 + 40,000 \times 0.3186$

$= -$ Rs. 5,38,103

Project B:

It requires 3 times repetition

PW(10%) $= -7,00,000 - 7,00,000 (P/F, 10\%, 4) - 7,00,000 (P/F, 10\%, 8) + 1,10,000 (P/A, 10\%, 12) + 40,000 (P/F, 10\%, 4) + 40,000 (P/F, 10\%, 8) + 40,000 (P/F, 10\%, 12)$

$= -7,00,000 - 7,00,000 \times 0.6830 - 7,00,000 \times 0.4665 + 1,10,000 \times 6.8137 + 40,000 \times 0.6830 + 40,000 \times 0.4665 + 40,000 \times 0.3186$

$= -$ Rs. 6,96,419

Both projects have negative value *i.e.*, both are not preferable but if decision is to be made we should go for lesser negative value *i.e.*, project A is best.

4. **Calculate PW of the following two mutually exclusive projects by using repeatability assumption when MARR is 10%.**

Project	A	B
Initial cost, Rs.	4,00,000	6,00,000
Annual revenue, Rs.	30,000	35,000
Annual costs, Rs.	3,000	4,000
Useful life (years)	3	9
Salvage value	4,000	7,000

[2014/Fall, 2016/Fall]

Solution:

Study period: LCM of 3 and 9 = 9 years

Project A:

It requires 3 time repetition

Net annual revenue = 30,000 – 3,000 = Rs. 27,000

PW (10%) $= -4,00,000 - 4,00,000 (P/F, 10\%, 3) - 4,00,000 (P/F, 10\%, 6) + 27,000 (P/A, 10\%, 9) + 4,000 (P/F, 10\%, 3) + 4,000 (P/F, 10\%, 6) + 4,000 (P/F, 10\%, 9)$

$= -4,00,000 - 4,00,000 \times 0.7513 - 4,00,000 \times 0.5645 + 27,000 \times 5.7590 + 4,000 \times 0.7513 + 4,000 \times 0.5645 + 4,000 \times 0.4241$

$= -$ Rs. 7,63,867.40

Project B:

It requires 1 time repetition

PW (10%) $= -6,00,000 + (35,000 - 4,000) (P/A, 10\%\ 9) + 7,000 (P/F, 10\%, 9)$

$= -6,00,000 + 31,000 \times 5.7590 + 7,000 \times 0.4241$

$= -$ Rs. 4,18,502

Both projects have negative value in which project B has less negative value. Hence, project B is best.

5. **A Company needs a machine and has found two quantities with cash flows shown below. Select the best quotation using PW method.**

Project	Quotation A	Quotation B
Purchase price	12,500	15,000
Annual O and M costs	5,000	4,000
Salvage value	2,000	1500
Life in years	3	4
MARR	10%	

[2015/Spring]

Solution:

Study period: LCM of 3 and 4 = 12 years

For project A:

It requires 4 time repetition.

PW (10%) $= -12,500 - 12,500 (P/F, 10\%, 3) - 12,500 (P/F, 10\%, 6) - 12500 (P/F, 10\%, 9) - 5,000 (P/A, 10\%, 12) + 2,000 (P/F, 10\%, 3) + (2,000 (P/F, 10\%, 6) + 2,000 (P/F, 10\%, 9) + 2,000 (P/F, 10\%, 12)$

$= -12,500 - 12,500 \times 0.7513 - 12,500 \times 0.5645 - 12500 \times 0.4241 - 5,000 \times 6.8137 + 2,000 \times 0.7513 + 2,000 \times 0.5645 + 2,000 \times 0.4241 + 2,000 \times 0.3186$

$= -$ Rs. 64,200

For project B:

It requires 3 times repetition.

PW (10%) $= -15,000 - 15,000 (P/F, 10\%, 4) - 15,000 (P/F, 10\%, 8) - 4,000 (P/A, 10\%, 12) + 1,500 (P/F, 10\%, 4) + 1,500 (P/F, 10\%, 8) + 1,500 (P/F, 10\%, 12)$

$= -15,000 - 15,000 \times 0.6830 - 15,000 \times 0.4665 - 4,000 \times 6.8137 + 1,500 \times 0.6830 + 1,500 \times 0.4665 + 1,500 \times 0.3186$

$= -$ Rs. 57, 295.15

Although both project has negative value, quotation B has less negative value, hence quotation B has best value.

6. **Following are the two independent projects, determine which project is worthful by using BCR. Assume that MARR is 10%.**

Project	A	B
Initial investment	3,50,000	4,50,000
Annual benefits	1,50,000	1,75,000
Annual O and M costs	30,000	40,000

Life in years	4	5
Salvage value	50,000	75,000

[2015/Spring]

Solution:

Given that;

MARR, i% = 10%

For project A:

Initial investment, I = Rs. 3,50,000

Annual benefits, B = Rs. 1,50,000

Annual O and M costs, O and M = Rs. 30,000

Life in years, N = 4 years

Salvage value, s = Rs. 50,000

Now, annual worth formulation,

AW (I) = I (A/ P, i%, N)

= 3,50,00 × (A/P, 10%, 4)

= 3,50,000 × 0.3155

= Rs. 1,10,425

AW (S) = S (A/E, i%, N)

= 50,000 (A/F, 10%,4)

= 50,000 × 0.2155

= Rs. 10,775

AW (B) = Rs. 1,50,000

AW (O and M) = Rs. 30,000

Hence,

$$BCR = \frac{AW\ (B)}{AW\ (I) - AW\ (S) + AW\ (O \text{ and } M)}$$

$$= \frac{1,50,000}{1,10,425 - 10,775 + 3,000}$$

$$= 1.157 > 1$$

For project B:

I = Rs. 4,50,000

B = Rs. 1,75000

O and m = Rs. 40,000

N = 5 years

S = Rs. 75,000

Now,

AW (I) = Rs. 4,50,000 (A/P, 10%, 5) = 4,50,000 × 0.2638

AW (S) = Rs. 75,000 (A/F, 10%, 5) = 75,000 × 0.1638 = Rs. 12,285

AW (B) = Rs. 1,75,000

AW (O and M) = Rs. 40,000

Then,

$$BCR = \frac{AW\ (B)}{AW\ (I) - AW\ (S) + AW\ (O \text{ and } M)}$$

$$= \frac{1,75,000}{1,18,710 - 12,285 + 40,000}$$

$$= 1.195 > 1$$

BCR of both project A and B is greater than 1. So, we use incremental analysis project B – A.

ΔAW (I) = Rs. 1,18,710 – 1,10,425 = Rs. 8,285

ΔAW (S) = Rs. 12,285 – 10,775 = Rs. 1,510

ΔAW (B) = Rs. 1,75,000 – 1,50,000 = Rs. 25,000

ΔAW (O and M) = Rs. 40,000 – Rs. 30,000 = Rs. 10,000

so,
$$BCR = \frac{\Delta AW\ (B)}{\Delta AW\ (I) - \Delta AW\ (S) + \Delta AW\ (O \text{ and } M)}$$

$$= \frac{25,000}{8,285 - 1,510 + 10,000}$$

$$= 1.49 > 1$$

Incremental BCR of B – A is greater than 1. So project B is better choice than A.

7. **Compare the following two mutually exclusive projects by using co-terminated assumption when MARR = 10%.**

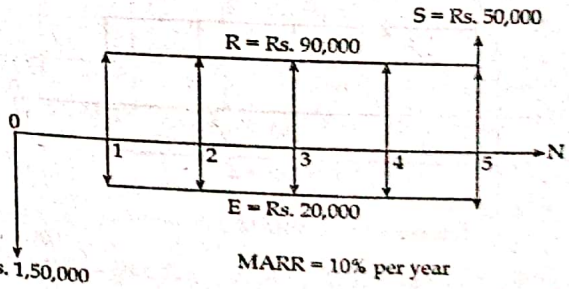
Project	A	B
Initial cost	1,50,000	2,00,000
Annual revenue	90,000	1,00,000
Annual O and M cost	20,000	22,000
Life in years	5	8
Salvage value	50,000	1,00,000

[2015/Fall]

Solution:

Using FW formulation,

FW of A:



FW (10%) = –1,50,000 (F/P, 10%, 5) + (90,000 – 20,000) (F/A, 10%, 5) + 50,000

= –1,50,000 × 1.6105 + 70,000 × 6.1051 + 50,000

= Rs. 2,35,782

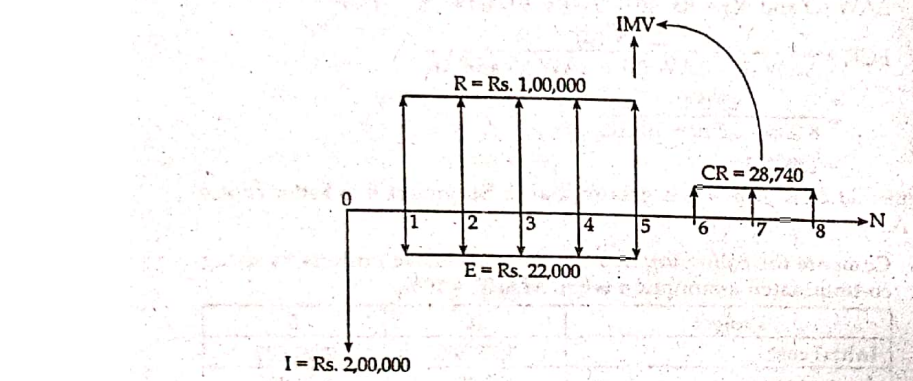
FW of B:

CR (10%) = 2,00,000 (A/P, 10%, 8) – 1,00,000 (A/F, 10%, 8)

$$= 2{,}00{,}000 \times 0.1874 - 1{,}00{,}000 \times \left[\frac{0.10}{(1+0.10)^8 - 1}\right]$$

= 2,00,000 × 0.1874 – 1,00,000 × 0.0874

= Rs. 28,740

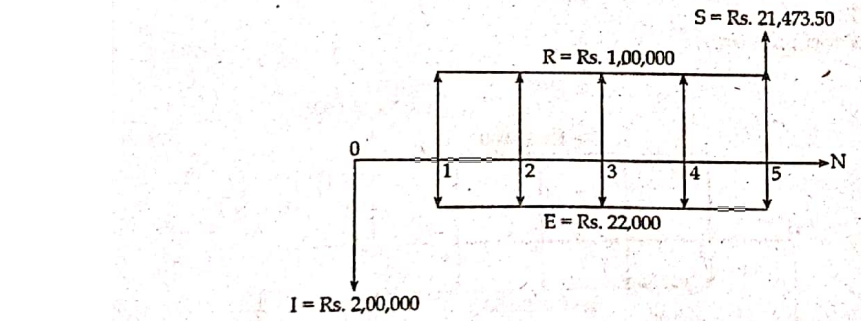


PW of CR at the end of 5th year remaining 3 years.

PW_{CR} (10%) = CR (P/A, 10%, 3)

= 28,740 × 2.4869

= Rs. 71,473.50



Hence,

FW of B = –2,00,000 (F/P, 10%, 5) + (1,00,000 – 22,000) (F/A, 10%, 5) + 71,473.50

= –2,00,000 × 1.6105 + 78,000 × 6.1051 + 71,473.50

= Rs. 2,25,571.30

Since, FW (10%) of A > FW (10%) of B, the project A is best.

8. Use repeatability assumptions to recommend the best project from following information.

Project	Quotation A	Quotation B
Initial investment, Rs.	40,000	50,000
Net annual revenue, Rs.	15,000	20,000
Salvage value, Rs.	5,000	6,000
Life in years	3	5
MARR	15%	

[2012/Spring, 2016/Spring]

Solution:

LCM of 3 and 5 years = 15 years

Project A:

It requires 5 times repetition.

PW (15%) = –40,000 – 40,000 (P/F, 15%, 3) – 40,000 (P/F, 15%, 6) – 40,000 (P/F, 15%, 9) – 40,000 (P/F, 15%, 12) + 15,000 (P/A, 15%, 15) + 5,000 (P/F, 15%, 3) + 5,000 (P/F, 15%, 6) + 5,000 (P/F, 15%, 9) + 5,000 (P/F, 15%, 12) + 5,000 (P/F, 15%, 15)

= –40,000 – 40,000 × 0.6575 – 40,000 × 0.4323 – 40,000 × 0.2843 – 40,000 × 0.1869 + 15,000 × 5.8474 + 5,000 × 0.6575 + 5,000 × 0.4323 + 5,000 × 0.2843 + 5,000 × 0.1869 + 5,000 × 0.1229

= –Rs. 6,309.50

Project B:

It requires 3 times repetition.

PW (15%) = –50,000 – 50,000 (P/F, 15%, 5) – 50,000 (P/F, 15%, 10) + 20,000 (P/A, 15%, 15) + 6,000 (P/F, 15%, 5) + 6,000 × (P/F, 15%, 10) + 6,000 (P/F, 15%, 15)

= –50,000 – 50,000 × 0.4972 – 50,000 × 0.2472 + 20,000 × 5.8474 + 6,000 × 0.4972 + 6,000 × 0.2472 + 6,000 × 0.1229

= Rs. 34,931.80

Since, PW(15%) of project B > PW (15%) of project A, hence project B is best project.

9. What are the methods used to analyze the projects of same useful life and different useful life?

Assuming infinite project life, recommend one of the following mutually exclusive projects if MARR = 10%.

Project	Quotation A	Quotation B
Initial investment, Rs.	7,50,000	18,00,000
Salvage value, Rs.	1,50,000	2,70,000
Annual costs, Rs.	1,35,000	90,000
Useful life in years	30	75

[2011/Spring, 2017/Spring]

Solution:

Methods used to analyze the projects having

a) **Same useful life**
   - i) Payback period method
   - ii) Equivalent worth method (PW, AW and FW)
   - iii) Rate of return method (IRR and ERR)
   - iv) Benefit cost ratio method

b) **Different useful life**
   - i) Repeatability assumption method
   - ii) Co-termination
   - iii) Capitalized worth method

AW of project A:

$AW_A (10\%) = -750{,}000 (A/P, 10\%, 30) - 135{,}000 + 150{,}000 (A/F, 10\%, 30)$
$= -750{,}000 \times 0.1061 - 135{,}000 + 150{,}000 \times 0.0061$
$= -$ Rs. 2,13,660

AW of project B:

$AW_B (10\%) = -18{,}00{,}000 (A/P, 10\%, 75) - 90{,}000 + 2{,}70{,}000 (A/F, 10\%, 75)$
$= -18{,}00{,}000 \times 0.1001 - 90{,}000 + 2{,}70{,}000 \times 0.0001$
$= -$ Rs. 2,70,153

Capitalized worth:

$$CW_A (10\%) = \frac{AW_A (10\%)}{i} = \frac{-2{,}13{,}660}{0.10} = -\text{ Rs. } 21{,}36{,}600$$

$$CW_B (10\%) = \frac{AW_B (10\%)}{i} = \frac{-2{,}70{,}153}{0.10} = -\text{ Rs. } 27{,}01{,}530$$

Here, $CW_A (10\%) > CW_B (10\%)$. Hence, select project A so as to reduce the cost.

10. Recommend the best project from the following two projects assume repeatability.

Project	Quotation A	Quotation B
Initial investment, Rs.	4,00,000	7,00,000
Annual revenue, Rs.	1,75,000	2,50,000
Annual cost, Rs.	25,000	35,000
Salvage value, Rs.	40,000	70,000
Useful life, year	6	8
MARR	12%	

[2014/Spring, 2018/Spring]

Solution:

Study period: LCM of 6 and 8 years = 24 years.

Project A:

It needs 4 times repetition

$PW(12\%) = -4{,}00{,}000 [1 + (P/F, 12\%, 6) (P/F, 12\%, 12) + (P/F, 12\%, 18)]$
$+ 40{,}000 [(P/F, 12\%, 6) + (P/F, 12\%, 12) + (P/F, 12\%, 18)$
$+ (P/F, 12\%, 24)] + (1{,}75{,}000 - 25{,}000) (P/A, 12\%, 24)$
$= -4{,}00{,}000 [1 + 0.5066 + 0.2567 + 0.1300] + 40{,}000 [0.5066$
$+ 0.2567 + 0.1300 + 0.0659] + [(1{,}75{,}000 - 25{,}000) \times 7.7843]$
$=$ Rs. 4,48,693

Project B:

It needs 3 times repetition

$PW(12\%) = -7{,}00{,}000 [1 + (P/F, 12\%, 8) + (P/F, 12\%, 16)] + 70{,}000$
$[(P/F, 12\%, 8) + (P/F, 12\%, 16) + (P/F, 12\%, 16)$
$+ (P/F, 12\%, 24) + [(2{,}50{,}000 - 35{,}000) (P/A, 12\%, 24)]$
$= -7{,}00{,}000 [1 + 0.4039 + 0.1631]$
$+ 70{,}000 [(0.4039 + 0.1631 + 0.0659] + 2{,}15{,}000 \times 7.7843$
$=$ Rs. 6,21,027.50

Here, PW_B (6,21,013.50) > PW_A (4,48,693), so project is best to select.

11. Consider the following three sets of mutually exclusive alternatives.

EOY	Alternatives		
	D_1 (Rs.)	D_2 (Rs.)	D_3 (Rs.)
0	-2,000	-1,000	-3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000

Which project would you select based on BCR method on incremental investment assuming that MARR = 15%.

[2018/Spring]

Solution:

PW of cost of alternative-1 (D_1) = 2,000
PW of cost of alternative-2 (D_2) = 1,000
PW of cost of alternative-3 (D_3) = 3,000

From above calculation, the order of alternatives from lowest equivalent cost to highest equivalent cost is D_2, D_1 and D_3.

The lowest equivalent cost alternative D_2 is first compared against do-noting alternative *i.e.*, the B/C ratio of alternative D_2 on its cash flow is calculated.

$$\text{B/C ratio of } D_2 = \frac{\text{PW of benefits}}{\text{Initial investments + PW of expenses}}$$

$$= \frac{800\,(1+0.15)^{-1} + 500(1+0.15)^{-2} + 500\,(1+0.15)^{-3}}{1{,}000}$$

$$= \frac{1{,}402.48}{1{,}000} = 1.403$$

As the B/C ratio of alternative D_2 is greater than 1.0, D_2 now becomes the base alternative and is compared against the next higher equivalent cost alternative. *i.e.*, Alternative D_1. Now the incremental benefits and incremental cost between D_2 and D_1 are calculated and incremental BCR is obtained.

Incremental annual benefits between D_2 and D_1:

PW (15%) benefits of D_1 $= 1{,}500 \times 1.15^{-1} + 1{,}000 \times 1.15^{-2} + 800 \times 1.15^{-3}$
$= \text{Rs. } 2{,}586.50$

PW (15%) benefits of D_2 $= 800 \times 1.15^{-1} + 500 \times 1.15^{-2} + 500 \times 1.15^{-3}$
$= \text{Rs. } 1{,}402.48$

PW (15%) benefits of D_3 $= 1{,}500 \times 1.15^{-1} + 2{,}000 \times 1.15^{-2} + 1{,}000 \times 1.15^{-3}$
$= \text{Rs. } 3{,}474.15$

Hence, PW of incremental benefits between D_2 and D_1
$= \text{Rs. } 2{,}586.40 - 1{,}402.48$
$\Delta B = \text{Rs. } 1{,}183.93$

PW of incremental costs between D_2 and D_1
$= \text{Rs. } (2{,}000 - 1{,}000)$
$\Delta C = \text{Rs. } 1{,}000$

Hence, incremental BCR between D_2 and D_1

$$= \frac{\Delta B}{\Delta C} = \frac{1{,}183.93}{1{,}000} = 1.183$$

As the incremental BCR is greater than 1.0, alternative D_1 becomes the new base alternative and alternative D_2 is removed from further analysis. Alternative D_1 is now compared against the next higher equivalent cost alternative. *i.e.*, alternative D_1 and D_3 is determined in the same manner as that was determined between the alternatives D_1 and D_2.

Now, PW of incremental benefits between D_1 and D_3
$= \text{Rs. } (3{,}474.15 - 2{,}586.50) = \text{Rs. } 887.65$

PW of incremental costs between D_1 and D_3
$= \text{Rs. } (3{,}000 - 2{,}000) = \text{Rs. } 1{,}000$

Hence, incremental BCR between D_1 and $D_3 = \frac{887.65}{1{,}000} = 0.8875$

Here, the incremental B/C ratio between alternatives D_1 and D_3 is 0.8875 which is less than 1.0. Thus, incremental cost associated with alternative D_3 is not justified. Hence, alternative D_1 is selected as the best alternative among D_1, D_2 and D_3 as no other alternative is left for comparison.

Alternative method

MARR = 15% per year

Costs:

PW of $D_1 = \text{Rs. } 2{,}000$
PW of $D_2 = \text{Rs. } 1{,}000$
PW of $D_3 = \text{Rs. } 3{,}000$

Benefits:

$$\text{PW of } D_1 = \frac{1{,}500}{1.15} + \frac{1{,}000}{(1.15)^2} + \frac{800}{(1.15)^3} = \text{Rs. } 2{,}586.5$$

$$\text{PW of } D_2 = \frac{800}{1.15} + \frac{500}{(1.15)^2} + \frac{500}{(1.15)^3} = \text{Rs. } 1{,}402.48$$

$$\text{PW of } D_3 = \frac{1{,}500}{1.15} + \frac{2{,}000}{(1.15)^2} + \frac{1{,}000}{(1.15)^3} = \text{Rs. } 3{,}474.15$$

BCR of D_1, D_2, and D_3:

	Alternatives		
	D_1	D_2	D_3
Cost	2,000	1,000	3,000
Benefit	2,586.5	1,402.48	3,474.5
B/C	1.293	1.402	1.158

B/C = Benefits/Costs

All alternatives have B/C greater than 1.

Now, Incremental B – C $\left(\frac{\Delta B}{\Delta C}\right)$ of the alternatives:

	$D_1 - D_2$	$D_3 - D_2$
ΔC	1,000	1,000
ΔB	1,184.02	888
$\frac{\Delta B}{\Delta C}$	1.184	0.888

Incremental B – C of $D_1 - D_2$ is greater than 1 but that of $D_3 - D_2$ is less than 1. So, D_1 is best alternative.

12. Use repeatability assumption to select the best project. MARR = 10%.

Project	A	B	C
Initial investment	2,000	3,000	4,000
Annual income	1,000	1,200	1,500
Life	3	5	7
Salvage value	20% of initial investment		

[2018/Fall]

Solution:

Study period: LCM of 3, 5 and 7 = 105 years.
Project A needs 35 times repetition.
Project B needs 21 times repetition.
Project C needs 15 times repetition.

'W cost of project A:

$$PW\ (10\%) = -2{,}000\ [1 + (1.10)^{-3} + (1.10)^{-6} + (1.10)^{-9} + (1.10)^{-12} + (1,10)^{-15} + (1.10)^{-18} + (1.10)^{-21} + (1,10)^{-24} + (1.10)^{-27} + (1.10)^{-30} + (1,10)^{-33} + (1.10)^{-36} + (1.10)^{-39} + (1,10)^{-42} + (1.10)^{-45} + (1.10)^{-48} + (1,10)^{-51} + (1.10)^{-54} + (1.10)^{-57} + (1,10)^{-60} + (1.10)^{-63} + (1.10)^{-66} + (1,10)^{-69} + (1.10)^{-72} + (1.10)^{-75} + (1,10)^{-78} + (1.10)^{-81} + (1.10)^{-84} + (1,10)^{-87} + (1.10)^{-90} + (1.10)^{-93} + (1.10)^{-96} + (1.10)^{-99} + (1.10)^{-102}] + 1{,}000 \times (P/A, 10\%, 105) + 400[(1.10)^{-3} + (1.10)^{-6} + (1.10)^{-9} + (1.10)^{-12} + \ldots\ldots\ldots + (1,10)^{-102} + (1.10)^{-105}]$$

$$= -2{,}000 \times 4.0116 + 1{,}000 \times \left[\frac{(1.10)^{105} - 1}{0.10 \times (1.10)^{105}}\right] + 4{,}000 \times 3.0116$$

$$= -\text{Rs. } 8{,}023 + 10{,}000 + 1{,}204$$

$$= \text{Rs. } 3{,}166.4$$

PW cost of project B:

$$PW\ (10\%) = -3{,}000\ [1 + (1.10)^{-5} + (1.10)^{-10} + (1.10)^{-15} + (1.10)^{-20} + \ldots\ldots\ldots + (1.10)^{-95} + (1.10)^{-100}] + 1{,}200\ (P/A, 10\%, 105) + 600\ [(1.10)^{-5} + (1.10)^{-10} + \ldots\ldots\ldots + (1.10)^{-100} + (1.10)^{-105}]$$

$$= -3{,}000 \times 2.638 + 120 \times 10 + 600 \times 1.638$$

$$= -\text{Rs. } 5{,}731.2$$

PW cost of project C:

$$PW\ (10\%) = -4{,}000\ [1 + (1.10)^{-7} + (1.10)^{-14} + \ldots\ldots\ldots + (1.10)^{-91} + (1.10)^{-98}] + 1{,}500(P/A, 10\%, 105) + 300 \times [(1.10)^{-7} + (1.10)^{-14} \ldots\ldots\ldots + (1.10)^{-105}]$$

$$= -4{,}000 \times 2.05 + 1500 \times 10 + 800 \times 1.054$$

$$= \text{Rs. } 7{,}627.2$$

Here, PW (10%) of project C > PW (10%) of project. A > PW (10%) of project B.

Hence, project C is the best project among three.

13. Select which project is terrible to invest among two following alternative projects whose cash flows are as follows. MARR is 12% per year. Use IRR method and incremental analysis.

Project	A	B
Investment	50,000	1,50,000
Net annual revenue	25,000	70,000
Net annual cost	3,000	2,000
Salvage value	15,000	40,000
Useful life	7 years	7 years
Repair and maintenance cost at 3rd and 5th year.	10,000	15,000

[2016/Fall, 2017/Fall]

Solution:

MARR = 12% per year

N = 7 years

IRR for project/investment A:

$$PW = -50{,}000 + (25{,}000 - 3{,}000)\ (P/A, i\%, 7) + 15{,}000\ (P/F, i\%, 7) - 10{,}000\ (P/F, i\%, 3) - 10{,}000\ (P/F, i\%, 5)$$

IRR is that rate of i' which makes PW = 0

Let, i'% = 20%

$$PW\ (20\%) = -50{,}000 + 22{,}000 \times 3.6046 + 15{,}000 \times 0.2797 - 10{,}000 \times 0.5787 - 10{,}000 \times 0.4019$$

$$= \text{Rs. } 23{,}681.7$$

Let, i'% = 25%

$$PW\ (25\%) = -50{,}000 + 22{,}000 \times 3.1611 + 15{,}000 \times 0.2097 - 10{,}000 \times 0.5120 - 10{,}000 \times 0.3277$$

$$= \text{Rs. } 14{,}292.7$$

Let, i'% = 30%

$$PW\ (30\%) = -50{,}000 + 22{,}000 \times 2.8021 + 15{,}000 \times 0.1594 - 10{,}000 \times 0.4552 - 10{,}000 \times 0.2693$$

$$= \text{Rs. } 6{,}792.2\ (+ve)$$

Let, i'% = 35%

$$PW\ (35\%) = -50{,}000 + 22{,}000 \times 2.5075 + 15{,}000 \times 0.1224 - 10{,}000 \times 0.4064 - 10{,}000 \times 0.2230$$

$$= \text{Rs. } 707\ (+ve)$$

Let, i'% = 40%

$$PW\ (40\%) = -50{,}000 + 22{,}000 \times 2.2628 + 15{,}000 \times 0.0949 - 10{,}000 \times 0.3644 - 10{,}000 \times 0.1859$$

$$= -\text{Rs. } 4298\ (-ve)$$

$$\text{Hence, IRR} = 35\% + \frac{707}{707 + 4{,}298} \times (40 - 35)\% = 35.71\%.$$

Since, IRR of project A > MARR of project, hence it is feasible project.

For project B:

$$PW = -1{,}50{,}000 + (70{,}000 - 2000)\ (P/A, i\%, 7) + 40{,}000\ (P/F, i\%, 7) - 15{,}000\ (P/F, i\%, 3) - 15{,}000\ (P/F, i\%, 5)$$

Let, i'% = 20%

$$PW\ (20\%) = -1{,}50{,}000 + 68{,}000 \times 3.6046 + 40{,}000 \times 0.2791 - 15{,}000 \times 0.5787 - 15{,}000 \times 0.4019$$

$$= \text{Rs. } 91{,}567.8$$

Let, i'% = 30%

$$PW\ (30\%) = -1{,}50{,}000 + 68{,}000 \times 2.8021 + 40{,}000 \times 0.1594 - 15{,}000 \times 0.4552 - 15{,}000 \times 0.2693$$

$$= \text{Rs. } 36{,}051.3$$

Let, i'% = 35%

PW (35%) = – 1,50,000 + 68,000 × 2.5075 + 40,000 × 0.1224 – 15,000 × 0.4044 – 15,000 × 0.2230

= Rs. 19,965 (+ve)

Let, i'% = 40%

PW (40%) = – 1,50,000 + 68,000 × 2.2628 + 40,000 × 0.0949 – 15,000 × 0.3644 – 15,000 × 0.1859

= – Rs. 588 (–ve)

Hence,

$$IRR = 35\% + \frac{19,965}{19,965 + 588} \times (40 - 35)\%$$

= 35% + 4.822%

= 39.822%

Since, IRR of project B > MARR, hence it is feasible project and IRR of project B > IRR of project A, it is feasible to invest on project B.

Performing the incremental analysis:

Selecting project A as base alternative. Incremental cash flows will be

Items	Project A	Project B	Project (B – A)
Investment	50,000	1,50,000	1,00,000
Net annual revenue	35,000	70,000	45,000
Net annual cost	3,000	2,000	–1000
Salvage value	15,000	40,000	35,000
Repair and maintenance cost 3rd and 5th year	10,000	15,000	5,000

Using PW formation,

PW = –1,00,000 + (45,000 + 1,000) (P/A, i'%, 7) + 25,000 × (P/F, i'%, 7) – 5,000 (P/F, i'%, 3) – 5,000 (P/F, i'%, 5)

Let, i'% = 20%

PW (20%) = –1,00,000 + 46,000 × (P/A, 20%, 7) + 25,000 × (P/F, 20%, 7) – 5,000 × (P/F, 20%, 3) – 5,000 × (P/F, 20%, 5)

= –1,00,000 + 46,000 × 3.6046 + 25,000 × 0.2791 – 5,000 × 0.5787 – 5,000 × 0.4019

= Rs. 67,887.60

Let, i' % = 30%

PW (30%) = –1,00,000 + 46,000 × 2.8021 + 25,000 × 0.1594 – 5,000 × 0.4552 – 5,000 × 0.1594

= Rs. 29,808.6

Let, i' % = 40%

PW (40%) = –1,00,000 + 46,000 × 2.2628 + 25,000 × 0.0949 – 5,000 × 0.3644 – 5,000 × 0.1859

= Rs. 3,709.8

Let, i' % = 50%

PW (30%) = –1,00,000 + 46,000 × 1.8829 + 25,000 × 0.585 – 5,000 × 0.2963 – 5,000 × 0.1317

= Rs. 901.60

$$\text{Hence IRR} = 40\% + \frac{3,709.8}{3,709.8 + 901.60} \times (50 - 40)\%$$

= 40% + 8.045%

= 48.045 % > MARR (12%)

Hence, the extra investment in the project B is justified. Hence, select project B for more profit.

14. Select the best project by using IRR method when MARR is 8%. Use incremental analysis if necessary.

	Project A	Project B
Initial investment	1,00,000	5,00,000
Annual revenue	50,000	1,75,000
Life year	6	6
Salvage value	10,000	1,00,000

[2020/Fall]

Solution:

Here;

MARR = 8%

For project A:

Initial investment, I = Rs. 1,00,000

Annual revenue, R = Rs. 50,000

Life year, N = 6 years

Salvage value, S = Rs. 10,000

Formula for net present worth:

PW (i%) = –I + R (P/A, i%, N) + S(P/F, i%, N)

= –10,000 + 50,000 (P/A, i%, 6) + 10,000 (P/F, i%, 6)

When i = 20%,

PW(20%) = –1,00,000 + 50,000 (P/A, 20%, 6) + 10,000 (P/F, 20%, 6)

= Rs. 69,624.48

When i = 45%,

PW (45%) = –1,00,000 + 50,000 (P/A, 45%, 6) + 10,000 (P/F, 45%, 6)

= Rs. 232.06

When i = 50%,

PW (50%) = –1,00,000 + 50,000 (P/A, 50%, 6) + 10,000 (P/F, 50%, 6)

= Rs. 7,901.23

Now,

$$\text{IRR of project A} = 45 + \frac{232.06}{232.06 - (-7,901.23)} \times (50 - 45) = 45.14\%$$

For project B:

Initial investment, I = Rs. 50,00,000
Annual revenue, R = Rs. 1,75,000
Life year, N = 6 years
Salvage value, S = Rs. 1,00,000

We know,

$$PW(i\%) = -5,00,000 + 1,75,000\,(P/A, i\%, 6) + 1,00,000\,(P/F, i\%, 6)$$

$$= -500000 + 175000 \times \frac{(1+i\%)^6 - 1}{i\% \times (1+i\%)^6} + \frac{100000}{(1+i\%)^6}$$

When i = 8%,

$$PW(8\%) = -500000 + 175000\,(P/A, 8\%, 6) + 100000\,(P/F, 8\%, 6)$$
$$= \text{Rs. } 372020.9$$

When i = 32%,

$$PW(32\%) = -500000 + 175000\,(P/A, 32\%, 6) + 100000\,(P/F, 32\%, 6)$$
$$= -\text{Rs. } 37602.78$$

When i = 28%,

$$PW(28\%) = -500000 + 175000\,(P/A, 28\%, 6) + 100000\,(P/F, 28\%, 6)$$
$$= \text{Rs. } 5628.82$$

so, IRR of project B $= 28 + \frac{5628.82}{5628.82 - (-37602.78)} \times (32 - 28) = 28.52\%$

Here, IRR of project A > IRR of project B. Both are greater than MARR = 8%. So, we choose project A.

Performing incremental analysis:

Selecting project A as base alternative:

	Project A	Project B	Project B – Project A
Initial investment	1,00,000	5,00,000	4,00,000
Annual revenue	50,000	1,75,000	1,25,000
Salvage value	10,000	1,00,000	90,000

Using PW formulation,

$$PW = -4,00,000 + 1,25,000\,(P/A, i\%, 6) + 90,000\,(P/F, i\%, 6)$$

$$= -4,00,000 + 1,25,000 \times \frac{(1+i\%)^6 - 1}{i\% \times (1+i\%)^6} + \frac{90,000}{(1+i\%)^6}$$

When i = 10%,

$$PW(10\%) = -4,00,000 + 125,000\,(P/A, 10\%, 6) + 90,000\,(P/F, 10\%, 6)$$
$$= \text{Rs. } 1,95,210.24$$

When i = 25%,

$$PW(25\%) = -4,00,000 + 1,25,000\,(P/A, 25\%, 6) + 90,000\,(P/F, 25\%, 6)$$
$$= -\text{Rs. } 7,479.04$$

When i = 20%,

$$PW(20\%) = -4,00,000 + 1,25,000\,(P/A, 20\%, 6) + 90,000\,(P/F, 20\%, 6)$$
$$= \text{Rs. } 45,829.58$$

Now,

$$IRR = 20\% + \frac{45,829.58}{45,829.58 - (-7,479.04)} \times (25 - 20) = 24.30\% > \text{MARR}$$

Here, we see that IRR of B – A is greater than MARR (8%). Hence, investing in project B provides more profit than project A. So the project B is the best choice.

15. Evaluate the following two projects A and B having different useful lives, if MARR is 15% per year. Use PW method with repeatability assumption.

	Project A (Rs.)	Project B (Rs.)
Initial investment	5,40,000	6,50,000
Annual revenue	2,50,000	2,80,000
Annual costs	50,000	60,000
Salvage value	1,00,000	1,20,0000
Useful file	4 years	6 years

[2020/Fall]

Solution:

Here; MARR, i% = 15%

Project A and B has unequal life, so we choose LCM of two life as study period.

So study period, N = 12 years.

For project A:

Initial investment, I = Rs. 5,40,000
Annual revenue, R = Rs. 2,50,000
Annual cost, E = Rs. 50,000
Salvage value, S = Rs. 1,00,000
Useful life, N = 4 years
Annual profit, R – E = Rs. 2,00,000

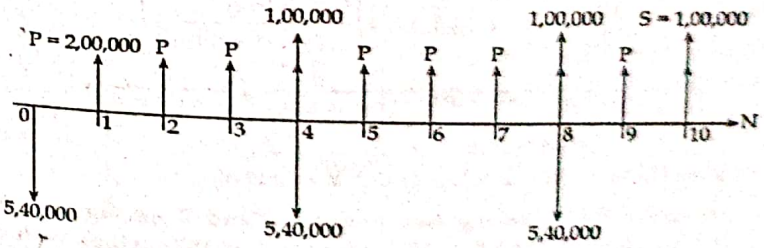


Figure: Cash flow diagram

so, $PW_A(15\%) = -I + I(P/F, 15\%, 4) - I(P/F, 15\%, 8) + (R - E)(P/A, 15\%, 12) + S(P/F, 15\%, 4) + S(P/F, 15\%, 8) + S(P/F, 15\%, 12)$

$$= -5,40,000\left(1 + \frac{1}{(1+0.15)^4} + \frac{1}{(1+0.15)^8}\right) + 20,000\left(\frac{(1+0.15)^{12}-1}{0.15(1+0.15)^{12}}\right) + 1,00,000\left(\frac{1}{(1+0.15)^4} + \frac{1}{(1+0.15)^8} + \frac{1}{(1+0.15)^{12}}\right)$$

$$= \text{Rs. } 1,67,406.3$$

For project B:

Initial investment, I = Rs. 6,50,000
Annual revenue, R = Rs. 2,80,000
Annual cost, E = Rs. 60,000
Salvage value, S = Rs. 1,20,000
Useful life, N = 6 years
Annual profit, R - E = Rs. 2,20,000

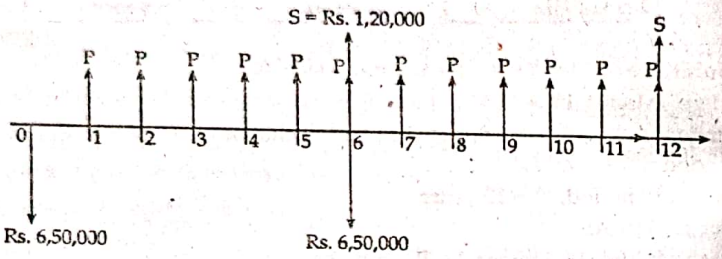


Figure: Cash flow diagram

so, $PW_B(15\%) = -I - I(P/F, 15\%, 6) + (R - E)(P/A, 15\%, 12) + S(P/F, 15\%, 6) + S(P/F, 15\%, 12)$

$$= -6,50,000\left(1 + \frac{1}{(1+0.15)^6}\right) + 2,20,000 \times \left[\frac{(1+0.15)^{12}-1}{0.15 \times (1+0.15)^{12}}\right] + 1,20,000\left[\frac{1}{(1+0.15)^6} + \frac{1}{(1+0.15)^{12}}\right]1$$

$$= \text{Rs. } 3,35,831.41$$

Since, $PW_B(15\%) > PW_A(15\%)$, project B is best choice.

16. Evaluate the following two projects A and B having different useful lives, if MARR is 15% per year. Use PW method with co-terminated assumptions; when MARR is 10 % per year.

	Project A	Project B
Initial investment	40,00,000	50,00,000
Annual revenue	15,00,000	20,00,000
Annual cost	5,00,000	7,00,000
Salvage value	5,00,000	6,00,000
Useful life	5 years	7 years

[2019/Spring]

Solution:

Let study period for both projects be 5 years which is useful life of project A.

$PW(i\%) = -I + (R - C)(P/A, i\%, N) + S(P/F, i\%, N)$

$PW(15\%) = -40,00,000 + (15,00,000 - 5,00,000)(P/A, 15\%, 5) + 5,00,000(P/F, 15\%, 5)$

$$= -40,00,000 + 10,00,000 \times \left[\frac{(1+0.15)^5 - 1}{0.15 \times (1+0.15)^5}\right] + \frac{5,00,000}{(1+0.15)^5}$$

$$= -\text{Rs. } 3,99,256.53$$

For project B:

It should terminate at the end of 5 years.

$$CR = 50,00,000(A/P, 15\%, 8) - 60,000(A/F, 15, 8)$$

$$= 5000000 \times \left[\frac{0.15 \times (1.15)^8}{((1.15)^8 - 1)}\right] - 6,00,000 \times \frac{0.15}{(1.15)^8 - 1}$$

$$= \text{Rs. } 10,70,540.394$$

PW of CR at end of 5th year.

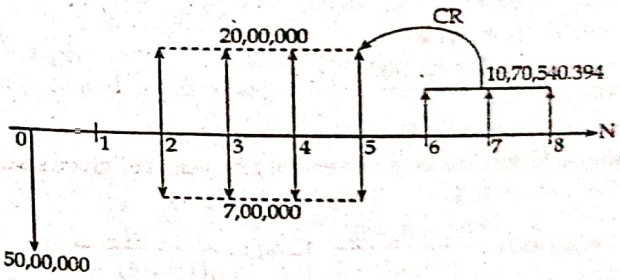
$$PW_{CR}(15\%) = \text{Rs. } 10,70,540.94(P/A, 15\%, 3) = 10,70,540.394 \times \left(\frac{1.15^3 - 1}{0.15 \times 1.15^3}\right)$$

$$= \text{Rs. } 24,44,284.72$$

PW of project B terminating at 5th year

$$PW(15\%) = -50,00,000 + (20,00,000 - 7,00,000)(P/A, 15\%, 5) + 24,44,284.72(P/F, 15\%, 5)$$

$$= \text{Rs. } 5,73,043.12$$



so, PW of project B > PW of project A. Hence, project B is the best alternative.

17. From the given information, select the best project using co-terminated assumption. Useful life = 5 years.

Items	X	Y	Z
Initial investment	50,000	40,000	30,000
Annual revenue	20,000	15,000	14,000
Annual expenses	15,000	10,000	8,000
Useful life	5 years	7 years	9 years
Salvage value	1,000	500	0
MARR	10%	10%	10%

[2019/Fall]

Solution:

Here; Study period, N = 5 years

For project X:

Initial investment, I = Rs. 50,000
Annual revenue, R = Rs. 20,000
Annual expenses, E = Rs. 15,000
Useful life, N = 5 years
Salvage value, S = Rs. 1,000
MARR, I = 10%
Here, study period = Useful life

So, present worth of project X:

$$PW(10\%) = -I + (R - E)(P/A, 10\%, 5) + S(P/F, 10\%, 5)$$

$$= -50{,}000 + (20{,}000 - 15{,}000) \times \left[\frac{(1+0.1)^5 - 1}{0.1 \times (1+0.1)^5}\right] + 1{,}000 \times \frac{1}{(1+0.1)^5}$$

$$= -\text{Rs. } 3{,}0425.14$$

For project Y:

Initial investment, I = Rs. 40,000
Annual revenue, R = Rs. 15,000
Annual expenses, E = Rs. 10,000
Useful life, N = 7 years
Salvage value, S = Rs. 500
MARR, i = 10%
Here, study period < Useful life

Project Y should be terminated at the end of fifth year, *i.e.*, given study period.

$$CR = I(A/P, 10\%, N) - S(A/F, 10\%, N)$$

$$= 40{,}000 \times \left[\frac{0.1 \times (1+0.1)^7}{(1+0.1)^7 - 1}\right] - 500 \times \frac{0.1}{[(1+0.1)^7 - 1]}$$

$$= \text{Rs. } 8{,}163.52$$

PW of CR at the end of fifth year;

$$PW_{CR}(10\%) = 8{,}163.52(P/A, 10\%, 2)$$

$$= 8{,}163.52 \times \left[\frac{(1+0.1)^2 - 1}{0.1 \times (1+0.1)^2}\right]$$

$$= \text{Rs. } 14{,}168.09$$

Now, PW of project Y terminating at end of 5th year:

$$PW(10\%) = -40{,}000 + (15{,}000 - 10{,}000)(P/A, 10\%, 5) + 14{,}168.09(P/F, 10\%, 5)$$

$$= -40{,}000 + 5{,}000 \times 3.79 + 14{,}168.09 \times 0.62$$

$$= -\text{Rs. } 12{,}248.8$$

For project Z:

Initial investment, I = Rs. 30,000
Annual revenue, R = Rs. 14,000
Annual expenses, E = Rs. 8,000
Useful life, N = 9 years
Salvage value, S = 0
MARR, i = 10%
Here, study period < Useful life

So, project Z should terminate at the end of 5 years *i.e.*, study period.

$$CR = 30{,}000(A/P, 10\%, 9) - 0(A/F, 10\%, 9)$$

$$= 30{,}000 \times 0.17$$

$$= \text{Rs. } 5{,}209.22$$

PW of CR at the end of fifth year

$$PW_{CR}(10\%) = 5{,}209.22(P/A, 10\%, 4)$$

$$= 5{,}209.22 \times 3.17$$

$$= \text{Rs. } 16{,}512.53$$

PW of project Z terminating at the end of fifth year:

$$PW(10\%) = -30{,}000 + (14{,}000 - 8{,}000)(P/A, 10\%, 5) + 16{,}512.53(P/F, 10\%, 5)$$

$$= \text{Rs. } 2{,}997.70$$

Among three projects X, Y and Z, project Z has highest present worth using co-terminated assumption. Hence project Z is selected as best alternative.

ADDITIONAL PROBLEMS

1. Select the best project using ERR when MARR = 18% and E = 12% per year.

Year	Project A	Project B
0	-40,000	-60,000
1	-38,000	25,000
2	35,000	40,000
3	35,000	-50,000
4	35,000	50,000
5	35,000	75,000

Solution:

Project A:

Discounting all cash outflows to year zero at 12%

= 40,000 + 38,000 (P/F, 12%, 1)

= 40,000 + 38,000 × 0.8929

ΣE_C = 7,3928.57

Compounding all cash inflows to year 5 at 12%

= 35,000 (F/A, 12%, 4)

= 35,000 × 4.7793

ΣR_C = Rs. 1,67,276.48

We know,

$\Sigma E_C (F/P, i'\%, N) = \Sigma R_C$

or, $7,390.20 \times (F/P, i'\%, 5) = 1,67,275.50$

or, $(1 + i')^5 = 2.2627$

or, $(1 + i') = 1.1774$

∴ $i'\% = 17.74\ \%$

Hence, ERR of project A = 17.74% < MARR.

Project B:

Discounting all cash outflows to year zero at 12%

= 60,000 + 50,000 (P/F, 12%, 3)

= 60,000 + 50,000 × 0.7118

ΣE_C = 95,590

Compounding all cash inflows to year 5 at 12%

= 25,000 (F/A, 12%, 4) + 40,000 (F/P, 12%, 3) + 50,000 × (F/P, 12%, 1) + 75000

= 25,000 ×1.8735 + 40,000 × 1.4049 + 50,000 × 1.12 + 75,000

ΣR_C = Rs. 2,26,533.50

We know,

$\Sigma E_C (F/P, i'\%, N) = \Sigma R_C$

or, $95,590 \times (F/P, i'\%, 5) = 2,26,533.50$

or, $(1 + i')^5 = 2.3698$

or, $(1 + i') = 1.188$

or, $i' = 0.188$

∴ $i'\% = 18.80\%$

ERR of project B = 18.80% > MARR

Hence, only project B is feasible and no incremental analysis is necessary. So, project B is best.

2. Select the best project using AW and ERR method. Useful life is 10 years and MARR = 10%. Perform incremental analysis if necessary.

Project	A	B
Initial investment	24,00,000	35,50,000
Annual revenue	8,20,000	12,00,000
Annual expresses	1,10,000	1,40,000
Salvage value	2,25,000	3,50,000

Solution:

Using AW method

Project A:

PW (10%) = -24,00,000 (A/P, 10%, 10) + (8,20,000 - 1,10,000) + 2,25,000 (A/F, 10%, 10)

= -24,00,000 × 0.1627 + 7,10,000 + 2,25,000 × 0.0627

= Rs. 3,33,627.50

Project B:

AW (10%) = -35,50,000 (A/P, 10%, 10) + (12,00,000 - 1,40,000) + 3,50,000 (A/F, 10%, 10)

= -35,50,000 × 0.1627 + 10,60,000 + 3,50,000 × 0.0627

= Rs. 5,04,360

Here, AW (10%) of project B > AW (10%) of project B. Hence, project B is best to select.

Using ERR method:

Assuming external reinvestment rate (∈) = 12%.

Project A:

Discounting all the cash outflows to present time at 12%

ΣE_C = Rs. 24,00,000

Compounding all the cash inflows to future time at 12%

$= (8,20,000 - 1,10,000)\ (F/A, 12\%, 10) + 2,25,000$

$= 7,10,000 \times 17.5487 + 2,25,000$

$\therefore \quad \Sigma R_C = \text{Rs. } 1,26,84,577$

We know,

$\Sigma E_C\ (F/P, i'\%, N) = \Sigma R_C$

or, $24,00,000\ (1 + i')^{10} = 1,26,84577$

or, $(1 + i')^{10} = 5.285$

or, $1 + i' = 5.285$

or, $1 + i' = 1.1811$

$\therefore \quad i'\% = 18.11$

Hence, ERR (18.11%) > MARR (10%), so acceptable.

Project B:

Discounting all the cash outflows to present time at 12%

$\Sigma E_C = \text{Rs. } 35,50,000$

Compounding all the cash inflows to future time at 12%.

$= (12,00,000 - 1,40,000)\ (F/A, 12\%, 10) + 3,50,000$

$= 10,60,000 \times 17.5487 + 3,50,000$

$\Sigma R_C = \text{Rs. } 1,89,51,659$

We know

$\Sigma E_C\ (F/P, i'\%, N) = \Sigma R_C$

or, $3,50,50,000\ (1 + i')^{10} = 1,89,51,659$

or, $(1 + i')^{10} = 5.3384$

or, $1 + i' = 1:1822$

$\therefore \quad i'\% = 18.22\%$

Hence, ERR (18.22%) > MARR (12%), So acceptable.

Performing the incremental analysis:

Selecting project A as base alternative.

Incremental cash flows:

Items	Project A	Project B	Project B – Project A
Initial investment	24,00,000	35,50,000	11,50,000
Annual revenue	8,20,000	12,00,000	380,000
Annual expenses	1,10,000	140,000	30,000
Salvage value	22,5000	3,50,000	1,25,000

Incrementing all the cash outflows to present time at 12%.

$\Sigma E_C = \text{Rs. } 11,50,000$

Compounding all the cash inflow to future time at 12%

$= (380,000 - 30,000)\ (F/A, 12\%, 10) + 1,25,000$

$= 350,000 \times 17.5487 + 1,25,000$

$\Sigma R_C = \text{Rs. } 62,045$

Establishing the equivalence between the equation at i'% by

$\Sigma E_C\ (F/P, i'\%, N) = \Sigma R_C$

or, $11,50,000\ (1 + i')^{10} = 62,67,045$

or, $(1 + i')^{10} = 5.449$

or, $1 + i' = 1.1847$

or, $i'\% = 18.47$

$\therefore \quad ERR = i' = 18.47\% > MARR$

The extra investment in the project B is justified. Hence, it is best to select project B for more profit.

3. **From the following 2 mutually exclusive projects, determine the best one project using discounted payback period method.**

Project	A	B
Initial investment (Rs.)	4,00,000	6,00,000
Net annual revenue (Rs.)	1,10,000	1,35,000

Salvage value = 20% of initial investment; N = 5 years.

Solution:

Using discounted playback period method:

Project A:

EOY	Cash flow (F)	PW of net cash flow at i = 15%	Cumulative cash flow
0	–4,00,000	–4,00,000	–4,00,000
1	1,10,000	95,652	–3,04,348
2	1,10,000	83,176	–2,21,172
3	1,10,000	72,327	–1,48,845
4	1,10,000	62,893	–85,952
5	1,90,000	94,464	8,512

PW (at i = 15%) = $F(1 + i)^{-N}$

PW for 1st year = $1,10,000 \times (1 + 0.15)^{-1} = 95,625$

PW for 2nd year = $1,10,000 \times (1 + 0.15)^{-2} = 83,176$

PW for 3rd year = $1,10,000 \times (1 + 0.15)^{-3} = 72,327$

PW for 4th year = $1,10,000 \times (1 + 0.15)^{-4} = 62,893$

PW for 5th year = $1,90,000 \times (1 + 0.15)^{-5} = 94,464$

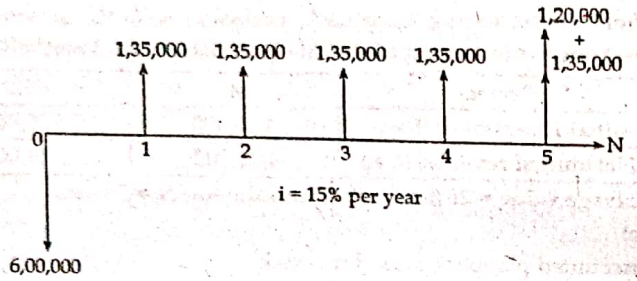
Here, cumulative cash flows turns into positive in the year 5. By interpolating

$\therefore$ Discounted playback period $(\theta') = 4 + \frac{85,952}{94,464} = 4.909$ years.

Project B:

EOY	Cash flow (F)	PW of net cash flow at i = 15%	Cumulative cash flow
0	-6,00,000	-6,00,000	-6,00,000
1	1,35,000	1,17,391	-4,82,609
2	1,35,000	1,02,079	-3,80,530
3	1,35,000	88,765	-2,91,765
4	1,35,000	77,187	-2,14,578
5	2,55,000	1,26,780	-87,798

$\because$ Salvage value = 20% of Rs. 6,00,000 = Rs. 1,20,000



Here, cumulative cash flow does not change into positive in 5 years. It indicates that the required payback period is more than 5 years.

Hence, we should choose project A as a best one which has payback period of 4.90 years.

CHAPTER **6**

RISK ANALYSIS

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6.1 ORIGINAL SOURCES OF PROJECT RISKS

Risk is the possibility of loss or injury. Project risk is an uncertain event or condition that, if it occurs, has an effect on atleast one project objective. The term project risk is used to refer variability in a project's NPW. A grater project risk means a greater variability in a project's NPW *i.e.*, risk is the potential for loss. There are four major sources of risk and uncertainty that affect project's Net present value *i.e.*, engineering economy studies.

1. Study Period

Longer study period generally increases the uncertainty of a capital investment and economic return. While shorter study period generally decreases the uncertainty of a capital investment and economic return. Hence, study period is a source of project risk.

2. Nature of Business

Nature of business is another source of risk and uncertainty. All business are of not same nature because some types of business operation are less stable than others while some are more stable than others. The level of risks depend upon the nature of business. *For example;* mining projects are more risky than horticulture projects with the expectation of income generation and stability.

3. Cash Flow Estimate

The accuracy of the cash inflow estimate is difficult to determine. A cash flow enables us to create a short term forecasts that enables us to determine how we are going to get money for the project and how we are going to pay for our expenses. The estimate would become uncertain of fair degree of reliance is not adopted.

4. Rate of Interest

Change in interest rate affect the public's demand for goods and services and thus, aggregates investment spending. A decrease in interest rates lowers the cost of borrowing, which encourages business to increase investment spending. Change in interest rate is a major rate of interest and its stability, current health of economy and future expectation of economic conditions are responsible.

6.2 METHOD OF DESCRIBING PROJECT RISKS

There are three methods for describing project risk:

i) Breakeven analysis
ii) Sensitivity analysis
iii) Scenario analysis

6.2.1 Breakeven Analysis (BEA)

Breakeven analysis in economics, business and cost accounting refers to the point in which total cost and total revenue are equal. A breakeven point analysis is used to determine the number of units or revenue needed to cover total costs (fixed and variable costs). It is the most common methodology used for economic evaluation of new investment alternatives.

Breakeven analysis for a single project

Total cost (C_T) = Fixed cost (C_F) + Variable cost (C_V)

The linear relationship assumed for the variable costs.

$$C_V = V_C \times X$$

where, V_C = Variable cost per unit output

X = Quantity of product

Hence, $C_T = C_F + V_C \times X$

The total sales revenue (S_T) during the same period is $S_T = S_P \times X$

where, S_P = Selling price per unit output.

The linear plots of the above two equations are shown in figure. The intersection point of the total sales revenue line and the total cost line is called the break-even point. The corresponding volume of production on the x-axis is known as the breakeven sales quantity. At the intersection point, the total cost is equal to the total revenue. This point is also called

the no-loss or no-gain situation. For any production quantity which is less than the break-even quantity, the total cost is more than the total revenue. Hence, the firm will be making loss. For any production quantity which is more than the break-even quantity, the total revenue will be more than the total cost. Hence, the firm will be making profit.

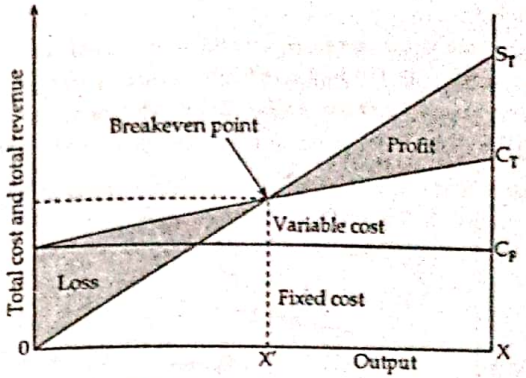


Figure: Breakeven analysis

In above figure, at breakeven output volume (X')

$$S_T = C_T$$

or, $S_P \times X' = C_F + V_C \times X'$

or, $C_F = X'(S_P - V_C)$

or, $$X' = \frac{C_F}{S_P - V_C}$$

Hence, to determine breakeven point or output volume.

$$\therefore \quad X' = \frac{C_F}{S_P - V_C}$$

The formula to find the break-even quality and breakeven sales quantity,

Break-even quantity

$$= \frac{\text{Fixed costs}}{\text{Selling price/unit} - \text{Variable cost/unit (in units)}}$$

$$= \frac{C_F}{S_P - V_C}$$

and, Break-even sales

$$= \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}} \times \text{Selling price/unit}$$

$$= \frac{C_F}{S_P - V_C} \times S_P$$

To reach a breakeven position, the selling price has to be greater than the variable cost per unit. At the breakeven point, the organization will neither make profit nor incur any loss.

Steps to determine breakeven point of common variable

- Define the common variable and its dimensional units
- Use PW or AW method to express the total cost of each alternative as a function of the common variable
- Equate the two relations and solve for the breakeven value of the variable
- If the predicted level is below than breakeven value, select alternative with the higher variable costs. If the predicted level is above than breakeven value, select the alternative with lower variable costs.

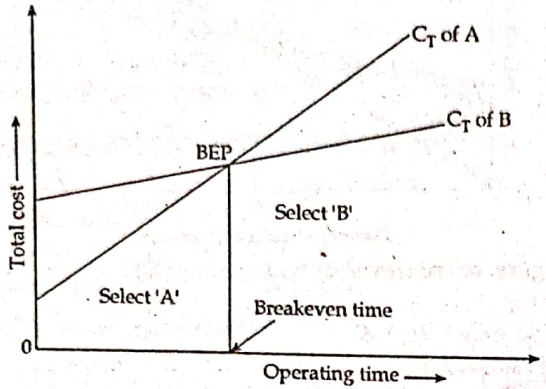


Figure: BEA and mutually exclusive project.

6.2.2 Sensitivity Analysis or "What If" Analysis

Sensitivity analysis reveals how much the NPW of a project will project change in response to a given change in one input parameter. A convenient and useful way to present the results of a sensitivity analysis is to plot sensitivity graphs. The slope of the line show how sensitive the NPW is to change in each of the input. The steeper the slope, the more sensitive the NPW is to a change in a particular variable/parameter.

In calculating cash flows, some items have a greater influence on the final result than others. In some problems, the most significant item may be easily identified. *For example*, the estimate of sales volume is often a major factor in a problem in which the quantity sold varies with the alternatives. In other problems, we may want to locate the items that have an important influence on the final results so that they can be subjected to subjected to special scrutiny.

Sensitivity analysis is sometimes called "What- if" analysis because it answers questions, such as "What if incremental sales are only 1,000 units, rather than 2,500 units? Then what will be the NPW?

Sensitivity analysis begins with a base-case situation, which is developed by using the most likely values for each input. We than change the specific variable of interest by several specified percentage points above and below the most likely value, while holding other variables constant. Next, we calculate a new NPW for each of the values obtained. A convenient and useful way to present the results of a sensitivity analysis is to plot sensitivity graphs. The slopes of the lines show how sensitive the NPW is to changes in each of the inputs. The steeper of the lines show how sensitive the NPW is to a change in a particular variable. Sensitivity graphs identify the crucial variables that affect the final outcome most.

Hence, sensitivity graph identify the crucial variables that affect the final outcome most. Graphical displays provides a useful means to communicate the relative sensitivities of the different variables to the corresponding NPW value.

6.3.3 Scenario Analysis

Although, both sensitivity and break-even analysis are useful, they have limitations. Often, it is difficult to specify precisely the relationship between a particular variable and the NPW. The relationship is further complicated by interdependencies among the variables. Holding operating costs do not behave in this manner. Yet, it may complicate the analysis too much to permit may complicate the analysis too much to permit movement in more than one variable at a time.

Scenario analysis is the process of calculating the value of a specific investments under a variety of scenarios *i.e.*, future possibilities. A more comprehensive approach to deal with uncertainty than sensitivity analysis and breakeven analysis is scenario analysis. It examines several possible situations, usually worst case, most likely case and best case.

For example, the decision maker may examine two extreme cases; a "worst-case" scenario (low unit sales, low unit price, high variable cost per unit, high fixed cost and so on) and a "best-case" scenario. The NPWs under the worst and the best conditions are then calculated and compared with the excepted or base-case, NPW.

Scenario analysis is a way of structuring thinking about the future of identifying potential problems and also of increasing preparedness handle them. It is a technique that considers that sensitivity of NPW due to change in key variables at a time and the range of likely values of those variables.

BOARD EXAM SOLVED PROBLEMS

1. What do you mean by project risk? Explain briefly about the methods of project risk management. [2017/Fall]

Answer:

There are host of external factors which may play a role in determining the outcome regarding whether the project has been successful or not. These are called project risk. Project risk is defined as, "An uncertain event or condition that if it occurs has a positive or negative effect on a project's objectives.

Method of project risk management are:

Risk management is a method of controlling the uncertainties in a project that is anything that may stop the project from achieving its goals. The aim of risk management is to minimize uncertainties and ensure that the project is delivered on time.

i) **Identify the risks**
Risk identification is the first and most important step. It involves an exchange of opinions about each step of the project.

ii) **Analyze the risks**
The analysis of identified risk is a lengthy process, but it forms the foundation of all ensuring risk management Determine what drives each of the identified risks. Readily available or specially desired software can facilitate this analysis.

iii) **Prioritize risks**
This is the process where a risk manager has to decide which risks he will take action against and those resources available to the project.

iv) **Create action plans**
Action plans begin by preventing risks or reducing the impact of identifiable risks. This ensures the use of correct materials instead of a cheap substitutes during a production process.

v) **Risk transfer**
Insurance coverage is the classic version of risk transfer. The insurance provider indemnities against any loss after an agreed deductible.

vi) **Monitor progress**
The final step is a continuous monitoring of project's starts to identify any changes in risks. Project managers should hole reviews revenues of ongoing risks and amend action or contingency plans accordingly.

2. Write the notes on Breakeven analysis. [2012/Fall, 2013/Fall, 2014/Fall]

Answer: See the topic 6.2.1.

Advantages of breakeven analysis:

- Simple to conduct and understand.
- Can cope with changing circumstances.
- Shows profit and loss at different level of output.

Disadvantages of breakeven analysis:

- Assumes that all output is sold at a given price.
- Assumes production and sales are the same.
- Breakeven charts may be time consuming to prepare.
- No semi-variable costs.
- Relies in accuracy and quality of data.

Factor affecting breakeven

Internal factors:

- Employing extra sales staff
- Price increases
- Automation replaces direct labour

External factors:

- Recession cuts demand.
- Price war forces price cut.
- Inflation pushes up direct costs.

3. Write short notes on sources of project risk. [2018/Spring]

Answer: See the topic 6.1 and 6.2.2.

Advantages of sensitivity analysis:

- It compels the decision maker to identify the variables which affect the cash flow forecasts. This helps him in understanding the investment project in totality.
- It helps to expose inappropriate forecasts and thus guides the decision maker to concentrate on relevant variables.

Disadvantages of sensitivity analysis:

- It does not provide clear cut results. The terms optimistic and pessimistic could mean different things to different people.
- It fails to focus on the interrelationship between underlying variables. *For example;* sales volume may be related to price and cost but we analyze each variable differently.

5. Define the sensitivity analysis. [2017/Spring]

Answer: See the topic 6.2.2.

6. From the following information, find how many hours/year would be the motors have to be operated at full load for annual cost to be equal? MARR = 15% per year.

Motor	A	B
Purchase cost	Rs. 1,25,000	Rs. 1,60,000
Efficiency	74%	92%
Life	10 years	10 years
Maintenance cost	Rs. 5000/year	Rs. 2500/year

Annual tax and insurance = 1.5% of investment for both motors.

Electricity cost = Rs. 5/kWhr

Power of both motors = 100 Hp. **[2011/Fall]**

Solution:

For motor A:

Capital recovery cost = 1,25,000 (A/P, 15%, 10)
= 1,25,000 × 0.1993
= Rs. 24,912.50

Maintenance cost = Rs. 5,000

Tax and insurance = 1.5% of 1,25,000 = Rs. 1,875

Operating expenses for power (electricity cost):

We know,

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}}$$

$$\therefore \quad \text{Input} = \frac{\text{Output}}{\eta}$$

Let, 'x' be the number of hours of operation per year

Operating expenses
= Input × Rate × Hours of operation

$$= \frac{\text{Output}}{\eta} \times \text{Rate} \times \text{Hours of operation}$$

$$= \frac{(100 \times 0.746)}{0.74} \times 5 \times x \qquad [\because 1 \text{ Hp} = 0.746 \text{ kW}]$$

$$= 504.054 \times x$$

Therefore, total annual cost for motor A
= Rs. (24,912.50 + 5,000 + 1,875 + 504.054 x)
= Rs. 31,787.50 + 504.054 x (1)

For motor B:

Capital recovery cost = 1,60,000 (A/P, 15%, 10)
= 1,60,000 × 0.1993
= Rs. 31,888

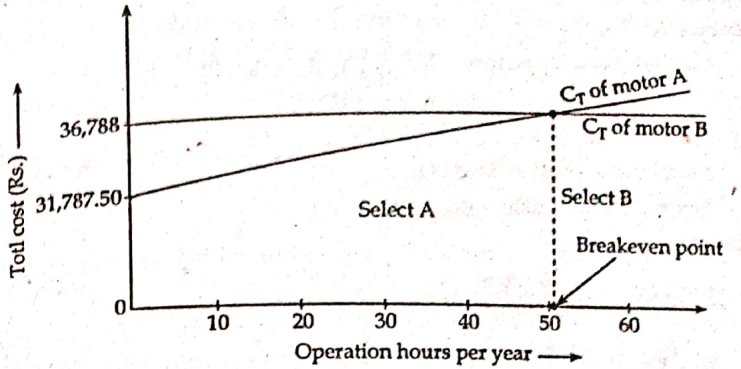
Maintenance cost = Rs. 2,500

Tax and insurance = 1.5% of 1,60,000 = Rs. 2,400

Now,

$$\text{Operating expenses} = \frac{(100 \times 0.746)}{0.92} \times 5 \times x$$

$$= 405.43x$$



Therefore, total annual cost for motor B,
= Rs. (31,888 + 2,400 + 2,500 + 405.43x)
= Rs. 36,788 + 405.43x (2)

To get breakeven point,

AW of A = AW of B

or, 31,787.50 + 504.054x = 36,788 + 405.44x

or, 98.614 x = 5,000.50

or, $x = \frac{5,000.50}{98.614}$

$\therefore$ x = 50.707 hours per year.

Hence, the motors have to be operated for 50.70 hours per year at full load for the annual costs to be equal.

7. **Following information has been obtained regarding two motors.**

Motor	A	B
Size	100 Hp	120 Hp
Cost (Rs.)	1,30,000	1,56,000
Life in year	20	20
Salvage value (Rs.)	0	0
Efficiency	89.50%	93%
Annual maintenance cost (Rs.)	8,000	250
Electricity cost (Rs.)	6/kWhr	6/kWhr

Annual tax and insurance = 2% of investment for both motors.

i) At what operating hour are they equivalent?

ii) If the motor have to be operated 55 hrs a year, which on should be selected? Take MARR = 10% per year. [2012/Fall]

Solution:

For motor A:

Capital recovery cost = 1,30,000 (A/P, 10%, 20)
= 1,30,000 × 0.1175
= Rs. 15,275

Maintenance cost = Rs. 8,000

Tax = 2% of 1,30,000 = Rs. 2,600

We know,

$$\text{Efficiency } (\eta) = \frac{\text{Output}}{\text{Input}}$$

or, $$\text{Input} = \frac{\text{Output}}{\text{Efficency}}$$

Let 'x' be the numbers of hours of operation per year.

Operating expenses
= Input × Rate × Hours of operation

$$= \frac{\text{Output}}{\eta} \times \text{Rate} \times \text{Hours of operation}$$

$$= \left(\frac{100 \times 0.746}{0.895}\right) \times 6 \times x$$

= 500.12 x

Total annual cost for motor A (AW of A)
= Rs. 15,275 + 8,000 + 0 + 2,600 + 500.12x
= Rs. 25,875 + 500.12x (1)

For motor B:

Capital recovery cost = 1,56,000 (A/P, 10%, 20)
= 1,56,000 × 0.1175
= Rs. 18,330

Maintenance cost = Rs. 250

Tax = 2% of 1,56,000 = Rs. 3,120

$$\text{Operating expenses} = \left(\frac{120 \times 0.746}{0.93}\right) \times 6 \times x = 577.55\,x$$

Total annual cost for motor B (AW of B):
= Rs. 18,330 + 250 + 3120 + 0 + 577.55x
= Rs. 21,700 + 577.55x (2)

To get breakeven point,
AW of A = AW of B

or, 25,875 + 500.12x = 21,700 + 577.55x

or, 77.43x = 4175

∴ x = 54 years

i) Hence the motors should be operated for 54 hours per year at full load for the annual costs to be equal.

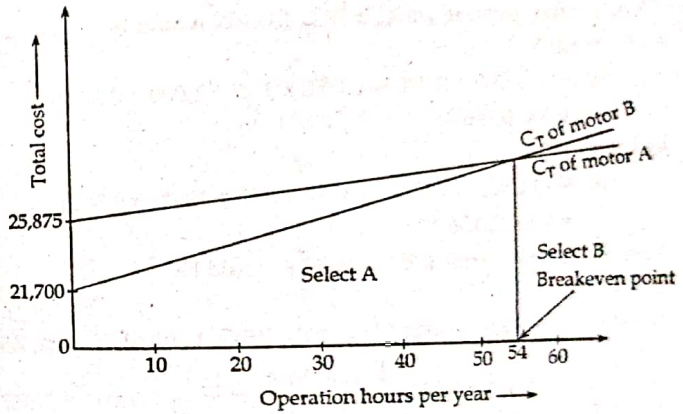
ii) If motor have to be operated 55 hours a year, then x = 55.

Now,

Total annual cost for motor A = 25,875 + 500 × x
= 25,875 + 500 × x
= Rs. 53,381.6

Total annual cost for motor B = 21,700 + 577.55 × 55
= Rs. 53,465.25

Since total annual cost for motor A is less than that of motor B, motor A should be selected. If the motor A have to be operated 55 hours a year.



8. Perform the sensitivity analysis of the following project over the range of ± 25% in (i) initial investment (ii) annual revenue (iii) useful life.

Initial investment = 11,500
Annual revenue = 3,000
Salvage value = 1,000
MARR = 10% per year
Useful life = 6 years **[2015/Fall]**

Solution:

Here;

N = 6 years
MARR = 20% per year

I = Rs. 11,500
R = Rs. 3,000
S = Rs. 1,000

We have,

$$PW(i\%) = -I + R(P/A, i\%, N) + S(P/F, i\%, N)$$
$$= -11,500 + 3,000\,(P/A, 10\%, 6) + 1,000\,(P/F, 10\%, 6)$$
$$= -11,500 + 3,000 \times 4.3552 + 1,000 \times 0.5644$$
$$= \text{Rs. } 2,130$$

i) When initial investment (I) varies ± 25%, the PW would be

At I = +25%

$$PW = -11,500 \times 1.25 + 3,000 \times 4.3552 + 1,000 \times 0.5644$$
$$= -\text{ Rs. } 745$$

At I = -25%

$$PW = -11,500 \times 0.75 + 3,000 \times 4.3552 + 1,000 \times 0.5644$$
$$= \text{Rs. } 5,005$$

ii) When annual revenue varies ± 25%, the PW would be

At AR = +25%

$$PW = -11,500 + 3,000 \times 4.3552 \times 1.25 + 1,000 \times 0.5644$$
$$= \text{Rs. } 5,396.40$$

At AR = -25%

$$PW = -11,500 + 3,000 \times 4.3552 \times 0.75 + 1,000 \times 0.5644$$
$$= -\text{ Rs. } 1,136.40$$

iii) When useful life varies ± 25%, the PW would be

At N = +25%

$$PW = -11,500 + 3,000\,(P/A, 10\%, 7.5) + 1,000\,(P/F, 10\%, 7.5)$$
$$= -11,500 + 3,000 \times \left[\frac{(1+0.10)^{7.5}-1}{0.10\,(1+0.10)^{7.5}}\right] + 1,000\,(1+0.10)^{-7.5}$$
$$= -11,500 + 3,000 \times 5.1073 + 1,000 \times 0.4892$$
$$= \text{Rs. } 4,310$$

At N = -25%

$$PW = -11,500 + 3,000\left[\frac{(1+0.10)^{4.5}-1}{0.10\,(1+0.10)^{4.5}}\right] + 100 \times (1+010)^{-4.5}$$
$$= -11,500 + 3,000 \times 3.4878 + 1,000 \times 0.65122$$
$$= -\text{ Rs. } 385.38$$

Sensitivity graph:

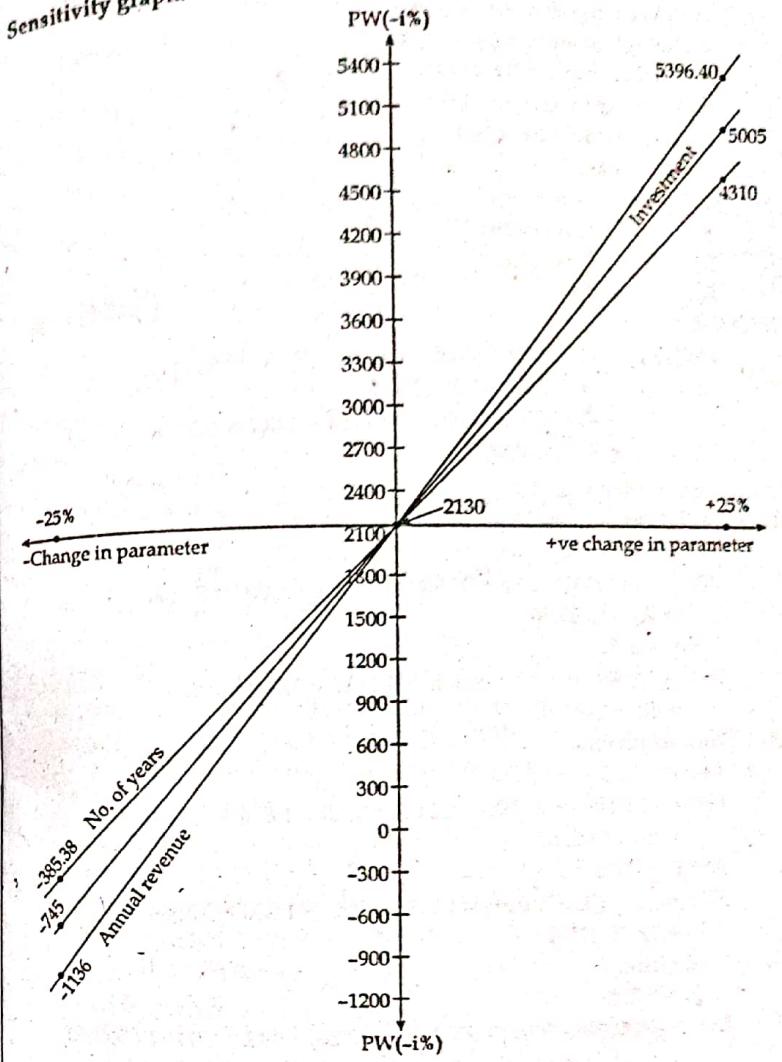


Figure: Sensitivity graph

Calculation table:

Parameters	PW (10%)		
	-25%	0	+ 25%
I	5,005	2,130	-745
AR	-1,136.40	2,130	5,396.40
N	-385.38	2,130	4,310

9. Perform the sensitivity analysing using PW method. (Choose the suitable range that you prefer).
Initial investment = Rs. 1,00,000
Annual revenue = Rs. 40,000
Annual expenses = Rs. 5,000
Salvage value = Rs. 1,000
Life = 6 years
MARR = 12% per year
i) Initial investment
ii) Annual revenue
iii) Useful life **[2015/Spring]**

Solution:

$$PW(12\%) = -1,00,000 + (40,000 - 5,000)\,(P/A, 12\%, 6) + 1,000 \times (P/F, 12\%, 6)$$
$$= -1,00,000 + 35,000 \times 4.1114 + 1,000 \times 0.5066$$
$$= \text{Rs. } 44,405.6$$

Choosing variation of ± 20%

i) Initial investment:

At I = + 20%

$$PW = -1,00,000 \times 1.20 + 35,000 \times 4.1114 + 1,000 \times 0.5066$$
$$= \text{Rs. } 24,405.60$$

At I = – 20%

$$PW = -1,00,000 \times 0.8 + 35,000 \times 4.1114 + 1,000 \times 0.5066$$
$$= \text{Rs. } 64,405.06$$

ii) Annual revenue

At AR = + 20%

$$PW = -1,00,000 + 35,000 \times 4.1114 \times 1.20 + 1,000 \times 0.5066$$
$$= \text{Rs. } 73,185.60$$

At AR = –20%

$$PW = -1,00,000 + 35,000 \times 4.1114 \times 0.80 + 1,000 \times 0.5066$$
$$= \text{Rs. } 15,625.8$$

iii) Useful life

At N = + 20%

$$PW = -1,00,000 + 35,000\,(P/A, 12\%, 7.20) + 1,000\,(P/F, 12\%, 7.20)$$
$$= -1,00,000 + 35,000\left[\frac{(1+0.12)^{7.20}-1}{0.12\,(1+0.12)^{7.2}}\right] + 1,000\,(1+0.12)^{-7.2}$$
$$= -1,00,000 + 35,000 \times 4.6483 + 1,000 \times 0.4422 = \text{Rs. } 63,143.7$$

At N = –20%

$$PW = -1,00,000 + 35,000\,(P/A, 12\%, 4.8) + 1,000\,(P/F, 12\%, 4,8)$$
$$= -1,00,000 + 35 \times \left[\frac{1.12^{4.8}}{0.12 \times 1.12^{4.8}}\right] + 1,000 \times 1.12^{-4.8}$$

$$= -1,00,000 + 35,000 \times 3.4964 + 1,000 \times 0.5804$$
$$= \text{Rs. } 22,954.40$$

Sensitivity table:

Parameters	PW (12%)		
	–20%	0	+20%
I	64,405.06	44405.6	24,405.6
AR	15625.8	44405.6	73185.4
N	22,954.40	44405.6	63,143.7

10. Analyze the sensitivity of present worth to ±40% deviation change of the project having investment = Rs. 11,500, useful life = 10 years; MARR = 10% on **[2017/Spring]**
i) Interest
ii) Life

Solution:

Here;
N = 10 years
MARR = 10%
R = Rs. 5,000
E = Rs. 2,000
I = Rs. 11,500
S = Rs. 1,000

Using PW formulation,

$$PW(i\%) = -I + (R - E)\,(P/A, i\%, N) + S(P/F, i\%, N)$$
$$W(10\%) = -11,500 + (5,000 - 2,000)\,(P/A, 10\%, 10) + 1,000\,(P/F, 10\%, 10)$$
$$= -11,500 + 3,000 \times 6.1446 + 1,000 \times 0.3855$$
$$= \text{Rs. } 7,319.3$$

i) N = ± 40%

At N = +40,

$$PW = -11,500 + 3,000\,(P/A, 10\%, 14) + 1,000\,(P/F, 10\%, 14)$$
$$= -11,500 + 3,000 \times 7.3667 + 1,000 \times 0.2633$$
$$= \text{Rs. } 10,863.40$$

At N = –40%,

$$PW = -11,500 + 3,000\,(P/A, 10\%, 6) + 1,000\,(P/F, 10\%, 6)$$
$$= -11,500 + 3,000 \times 4.3553 + 1,000 \times 0.5645$$
$$= \text{Rs. } 2,130.40$$

ii) Interest

At MARR = +40%,

$$PW = -11,500 + 3,000\,(P/A, 14\%, 10) + 1,000\,(P/F, 14\%, 10)$$
$$= -11,500 + 3,000 \times 5.216 + 1,000 \times 0.2697$$
$$= \text{Rs. } 4,417.70$$

At MARR = –40%,

PW = –11,500 + 3,000 (P/A, 6%, 10) + 1,000 (P/F, 6%, 10)

= –11,500 + 3,000 × 7.3601 + 1,000 × 0.5584 = Rs. 11,138.70

Sensitivity table:

Parameters	PW (10%)		
	–40%	0	40%
MARR	11,138.70	7,319.3	4,417.70
N	2,130.40	7,319.3	10,863.40

11. If sales = Rs. 80000, fixed cost = Rs. 15,000, variable cost = Rs. 35,000, find profit and break even volume. [2017/Spring]

Solution:

Here; Fixed costs, C_F = Rs. 15,000

Variable costs, C_V = Rs. 35,000

Sales costs, S_T = Rs. 80,000

Profit = ?

Breakeven volume = ?

We know

Let production unit = 1,000 units = Q

Total cost, $C_T = C_F + C_V$ = 15,000 + 35,000 = Rs. 50,000

Variable cost per unit, $V_C = \frac{C_V}{Q} = \frac{35,000}{1,000}$ = 35 per unit.

Selling price per unit (S_P) = $\frac{ST}{Q} = \frac{80,000}{1,000}$ = 80 per unit.

∴ Breakeven output volume = $\frac{C_F}{S_P - V_C} = \frac{15,000}{(80 - 35)}$ = 333.33 units

and, Profit = Total sales amount – Total cost

= Rs. (80,000 – 50,000)

= Rs. 30,000

12. From the following information, conduct scenario analysis based on FW formation. Assume I = 2,25,000, MARR = 13.5%, life of project is 5 years. Also give your remarks based on results of different scenarios.

Variable considered	Worst case scenario	Most likely scenario	Best case scenario
Annual sales	86,000	1,10,000	1,37,000
Annual variable costs	37,000	40,000	38,000
Annual fixed costs	21,000	20,000	18,000
Salvage value	40,000	50,000	60,000

[2018/Spring]

Solution:

I = Rs. 2,25,000

MARR = 13.5%

N = 5 years

i) **Worst case scenario**

FW(13.5%) = –2,25,000 (F/P, 13.5%, 5) + (86,000 – 37,000 – 21,000) (F/A, 13.5%, 5) + 40,000

= –2,25,000 × 1.8836 + 28,000 × 6.5448 + 40,000

= – Rs. 2,00,556.60 (loss)

ii) **Most likely scenario**

FW(13.5%) = –2,25,000 (F/P, 13.5%, 5) + (1,10,000 – 40,000 – 20,000) (F/A, 13.5%, 5) + 50,000

= –2,25,000 × 1.8836 + 50,000 × 6.5448 + 50,000

= – Rs. 46,570 (loss)

iii) **Best case scenario**

FW (13.5%) = –2,25,000 (F/P, 13.5%, 5) + (1,37,000 – 38,000 – 18,000) (F/A, 13.5%, 5) + 60,000

= Rs. 1,66,318.80 (profit)

Scenario analysis indicates that there is risk for investment in worst case and most likely scenario while there is no risk for investment in best case scenario.

13. Perform sensitivity analysis of the following over a range of –10 to + 30% in initial investment, –10 to +10% in useful life and –20% to 20% in MARR. Draw sensitivity diagram and decide the most sensitive parameter.

Initial cost	Annual income	Useful life	Salvage value	MARR
20 crore	3 crore	30 years	0	10%

[2018/Fall]

Solution:

Here; I = Rs. 20 crore

AR = Rs 3 crore

N = 30 years

S = Rs. 0

MARR = 10% per year

We know,

PW (10%) = –20,00,00,000 + 3,00,00,000 (P/A, 10%, 30)

= –20,00,00,000 + 3,00,00,000 × 9.4269

= Rs. 8,28,07,000

i) **Change in I**

If I = –10%

PW = –20,00,00,000 × 0.9 + 3,00,00,000 × 9.4269 = Rs. 1,02,807,000

If I = +10%

PW = -200000,000 × 1.1 + 3,00,00,000 × 9.4269 = Rs. 6,28,07,000

If I = -30%, PW = Rs. 14,28,07,000

If I = 30%, PW = Rs. 2,28,07,000

ii) Change in N

If N = -10%

PW = -20,00,00,000 + 3,00,00,000 (P/A, 10%, 27)

= -20,00,00,000 + 3,00,00,000 × 9.2372 = Rs. 77116000

If N = +10%

PW = -20,00,00,000 + 3,00,00,000 × (P/A, 10%, 33)

= -20,00,00,000 + 3,00,00,000 × 9.5694

= Rs. 8,70,82,000

iii) Change in MARR

If MARR declines by 20%. *i.e.*, MARR = 8%

PW(8%) = -20,00,00,000 + 3,00,00,000 × (P/A, 8%, 30)

= -20,00,00,000 + 3,00,00,000 × 11.2578

= Rs. 13,77,34,000

If MARR increases by 20%, *i.e.*, MARR = 12%

PW(12%) = -20,00,00,000 + 3,00,00,000 × (P/A, 12%, 30)

= -20,00,00,000 + 3,00,00,000 × 8.0552

= Rs. 4,16,56,000

Parameters	PW (10%)						
	-10%	-20%	-30%	0	10%	20%	30%
I	10.28	12.28	14.28	8.28	6.28	4.28	2.28
N	7.71	-	-	8.28	8.70	-	-
MARR	-	13.77	-	8.28	-	4.16	-

Hence, MARR parameter of the project is highly sensitive compare to other parameter. It is followed by investment parameter and useful life parameter respectively.

14. Explain with example why the decision criteria of present worth (*i.e.*, PW or net present value) conflicts with decision criteria of IRR and how this can be overcome? [2016/Spring]

Answer:

Net present value (NPV) is an absolute measure, *i.e.*, it represents the dollar amount of value added or lost by undertaking a project. IRR on the other hand is a relative measure *i.e.*, it is the rate of return a project offers over its life span.

NPV and IRR are two of the most widely used investment analysis and capital budgeting decision tools. Both are discounting models *i.e.*, they take into account the time value of money phenomena. However, each method has its strength and weakness and there are situations in which they do not agree on the ranking of acceptability of projects.

For example; There might be a situation in which project A has higher NPV but low IRR than project B. This NPV and IRR conflict depends on whether the projects are independent or mutually exclusive.

NPV Vs IRR conflict also arises due to different cash flow distribution. IRR inherently assume that any cash flows can be reinvested at the IRR. This assumptions is unrealistic be achieved. NPV on the other hand assumes reinvestment at the cost of capital, which is conservative and realistic.

For example; Project A requires $10 million investments and generates $10 million each in year 1 and year 2. It has NPV of $7.4 million @ 10% discount rate and IRR of 61.8%. Project B requires $1 million investment and generates @2 million in year 1 and $1 million in year 2. It's NPV @10% and IRR turn out to be $1.6 million and 141.4%. Based on NPV on would conclude that project A is better but IRR offers a contradictory view. This conflict arose mainly due to the size of the project.

Comparison of Strength and Weakness

NPV is theoretically sound because it has realistic investment assumption. It consists the cost of capital and provides a dollar value estimate of value added, which is easier to understand. Another very important feature of NPV analysis is its ability to notch the discount rate up and down to allow for different risk level of the project. However, NPV is dependent on the size of the project. IRR is not affected by the size of the project.

IRR is also easier to calculate because it does not require estimation of cost of capital or hurdle rate. It just requires initial investment and cash flow. However, this same convenience can become a disadvantage if projects are accepted without comparison to cost of capital. However, IRR's assumption of reinvestment at IRR is unrealistic and could result in inaccurate ranking of projects.

Conclusion

Whenever there is a conflict in ranking of project based on NPV and IRR, it is safer to always prefer the NPV ranking. This is due to the realistic assumption and theoretical soundness of the method. However, IRR is a greater complement to the NPV.

15. Which motor would you select if you have to operate 12 hours a day?

	Motor A	Motor B
Purchase price	Rs. 3,00,000	Rs. 4,00,0000
Capacity	2 PH	2 HP
Efficiency	75%	90%
Annual cost	Rs. 30,000	Rs. 25,500
Electricity cost	Rs. 10 per kWh	Rs. 10 per kWh
Life in years	5	7

[2019/Fall]

Solution:

Here; Time of operation = 12 hours a day.

For motor A:

Purchase price, I = Rs. 3,00,000

Capacity = 2 HP = 2 × 0.746 kW = 1.5 kW

Efficiency, η = 75%

Annual cost, C = Rs. 30,000

Electricity cost, E = Rs. 10 per kWh

Life in years, N = 5 years

Let, MARR, i = 10%

We use co-terminated assumption with salary period,

N = 7 years.

$$\text{Input power} = \frac{\text{Output}}{\eta} = \frac{1.5}{0.75} = 2 \text{ kW}$$

Now,

Electricity cost per year, E = 2 × 12 × 365 × 10 = Rs. 87,600

Now, future worth at end of year 7:

FW (10%) = [-3,00,000 (F/P, 10%, 5)
- (3,00,000 + 87,600) (F/A, 10%, 5) (F/P, 10%, 2)]
= (-3,00,000 × 1.61 - 1,17,600 × 6.11) × 1.21
= -Rs. 14,53,858.56

For motor B:

Purchase price, I = Rs. 4,00,000

Capacity = 2 HP = 1.5 kW

Efficiency, η = 90%

Annual cost, C = Rs. 25,500

Electricity cost, EC = Rs. 10 per kWh

Life in years, N = 7

$$\text{Input power} = \frac{\text{Output}}{\eta} = \frac{1.5}{0.9} = 1.67 \text{ kW}$$

Electricity cost per year, E = 1.67 × 12 × 365 × 10 = Rs. 73,146

Now, future worth at end of year 7:

FW(10%) = -4,00,000 (F/P, 10%, 7) - (25,500 + 73,146) (F/A, 10%, 7)
= -4,00,000 × 1.95 - 98,646 × 9.49
= -Rs. 17,16,150.54

Since, FW of motor A is greater than motor B, I would select motor A.

16. Flower shop keeper want to a bunch of rose on Rs. 100, the shop need to pay Rs. 10,000 for a rent and Rs. 15,000 for the helper, 98 he could sold the bunch of rose on Rs. 125. How much quantity the bunch of flowers need to sold to meet break-even point?

[2020/Fall]

Solution:

Here; Variable cost, $C_V = 100 \times D$

where, D = Quantity of rose

Fixed cost, C_F = Rs. 10,000 + Rs. 15,000 = Rs. 25,000

Total cost, $C_T = C_V + C_F$ = Rs. 100 × D + Rs. 25,000

Total revenue, TR = Rs. 125 × D

For break-even point, we have,

$TR = C_T$

or $125 \times D = 1{,}000 \times D + 25{,}000$

or $25 \times D = 25{,}000$

or $D = \frac{25{,}000}{25}$

$\therefore D = 1000$

Hence, the shopkeeper must sell 1,000 roses to meet the break-even point.

17. Perform sensitivity analysis using PW method over a range of (+ or -) 20% in

a) Initial investment

b) Net annual revenue

c) Salvage valve

Initial investment = Rs. 2,00,000

Annual revenue = Rs. 50,000

Annual expenses = Rs. 5,000

Salvage value = Rs. 25,000

Useful life = 10 years

MARR = 12% per year

Draw also sensitivity graph. **[2019/Spring]**

Solution:

Here;

Initial investment, I = Rs. 2,00,000

Annual revenues, R = Rs. 50,000

Annual expenses, E = Rs. 5,000

MARR, i% = 12% per year

Useful life, N = 10 years

Salvage value, S = Rs. 25,000

We know,

PW (i%) = -I + (R - E) (P/A, i%, N) + S (P/F, i%, N)

so, PW(12%) = -2,00,000 + (50,000 - 5,000) (P/A, 12%, 10)
+ 25,000 (P/F, 12%, 10)

$$= -2{,}00{,}000 + 45{,}000 \times \left[\frac{(1.12)^{10} - 1}{0.12 \times (1.22)^{10}}\right] + \frac{25{,}000}{(1.12)^{10}}$$

= Rs. 62,309.37

a) When initial investment (I) varies ± 20%,

At I = –20%,

$$PW = -2,00,000\,(1 - 0.2) + 45,000 \times \frac{(1.12)^{10} - 1}{0.12 \times (1.12)^{10}} + \frac{25,000}{(1.12)^{10}}$$

= Rs. 1,02,309.37

At I = + 20%,

$$PW = -2,00,000\,(1 + 0.2) + 45000 \times \frac{(1.12)^{10} - 1}{0.12 \times (1.12)^{10}} + \frac{25,000}{(1.12)^{10}}$$

= Rs. 22,309.37

b) When net annual revenue varies ± 20%,

At (R – E) = + 20%

$$PW = -2,00,000 + (1 + 0.2) \times 45,000 \times \frac{(1.12)^{10} - 1}{0.12 \times (1.12)^{10}} + \frac{25,000}{(1.12)^{10}}$$

= Rs. 1,13,161.37

At (R – E) = –20%

$$PW = -2,00,000 + 45,000 \times (1 - 0.2) \times \frac{(1.12)^{10} - 1}{0.12 \times (1.12)^{10}} + \frac{25,000}{(1.12)^{10}}$$

= Rs. 11,457.36

c) When salvage value (S) varies ± 20%,

At S = –20%

$$PW = -2,00,000 + 45,000 \times \frac{(1.12)^{10} - 1}{0.12 \times (1.12)^{10}} + \frac{2,50,000 \times (1 - 0.2)}{(1.12)^{10}}$$

= Rs. 60,699.5

At S = + 20%,

$$PW = -2,00,000 + 45,000 \times \frac{(1.12)^{10} - 1}{0.12 \times (1.12)^{10}} + \frac{2,500 \times (1 + 0.2)}{(1.12)^{10}}$$

= Rs. 63,919.23

Sensitivity table:

Parameters	PW (12%) (Rs.)		
	–20%	0	+20%
I	1,02,309.37	62,309.37	22,309.37
R – E	11,457.36	62,309.37	1,13,161.37
S	60,699.5	62,309.37	63,919.23

ADDITIONAL PROBLEMS

1. **The details of production costs and revenues of a project are as under:**
Total cost = Rs. 90,000
Fixed cost = Rs. 30,000
Sales cost = Rs. 1,10,000
Sales volume = 10,000
Find breakeven point in terms of number of units. What should be the output if the profit desired is Rs. 50,000?

Solution:

We know

Total cost (C_T) = Fixed cost (C_F) + Variable cost (C_V)

or, $C_V = C_T - C_F$

or, $C_V = 90,000 - 30,000$

∴ C_V = Rs. 60,000

Now, variable cost per unit,

$$C = \frac{60,000}{10,000} = \text{Rs. 6 per unit}$$

Selling price per unit,

$$S_P = \frac{1,10,000}{10,000} = \text{Rs. 11 per unit}$$

so,

i) $$Q_{BEP} = \frac{C_F}{(S_P - V_C)}$$

$$= \frac{30,000}{(11 - 6)}$$

= 6,000 units

ii) If the desired profit is Rs. 5,000

Profit = Total sales – Total cost

or, $50,000 = S_P \cdot Q - (C_F + C_V)$

or, $50,000 = S_P \cdot Q - (30,000 + V_C \times Q)$

or, $50,000 = S_P \cdot Q - 30,000 - V_C \cdot Q$

or, $50,000 = (S_P - V_C)Q - 30,000$

or, $80,000 = (11 - 6)Q$

or, $$Q = \frac{80,000}{5}$$

∴ Q = 16,000

Hence, required units for the profit to be Rs. 50,000 is 16,000 units.

2. **From the following information, calculate NPW for each scenario by assuming I = 1,25,000, MARR = 15% and N = 7 years.**

Variable considered	Worst case scenario	Most-likely scenario	Best-case scenario
Unit demand/yr	1,600	2,000	2,400
Unit price (Rs.)	48	50	53
Variable cost (Rs.)/unit	17	15	12
Fixed cost (Rs.)/yr	11,000	10,000	8,000
Salvage value	30,000	40,000	50,000

Solution:

i) Worst-case scenario

$$NPW(15\%) = -1,25,000 + (1,600 \times 48 - 1,600 \times 17 - 11,000) \times (P/A, 15\%, 7) + 30,000\ (P/F, 15\%, 7)$$
$$= -1,25,000 + 38,600 \times 4.1604 + 30,000 \times 0.3759$$
$$= \text{Rs. } 46,868.44 \text{ (Profit)}$$

ii) Most-likely scenario

$$NPW(15\%) = -1,25,000 + (2,000 \times 50 - 2,000 \times 15 - 1,00,000) \times (P/A, 15\%, 7) + 40,000\ (P/F, 15\%, 7)$$
$$= -1,25,000 + 60,000 \times 4.1604 + 40,000 \times 0.3759$$
$$= \text{Rs. } 1,39,660 \text{ (Profit)}$$

iii) Best-case scenario

$$NPW(15\%) = -1,25,000 + (2,400 \times 53 - 2,400 \times 12 - 8,000) \times (P/A, 15\%, 7) + 50,000\ (P/F, 15\%, 7)$$
$$= -1,25,000 + 90400 \times 4.1604 + 50,000 \times 0.3759$$
$$= \text{Rs. } 2,69,895 \text{ (Profit)}$$

Scenario analysis indicates that there is no risk for investment.

3. An estimation of a new model generator has the following information.

Purchase cost (Rs.)	3,00,000
Annual maintenance cost (Rs.)	8,000
Annual energy at full load	12,000 kW
Value of energy generated	Rs. 4/kWhr
Salvage value (Rs.)	80,000
MARR	10% per year

Determine how long will it operate for breakeven point?

Solution:

Here;

MARR = 10% per year
I = Rs. 3,00,000
S = Rs. 80,000
E = Rs. 8,000
R = 12,000 × 4 = Rs. 48,000
N = ?

We know, Using FW formulation,

$$-3,00,000\ (F/P, 10\%, N) + (48,000 - 8,000)\ (F/A, 10\%, N) + 80,000 = 0$$

or, $$-3,00,000 \times (1 + 0.10)^N + 40,000 \times \left[\frac{(1 + 0.10)^N - 1}{0.10}\right] + 80,000 = 0$$

or, $$-3,00,000 \times 1.10^N + 40,000 \times 1.10^N - 4,00,000 \times 1 + 80,000 = 0$$

or, $$1.10^N\ [4,00,000 - 3,00,000] = 3,20,000$$

or, $$(1,10)^N = \frac{3,20,000}{1,00,000}$$

or, $$(1.10)^N = 3.2$$

Taking log on both sides,

or, $$N \log (1.10) = \log (3.2)$$

or, $$N = \frac{0.5052}{0.04139}$$

∴ $$N = 12.205 \text{ years.}$$

Hence, the new model generator has to operate for 12.205 years for breakeven point.

4. After what time we can obtain the breakeven point of the following project?

Initial investment	Rs. 5,00,000
Annual revenue	Rs. 1,20,000
Annual expenses	Rs. 20,000
Interest rate	10%

Solution:

Using FW formulation

or, $$-5,00,000\ (F/P, 10\%) + (1,20,000 - 20,000)\ (F/A, 10\%, N) = 0$$

or, $$-5,00,000 \times (1 + 0.10)^N + 1,00,000 \times \left[\frac{(1 + 0.10)^N - 1}{0.10}\right] = 0$$

or, $$-5,00,000 \times (1.10)^N + 10,00,000 \times [(1.10)^N - 1] = 0$$

or, $$-5,00,000 \times 1.10^N + 10,00,000 \times 1.10^N - 10,00,000 = 0$$

or, $$1.10^N\ [10,00,000 - 5,00,000] = 10,00,000$$

or, $$1.10^N = \frac{10,00,000}{5,00,000}$$

or, $$1.10^N = 2$$

Taking log on both sides,

or, $$N \log (1.10) = \log (2)$$

or, $$N = \frac{0.3010}{0.04139}$$

∴ $$N = 7.28 \text{ years.}$$

Hence, we can obtain breakeven point on 7.28 years.

5. Perform sensitivity analysis of the following project over the range of ±40% for

i) initial investment

ii) annual revenue

iii) useful life

Draw diagram also

Initial investment = Rs. 1,00,000

Annual revenue = Rs. 40,000

Salvage revenue = Rs. 1,000

MARR = 12%

Useful life = 6 years

Solution:

Using PW formulation

PW (12%) = –1,00,000 + 40,000 (P/A, 12%, 6) + 1,000 (P/F, 12%, 6)

= –1,00,000 + 40,000 × 4.1124 + 1,000 × 0.5066

= Rs. 64,962.60

i) When initial investment (I) varies ± 40%

At I = +40%

PW = –1,00,000 × 1.4 + 40,000 × 4.1124 + 1,000 × 0.5066

= Rs. 24,962.6

At I = –40%

PW = –1,00,000 × 0.6 + 40,000 × 4.1124 + 10,000 × 0.5066

= Rs. 10,4962.60

ii) When annual revenue (AR) varies ± 40%

At AR = +40%

PW = –1,00,000- + 40,000 × 4.1114 × 1.4 + 1,000 × 0.5066

= Rs. 130745

At AR = –40%

PW, = –1,00,000 + 40,000 × 4.1114 × 0.6 + 1,000 × 0.5066

= – Rs. 819.80

iii) When useful life (N) varies ± 40%

At N = +40%

PW = –1,00,000 + 40,000 (P/A, 12%, 8.4) + 1,000 × (P/F, 12%, 8.4)

$$= -1{,}00{,}000 + 40{,}000\left[\frac{(1.12)^{8.4}-1}{0.12\times(1.12)^{8.4}}\right] + 1{,}000\times(1.12)^{-8.4}$$

= –1,00,000 + 40,000 × 5.1168 + 1,000 × 0.3859

= Rs. 1,05,057.90

At N = –40%

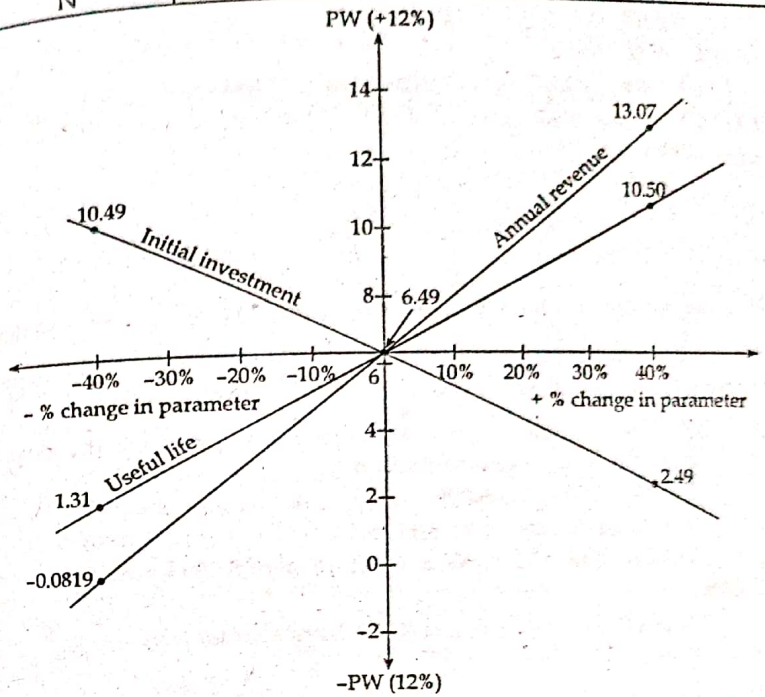
PW = –1,00,000 + 40,000 (P/A, 12%, 3.6) + 1,000 × (P/F, 12%, 3.6)

= –1,00,000 + 40,000 × 2.7917 + 1,000 × 0.665

= Rs. 13,171.70

Sensitivity table (In X 0000):

Parameters	PW (10%)		
	–40%	0	40%
I	10.49	6.49	2.49
AR	–0.081	6.49	13.07
N	1.31	6.49	10.50



From diagram, annual revenue (AR) parameter of the project is highly sensitive when compared to other parameters. *i.e.*, AR curve is more steeper than other in diagram. It is followed by investment (I) parameter and useful life (N) parameter respectively.

6. If cost of 20 watt CFL lamp is Rs. 3,000 whereas cost of 100 watt filament lamp is Rs. 40, but these bulbs have equal lighting power, which bulb do you prefer in your house? Why? Electricity cost is Rs. 15 per unit (KW-hour).

Solution:

For CFL bulb:

Initial cost = Rs. 300

Cost of use = $\frac{20}{1000} \times x \times 15 = 0.3x$

Since, 1 kW = 1,000 watt

x = Number of hours lighting the lamp

∴ Total cost = Rs. (300 + 0.3x)

For filament lamp:

Initial cost = Rs. 40

Cost of use = $\frac{100}{1,000} \times x \times 15 = 1.5x$

∴ Total cost = Rs. (40 + 1.5x)

At breakeven situation,

Total cost of CFL lamp = Total cost of filament lamp

or, $300 + 0.3x = 40 + 1.5x$

or, $260 = 1.2x$

or, $x = \frac{260}{1.2}$

∴ $x = 216$ hours

In a day, we light the bulb for 4 hours in average. Hence, $x = \frac{216}{4} = 54$ days.

Since, I have to use for more than 54 days, I prefer to use CFL lamp in my house which will be less expressive in comparison to filament lamp.

7. **Find breakeven output level and current profitability position from the following information.**

Total cost = Rs. 12,00,000

Total variable cost = Rs. 4,00,000

Total income = 15,00,000 at production of 5,000 units.

Solution:

Total cost (C_T) = Fixed cost (C_F) + Variable cost (C_V)

or, $C_F = C_T - C_V$

or, $C_F = 12,00,000 - 4,00,000$

∴ C_F = Rs. 8,00,000

Now,

Variable cost per unit, $V_C = \frac{C_V}{Q} = \frac{4,00,000}{5,000} = 80$ per unit

Selling price per unit, $S_P = \frac{S_T}{Q} = \frac{15,00,000}{5,000} = 300$ per unit

∴ Breakeven output volume = $\frac{C_F}{S_P - V_C} = \frac{8,00,000}{(300 - 80)} = 3,636.36$ units

and,

Profit = Total sales amount – Total cost

= Rs. 15,00,000 – Rs. 12,00,000

= Rs. 3,00,000

Hence, breakeven output level is 3,636.36 units and profit is Rs. 3,00,000.

Chapter 7

ECOLOGICAL LIMITS AND ECONOMIC DEVELOPMENT

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7.1 ECONOMIC THEORY AND ECOLOGICAL LIMIT

7.1.1 Economic Theory

Economic theory is a particular way of explaining economics or an economic activity and its impact on various grounds. Economic theory involves generalization which are statements of general tendencies or uniformities of relationships among various elements of economic phenomena.

Economic theory is a broad concept for the explanation and understanding of the movement of goods in a market. Most of the traditional concept of economics is based on micro behaviour by explaining effect of individual economic event. Modern economic theory

has tended to overlook about the importance of ecosystem, the use of land and environment for balancing ecological constraints that refers ecological economics.

7.1.2 Ecological Limit

The carrying capacity perspectives in particular holds that human impact on the biosphere should stay within some identifiable limits if we are to avoid abrupt shifts in its functioning and the loss of valuable life-support services. There are nine bio-physical thresholds (corresponding to domains of human influence over the earth system) that should not be transgressed. These includes biodiversity loss, climate change, biochemical flows, land system change and so on. Staying within these limits would help secure the stability of the earth system in a state as close as possible to the favourable Holocene condition which is of paramount importance for the flourishing of human civilization. Hence, the suggestion that these limits define a 'safe operating space for humanity'.

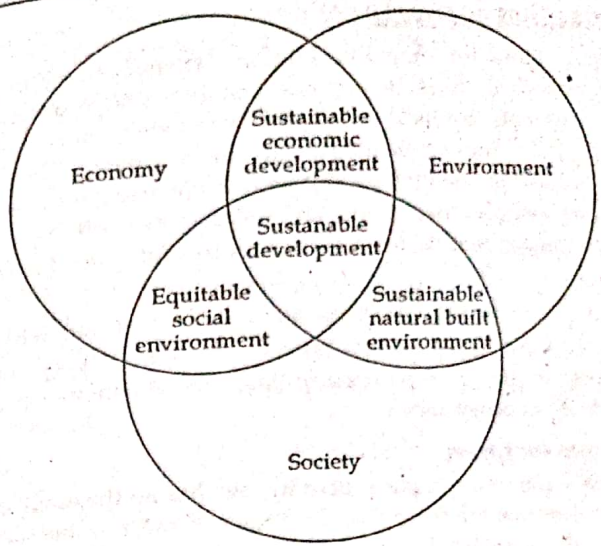
There are limits to the earth's capacity to act as a source of material inputs feeding for the economic system and sink for the wastage products. Hence functioning ecological mechanism of the earth to keep economic development with sustainable manner, there must be limiting availability of natural resources that technically refers ecological limit.

Economical constraints are limiting the availability of natural resources. In response to classical and neo-classical economics, the approaches of the two schools of thought-environmental economics and ecological economics was emerged. These two schools of thoughts are emerged in response to the challenges posed by the ecological limits on economic development. The environmental economics takes a more interdisciplinary approach of integrating ecological factors governing resources regeneration and waste absorption into the economic models.

7.2 CONCEPT OF SUSTAINABLE DEVELOPMENT

Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. Sustainable development is a process for meeting human development goals while sustaining the ability of natural systems to continue to provide the natural resources and ecosystem services upon which the economy and society depends. Sustainable development ties together concern for the carrying capacity of natural systems with the social, political and economic challenges faced by humanity.

Sustainable development is a continuous process. There is an additional focus on the present generation's responsibility to regenerate, maintain and improve planetary resources for use by future generations.



Sustainable development has continued to evolved as that of protecting the world's resources while it's true agenda is to control the world's resources. Environmentally sustainable economic growth refers to economic development that meets the needs of all without leaving future generations with fewer natural resources than those we enjoy today. The essence of this form of development is a stable relationship between human activities and the nature world, which does not diminish the prospects for future generations to enjoy a quality of life at least as good as our own.

There are 3 primary objectives of sustainable development. They are;

1. **Economic growth**

Building a strong, competitive economy by ensuring that sufficient land of the right type is available in the right places and at the right time to support growth and innovation, and indentifying then coordinating development requirement.

2. **Environmental protection**

Contributing to protecting and enhancing our natural and developed environment, while helping to improve biodiversity, use natural resources wisely, minimizing waste and pollution and adapting to and helping to decrease climate change, including a global shift to low carbon economy.

3. **Social inclusion**

Supporting strong, vibrant and health communities by providing the supply of housing required to meet the needs of present and future generations, and by creating local services that reflect the community's needs and support its health, social and culture well-being.

7.3 ECOLOGICAL FOOTPRINT

The ecological footprint measures human demand on nature *i.e.*, the quantity of nature it takes to support people or an economy. It is a measure of human impact on earth's ecosystem and reveals the dependence of the human economy on nature capital. The ecological footprint is defined as the biologically productive area needed to provide for everything people use: fruits and vegetables, fish, wood, fibers, absorption of carbondioxide from fossil fuel us and space for roads and buildings.

Bio capacity is the productive area that can regenerate what people demand from nature. Footprint and bio capacity change every year with a number of people, per person consumption, the efficiency of production and productivity of ecosystems.

Environmental footprint

The effect that a person, company, activity, etc. has no the environment is called an environmental footprint. *For example;* the number of natural resources that they use and the amount of harmful gases that they produce.

Global footprint

At a global scale, footprint assessments show how big humanity's demand is compared to what planet earth can renew.

Advantages of ecological foot prints;

i) It is a single unit that allows disaggregation of indicators.
ii) It could be widely applied to various programs/activities.
iii) It may be top down or bottom up.
iv) It created credibility as a policity tool.
v) It is useful as a means of communication policy.
vi) It is able to indicate the nature of limited natural capital.
vii) It has an ability to analyze sustainable development of a vague concept into measurable objectives.

Disadvantages of ecological footprints;

i) Ecological footprint analysis uses hypothetical land which does not represent the actual land use.
ii) It risks as having double counting.
iii) Most are portraits of consumption.
iv) If does not provide clear policy guidance except for the subtraction of consumption.

Ecological footprint standards use global hectares as a measurement unit. This makes ecological footprint globally comparable just as financial assessments. At a global scale, it is used to estimate how rapidly we are depleting natural capital. The world average ecological footprint in 2013 was 2.8 global hectares per person. The average per country ranges from

over 10 to under 1 global hectares per person. There is also a high variation within countries, based on individual lifestyle and economic possibilities.

The formula of the ecological footprint is,

$$EF = \left(\frac{P}{YN}\right) \times YF \times EQF$$

where, EF = Ecological footprint
P = Amount of a product harvested or waste emitted
YN = National average yield for P
YF = Yield factors for the country and land use type
EQF = Equivalence factor for the country and land use type

The ecological footprint, in its most basic from, is calculated using the following equation:

$$EF = \frac{D}{Y}$$

where, D = Annual demand of a product
Y = Annual yield of the same product which is expressed in global-hectares

For example;

- A person who walks or takes public transportation has a smaller footprint than someone who travels 50 km by a smart car.
- A family of 6 living in 120 m^2 area has a smaller footprint than a family of two living in 250 m^2 area.

7.4 OVERCOMING ECOLOGICAL LIMITS

If sustainable development is considered possible as per the holistic approach which takes account of the interactive relation between the human system and the ecosystem, overcoming ecological limit is possible. Technology and human values can also play a significant role in creating space for economic development by relaxing the ecological constraints. Also, delinking economic growth and the environment through dematerialization of development, decarbonization of energy, development of the renewable as alternative energy sources, recycling of wastes, etc. taking an organic view of technology and resource development also help on overcoming ecological limit.

The role of institutions in shaping the character and implementation of sustainable development by realizing the potential of such delinking of economic growth and the natural environment is important for overcoming ecological limit. Also, to overcome ecological limit, the integration of the concerned ecological factors into primarily economic models of development should be done. Future, the framework and the methodological approach to neoclassical economics should be a mended to appropriate modification required for addressing such issues.

BOARD EXAM SOLVED PROBLEMS

1. **Define ecological footprint. Explain the concept of sustainable development.** [2015/Spring, 2018/Spring, 2019/Spring]

Answer: See the topic 7.3 and 7.2.

2. **Write short notes on ecological limit and ecological footprint.** [2016/ Spring]

Answer: See the topic 7.1.2 and 7.3.

3. **Write short notes on ecological limit and sustainable development.** [2017/F, 2018/F, 2020/Fall]

Answer: See the topic 7.1.2 and 7.2.

4. **Define ecological footprint.** [2017/ Spring]

Answer: See the topic 7.3.

5. **Briefly explain about ecological limit, overcoming ecological limit and sustainable development.** [2019/Fall]

Answer: See the topic 7.1.2, 7.4 and 7.2.

6. **Write short notes on ecological footprint.** [2019/Spring]

Answer: See the topic 7.3.

CHAPTER 8

DEPRECIATION AND CORPORATE INCOME TAXES

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8.1 DEPRECIATION

8.1.1 Depreciation and its Causes

Any equipment which is purchased today will not work for ever. This may be due to wear and tear of the equipment or obsolescence of technology. Hence, it is to be replaced at the proper time for continuance of any business. The replacement of the equipment at the end of its life involves money. This must be internally generated from the earnings of the equipment. The recovery of money from the earnings of an equipment

for its replacement purpose is called depreciation fund since we make an assumption that the value of the equipment decreases with the passage of time. Thus, the word depreciation means decrease in value of any physical asset with the passage of time.

"Depreciation can be defined as a gradual decrease in utility of fixed assets with use and time" -Chan S. Park.

"Depreciation is the gradual and permanent decrease in the value of an capital assets from any cause". -R.N. Carter.

Depreciation is a process of allocation not of valuation.

8.1.1.1 Causes of Depreciation

a) Physical depreciation

Physical depreciation can be defined as the reduction in an asset's capacity to perform its intended service due to physical impairment. It can occur in any fixed assets in the form of:

i) Wear and Tear
The continuous use of an asset makes it old and decrease in working capacity and hence the value of asset depreciates.

ii) Accidental causes due to natural disaster or by manmade disaster.

iii) Deterioration from interaction with the environment, including such agents as corrosion, rotting and other chemical change.

b) Functional depreciation

Functional depreciation occurs as a result of changes in the organization or in technology that decreases or eliminates the need for an asset. It is due to

i) **Obsolescence**
An existing asset may loss its usefulness due to an improvement in technology, new invention, change in style etc. It is also called technological depreciation.

ii) **Effusion of time**
With the passage of time, the value of some asset diminishes even if they are not used in the business.

iii) **Fall in market value**
The value of an asset may decrease due to fall in market price of the assets.

8.1.2 Asset Depreciation or Economic Depreciation

Fixed assets, such as equipment and real estate, are economic resources that are acquired to provide future cash flows. Generally, depreciation can be defined as a gradual decrease in the utility of fixed assets with use and time. While this general definition does not capture the subtleties inherent in a more specific definition of depreciation, it does not provide us with a starting point for examining the variety of underlying ideas and practices.

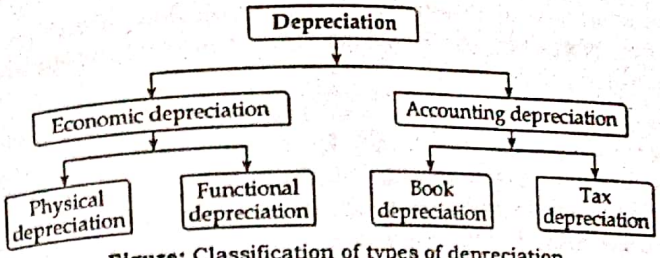
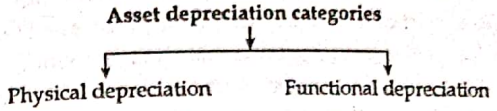


Figure: Classification of types of depreciation

Economic depreciation is a measure of the decrease in value of an asset over time. Economic depreciation is different deterioration and technological obsolescence.



We can classify depreciation into the categories of physical or functional depreciation. Physical depreciation can be defined as a reduction in an asset's capacity to perform its intended service due to physical impairment. Physical depreciation can occur in any fixed asset in the form of;

i) Deterioration from interaction with the environment, including such agents as corrosion, rotting and other chemical changes, and,

ii) Wear and tear from use. Physical depreciation leads to a decline in performance and high maintenance costs.

Functional depreciation occurs as a result of changes in the organization or in technology that decrease or eliminate the need for an asset. Examples of functional depreciation include obsolescence attributable to advances in technology, a declining need for the services performed quantity or quality demands.

Physical and functional depreciation are categories of economic depreciation. The measurement of economic depreciation does not require that an asset be sold. The market value of the asset can be closely estimated without actually testing it in the market place. The need to have a precise scheme for recording the ongoing decline in the value of an asset as a part of the accounting process leads us to an exploration of how organizations account for depreciation.

Economic depreciation = Purchase price – Current market value
= Purchase price – Salvage value
= Initial cost – Salvage value

8.1.3 Accounting Depreciation

Accounting depreciation is the systematic allocation of asset's value into book value in proportion of its total depreciable cost. Basically accounting

depreciation is used to estimate after tax cash flow of any project. It provides information to make financial statement *i.e.*, profit and loss account of any organization.

The acquisition of fixed assets is an important activity for a business organization, whether the organization is starting up or acquiring new assets to remain competitive. Like other disbursements, the cost of these fixed assets must be recorded as expenses on a firm's balance sheet and income statement. However, unlike costs such as maintenance, material and labor costs, the costs of fixed assets are not treated simply as expenses to be accounted for in the year that they are acquired. Rather, these assets are capitalized; that is, their costs are distributed by subtracting them as expenses from gross income, one part at a time over a number of periods. The systematic allocations of the initial cost of an asset in parts over a time, known as the asset's depreciable life, is what we mean by accounting depreciation. Because accounting depreciation is the business world, we sometimes refer to it more generally as asset depreciation.

Accounting depreciation is based on the matching concept: A fraction of the cost of the asset is chargeable as an expense in each of the accounting periods in which the asset provides service to the firm and each charge is meant to be a percentage of the whole cost that matches the percentage of the value utilized in the given period. The matching concept suggests that the accounting depreciation allowance generally reflects, at least to some extent, the actual economic depreciation of the asset. In engineering economic analysis, we use the concept of accounting depreciation exclusively. This is because accounting depreciation provides a basis for determining the income taxes associated with any project under taken.

8.2 DEPRECIATION METHOD

There are several methods of depreciation. Some important methods of depreciation are:

a) Straight line method
b) Declining balance method
c) Double rate declining balance method
d) Sinking fund method
e) Sum of the year method
f) Service output method
g) Straight line plus average interest method
h) Modified accelerated cost recovery system (MACRs)

a) Straight Line Method

It is also known as fixed installment method. In this method, the equal or fixed amount of depreciation is charged every year throughout the life of the asset so that the book value of asset may be reduced to zero.

$$\text{Annual depreciation} = \frac{\text{Original cost} - \text{Salvage value}}{\text{Life of the asset}}$$

$$= \frac{\text{Total depreciatin value}}{\text{Life of asset}} = \frac{I - S}{N}$$

If rate (R) is given,

$$\text{Annual depreciation} = \text{Total depreciable value} \times \frac{\text{Rate (\%)}}{100}$$

$$\therefore \quad \text{Rate of depreciation} = \frac{\text{Amont of anual depreciation}}{\text{Total depreciable value}} \times 100$$

$$= \frac{1}{N} \times 100 \qquad [\because N = \text{life at asset}]$$

and, Total depreciable value = Initial cost + All other expenses − Salvage value

Also, we can use the following formula to calculate depreciation by straight line method.

$$d_K = \frac{B - S}{N}$$

$$D_K = K\frac{(B - S)}{N}$$

$$BV_K = B - D_K$$

where, d_K = Annual depreciation in the year K, B = Original cost
S = Salvage value, N = Life of the asset
D_K = Cumulative depreciation through year K
BV_K = Book value of the end of year K.

b) Declining Balance Method or Diminishing Balance Method

The declining balance method, also known as the reducing balance method, is an accelerated depreciation method that records larger depreciation expenses during the earlier years of an asset's useful life and smaller ones in later years. Usually, this method is suitable for the assets having long life and are subject to addition and extension from time to time, such as building plant, equipment, etc. We cannot reduce book value of asset to zero under this method.

$$\text{Percentage rate, } R = 1 - \sqrt[N]{\frac{S}{B}}$$

where, R = Rate of depreciation, N = Estimated life of asset
S = Salvage value, B = Original cost

If salvage value (S) = 0, this formula cannot be applied.

Formula to calculate depreciation of Matheson method:

$$d_K = B(1 - R)^{K-1} \cdot (R)$$

$$D_K = B[1 - (1 - R)^K]$$

$$BV_K = B(1 - R)^K$$

c) Double rate Declining Balance Method

The double declining balance depreciation method is a form of accelerated depreciation that doubles the regulator declining balance approach. It is also known as the 200% declining balance method of depreciation.

$$\text{Depreciation rate, } R = \frac{100}{N} \times 2$$

As N increases, depreciation expenses decreases resulting in a situation in which depreciation is highest in the first year and then decreases over the assets depreciable life.

d) Sum of the Year (SOYD) Method

According to this method, per year depreciation charge is calculated from the ratio of the sum of the years digit for the total useful life and remaining useful life at the beginning of the particular year.

SOYD depreciation

$$= \frac{\text{Remaining useful life at the begining of particular year} \times (I - S)}{\text{SOYD for the total useful life}}$$

This methods gives larger depreciation amount during the beginning years of assets and smaller depreciation amount as assets getting old.

e) Straight Line Plus Average Interest Method

It is also called as declining balance with switchover to straight line method. According to this method, the switchover occurs in the year when larger or equal depreciation amount is obtained from the straight line method in comparison to declining balance method.

f) Sinking Fund Method

The sinking fund method is a depreciation method that provides fund for the replacement of an asset at the end of its useful life. Under depreciation fund method or sinking fund method, a fund is created with the amount of annual depreciation. This method ensures that the full capital invested in a project is recovered at the end of the project's life. According to this, the book value of asset decreases at increasing rate with respect to the life of the asset.

Let, i = Compound interest rate

A = Annual equivalent amount

To find fixed annual equivalent amount, A;

$$A = (I - S)\,(A/F, i\%, N)$$

To find net depreciation charges in the year K;

$$d_K = (I - S)\,(A/F, i\%, N) \times (F/P, i\%, K - 1)$$

To find book value at the end of year K;

$$BV_K = I - (I - S)\,(A/F, i\%, N) \times (F/A, i\%, K)$$

g) Service Output Method

This method assumes that the total depreciation that has taken place is directly proportional to the quantity of output of the properly upto that time. This method has the advantage of making the unit cost of depreciation constant and giving low depreciation expenses during periods of low production. This method is also known as unit of production method.

$$\text{Depreciation} = \frac{\text{Units of production used} \times (I - S)}{\text{Total working hours or production unit}}$$

This method is similar to straight line method except that life of the asset is estimated in terms of number of operations or number of machine hours. Such a method is useful where a company has many fixed assets with varying usage.

h) Modified Accelerated Cost Recovery System (MACRS)

The MACRS is a depreciation system which allows the capitalized cost basis of assets to be recovered over a specified life of the asset by annual dedications for valued deprecation and went into effect in 1986.

Only by declining balance method and straight line method of computing depreciation are allowed depreciation; the main system is called the general depreciation system (GDS) and the second system is called the alternative depreciation system (ADS). ADS provides a longer recovery period and used only the straight line method of depreciation. Under the MACRS, the salvage value of property is always treated as zero.

Three assumptions are important

- Salvage value are zero for all assets.
- The first and last years of the recovery period each 1/2 years.
- The declining balance rate is 200% for 3, 5, 7 and 10 year property; it is 150% for 15 and 20 year property.

For personal property; the following MACRS, recovery period apply 3, 5, 7, 10, 15 and 20 years

Property class

- 27.5 year property (Real property) – Residential rental property (homes and mobiles homes)
- 39 year property (Real property) – non-residential real property attached to the land, but not the land itself.

8.3 CORPORATE INCOME TAXES

8.3.1 Introduction to Corporate Income Tax

Corporate income tax is levied by government to organization for their taxable income. The corporate income tax allow dedications of the costs of goods sold, salaries and wages, rent, interest, advertising, depreciation amortization, depletion etc as expenses. This is an assessment levied by a government on the profits of a company. The rate of corporation income tax paid by a business varies between countries, although since corporation are legal entities distinct from their owners and operators,

they are typically taxed as if they were people. Companies use everything in the tax code to lower the cost of taxes paid by reducing taxable income.

8.3.2 Taxation Law in Nepal

Nepal has long history in the taxation. The political history of Nepal shows the existence of various forms of tax since the ancient period. "Taxes were imposed as per the Shastras, Kavtilya Nitee, Manu Smiriti, Yagya Valjy Smiriti during the ancient period. The Lichhavi rules entered Nepal around the middle of the fifth century B.C. and ruled Nepal. They imposed three forms of Karas (taxes), Bhaga-tax on agriculture, Bhoga-tax on livestock and Kara-tax on trade.

The Mallas replaced Lichhavi rulers and rules Kathmandu valley for almost three centuries from 1200 to 1228 B.C. They seem to be the first rulers who started imposing taxes on land. The subsequent Shah regime also continued the tax system of the Mallas, which was based on land and trade. After the unification of the country, different types of taxes *i.e.*, land tax, transit tax, forest product tax, mining tax and market duties were levied. For Ranas, the main source of government revenue was land tax, customs duty and excise duty. The tax system was based on a contract and Amanat, Jimmal, Mukhiya, Dittha etc were the persons who used to collect taxes.

The modern tax system, however, began only with the advent of democracy in the 1950, with an overhaul of the tax system in 1951 as one of the first steps. The first income tax was introduced by first elected government in 2016 under finance act. Business and employment tax act 2019 enacted till 2031. The income tax act 2031 replaces it, which was further replaced by income tax act 2058, which is the modern tax regulation in Nepal.

Today, the Inland Revenue Department (IRD) oversees the enforcement of tax laws and administration and also monitors the non-tax revenues such as dividends, royalties, etc. Income tax, VAT, custom duty and excise duty are the major sources of government revenue in Nepal. Besides, the IRD taxes are also collected at the local level by the local bodies as per the local government Act 1999.

At present, following tax acts are enacting in Nepal.

VAT act - 2052
Income tax act - 2058
Excise tax act - 2058
Custom tax act - 2058

8.3.3 Depreciation Rates

According to income tax act-2058, depreciation allowance rates have been granted on various assets like building, furniture, vehicles, etc., which are as follows:

Block	Assets included	Rate applied
A	Buildings, structures and similar assets of permanent nature.	5% per year on base amount.
B	Office equipments, fixture, furniture computer and data processing equipments.	25% per year on base amount.
C	Automobiles, buses, minibuses and all other transport assets.	20% per year on base amount.
D	Construction and earth moving equipments and any other depreciable assets that is not included in other blocks.	15% per year on base amount.
E	All intangible assets including software.	$\frac{\text{Cost price - Salvage value}}{\text{Useful life}}$ for each year.

For the first accounting year, the asset may be purchased at different time periods. In such cases, base amount is determined as follows:

Purchase from 1st Shrawan to 30th Poush = 100% of value.

Purchase from 1st Magh to 31st Chaitra = $\frac{2}{3}$ of value.

Purchase from 1st Baisakh to 31st Ashadh = $\frac{1}{3}$ of value.

8.3.4 Personal Tax

Personal tax is a direct tax and it is generally imposed by the central authority and collected through the local authorities. It is mostly levied on income and property in Nepal.

a) Income Tax

An income tax is a tax imposed in individuals or entities (tax payers) that varies with respective income or profits (taxable income). Income tax is computed as the product of a tax rate times taxable income. Taxation rates may vary by type or characteristics of the tax payer. The tax rate may increase as taxable income increases (referred to as the graduated or progressive rates).

Income tax rates Nepal according to 2078/079 budget:

Income Tax Rate in Nepal for Individual (Unmarried):

Annual Income	Income Tax Rate
Up to Rs. 4,00,000	1%
Additional Rs. 1,00,000	10%
Additional Rs. 2,00,000	20%
Additional Rs. 1,300,000	30%
Additional Tax Above Rs. 2,000,000	36%

Income Tax Rate in Nepal for Married Couple:

Annual Income	Tax Rate In Nepal
Up to Rs. 450,000	1%
For Additional Rs. 100,000	10%
For Additional Rs. 200,000	20%
For Additional Rs. 1,250,000	30%
Additional Tax Above Rs. 2,000,000	36%

Tax Rate in Nepal for Non-Residents:

Source of Income	Tax Rate
Normal transactions	25%
Through shipping, air or telecom services, postage, satellite, and optical fiber project	5%
Shipping, air, or telecom services through the territory of Nepal	2%
Repatriation of profit by Foreign Permanent Establishment	5%

To calculate your tax liability, add all the personal incomes and check for the slabs to find out rates. *For example;* if you are unmarried and you earn NRs. 5,50,000 per year, your tax is calculated as:

4,00,000 × 1% = Rs. 4,000 Plus
1,00,000 × 10% = Rs. 10,000 Plus
50,000 × 20% = Rs. 10,000
Total Tax = 4,000 + 10,000 + 10000 = NRs. 24,000

b) Property tax

Property tax is an annual tax on real property. Taxes are compulsory levied on the ownership of building, weapons, land, vehicles and so on. It is common throughout the world and has often been the subject of political issues. Collected property taxes are used by the governing body of the jurisdiction in which is property is located. Property tax rates and the types of properties taxed vary by jurisdiction.

In Nepal, there are two types of property tax:

- House and land tax
- Unified property tax

8.3.5 Corporate Tax

A corporate tax also called corporation tax or company tax imposed by a jurisdiction on the income or capital of corporations or analogous legal entities. Many countries impose such taxes at the national level and a similar tax may be imposed at state or local levels. Nepal government has followed book profit as the tax base for the purpose of corporate income tax.

A corporate tax is a levy placed on the profit of a firm to raise taxes. After operating earnings are calculated by dedicating expenses, including the

cost of goods sold (COGS) and depreciation from revenues, enacted tax rates are applies to generate a legal obligation the business owes the government. Rules surrounding corporate taxation vary greatly around the world and must be voted upon and approved by the government to be enacted.

Corporation are permitted to reduce taxable income by certain necessary and ordinary business expenditures. All current expenses required for the operation of the business are fully tax deductible. A corporation can deduct employee salaries, health benefits, tuition reimbursement and bonuses. In addition, a corporation can reduce its taxable income by insurance premium, travel expenses, bad debts, interest payments, sales taxes, fuel taxes and excise taxes.

Corporate Income Tax in Nepal for Fiscal year 2078/079:

Type of business	Normal tax rate	Rebate	Applicable tax rate
Normal business	25%	-	25%
Special industry under section 11 for the whole year	25%	20%	20%
Constructing and operating ropeway, cable car, railway, tunnel or sky bridge	25%	40%	15%
Constructing and operating roads, bridges, tunnel, railway, and airports	25%	50%	12.50%
Trolley bus or trams	25%	40%	15.00%
Entities with export income from a source in Nepal	25%	20%	20%
Banks and financial institutions (A, B and C class)	30%	-	30%
General Insurance (Non-life insurance)	30%	-	30%
Tobacco, alcohol, cigarette and related products	30%	-	30%
Telecom and internet services	30%	-	30%
Capital market, securities, merchant banking, commodity futures market, securities and commodity broker	30%	-	30%
Money transfer ,	30%	-	30%
Petroleum business under Nepal Petroleum Act, 2040	30%	-	30%

8.3.6 VAT (Value Added Tax)

A value added tax (VAT) is a consumption tax placed on a product whenever value is added at each stage of the supply chain, from production to the point of sale. The amount of VAT that the user pays is

the cost of the product less any of the costs of materials used in the product less any of the costs of materials used in the product that have already been taxed. It is a scientific tax system which was first introduced in 1954 A.D. in France. Now, this tax system is in over 158 countries. In Nepal, VAT has come into-consideration to replace of old indirect taxes. It was introduced on 16th November, 1997.

There are two rates of VAT: Normal VAT rate is 13% whereas some goods or services are subjected to VAT at 0%. In addition some goods or services are exempt from VAT, Suppliers of taxable goods and services are required to register under the VAT act and collect this tax. It however, not necessary for them to register if they deal with only tax exempt goods and services. Similarly, small vendors falling in below the register for VAT. Government offices should purchase taxable goods and service having price more than Rs. 5,000 from the VAT registered trader compulsory.

Objectives of VAT

- To increase in government revenue
- To reduce tax evasion practice
- To avoid cascading effect
- To make the tax system more transparent
- To make flexible tax system
- To increase in exports
- To abolish several other taxes like turnover tax, surcharge etc.
- Avoids double taxation.

Features of VAT

- VAT avoid cascading effect existed in sales tax and contains catch-up effect
- VAT is based on self assessment system and provides the facility of tax credit and tax refund.
- Wider scope
- Indirect tax levied on goods and services.
- Expert promotional due to zero VAT on expert of goods and services.

Advantages of VAT

- VAT is a natural tax so it can be imposed on all types of business.
- As the VAT is collected in small installments so the consumers has minimum burden.
- Due to catch-up effect of VAT, it minimizes avoidance.
- Compared to other tax, VAT is easy to manage.
- Huge amount of revenue is generated on a low tax rate through VAT.
- Improves economic efficiency.
- Widening of tax base and reduction in rates of tax.
- Removes the cascading of taxes.

Disadvantages of VAT

- Tax evasion is possible through fake invoices.
- Increased compliance cost as records of all purchases and sales requires to be maintained.
- VAT encourages cash transactions to avoid VAT.
- VAT strength inflation.
- It is one kind of regressive tax. (The poor pay more in comparison than the rich).
- Relatively complex to understand.

8.4 GENERAL PROCEDURE FOR MAKING AFTER TAX ECONOMIC ANALYSIS

After tax economic analysis refers to the profitability measurement of any project including all income taxes. In another word, it is the analysis of after tax cash flow estimates and profitability analysis of the project.

General procedure for ATCF estimation

i) Find gross income before depreciation expenses also known as BTCF
ii) Calculate depreciation expenses for each year
iii) Find taxable income (deduct depreciation from BTCF)
iv) Find taxes for each period.
v) Deduct tax amount from BTCF to get ATCF for each year.
vi) Find NPW or NFW considering time value of money and make economic analysis.

End of year	BTCF	Depreciation	Total taxable income	Tax amount	ATCF
K	$R_K - E_K$	d_K	$R_K - E_K - d_K$	$t(R_K - E_K - d_K)$	$(1-t)(R_K - E_K) + td_K$

where, R_K = Revenues from the project during project K.
E_K = Expenses from the project during project K.
d_K = Depreciation amount of the project during project K.
t = Income tax rate.
T_K = Income tax amount during period K.

8.5 TAXATION DETAILS

1. Introduction

In Nepal, Value Added Tax (VAT) was introduced on 16 Nov. 1997. This tax was levied in place of the Sales Tax, Hotel Tax, Contract Tax and Entertainment Tax. However, it could not be implemented fully until the FY 1998/99 due to political instability and strong opposition from the business community. VAT replaces the old Sales Tax, Contract Tax, Hotel Tax and Entertainment Tax. It has been designed to collect the same revenue as the four taxes it replaced. Since the collection of both customs duties and income tax depends, to a great extent, upon the effectiveness of VAT, it is expected to help enhance revenue collection.

(VAT) is a broad-based tax as it also covers the value added to each commodity by a firm during all stages of production and distribution. It is a modern tax system to improve the collection of taxes, to increase efficiency and to lessen tax evasion. It is also regarded as the backbone of income tax system in Nepal. The current threshold for VAT registration is Rs. 2 million. Those vendors whose annual turnover is below the threshold can, however, register voluntarily. There is a persistent increase in the number of VAT registrants. IT has crossed the 40,000 mark. At the time of conversion from the then existing sales tax to VAT, a total of 2045 taxpayers were converted as VAT registrants. As the taxpayers are increasing, the amount of revenue collection and the level of tax compliance is improving today.

2. How does it Work?

VAT is a tax imposed on the value added to goods and services consumed in Nepal or exported outside. The tax is based on the principle that each producer or distributor adds value, in some way, to the materials they have purchased and it is this added value that is taxed at each stage of the production and distribution chain. There is the presumption that VAT is shifted forward completely to the Consumer. In the VAT system, producers, distributors and people providing services raise VAT on the products or services sold or provided. The difference between the VAT collected on sales and the VAT charged on purchases determines the amount a registrant must remit or the amount that may be claimed as a refund. In other words, if the tax on sales is more than the tax on purchases, the dealer/businessman remits the difference. If the tax on sales is less than the tax on purchases, the dealer may carry forward this credit to the next month.

3. What is an input tax credit?

Registered businessmen are obliged to collect and remit VAT on their taxable transactions. These registrants are entitled to recover the tax paid on their purchases. This recovery or refund is known as input tax credit. The Input Tax Credit (ITC) is the total of the tax paid or payable on taxable goods and services purchased in Nepal, and the tax paid on taxable goods imported into Nepal. Where VAT is paid or payable by a registrant on a purchase or on imports, the registrant is allowed to claim input tax credit for those purchases made by the registrant in his commercial activities. However, even though the purchase of goods or services may relate in part to a commercial activity, in certain circumstances the purchase may not generate input tax credit entitlement when he sells goods or services which are tax-exempt.

Most registrants are entitled to claim input tax credits for the tax paid on acquisitions of capital goods for use primarily in commercial activities. Unlike the income tax deduction rules, the input tax credit for any VAT paid on capital goods is not amortized over the life of the asset. It can be

claimed in full in the period in which it is acquired. If the capital goods are later put to a non-commercial use, special change-of-use rules will apply. Taxpayers whose export is more than 50% of total sales or those who are continuously on 6 months credit may claim for refund. Refund shall be made within 30 days from the receipt of refund claim.

4. What is taxed and what is not?

VAT divides all goods and services into two basic categories, taxable and tax-exempt. Goods and services are either taxed at the standard rate of 13 percent or they are taxed at 0%. Those taxed at the standard rate include all goods and services except those which are specified as taxed at 0% or tax-exempt.

5. What is Tax-Exempt?

The purchaser will NOT pay VAT on tax-exempt goods and services and the supplier is not allowed input tax credits on purchases related to the following goods and services:

a) Goods and services of basic needs which include rice, pulses flour, fresh fish, meat, eggs, fruits, flowers, edible oil, piped water, wood fuel.

b) Basic agricultural products are also tax-exempt, for example, paddy, wheat, maize, millet, cereals and vegetables.

c) The expense of buying goods and services required to grow basic agricultural products are tax-exempt. This includes live animals, agricultural inputs including machinery, manure, fertilizer, seeds, and pesticides.

d) Social welfare services including medicine, medical services, veterinary services and educational services.

e) Goods made for the use of disabled persons.

f) Air Transport.

g) Educational and cultural goods and services such as books and other printed materials, radio and television transmissions, artistic goods, cultural programmes, non-professional sporting events and admissions to educational and cultural facilities.

h) Personal services are also tax-exempt. These are services provided, for example, by actors and other entertainers, sportsmen, writers, translators and manpower supplies agents.

i) Exemption from VAT is also extended to the purchase and renting of land and buildings.

j) Financial and insurance services.

k) Postage and revenue stamps, bank notes, cheque books.

6. Are Imported Goods Taxed?

The VAT Act, Schedule I lists imports which are tax-exempt. Some of these include prescription drugs, basic groceries, medical devices and agricultural products. Most imports, however, are fully taxable at customs

point. Thereafter these are treated on the same basis as domestically produced goods. The VAT on imported goods is collected by Customs. It is calculated on the dutiable value of the goods, in other words, on the value of the goods including transportation, insurance, freight and commissions PLUS any duty or other taxes (other than VAT) payable on the goods. The value for the duty of the goods is determined in accordance with the valuation provisions contained in the Customs Act.

Registrants may claim input tax credit for the VAT paid on imported goods used in their commercial activities.

7. Are Exports Taxed?

VAT is applicable only to the consumption of goods and services in Nepal. However, supplies made in Nepal that are exported are taxable at 0%. Exporters are allowed to claim input tax credits for VAT paid or payable on purchase of goods and services relating to their commercial activities. Exports taxed at 0% include exports of both goods and services.

8. Who must register?

Registration is required for any business:

a) With an annual taxable turnover of more than 2 million rupees.

b) Belonging to a conglomerate which has an aggregate annual taxable turnover exceeding 2 million rupees.

9. Who is affected by VAT?

In addition to consumers, persons involved in commercial activities are affected by VAT. A person means an individual, firm, company, association, cooperative, institution, joint business, partnership, trust, government body or religious organization.

There are categories of persons and organizations which are not required to collect VAT nor allowed to claim a refund of the VAT they have paid in producing their goods and services for sale. These would include unregistered small suppliers, that is, persons with annual sales of taxable goods and services of Rs. 2 million or less. But even such people can voluntarily register for VAT purposes.

10. What factors are taken into account when determining annual sales of taxable goods and services?

In assessing the value of taxable supplies a vendor must include the value of the supplies taxed at the standard VAT rate and his sales of supplies sold at 0% (the zero rated supplies). Sales of exempt supplies will not be included. If he has had or expects an abnormally large sale he should contact his IRO to determine if this sale should be taken into account when calculating his need to register.

11. Can Small Businesses Register?

Businesses with taxable annual sales of under 2 million rupees may apply to register. If a business chooses to register, it must remain registered for a full fiscal year.

12. Who May Apply to Cancel Their Registration?

A VAT registration may be cancelled by anyone whose total taxable sales for four consecutive calendar quarters is not more than 2 million rupees and who has been registered for a full fiscal year or by persons who no longer has a commercial activity because of bankruptcy, receivership, or cessation of the business.

13. What are the obligations of VAT registrants?

VAT registrants are required to:

a) Submit VAT return and pay tax within the 25th day of the following month.

b) Provide their customers with a tax invoice.

c) Maintain Purchase Book, Sales Book, VAT Account.

d) Keep their VAT records for a period of 6 years.

e) Inform the IRO of changes to the business including new address, telephone number or a reorganization of a partnership within 15 days.

f) Put their Certificate of Registration in the premises where customers may easily see and read it.

g) Allow tax officers to enter the business to examine the business records and the stock on hand.

14. What is a taxpayer required to put on his tax invoice?

The tax invoice will require the name and address of the seller and the purchaser, the seller's PAN number and invoice number, the date of the transaction and a description of the sale including the number of items purchased, the unit cost of each item and a mention of any discounts given.

The tax invoice must be prepared in three copies and the first copy should be clearly identified as a tax invoice. The original copy is to be given to the purchaser, the second copy is to be retained for audit purposes while the bottom copy is for use by the seller in preparing a record of the transaction.

15. What is an abbreviated invoice?

Tax Officers may grant permission for a VAT registrant to issue an abbreviated invoice for retail sales below the value of Rs. 5000. The chief difference between the two tax invoices is that an abbreviated invoice does not require the name and address of the purchaser. The registrants have the right to request a detailed tax invoice as they will not be able to claim input tax credits with abbreviated invoices. IRD may order taxpayer to issue invoices by using cash machine or computer. The procedure in such case shall be as prescribed by the DG of IRD.

IRD will have anytime access to the database of the taxpayer.

16. Is it necessary to maintain a special accounting system?

Most businesses will require only minor modifications to their record keeping. In order to complete his VAT return a taxpayer will need to ensure that his books and records provide:

a) The amount of VAT paid on purchases.
b) The amount of VAT collected on sales.
c) A method of distinguishing between taxable and exempt sales.
d) The time the goods and services were supplied.
e) Evidences that goods were exported, if any.

17. What books and records must be kept?

A taxpayer must keep the following books and records:

a) A purchase book
b) A sales book, and
c) A VAT accounts.

18. What is to be included in the purchase and sales book?

Purchase and sales books include:

a) The invoice number.
b) The invoice date.
c) The supplier's name and PAN number in the purchase book.
d) The customer's name and PAN number in the sales book.
e) The taxable value, and
f) The amount of VAT.

Businesses which sell both taxable and exempt goods will need to complete additional columns of information to separate exempt sales and the purchases related to them. IROs will be pleased to provide taxpayers with a sample Purchase Book (Schedule 8) and a sample Sales Book (Schedule 9) format. PAN number refers to the Taxpayer Identification Number which will be allocated to each registrant after completion of the registration process.

19. What is the VAT account?

The VAT account is a monthly summary showing the source of the figures used in the VAT return. This account contains purchases and sales and the VAT spent and collected. A sample VAT Account (Schedule 7) may be obtained from the concerned IRO.

20. Offences

Fines will be imposed if the taxpayer fails to file his return within the specified time. A registrant will make his VAT payment at a bank where he will receive a bank voucher as proof of payment. This number is to be entered on his VAT return to be submitted within 25 days after the end of the month of business transaction.

The VAT Act imposes fines for failing to register. Similarly, if a registrant fails to use his registration number or clearly display his registration certificate in the business premises, he is liable to fine.

Other penalties may be imposed if, for example, a registrant fails to file a return, issue invoices, keep an up-to-date account of transactions, obstructs visits by a tax officer in investigation, prepares false accounts and invoices or attempts to evade tax. Similarly, IRO/IRD may purchase or cause to purchase under invoiced goods.

21. Administrative Review

A taxpayer who is not satisfied with the Tax Assessment of a Tax Officer may submit an application to the DG of IRD for administrative review within 30 days from the time of receiving such decision.

22. Tax Plate

In order to make public the status of the registered taxpayer automatically, each registered taxpayer is required to display the tax plate which is visible from outside in the premises of the taxpayer. The size and the colour of the tax plate should be as follows:

A. Size

The size of the tax plate should be 30 cm long and 10 cm width.

B. Information to be shown on the tax plate:

i) Permanent Account Number (PAN)
ii) Taxpayer's Name.

C. Colour

The tax status of the taxpayer will be identified on the basis of the following background and the colour of the tax plate:

i) Bush green background and letters in white colour: VAT registered tax payers.
ii) Lemon yellow background and letters in black colour: Tax payers dealing in VATable goods and services but whose transaction is the below threshold *i.e.*, Rs. 2 million per year.
iii) Signal red background and letters in red colour: Tax payers dealing in non-VATable goods and services.

BOARD EXAM SOLVED PROBLEMS

1. Write short notes on VAT. [2011/Fall, 2012/Spring, 2013/Spring, 2013/Fall, 2014/Spring, 2014/Fall, 2015/Fall, 2016/Fall, 2019/Fall]

Answer: See the topic 8.3.6.

2. Write short notes on personal tax and corporate tax. [2011/Fall 2013/Fall, 2014/Spring, 2014/Fall, 2018/Spring, 2019/Spring]

Answer: See the topic 8.3.5 and 8.3.4.

3. Write short notes on taxation structures in Nepal. [2013/Spring]

Answer:

Nepal's tax structure is composed of three categories of revenues. These are direct taxes, indirect taxes and non-taxes. Direct tax include income tax, property tax, house rent tax, loan tax, corporate tax, interest tax, etc. and indirect tax includes VAT, custom duties, excise duties; tax is a compulsory contribution or payment to government by the people and institute without any expectation of direct benefit.

a) Direct tax

Direct tax is levied on individual persons or corporations and the tax burden ultimately lies on the tax payer. These taxes are also known as personal tax. A tax is paid to be direct when impact and incidence of a tax are on one and some person. The tax rates are determined by parliament through government budget in Nepal.

b) Indirect tax

Indirect tax is imposed on commodities and allows the tax burden to shift. The consumers are the ultimate payers of the tax through it is imposed on the producer and sellers. These taxes are also known as impersonal taxes. Indirect tax is a major sources of tax revenue in Nepal. Indirect taxes promotes inflation. It is of regressive in nature.

4. Write short notes on types of depreciation. [2014/Spring]

Answer: See the topic 8.1.2 and 8.1.3.

5. Write short notes on direct and indirect tax. [2015/Spring]

Answer:

S.N.	Direct tax	Indirect tax
i)	A tax is said to be direct when impact and incident of tax are on one and same person.	If impact of tax is on one person and incidence on the another, then that tax is called indirect tax.
ii)	Direct tax is imposed on the individual organization and burden of tax cannot be shifted to others.	Indirect tax is imposed on commodities and allows the tax burden to shift.
iii)	Direct taxes are lesser burden than indirect taxes to people as direct taxes are based on income earning ability of people.	Indirect taxes are borne by the consumer of commodities and service irrespective of financial ability as the MRP includes all taxes.

iv)	Tax evasion is possible.	Tax evasion is hardly possible because it is included in the price of goods and services.
v)	Direct tax helps in reducing inflation.	Indirect taxes promotes inflation.
vi)	Its nature is progressive.	Its nature is regressive.
vii)	It is levied on persons, company, firm.	It is levied on consumers of goods and services.
viii)	It includes income tax, wealth tax, property tax, corporate tax, import and export duties.	It includes VAT, service tax, exercise duty, custom duty, security transaction tax, etc.

Merits of direct tax are:

- equality
- elastic
- progressive
- civic consciousness
- certainty
- economy
- anti-inflationary

Demerits of direct tax are

- lack of inspiration
- arbitary
- possibility of evasion
- unpopular
- inconvenient

Merits of indirect tax are:

- convenient
- difficulty of evasion
- wider scope
- promote social welfare

Demerits of indirect tax are

- inequality
- unproductive
- uncertainty
- inflation

6. Write short notes on causes of depreciation. [2015/Fall, 2018/Spring, 2014/Fall]

Answer: See the topic 8.1.1.1.

7. What is value added tax? Explain briefly the effectiveness of VAT in Nepal. [2012/Fall]

Answer:

A value added tax (VAT) is a consumption tax placed on a project whenever value is added at each stage of the supply chain, from production to the point of sale. Also, VAT is a consumption tax levied on products at every point of sale where value has been added, starting from raw materials and going all the way to final retail purchase.

For *example*; If a product cost Rs. 1,000 and there is a 13% VAT, the consumer pays Rs. 1,130 to merchant. The merchant keeps Rs. 1,000 and remits Rs. 130 to the government.

Effectiveness of VAT in Nepal:

VAT implementation is seen extremely challenging in Nepal. Resistance from the business community, ignorance of general people, lack of full support and commitments from the politicians and government officials forced the authority responsible for implementing VAT to make compromises on various aspects of VAT which has weekend the process of its implementation right from the beginning. The attitude of businessman and tax administration also appears hostile to the effective implementation of VAT in Nepal. The culture of doing business without maintaining proper books of accounts or maintaining multiple sets of books of accounts have of accounts have made implementation of VAT difficult. Due to traction, the auditing system, one of the most important aspects of VAT operation, is not effective.

The existing large amount of unauthorized trade with India has been posing a great threat for proper implementation of VAT. The illegal import is helping to black market channel resulting in a large scale tax evasion; the scope for illegal trade and tax evasion has not been decreased even after the implementation of VAT because tax administration is not strong and efficient enough to check this situation. No billing, lack of invoicing, incorrect value in billings is the main problems observed in invoicing system, leading weak VAT implementation. There is a high level of corruption and tax evasion by the means of illegal alliance between tax payer and tax administration. One of the best feature of VAT is the catch up effect which makes tax evasion impossible but this effect is not achieved because of illegal trade, undervalued transactions without invoices and lack of administrative capabilities to catch and destroy the illegal channels.

8. Define depreciation. **[2012/Fall]**

Answer: See the topic 8.1.1.

9. Define tax. **[2014/Spring]**

Answer:

A tax is a mandatory financial charge or some other type of levy imposed upon a taxpayer by a government in order to fund various public expenditures. A failure to pay, along with evasion of or resistance to taxation is punishable by law. Taxes are imposed upon all those people who are able to pay or earn who owns property, who does consume or expenses. It is not any fee or fine or price under taken by the government.

10. A machine costing of Rs. 1,00,000 is estimated to have life of 10 years. The salvage value of the machine at the end of life is Rs. 20000. Find deprecation charge and book value of each year and tabulate it. Use straight line method and sum of years digit (SOYD) method. **[2012/Fall, 2016/Fall]**

Solution:

Here;

Original cost = Rs. 1,00,000

Salvage value = Rs. 20,000

Estimated life = 10 year

We know

i) **Straight line method:**

$$\therefore \text{Annual depreciation} = \frac{\text{Original cost} - \text{Salvage value}}{\text{Estimted life}}$$

$$= \frac{1,00,000 - 20,000}{10}$$

= Rs. 8,000 per year.

$$\therefore \text{Rate of depreciation} = \frac{\text{Annual depreciation}}{\text{Total depreciation value}} \times 100$$

$$= \frac{8,000}{(1,00,000 - 20,000)} \times 100$$

= 10% per year

[∴ Total depreciable value = original cost + all other expenses – salvages value]

Year	Depreciation (Rs.)	Book Value (Rs.)
1	8,000	1,00,000 – 8,000 = 92,000
2	8,000	92,000 – 8,000 = 84,000
3	8,000	84,000 – 8,000 = 76,000
4	8,000	76,000 – 8,000 = 68,000
5	8,000	68,000 – 8,000 = 60,000
6	8,000	60,000 – 8,000 = 52,000
7	8,000	52,000 – 8,000 = 44,000
8	8,000	44,000 – 8,000 = 36,000
9	8,000	36,000 – 8,000 = 28,000
10	8,000	28,000 – 8,000 = 20,000

ii) **Sum of years digit (SOYD) method**

I = Rs. 100000

S = Rs. 20000

N = 10 years

Total depreciable value = I – S = Rs. (1,00,000 – 20,000) = Rs. 80,000

$$\text{Sum of the year digit} = N\frac{(N+1)}{2} = 10\frac{(10+1)}{2} = 55$$

Proportion of the year = 10 : 9 : 8 : 7 : 6 : 5 : 4 : 3 : 2 : 1

$$\text{SOYD depreciation} = \frac{\text{Remaining useful life}}{\text{Total useful life}} \times (I - S)$$

Book value = I – Accumulated depreciation

Year	Depreciation (Rs.)	Book Value (Rs.)
1	$\frac{10}{55} \times 80{,}000 = 14{,}545.45$	85,454.55
2	$\frac{9}{55} \times 80{,}000 = 13{,}090.90$	72,363.65
3	$\frac{8}{55} \times 80{,}000 = 11{,}636.36$	60,727.29
4	$\frac{7}{55} \times 80{,}000 = 10{,}181.81$	50,545.48
5	$\frac{6}{55} \times 80{,}000 = 8{,}727.27$	41,818.21
6	$\frac{5}{55} \times 80{,}000 = 7{,}272.72$	34,545.49
7	$\frac{4}{55} \times 80{,}000 = 5{,}818.18$	28,727.31
8	$\frac{3}{55} \times 80{,}000 = 4{,}363.63$	24,363.68
9	$\frac{2}{55} \times 80{,}000 = 2{,}909.09$	21,454.59
10	$\frac{1}{55} \times 80{,}000 = 1{,}454.54$	20,000
Total	Rs. 80,000	

Hence, depreciable value = Rs. 80,000

Book value = Rs. 20,000

11. A company has purchased equipment whose first cost is Rs. 10,000 with an estimated life of 5 years. The estimated salvage of the equipment at the end of its life time is Rs. 2,000. Determine the depreciation charge and book value at 3 and 4 years using the straight line and sum of years digit (SOYD) method of depreciation. [2018/Spring]

Solution:

Original cost B = Rs. 10,000

Salvage value = Rs. 2,000

Estimate life, N = 5 years 2,000

i) Straight line method

$$\text{Annual depreciation} = \frac{B - S}{N} = \frac{10{,}000 - 2{,}000}{5} = \text{Rs. } 1{,}600 \text{ per year.}$$

Cumulative depreciation throughout the 3rd year:

$$D_3 = 3\frac{(10{,}000 - 2{,}000)}{5} = \text{Rs. } 4{,}800$$

∴ Book value at the 3rd year,

BV_3 = Rs. (10,000 – 4,800) = Rs. 5,200

∴ Cumulative depreciation throughout the 4th year:

$$D_4 = 4\frac{(10{,}000 - 2{,}000)}{5} = \text{Rs. } 6{,}400$$

∴ Book value at the 4th year,

BV_4 = Rs. (10,000 – 6,400) = Rs. 3,600

Another method:

$$\text{Annual depreciation} = \frac{B - S}{N} = \frac{10{,}000 - 2{,}000}{5} = \text{Rs. } 1{,}600 \text{ per year}$$

Now,

EOY	Depreciation charge	Book value
0	–	10,000
1	1,600	8,400
2	1,600	6,800
3	1,600	5,200
4	1,600	3,600
5	1,600	2,000

For year 3,

∴ Depreciation charge = Rs. 1,600

Book value = Rs. 5,200

For years 4,

∴ Depreciation charge = Rs. 1,600

Book value = Rs. 3,600

ii) SOYD method:

N = 5 years

Sum of the year digit = 5 + 4 + 3 + 2 + 1 = 15

or, $N\frac{(N+1)}{2} = 5\frac{(5+1)}{2} = 15$

Proportion of year = 5 : 4 : 3 : 2 : 1

Depreciation charges:

Year	Depreciation (Rs.)	Book Value (Rs.)
1	$\frac{5}{15} \times (10{,}000 - 2{,}000) = 2{,}666.67$	7,333.33
2	$\frac{4}{15} \times 8{,}000 = 2{,}133.33$	5,200
3	$\frac{3}{15} \times 8{,}000 = 1{,}600$	3,600
4	$\frac{2}{15} \times 8{,}000 = 1{,}066.66$	2,533.34
5	$\frac{1}{15} \times 8{,}000 = 533.34$	2,000
Total	Rs. 8,000	

∴ Depreciation charge at 3rd year = Rs. 1,600

∴ Depreciation charge at 4th year = Rs. 1,066.67

∴ Book value at 3rd year = Rs. 3,600

∴ Book value at 4th year = Rs. 2,533.34

12. A company has purchased an equipment whose first cost is Rs. 100,000 with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Determine the depreciation using:

i) Double declining methods of deprecation

ii) SOYD method **[2013/Spring, 2013/Fall, 2017/Spring]**

Solution:

Original cost, B = Rs. 1,00,000

Salvage value, S = Rs. 20,000

Estimated life, N = 8 years

i) Double declining method or 200% declining method

Double declining rate

$$R = \frac{1}{N} \times 100 \times 2 = \frac{1}{8} \times 100 \times 2$$

∴ R = 25%

so, Depreciation charge = $0.25 \times BV_{n-1}$

where, BV = Book value of previous EOY.

EOY	Depreciation charge	Book value
0	–	1,00,000
1	0.25 × 1,00000 = 25000	75000
2	0.25 × 75000 = 18750	56250
3	0.25 × 56250 = 14062.5	42187.5
4	0.25 × 42187.5 = 10546.875	31640.625
5	0.25 × 31640.625 = 7910.156	23730.469
6	0.25 × 23730.469 = 5932.617	17797.852
7	0.25 × 17797.852 = 4449.463	13348.389
8	0.25 × 13348.389 = 3337.097	10011.292
Total = 89988.708		

ii) SOYD method:

$$SOYD = N\frac{(N+1)}{2} = 8\frac{(8+1)}{2} = 36$$

$$\text{SOYD depreciation} = \frac{\text{Remaining useful life}}{\text{Total useful life}} \times (B - S)$$

Depreciation charge and book value:

Year	SOYD Depreciation	Book Value
1	$\frac{8}{36} \times 80{,}000 = 17{,}777.78$	82,222.22
2	$\frac{7}{36} \times 80{,}000 = 15{,}555.56$	66,666.66
3	$\frac{6}{36} \times 80{,}000 = 13{,}333.33$	53,333.33
4	$\frac{5}{36} \times 80{,}000 = 11{,}111.11$	42,222.22
5	$\frac{4}{36} \times 80{,}000 = 8{,}888.89$	33,333.33
6	$\frac{3}{36} \times 80{,}000 = 6{,}666.66$	26,666.67
7	$\frac{2}{36} \times 80{,}000 = 4{,}444.44$	22,222.23
8	$\frac{1}{36} \times 80{,}000 = 2{,}222.23$	20,000
Total	Rs. 80,000	

Hence, total depreciable amount is Rs. 80,000 and book value is Rs. 20,000 at the end of 8th year.

13. A machine is costing of Rs. 5,00,000 with estimated salvage value of Rs. 50,000 at the end of 5th year. Find deprecation amount by,

i) Straight line method for each year

ii) Declining balance methods for each year **[2015/Fall]**

Solution:

B = Rs. 5,00,000

S = Rs. 5,000

N = 5 years

i) Straight line method:

$$\text{Annual depreciation} = \frac{B - S}{N} = \frac{5{,}00{,}000 - 50{,}000}{5}$$

= Rs. 90,000 per year

$$\text{Depreciation rate} = \frac{90{,}000}{(5{,}00{,}000 - 50{,}000)} \times 1{,}000 = 20\% \text{ per year}$$

Year	Depreciation	Book Value
1	90,000	5,00,000 – 90,000 = 4,10,000
2	90,000	4,10,000 – 90,000 = 3,20,000
3	90,000	3,20,000 – 90,000 = 2,30,000
4	90,000	2,30,000 – 90,000 = 1,40,000
5	90,000	1,40,000 – 90,000 = 5,00,00

ii) **Declining balance method:**

Rate of deprecation = $1 - \sqrt[N]{\frac{S}{B}} = 1 - \sqrt[5]{\frac{50,000}{5,00,000}} = 1 - 0.6309 = 37\%$

EOY		Book value
0	-	5,00,000
1	0.37 × 5,00,000 = 1,85,000	3,15,000
2	0.37 × 3,15,000 = 1,16,550	1,98,450
3	0.37 × 1,98,450 = 73,426.5	1,25,023.5
4	0.37 × 1,25,023.5 = 46,258.7	78,764.8
5	0.37 × 78,764.8 = 29,142.98	49,621.82

14. Consider the following accounting information for a computer system:
Cost basis of the assets, I = Rs. 10,000
Useful life, N = 5 years
Estimated salvage life, S = 0
Use double declining balance method to compute annual depreciation allowances and resulting book values. [2015/Spring]

Solution:

I = Rs. 10,000
N = 5 years
S = Rs. 0

Using double declining balance depreciation method,

Rate = $\frac{1}{N} \times 100 \times 2 = \frac{1}{5} \times 100 \times 2 = 40\%$

Year	Book value (Rs.)	Depreciation amount (Rs.)
1	10000	0.40 × 10,000 = 4000
2	10,000 – 4,000 = 6000	0.40 × 6,000 = 2400
3	6,000 – 2400 = 3600	0.40 × 3600 = 1440
4	3600 – 1440 = 2160	0.40 × 2160 = 864
5	2160 – 864 = 1296	0.40 × 1296 = 518.4
	Total	Rs. 9222.4

Total depreciated amount at the end of 5th year is Rs. 9,222.40.

15. Consider following accounting information for a computer system.
Cost basis of the asset = Rs. 40,000
Useful life = 5 years
Estimated salvage value = Rs. 2,500
Compute annual depreciation and resulting book value using double declining balance method. [2014/Spring]

Solution:

I = Rs. 40,000 N = 5 year
S = Rs. 2,500

Rate of depreciation = $\frac{1}{N} \times 100 \times 2 = \frac{1}{5} \times 100 \times 2 = 40\%$

Year	Book value (Rs.)	Depreciation amount @ 40%
1	40,000	0.40 × 40,000 = 16,000
2	24,000	0.40 × 24,000 = 9,600
3	14,400	0.40 × 14,400 = 5,760
4	8,640	0.40 × 8,640 = 3,456
5	5,184	0.40 × 5,184 = 2,073.6
	Total	Rs. 36,889.60

Book value = Rs. 5,184
Total depreciation amount = Rs. 36,889.60

16. If a machine costing is Rs 4,00,000 is estimated 10 years useful life and Rs. 50,000 salvage value. Find depreciation amount for each year by using declining balance and sinking fund methods. [2014/Fall]

Solution:

Here; Machine cost, B = Rs. 4,00,000
Salvage value, S = Rs. 50,000
Estimated life, N = 10 years
Let, MARR = 10% per year

i) **Declining balance method:**

Rate of depreciation = $1 - \sqrt[N]{\frac{S}{B}} = 1 - \sqrt[10]{\frac{50,000}{4,00,000}} = 1 - 0.812 = 19\%$

Year	Book value	19% declining balancing depreciation
1	4,00,000	76,000
2	3,24,000	61,560
3	2,62,440	49,863.60
4	2,12,576.40	40,389.51
5	1,72,186.88	32,715.50
6	1,39,471.37	26,499.56
7	1,12,971.80	21,464.643
8	91,506.35	17,386.207
9	74,120.14	14,082.82
10	60,037.32	11,407
	Total	Rs. 3,51,368.84

Hence, depreciation amount is Rs. 3,51,368.84 at the end of 10th year.

ii) **Sinking fund method:**

Sinking fund depreciation = (B – S) (A/F, i%, N)
= (4,00,000 – 50,000) (A/F, 10%, 10)
= 35,00,000 × 0.0627
= Rs. 21,945 per year

Year	Depreciation with MARR = 10%	Cumulative depreciation	Book value (I–cumulative depreciation)
1	21,945	21,945	3,78,055
2	21,945 × 1.1 = 24,139.5	46,084.5	3,53,915.5
3	24,139.5 × 1.1 = 26,553.45	72,637.95	3,27,362.05
4	26,553.45 × 1.1 = 29,208.8	1,01,846.8	2,98,153.26
5	29,208.8 × 1.1 = 32,129.68	1,33,976.48	2,66,023.52
6	32,129.68 × 1.1 = 35,342.7	1,69,319.18	2,30,680.82
7	35,342.7 × 1.1 = 38,876.97	2,08,196.15	1,91,803.85
8	38,876.97 × 1.1 = 42,764.66	2,50,960.82	1,49,039.19
9	42,764.66 × 1.1 = 47,041.12	2,98,001.95	1,01,998.07
10	4,704.12 × 1.1 = 51,745.23	3,49,747.182	50,252.84

Hence, depreciation amount is Rs. 3,49,747.182 at the end of 10th year.

17. From the following information find annual depreciation and the book value of each year by straight line, declining balance, SOYD and sinking fund method.

Initial cost = Rs. 7,000

MARR = 10%

Useful life = 5 year

Salvage value = Rs. 2,000 **[2016/Spring]**

Solution:

B = Rs. 7,000 S = Rs. 2,000

N = 5 year MARR = 10%

i) Straight line method

$$\text{Depreciation for each year} = \frac{I - S}{N} = \frac{7,000 - 2,000}{5} = \text{Rs. } 1,000 \text{ per year}$$

Year	Depreciation	Book value
1	1,000	7,000 – 1,000 = 6,000
2	1,000	6,000 – 1,000 = 5,000
3	1,000	5,000 – 1,000 = 4,000
4	1,000	4,000 – 1,000 = 3,000
5	1,000	3,000 – 1,000 = 2,000
Total	Rs. 5,000	

Total depreciated value = Rs. 5,000

Total book value = Rs. 2,000

ii) Declining balance method:

Rate of depreciation,

$$R = 1 - \sqrt[N]{\frac{S}{I}} = 1 - \sqrt[5]{\frac{2,000}{7,000}} = 1 - 0.7784 = 22.17\% \text{ per year}$$

Annual depreciation, $D_k = r \times B_{k-1}$

where, k = End of year value = 1, 2, 3, 4, 5

Book value, $B_k = B_{k-1} - D_k$

EOY	Annual depreciation (Rs.)	Book value
0	–	7000
1	0.2217 × 7000 = 1551.9	5448.1
2	0.2217 × 5448.1 = 1207.8	4240.3
3	0.2217 × 4240.3 = 940.1	3300.2
4	0.2217 × 3300.2 = 731.7	2568.5
5	0.2217 × 2568.5 = 569.4	1999.1

iii) Sinking fund method

Sinking fund depreciation = (I – S) (A/F, i%, N)

= (7,000 – 2,000) (A/F, 10%, 5)

= 5,000 × 0.1638

= Rs. 819 per year

Year	Depreciation amount	Cumulative depreciation	Book value
1	819	819	6,181
2	1.1 × 819 = 901	1,720	5,280
3	1.1 × 901 = 991	2,712	4,289
4	1.1 × 991 = 1,090	3,802	3,199
5	1.1 × 1,090 = 1,199	5,000	2,000
Total		Rs. 5,000	

iv) SOYD method:

$$\text{SOYD} = N\frac{(N+1)}{2} = 5\frac{(5+1)}{2} = 15$$

$$\text{Depreciation} = \frac{\text{Remaining useful life}}{\text{Total useful life}} \times (B - S)$$

Year	SOYD depreciation	Book value
1	$\frac{5}{15} \times 5,000 = 1,666.67$	5,333.34
2.	$\frac{4}{15} \times 5,000 = 1,333.34$	4,000
3.	$\frac{3}{15} \times 5,000 = 1,000$	3,000
4.	$\frac{2}{15} \times 5,000 = 666.66$	2,333.33
5	$\frac{1}{15} \times 5,000 = 333.33$	2,000
Total	Rs. 5,000	

18. A machine costs Rs. 12,00,000 now and its useful life is 5 years. Its salvage value is expected to be Rs. 5,00,000. Calculate depreciation in each year and corresponding salvage value using sinking fund method and double declining balance method. [2018/Fall]

Solution:

Here;

Machine initial costs, B = Rs. 12,00,000
Estimated life, N = 5 years
Salvage value, S = Rs. 5,00,000

i) Double declining balance method:

$$\text{Declining balance rate (R)} = \frac{1}{N} \times 100 \times 2$$

$$= \frac{1}{5} \times 100 \times 2 = 40\%$$

Annual depreciation, $D_k = r \times B_{k-1}$
Book value, $B_k = B_{k-1} - D_k$

EOY (k)	Annual depreciation, D_k (Rs.)	Book value, B_k (Rs.)
0	-	12,00,000
1	0.4 × 12,00,000 = 4,80,000	7,20,000
2	0.4 × 7,20,000 = 2,88,000	4,32,000
3	0.4 × 4,32,000 = 1,72,800	2,59,200
4	0.4 × 2,59,200 = 1,03,680	1,55,520
5	0.4 × 1,55,520 = 62,208	93,312
Total = Rs. 11,06,688		

Salvage value is Rs. 5,00,000.
Total depreciation at the end of 5th year = Rs. 11,06,688.

ii) Sinking fund method

Let MARR = 10%

Sinking fund depreciation = (B – S) (A/F, i%, N)
= (12,00,000 – 5,00,000) (A/F, 10%, 5)
= 7,00,000 × 0.1638
= Rs. 1,14,660 per year

Year	Depreciation amount with i = 10%	Cumulative depreciation	Book value
1	1,14,660	1,14,660	1,14,660
2	1,14,660 × 1.1 = 1,26,126	2,40,786	9,59,214
3	1,23,126 × 1.1. = 1,38,738.6	3,79,524.6	8,20,475.4
4	1,38,738.6 × 1.1 = 1,52,612.46	5,32,137.06	6,67,862.94
5	1,52,612.46 × 1.1 = 1,67,873.30	7,00,000	5,00,000

19. A construction equipment has initial cost and annual saving per year are of Rs. 40,000 and Rs. 20,000 respectively. With annual operating and maintenance cost of Rs. 7,000. If will depreciate by MACRS method and will have no savage value. The useful life of equipment is 5 years. Estimate before and after tax cash flow. The company pays income tax @ 40%. [2017/Fall]

Solution:

Here;

Tax rate = 40%
N = 5 years
I = Rs. 40,000
S = Rs. 0
R = Rs. 10,000 per year
E = Rs. 7,000 per year

We know,

$BTCF_K = R_K - E_K$
$ATCF_K = BTCF_K - T_K$
$= R_K - E_K - t(R_K - E_K + d_K)$
$= (1 - t)(R_K - E_K) + td_K$

Year	BTCF = Revenue – Cost ($R_K - E_K$)	Depreciation amount with MACRS (d_K)	Net taxable income = BTCF – d_K	Tax amount @ 40%	ATCF = BTCF – Tax paid
0	-40000	-	-	-	-40000
1.	$\frac{1}{2}$(20,000 – 7,000) = 6,500	20% of 40,000 = 8,000	6,500 – 8,000 = -1,500	No tax	6,500
2.	20,000 – 7,000 = 13,000	32% of 40,000 = 12,800	200	0.40 × 200 = 80	1,300 – 80 = 12,920
3.	13,000	19.2% of 40,000 = 7,680	5,320	0.40 × 5,320 = 2128	10,872
4.	13,000	11.52% of 40,000 = 4,608.0	8,392	0.40 × 8,392 = 3,356	9,643.20
5.	13,000	11.52% of 4,000 = 4608.0	8,392	0.40 × 8,392 = 3,356	9,643.20
6.	$\frac{1}{2}$(20,000 – 7,000) = 6,500	5.76% of 40,000 = 2304	4,196	0.40 × 4,196 = 1,678	4,822

NOTE:
When $R_K > E_K + d_K \Rightarrow$ Tax liability
When $R_K < E_K + d_K \Rightarrow$ No tax liability

MACRS % calculation is as follows for 5 year class.

Straight line rate = $\frac{1}{N}$ × book value

Declining balance rate = $\frac{R}{N} = \frac{200\%}{5} = 40\%$

[∵ For 5 years life, R = 200% according to IRS guide line]

= $\frac{40}{1,000}$ × book value = 0.4 × book value

Year	MACRS % calculation	Selected MACRS
1.	$\frac{1}{2}$ year DB = $\frac{1}{2}$ × 0.40 × 100% = 20% $\frac{1}{2}$ year SL = $\frac{1}{2} \times \frac{1}{5}$ × 100% = 10%	20%
2.	DB = 0.40 × (100 – 20)% = 32% SL = $\frac{1}{4.5}$ × (100 – 20)% = 17.78%	32%
3.	DB = 0.40 × (100 – 20 – 32)% = 19.2% SL = $\frac{1}{3.5}$ (100 – 52)% = 13.7%	19.20%
4.	DB = 0.40 × (100 – 52 – 19.20)% = 11.52% SL = $\frac{1}{2.5}$ (100 – 71.20)% = 11.52%	11.52%
5.	DB = 0.40 × 17.28% = 6.9% SL = $\frac{1}{1.5}$ × 17.28% = 11.52%	11.52%
6.	DB = $\frac{1}{2}$ × 0.40 × 5.76% = 1.15% $\frac{1}{2}$ year SL = $\frac{1}{2} \times \frac{1}{0.5}$ × 5.76% = 5.76%	5.76%
	Total	100%

20. We are considering the purchase of bike at a cost of Rs. 1,10,000 with an estimated salvage value of Rs. 2,000 and a project useful life of 5 years. Interest is 10%, determine declining balance with conversion to straight line depreciation. [2011/Fall]

Solution:

B = Rs. 1,10,000

S = Rs. 2,000

N = 5 years

Straight line depreciation, $D_{ks} = \frac{B_{k-1} - S}{N - (k-1)}$

k = end of year (EOY) = 1, 2, 3, 4, 5

Declining method rate,

$r = 1 - \sqrt[N]{\frac{S}{B}} = 1 - \sqrt[5]{\frac{2,000}{1,10,000}} = 0.55 = 55\%$

Declining method depreciation, $D_{kd} = r\% \times B_{k-1}$

EOY	Declining method depreciation, D_{kd} (Rs.)	Straight line depreciation, D_{ks} (Rs.)	Actual depreciation	Book value B_k (Rs.)
0	–	–	–	1,10,000
1	0.55 × 11,000 = 60,500	$\frac{1,10,000 - 2,000}{5 - 1 + 1}$ = 21,600	60,500	49,500
2	2.55 × 49,500 = 27,225	$\frac{49,500 - 2,000}{5 - 2 + 1}$ = 11,875	27,225	22,275
3	0.55 × 22,275 = 12,251.25	$\frac{22,275 - 2,000}{5 - 3 + 1}$ = 6,758.3	12,251.25	10,023.75
4	0.55 × 10,023.75 = 5,513.1	$\frac{10,023.75 - 2,000}{5 - 4 + 1}$ = 4,011.9	5,513.1	4,510.65
5	0.55 × 4,510.65 = 2,480.9	$\frac{4,510.65 - 2,000}{5 - 5 + 1}$ = 2510.65	2,510.65	2,000

Here, switchover occurs during the final year.

21. An organization wants to purchase of Rs. 10,00,000 machine that is assigned to 5 years useful life and expected salvage Rs. 2,00,000. Compute depreciation by SOYD and MACRs method for each year. [2020/Spring]

Solution:

Here;

Initial cost, I = Rs. 1,00,0000

Salvage value, S = Rs. 2,00,000

Useful life, N = 5 years

a) **SOYD method**

Sum of the year digit = $\frac{N(N+1)}{2} = \frac{5(5+1)}{2} = 15$

Proportion of the year = 5 : 4 : 3 : 2 : 1

SOYD depreciation = $\frac{\text{Remaoning useful life}}{\text{Total useful life}} \times (I - S)$

Year	Remaining estimated useful life at beginning of year	SYD	Applicable percentage	Annual depreciation	Book value
1	5	$\frac{5}{15}$	33.33	2,66,640	7,33,360
2	4	$\frac{4}{15}$	26.67	2,13,360	5,20,000
3	3	$\frac{3}{15}$	20	1,60,000	3,60,000
4	2	$\frac{2}{15}$	13.33	1,06,640	2,53,360
5	1	$\frac{1}{15}$	6.67	53,360	2,00,000
Total	15			Rs. 8,00,000	

b) MACRs method

D = I – S = Rs. 10,00,000 – Rs. 2,00,000 = Rs. 8,00,000

Year	Depreciation percentage (x%), MACRs	Depreciation amount, MACRs	Book value
0	-	-	10,00,000
1	20%	1,60,000	8,40,000
2	32%	2,56,000	5,84,000
3	19.2%	1,53,600	4,30,400
4	11.52%	92,160	3,38,240
5	11.52%	92,160	2,46,080
6	5.76%	46,080	2,00,000

Depreciation amount = x% × D = x% × 8,00,000

22. A company is considering the purchase of second hand computers at a cost of Rs. 10,500 each with an estimated salvage value of Rs. 500 and a projected useful life of four years. Determine the annual depreciation and book values using double declining balance with conversion to straight line depreciation method. [2019/Fall]

Solution:

Here;

Initial investment, I = Rs. 10,500

Salvage value, S = Rs. 500

Useful life N = 4 years

Now, Double declining rate,

$$R = \frac{1}{N} \times 100 \times 2 = \frac{100}{4} \times 2 = 50\%$$

$$\therefore r = 0.50$$

Straight line depreciation method $= \frac{BV_N - S}{N}$

BV = Book value

200% double declining method

Depreciation = $r\ BV_{N-1}$

Initially, book value (BV) = I = Rs. 10,500

EOY	200% DB method	SL method (S = Rs. 500)	Depreciation selected	Book value, BV
0	-	-	-	10,500
1	5,250	2500	5,250	5,250
2	2,625	1,583.33	2,625	2,625
3	1,312.5	1,062.5	1,312.5	1,312.5
4	656.25	812.5	812.5	500

The switchover to SL occurs in last year *i.e.*, 4th year,

23. What is depreciation? A photocopy machine is costing of Rs. 4,60,000 with estimated salvage value Rs. 12,000 at the end of 6th year. Find yearly depreciation amount and book value by,

i) Double declining balance conversion to straight line method.

ii) Sum of years digit (SOYD) method. [2019/Spring]

Solution:

Here;

Initial cost, I = Rs. 4,60,000

Salvage value, S = Rs. 12,000

Useful life, N = 6 years

i) **Double declining switchover to straight line method**

For double declining,

$$r = \frac{1}{N} \times 100 \times 2 = \frac{200}{6} = 33.33\% = 0.333$$

Straight line method depreciation,

$$SL = \frac{BV_N - S}{N}$$

BV_N = Book value at year N

Double declining method depreciation = $r\ BV_{N-1}$

Initial book value (BV) = I = Rs. 4,60,000

EOY	200% DB method	SL method	Depreciation method	Book Value (BV)
0	–	–	–	4,60,000
1	1,53,333.333	74,666.67	1,53,333.333	3,06,666.667
2	1,02,222.222	58,933.333	1,02,222.222	2,04,444.445
3	68,148.148	48,111.111	68,148.148	1,36,296.297
4	45,432.099	41,432.099	45,432.099	90,864.198
5	30,288.066	39,432.099	39,432.099	51,432.099
6	17,144.033	39,432.033	39,432.099	12,000

The switchover occurs in 5th year.

ii) SOYD method

Sum of the year digit, $SYD = \frac{N(N+1)}{2} = \frac{6(6+1)}{2} = 21$

Annual depreciation = Applicable percentage × $(BV_{N-1} - S)$

Proportion of the year = 6 : 5 : 4 : 3 : 2 : 1

Year	Remaining estimated useful life	SYD	Applicable percentage	Annual depreciation	Book value
0	–	–	–	–	4,60,000
1	6	$\frac{6}{21}$	28.57	1,27,993.6	3,32,006.4
2	5	$\frac{5}{21}$	23.81	1,06,668.8	2,25,337.6
3	4	$\frac{4}{21}$	19.05	85,344	1,39,993.6
4	3	$\frac{3}{21}$	14.28	63,974.4	76,019.2
5	2	$\frac{2}{21}$	9.52	42,649.6	33,369.6
6	1	$\frac{1}{21}$	4.76	21,324.8	12,044.8

24. Write short notes on depreciation by sinking fund method. [2020/Fall]

Solution: See the topic 8.1.1.1 (f)

ADDITIONAL PROBLEMS

1. Compute the annual depreciation allowances and the resulting book value using declining method with switchover to straight line method. *i.e.*, straight line plus average interest method.
Cost of asset = Rs. 30,000
Estimated life = 5 years
Salvage value = 0

Solution:
Here; I = Rs. 30,000
N = 5 years
S = Rs. 0

Straight line depreciation, $D_k = \frac{B_{k-1} - S}{N - (k-1)}$

K = End of year (EOY) $= 1, 2, 3, 4, 5 = \frac{B_{k-1} - 0}{N - k + 1} = \frac{B_{k-1}}{N - k + 1}$

For first year,

$$D_{1s} = \frac{B_0}{5 - 1 + 1} = \frac{30,000}{5} = \text{Rs. } 6,000$$

For declining balance, we use double declining, so,
Rate of depreciation,

$$r = \frac{1}{N} \times 100 \times 2 = \frac{1}{5} \times 100 \times 2 = 40\%$$

Depreciation amount, $D_{kd} = r\% \times B_{k-1}$

For first year,

$$D_{1d} = 40\% \times B_0 = 0.40 \times 3,000 = \text{Rs. } 12,000$$

All values are tabulated as follows.

EOY	Double declining depreciation, D_{kd} (Rs.)	Straight line depreciation, D_{ks} (Rs.)	Actual depreciation (Rs.)	Book value B_k (Rs.)
0	–	–	–	30,000
1	0.4 × 3,00,00 = 12,000	$\frac{30,000}{5 - 1 + 1} = 6,000$	12,000	18,000
2	0.4 × 18,000 = 7,200	$\frac{18,000}{5 - 2 + 1} = 4,500$	7,200	10,800
3	0.4 × 10,800 = 4,320	$\frac{10,800}{5 - 3 + 1} = 3,600$	4,320	6,480
4	0.4 × 6,480 = 2,592	$\frac{6,480}{5 - 4 + 1} = 3,240$	3,240	3,240
5	0.4 × 3,240 = 1,296	$\frac{3,240}{5 - 5 + 1} = 3,240$	3,240	0

Here, the switchover occurs in year 4.

2. Consider the following information for machine operation system.

Initial cost = Rs. 4,000

Useful life = 10 years

Salvage value = Rs. 0

Depreciation rate = 20% per year

Compute the annual depreciation amount and book values by using straight line plus average interest method.

Solution:

Here;

B = Rs. 4,000

N = 10 years

R = 20%

S = 0

Switch from 20% declining balancing to straight line method.

Year	BOY book value	20% declining balance method	Switching decision	Straight line method	Depreciation amount select
1.	4,000	800	>	400	800
2.	3,200	640	>	355.56	640
3.	2,560	512	>	320	512
4.	2,048	409.6	>	292.57	409.6
5.	1,638.4	327.68	>	273.07	327.68
6.	1,310.72	262.14	=	262.14	262.14(switch)
7.	1,048.58	209.72	<	262.14	262.14
8.	786.44	167.77	<	262.14	262.14
9.	524.30	134.22	<	262.14	262.14
10.	262.16	107.37	<	262.14	262.14
Total		Rs. 3,570.50			Rs. 4,000

From above table, we see the switchover occurs in year 6.

3. Machine is purchased at Rs. 40,000 and its scarp value is Rs. 10,480 and its useful life is 6 years. Find:

i) Rate of depreciation under diminishing balance method.

ii) Amount of deprecation for 6 years

Solution:

Here;

B = Rs. 40,000

S = Rs. 10,480

N = 6 years

We know,

i) Rate of depreciation, $(R) = 1 - \sqrt[N]{\frac{S}{B}} = 1 - \sqrt[6]{\frac{10,480}{40,000}} = 20\%$

ii) Amount of depreciation for 6 years.

EOY	Annual depreciation	Book value (Rs.)
0	-	40,000
1	0.2 × 40,000 = 8,000	32,000
2	0.2 × 32,000 = 6,400	25,600
3	0.2 × 25,600 = 5,120	20,480
4	0.2 × 20,480 = 4,096	16,384
5	0.2 × 16,384 = 3,276.8	13,107.2
6	0.2 × 13,107.2 = 2,621.44	10,485.76
Total = Rs. 29,514.24		

Depreciated amount for 6 years = Rs. 29,514.24

4. A laptop costing of Rs. 40,000 is estimated to have life of 10 years and salvages value is 0. Find

i) Depreciation amount for the 7th year.

ii) Accumulated depreciation throughout 7th year.

iii) Book value at the end of 7th year.

Solution:

Here;

B = Rs. 40,000

S = Rs. 0

N = 10 years

Using straight line depreciation method,

i) Depreciation amount of the 7th year:

$$d_7 = \frac{B - S}{N} = \frac{40,000 - 0}{10} = \text{Rs. } 4,000$$

ii) Accumulated depreciation throughout 7th year:

$D_7 = 7 \times d_7 = 7 \times 4,000 = 28,000$

iii) Book value at the end of 7th year:

$BV_7 = B - D_7 = 40,000 - 28,000 = \text{Rs. } 12,000$

5. Suppose an equipment purchased for Rs. 10,00,000. It is expected to generate income of Rs. 350,000 per year during 5 years and corporate income tax rate is 25% per year. Under the recovery periods, depreciation are as follows:

Year	1	2	3	4	5
Depreciation	1,00,000	20,00,00	2,00,000	2,00,000	1,00,000

Estimate before and after tax flow.

Solution:

EOY	BTCF	d_K	Taxable income	Income tax @25%	ATCF
0	-1,00,000	-	-	-	-1,00,000
1.	3,50,000	1,00,000	2,50,000	62,500	2,87,500
2.	3,50,000	2,00,000	1,50,000	37,500	3,12,500
3.	3,50,000	2,00,000	1,50,000	37,500	3,12,500
4.	3,50,000	2,00,000	1,50,000	37,500	3,12,500
5.	3,50,000	1,00,000	1,50,000	62,500	2,87,500

6. What are the factors affecting to the amount of depreciation?

Solution:

Factors affecting to the amount of depreciation are:

i) Cost of assets

The cost of asset include the purchase price less any trade discount plus all the costs essential to bring the asset to a usable condition. It is taken as a main basis for determining the depreciation rate and amount.

ii) Estimated scrap value

Scrap value refers to the value estimated to be realize after the expiry of the useful working life of the asset. This is also known as salvage or residual value. It should be determined for correct ascertainment of depreciation.

iii) Estimated useful life

An asset cannot work forever. Every asset has a certain useful life of an asset is generally to be taken in terms of asset's expected use. The service life may be measured in terms of time, unit of output, working hours, etc.

iv) Legal provision

The amount of depreciation depends upon the statutory and legal provisions prescribing the admissible rate of depreciation on fixed assets. All the statutory and legal provisions must be carefully evaluated.

CHAPTER **9**

ENTERPRISE FINANCING AND CAPITAL INVESTMENT

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9.1 METHODS OF FINANCING

Financing is the process of providing funds for business activities, making purchases or investing. Financial institutions, such as banks, are in the business of providing capital to businesses, consumers and investors to help them achieve their goals. The use of financing is vital in any economic system, as it allows companies to purchase products out of their immediate reach.

The two broad choices a firm has for financing an investment project are equity financing and debt financing.

9.1.1 Equity Financing

It is first type of financing. Equity financing is the process of raising capital through the sales of share in an enterprise. Equity financing essentially refers to the sale of an ownership to raise funds for business purposes. It has two forms/models:

i) The uses of retained earning into company development.

ii) Issuing share to outside/creating new owners.

If a company does not have sufficient cash on hand to make an investment and does not wish to borrow in order to fund the investment, financing can be arranged by selling common stock to raise the required funds. To do this, the company has to decide how much money to raise, the type of securities to issue (common stocks or preferred stock) and the basis for pricing the issue. Once the company has decided to issue common stock, it must estimate flotation costs the expenses it will incur in connection with the issue, such as investment banker's fees, lawyer's fee, accountant's costs and cost of printing and engraving. Generally investment bankers will buy the issue from the company at a discount below the price at which the stock is to be offered to the public. If the company is already public owned, the offering price will commonly be based on the existing market price of the stock.

If the company is going public for the first time, no established price will exist, so investment bankers have to estimate the expected market price at which the stocks will sell after the stock issue.

Pros:

- No obligation to pay dividends on equity.
- Possible industry experience and connections from right investors.
- Investors' money doesn't have to be returned if business fails.
- Improves financial health of business by reducing leverage.

Cons:

- Having to give up a portion of ownership.
- Equity costs more than debt because investors risk more.
- Attracting investors can be harder than getting a loan.

9.1.2 Debt Financing

The second major type of financing a company can select is debt financing, which includes both short term borrowing from financial institutions and the sale of long term bonds, where in money is borrowed from investors for a fixed period. Hence, a debt financing occurs when a firm raises money for working capital or capital expenditures by selling debt instruments to individuals and for institutional investors. In return

for lending credits and receive a promise that the principal and interest on the debt will be repaid.

With debt financing, the interest paid on the loans or bonds is treated as an expense for income-tax purposes. Since interest is a tax-deductible expense, companies in high tax brackets may incur lower after-tax financing costs with a debt. In addition to influencing the borrowing interest rate and tax bracket, a loan-repayment method can affect financing costs.

When the debt-financing option is used, we need to separate the interest payments from the repayment of the loan for our analysis. The interest-payment schedule depends on the repayment schedule established at the time the money is borrowed. The two common debt-financing methods are as follows:

1. **Bond Financing**

This type of debt financing does not involve the partial payment of principal; only interest is paid each year (or semiannually). The principal is paid in a lump sum when the bond matures. Bond financing is similar to equity financing in that flotation costs are involved when bonds are issued.

2. **Term Loans**

Term loans involve an equal repayment arrangement according to which the sum of the interest payments and the principal payments is uniform; interest payments decrease, while principal payments increase, over the life of the loan. Term loans are usually negotiated directly between the borrowing company and a financial institution, generally a commercial bank, an insurance company, or a pension fund.

Pros:

- Doesn't dilute owner's portion of ownership.
- Lender doesn't have claim on future profits.
- Debt obligations are predictable and can be planned.
- Interest is tax deductible.
- Debt financing offers flexible alternatives for collateral and repayment options.

Cons:

- Debt must be repaid.
- Can be difficult to qualify for, depending on financial status and credit score.
- Some debt instruments restrict businesses from pursuing alternative financing options.
- Higher debt-equity ratios increase the financial risk of the company.
- Owners may be required to personally guarantee the debt.
- Assets could be seized in case of default.

Difference between debt financing and equity financing:

S.N.	Debt financing	Equity financing
i)	Must be repaid or refinanced.	Can usually be kept permanently.
ii)	Debts provides are conservator.	Equity providers are aggressive.
iii)	Does not grant ownership of the firm.	Grants ownership of the firm.
iv)	Interest payments are tax deductible.	Dividend payments are not tax deductible
v)	Collateral assets must usually be available.	No collateral required.
vi)	Requires regular interest payments.	No payments requirement.
vii)	Debts has little or no impact on control of the company.	Equity requires shared control of the company and may impose restrictions.
viii)	Debts holders cannot share any upside or profits.	Shareholders can share any upside or profits.
ix)	Debts has a maturity date.	Stock has no maturity date.
x)	Debt finance is comparatively short term finance.	Equity, on the other hand, is long term finance for the company.
xi)	Debt financing can be categorized by Term Loan, Debentures, Bonds, etc.	Shares and Stocks can categorize equity.
xii)	Debt falls under low-risk investments.	Equity falls under high-risk investments.
xiii)	Lenders get paid to interest over and above the principal amount financed.	Shareholders of the company get a dividend on the ratio of shares held/profit earned by the company.
xiv)	The interest payable to the lenders is fixed and regular and also mandatory.	Dividend paid to the shareholders is variable, irregular as it completely depends on the profit earnings of the company.
xv)	Debt finance is a loan or a liability of the company.	Equity finance is an asset of the company, or the companies own funds.

9.1.3 Capital Structure or Debt Ratio

The ratio of total debts to total capital; generally called the debts ratio or capital structure represents the percentage of the total capital provided by borrowed funds.

$$\text{Capital structure} = \frac{\text{Total debt}}{\text{Total capital}}$$

i.e.,

For example; A debt ratio of 0.30 indicates that 30% of the capital is borrowed and 70% funds are provided by equity.

This types of financing is called mixed financing.

$$\text{Also, debt ratio} = \frac{\text{Debt}}{\text{Total assets}} = \frac{\text{Debt}}{\text{Debts + Equity}}$$

A firm usually establishes a target capital structure or target debt ratio, after considering the effects of various financing methods. This target may change over time as business conditions vary, but a firm's management always strives to achieve the target whenever individual financing decisions are considered.

On the one hand, if the actual debt ratio is below the target level, any new capital will probably be raised by issuing debt. On the other hand, if the debt ratio is above the target, expansion capital will be raised by issuing stocks.

9.2 COST OF CAPITAL

Cost of capital is the required return necessary to make a capital budgeting such as new factory, worthwhile. Cost of capital includes the cost of debt and the cost of equity and is used by companies internally to judge whether a capital project is worth the expenditure of resources and by investor who use it to determine whether an investment is worth the risk compared to the return.

"The cost of capital is the minimum required rate of earnings or the cutoff rate of expenditure". -Solomon Ezra

9.2.1 Cost of Equity

Cost of equity is the minimum rate of return a firm must offer shareholders to compensate for waiting for their returns and for bearing some risk.

$$\therefore \text{ Cost of equity} = \frac{\text{Next year's Dividend per share}}{\text{Current market stock value}} + \text{Growth rate of dividends}$$

Cost of equity calculation is based on sources of capital:

i) **Cost of retained earnings**

$$k_r = \frac{D_1}{P_0} + g$$

where, D_1 = First year dividend or $D_0 (1 + g)$

P_0 = Present cost of stocks

g = Grow rate of stock

ii) Issuing new common stock

$$k_p = \frac{D_1}{P_0(1 - f_c)} + g$$

where, f_c = Flotation cost

iii) Cost of preferred stock

$$k_p = \frac{D^*}{P^*(1 - f_c)}$$

where, D^* = Fixed annual dividend

P^* = Issuing price

iv) Cost of equity

Weight average cost of equity,

i.e., $\frac{C_r}{C_e} k_r + \frac{C_c}{C_e} k_e + \frac{C_p}{C_e} k_p$

where, C_r = Amount of equity financed from retained earnings.

C_p = Amount of equity financed from preferred stock.

C_c = Amount of equity financed from issuing new common stock

$C_e = C_r + C_p + C_c$

Another method of determining cost of equity is,

$$i_e = r_f + \beta[r_m - r_f]$$

where, i_e = Cost of equity

r_f = Risk free rate

β = Firms beta risk

r_m = Market rate of return

If $\beta > 1$; stock is more volatile than the market.

If $\beta < 1$; stock is more volatile than the market.

An Alternative Way of Determining the Cost of Equity

Whereas debt and preferred stocks are contractual obligations that have easily determined costs, it is not easy to measure the cost of equity. In principle, the cost of equity capital involves an **opportunity cost**. In fact, the firm's after-tax cash flows belong to the stockholders. Management may either pay out these earnings in the form of dividends or retain the earnings and reinvest them in the business. If management decides to retain the earnings, an opportunity cost is involved.

Stockholders could have received the earnings as dividends and invested that money in other financial assets. Therefore, the firm should earn on its retained earnings at least as much as the stockholders themselves could earn in alternative, but comparable, investments.

What rate of return can stockholders expect to earn on retained earnings? This question is difficult to answer, but the value sought is often regarded as the rate of return stockholders require on a firm's common stock. If a firm cannot invest retained earnings so as to earn at least the rate of

return on equity, it should pay these funds to its stockholders and let them invest directly in other assets that do provide that rate of return. In general, the expected return on any risky asset is composed of three factors:

$$\begin{pmatrix}\text{Expected return}\\ \text{on ricky asset}\end{pmatrix} = \begin{pmatrix}\text{Risk-free}\\ \text{interest rate}\end{pmatrix} + \begin{pmatrix}\text{Inflation}\\ \text{premium}\end{pmatrix} + \begin{pmatrix}\text{Risk}\\ \text{premium}\end{pmatrix}$$

This equation says that the owner of a risky asset should expect to earn a return from three sources:

- Compensation from the opportunity cost incurred in holding the asset, known as the risk-free interest rate.
- Compensation for the declining purchasing power of the investment over time, known as the inflation premium.
- Compensation for bearing risk, known as the risk premium.

Fortunately, we do not need to treat the first two terms as separate factors because together they equal the expected return on a default-free bond such as a government bond. In other words, owners of government bonds expect a return from the first two sources, but not the third-a state of affairs we may express as

$$\begin{pmatrix}\text{Expected return}\\ \text{on ricky asset}\end{pmatrix} = \begin{pmatrix}\text{Interest rate on}\\ \text{government bond}\end{pmatrix} + \begin{pmatrix}\text{Risk}\\ \text{premium}\end{pmatrix}$$

When investors are contemplating buying a firm's stock, they have two primary things in mind:

i) cash dividends, and

ii) gains (appreciation of shares) at the time of sale.

From a conceptual standpoint, investors determine market values of stocks by discounting expected future dividends at a rate that takes into account any future growth. Since investors seek growth companies, a desired growth factor for future dividends is usually included in the calculation.

9.2.2 Cost of Debt

Cost of debt is the effective rate that a company pays on its current debt. The two types of debt financing are term loans and bonds. Because the interest payments on both are tax deductible, the effective cost of debt will be reduced. To determine the after tax cost of debt (id), we use formula,

$$i_d = \left(\frac{C_s}{C_d}\right) k_s (1 - t_m) + \left(\frac{C_b}{C_d}\right) k_b (1 - t_m)$$

where, i_d = Cost of debt

C_s = Amount of short term loan

C_d = Amount of bond financing

$C_d = C_s + C_b$ = total amount of debt

k_s = Before tax interest rate short term loan
k_b = Before tax interest rate on bond
t_m = Firm's marginal tax rate

9.2.3 Calculating the Cost of Capital

Weighted-Average Cost of Capital

Assuming that a firm raises capital on the basis of the target capital structure and that the target capital structure remains unchanged in the future, we can determine a **tax-adjusted weighted-average cost of capital** (or, simply stated, the **cost of capital**). This cost of capital represents a composite index reflecting the cost of raising funds from different sources.

We can determine tax weighted average cost of capital (simply stated as the cost of capital) by reflecting the cost of raising funds from different sources.

$$\therefore \quad \text{Cost of capital; } k = \frac{i_d \cdot C_d}{V} + \frac{i_e \cdot C_e}{V}$$

where, C_d = Total debt capital

$C_d = C_s + C_b$

C_e = Total equity capital = $C_r + C_p + C_c$

i_e = Average equity interest rate per period

i_d = After tax average borrowing interest rate per period.

k = Tax adjustment weighted average cost of capital.

$V = C_d + C_e$

Marginal cost of capital

It is define as the cost of obtaining another dollar of new capital and the marginal cost rises as more and more capital is raised during a given period.

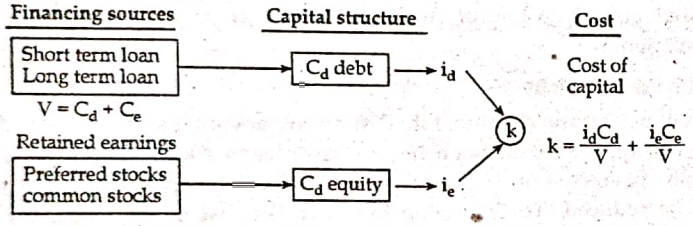

Figure: Process of calculating the cost of capital

9.3 PROJECT FUNDING MECHANISM

In life cycle of project, various nature of cost will be incurred and such cost is funded by mainly three mechanism:

- Government budget
- Private investment
- Public private partnership

9.3.1 Government Budget

A government budget is an annual financial statement presenting the revenues and spending for a financial year that is often passed by the legislature. The budget is also known as the annual financing statement of the country. Most of the development and public utility projects like road, water supply, electricity, gas, hospitals etc are established and run by government funding through budget. The funding made by the local and central government agencies to produce and delivery various public utilities include in this heading.

9.3.2 Private Investment

Private investment includes domestic and foreign private organization or people having financial and technical capabilities. All private companies and business projects are established and run by the private investment. Generally, private sector choose low risk and high return low investment as well as short run projects. Government also emphasized globalization and liberalization policies to encourage private sectors for the development works. Its main objective is to maximize the profit rather than doing social welfare.

9.3.3 Public Private Partnership (PPP)

Public private partnership is a contractual agreement between a public agency (federal, state or local) and a private sector entity. Through this agreement, the skills and assets of each sector (public and private) are shared in delivering a service or facility for the use of general public. In addition to the sharing of resources, each party shares in the potential risks and rewards in the delivery of the services and/ or facility.

Public = Government institutions like ministries, department, VDC, municipalities, etc.

Private = Local and international body that may include financial and technical expertise relevant to the project.

Partnership = Formal agreement in which two or more parties cooperate to manage and operate a business or to provide services.

In PPP, government as well as private sector both are active on commercial functions of the projects and work for their mutual benefits. In PPP, government remains actively throughout the life of project and private sectors is responsible for the more commercial functions. Like the project design, construction, finance and operations. PPP is widely accepted project funding mechanism (development approach) throughout the world. Through PPP, government can reduce cost and risk and private sectors can generate business opportunities ad general people can receive better and more accessible services.

Success factors of PPP models

- Political will
- Security and privacy
- Legal framework
- Project address a real need
- Transparent selection procedures
- Clear definition of the scopes, roles, risk and obligations
- Consultation with stakeholders.

Benefits from PPP

- Cost effectiveness
- Project acceleration
- Better public sector risk
- Reduce public sectors risk
- Improve service delivery
- Increase investment in public infrastructure.

9.4 FIRR, EIRR AND RETURN ON EQUITY

9.4.1 Financial Internal Rate of Return (FIRR)

The FIRR is an indicator to measure the financial return on investment of an income project and is used to make the investment decision. Some mega projects may generate different financial inflows and outflows for different stakeholders and their internal rate of return for that investment project may be different. FIRR can be calculated by various stakeholders based on their investment (cash inflow and outflow) projects. This can be

- FIRR for project entity (for total investment)
- FIRR for stockholder (for share/equity capital)
- FIRR for lenders/banks (for debt capital)
- FIRR for government
- FIRR for lender and contractor

FIRR is obtained by equating the present value of investment costs (cash outflows) and present value of net incomes (cash inflows).

$$\sum_{n=0}^{m} \frac{In}{(1+r)^n} = \sum_{n=0}^{m} \frac{Bn}{(1+r)^n}$$

where, In = Investment expenditure for the entire project life period from year o to m.

Bn = Annual net incomes for the entire project life period from year o to m.

Solving above equation, we can obtain the value of r which is called FIRR.

Decision rule:

If FIRR > MARR; accept the project.

If FIRR < MARR; reject the project.

If FIRR = MARR; remain indifferent.

9.4.2 Economic Internal Rate of Return (EIRR)

Economic internal rate of return is calculated as the rate of discount for which the present value of the net benefit stream becomes zero or at which the present value of the benefit stream is equal to the present value of the cost stream. The EIRR determines the discount rate where the present value of benefits is just equal to the present value of costs.

If there is no project boundary and externalities and price and policy distribution is zero, then, FIRR can be used as proxy for EIRR.

EIRR = FIRR + Profitability associated with consumers due to market price distortion + Profitability associate with externalities expressed in market price + Profitability associated with environment distraction.

For a project to be acceptable, the EIRR should be greater than the economic cost of capital.

9.4.3 Return on Equity (ROE)

Return on equity (ROE) is a measure of financial performance calculated by dividing net income by shareholder's equity. Because shareholder's equity is equal of a company's assets minus its debt, ROE could be though as the return on the net assets. ROE is expressed as a percentage and can be calculated for any company if net income and equity are both positive number *i.e.*,

$$ROE = \frac{\text{Net income} - \text{Preferred dividend}}{\text{Shareholder's equity or common equity}} \times 100$$

$$= \frac{\text{Net income}}{\text{Shareholder's equity}} \times 100$$

ROE is sometimes called "return of net worth". ROE is more than a measure of profit; it is a measure of efficiency. A rising ROE suggest that a company is increasing its ability to generate profit without needing as much capital. It also indicates how well a company's management is deploying the shareholder's capital. Falling ROE is usually a problem. Many investors choose to calculate the ROE at the beginning and end of period to see the change in return ratio.

$$\text{Return on equity} = \text{Return of assets} \times \text{Leverage}$$

$$= \frac{\text{Net income}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Average equity}}$$

$$\therefore \quad \text{Return on equity} = \frac{\text{Net income}}{\text{Average equity}}$$

Hence, ROE is a profitability ratio from the investors point of view not the company point of view.

BOARD EXAM SOLVED PROBLEMS

1. What are the sources of project finance? Explain the advantages of debt financing. If interest on debt is 12%, dividend to share holders over 15%, calculate weighted average cost of capital if debt is 70% and equity is 30%, tax rate is 20%. [2018/Fall, 2019/Spring]

Answer:

Source of project finance are:

i) Debt finance
ii) Equity finance
iii) Public finance
iv) Personal capital
v) Retained profits

Advantages of debt financing are

i) **Utilization of resources**
When a business use debt finance its operation, they got no option than to fully utilize their resources because they will have to pay back the debt and interest to their creditor.

ii) **Tax advantage**
Debt financing also offers tax advantage to business as interest is deductible for income tax purpose.

iv) **No future lender claims**
Lenders has no direct claim on future earnings.

v) **Non dilutive**
Debt does not dilute the ownership of small business.

vi) **Future impact forecasting**
Interest and principal repayment are based on fixed percentage and can be forecast.

vii) **Simple loan repayment:**
Lenders are only entitled to loan repayment and interest on loan.

Solution:

Interest on debt = 12%
Tax rate, T_C = 20%
Dividend to shareholder ratio = 15%
Weighted average cost of capital = ?
Debt (D/V) = 70%
Equity (E/V) = 30%

We know,

Weighted average cost of capital, WACC

$$= \frac{E}{V} \times Re + \frac{D}{V} \times R_C \times (1 - T_C)$$

Also,
Cost of debt, Rs = 10%
Cost of equity, Re = 15% = dividend to shareholder ratio.

Hence,
$$\text{WACC} = 0.30 \times 0.15 + 0.70 \times 0.12 \times (1 - 0.20)$$
$$= 0.045 + 0.06720$$
$$= 11.22\%$$

Thus, weighted average cost to capital is 11.22%.

2. Write short notes on capital structure. [2017/Spring]

Answer: See the topic 9.1.3.

3. What are the factors to be considered while determining the cost of debt? Calculate the after tax cost of debt while the interest rate = 10% and tax rate = 40%. [2017/Spring]

Answer:

Factors to be considered while determining the cost of debt are:

- Interest risk
- Nominal interest rate
- Real interest rate
- Risk free interest rate
- Inflation
- Inflation premium
- Maturity premium

Solution:

Interest rate = 10%
Tax rate = 40%

∴ After tax cost of debt = Interest rate × (100% – Tax rate)
= 10% × (100% – 40%)
= 10% × 60%
= 6%

Hence, after tax cost of debt is 6%.

4. Define funding and financing with examples. [2017/Spring]

Answer:

Funding is an amount of money provided by the organization or government on the basis of an agreement. It is usually free of charge.

For example; Fund for building roads provided by government.

Financing is an amount of capital or the sum of money provided to an organization with the expectation to reply, and organizations are liable to pay back the capital amount along with a certain percentage of interest.

For example; Capital provided by bank to open new company.

5. Write short notes on project funding mechanism. [2016/Spring, 2017/Fall]

Answer: See the topic 9.3, 9.3.1, 9.3.2, 9.3.3.

6. What do you mean by equity financing and debt financing? Explain ways to project funding mechanism by giving example. [2017/Fall]

Answer: See the topic 9.3, 9.1.1 and 9.1.2.

7. Write short notes on method of financing. [2016/Fall]

Answer: See the topic 9.1, 9.1.1, 9.1.2 and 9.1.3.

8. Write short notes on stock and bond. [2011/Spring, 2012/Fall, 2014/Fall, 2016/Fall]

Answer:

S.N.	Stocks/Share	Bonds/debenture
i)	Stocks are the financial instrument that carries ownership interest, issued by the company in exchanged for cash.	Bounds are the debt instrument issued by the company to raise capital with a promise to pay back the money after some period along with interest.
ii)	Stocks represent ownership.	Bonds represent debt.
iii)	Stocks do not have a maturity date.	Bonds have a maturity date.
iv)	High risk	Comparative low risk.
v)	All corporations issue or after to sell stocks. That is what makes them corporation.	Corporations are not required to issue bonds.
vi)	Stockholder's can elect a board of directors, which controls the corporation.	Bondholder's usually have no voice in or over management of the corporation.
vii)	Return is dividend.	Return is interest.
viii)	No return guarantee.	Return guaranteed.
ix)	It is not convertible into bond.	It may be convertible to bond.
x)	Issue of shares at a premium or discount has to be made with the company.	There is no any restriction on the issue of bond at a premium or discount.

9. Define debt and equity financing. What is difference between them? If Asian electric company presently pays a dividend of Rs. 12 per share and has a share price of Rs. 110. The expected growth was 8% forever then what is the required return on equity? [2015/Fall]

Answer: See the topic 9.1.1 and 9.2.2.

Present share price including dividend = Rs. 110

Initial price of share before dividend = Rs. (110 – 12) = Rs. 98

Expected growth on share = 8%

$$\text{Required return on equity} = \frac{\text{Initial price} \times \text{Expected growth rate}}{100}$$

$$\therefore = \frac{98}{100} \times 8 = \text{Rs. } 7.84$$

Hence, required return on equity = Rs. 7.84.

10. Write short notes on FIRR and EIRR. [2015/Spring, 2019/Fall, 2020/Fall]

Answer: See the topic 9.4.1 and 9.4.2.

S.N.	FIRR	EIRR
i)	FIRR is calculated by financial analysis.	EIRR is calculated by economic analysis.
ii)	FIRR is mainly carried out by private sector.	EIRR is carried out by public sector.
iii)	Tax and subsidies are incorporated in FIRR.	Tax and subsidies are ignored in EIRR.
iv)	Market price is used.	Shadow price is used.
v)	Direct costs of project and benefits are calculated by investors point of view.	Direct as well as indirect cost and benefits are calculated by social point of view.

11. Explain cost of capital. Briefly explain the equity financing and debt financing. [2015/Spring]

Answer: See the topic 9.1.1, 9.1.2 and 9.2.

12. Write short notes on public private partnership. [2014/Spring]

Answer: See the topic 9.3.3.

13. Bond issued by Everest Bank Ltd. has a coupon rate of 8%. Interest is paid quarterly and the bond mature in 5 years. The face value of the bond is Rs. 1,000. What is the present value of the bond if market interest rate is 7.5%? [2013/Fall, 2012/Spring]

Solution:

Coupon rate = 8% compounded quarterly.

$$\text{Effective coupon rate} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.08}{4}\right)^4 - 1 = 8.240\%$$

Yearly return amount = Rate × Face value

= 0.0824 × 1,000

= Rs. 82.40

Present value of bond:

$$B_0 = \frac{82.4}{(1+0.075)^1} + \frac{82.4}{(1+0.075)^2} + \frac{82.4}{(1+0.075)^3} + \frac{82.4}{(1+0.075)^4} + \frac{82.4}{(1+0.075)^5} + \frac{1,000}{(1+0.075)^5}$$

$$= 76.65 + 71.3 + 66.3 + 61.7 + 57.4 + 696.6$$

$$= \text{Rs. } 1,030$$

∴ Present value of bond = Rs. 1,030.

14. Difference between common stock and preferred stock.

[2012/Spring, 2013/Fall]

Answer:

S.N.	Common stock	Preferred stock
i)	Common stock refers to the ordinary stock representing part ownership and confers voting rights to the person holding it.	Preferred stock represents that part of company's capital that carry preferential right to be paid, when the company goes bankrupt or wound up.
ii)	Differential rights	Preferential rights.
iii)	Return on capital is not guaranteed.	Return on capital is guaranteed.
iv)	Cannot be redeemed.	Can be redeemed.
v)	It is not convertible.	It is convertible.
vi)	Payment to common stock holder are made at the end.	Preferred stockholders are paid before common stockholders.
vii)	It has voting right on general meeting.	It do not have any voting rights.
viii)	Growth potential is high.	Low growth potential.
ix)	Rate of dividend may vary over the years.	Rate of dividend is fixed.
x)	If there is no profit, common stockholders receive nothing.	Irrespective of making profit/ incurring losses, preferred stockholders receive the dividend.
xi)	Don't receive arrears in the next year.	Receives arrears in the next year.

15. What do you mean by financing and method of financing?

[2020/Fall]

Solution: See the topic 9.1, 9.1.1 and 9.1.2.

CHAPTER **10**

BASIC ACCOUNTING PROCEDURE

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10.1 ACCOUNTING

Accounting is a system of collecting, summarizing, analyzing, classifying and interpreting business information in financial and monetary terms.

Accounting is the systematic and comprehensive recording of financial transactions pertaining to a business.

10.1.1 Assets

An asset is any resource owned by the business organization to generate economic benefit. Anything tangible or intangible that can be owned or controlled to produce value and that is held by a company to produce positive economic value is an asset.

For example; Cash in hand, stock, land, building, goodwill, loan.

Types of assets are:

a) Fixed assets

Fixed assets are of a fixed nature in the context that they are not readily convertible into cash. They require elaborate procedure and time for their sale and converted into cash.

For example; Land, building, plant and machinery equipment, furniture and fixtures, vehicles, etc.

b) Current assets

Current asset is any asset which can reasonably be expected to be sold, consumed or exhausted through the normal operation of a business within the current fiscal year or operating cycle.

For example; Sundry debtors, accrued incomes, cash in hand, cash at bank, Bills receivable, etc.

c) Fictitious assets

Fictitious assets are those assets which are neither tangible assets nor intangible assets but represent loss or expenses yet to be written off. They are taken as the assets on legal and technical grounds.

For example; Advertisement expenditure, deferred revenue expenditure, discount on shares and debentures, debit balance of profit and loss account, etc.

d) Intangible assets

An intangible asset is an asset that is not physical in nature. It includes patent, copyrights, franchises, goodwill, trademarks and trade names. Intangible assets are categorized as limited life and indefinite life. While intangible assets don't have the obvious physical value of a factory or equipment, they can prove valuable for a firm and be critical to its long term success or failure.

For example; Goodwill, brand recognition, patents, trademarks, customer lists, etc.

10.1.2 Liabilities

Liabilities are debts payable in the future by the firm to its creditors. It denotes economic obligation to pay cash or to provide goods or services in the accounting records with a credit and decreased with a debt. A liability can be considered as a source of fund. Liabilities are aggregated on the balance sheet with two general classifications, which are current liabilities and long term liabilities.

a) Current liabilities

Current liabilities are a company's debts or obligations that are due within one year. Current assets are converted into cash to pay current liabilities.

For example; Account payable, notes payable, advance income, short term loan, Bank overdraft, sales tax payable, etc.

b) Long term liabilities

Long term liabilities are financial obligations of a company that become due more than one year. It is also known as permanent liabilities.

For example; Deferred revenues, capital leases, long term loans, bounds payable, bank loan, pension liabilities, etc.

10.1.3 Fundamental Equation of Accounting

The fundamental accounting equation also called the balanced sheet equation, represents the relationship between balanced sheet equation, represents the relationship between the assets, liabilities and owner's equity of a business or a person. It is the fundamental of the double entry book-keeping system. Assets denote resources, and capital and liabilities are source of financial. The value of resources and sources must be equal.

It can be expressed as

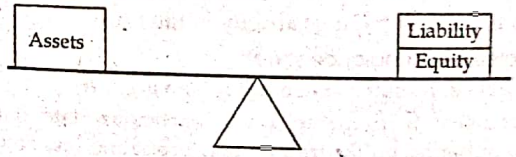
Assets = Liabilities + Equity

A = L + E

Capital = Assets – Liabilities

Liabilities = Assets – Capital

In a corporation, capital represents the shareholders equity. Since, every business transaction affects at least two of a company's account the accounting equation will be in balance, meaning the left side should always equal the right side.



10.2 FINANCIAL STATEMENTS

Financial statements are written records that convey the financial activities and conditions of a business or entity and consist of four major components. Relevant financial information is presented in a structured manner and in a form easy to understand. They are useful for the following reasons:

- To determine the ability of a business to generate cash and the source and uses of that cash.
- To determine whether a business has the capability to pay back its debt.
- To track financial results on a trend line to stop any looming profitability issues.
- To derive financial ratios from the statement that can indicate the condition of the business.
- To ascertain the results of various business operation such as debtors, profit, loss, payables, incomes etc.
- To promote effective decision making.

Basic financial statements are:

- Balance sheet
- Cash flow statement
- Income statement
- Statement of retained earnings

10.2.1 The Balance Sheet

A balance sheet is a statement of the financial position of a business which states the assets, liabilities and owner's equity at a particular point of time. Balance sheet is the final step of final account. Balance sheet is not an account but a statement of assets, liabilities and capital of the business enterprises at a given date. In balance sheet, total of all assets will always be equal to the total of all capital and liabilities.

i.e., Assets = Capital + Liabilities

a) Objective/Importance of Balance sheet:

i) It reveals the financial position. *i.e.*, state of affairs of a business.
ii) It ascertains composition of assets and liabilities.
iii) It tells about the balance of debtors and creditors.
iv) It tells about the liabilities and assets of the firm.
v) It tells about liquidity position of a firm.
vi) It depicts the debt paying capacity of the firm.

b) Characteristics of balance sheet:

i) Balance sheet is only a statement not an account.
ii) Balance sheet is prepared on a particular date but not for a particular period unlike trading and profit and loss account.
iii) Balance sheet shows the nature and value of assets and the nature and the amount of liabilities at a given date.
iv) The totals of liabilities and assets are always equal.
v) It shows the financial position of the business concern.
vi) It shows what the firm owes to others and also what others owe to the firm.

c) General format of balance sheet:

Assets		Liabilities
Current assets	=	Current liabilities
		Long term liabilities
Long term assets		Equity i) Owner contribution ii) Retained earnings

There are several balance sheet formats available and generally, it is categorized as classified, common size, comparative and vertical balance sheets. In account format, the balance sheets is divided into left and right sides like a T-account. The assets are listed on the left hand side whereas both liabilities and owner's equity are listed on the right hand side of the balance sheet. If all the elements of balance sheet are correctly listed, the total asset side must be equal to the total of liabilities and owner' equity side.

..... Company

Balance Sheet

As on .....

Assets	Rs.	Liabilities and Capital	Rs.
Fixed assets:		Opening capital: × ×	
Land and building	× ×	Less: Drawing : × ×	
Plant and machinery	× ×	Add: Net profit : × ×	
Equipments	× ×	Less: Net loss : × ×	× ×
Furniture and fixtures	× ×	**Reserve and funds:**	
vehicles	× ×	General reserve	× ×
live stocks	× ×	Capital reserve	× ×
Goodwill	× ×	Specific reserve	× ×
Patent, trademark and copyright	× ×	Sinking fund	× ×
Investment:		Other reserve and funds	× ×
Investment in share and debentures	× ×	**Secured loans:**	
Investment in government securities	× ×	Debenture	× ×
Current assets:		Mortgaged loan	× ×
Sundry debtors	× ×	Bonds	× ×
Cash in hand	× ×	other secured loans and advance	× ×
Cash at bank	× ×	**Unsecured loan:**	
Loss tools	× ×	Fixed deposit	× ×
		Bank overdraft	× ×
		Short term loan from bank	× ×

Stores and spare parts	× ×	**Current liabilities:**	
Closing stock	× ×	Bills payable	× ×
Loan and advance:		Advance receipts	× ×
Deposit with suppliers	× ×	Sundry creditors	× ×
Bills receivable	× ×	Accounts payable	× ×
Prepaid expenses	× ×	Outstanding payable	× ×
Advance paid(tax, rate etc)	× ×	Outstanding expenses	× ×
Miscellaneous expenses:		Other current liabilities	× ×
Preliminary expenses	× ×	**Provisions:**	
Advertisement expenses	× ×	Provision for taxation	× ×
unadjusted development		Provision for staff	× ×
expenses	× ×	Provided fund	× ×
		Pension fund	× ×
		Other provisions	× ×
	× ×		× ×

10.2.2 Income Statement

The income statement is one of the financial statement used to assess a company's performance and financial position. The income statement summarizes the revenues and expenses generated by the company over the entire reporting period. The income statement is also called as profit and loss (P and L) statement. Statement of earnings, statement of income or statement of operations. The basic equation on which an income statement is based is:

Revenue – Expenses = Net income (Loss)

In case of trading business, it is prepared in two steps.

i) Trading account
ii) Profit and loss account

Importance of income statement

- To ascertain gross and net profit
- To keep control on indirect expenses
- Basis for managerial decision.
- Basis for preparing balance sheet.

10.2.2.1 Trading Account

The account which is prepared to determine the gross profit or gross loss of a business concern is called trading account. In equation, gross profit = sales–cost of goods sold. It should be noted that the result of the business determined through trading account is not true result. The true result is the net profit or the net loss which is determined through profit and loss account.

1. Features of trading account are:

i) It is the first stage of final accounts of a trading concern.

ii) It is prepared on the last day of an accounting period.
iii) Only direct revenue and expenses are considered in it.
iv) Direct revenues are recorded on its credit side and direct expenses are recorded on its debit side.
v) All items of direct expenses and direct revenue concerning current year are taken into account but no item relating to past or next year is considered in it.

2. Importance of trading account are:

i) It provides information related to gross profit and loss and helps in defining the upper limits for the operating expenses of the business.
ii) It helps in computation of gross profit ratio. A decrees in the gross profit indicates increase in the purchased cost or decrease in selling price.
iii) To ascertain the performance of different years of business through the gross profit ratio which is calculated by dividing the gross profit by sales.

3. Format of trading account

All the direct expenses are debited and all direct incomes are credited to trading account being a special nominal account. Trial balance is a list of all ledger accounts balance. So all the necessary information for preparation of a trading account is available from the trial balance. If the credit side of the trading account exceeds the debit side, the result is "gross profit" and it debit side exceeds the credit side, the result is "gross loss". The format of a trading accounting is shows below:

Trading account of .....
For the year ended .....

Dr. Cr.

Particulars	Rs.	Particulars	Rs.
To opening stock	× ×	By sales : × ×	
To purchase : × ×		Less: Sales return	
Less: Purchase return		or, return inward	
or, return outward		or, return from debtors: × ×	× ×
or, return to creditors : × ×	× ×		
To purchase expenses:		By closing stock	× ×
Carriage/Carriage inward	× ×	(By gross loss transferred	
Freight/Freight inward	× ×	to profit and loss account)	× ×
Freight on purchase	× ×		
Dock charges	× ×		
Coolie and cartage	× ×		
Import duty	× ×		

Custom duty on import	× ×		
Clearing charges	× ×		
Packing on purchase	× ×		
To factory/Manufacturing expenses:			
Wages/Salaries	× ×		
Factory rent	× ×		
Fuel and power	× ×		
Coal, gas, water	× ×		
Heating and lighting	× ×		
Store consumed	× ×		
Excise duty	× ×		
Royalties	× ×		
All other factory expenses	× ×		
To gross profit expenses	× ×		
To profit loss account	× ×		
	× ×		× ×

10.2.2.2 Profit and Loss Account

Profit and loss account is prepared after the completion of trading account. The profit and loss account is a financial statement that summarizes the revenues, costs and expenses during a specified period usually a fiscal year. Gross profit or loss of a business is ascertained through trading account and net profit is determined by deducting all the indirect expense from the gross profit through profit and loss account. Thus, profit and loss account starts with the result provided by trading account.

1. **Importance of profit and loss account are:**

i) It provides information about net profit earned or net loss suffered by the business.

ii) It helps in preparing balance sheet after finding out net profit or net loss.

iii) It helps in controlling indirect expenses and in improving profitability.

iv) It follows in the estimation of profits for the coming years by comparing the profits of previous year.

v) It helps for the calculation of important financial ratios.

2. **Format of profit and loss account**

Profit and loss account is also prepared as the trading account. Profit and loss account keeps all indirect income in the credit side with gross profit and all indirect expenses including gross loss are kept on debt side. The format of profit and loss account is shown below:

Profit and Loss Account of ..... Company
For the year ended .....

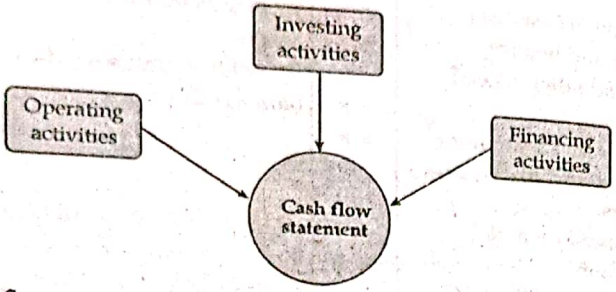
Dr. Particulars	Rs.	Particulars	Rs.	Cr.
To gross loss	× ×	By gross profit	× ×	
To office & administrative expenses:		By interest received	× ×	
Salary and wages	× ×	By bad debt recovered	× ×	
Printing and stationary	× ×	By dividend received	× ×	
Rent and taxes	× ×	By discount received	× ×	
Insurance and interest	× ×	By commission received	× ×	
Audit and license fee	× ×	By profit on sale of assets	× ×	
Lighting and heating	× ×	By appreciation on fixed assets	× ×	
Bonus and other official expenses	× ×	By net loss transferred to balance sheet	× ×	
Electricity and Telephone	× ×			
To selling and distribution expenses:				
Salesman salary and commission	× ×			
Packing & travelling expenses	× ×			
Advertising & publishing	× ×			
Free sample & ware house	× ×			
Carriage & freight outward	× ×			
Export duty and discount allowed	× ×			
Research and development expenses	× ×			
VAT and bad debts	× ×			
To other expenses and loses:				
Depreciation	× ×			
Repair & maintenance	× ×			
Abnormal loss on goods or machines	× ×			
To net profit transferred to balance sheet	× ×			
	× ×			× ×

It debit side exceeds credit side, the result shall be net loss. The net loss is then transferred to asset side of balance sheet. If credit side is more than

debit side, the result will be net profit and it is then transferred to liabilities side of balance sheet.

10.2.3 Cash Flow Statement

A cash flow statement also known as statement of cash flows is a financial statement that shows how changes in balance sheet accounts and income affect cash and cash equivalents, and breaks the analysis down to operating, investing and financial activities. Cash flow is the statement of sources and uses of cash funds that shows where from cash came and where cash was used during the given period. The difference between inflow and outflow of cash and cash equivalent is termed as net increase or decrease in cash and cash equivalent.



The cash flow statement reports cash flows during the period in 3 category.

- Operating activities
- Financing activities
- Investing activities

1. **Objectives of cash flow statement:**

i) To asset short term cash planning of the firm
ii) To evaluate the financial policies of the firm
iii) To know the liquidity position of the firm
iv) To know the company's operating, inverting and financing activities during accounting period.
v) To understand the reasons for variation in cash position of the firm.
vi) To help in planning of the repayment of loan replacement of fixed assets and other long term planning, etc.
vii) To know the cash payment and cash receipt of the firm during the accounting period.
viii) To evaluate the financial policies of the firm.

It is an important financial tool for the management internally and for the investors of creditors externally and for the investors or creditors in assessing the company's ability to manage cash flows, to generate positive future cash flow, to pay dividend and interest, etc.

10.3 RATIO ANALYSIS

Ratio is the mathematical relationship between two figure where one number is expressed in terms of another. Ratio analysis can evaluate the performance of any analysis of financial statement by the help of ratio between two accounting figures.

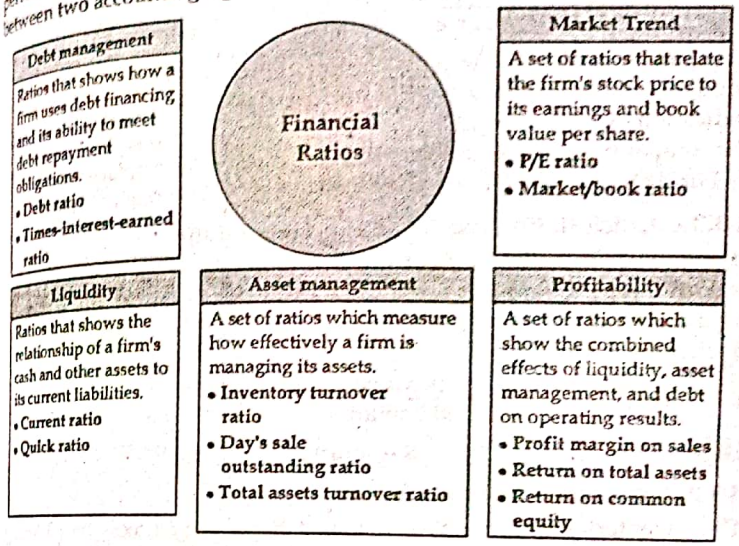


Figure: Types of ratios used in evaluating a firm's financial health.

Importance of ratio analysis:

- Helpful in decision making
- Helpful in future forecasting
- Helpful in corrective action
- Helpful in cost control
- Helpful in comparing inter-firm performance
- Helpful in measuring financial solvency
- Helpful in assessing operating efficiency of the business

10.3.1 Debt Ratio

It is also known as solvency/leverage/capital structure ratio. The ratio between total liabilities and total assets, generally called the debt ratio tells us the proportion of the company's assets that it has financed with debt

$$\therefore \text{Debt ratio} = \frac{\text{Total debt}}{\text{Total asset}}$$

Total debt includes both current liabilities and long term debt. If the debt ratio is unity, then the company has used debt to finance all of its assets. Debt ratio measures the ability of firm for paying the interest regularly and to repay the principal on agreed time.

10.3.2 Current Ratio

The current ratio measures a company's ability to pay its short term obligations. Current ratio is the ratio of current assets to current liabilities which is calculated by dividing current assets by current liabilities.

$$\therefore \quad \text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

Companies with a seemingly current ratio may not be safer than a company with a relatively low current ratio. The current ratio is an indication of a firm's liquidity. A current ratio of less than 1 indicates that the company may have problems meeting its short term obligations. A ratio of 1 or higher shows that company has enough assets to cover liabilities.

10.3.3 Quick Ratio/Acid-Test Ratio/Liquid Ratio

It measures the ability of firs to pay current liability immediately. Quick ratio is a type of liquidity ratio which measures the ability of a company to use its near cash or quick assets to extinguish or retire its current liabilities immediately.

$$\therefore \quad \text{Quick ratio} = \frac{\text{Quick assets}}{\text{Current liabilities}}$$

Higher the quick ratio, better the liquidity position.

10.3.4 Inventory Turnover Ratio

The inventory turnover ratio measures how many times the company sold and replaced its inventory over a specific period. It is also called stock turnover ratio. It is calculated by dividing the cost of goods sold by average inventory.

$$\therefore \quad \text{Inventory turnover ratio} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Cost of goods sold} = \text{Opening stock} + \text{Purchase} + \text{Direct expenses} - \text{Closing inventory}$$

$$= \text{Sales} - \text{Gross profit}$$

$$\text{Average inventory} = \frac{\text{Opening stock} + \text{Closing stock}}{2}$$

$$\text{Also, Inventory turnover ratio} = \frac{\text{Sales}}{\text{Averge inventory}} = \frac{\text{Sales}}{\text{Closing inventory}}$$

Hence, high inventory turnover indicates efficient inventory management. Low inventory turnover indicates stocks are kept for long time in store.

10.3.5 Total Asset Turnover (Investment Turnover Ratio)

The total assets turnover ratio measure how effectively the firm uses its total assets in generating its revenues. It is the ratio of sales to all the firm's assets.

$$\therefore \quad \text{Total assets turnover ratio} = \frac{\text{Net sales}}{\text{Total assets}}$$

Total assets = Fixed assets + investment + current assets, loan and advance – fictitious assets.

Net sales = Total sales – Sales return.

Higher the investment turnover ratio implies better utilization of total assets for making the sales and vice-versa.

10.3.6 Profit Margin on Sales

Profit margin is the direct expression of gross profit in relation of sales.

$$\therefore \quad \text{Gross profit margin} = \frac{\text{Gross profit amount}}{\text{Sales quantities}}$$

Higher ratio of net profit to net sales quantity is a indication of efficient management that reflects lower cost of goods sold and maximum profit.

$$\therefore \quad \text{Net profit margin} = \frac{\text{Net profit amount}}{\text{Sales quantities}}$$

10.3.7 Return on Total Assets

It is also called profit to assets ratio. The ratio of net income to total assets measures the return on total assets after interest and taxes.

$$\therefore \quad \text{Return on total assets} = \frac{\text{Net profit}}{\text{Total asets}}$$

$$= \frac{\text{Net income} + \text{Interest expense (1 – Tax rate)}}{\text{Average total asets}}$$

Higher ratio implies available resources are employed efficiently.

10.3.8 Price Earning Ratio

The price earnings ratio is the ratio for valuing a company that measures its current share price relative to its per share earnings. It is also known as price multiple or the earning multiple. It shows that how much investors are ready to pay per rupee of reported profit.

$$\therefore \quad \text{Price sharing ratio} = \frac{\text{Initial market price per share}}{\text{Current earning per share}}$$

10.3.9 Book Value Per Share

Book value per share measures the amount that would be distributed to holders of each share of common stock if all assets were sold at their balance sheet carrying amounts and if all creditors were paid off. It is the ratio between common shareholder's equity and number of common shares outstanding.

$$\therefore \quad \text{Book value per share} = \frac{\text{Common shareholdr's equity}}{\text{Number of common shares outstanding}}$$

$$= \frac{\text{Total common stockholder's equity} - \text{Preferred stock}}{\text{Number of common shares}}$$

A relatively high book value per share in relation to stock price often occurs when a stock is undervalued. Book value per share indicate the book value of each share of stock.

BOARD EXAM SOLVED PROBLEMS

1. What is financial ratios? Explain major financial ratios that can be applied in decision making process in business. [2015/Fall, 2016/Spring, 2017/Fall, 2018/Spring, 2019/Spring]

Answer:

A financial ratio or accounting ratio is a relative magnitude of two selected numerical values taken from an enterprise's financial statements. Financial ratios are relationship determined from a company's financial information and used for comparison purpose.

See the topic 10.3, 10.3.1, 10.3.2, 10.3.3, 10.3.4, 10.3.5, 10.3.6, 10.3.7, 10.3.8 and 10.3.9.

2. Difference between net profit and gross profit. [2018/Fall]

Answer:

S.N.	Gross profit	Net profit
i)	Gross profit is the difference between sales and cost of goods sold.	Net profit is the difference between gross profit and all other income less all indirect expenses and losses.
ii)	It is ascertained from the trading account.	It is ascertained from the profit and loss account.
iii)	It is transferred to profit and loss account.	It is transferred to capital account.
iv)	It does not include any income from other sources.	It may include income from other sources.
v)	**Formula:** Gross profit = Revenue – Costs of goods sold	**Formula:** Net profit = Gross profit – Expenses
vi)	It shows the credit balance of the trading account.	It shows the credit balance of the profit and loss account.
vii)	It controls excess costs.	It shows a company's performance in a year.
viii)	It is difficult to make financial decisions using gross profit as it does not include expenses, taxes and interest on loans.	This is true profit that a company can use to make business decisions for its development.

3. Difference between general accounting and cost accounting. [2011/Fall, 2014/Spring, 2016/Fall, 2017/Spring]

Answer:

S.N.	General/Financial accounting	Cost accounting
i)	It provides information about the business in a general way. It tells about the profit and loss and financial position of the business to owners and other outside parties.	It provides information to management for proper planning, operation control and decision making.
ii)	These accounts are kept in such a way as to meet the requirements of companies act and income tax act.	These accounts are generally kept voluntarily to meet the requirements of management. But now companies act has made it obligatory to keep cost records in some manufacturing industries.
iii)	It classifies, records and analyzes the transactions in a subjective manner *i.e.*, according to the nature of expenses.	It records the expenditure in an objective manner *i.e.*, according to the purpose for which the costs are incurred
iv)	Stocks are valued at cost or market price whichever is less.	Stocks are valued at cost.
v)	Financial accounts do not provide information on the relative efficiencies of various workers, plans and machinery.	Cost accounts provide valuable information on the relative efficiencies of various workers, plants and machinery.
vi)	The costs are reported in aggregate in financial accounts.	The costs are broken down on a unit basis in cost accounts.
vii)	It reports operating results and financial position usually at the end of the year.	It gives information through cost reports to management as and when desired.
viii)	Monetary information is only used *i.e.*, only monetary transactions are recorded.	Non monetary information like units is also used *i.e.*, it deals with monetary as well as non-monetary information.
ix)	It is the account of whole business and discloses the net profit or less of the business as a whole.	It is only a part of general account and disclose profit and loss of each product (job or service)

4. Define accounting. How do you formulate an accounting equation? [2014/Spring, 15/Spring, 2017/Fall]

Answer: See the topic 10.1 and 10.1.3.

5. Explain the golden role of accounting. [2011/Fall, 2014/Fall, 2016/Fall]

Answer:

Golden rules of accounting are the basic accounting rules on the basis of which accounting entries are recorded. It is the traditional approach for the rules of debit and credit. There are 3 types of account *i.e.*, real, personal and nominal account.

i) Personal account

Personal accounts are related to individuals, firms, companies etc. The person who receives a benefit is debited and who gives the benefit is credited. The rule for personal accounts is:

"Debit the receiver, and credit the giver"

ii) Real account

All assets of a firm, which are tangible or intangible fall under the category real account. The golden rule for real account is:

"Debit what comes in and credit what goes out"

iii) Nominal account

Accounts which are related to incomes, expenses, gains and losses of the business are called nominal account. The golden rule of debit and credit for nominal accounts is rule of debit and credit for real account is,

"All expenses and losses are debited and all incomes and gains are credited."

6. What are the significance of financial statements in business? Explain the financial statement with its type. [2016/Spring]

Answer: See the topic 10.2, 10.2.1, 10.2.2 and 10.2.3.

7. What is income statement and balance sheet? How are they related to each other? [2015/Spring, 2018/Spring, 2019/Fall]

Answer: See the topic 10.2.2 and 10.2.1.

The relationship between balance sheet and income statement is that the profit of the business shown in the income statement belongs to the owners and this is shown by a movement in equity between the opening and closing balance sheets of the business. The income statement reports a company's financial performance while the balance sheet reports its financial health. One proceeds from the another. A company financial performance regulates its financial health. Performance and health are liked through the net income account on the income statement and the equity account on the balance sheet.

So, the relationship between balance sheet and income statement is that the profit for the period which comes from the income statement represents the movement on equity which is the difference between the opening and closing equity in the balance sheet of the business.

8. What are the elements used in debit and credit side of trading account and profit and loss account? [2011/Fall]

Answer:

Elements that are used in debit and credit side of

a) **Trading account:**

On debit side:

i) opening stock
ii) purchase and purchase return
iii) purchase related expenses
iv) factory or manufacturing expenses

On credit side:

i) sales and sales return
ii) closing stock

b) **Profit and loss account**

On debit side:

i) financial expenses
ii) depreciation and repair and maintenance
iii) selling and distributing expenses
iv) office and administrative expenses
v) other expenses and losses

On credit side:

i) indirect incomes
ii) other incomes and profits

9. Write the short notes on balance sheet. [2013/Spring, 2017/Fall]

Answer: See the topic 10.2.1.

10. Difference between trading account and profit and loss account. What are the elements used in balance sheet? [2014/Fall]

Answer:

Elements used in balance sheet

The balance sheet consists of three major elements: assets, liabilities and owner's equity.

i.e., Assets = Liabilities + Owner's equity.

S.N.	Trading account	Profit and loss account
i)	Trading account is the part of profit and loss account.	Profit and loss account is the main account.
ii)	Gross loss or gross profit is ascertained from trading account.	Net profit or net loss is ascertained from profit and loss account.
iii)	Balance of trading account is transferred to profit and loss account.	Balance of profit and loss account is transferred to capital account.

iv)	It does not start with the balance of any account.	It always start with the balance of a trading account (gross profit or gross loss)
v)	All direct expenses are considered in it.	All expenses connected with sales and administration of business are recorded.
vi)	It is the first stage of final account.	It is the second stage of final account.

11. Prepare one sample of balance sheet including the components of assets and liability. [2013/Fall, 2017/Spring]

Answer: See the topic 10.2.1. (3)

12. Define asset and liability. [2013/Fall]

Answer: See the topic 10.1.1 and 10.1.2.

13. List out the major financial statements. Explain the main purpose of preparing the profit and loss account along with providing its appropriate format. [2012/Spring]

Answer:

Major financial statements are:

i) Balance sheet
ii) Income statement
iii) Cash flow statement

Main purpose of preparing the profit and loss account are:

See the topic 10.2.2.2 (1)

General format:

See the topic 10.2.2.2 (2)

14. Define the financial ratios and their importance for financing. [2017/Spring]

Answer:

Financial ratio is defined as the mathematical expression of the relationship between two accounting figures.

Importance of financial ratios are:

i) Helpful in assessing operating efficiency of the business.
ii) Helpful in measuring financial solvency.
iii) Helpful in decision making
iv) Helpful in corrective action
v) Helpful in cost control.
vi) Helpful in future forecasting
vii) Helpful in comparing inter-firm preference.

Ratios are used for efficiency and profitability measurement of any organization by the relationship between figures and evaluating financial performance. The popular used ratios are:

i) Debt ratios
ii) Operating ratios
iii) Liquidity ratios

15. The following is trial balance of Acharya Company. [2015/Fall]

Particulars	Debit (Rs.)	Credit (Rs.)
Capitals		2,00,000
Plants and machinery	50,000	
Furniture and fitting	75,000	
Motor van	24,000	
Sundry debtors	40,000	
Cash at bank	7,000	
Wages	1,50,000	
Purchases and sales	2,13,000	4,00,000
Bills receivable and payable	35,000	15,000
Sundry creditors		20,000
Salaries	36,000	
Drawings	20,000	
Discount received		10,000
Bank loan		30,000
General reserve		20,000
Opening stock	40,000	
Bad debt	5,000	
Total	6,95,000	6,95,000

Value of closing stock = Rs. 52,000
Net Profit = Rs. 18,600
Required: Balance sheet.

Solution:

Profit and loss account

For the year ended 30th Chaitra 2074

Dr. Cr.

Particulars	Amount	Particulars	Amount
To opening stock	40,000	By gross profit	49,000
To purchases	2,13,000	By discount received	10,000
To wages	1,50,000	By sales	4,00,000
To gross profit	49,000	By closing stock	52,000
To salaries	36,000		
To bad debt	5,000		
To net profit	18,000		
Total	59,000		59,000

Balance sheet of Acharya Company

Assets	Amount	Capital and liabilities	Amount
Plant and machinery	50,000	Capital : 2,00,000	
Furniture and fitting	75,000	Add: Net profit : 18,000	
Motor van	24,000	Less: Drawing : 20,000	1,98,000
Bills receivable	35,000	General reserve	20,000
Sundry debtors	40,000	Bank loan	30,000
Closing stock	52,000	Bills receivable	15,000
Cash at bank	7,000	Sunday creditors	20,000
Total	2,83,000		2,83,000

16. Prepare balance sheet of ABC Company from the following information as on 31st December 2014. [2014/Spring]

Capital	40,000	Building	3,00,000
Sundry debtors	15,000	Furniture	12,500
Cash in hand	4,000	Cash at bank	7,500
Bank overdraft	85,200	Bills receivable	4,500
Sundry creditors	3,500	Closing stock	3,500
Reserve funds	4,500	Net profit	20,500

Solution:

Balance sheet of ABC Company

as on 31st December 2014

Assets	Amount	Capital and liabilities	Amount
Building	30,000	Capital : 40,000	
Furniture	12,500	Add: Net profit : 20,500	60,500
Bills receivable	4,500	Bank overdraft	85,200
Closing stock	3,500	Sunday creditors	3,500
Sundry debtors	15,000	Reserve fund	4,500
Cash at bank	7,500		
Cash in hand	4,000		
Total	77,000		77,000

17. From the following trial balance, prepare P/L account and balance sheet. [2018/Fall]

Debit balance	Amount	Credit balance	Amount
Closing stock	30,000	Capital	2,50,000
Sundry debtors	50,000	Gross profit	1,22,000
Plant and machinery	2,25,000	Dividend received	1,250
Goodwill	14,500	Interest received	750
Land and building	1,35,000	Sundry creditors	39,000

Salaries and wages	27,500	Reserve fund	50,000
Rent	7,500	Bank loan	50,000
Selling expenses	12,500	Bank overdraft	23,500
Cash at bank	10,000		
Deposit with custom	7,500		
Advertisement	5,000		
Investment	12,000		
Total	5,36,500		5,36,500

Solution:

Profit and loss account of ..... Company

For the year ended 2074/12/30

Dr. Cr.

Particulars	Amount	Particulars	Amount
To rent	7,500	By gross profit	1,22,000
To salaries and wages	27,500	By interest	750
To selling expenses	12,500	By dividend received	1,250
To advertisement	5,000		
To net profit	71,500		
Total	1,24,000		1,24,000

Balance sheet of ..... Company

as on 2074/12/30

Assets	Amount	Capital and liabilities	Amount
Closing stock	30,000	Capital : 2,50,000	
Cash at bank	10,000	Add: Net profit : 71,500	3,21,500
Sundry debtors	50,000	Bank loan	50,000
Plant and machinery	2,25,000	Bank over draft	23,500
Land and building	1,35,000	Sunday creditors	39,000
Goodwill	14,500	Reserve fund	50,000
Investment	12,000		
Deposit with custom	7,500		
Total	4,84,000		4,84,000

18. What do you mean by profit and loss account and balance sheet? [2012/Fall]

Answer: See the topic 10.2.2.2 and 10.2.1.

19. Write short notes on: Ratio analysis for making decision. [2019/Fall]

Solution: See the topic 10.3.

20. Explain analytically the following ratios

i) Debt ratio
ii) Current ratio
iii) Quick ratio/acid test ratio
iv) Cost of capital

Solution:

i) See the topic 10.3.1
ii) See the topic 10.3.2
iii) See the topic 10.3.3
iv) See the topic 9.2

21. Write short notes on: Quick acid test ratio. [2019/Spring]

Solution: See the topic 10.3.3.

22. Write short notes on: Asset and liabilities. [2020/Fall]

Solution: See the topic 10.1.1 and 10.1.2.

23. Explain: [2020/Fall]

i) Debt ratio
ii) Current ratio
iii) Cost of equity
iv) Cost of debt

Solution:

i) See the topic 10.3.1.
ii) See the topic 10.3.2.
iii) See the topic 9.2.1.
iv) See the topic 9.2.2.

24. Fill up the following data on standard balance sheet: cash = 2000, Account receival = $ 3000, Inventory = $ 5000, Land and building = $ 13000, equipment = $ 7000, depreciation = $ 1000, Bank overdraft = $ 500, Account payable = $ 1000 and returned earning = $ 6500? [2020/Fall]

Solution:

Balance sheet of ....., Company
as on 1-Jan-2022

Assets	Amount	Capital and liabilities	Amount
Cash	$ 2000	Depreciation	$ 1000
Inventory	$ 5000	Bank overdraft	$ 500
Account receivable	$ 3000	Account payable	$ 1000
Land and building	$ 13000	Returned earning	$ 6500
Equipment	$ 7000		

ADDITIONAL PROBLEMS

1. Difference between trial balance and balance sheet.

Answer:

S.N.	Trial balance	Balance sheet
i)	It shows the balance of all ledger account.	It shows the balances of personal and real accounts only.
ii)	It is prepared after the completion of the ledger account.	It is prepared after the completion of trading and P and L account.
iii)	It shows the opening stock.	It shows the closing stock.
iv)	Its objective is to check the arithmetical accuracy.	Its objective is to reveal the financial position of the business.
v)	The preparation of trial balance is not compulsory.	The preparation of balance sheet is must.
vi)	Trial balance can't be used as evidence in the court of law.	Balance sheet can be presented as documentary evidence in the court of law.
vii)	It contains all ledgers *i.e.*, capital, liabilities, incomes, gains, assets, expenses and losses.	It contains only assets, liabilities and capital including net profit and net loss.

2. Difference between journal and ledger.

Answer:

S.N.	Journal	Ledger
i)	Journal is a subsidiary book of account. It is the store house of recording transitions.	Ledger is the permanent and final book of accounts. It is termed as the means of classified transactions.
ii)	Journal helps in preparing ledger account correctly.	The objectives of ledger is to know income and expenditure of different heads.
iii)	Transactions are recorded in journal in chronological order of dates.	Ledger is prepared according to nature of accounts.
iv)	The total results of transitions cannot be known from journal.	Results of particular head of accounts can be known from ledger.
v)	It is a book of primary entry.	It is a book of final entry.
vi)	The process of recording of entries in these book is called journalizing.	The process of recording of entries in the ledger is called posting.
vii)	Journal is not balanced.	Every account is balanced.
viii)	The narration is given after every entry.	The narration is not given.

3. **Moon and Moon Company has just completed preparing its trial balance as of 30 June 2018. Prepared income statement and balance sheet.**

Moon and Moon Company
Trial balance
As of 30 June 2018

Accounts	Debit (Rs.)	Credit (Rs.)
Cash	6,200	
Accounts receivable	10,400	
Prepaid rent	4,400	
Chemical inventory	9,400	
Accumulated depreciation		1,050
Accounts payable		1,180
Capital stock		5,000
Retained earnings		25,370
Treatment revenue		40,600
Wages and salary expenses	22,500	
Utility expense	1,240	
Advertising expenses	860	
Equipment	18,200	
Total	73,200	73,200

Solution:

Income statement of Moon and Moon Company
As of 30 June 2018

Dr. Cr.

Particulars	Amount	Particulars	Amount
To wages and salaries	22,500	By treatment revenue	40,600
To advertising expenses	860		
To utility expenses	1,240		
To net profit	16,000		
Total	40,600		40,600

Balance sheet of Moon and Moon Company
As of 30 June 2018

Assets	Amount	Capital and liabilities	Amount
Cash	6,200	Capital : 5,000	
Account receivable	10,400	Add: Net profit : 16,000	
Prepaid rent	4,400	Add: retain earning : 25,370	46,370
Chemical inventory	9,400	Accumulated depreciation	1,050
Equipment	18,200	Account payable	1,180
Total	48,600		48,600

PRACTICE QUESTIONS

1. Define economics. Also discuss the flow of goods, services, resources and money payments in a simple economy with the help of a suitable diagram.
2. Illustrate the effect of price on demand and supply; illustrate with the help of a diagram.
3. Discuss the factors which influence demand and supply.
4. Give the definition and scope of engineering economics.
5. Define the following costs with examples:
   a) Sunk cost
   b) Opportunity cost
   c) Marginal cost
   d) Marginal revenue
6. Define break-even point. Draw a break-even chart and explain its components.
7. Consider the following data of a company for the year 1998.
   Sales = Rs. 3,40,000
   Fixed cost = Rs. 70,000
   Variable cost = Rs. 55,000
   Find the following:
   a) Profit
   b) BEP
8. Explain the time value of money.
9. Give practical applications of various interest formulas.
10. A person deposits a sum of Rs. 4,00,000 in a bank for his son's education who will be admitted to a professional course after 7 years. The bank pays 17% interest rate, compounded annually. Find the future amount of the deposited money at the time of admitting his son in the professional course.
11. A person needs a sum of Rs. 3,00,000 for his daughter's marriage which will take place 14 years from now. Find the amount of money that he should deposit now in a bank if the bank gives 19% interest, compounded annually.
12. A person who is just 30 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 30 years starting from the end of next year. The bank gives 15% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.
13. A company is planning to expand its business after 5 years from now. The expected money required for the expansion programme is Rs. 8,00,00,000. The company can invest Rs. 80,00,000 at the end

of every year for the next five years. If the assured rate of return of investment is 18% for the company, check whether the accumulated sum in the account would be sufficient to meet the fund for the expansion programme. If not, find the difference in amounts for which the company should make some other arrangement after 5 years.

14. A financial institution introduces a plan to pay a sum of Rs. 15,00,000 after 10 years at the rate of 18%, compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10 years to receive Rs. 15,00,000 after 10 years from the institution.

15. A company is planning to expand its business after 5 years from now. The money required for the expansion programme is Rs. 4,00,00,000. What annual equivalent amount should the company deposit at the end of every year at an interest rate of 17 % compounded annually to get Rs. 4,00,00,000 after 5 years from now?

16. A company wants to set-up a reserve which will help it to have an annual equivalent amount of Rs. 15,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made as the reserve amount now.

17. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually. Suggest the best alternative to the customers.

18. A company takes a loan of Rs. 25,00,000 to modernize its boiler section. The loan is to be repaid in 20 equal installments at 12% interest rate, compounded annually. Find the equal installment amount that should be paid for the next 20 years.

19. A bank gives loan to a company to purchase an equipment which is worth of Rs. 5,00,000, at an interest rate of 18% compounded annually. This amount should be repaid in 25 yearly equal installments. Find the installment amount that the company has to pay to the bank.

20. A working woman is planning for her retired life. She has 20 more years of service. She would like to deposit 12% of her salary which is Rs. 5,000 at the end of the first year and thereafter she wishes to deposit the same amount (Rs. 5,000) with an annual increase of Rs. 1,000 for the next 14 years with an interest rate of 18%. Find the total amount at the end of the 15^{th} year of the above series.

21. A person invests a sum of Rs. 50,000 in a bank at a nominal interest rate of 18% for 15 years. The compounding is monthly. Find the maturity amount of the deposit after 15 years.

22. The cost of erecting an oil well is Rs. 1,50,00,000. The annual equivalent yield from the oil well is Rs. 30,00,000. The salvage value after its useful life of 10 years is Rs. 2,00,000. Assuming an interest rate of 18%, compounded annually, find out whether the erection of the oil well is financially feasible, based on the present worth method.

23. The details of the feasibility report of a project are as shown below. Check the feasibility of the project based on present worth method, using $i = 20\%$.

    Initial outlay = Rs. 50,00,000
    Life of the project = 20 years.
    Annual equivalent revenue = Rs. 15,00,000
    Modernizing cost at the end of the 10th year = Rs. 20,00,000
    Salvage value at the end of project life = Rs. 5,00,000.

24. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually. You are asked to advise the best alternative for the customers based on the present worth method of comparison.

25. A company has two alternatives for satisfying its daily travel requirements of its employees for the next five years:
    Alternative 1: Renting a vehicle at a cost of Rs. 10,00,000 per year.
    Alternative 2: Buying a vehicle for Rs. 5,00,000 with an operating and maintenance cost of Rs. 3,50,000 per year. The salvage value of the vehicle after five years is Rs. 1,00,000.
    Select the best alternative based on the present worth method of comparison using the interest rate of 20%, compounded annually.

26. A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs. 3,00,000, starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume $i = 15\%$, compounded annually.

27. A person invests a sum of Rs. 2,00,000 in a business and receives equal net revenue of Rs. 50,000 for the next 10 years. At the end of the 10th year, the salvage value of the business is Rs. 25,000. Find the rate of return of the business.

28. Define the following:
    a) Depreciation
    b) Book value
29. Distinguish between declining balance method of depreciation and double declining balance method of depreciation.
30. The Alpha Drug Company has just purchased a capsulating machine for Rs. 20,00,000. The plant engineer estimates that the machine has a useful life of five years and a salvage value of Rs. 25,000 at the end of its useful life. Compute the depreciation schedule for the machine by each of the following depreciation methods:
    a) Straight line method of depreciation
    b) Sum-of-the-years digits method of depreciation
    c) Double declining balance method of depreciation
31. A company has recently purchased an overhead travelling crane for Rs. 25,00,000. Its expected life is seven years and the salvage value at the end of the life of the overhead travelling crane is Rs. 1,00,000. Using the straight line method of depreciation, find the depreciation and the book value at the end of third and fourth year after the crane is purchased.
32. An automobile company has purchased a wheel alignment device for Rs. 10,00,000. The device can be used for 15 years. The salvage value at the end of the life of the device is 10% of the purchase value. Find the following using the double declining balance method of depreciation:
    a) Depreciation at the end of the seventh year
    b) Depreciation at the end of the twelfth year
    c) Book value at the end of the eighth year
33. A company has purchased a bus for its officers for Rs. 10,00,000. The expected life of the bus is eight years. The salvage value of the bus at the end of its life is Rs. 1,50,000. Find the following using the sinking fund method of depreciation:
    a) Depreciation at the end of the third and fifth year
    b) Book value at the end of the second year and sixth year
34. Consider Problem 4 and find the following using the sum-of-the-years digits method of depreciation:
    a) Depreciation at the end of the fourth year
    b) Depreciation at the end of the seventh year
    c) Book value at the end of the fifth year
    d) Book value at the end of the eighth year
35. A company has purchased a Xerox machine for Rs. 2,00,000. The salvage value of the machine at the end of its useful life would be insignificant. The maximum number of copies that can be taken during its lifetime is 1,00,00,000. During the fourth year of its operation, the number of copies taken is 9,00,000. Find the depreciation for the fourth year of operation of the Xerox machine using the service output method of depreciation.

36. A manufacturer of TV buys TV cabinet at Rs. 500 each. In case the company makes it within the factory, the fixed and variable costs would be Rs. 4,00,000 and Rs. 300 per cabinet respectively. Should the manufacturer make or buy the cabinet if the demand is 1,500 TV cabinets?
37. A manufacturer of motor cycles buys side boxes at Rs. 240 each. In case he makes it himself, the fixed and variable costs would be Rs. 30,00,000 and Rs. 90 per side box respectively. Should the manufacturer make or buy the side boxes if the demand is 2,500 side boxes?
38. A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.
39. You deposit Rs. 5,000 in a savings account that earns 8% simple interest per year. What is the minimum number of years you must wait to double your balance? Suppose instead that you deposit the Rs. 5,000 in another savings account that earns 7% interest compounded yearly. How many years will it take now to double your balance?
40. Compare the interest earned by Rs. 1,000 for five years at 8% simple interest with that earned by the same amount for five years at 8% compounded annually.
41. You are considering investing Rs. 3,000 at an interest rate of 8% compounded annually for five years or investing the Rs. 3,000 at 9% per year simple interest for five years. Which option is better?
42. You are about to borrow Rs. 10,000 from a bank at an interest rate of 9% compounded annually. You are required to make five equal annual repayments in the amount of Rs. 2,571 per year, with the first repayment occurring at the end of year 1. Show the interest payment and principal payment in each year.
43. Suppose that you are obtaining a personal loan from your uncle in the amount of Rs. 20,000 (now) to be repaid in two years to cover some of your college expenses. If your uncle usually earns 8% interest (annually) on his money, which is invested in various sources, what minimum lump-sum payment two years from now would/make your uncle happy?
44. What will be the amount accumulated by each of these present investments?

a) Rs. 5,000 in 8 years at 5% compounded annually
b) Rs. 2,250 in 12 years at 3% compounded annually
c) Rs. 8,000 in 31 years at 7% compounded annually
d) Rs. 25,000 in 7 years at 9% compounded annually

45. What is the present worth of these future payments?
a) Rs. 5,500 6 years from now at 10% compounded annually
b) Rs. 8,000 15 years from now at 6% compounded annually
c) Rs. 30,000 5 years from now at 8% compounded annually
d) Rs. 15,000 8 years from now at 12% compounded annually

46. For an interest rate of 13% compounded annually, find
a) How much can be lent now if Rs. 10,000 will be repaid at the end of five years?
b) How much will be required in four years to repay a Rs. 25,000 loan received now?

47. How many years will it take an investment to triple itself if the interest rate is 12% compounded annually?

48. What is the future worth of a series of equal year-end deposits of Rs. 1,000 for 10 years in a savings account that earns 7%, annual interest if
a) All deposits are made at the *end* of each year?
b) All deposits are made at the *beginning* of each year?

49. What equal annual series of payments must be paid into a sinking fund to accumulate the following amounts?
a) Rs. 22,000 in 13 years at 6% compounded annually
b) Rs. 65,000 in 8 years at 7% compounded annually

50. A no-load (commission-free) mutual fund has grown at a rate of 11% compounded annually since its beginning. If it is anticipated that it will continue to grow at that rate, how much must be invested every year so that Rs. 15,000 will be accumulated at the end of five years?

51. You have borrowed Rs. 25,000 at an interest rate of .16%. Equal payments will be made over a three-year period. (The first payment will be made at the end of the first year.) What will the annual payment be, and what will the interest payment be for the second year?

52. What single amount at the end of the fifth year is equivalent to a uniform annual series of Rs. 3,000 per year for 10 years if the interest rate is 9% compounded annually?

53. You have Rs. 10,000 available for investment in stock. You are looking for a growth stock whose value can grow to Rs. 35,000 over five years. What kind of growth rate are you looking for?

54. A department store has offered you a credit card that charges interest at 1.05% per month, compounded monthly. What is the nominal interest (annual percentage) rate for this credit card? What is the effective annual interest rate?

55. A local bank advertised the following information: Interest 6.89%-effective annual yield 7.128%. No mention was made of the interest period in the advertisement. Can you figure out the compounding scheme used by the bank?

56. College Financial Sources, which makes small loans to college students, offers to lend Rs. 500. The borrower is required to pay Rs. 400 at the end of each week for 16 weeks. Find the interest rate per week. What is the nominal interest rate per year? What is the effective interest rate per year?

57. A loan company offers money at 1.8% per month, compounded monthly.
a) What is the nominal interest rate?
b) What is the effective annual interest rate?
c) How many years will it take an investment to triple if interest is compounded monthly?
d) How many years will it take an investment to triple if the nominal rate is compounded continuously?

58. Suppose your savings account pays 9% interest compounded quarterly. If you deposit Rs. 10,000 for one year, how much would you have?

59. What will be the amount accumulated by each of these present investments?
a) Rs. 5,635 in 10 years at 5% compounded semiannually.
b) Rs. 7,500 in 15 years at 6% compounded quarterly.
c) Rs. 38,300 in 7 years at 9% compounded monthly.

60. How many years will it take an investment to triple if the interest rate is 9% Compounded (a) Quarterly? (b) Monthly? (c) Continuously?

61. A series of equal quarterly payments of Rs. 5,000 for 12 years is equivalent to what present amount at an interest rate of 9% compounded (a) Quarterly? (b) Monthly? (c) Continuously?

62. What is the future worth of an equal payment series of Rs. 3,000 each quarter for five years if the interest rate is 8% compounded continuously.

63. Suppose that Rs. 2,000 is placed in a bank account at the end of each quarter over the next 15 years. What is the future worth at the end of 15 years when the interest rate is 6% compounded (a) Quarterly? (b) Monthly? (c) Continuously?

64. If the interest rate is 8.5% compounded continuously, what is the required quarterly payment to repay a loan of Rs. 12,000 in five years?

65. What is the future worth of a series of equal monthly payments of Rs. 2,500 if the series extends over a period of eight years at 12% interest compounded
a) Quarterly? b) Monthly? c) Continuously?

66. A series of equal quarterly payments of Rs. 2,000 for 15 years is equivalent to what future lump-sum amount at the end of 10 years at an interest rate of 8% compounded continuously?

67. What will be the required quarterly payment to repay a loan of Rs. 32,000 in five years, if the interest rate is 7.8% compounded continuously?

68. A series of equal quarterly payments of Rs. 4,000 extends over a period of three years. What is the present worth of this quarterly payment series at 8.75% interest compounded continuously?

69. Raj is purchasing a Rs. 24,000 automobile, which is to be paid for in 48 monthly installments of Rs. 543.35. What effective annual interest is he paying for this financing arrangement?

70. What is the present worth of the following series of payments?
a) Rs. 1,500 at the end of each six-month period for 12 years at 8% compounded semiannually.
b) Rs. 2,500 at the end of each quarter for 8 years at 8% compounded quarterly.
c) Rs. 3,800 at the end of each month for 5 years at 9% compounded monthly.

71. Shyam borrowed Rs. 25,000 from a bank at an interest rate of 10% compounded monthly. The loan will be repaid in 36 equal monthly installments over three years. Immediately after his 20^{th} payment, Bob desires to pay the remainder of the loan in a single payment. Compute the total amount he must pay.

72. Sagarmatha Corporation purchased a vibratory finishing machine for Rs. 20,000 in year 0. The useful life of the machine is 10 years, at the end of which the machine is estimated to have a salvage value of zero. The machine generates net annual revenues of Rs. 6,000. The annual operating and maintenance expenses are estimated to be Rs. 1,000. If Covington's MARR is 15%, how many years will it take before this machine becomes profitable?

73. A group of concerned citizens has established a trust fund that pays 6% interest, compounded monthly, to preserve a historical building by providing annual maintenance funds of Rs. 30,000 forever. Compute the capitalized equivalent amount for these building maintenance expenses.

74. You are considering purchasing a dump truck. The truck will cost Rs. 45,000 and have an operating and maintenance cost that starts at Rs. 15,000 the first year and increases by Rs. 2,000 per year. Assume that the salvage value at the end of five years is Rs. 9,000 and interest rate is 12%. What is the equivalent annual cost of owning and operating the truck?

75. Emerson Electronics Company just purchased a soldering machine to be used in its assembly cell for flexible disk drives. The soldering machine cost Rs. 250,000. Because of the specialized function it performs, its useful life is estimated to be five years. It is also estimated that at that time its salvage value will be Rs. 40,000. What is the capital cost for this investment if the firm's interest rate is 18%?

76. You wish to sell a bond that has a face value of Rs. 1,000. The bond bears an interest rate of 6%, payable semiannually. Four years ago, the bond was purchased at Rs. 900. At least an 8% annual return on the investment is desired. What must be the minimum selling price of the bond now in order to make the desired return on the investment?

77. With Rs. 10,000 available, you have two investment options. The first is to buy a certificate of deposit from a bank at an interest rate of 10% annually for five years.
The second choice is to purchase a bond for Rs. 10,000 and invest the bond's interest in the bank at an interest rate of 9%. The bond pays 10% interest annually and will mature to its face value of Rs. 10,000 in five years. Which option is better? Assume that your MARR is 9% per year.

78. You are considering two types of automobiles. Model A costs Rs. 18,000 and model B costs Rs. 15,624. Although the two models are essentially the same, after four years of use model A can be sold for Rs. 9,000, while model B can be sold for Rs. 6,500. Model A commands a better resale value because its styling is popular among young college students. Determine the rate of return on the incremental investment of Rs. 2,376. For what range of values of your MARR is model A preferable?

79. Identify which of the following transactions and events are product costs and which are period costs:
- Storage and material handling costs for raw materials.
- Gains or losses on the disposal of factory equipment.
- Lubricants for machinery and equipment used in production.
- Depreciation of a factory building.
- Depreciation of manufacturing equipment.
- Depreciation of the company president's automobile.

- Leasehold costs for land on which factory buildings stand.
- Inspection costs of finished goods.
- Direct labor cost.
- Raw-materials cost.
- Advertising expenses.

80. Identify which of the following costs are fixed and which are variable:
- Wages paid to temporary workers.
- Property taxes on a factory building.
- Property taxes on an administrative building.
- Sales commission.
- Electricity for machinery and equipment in the plant.
- Heating and air-conditioning for the plant.
- Salaries paid to design engineers.
- Regular maintenance on machinery and equipment.
- Basic raw materials used in production.
- Factory fire insurance.

81. An executive from a large merchandising firm has called your vice-president for production to get a price quote for an additional 100 units of a given product. The vice-president has asked you to prepare a cost estimate. The number of hours required to produce a unit is 5. The average labor rate is Rs. 12 per hour. The materials cost Rs. 14 per unit. Overhead for an additional 100 units is estimated at 50% of the direct labor cost. If the company wants to have a 30% profit margin, what should be the unit price to quote?

82. A machine now in use was purchased four years ago at a cost of Rs. 20,000. It has a book value of Rs. 6,246. It can be sold for Rs. 7,000, but could be used for three more years, at the end of which time it would have no salvage value. What is the current amount of economic depreciation for this asset?

83. General Service Contractor Company paid Rs. 200,000 for a house and lot. The value of the land was appraised at Rs. 65,000 and the value of the house at Rs. 135,000. The house was then torn down at an additional cost of Rs. 5,000 so that a warehouse could be built on the lot at a cost of Rs. 250,000. What is the total value of the property with the warehouse? For depreciation purposes, what is the cost basis for the warehouse?

84. A new drill press was purchased for Rs. 126,000 by trading in a similar machine that had a book value of Rs. 39,000. Assuming that the trade-in allowance is Rs. 40,000 and that Rs. 86,000 cash is to be paid for the new asset, what is the cost basis of the new asset for depreciation purposes?

85. A firm is trying to decide whether to keep an item of construction equipment for another year. The firm is using DDB for book purposes, and this is the fourth year of ownership of the equipment, which cost Rs. 150,000 new. What is the depreciation in year 3?

86. The double-declining-balance method is to be used for an asset with a cost of Rs. 68,000, an estimated salvage value of Rs. 12,000, and an estimated useful life of six years.
a) What is the depreciation for the first three fiscal years, assuming that the asset was placed in service at the beginning of the year?
b) If switching to the straight-line method is allowed, when is the optimal time to switch?

87. If a truck for hauling coal has an estimated net cost of Rs. 100,000 and is expected to give service for 250,000 miles, resulting in a salvage value of Rs. 5,000, depreciation would be charged at a rate of 38 cents per mile. Compute the allowed depreciation amount for the same truck's usage amounting to 55,000 miles.

88. Nepal Paving Company purchased a hauling truck on January 1, 2005, at a cost of Rs. 32,000. The truck has a useful life of eight years with an estimated salvage value of Rs. 5,000. The straight-line method is used for book purposes. For tax purposes, the truck would be depreciated with the MACRS method over its five-year class life. Determine the annual depreciation amount to be taken over the useful life of the hauling truck for both book and tax purposes.

89. A machine is classified as seven-year MACRS property. Compute the book value for tax purposes at the end of three years. The cost basis is Rs. 145,000.

90. A piece of machinery purchased at a cost of Rs. 86,000 has an estimated salvage value of Rs. 12,000 and an estimated useful life of five years. It was placed in service on May 1 of the current fiscal year, which ends on December 31. The asset falls into a seven-year MACRS property category. Determine the depreciation amounts over the useful life.

91. Suppose that a taxpayer places in service a Rs. 20,000 asset that is assigned to the six year class (say, a new property class) with a half-year convention. Develop the MACRS deductions, assuming a 200% declining-balance rate followed by switching to straight line.

92. Consider the following data of a company for the year 2021:
Sales = Rs. 1,20,000
Fixed cost = Rs. 25,000

Variable cost = Rs. 45,000

Find the following:

a) Contribution

b) Profit

c) BEP

93. Consider the following data of a company for the year 2020:

Sales = Rs. 80,000

Fixed cost = Rs. 15,000

Variable cost = 35,000

Find the following:

a) Contribution

b) Profit

c) BEP

94. Define opportunity cost. Why engineering economics is considered as important aspect for making decision for engineers? Explain.

95. A bank is starting its nominal interest rate of 9% p.a. and compounding quarterly. Calculate the effective interest rate (i) a year (ii) a quarter (iii) a month (iv) half year.

96. If you wish to withdraw Rs. 2000 at the end of 1" year and expecting to be increased by 15% pa then after till end of 8 years, what amount need to be deposited in a bank right now which has an interest of 15% per annum.

97. Define mutually exclusive, independent and contingent projects. How much should you deposit at present that earns 12% interest per year when you can draw Rs. 10,000 per month for (i) 50 years (ii) Forever

98. Explain the terms depreciation, corporate tax, personal income tax and book value. Show the depreciation and book values in each year for an equipment having following details using MACRS method.

Investment (I) : 25,00,000

Useful life (n) : 7 years

99. Define time value of money, nominal and effective interest rate. Calculate future sum at the end of 5 m year when monthly deposit is Rs. 6,000 for 3 years that earns 7% interest per year.

100. What do you mean by tax, personal tax and corporate tax? Develop a model to calculate after tax cash flow.

101. Explain any two drawbacks of IRR with example. Differentiate between Economic analysis and financial analysis.

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