Chapter-1

Introduction

Simulation - immitation of reality

System - Entities
- Attributes
- Activities

System Boundary

System Environment

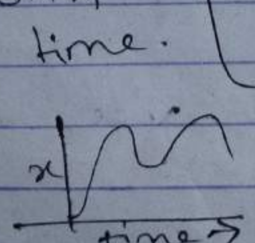
System Types

(1) Continuous & Discreete

Continuous: changes are predominantly smooth with respect to time.

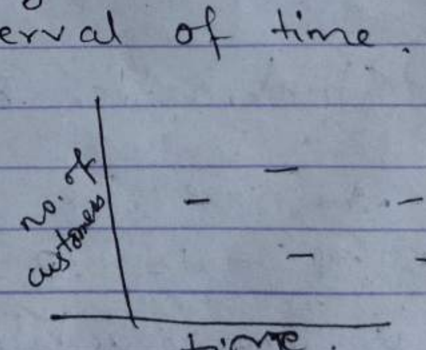
eg:

$\frac{dx}{dt}$



flow of water in ~~tap~~ dam.

Discreete: changes are at discreete interval of time.



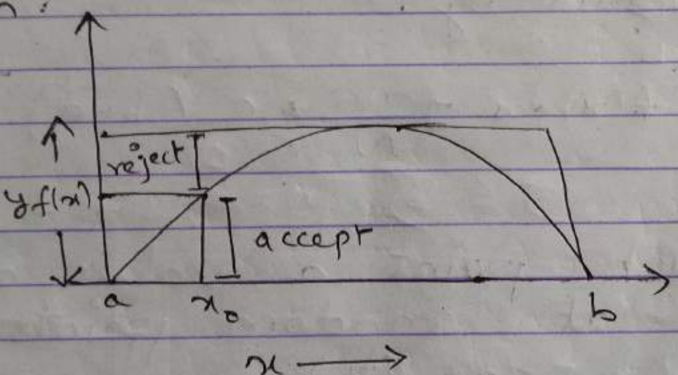
eg: arrival of people in a room at different time intervals.

Chapter-2

System Simulation

V. Imp! # Monte-Carlo Simulation Method

Monte-Carlo method consists of experimental sampling of with random numbers. This technique is a computational technique applied to static models.
eg: the integral of a single variable over a given range corresponds to finding the area under the graph representing the function:



Here, $f(x)$ is the positive f^n bounded with lower and upper bounds 'a' and 'b' respectively. The function is bounded above by 'c'. As shown in the figure, the f^n is contained within a rectangle with length 'c' and $(b-a)$. We pick the random points within the rectangle and determine whether they lie under the curve or not. The fraction of points falling on or below the curve to the area of rectangle is given by:

$$\left[\frac{n}{N} = \frac{\int_a^b f(x)\,dx}{c(b-a)}\right]$$

where:

$n \rightarrow$ no. of points on or below curve

$N \rightarrow$ No. of generated points.

As the number N increases, the accuracy of result increases.

For sufficient points, the value of integral is estimated by:

$$\left[\int_a^b f(x)\,dx = \frac{n}{N} * c\,(b-a).\right]$$

For each point, a value of x is selected at random points between a and b, say (x_0).

A second random number is between 0 to c to given 'y'. If, y is $[y \leq f(x_0)]$, the point is accepted in n, else rejected.

eg: Solve: $\int_0^4 x^2 dx$

Using monte - Carlo and find the error percentage:

Soln:

Given:

$$\int_0^4 x^2 dx$$

Here,

$$y = f(x)$$
$$= x^2$$

Lower limit = 0

Upper limit = 4

x	0	1	2	3	4
y	0	1	4	9	16

Area of rectangle = (4 - 0) * 16
= 64

Using Monte - Carlo Method

$$\int_0^4 x^2 dx = \frac{n}{N} * 64$$

$$= \left[\frac{64 * n}{N}\right]$$

Now, to get the value of n & N;
x ranges from 0 to 4, i.e $0 \leq x \leq 4$

y ranges from 0 to 16 i.e $0 \leq y \leq 16$

Suppose, we take $N = 7$, random points (x, y) inside rectangle such that $0 \leq x \leq 4$ and $0 \leq y \leq 16$.

let

$$(x_1, y_1) = (2, 5)$$

$$f(x_0) = x^2 = (2)^2$$
$$= 4$$

If $y \leq f(x_0)$

$\cancel{5} \leq \cancel{4}, 4$ $n = \cancel{0}$, $N = 1$

$$(x_2, y_2) = (0, 7)$$

$$f(x_0) = 0$$

If $y \leq f(x_0)$

$7 \leq 0$, $n = 0$, $N = 2$

So;

$(x_1, y_1) = (2,5)$, $5 \leq 4$, rejected

$(x_2, y_2) = (0,7)$, $7 \leq 0$, rejected

$(x_3, y_3) = (2,2)$, $2 \leq 4$, accepted

$(x_4, y_4) = (3,1)$, $1 \leq 9$, accepted

$(x_5, y_5) = (4,5)$, $5 \leq 16$, accepted

$(x_6, y_6) = (2,16)$, $16 \leq 4$, rejected

$(x_7, y_7) = (4,16)$, $16 \leq 16$, accepted

Now;
From Monte-Carlo method;

$$n = 4$$

$$\int_0^4 x^2 dx = \frac{n}{N} \times 64$$

$$= \frac{4}{7} \times 64$$

$$= 36.57$$

But actual area $= \int_0^4 x^2 dx$

$$= \left[\frac{x^3}{3}\right]_0^4$$

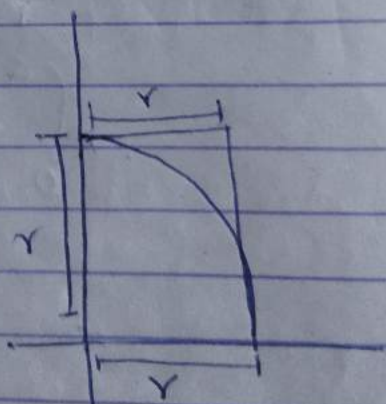
$$= \frac{4^3}{3} = \frac{64}{3} = 21.33$$

$$\% \text{ error} = \left| \frac{\text{actual value} - \text{simulated value}}{\text{actual value}} \times 100\% \right|$$

#

$$= \frac{21.33 - 36.57}{21.33} \times 100\%$$

$$= 71.4\% \;//$$



Determine the value of pi(π) using Monte-Carlo method.

Soln:

Here;

Let us suppose $N = 7$.

Equation of circle be at origin; (0,0)

$$x^2+y^2=r^2$$

$$\text{or, } f(x,y) = x^2+y^2-r^2$$

Let $r = 1$,

$$\left[f(x,y) = x^2+y^2-1\right]$$

Here,

If $f(x,y) \leq 0$, accepted
else rejected:

$(x_1, y_1) = (0.1, 0.2)$, $-0.95 \leq 0$ (accepted)

$(x_2, y_2) = (0.2, 0.3)$, $-0.87 \leq 0$ (accepted)

$(x_3, y_3) = (0.4, 0.1)$, $-0.83 \leq 0$ (accepted)

$(x_4, y_4) = (0.3, 0.5)$, $-0.66 \leq 0$ (accepted)

$(x_5, y_5) = (0.5, 06)$, $-0.39 \leq 0$ (accepted)

$(x_6, y_6) = (0.66, 0.77)$, $0.02 \leq 0$ (accepted)

$(x_7, y_7) = (0, 1)$ $0 \leq 0$ (accepted)

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For value of π ;

$$\pi = \frac{7}{N} \times 4$$

$$= \frac{7}{7} \times 4 = 4$$

And

actual value = 3.14

$$\text{Error} = \left| \frac{\text{actual value} - \text{simulated value}}{\text{actual value}} \times 100\% \right|$$

$$= \left| \frac{3.14 - 4}{3.14} \times 100\% \right|$$

$$=$$

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Soln:

To determine the value of pi we have circular curve that gives area $= \pi r^2$.

Let us consider first quadrant of the circle. Then,

$$\left[\text{Area of first quadrant} = \frac{1}{4}\pi r^2\right]$$

Enclose the circle in a square with length r.

$\therefore$ Area of square $= r^2$

Using Monte-Carlo method;

$$\frac{\text{Area of curve}}{\text{Area of square}} = \frac{\text{No. of points on or below curve}}{\text{Total no. of points}}$$

i.e $$\frac{\frac{1}{4}\pi \cancel{r^2}}{\cancel{r^2}} = \frac{n}{N}$$

or, $$\frac{1}{4}\pi = \frac{n}{N}$$

$$\boxed{\pi = \frac{4n}{N}}$$

or, $$\left[\pi = \frac{4n}{N}\right]$$

Now, to calculate no. of points on or below curve, let us take equation of circle at origin (0,0) i.e $x^2+y^2=r^2$

Assuming it as function $f(x,y) = x^2+y^2-r^2$

Let $r = 1$, then;

$$f(x,y) = x^2+y^2-1$$

The random points for x & y in bet^n (0-1). If $f(x,y) \leq 0$, the point is accepted in else the point is rejected.

$$(x_1,y_1) = (0.8, 0.9)$$

$$\rightarrow f(x_1,y_1) = (0.8)^2+(0.9)^2-1$$
$$= 0.64+0.81-1$$
$$= 1.45-1$$
$$= 0.45$$

Here,

$0.45 \leq 0$, not true, hence rejected.

i.e, $n=0$, $N=1$

$\rightarrow (x_2,y_2) = (0.1, 0.2)$, $n=1$, $N=2$

$\rightarrow (x_3,y_3) = (0.7, 0.9)$, $n=1$, $N=3$

$\rightarrow (x_4,y_4) = (0.2, 0.3)$, $n=2$, $N=4$

$\rightarrow (x_5,y_5)$ $(0.01, 0.02)$, $n=3$, $N=5$

$\rightarrow (x_6, y_6) = (0.31, 0.41),\ n = 4, N = 6$

$\rightarrow (x_7, y_7) = (0.62, 0.01),\ n = 5, N = 7$

Here; we got values of $n = 5$ & $N = 7$

For value of π;

$$\pi = \frac{4 \times 5}{7}$$

$$= \frac{20}{7}$$

$$= 2.85$$

Actual value of $\pi = 3.14$

$$\%\ \text{error} = \left| \frac{\text{actual value} - \text{obtained value}}{\text{actual value}} \times 100\% \right|$$

$$= \frac{3.14 - 2.85}{3.14} \times 100\%$$

$$= 9.01\% \ //$$

Experimental nature of Simulation

by another method;

$$\pi = \frac{4 \times n}{N} \quad \text{---- (i)}$$

To calculate no of points on or below curve, we have equation of circle

$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2} \quad \text{---- (ii)}$$

let $r = 3$, then eqn (ii) becomes

$y = \sqrt{9 - x^2}$ and the range of x & y is;

$$0 \le x \le 3$$
$$0 \le y \le 3$$

Now, selecting random points for x & y;

$(x_1, y_1) = (0, 3) \Rightarrow 3 \le \sqrt{9}$ accepted

$(x_2, y_2) = (1, 2) \Rightarrow 2 \le \sqrt{8}$ accepted

$(x_3, y_3) = (2, 1) \Rightarrow 1 \le \sqrt{5}$ accepted

$(x_4, y_4) = (3,0) \Rightarrow 0 \leq \sqrt{0}$, accepted

$(x_5, y_5) = (2,3) \Rightarrow 3 \leq \sqrt{5}$, rejected

$(x_6, y_6) = (1,3) \Rightarrow 3 \leq \sqrt{8}$, rejected

Now;

$n = 4, \quad N = 7$

$$\pi = \frac{4 \times 4}{7}$$

$$= 2.28$$

% error =

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Types of System Simulation

We distinguish betⁿ continuous and discreete system as being in which smooth or sudden changes occur. The distinction betⁿ continuous & discreete model is not made during the classification of model, because whether mathematical or analytical technique will be applied to the model can't be made at the time of classification. The distinction is important when it is decided to perform simulation. The computation technique vary for two kind of models:

(i.) Numerical computation technique for discrete system

(ii.) Numerical computation technique for continuous system.

→ (i.) To illustrate the general computation of simulation in discreete model, consider the below example:

eg. A clerk begins his work with pile of documents to be processed in a day. He begins the work as soon as he finishes the previous one. He takes 5 minute break after he works for an hour.

Suppose there are 7 documents with their processing time provided. Explain how simulation takes place to ~~prove~~ calculate the total time required to finish the entire processing. The numerical computation is shown below.

S.No	Start time (t_s)	Processing Time (t_p)	Finish Time (t_f)	Cumulative time	Break	Remaining Job
1.	0	20	20	20	0	7
2.	20	25	45	45	0	6
3.	45	30	75	5	1	5
4.	80	20	100	25	0	4
5.	100	20	120	45	0	3
6.	125	20	145	5	1	2
7.	165	17	162	22	0	1

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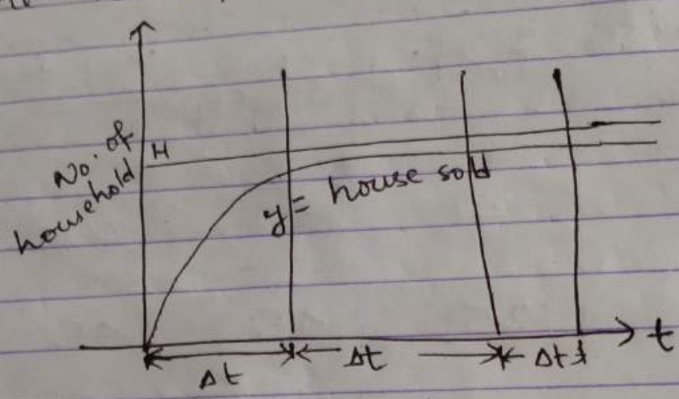
→ (ii.) To illustrate this technique, let us consider below example:

\# A builder observes that the rate at which he sells house depends directly upon the no. of families



In above figure, the horizontal line 'H' represent potential market of house and y represents the rate at which house is sold. Mathematically, the rate of change of selling house can be represented as: $\left[\frac{dy}{dt} \propto H - y\right]$. To solve the problem using simulation, we follow step by step procedure. Suppose computation is made at uniform interval of time, Δt, and calculation is proceeded to t_i. So, rate of change of y can be interpreted as amount of change per unit time.

i.e rate of change of 'y' = $\frac{\Delta y_i}{\Delta t}$

So, from eqn (i);

$$\frac{\Delta y_i}{\Delta t} = k_1(H-y) \text{ ---- (i)}$$

Repitition of calculation for each new value of y produces the output at the end of next interval. Thus, simulation output is the series of line segments approximate to the curve.

Comparision of Simulation & Analytic Method.

Analytic Method:

If the model is simple, it may be possible to work with relationship and quantities to get an exact analytical method. Analytic method are expensive and time consuming. It gives generic solution. This method provides good accuracy. There is limited problem that can be solved analytically.

Simulation Method:

Most real world system are too complex to allow to be evaluated analytically. In this case, we study the model by the means of simulation. The model uses computer to evaluate a model numerically i.e. data are gathered in order to estimate the desired true property. Simulation gives fast results. It gives specific solution. Simulation model is an approximation where we compromise accuracy. It is an extension of analytical method.

Distributed Lag Model

Models that have the properties of changing only at fix interval of time and basing current values of the variables on the basis of current values and the values that occur in previous intervals are called distributed lag models. These models are used for economic studies. These methods consist of linear algebraic equations. They represent continuous system but the data is available at fixed interval of time.

eg:

let,

C = Consumption

I = Investment

T = Tax

G = Government Expenditure

Y = National Income.

$$C = 20 + 0.7(Y - T)$$
$$I = 2 + 0.1Y$$
$$T = 0.2Y$$
$$Y = C + I + G$$

This is static mathematical model but, it can be made dynamic by picking a fixed time interval and expressing the current values of variables in terms of values at previous intervals. These variables are lag variables and is denoted by attaching the suffix minus n to the variables where 'n' indicates the variable. The above set of equations could be made dynamic by lagging all the variable as follows;

$$C = 20 + 0.7(Y_{-1} - T_{-1})$$

$I =$

$T =$

$Y = C_{-1} + I_{-1} + G_{-1}$

Q. Given following set of equations, find the growth in national consumption for 5 years. Assume the initial Y_{-1} is 80 and take government expenditure as follows;

Year	G. exp	equations
1	25	$I = 2 + 0.2Y_{-1}$
2	30	$Y = 45.45 + 2.27(I+G)$
3	35	$T = 0.2Y$
4	40	$C = 20 + 0.7(Y-T)$
5	45	

Soln:

Given,

$Y_{-1} = 80$

i) For 1st Year;

$I = 2 + 0.2 \times 80 = 18$

$Y = 45.45 + 2.27(I+G)$
$= \underline{143.06}$

$T = 0.2 \times 143.00$
$= 28.61$

$C = 20 + 0.7(143.06 - 28.61)$
$=$

ii) For 2nd Year;

$I = 2 + 0.2 \times (143.06)$
$= 30.612$

$Y = 45.45 + 2.27(30.612 + 30)$
$= 183.03$

$T = 0.2 \times (183.03$
$= 36.606$

$C = 20 + 0.7(183.03 - 36.606)$
$= 122.4968$

iii) For 3rd Year;

$I = 2 + 0.2(183.03)$
$= 38.606$

$Y = 45.45 + 2.27(38.606 + 35)$
$= 212.53$

$T = 0.2 \times (212.53)$
$= 42.50$

$C = 20 + 0.7(212.53 - 42.50)$
$= 139.021$

iv) For 4th Year;

$I = 2 + 0.2(212.53)$
$= 44.506$

$Y = 45.45 + 2.27(44.506 + 40)$
$= 237.278$

$T = 0.2 \times 237.278$
$= 47.45$

$C = 20 + 0.7(237.278 - 44.506)$
$= 152.879$

v) For 5th Year;

$$I = 2 + 0.2(237.27)$$
$$= 49.454$$

$$T = 0.2 \times 259.86$$
$$= 51.972$$

$$Y = 45.45 + 2.27(49.454 + 49.\ldots)$$
$$= 259.86$$

$$C = 20 + 0.7(259.86 - 51.972)$$
$$= 165.5216$$

Cobweb Model

Distributed lag model can be constructed from the static market model. From the example of supply demand behaviour of market model that depend upon price, the supply should be dependent upon price from the previous marketing period. The demand responds to current price. Assuming market is clear, the market model in distributed lag form can be represented as follows;

$$Q = a - bP$$
$$S = c + dP_{-1}$$
$$Q = S$$

For given initial value of price, the value of S at the end of first interval can be derived. This determines the value of Q. Since the market is clear, the new value of P can be derived. This P becomes the previous value for second interval and shown as figure below shows the flustuation of price for following 2 cases;

a) $P_0 = 1.0$
$a = 12.4$
$b = 1.2$
$c = 1.0$
$d = 0.9$

b) $P_0 = 5.0$
$a = 10.9$
$b = 0.9$
$c = 2.4$
$d = 1.2$

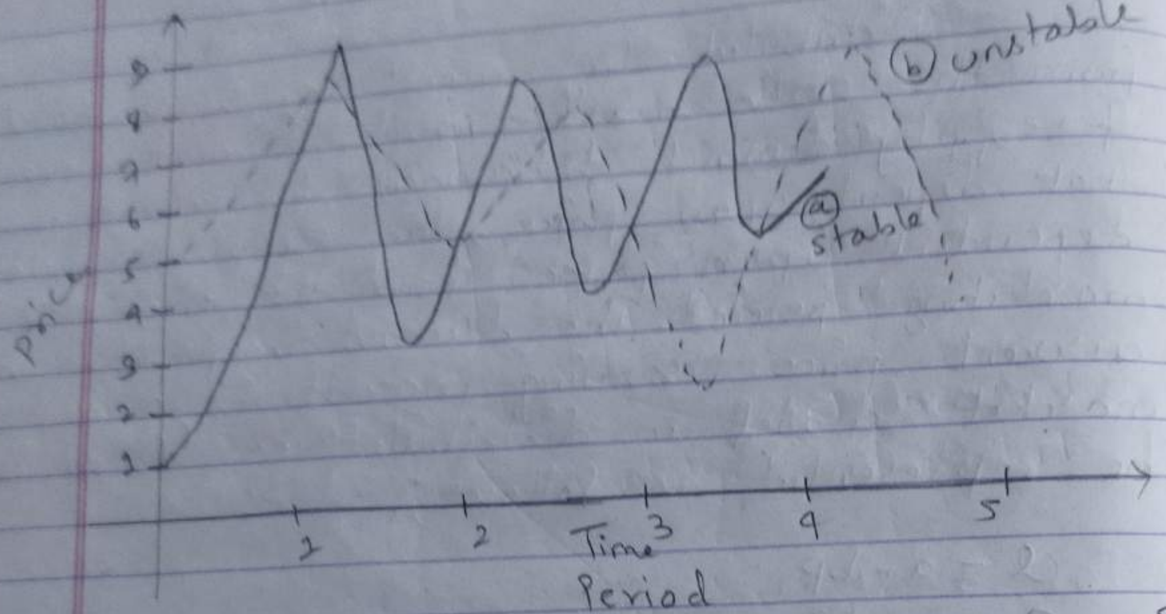


fig: Fluctuation of market price

Case (a) represents stable market in which the market settles to a price of 5.43. Case (b) is unstable with price fluctuating and increasing amplitude.

Models of this type are called Cobweb Model because they can be solved graphically. For the stable case, the market model is illustrated as below;

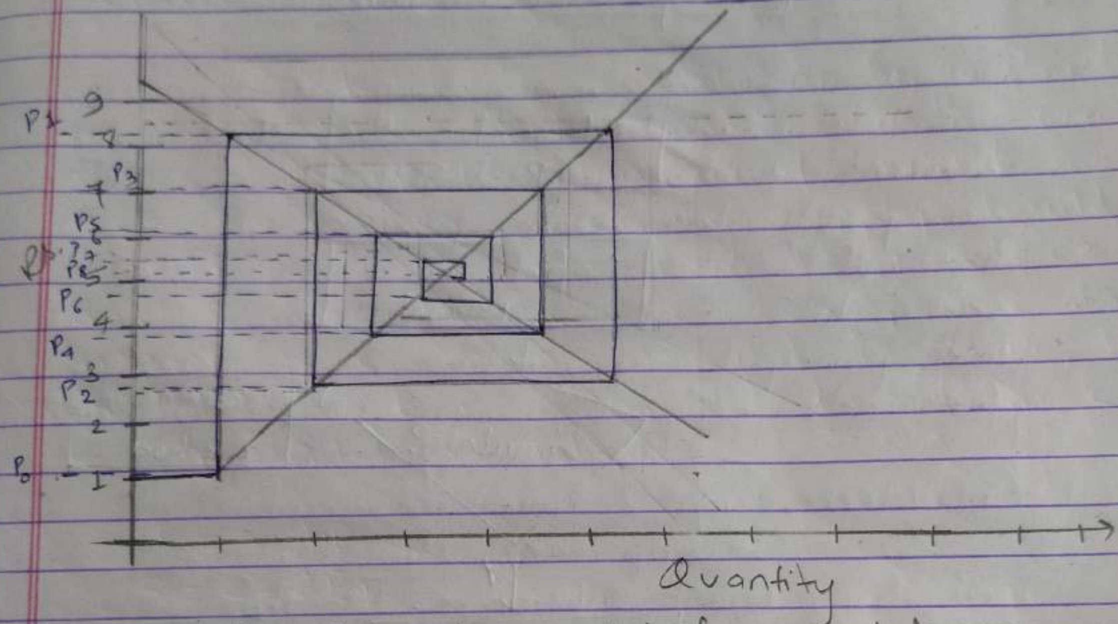


fig. Cobweb model for market economy

(a)

$P_0 = 1.0$

$a = 12.4$

$b = 1.2$

$c = 1.0$

$d = 0.9$

(b)

$P_0 = 5.0$

$a = 10.9$

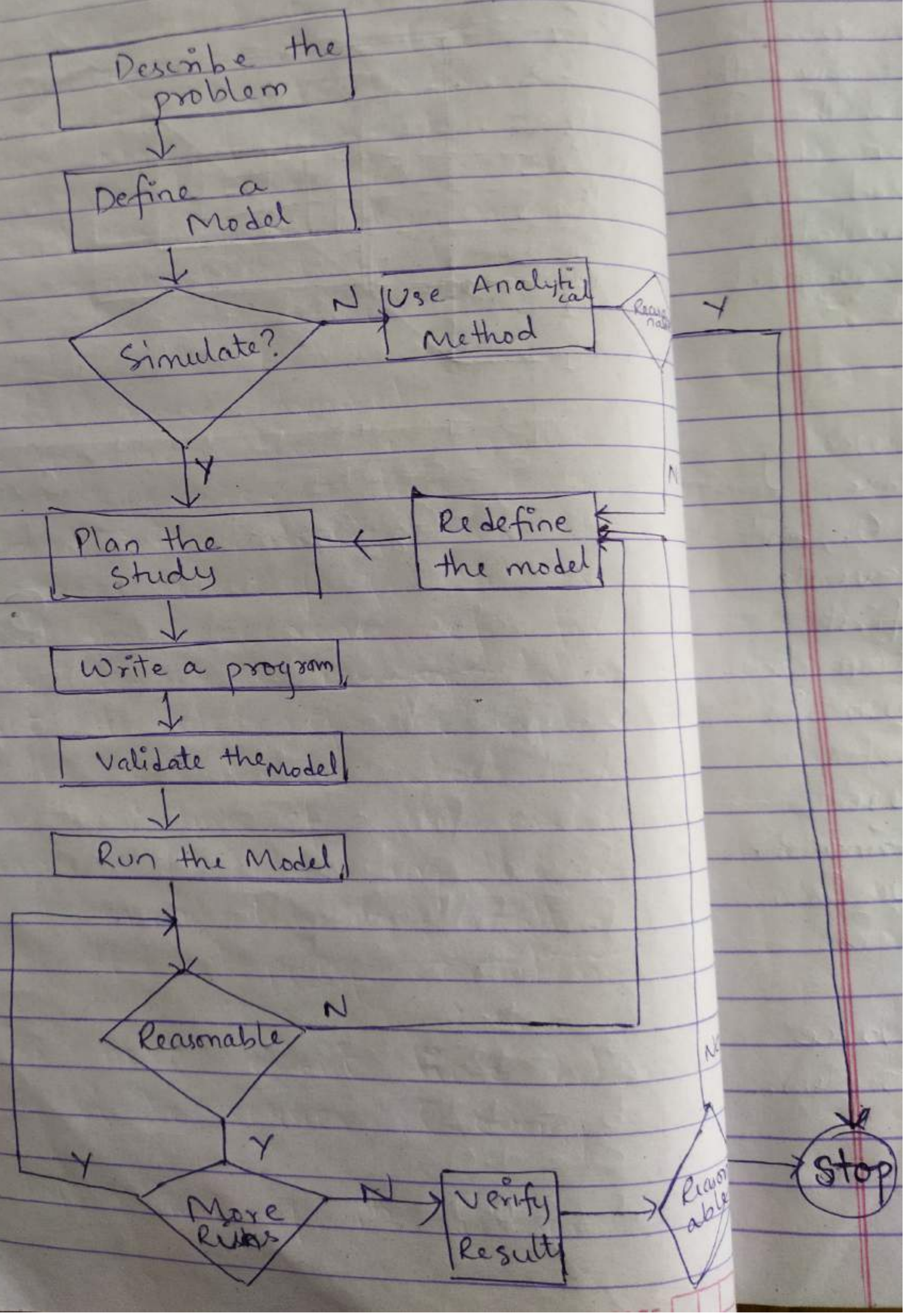
$b = 0.9$

$c = 2.4$

$d = 1.2$

Imp!!

Steps of Simulation



Queue Characteristics (SSQM)

1. Calling Population
2. System Capacity
3. Arrival Process
4. Queue behaviour & Discipline
5. Service time & Service Mechanism

Imp!

Time advanced Mechanism.

Simulation clock is the variable in a simulation model that gives the current value of simulated time. There is no particular unit for simulation clock time. It is assumed and is used as same time unit throughout the simulation. Two principal approaches have been suggested for advancing the simulation clock :-

(i) Next event time advanced

- Approach time is generally used by most software. In this case, simulation clock is initialized to 0., and the time of occurence of future events are determined. The occurence of first event time is advanced in simulation clock. For this point, the state of the system is updated. Then the simulation clock is advanced to the time of next occurent event. This process of advancing the simulation clock from 1 event time to another is continued till the stopping condition is satisfied.

eg: for next event time advance approach for single server Queueing system.

t_i = time of arrival of ith customer.

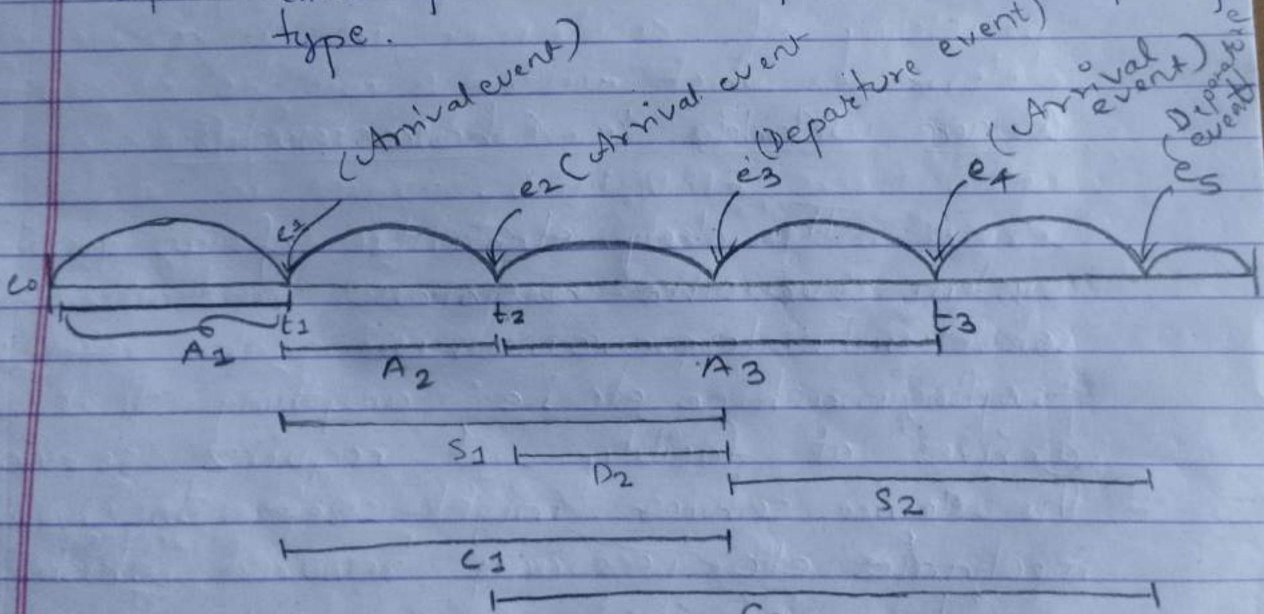
$A_i = t_i - t_{i-1}$ (Inter arrival time between i^{th} and $(i-1)^{th}$ customer.

S_i = time that the server actually spends serving i^{th} customer.

D_i = delay of i^{th} customer

$C_i = t_i + D_i + S_i \Rightarrow$ total time spent by i^{th} customer in server.

e_i = time of occurence of i^{th} event of any type.



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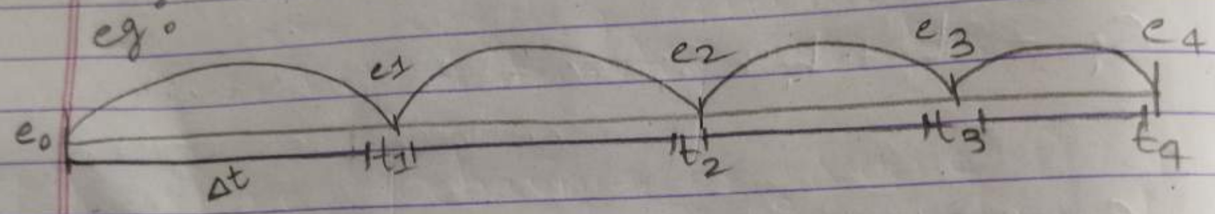
interval oriented

(ii) Fixed Increment Time Advance

In this case, simulation clock is divided into fixed in interval of time. Simulation clock is advance at the start time of the interval. Statistics is gathered at the end of interval.

eg:



Queue Models & Characteristics

The key elements of a queuing system are the customers and the servers. The term customers can refer to people, machine etc i.e anything that arrives at facility and requires service. The term server might refer to reception, mechanics etc i.e any resource which provides required service.

Elements of queue

(i) Calling Population

It is the population of potential customer. It may be finite or infinite. eg:- a bank of 5 machines that are curing parts. After an interval of time, a machine automatically opens and must be attended by a worker who removes the cured one and puts uncured into the machine. Here, machines are the customers and the workers are server. In this case, the calling population is finite & consists of 5 machines.

For large population of customers the calling population is infinite. eg:- customers of bank, restaurant.

(ii) System Capacity

- In many queuing system, there is a limit to the number of customers that may be in the waiting line on system. eg: A car washroom with 10 cars' capacity, an arriving car which finds the queue full returns to calling population. For unlimited capacity, eg:- concert ticket sale.

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(iii) Arrival Process

The arrival process of infinite population is usually characterized in terms of inter-arrival time of successive customers. Arrival may be at scheduled time or at random time. At random time, the inter-arrival times are usually characterized by probability distribution function.

(iv) Queue Behaviour or Queue Discipline

- Queue discipline refers to logical ordering of customer in a queue and determines which customer will be chosen for service when a server becomes free. Common methods are; FIFO, LIFO, service in random order, service in priority, shortest processing time.

Queue behaviour is customer action while in a queue waiting for service to begin.

Service time & service mechanism

- The service time of successive arrivals are denoted by $S_1, S_2 \ldots$. They may be constant or of random duration. Sometimes services may be identically distributed for all customers of a given type or class. In some system,

PAGE

Service time depends upon the time of the day or the length of waiting time. Server's may work faster than usual when the waiting time is long. Thus, reducing the service time.

A queuing system consists of a no of serving centers and interconnecting queues. Service centers consist of no of servers, denoted by c, working in parallel. Parallel service mechanisms are either single ($c = 1$), multiple servers give the range ($1 < c < \infty$) or unlimited servers ($c = \infty$)

Queue Notation

A/B/C/N/K

where;

A ⇒ Inter arrival time duration

B ⇒ Service-time durations

C ⇒ No. of parallel servers

N ⇒ System capacity

K ⇒ Calling population

eg:

M/M/1/∞/∞

which means inter-arrival and service time is exponentially distributed with single server.

Here;

System capacity and calling population is infinitely. This can also be represented as M/M/1.

Performance measure of Queue.

To measure the performance of queue, we estimate on the basis of 3 parameters.

(i.) First measure is to measure expected avg. delay in queue of an customer completing their delays during simulation. This is denoted by $d(n)$. The expected in the definition of $d(n)$ mean that on a given run simulation, the average delay of n customer depend upon inter arrival and service time random variables. On another run, inter arrival and service time may be different which will change delay for n customer. From a single run of simulation, let, $D_1, D_2 \ldots D_n$ be the delays of n customer. The estimator $d(n)$ is :-

$$\hat{d}(n) = \frac{\sum_{i=0}^{n} D_i}{n}$$

This gives the system performance from customer point of view.

(ii.) Second measure is the expected average no. of customer in queue. It is denoted by $q(n)$. This is different type of average as it is taken over continuous time rather than customers. Thus, we define it by time average no. of customers in queue. Let, $Q(t)$ denote the no. of customer in queue at time 't'. Let $T(n)$ be the time required to observe n delays in queue. For any time between 0 and $T(n)$, $Q(t)$ is non-negative integer.

Let, P_i be the expected proportion of time. Then, $q(n)$ is represented as:

$$q(n) = \sum_{i=0}^{\infty} i P_i$$

To estimate $q(n)$ for a simulation, we replace P_i with estimate of it i.e;

$$q(n) = \sum_{i=0}^{\infty} i \hat{P}_i \text{ ------ (i)}$$

If we assume T_i be the total time during simulation and queue is of length i, then, $T(n) = T_0 + T_1 + T_2 + \cdots$ and

$\hat{P}_i = \frac{T_i}{T_n}$. Thus, from eqn (i)

$$\hat{q}(n) = \frac{\sum_{i=0}^{\infty} i T_i}{T(n)} \text{ ------ (ii)}$$

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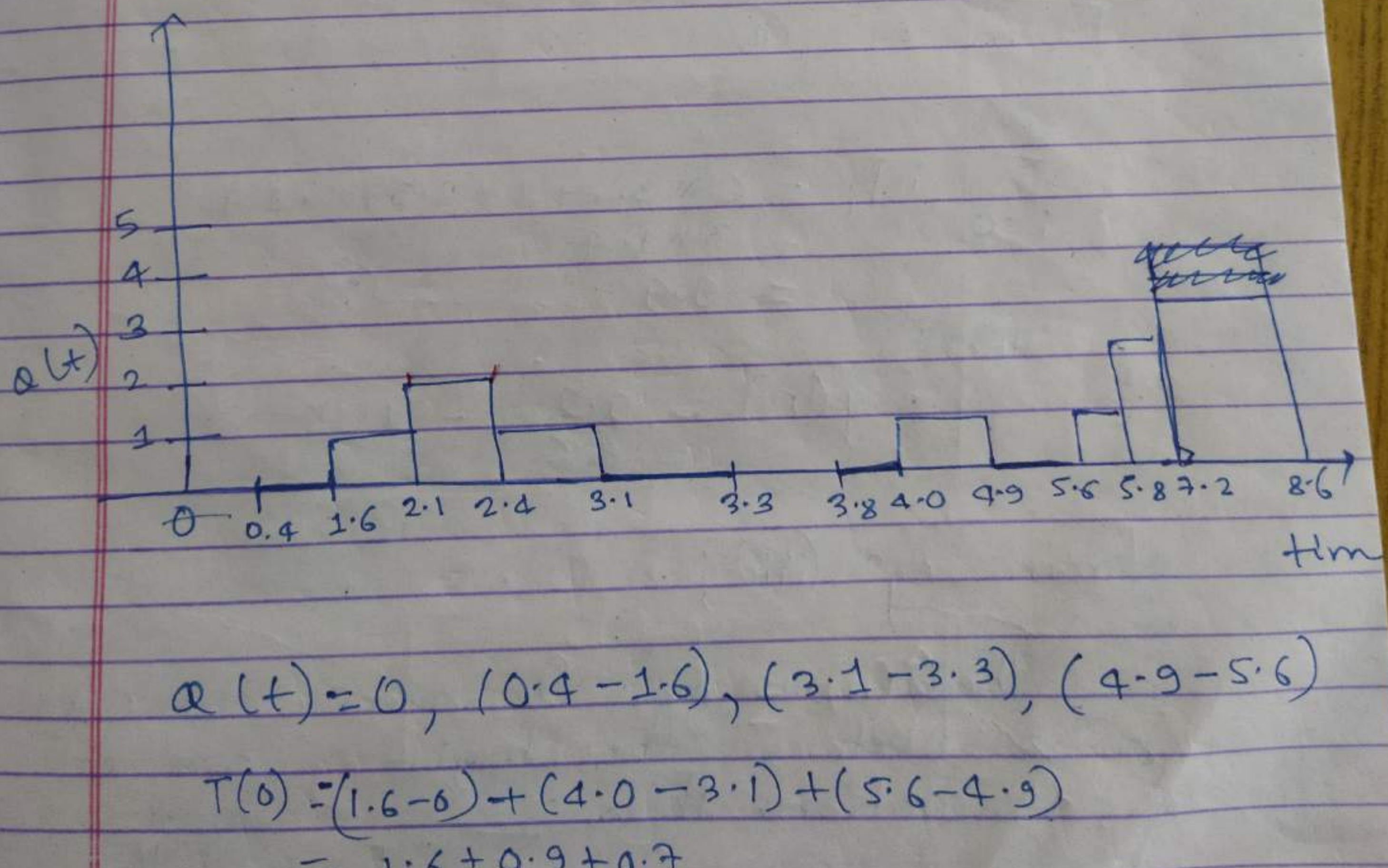
eg: n = 6   Q. Imp!!

~~Arrival time~~

Arrival time	0.4	1.6	2.1	3.8	4.0	5.6	5.8	7.2
Departure time	2.4	3.1	3.3	4.9	8.6			

Simulation ends at $\underline{T(6) = 8.6}$ *



$Q(t) = 0,\ (0.4 - 1.6),\ (3.1 - 3.3),\ (4.9 - 5.6)$

$T(0) = (1.6 - 0) + (4.0 - 3.1) + (5.6 - 4.9)$

$= 1.6 + 0.9 + 0.7$

$= 3.2$

PAGE

$T(1) = (2.1-1.6)+(3.1-2.4)+(4.9-4.0)+(5.8-5.6)$
$= 2.3$

$T(2) = (2.4-2.1)+(7.2-5.8)$
$= 1.7$

$T(3) = (8.6-7.2) \times 1.$
$= 1.4$

From eqⁿ (ii);

$$\sum_{i=0}^{\infty} iT_i = 0*3.2+1*2.3+2*1.7+3*1.4$$
$$= 9.9 \text{ -------- (iii)}$$

Then,

$$q(n) = \frac{9.9}{8.6} = 1.15$$

From eqⁿ (iii);

The summation is area under Q(t) curve between the beginning and end of simulation. Thus, $\sum_{i=0}^{\infty} iT_i$ can be written as;

$$\int_0^{T(n)} Q(t)\,dt.$$

So, the estimator of $q(n)$ can be written as;

$$\left\{ \hat{q}(n) = \frac{\int_0^{T(n)} Q(t)\,dt}{T(n)} . \right\}$$

(iii)

$$\hat{u}(n) = \frac{(3.3-0.4)+(8.6-3.8)}{8.6}$$

$$= 0.90 = 90\%$$

The third output performance measure is in terms of how busy the server is. The expected utilization of the server is the expected proportion of time during the simulation that the server is busy. i.e a number between 0 and 1. It is denoted by $u(n)$. For single simulation estimate of $\hat{u}(n)$ - the observed proportion of time during the simulation that the server is busy. It can be viewed as

busy function.
i.e;

$B(t) = 1$ if the server is busy at time t.

$\quad\;\; = 0$ if " " " idle " " ".

So, $\hat{u}(n)$ can be expressed as;

$$\hat{u}(n) = \frac{(3.3-0.4)+(8.6-3.8)}{8.6}$$

$$= \frac{7.7}{8.6}$$

$$= 0.90$$

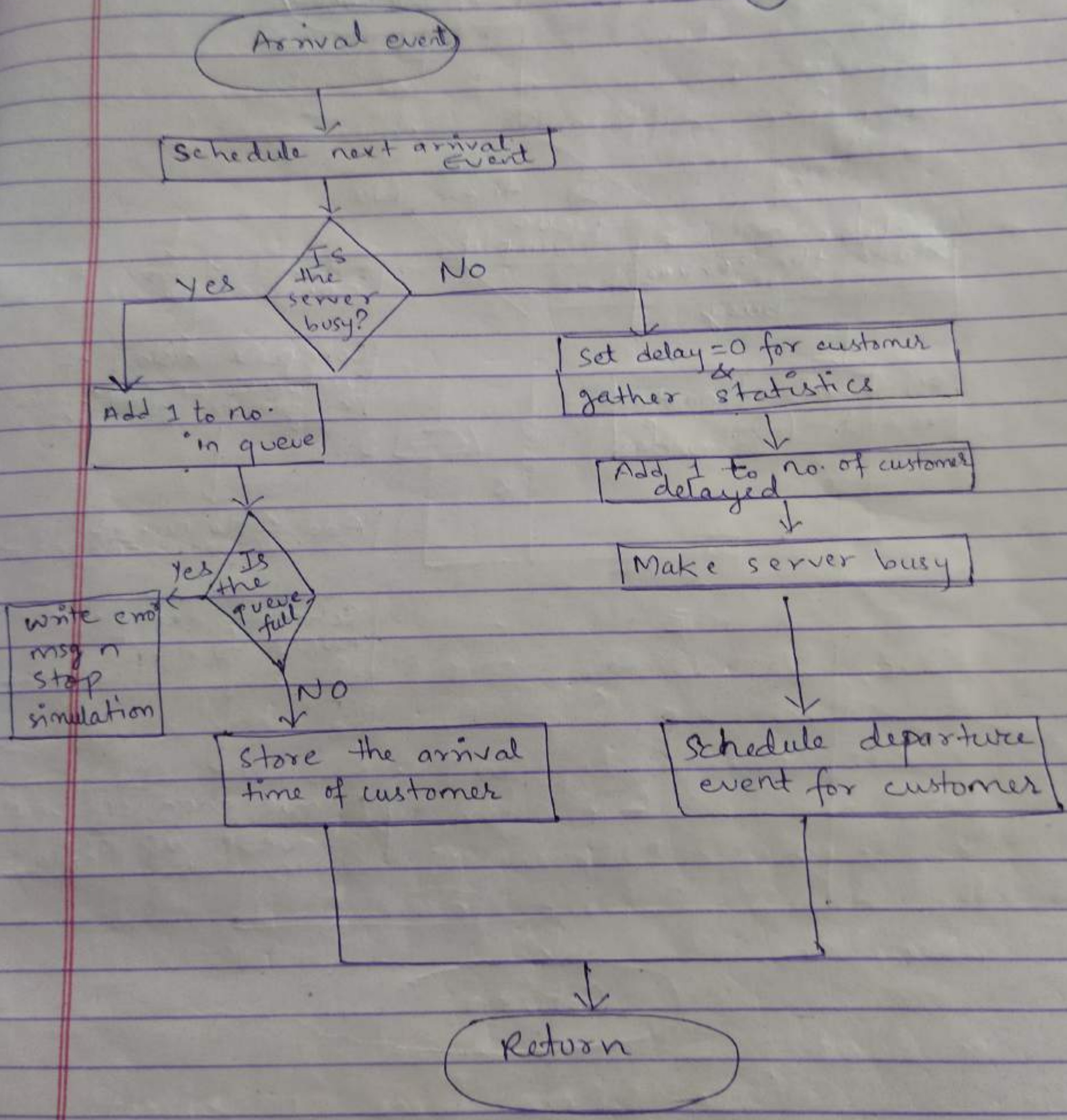
$$= 90\%$$

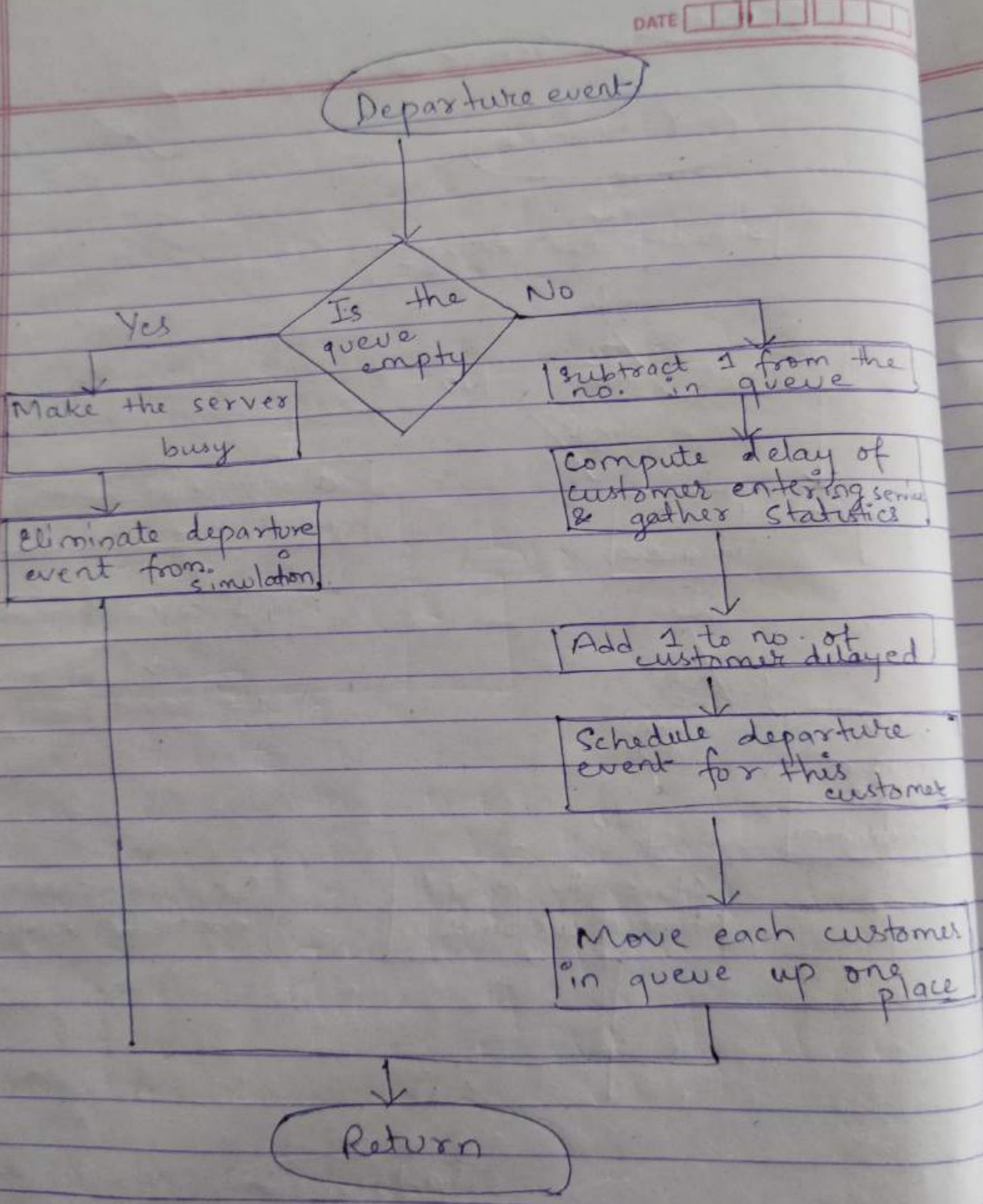
This means server was busy about 90% during simulation.

The numerator can be written as area under B(t) function i.e

$$\boxed{\hat{u}(n) = \frac{\int_0^{T(n)} B(t)\,dt}{T(n)}}$$

SSQM (Single Server Queuing Model)





DATE
Departure event
Is the queue empty
Yes
No
Make the server busy
Eliminate departure event from simulation
Subtract 1 from the no. in queue
Compute delay of customer entering service & gather statistics
Add 1 to no. of customer delayed
Schedule departure event for this customer
Move each customer in queue up one place
Return

Chapter-3 Continuous System

A continuous system is one in which pre-dominant activities of the system cause smooth changes in the attributes of the system entities. When such system is modelled, mathematically, the variable of the model representing the attributes are controlled by continuous functions. In general, in continuous system, the relationship describes the rate at which attribute changes so that the model consists of differential equation. We can use differential equation to represent the behaviour of continuous system.

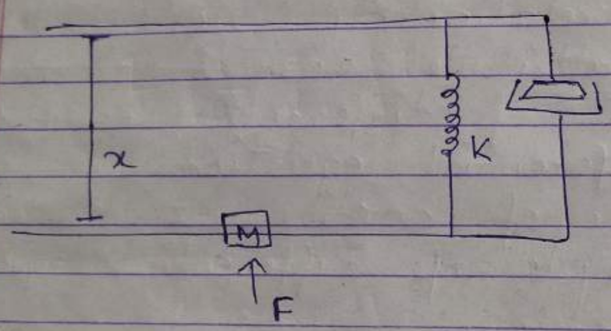
An example of differential equation is :-

$$\left[M \frac{d^2x}{dt^2} = kF(t) - D\frac{dx}{dt} - kx \right]$$

Automobile suspension wheel's equation.

Above equation represents a linear differential equation. Linear means the variable in an equation appears only with a power 1. Non-linear equations are equation having the power of variable more than 1. When more than one independent variables occur in a differential equation, the equation is said to be partial differential equation. Differential equation occurs repeatedly in scientific & engineering studies. The reason for this, is that most physical and chemical process involves rate of change, which require differential equations for their mathematical description.

Differential Equation for Automobile Suspension Wheel.



Here, M is the mass of body. F(t) is the force applied to the mass, which moves the body to upward direction. Let, x be (+ve) for upward displacement 'K' is the stiffness of string and 'D' is the damping force exerted by shock absorber.

Now:

We know force is directly proportional to acceleration of the body i.e

$F \propto a$

$$K F(t) = Ma$$

where, K is proportionality constant

i.e $K F(t) = M \frac{d^2x}{dt^2}$ ------ (i)

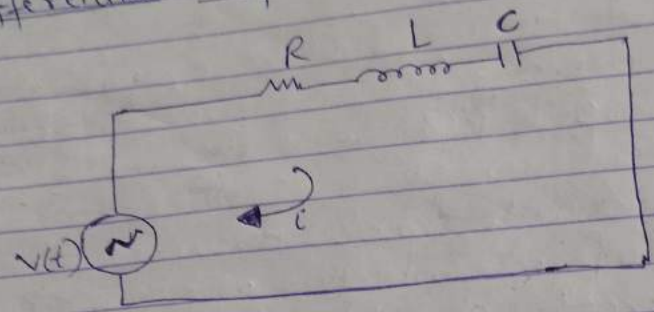
Spring and shock absorber exerts a resisting force which opposes the force. So, these forces should ~~force~~ be subtracted from total force. Resisting force exerted by spring is given by kx and by shock absorber is given by rate of change of position ~~is~~ i.e $D \frac{dx}{dt}$

From eqn (i);

$$\boxed{M \frac{d^2x}{dt^2} = K F(t) - D \frac{dx}{dt} - kx}$$

which is the required equation. //

Differential equation for Electrical System



$$L\frac{di}{dt} \cdot \frac{1}{L} * \int i$$

$$\frac{dq}{dt}$$

Here; from the combination above;

Voltage across $R (V_R) = iR$

Voltage across $L (V_L) = L\frac{di}{dt}$

Voltage across $C (V_C) = \frac{1}{C}\int i$

We know that;

$i =$ rate of change of charge

i.e $\left[\frac{dq}{dt}\right]$

Then; using Kirchov's voltage law; We can say that;

$$V(t) = V_L + V_C + V_R$$

Putting the values of V_L, V_C & V_R.

$$V(t) = L\frac{di}{dt} + \frac{1}{C}\int i + iR$$

$$= L\frac{d^2q}{dt^2} + \frac{1}{C}\int \frac{dq}{dt} + R\frac{dq}{dt}$$

$$= R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$\therefore V(t)$

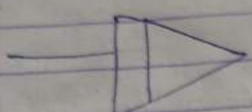
$$= L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

Hence;

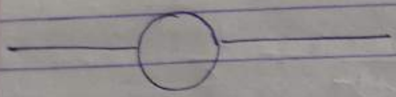
$$\boxed{V(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}}$$

is the required differential eqn for electrical system.

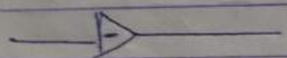
Analog Computer



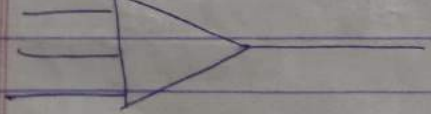
Integrator



Scale factor



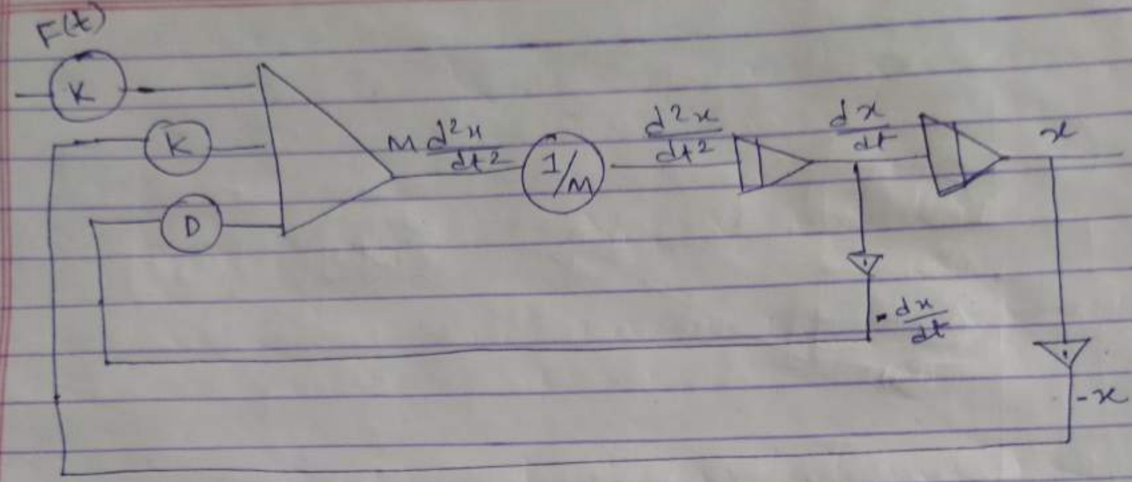
Inverter



Summer

1. $$M\frac{d^2x}{dt^2} = kF(t) + \left(-D\frac{dx}{dt}\right) + (-kx)$$

$$\frac{d^2x}{dt^2} = \frac{1}{M}\left(kF(t) + \left(-D\frac{dx}{dt}\right) + -kx\right)$$



Analog Computer

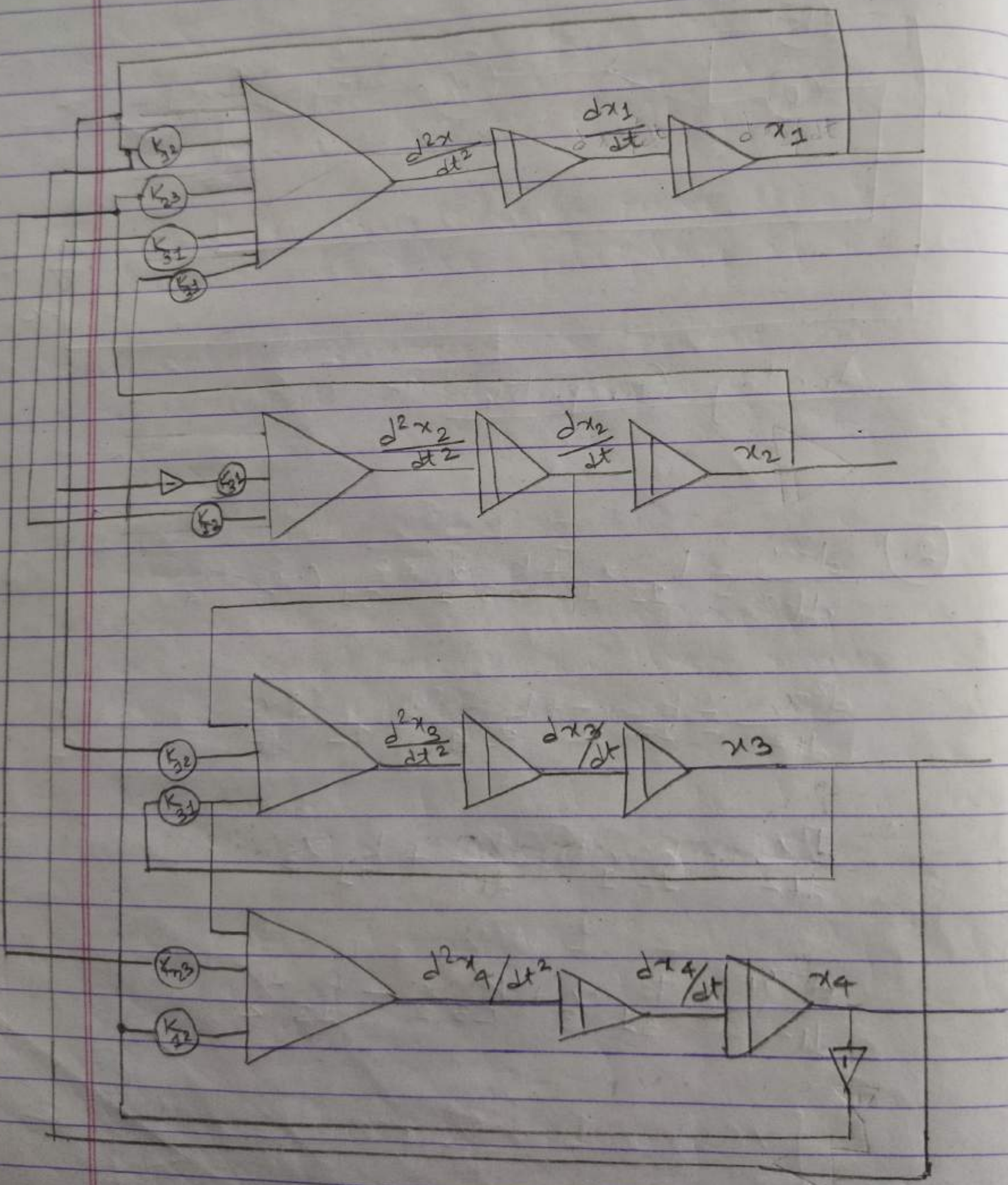
(2.) $$\frac{d^2x}{dt^2} = k_{12}x_1 + k_{23}x_2 + k_{31}(x_3 - x_4)$$

$$\frac{d^2x_2}{dt^2} = k_{23}x_2 + k_{31}x_4 + k_{12}\frac{dx_1}{dt}$$

$$\frac{d^2x_3}{dt^2} = k_{12}x_1 + \frac{dx_2}{dt} + k_{31}x_3$$

$$\frac{d^2x_4}{dt^2} = k_{31}x_3 + k_{23}x_2 - k_{12}x_4$$

DATE

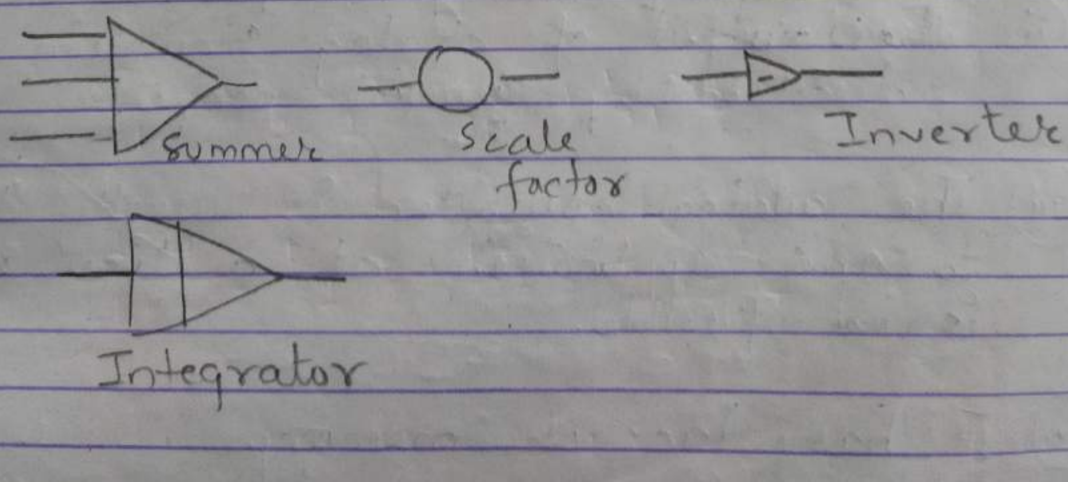


d2x/dt2
dx1/dt
x1
d2x2/dt2
dx2/dt
x2
d2x3/dt2
dx3/dt
x3
d2x4/dt2
dx4/dt
x4

~~Digital Analog~~

Analog Computers

- Analog computers are generally used to solve continuous model. Sometimes also used to solve static model. Here, behaviour of system represented by mathematical expressions are combined together using amplifiers such as adder or integrator to represent the mathematical model. This combination of a continuous system is referred as analog computer or when they are used to solve differential equation, they are referred as differential analyzer. The most widely used form of analog computers is the electronic analog computers based on operational amplifiers. The op-amp used are :-



Advantage of analog simulation

(i.) Analog computers have higher speed of solution than that of digital simulation.

(ii.) Analog simulations have direct access to an immediate display of result.

(iii.) Analog simulation is more natural so can reflect system structure and interpret the result.

Disadvantge

(i.) The result obtained from analog simulation has limited accuracy than implementing digital simulation.

(ii.) The output of analog simulation is not understood by general people.

(iii.) The analog computer is usually dedicated to one application at a time & hence is not flexible.

(iv.) It has lack of memory.

(v.) In analog simulation, hardware elements have to be combined to simulate the system, they have to be tested and calibrated which is not required in digital simulation.

Digital Analog Simulation

→ To avoid disadvantage of analog computer, digital analog simulator were produced.

→ It allowed solving continuous system similar to analog computer.

These languages contain macro-instructions that carry out the actions of adders, integrators and sign changer.

These macro-instructions were linked together by a program in the similar manner as operational amplifiers connected in analog computer. These simulators have the drawback of system being specific. Thus, programming languages were developed.

Continuous System Simulation Languages (CSSLs).

To overcome the restriction (CSSLs) have been developed. They use the familiar statements allowing a problem to be programmed directly from the equation broken into functional element. CSSL include macros or sub-routines which perform function of specific analog element. It also includes varieties of algebraic and logical expressions to describe the relationship between variables.

eg: Continuous System Model Program (CSMP-3)

↳ Three statements

(i.) Data Statements →

(ii) Structural Statements

(iii) Control Statements

This program is constructed using three statements as mentioned above.

(i.) → These statements assign numeric values to constants, parameters and set initial conditions.

CONST - is used to set constant values

eg: CONST M = 2.0, K = 400.0

- PARAM :- used to set parameters for variables.

eg; PARAM D = (2.0, 7.8, 6.2)

- INCON :- used to set the initial value of integration

eg; INCON E = 0.5

(ii) Structural Statements

- These statements are used to define the body of the system. It uses FORTRAN like statements.

eg:

$$M\frac{d^2x}{dt^2} = K\,F(t) - D\frac{dx}{dt} - Kx \quad \text{---- (i)}$$

To represent eq^n (i) for eg:-

{ X2DOT = $\frac{1.0}{M}$ * (K * F(t) - D * X1DOT - K*X)

eg: (X² = X**2.0)

Som

Functio

Gener

1) Y = I

2) Y = L

3) Y =

4. Y =

5. Y

6. Y

7. Y

8. Y

9. Y

10. Y

11.

Some structural Statements

Functions Used in CSMP

General form	Function
1) $Y = INTGRL(C, x)$	$y = \int_0^1 x\,dt + C$
2) $Y = LIMIT(P_1, P_2, x)$	$y = P_1, \; x < P_1$ $y = P_2, \; x > P_2$ $y = x, \; P_1 \le x \le P_2$
3) $Y = STEP(P)$	$y = 0 \quad t < P$ $y = 1 \quad t \ge P$
4. $Y = EXP(x)$	$y = e^x$
5. $Y = ALOG(x)$	$y = \ln(x)$
6. $Y = SIN(x)$	$y = \sin(x)$
7. $Y = Cos(x)$	$y = \cos(x)$
8. $Y = SQRT(x)$	$y = \sqrt{x}$
9. $Y = ABS(x)$	$y = \lvert x \rvert$
10. $Y = AMAX1(X_1, X_2, ----)$	$y = Max(X_1, X_2 ----)$
11. $Y = AMIN1(X_1, X_2, -----)$	$y = Min(X_1, X_2 -----)$

DATE

(iii) Control Statements - Specifies the option in the execution of the program and the choice of output.

eg:- TIMER ⇒ it is used to specify time interval.

TIMER DELT = 0.05, FINTIM = 2.5, PRDEL = 1.5, OUTDEL = 0.05

DELT = Integration Interval
FINTIM = Finish time

PRDEL = Interval set for element to be printed.

OUTDEL = Interval set for result to be plotted.

PRINT = To print the result
PRTPLT → Plot the result
TITLE → To write the title.
LABEL → To label the output.

CSMP Code for Automobile Suspension Model

$$\left\{ \frac{d^2x}{dt^2} = \frac{1}{M}\left(k \cdot F(t) - D\frac{dx}{dt} - kx\right) \right\}$$

```
TITLE    AUMOBILE   SUSPENSION   WHEEL

CONST    M=2.0, K=400.0, F=20.0

PARAM    D=(5.6, 7.8, 15.2, 35.6)

X2DOT   = 1.0/M * ( K * F(t) - D * X1DOT - K * X)

X1DOT   = INTGRL (0.0, X2DOT)
  X     = INTGRL (0.0, X1DOT)
TIMER    DELT = 0.05, FINTIM = 2.5, PRDEL = 0.5,
         OUTDEL = 0.05

PRINT   X2DOT, X1DOT, X

PRTPLT   X
LABEL    DISPLACEMENT VS TIME
END
STOP
```

$$\# \left\{ E(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} \right\}$$

~~TITLE~~

The above eq^n can be written as;

$$\frac{d^2q}{dt^2} = \frac{1}{L}\left(E(t) - R\frac{dq}{dt} - \frac{q}{C} \right)$$

CSMP Program

```
TITLE   Electrical System

CONST   L = 35.0, R = 5.0, C = 2.0, E = 1.0
Q2DOT   = (1.0/L)*(E-R*Q1DOT-1.0/C*q)
Q1DOT   = INTGRL (0.0, Q2DOT)
  Q     = INTGRL (0.0, Q1DOT)

TIMER  DELT = 0.005, FINTIM = 1.5, PRDEL = 0.5,
                                    OUTDEL = 0.5

PRINT  Q, Q1DOT, Q2DOT
PRTPLT  Q
LABEL   Charge Vs Time
END
STOP
```

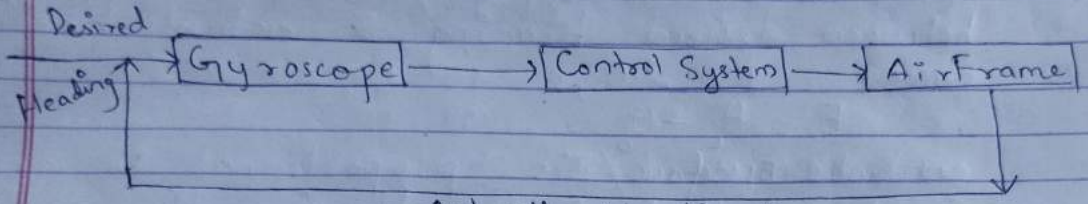
$$v(t) = 20.0$$

$$Q2DOT = \frac{1.0}{L} * \left(v(t) - R * Q1DOT - Q/C \right)$$

Hybrid Simulation (short notes)

Analog & digital computers are combined to provide hybrid simulation. One computer simulates the system being studied whereas the other computer simulates the environment in which the system is being studied. The system maybe the combination of continuous & discrete sub-system. These type of system can be simulated by analog & digital computer linked together. Hybrid Simulation requires technically developed systems. High Speed converters are used to transform signals from one form of representation to another.

Feedback System



Actually Heading θ_o

Feedback Systems are those system which have coupling between inputs and outputs. An example of feedback system is aircraft system which has continuous control. Here, as shown in figure, input is desired unit and output is actual heading. The output of the aircraft system is coupled with inputs. The gyroscope in the system is used to detect the difference between two headings. The difference is being used to operate the control surfaces as change in the headings affects the signal used for controlling the aircraft. A feedback is established by using the difference. The difference $(\theta_i - \theta_o)$ is the error signal and is denoted by 'E'. If control surface is proportional to error, then force changing the heading is also proportional to error signal. The feedback in the autopilot system is said to be negative feedback. For positive feedback, the force tends to increase the deviation and system becomes unstable.

DATE

Simulation of Autopilot System

The error signal 'E' is the difference between desired heading & actual heading i.e.

$$E = \theta_i - \theta_o \quad \text{------} (i)$$

control surface is proportional to error signal so that force changing the system is also proportional to error signal. Turning of aircraft produces a resisting force which is proportional to angular velocity. The force applies torque to aircraft which turns aircraft i.e;

$$T = K \cdot E - D\frac{d\theta_o}{dt} \quad \text{------} (ii)$$

For turning motion, the angular acceleration of a body is proportional to applied torque. I is the inertia of body.

Angular acceleration is second derivative of the heading.

$$I\frac{d^2\theta_o}{dt^2} = \text{torque} \quad \text{------} (iii)$$

from (i), (ii) & (iii)

$$I\frac{d^2\theta_o}{dt^2} = K(\theta_i - \theta_o) - D\frac{d\theta_o}{dt}$$

Dividing both sides by 'I',

PAGE

$$\frac{d^2\theta_0}{dt^2} = \omega^2\theta_i - 2\alpha\frac{d\theta_0}{dt} - \omega^2\theta_0$$

where;

$$\frac{K}{I} = \omega^2 \quad \text{and}$$

$$\frac{D}{I} = 2\alpha\omega$$

CSMP Program:

TITLE Autopilot System

CONST K = 2000 I = 13.6 ~~D = 26.2~~

PARAM α = (

TITLE SIMULATION OF AUTOPILOT SYSTEM

PARAM D = (5.65, 6.8, 7.6, 56.5, 113.2)

CONST I = 2.0, K = 100.0, A = 0.05

ω * * 2.0 = K/I

α = D/(2*I*ω)

INPUT = A * time

ANGACC = ω ** 2.0 * INPUT - 2.0 * α * ω * ANGVEL
- ω * * 2.0 ANGDIS

ANGVEL = INTGRL (0.0, ANGACC)
ANGDIS = INTGRL (0.0, ANGVEL)

TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.5,
OUTDEL = 0.5

PRINT ANGACC, ANGVEL, ANGDIS
PRTPLT ANGDIS
LABEL ANGULAR DISPLACEMENT VS ~~[illegible]~~ TIME
END
STOP

short notes

DATE

Predator Prey Model (Lotka-Volterra Model)

→ represents first order non linear differential equation.

Preditor Prey equations are a pair of 1st order non-linear differential equations frequently used to describe the dynamics of biological system in which 2 species interact one as a predator and other as a prey. The population changes through time according to a pair of equations.

$$\frac{dx}{dt} = \alpha x - \beta xy \text{ ---- (i)}$$

$$\frac{dy}{dt} = \delta xy - \gamma y \text{ ---- (ii)}$$

where, x = no. of prey
y = no. of predators
$\frac{dy}{dt}$ and $\frac{dx}{dt}$ represents growth rate of two populations over time.

$\alpha, \beta, \delta, \gamma$ are parameters describing the interaction of two species.

The above equation is a framework that can model the dynamics of ecological system. with prediter prey interaction, competition, disease & mutualism.

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It makes no. of assumptions :-
(i) The preditory popⁿ finds sufficient food all the time.

(ii) The food supply of preditor popⁿ depends on size of prey popⁿ.

(iii) The rate of change of popⁿ & is proportional to its size.

(iv) During this process, environment doesn't change in favour of any one species.

(v) Preditors have limitless appetitte.

eqⁿ (i) can be represented as the change in prey's number is given by its own growth - the rate at which it is preyed upon.

Eqⁿ (ii) represents change in preditor popⁿ is growth fueled by food supply - its natural death.

DATE

Chapter - 4 Discrete Event System Simulation

Components and Organization of Discrete Event Simulation.

1) System state
- The collection of state variables necessary to describe the system at a particular time.

2) Simulation Clock
- A variable giving the current value of simulated time.

3) Event List
- A list containing the next time ~~with~~ when each type of event will occur.

4) Statistical Counters
- Variables used for storing statistical information about system performance.

5) Initialization routine :- A sub-program that initializes the simulation model at time 0.

6) Timing Routine :- A sub-program that determines the next event from the event

PAGE

list and then advances the simulation clock to the time when the event is to occur.

(7.) Event Routine :- A sub-program at ~~a base~~ that updates the system state when a particular type of event occurs.

(8.) Library Routines :- A set of sub-programs used to generate random observations that were determined as part of simulation model.

(9.) Report Generator :- A sub-program that computes estimates of the desired measures of performance and produces a report when the simulation ends.

(10.) Main Program :- A sub-program that invokes the timing routine to determine the next event and then transfers control to the corresponding event routine to update the system state appropriately. It also checks the termination and invokes the report generator when the simulation is over.

list and then advances the simulation clock to the time when the event is to occur.

(7.) Event Routine :- A sub-program at ~~a base~~ that updates the system state when a particular type of event occurs.

(8.) Library Routines :- A set of sub-programs used to generate random observations that were determined as part of simulation model.

(9.) Report Generator :- A sub-program that computer estimates of the desired measures of performance and produces a report when the simulation ends.

(10.) Main Program :- A sub-program that invokes the timing routine to determine the next event and then transfers control to the corresponding event routine to update the system state appropriately. It also checks the termination and invokes the report generator when the simulation is over.

Generation of Arrival Patterns

Arrival pattern for particular system is specified for simulation. The exogeneous arrivals can be designed for simulation. Two basic arrival patterns are :-

(i) Trace Driven

- The sequence of inputs can be generated from the observation on a particular system are tested from records gathered from a running system. This method is trace driven simulation. Here, program monitors can be attached to the running system to extract data with no or little disturbance to running system.

(ii) Bootstraping method

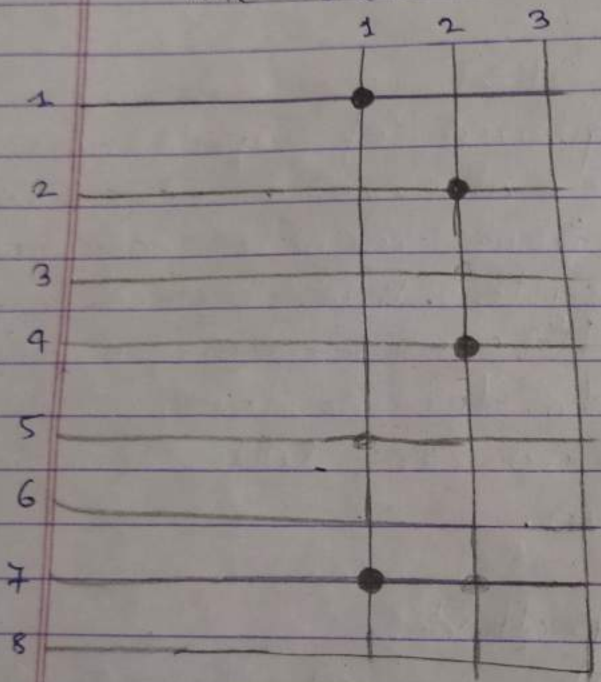
- The arrival time of an entity is recorded as one of the event times. When simulation clock time reaches this event time, the event entering the system is executed and the arrival time of next entity is calculated. This method is called bootstrapping method. Here, one entity creates its successor.

Imp!!

Telephone Call Simulation as Lost Call System

link = 3

line = 8 $(2^{n}+2)$



Line

1	1
2	1
3	0
4	1
5	0
6	0
7	1
8	0

Link

Max^m	3
In Use	2

let initial simulation clock be 15

Simulation Clock

15

DATE

Call-in process

From	To	End
2	4	30
1	7	25

Processed	Completed	Block	Busy
2	0	0	0

Counter

System state = 1

Next Arrival

From	To	Length
3	5	15

1	22

Arrival time

Next Arrival

From	To	Length
3	7	15

20

Arrival time

From	To	End
3	5	37
2	4	30
1	7	25

Call-in progress

Line

	Line
1	1
2	1
3	0
4	1
5	0
6	0
7	1
8	0

Link

Max^m	3
InUse	2

Processed	Completed	Block	Busy
13	0	0	1

Counter

System state - 2 (Busy)

		From	To	Length
22		3	5	15
Arrival time		Next Arrival		

Call in progress

From	To	End
3	5	37
2	4	30
1	7	25

Link

Max^m	3
In use	3

Processed	C	B	B
4	0	0	1

Counters

System state → 3

Next Arrival

F	T	L
6	8	20

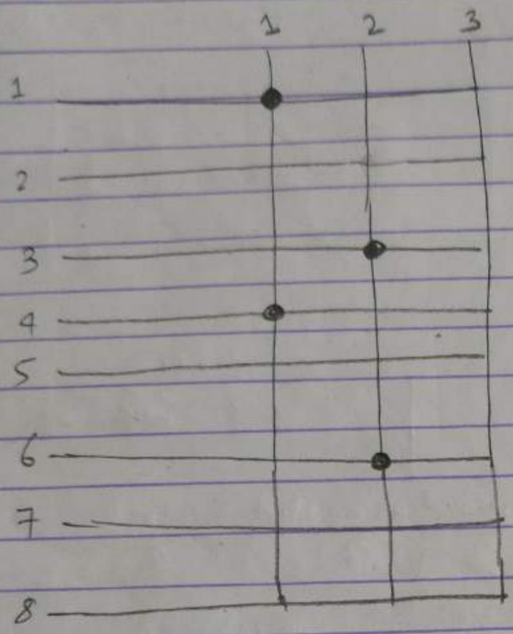
Here, link is full, so call will be blocked.

Processed	Completed	Block	Busy
5	0	1	1

Counters

System state → 4 // Block case.

Delayed Call System



Line

	Line
1	1
2	0
3	1
4	1
5	0
6	1
7	0
8	0

Link

	Link
Max^m	3
In Use	2

Clock

Clock
10

Processed	completed	Block	Busy
2	0	0	0

Counters

Next Arrival

From	To	Length
2	4	15

system State-1

Arrival time
15

Call-in-progress

From	To	End
3	6	30
1	4	25

Delayed Call List

From	To	length

Line

1	1
2	0
3	1
4	1
5	0
6	1
7	0
8	0

Link

Maxm	3
Inuse	2

Clock

15

Processed	completed	Block	Busy
3	0	0	1

Counters

System state - 2

Next Arrival

From	To	length
7	8	18

20

Arrival time

Call in progress

From	To	Length
3	6	30
1	4	25

Delayed call list

From	To	length
2	4	15

	Line
1	0
2	1
3	1
4	1
5	0
6	1
7	0
8	0

Link

Max	3
In use	2

clock

25

Processed	completed	Block	Busy
3	1	0	1

Counters

System state - 3

Next Arrival

From	To	length
7	8	18

27

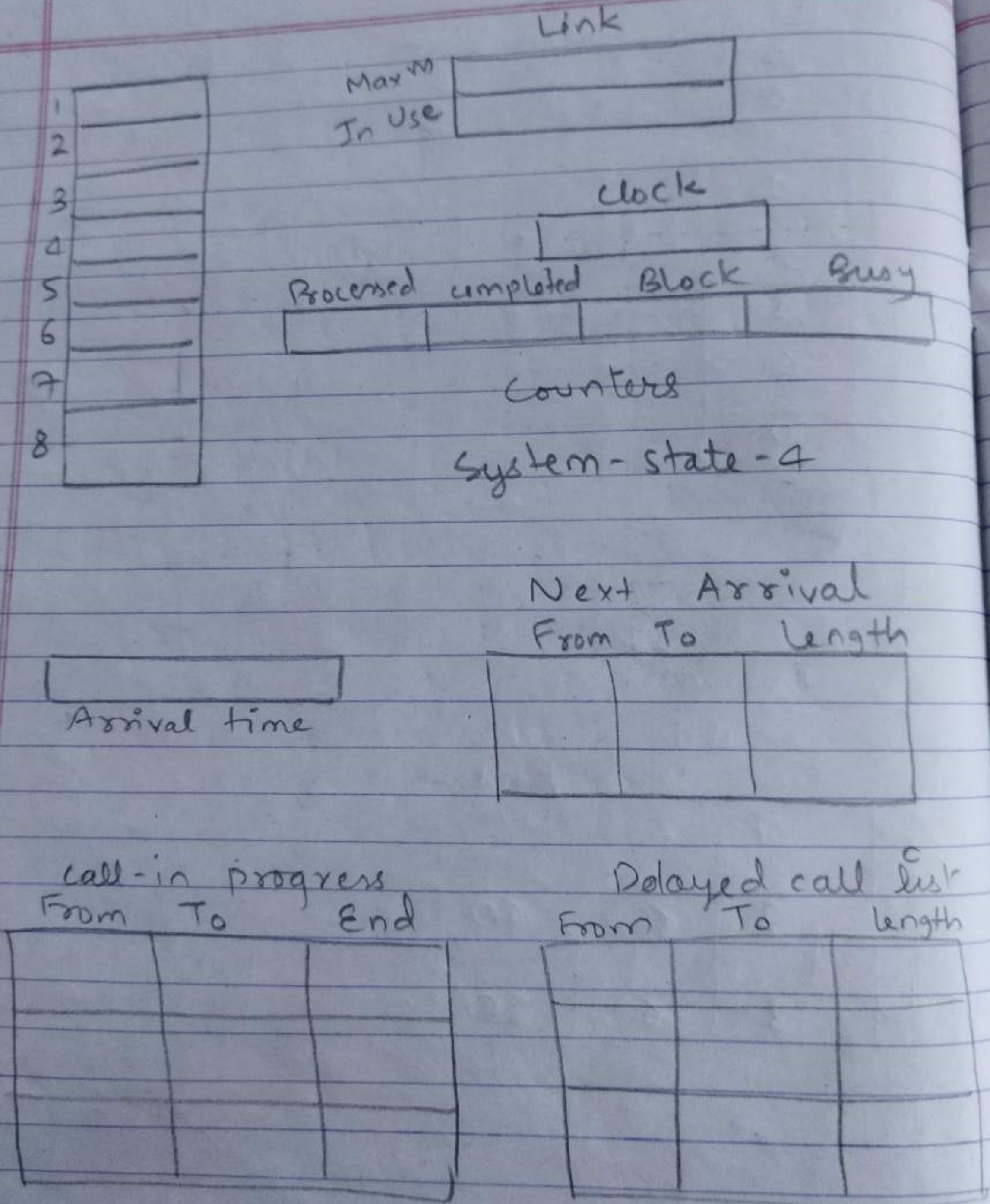
Arrival time

call-in progress

From	To	End
2	4	40
3	6	30

Delayed call list

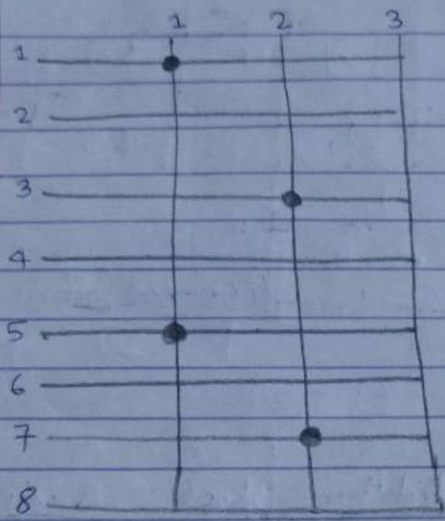
From	To	length


Link
Max^m
In Use
1
2
3
4
5
6
7
8
clock
Processed
completed
Block
Busy
Counters
System - state - 4
Next Arrival
From
To
Length
Arrival time
call-in progress
From
To
End
Delayed call list
From
To
length

Telephone call Simulation as lost Call system

DATE

Link = 3
Line = 8



Line

1	1
2	0
3	1
4	0
5	1
6	0
7	1
8	0

Link

Max	3
Inuse	2

Simulation clock

10

Call-in progress

From	To	Length
3	7	30
1	5	25

Next Arrival

From	To	length
2	7	10

Arrival time

15

Processed	Completed	Block	Busy
2	0	0	0

Counters

System clock → 1

DATE

1	1
2	0
3	1
4	0
5	1
6	0
7	1
8	0

Link

Maxm	3
InUse	2

Call in progress

From	To	End
3	7	30
1	5	25

Simulation Clock

15

Next Arrival

From	To	Length
2	6	15

Arrival time

20

Processed	completed	Block	Busy
3	0	0	1

Counters

system stadte→2

Line

1	1
2	1
3	1
4	0
5	1
6	1
7	1
8	0

Link

Max^m	3
InUse	3

Simulation clock

20

Arrival time

21

Call-in progress

From	To	End
2	6	35
3	7	30
1	5	25

Next Arrival

From	To	length
4	8	10

Processed	Completed	Block	Busy
4	0	0	1

Counters

System state → 3

Line

1	1
2	1
3	1
4	0
5	1
6	1
7	1
8	0

Link

Max^m	3
InUse	3

Call-in progress

From	To	End
2	6	35
3	7	30
1	5	25

simulation clock

21

Arrival time

Next Arrival

From	To	Length

processed	completed	Block	Busy
5	0	1	1

Counters

system state → 4

Simulation Programming Task

(i) Generate Model

(ii) Simulate

(iii) Report

Entities
Attributes

Activities

SystemImage	Routine

Simulation Algorithm

Report Generations

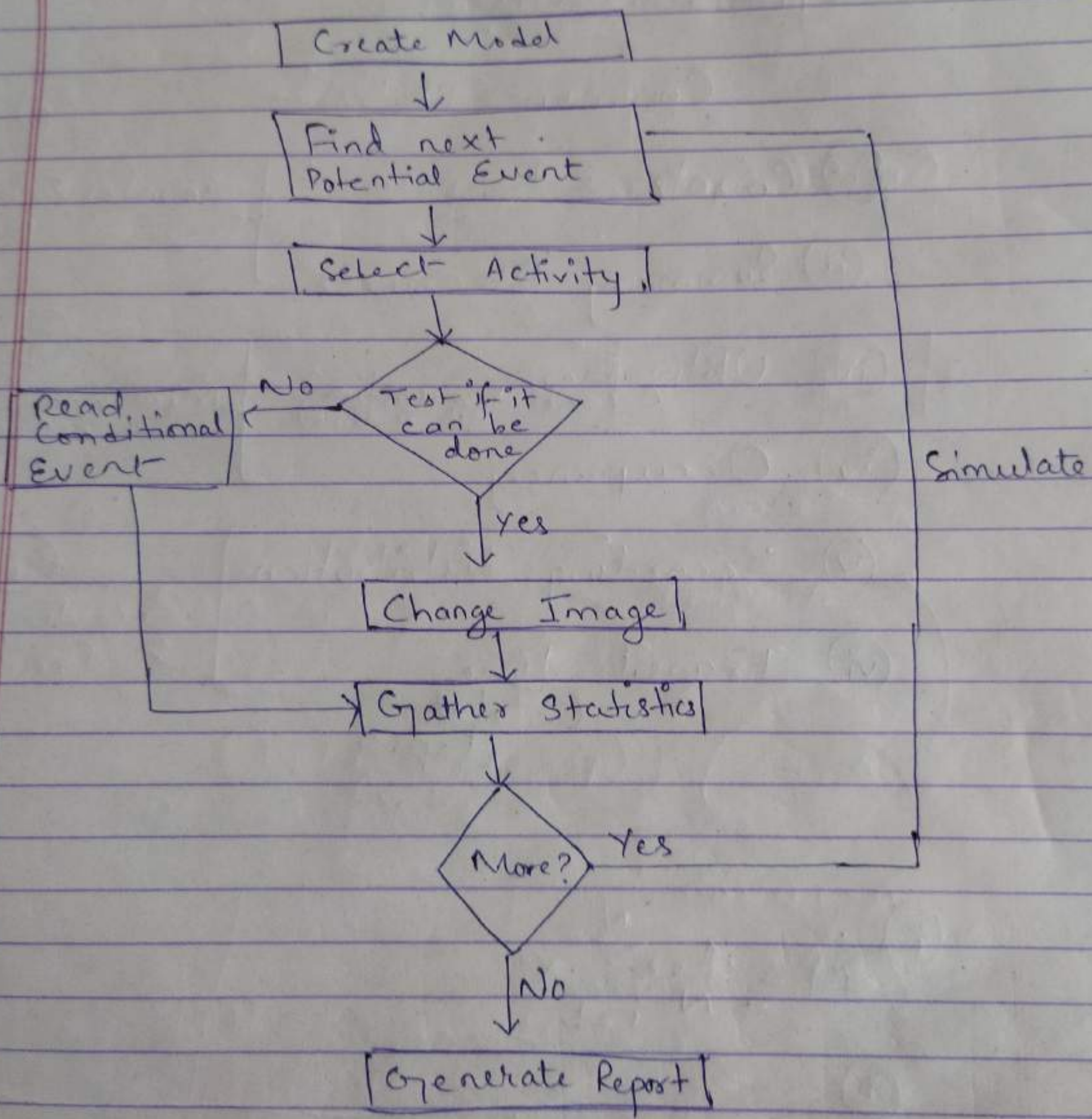


fig: Flow Chart

DATE

Gather Statistics

Common statistics

(i) Counts
(ii) Summary Measures

— (i) Counter & Summary Measure

(iii) Utilization (one transaction at a time)
(iv) Occupancy

— (ii) Measuring Utilization & occupancy

(v) Recording distribution
(vi) Transit time (Total elapse time)

— (iii) Recording distribution & Transit time.

(i)

$$M = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$S = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(M - x_i)^2}$$

DATE

(ii) $t_f \Rightarrow$ free time

$t_b \Rightarrow$ busy time

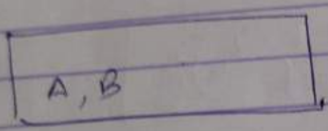
$$(t_f - t_b)_{1^{st}\,call} + (t_f - t_b)_{2^{nd}\,call}$$

$$\boxed{U = \frac{1}{T} \sum_{i=1}^{N} (t_f - t_b)_i}$$

$$\boxed{B = \frac{1}{NM} \sum_{i=1}^{N} n_r (t_{r+1} - t_r)}$$

Chapter-6 Simulation Languages

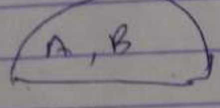
GPSS (General Purpose System Simulation)



(Advance block)

A ⇒ Mean

B ⇒ Modifier



(Generate block)

A ⇒ Mean

B ⇒ Modifier

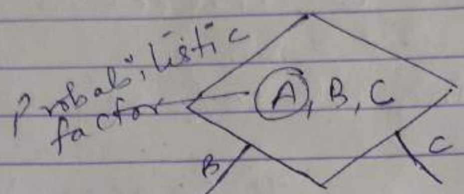
(:)

GENERATE

ADVANCE

(ii) Selection Factor

(iii) TRANSFER



A ⇒ Selection Factor (S)

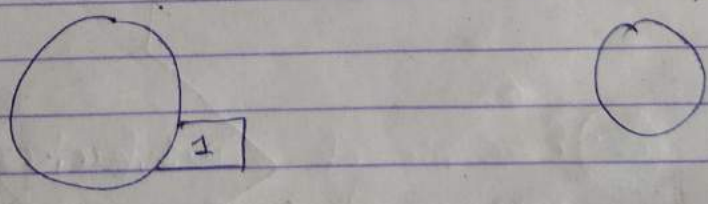
B ⇒ Next block (1-S)

C ⇒ Next block (S)

(iv) Termination

(a) TERMINATE

not terminate/Lost



DATE

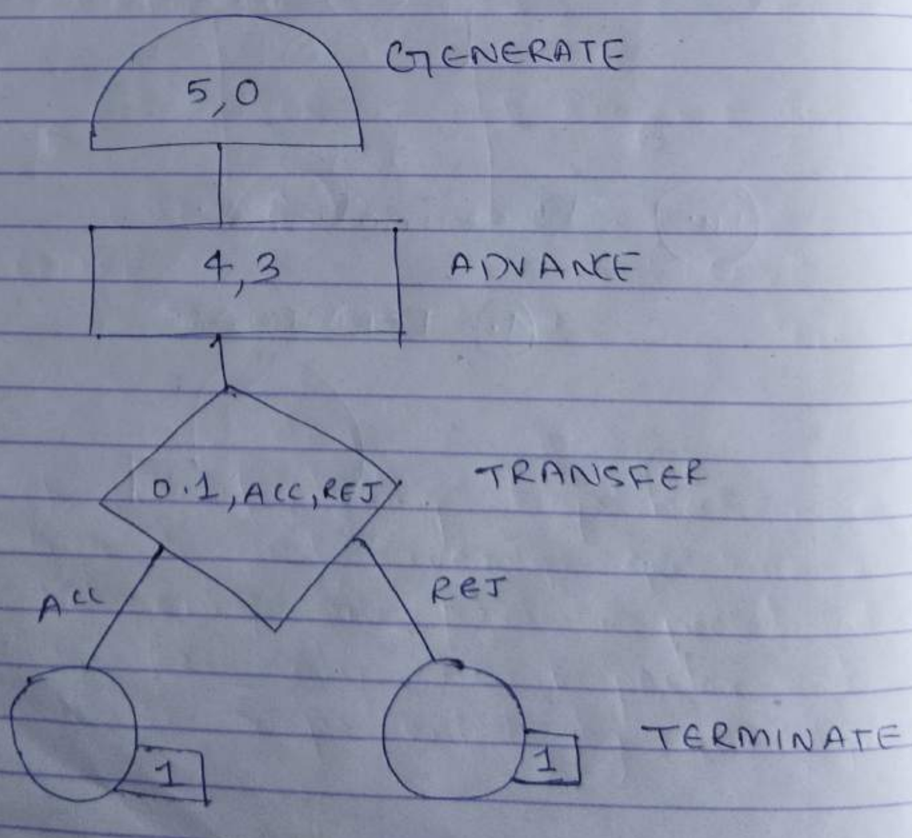
(Q) Manufacturing shop Model - 1

Generate [1 product - 5 minutes
Advance [Inspection - 4 ± 3 minutes

Selection [Selecting - 10% rejected
90% accepted

Terminate [Simulate - 1000 parts

Soln:



DATE

Location	Operation	Operation field	Comment
	SIMULATE		
*			
*	MANUFACTURING SHOP MODEL-1		
*			
	GENERATE	5,0	Creating Part
	ADVANCE	4,3	Inspecting parts
	TRANSFER	0.1, ACC,REJ	Selecting
ACC	TERMINATE	1	Accepting
REJ	TERMINATE	1	Rejecting
*	START	1000	

Relative clock 5005 Absolute clock 5005

Block Count

Block No.	Current	Total
1	0	1001
2	1	1001
3	0	1000
4	0	900
5	0	900
	0	100

Facilities and Storage

1. SEIZE
2. RELEASE
3. ENTER
4. LEAVE

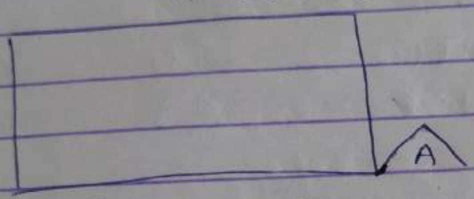


fig: SEIZE

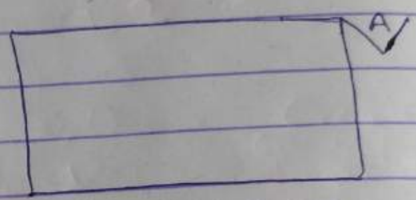


fig: RELEASE

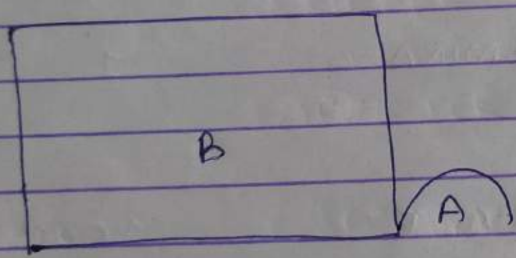


fig: ENTER

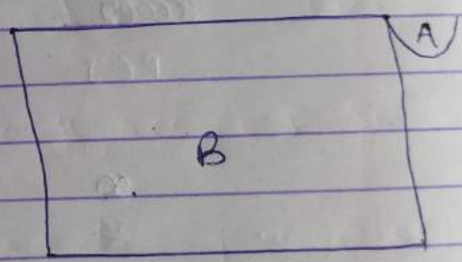
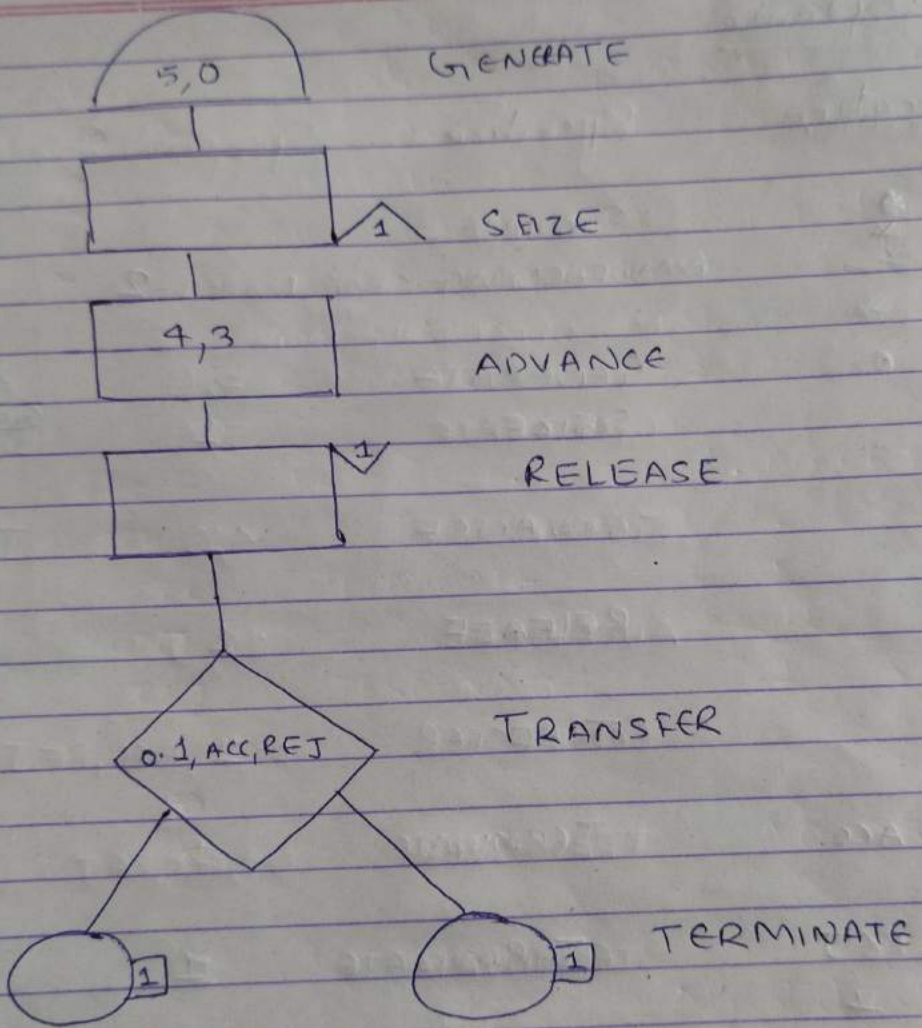


fig: LEAVE

Q. Example of manufactoring shop has average inspection time 4 and average generate rates 5. There will be only one part inspected at a time. Assuming for only one inspector and the inspector should be represented by facility.

DATE


5,0
GENERATE
1
SEIZE
4,3
ADVANCE
1
RELEASE
0.1, ACC, REJ
TRANSFER
1
1
TERMINATE

Program:

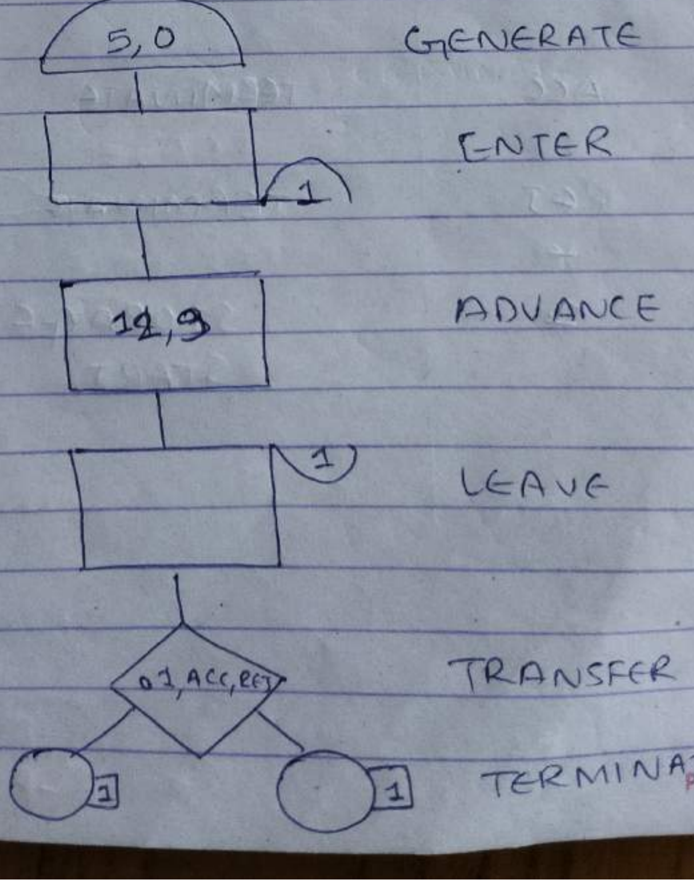
Location	Operation	Operation Field	Comment
	SIMULATE		
*			
*	MANUFACTURING-SHOP MODEL-2		
*			
*	GENERATE	5,0	Creating par
	SEIZE	1	Getting Inspecting par
	ADVANCE	4,3	Inspecting par
	RELEASE	1	Free Inspector
	TRANSFER	0.1, ACC, REJ	Selecting par
ACC	TERMINATE	1	Accepting
REJ	TERMINATE	1	Rejecting
*			
*	START	1000	

Relative Clock: 5430 Absolute clock: 5430

Block Count

Block no.	Current	Total
1	1	1001
2	0	1000
3	0	1000
4	0	1000
5	0	1000
6	0	900
7	0	100

(Q.) Manufacturing shop model is modified such that the average inspection time is 3 times as long as before and 3 inspections are possible at a time. Here, storage is used instead of facility. Ans:



DATE

Program

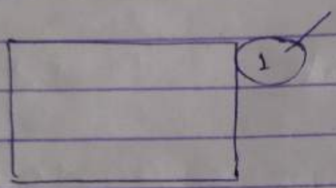
Location	Operation	Operation field	Comments
	SIMULATE		
*	MANUFACTURING SHOP MODEL -3		
*			
*			
	GENERATE	5, 0	Create parts
	ENTER	1	
	ADVANCE	12, 9	Inspect parts
	LEAVE	1	
	TRANSFER	0.1, ACC, REJ	Selecting parts
ACC	TERMINATE	1	Accepting
REJ	TERMINATE	1	Rejecting
*			
1	STORAGE	3	control statement used for enter & leave, that shows the capacity
	START	1000	

Relative clock: 5430 Absolute clock: 5430

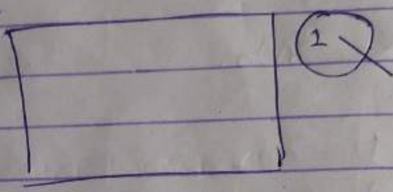
Block Count:

Block No.	Current	Total
1	1	1003
2	0	1002
3	2	1002
4	0	1000
5	0	1000
6	0	900
7	0	100

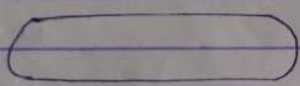
Gathering Statistics



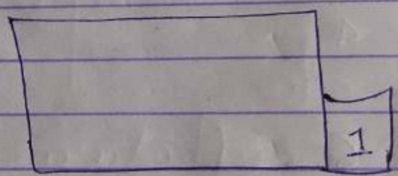
Queue



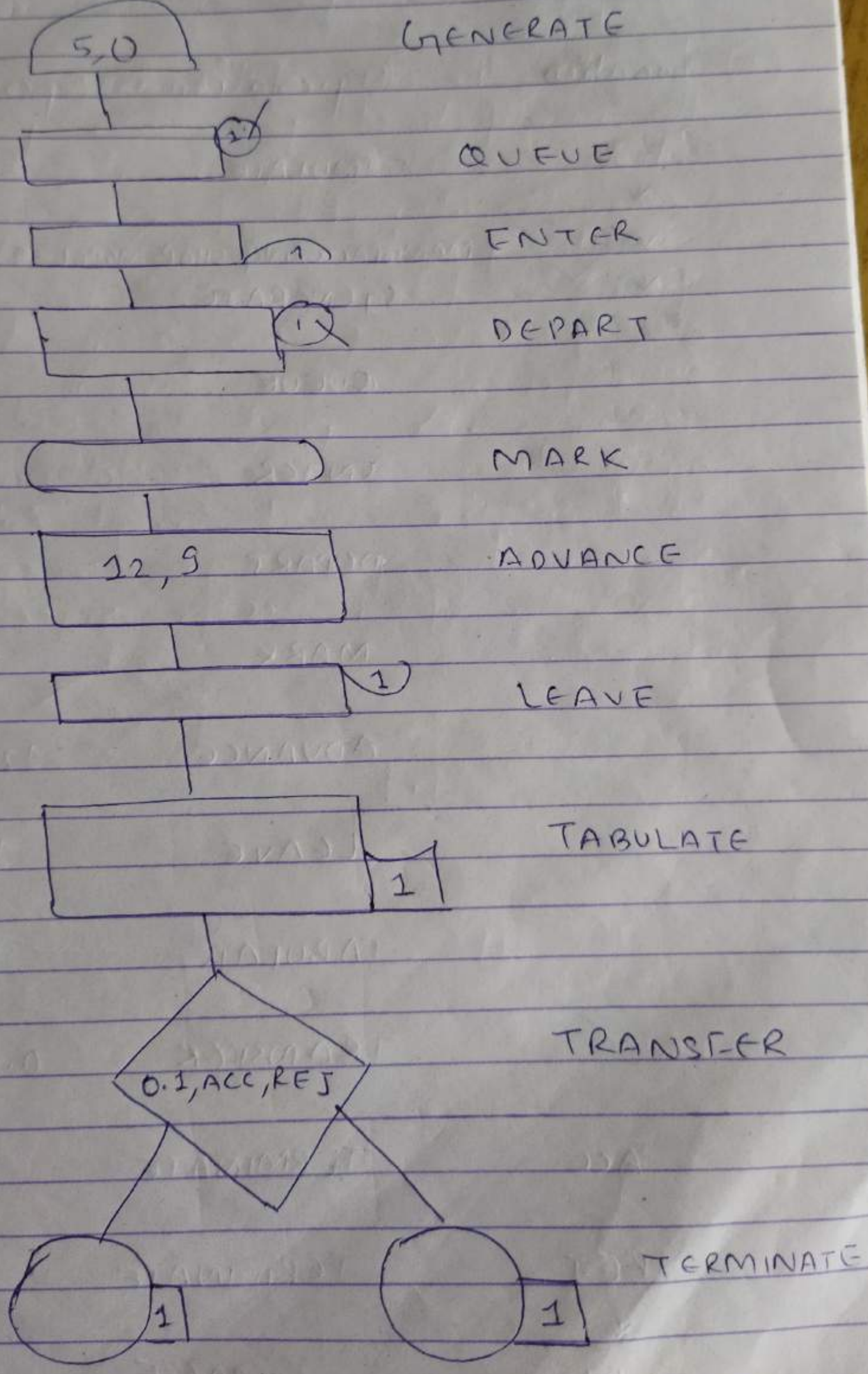
DEPART



MARK



TABULATE


5,0
GENERATE
1
QUEUE
1
ENTER
1
DEPART
MARK
12,9
ADVANCE
1
LEAVE
1
TABULATE
0.1,ACC,REJ
TRANSFER
1
1
TERMINATE

Program

Location	Operation	Operation Field	Comment
	SIMULATE		
*			
*	MANUFACTURING SHOP MODEL-4		
*	GENERATE	5,0	
	QUEUE	1	
	ENTER	1	
	DEPART	1	
	MARK		Time load
	ADVANCE	12,9	
	LEAVE	1	
	TABULATE	1	Measures transit
	TRANSFER	0.1,ACC,REJ	
ACC	TERMINATE	1	
REJ	TERMINATE	1	
*			
1	STORAGE	3	Lower inter no of
	TABLE	M1,5,5,10	
	START	1000	

Relative clock 5005 Absolute clock 50us

Block Count

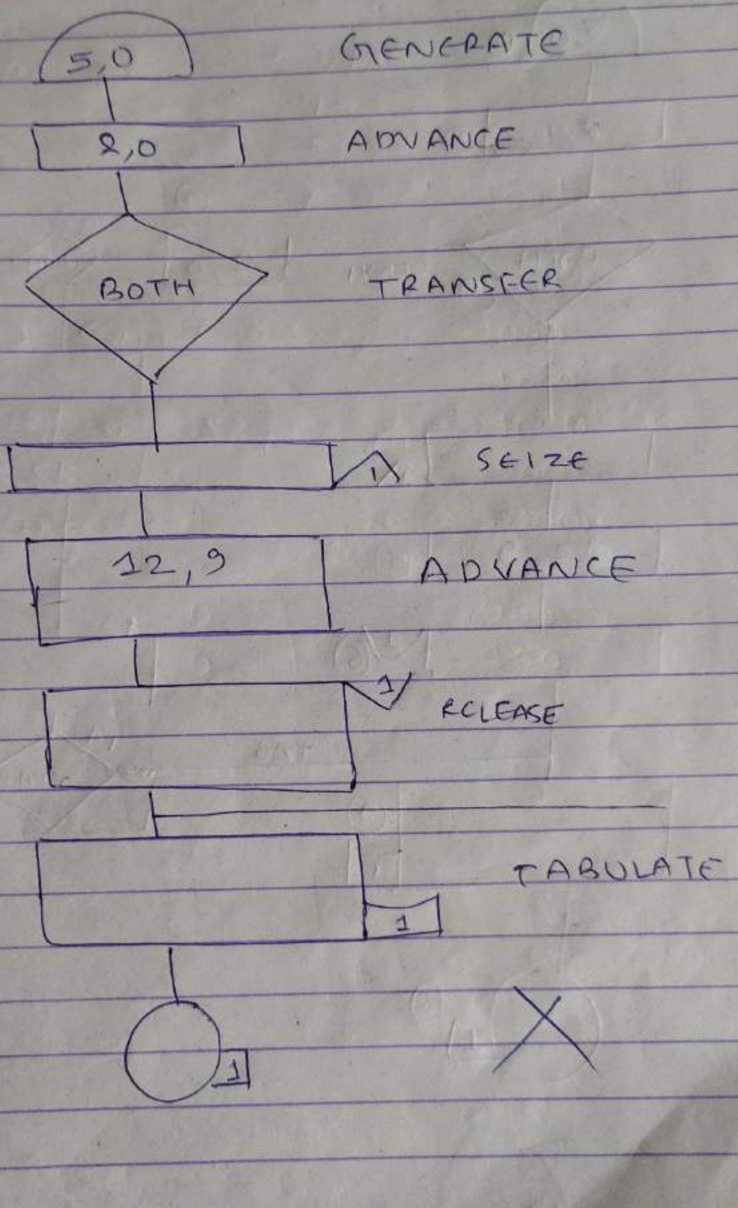
Block no.	Current	Total
1	0	1001
2	1	1001
3	0	1000
4	0	1000
5	0	1000
6	0	1000
7	0	1000
8	0	1000
9	0	1000
10	0	900
11	0	100

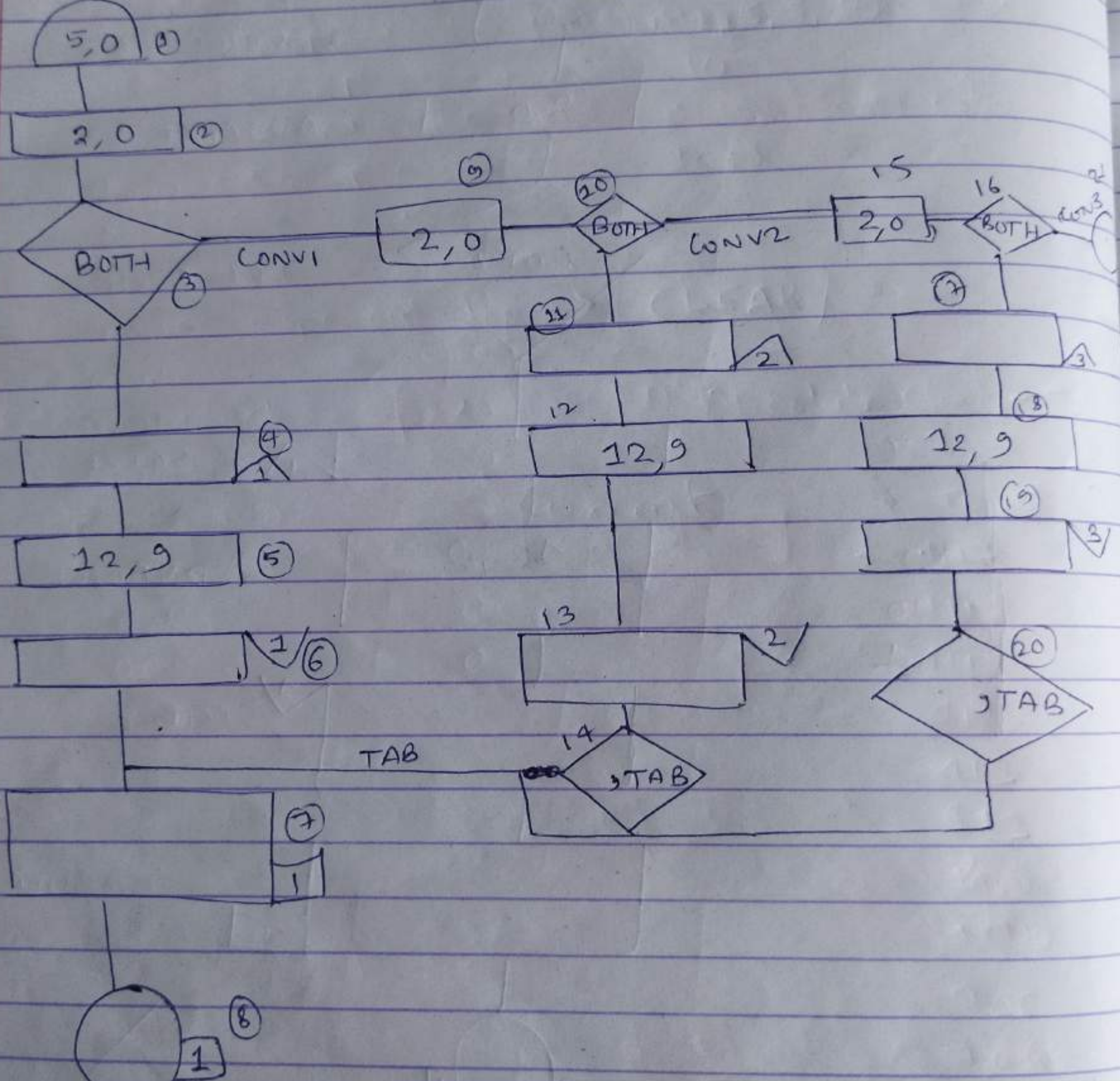
lower limit
interval size
no of intervals

Conditional transfer

Q. Consider the case of 3 inspectors and ~~case of~~ the manufactured parts are put on a conveyer which carries the parts to the inspector at intervals along the conveyer. It takes 2 minutes for a part to reach the 1st inspector. If it is free, it takes the part for inspection else it takes forther 2 minutes to reach to 2nd inspector, who will take if it is free else, passages the part to 3rd inspector which again takes 2 minutes otherwise they're lost. To keep the model small, only the transit time of the parts are recorded and the possibility of inspector rejecting parts are ignored.

DATE


5,0
GENERATE
2,0
ADVANCE
BOTH
TRANSFER
1
SEIZE
12,9
ADVANCE
1
RELEASE
TABULATE
1
1



Location

9 CONV1

Program Location	Operation	Operation Field	Comments
	SIMULATE		
*			
*	MANUFACTURING SHOP MODEL-5		
*	GENERATE	5,0	
	ADVANCE	2,0	
	TRANSFER	BOTH, CON1	
	SEIZE	1	
	ADVANCE	12,9	
	RELEASE	1	
	TABULATE	1	
	TERMINATE	1	
CONV1	ADVANCE	2,0	
	TRANSFER	BOTH,CONV2	
	SEIZE	2	
	ADVANCE	12,9	
	RELEASE	2	
	TRANSFER	,TAB	
CONV2	ADVANCE	2,0	
	~~SEIZE~~	~~BOTH,CONV3~~	
	~~ADVANCE~~	~~3~~	
	~~RELEASE~~	~~12,9~~	
	~~TRANSFER~~	~~3~~	
	TRANSFER	, ~~[illegible]~~ BOTH, CONV3	
	SEIZE	3	
	ADVANCE	12,9	
	RELEASE	3	
	TRANSFER	,TAB	
CONV3	TERMINATE		

	m1, 5, 5, 10
TABLE	10
START	
RESET	1000
START	

wipes out statistics

Transaction wipe out by → $\boxed{\text{CLEAR}}$

Absolute Clock - 5660

Relative clock - 5588

Block No:	Current	Total
1	-	1117
2		1118
3		1118
4		412
5		412
6		412
7		1000
8		1000
9		706
10		706
11		340
12		340
13		340
14		340
15		366
16		366
17		248
18		248
19		248
20		248
21		118

Program Control Statements

SIMULATE → Used to run the simulation
- Without this program will be assembled but not in use.

START → To begin the simulation or start the simulation.

STORAGE → Used to specify the capacity of Enter block.

RESET → Used to wipe out all the statistics gathered. It will leave the system loaded with transaction. It also sets relative clock to zero.

CLEAR → It wipes out statistics and ~~the~~ transactions in the system i.e return the system from beginning.

eg:

START	START
RESET	CLEAR
START	START

JOB → Wipes out the entire model preceding the statement and proceed with following statement.

END → Terminates all simulation.

SIMSCRIPT

Imp.!! 7 marks* with explain

Organization of SIMSCRIPT

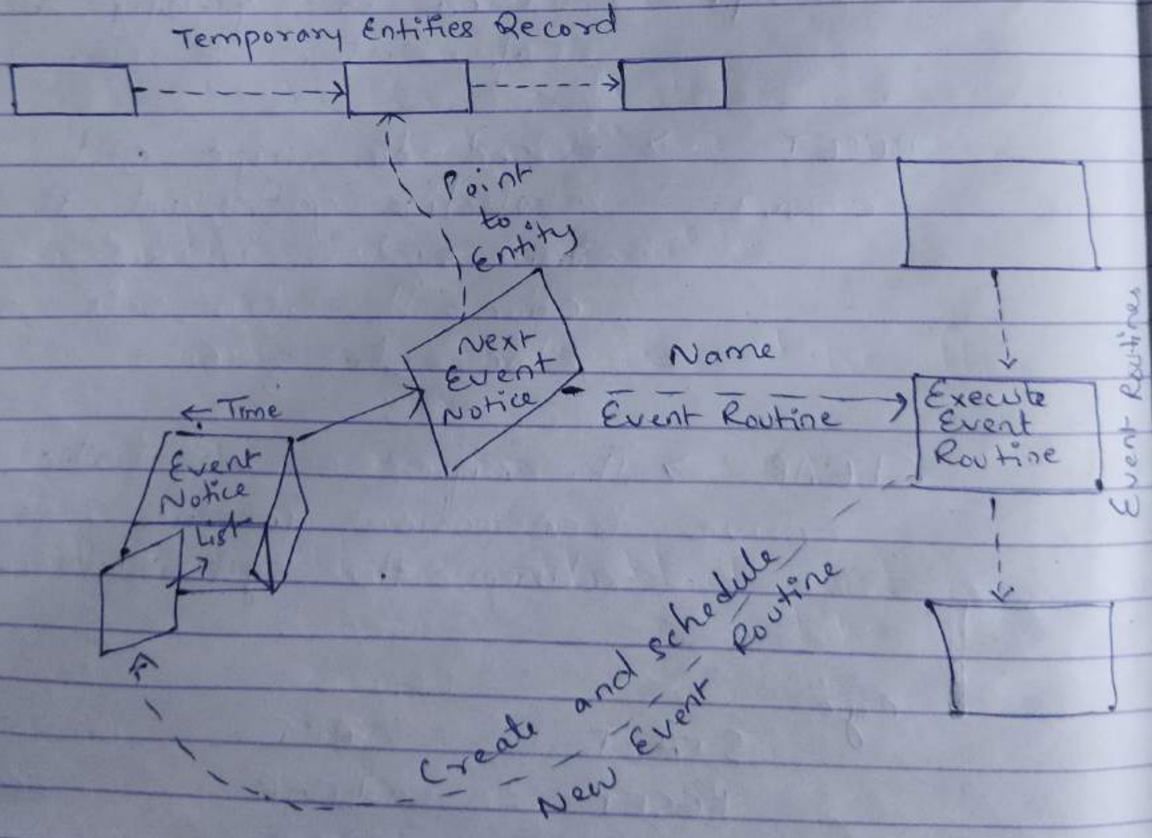


fig: Organization of SIMSCRIPT

⇒ Exogeneous events - need to be scheduled
Endogeneous " → are automatically scheduled.

DATE

Name and Labels

Variable Name, attributes

eg:-

Person. Age

Person. Education

Statements

Print n lines as Follows
or
Print n Lines Thus

eg: Print 1 Lines as Follows
"Welcome"

→ Print lines using variables

Print n Lines with x and y Like this

x ***

y ** . **

eg:-

Print 2 Lines with block and mean Like this

Block *

Mean * . *

DATE

→ To define variables

Define variable-name as variables

Define variable-name as Real variables.

#

→ To define temporary & permanent entities.

Every T. Line has state

→ Temporary Entities

Every Call has Origin and Destination.

→ Event NAME

eg: Arrival Event

Closing Event

PREAMBLE SECTION

→ For comments = single quotation mark twice

eg: " Welcome "

Preamble Section

"Telephone Call Simulation"

PREAMBLE

Normally, Mode is Integer

Event Notices Includes Arrival and Closing

Every Disconnect has Dis.call

Temporary Entities

Every Call has Origin and Destination

Permanent Entities

Every TLine has state

Define link.in.use, max.link, block, busy, finished and stop.time as variables.

Define inter.arrival.time and Mean.length as Real variables.

END

Referencing Variables

→ Single variables are referenced by their names.

→ Permanent entities are referenced by array.

eg: Reference to state of telephone line (I) where I is integer or expression.

STATE (ORIGIN CALL)
STATE (DESTINATION CALL)

→ Temporary Entities
CREATE A CALL

DESTROY A CALL

The Main Routine

(1)

Main
Read N. TLine, Max-links, Inter. Arrival. Time,
Mean. length And STop. Time
Create Every T. line
If N. TLine < 2 * Max. Links + 2
Print 1 Line Thus
Too few lines Specified Simulation Abandoned
stop
Else
Schedule An Arrival Now
Schedule A closing In Stop. Time Minutes
Start Simulation.

(3)

Skip 3 Output Lines
Print 3 Lines with
Finished, Busy, Blocked Thus
Call Processed *
Busy Calls *
Blocked Calls *
Stop
End

DATE

(2.)

"Reports To be Written At End of Simulation"

Print 1 line Thus
Simulation OF Telephone System
Skip 3 Output Lines

Print 5 lines With N.TLine, Max.Links, Inter. Arrival Time, Max.Length, Stop.Time Thus

No. of Lines *
No. of Links *
Mean.Inter.Arrival.Time *.*/seconds
Mean.Call Length *.* / seconds

Simulated Time *

Arrival Routine

Arrival Event
Create Call

Let Origin (call) = Randi.F(1, N.TLine, 1)
Untill State (Origin (call)) = 0,
Let Origin(call) = Randi. F (1, N.TLine, 1)

Let Destination (call) = Randi.F(1, N.TLine, 1)
Until Origin(call) NE Destination(call),

Let Destination (call) = Randi. F (1, N.TLine, 1)

If State Destination (call) = 0 and
Link.in.use < Max.Link

Let State (Origin(call) = 1

Let State (Destination (call) = 1

Add 1 to link.in.use
Schedule A Disconnect Given Call in
Exponential (Mean.Length, 1) secs
Else
If link.in.use = Max.link
Add 1 to Blocked
Else
Add 1 to Busy

Always
Add 1 to Finished
Destroy Call
Always
Schedule An Arrival in Exponential
(Inter.Arrival time, 1) secs

Return
End

The Disconnect Event

Event Disconnect Given Call
Let State (Origin (call)) = 0
Let State (Destination (call)) = 0
Destroy Call

Subtract 1 From Link.in.use
ADD 1 to Finished

Return
~~Stop~~ End

The Closing Event

Event Closing
Cancel the Arrival
Destroy the Arrival

Return
End

Chapter-5

Probability Concepts and Random Number Generation

Uniformity
- chi-square Test
- K-S Test

Independent

⇓

① Run Test
- Run up & Run down
- Run above & below mean
- Length of Runs

② Test for auto-correlation.

chapter-5.

Random Numbers
- The numbers generated by certain process that cannot be regenerated are called random no.s eg; Numbers generated by Noise sampling.

Properties of random nos.

1. Uniformity - All the generated numbers in the given range have equal probability of occurence.

2. Independence - The numbers in the given range or the sequence of numbers are independent i.e they are not autocorrelated.

Pseudo-random numbers

- Computer generated random numbers are called pseudorandom numbers.

- These numbers are generated using algorithm and the no. of generation can be repeated after certain interval of time. Thus called pseudorandom nos.

Random Number Generation Technique

1) Linear Congruential Generator (LCG)

$$Z_i = (aZ_{i-1} + c) \% M$$

↳ $a \Rightarrow$ Multiplicative factor
$c \Rightarrow$ Incremental factor
$Z_{i-1} \Rightarrow$ Seed value
$M \Rightarrow$ Modulus value.

when $a = 1$,

$$Z_i = (Z_{i-1} + c) \% M$$

↳ Incremental generator

Random no. U_i on $(0,1)$ is given by

$$U_i = \frac{Z_i}{M}$$

$c = 0$

$$Z_i = (aZ_{i-1}) \% M \Rightarrow \text{Multiplicative generator.}$$

let, $a = 5$, $c = 3$, $Z_0 = 11$, $M = 16$

$Z_1 = (5 * 11 + 3) \% 16 = 58 \% 16 = 10$

$u_1 = \frac{10}{16} = 0.6$

$Z_2 = (5 * 10 + 3) \% 16 = 53 \% 16 = 5$

$u_2 = \frac{5}{16} = 0.3$

$Z_3 = (5 * 5 + 3) \% 16 = 28 \% 16 = 12$

$u_3 = \frac{12}{16} = 0.7$

$Z_4 = (5 * 12 + 3) \% 16 = 63 \% 16 = 15$

$Z_4 = \frac{15}{16} = 0.9$

$Z_5 = (5 * 15 + 3) \% 16 = 78 \% 16 = 14$

$Z_5 = \frac{14}{} = 0.8$

$$z_6 = (5 * 14 + 3) \% 16 = 73 \% 16 = 9$$

$$u_6 = \frac{9}{16} = 0.5$$

$$z_7 = (5 * 9 + 3) \% 16 = 48 \% 16 = 0$$

$$u_7 = \frac{0}{16} = 0$$

$$z_8 = (5 * 0 + 3) \% 16 = 3 \% 16 = 3$$

$$u_8 = \frac{3}{16} = 0.1$$

$$z_9 = (5 * 3 + 3) \% 16 = 18 \% 16 = 2$$

$$u_9 = \frac{2}{16} = 0.1$$

$$z_{10} = (5 * 2 + 3) \% 16 = 13 \% 16 = 13$$

$$u_{10} = \frac{13}{16} = 0.81$$

$$z_{11} = (5 * 13 + 3) \% 16 = 68 \% 16 = 4$$

$$u_{11} = \frac{4}{16} = 0.25$$

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$z_{12} = (5 * 4 + 3) \% 16 = 23 \% 16 = 7$

$u_{12} = \frac{7}{16} = 0.43$

$z_{13} = (5 * 7 + 3) \% 16 = 38 \% 16 = 6$

$u_{13} = \frac{6}{16} = 0.3$

$z_{14} = (5 * 6 + 3) \% 16 = 33 \% 16 = 1$

$u_{14} = \frac{1}{16} = 0.06$

$z_{15} = (5 * 1 + 3) \% 16 = 8 \% 16 = 8$

$u_{15} = \frac{8}{16} = 0.5$

$z_{16} = (5 * 8 + 3) \% 16 = 43 \% 16 = 11$

$u_{16} = \frac{11}{16} = 0.68$

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→ For full period we have a theorem.

(1) The only positive value that exactly divides c & M is 1.

(2) If q is a prime no. that divides m then q divides a − 1.

(3) If 4 divides M then 4 divides a − 1.

2 # Mid-Square Method

(1) Take static number (seed) say n digits

(2) Square it to get 2n digits, if required add leading zeros.

(3) Next random no. is the mid n digits of the squarred number.

(4) Put decimal before this number to get next v_i.

$Z_i = 12 \qquad n = 2$

$(Z_i)^2 = 0\underline{14}4$

$Z_i = 12 \quad n = 2$

i	Z_i	Z_i^2	U_i
0	12	0144	0.12
1	14	0196	0.14
2	19	0361	0.19
3	36	1296	0.36
4	29	0841	0.29
5	84	7056	0.84
6	05	0025	0.05
7	02	0004	0.02
8	00	0000	0.00

Drawback

1. May generate same type of random numbers repeatedly (i.e no randomness)

2. Prediction is easy for next random number as the method is based on mid-square which is not efficient method of generation.

3. In some cases $z_i^2 = 0\,0$, the next number will be zero forever.

Mixed Generators

⇒ For a large period any high density of u_i's on $[0, 1]$ we want M to be large. Choice of M is 2^b where b is number of binary digits in a word on the computer for data storage.

eg: for 32-bit compilers, the left most bit is sign bit, $b = 31$

$\therefore m = 2^{31}$.

The Largest integer that can be represented is $(2^b - 1)$.

eg: $b = 4$ bits per word

$m = 16$

$z_3 = 12$ from $z_2 = 5$

Now,

$5 * z_{20} + 3 = 28$ which in binary representation, is 11100. Since 4-bit computer can store only four binary digits. the leftmost bit is dropped, leaving the binary 1100 which is representation of $z_3 = 12$.

Thus, $m = 2^b$ appears a good choice of for M.

We obtain full period if c is odd and $a-1$ is divisible by 4, z_0 can be any value between 0 to $m-1$.

For the choice of a.

$a = 2^l + 1$ where 'l' is some +ve integer.

Then,

$$a z_{i-1} = 2^l z_{i-1} + z_{i-1}$$

So that $a z_{i-1}$ can be obtained by "shifting" binary representation of z_{i-1} to the left by l bits and adding z_{i-1}. Thus multiplication can be replaced by and add operations.

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Test for Random Numbers

Random numbers have desirable properties i.e uniformity and independance. To achieve the desirable properties, number of tests can be performed. Five tests for random numbers are:-

(i.) Frequency Test - Uniformity

(ii.) Run Test.

(iii.) Test for autocorrelation

(iv.) Gap Test

(v.) Poker Test

Independence.

For uniformity test, the hypothesis are as follows:-

$$H_0 : R_i \sim U[0,1]$$

$$H_1 : R_i \nsim U[0,1]$$

The null hypothesis, H_0 reads the numbers generated are uniformly distributed. Failure to reject the hypothesis means that there is no evidence of non uniformity on the basis of test.

For independence test, the hypothesis are as follows:-

$$H_0 : R_i \sim \text{independent}$$

$$H_1 : R_i \nsim \text{independent}$$

The null hypothesis, H_0 reads the numbers generated are independent. Failure to reject the hypothesis mean that there is no evidence that the numbers are dependent on the basis of test.

For testing, level of significance 'α' is specified which gives the probability to accept or reject.

i.e

$$\alpha = P(H_0 \text{ reject} / H_0 \text{ true})$$

commonly α is 0.05.

1) Frequency Test

(i) K-S Test

The procedure used to perform K-S test are as follows:-

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Step 1: Rank the random number from smaller to larger i.e;

$$R_{(1)} \leq R_{(2)} \leq \ldots\ldots R_{(n)}$$

Step 2: Compute

$$D^{+} = \max_{1 \leq i \leq N} \left(i/N - R_{(i)} \right)$$

$$D^{-} = \max_{1 \leq i \leq N} \left(R_{(i)} - \frac{i-1}{N} \right)$$

Step 3: Compute

$$D = \max\left(D^{+}, D^{-}\right)$$

Step 4: For specified level of significance, α and given sample size N, take the table value D_α.

Step 5: Compare the table value D_α with D. If $D \leq D_\alpha$, the null hypothesis cannot be rejected ie the numbers are uniformly distributed.

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Q. Suppose 0.44, 0.81, 0.14, 0.05, 0.93 were generated. Perform K-S test to check the uniformity property for $\alpha = 0.05$. The table value $D_\alpha = 0.565$.

Soln:

Given, Sample size = 5

0.44, 0.81, 0.14, 0.05, 0.93

Step 1:

0.05, 0.14, 0.44, 0.81, 0.93.

Step 2:

For $D^+ = \max_{1 \le i \le N} \left(i/N - R_{(i)} \right)$

$R_{(1)}$ 0.05 $= \left(\frac{1}{5} - 0.05 \right)$

$= 0.15$

$R_{(2)}$ 0.14 $= \left(\frac{2}{5} - 0.14 \right)$

$= 0.26$

$R_{(3)}$ 0.44. $= \left(\frac{3}{5} - 0.44 \right)$

$= 0.16$

$R_{(4)}$ 0.81 $= \left(\frac{4}{5} - 0.81 \right)$

$= -0.01$

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$R_{(5)}$ $0.93 = \left(\frac{5}{5} - 0.93\right)$

$= 0.07$

R_i	0.05	0.14	0.44	0.81	0.93
i/N	0.2	0.4	0.6	0.8	1.0
D^+ $i/N - R_{(i)}$	0.15	0.26	0.16	-0.01	0.07
D^- $R_{(i)} - \frac{i-1}{N}$	0.05	-0.06	0.04	0.21	0.13

step 3:

Now,

$D^+ = 0.26$

$D^- = 0.21$

Step 4:

$D = \text{Max}(D^+, D^-)$

$= \text{Max}(0.26, 0.21)$

$= 0.26$

Since, $D(0.26) < D_\alpha(0.565)$, the null hypothesis cannot be rejected on the basis of this test i.e the numbers are uniformly distributed.

(ii) Chi-square Test

Steps:

(1) Divide the sample into n intervals of equal size. (0-0.1, 0.11-0.2, ---0.91-1.0)

(2) Count the observed frequency on the given range.

(3) Calculate the estimated frequency using

$$E(i) = \frac{N}{n}$$

where,

N ⇒ total no. of observation

n ⇒ Number of intervals.

(4) Perform

$$\chi_0^2 = \frac{\sum_{i=1}^{n} (O_i - E_i)^2}{E_i}$$

(5) Generate critical value, $\chi^2_{\alpha, n-1}$ from the table for level of significance α and n-1 degree of freedom.

(6) If $\chi_0^2 \le \chi^2_{\alpha, n-1}$, the null hypothesis that the numbers are uniformly distributed cannot be rejected.

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Q. Use the chi-square test with $\alpha = 0.05$ to test whether the data shown below are uniformly distributed. Let $n = 10$ with equal length. The critical value ~~= 0~~ is $\chi^2_{0.05,9} = 16.9$.

0.34	0.90 ✓	0.25	0.89	0.87	0.44	0.12
0.83	0.76	0.79	0.64	0.70	0.81	0.94 ✓
0.96	0.99 ✓	0.77	0.67	0.56	0.41	0.52
0.47	0.30	0.17	0.82	0.56	0.05	0.45
0.79	0.71	0.23	0.19	0.82	0.93 ✓	0.65
0.99 ✓	0.17	0.99 ✓	0.46	0.05	0.66	0.10
0.37	0.51	0.54	0.01	0.81	0.28	0.69
0.72	0.43	0.56	0.97 ✓	0.30	0.94 ✓	0.96 ✓
0.06	0.39	0.84	0.24	0.40	0.64	0.40
0.18	0.26	0.97 ✓	0.88	0.64	0.47	0.67

0.21	0.46	0.67
0.74	0.22	0.74
0.73	0.99 ✓	0.62
0.31	0.78	0.05
0.37	0.39	0.42
0.42	0.18	0.49
0.34	0.75	0.49
0.58	0.73	0.05
0.19	0.79	0.62
0.11	0.29	0.78

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	i	O_i	E_i	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
(0-0.1)	1	8	10	4	0.4
(0.11-0.2)	2	8	10	4	0.4
(0.21-0.3)	3	10	10	0	0
(0.31-0.4)	4	9	10	1	0.1
(0.41-0.5)	5	12	10	4	0.4
(0.51-0.6)	6	8	10	4	0.4
(0.61-0.7)	7	10	10	0	0
(0.71-0.8)	8	14	10	16	1.6
(0.81-0.9)	9	10	10	0	0
(0.91-1)	10	11	10	1	0.1

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i -}{E}$$

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Since, $\chi^2_0 (3.4) < \chi^2_{0.05,9} (16.9)$, the null hypothesis that the random numbers are uniformly distributed cannot be rejected on the basis of this test.

$$\chi^2_0 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

ADA
SP

$i \quad O_i \quad E_i \quad (O_i - E_i)^2 \quad (O_i - E_i)^2/E_i$

$$\chi^2_0 = \frac{\sum_{i=1}^{n} (\quad)}{E_i}$$

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Simulation

→ Simulation is the immitation of operation of the real world process or system over time.

→ Simulation can be done using a digital computer or by any other equipments.

→ In Simulation, we generate the artificial history of the system & observe the history to find out the different operational characteristics of the real system.

The behaviour of the system according to time is studied by creating a system model. There are many assumptions to be made when we develop a model. These assumptions can be expressed in terms of mathematical, logical, symbolic or other relationships. After the development of the model, it can be used to predict the changes in output by different changes in parameters of the system. Or, it can be used in design stage to build any real world system. Thus, simulation is a tool that can be used to predict the changes to existing system and as design tool to predict the performance of a new system under varying sets of circumstances.

Many times, the model can be made in terms of mathematical expressions. Thus, different mathematical techniques can be used to study the system. In these cases, a numerical computer can be used to immitate the behaviour of the system over time. Data collected by the process of simulation can be used to estimate the measure of performance of the system as if it was collected from the system.

<u>System</u> : A system is defined as group of objects that are joined together in some regular interaction or interdependence towards the accomplishment of same purpose. for eg: in a production system the machines, component parts and workers work together to produce a high quality machine.

<u>System Boundary</u>: It separates the system from the external environment. Thus, all the entities that lie outside the system boundary are said to be in the environment. S.B. is not a concrete term, it can change according to the system we consider. eg: in case of production line, the change in price of raw materials can be considered as a part of environment, but if we want to study effect of supply & demand, then it must be part of the system.

Model

Body of information of a system gathered for the purpose of studying system.

Entities, objects, ~~attributes~~. activity
- The different objects within the system that work among one another are system entities.

The property of entity is called an attribute of the entity. A specified length of time within the system is called an activity.

State : The state of a system is defined as the collection of variables that must be used to describe the system at any one time in relation to the object of study.

Events : Event is the occurence that may change the state of the system. If an event occurs, inside the system then it is called endogeneous and if it occurs outside the system, then it is exogeneous.

Types of Systems

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(1.) Continuous & Discreete system

→ The system in which the changes are predominantly smooth are called continuous system.

eg: in an aircraft system, with automatic pilot, the movement of air craft occurs smoothly.

We can say that continuous system is one in which the state variables change continuously over time.

→ A discreete system is one in which the state variables change only at a discreete set of points in time.

Bank is an example of discreete systm; Here the state variable i.e the no. of customers in a bank change only when customer arrives or when the service provided to a customer is completed.

Few systems are wholly continuous or discreete. The aircraft for example, may make discreete adjustment to its trim as altitude change. In a factory system, which can be considered as discreete, the machining operation is continuous even though start & finish of a job are discreete.

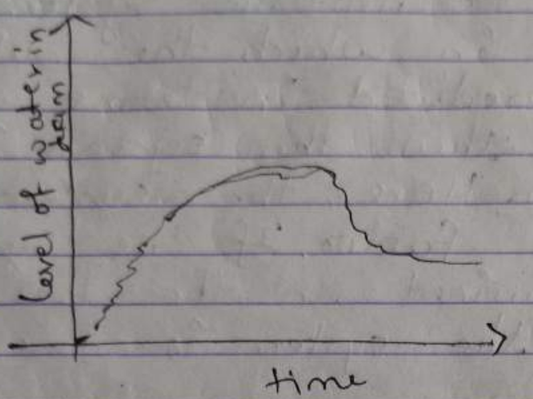

fig: Level of water behind dam to show example of continuous system

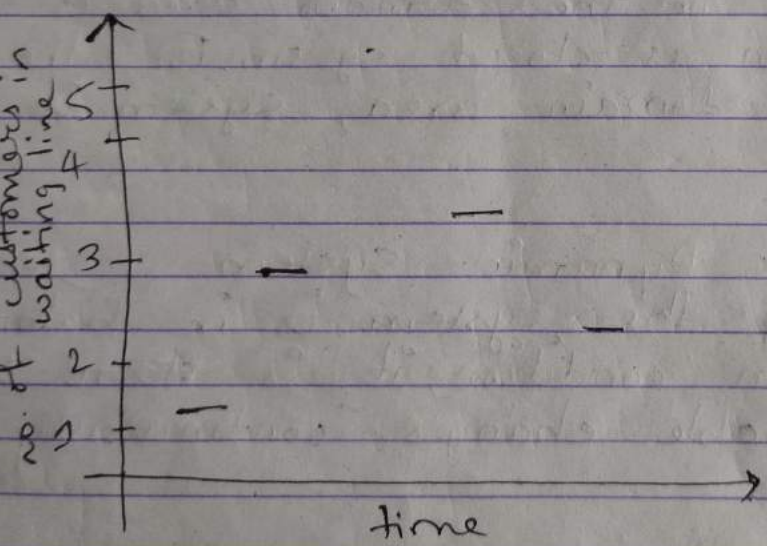

fig: No. of customers in line in a bank to show example of discreete system

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(2.) Stochastic & Deterministic System

→ The system in which the output of ~~of~~ can't be predicted in terms of set of inputs then the system is stochastic system. Here, the system output has probabilistic factor. If the system output is determined on the basis of set of inputs, the system is deterministic system.

(3.) Close & Open System

→ The system that has closed system boundary or the domain area is fixed is known as closed system. For the variable domain area, system is open system.

(4.) Static & Dynamic System

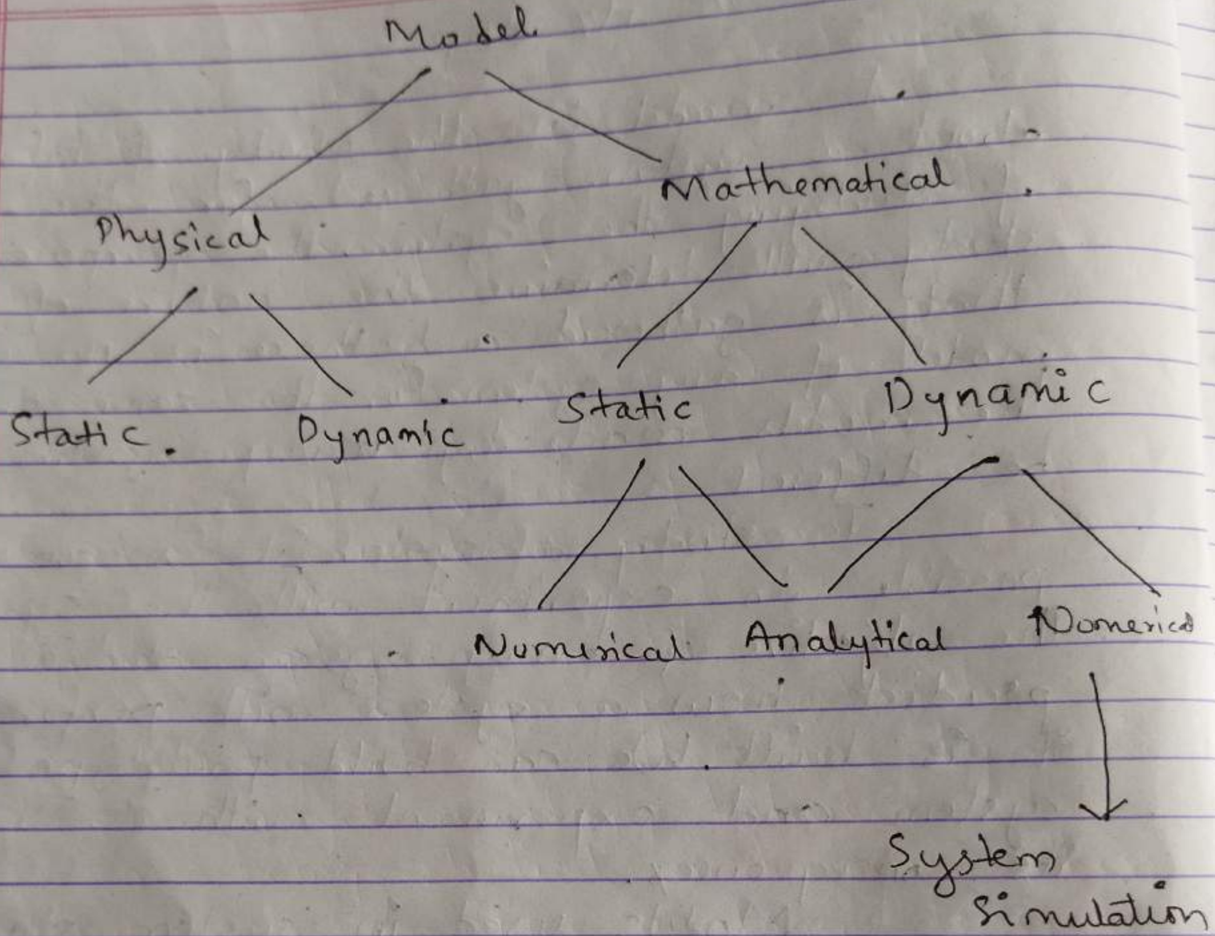
→ If the system is in balanced or equillibrium condition, it is static & if the variable changes continuously it is dynamic.

Model & its types

A model is the body of information about a system gathered for the purpose of studying the system. The purpose of study will determine the nature of informatⁿ, that is gathered and there is no unique model of any system. For the same system, the models made by many system may be different.

To study a system, it is sometimes possible to experiment with the system itself. The objective of many system studies is to predict how a system will perform before it is built. We can build prototype of a system and experiment with it, To study its consequences.

Models used in system studies is classified in many ways. One of such classification is shown below: refer fig: 1



In this scheme, models are divided first into two types: physical & mathematical. Physical models are based on analogy betn systems.

eg: analogy betn electrical & mechanical system.

In a physical model of a system the system attributes are represented by measurements such as voltage or position of a shaft. The system activities are shown in terms of physical laws of another system. for eg: the rate at which the shaft of a DC motor turns depends on voltage applied. Thus, if we have to model a car system then we can use voltage applied as velocity of car, the no. of revolution of shaft will give the distance travelled in given time.

Mathematical model use mathematical expressions and equations to represent the system. The system attributes are represented by variables and the activities are represented by mathematical functions that inter relate the variables.

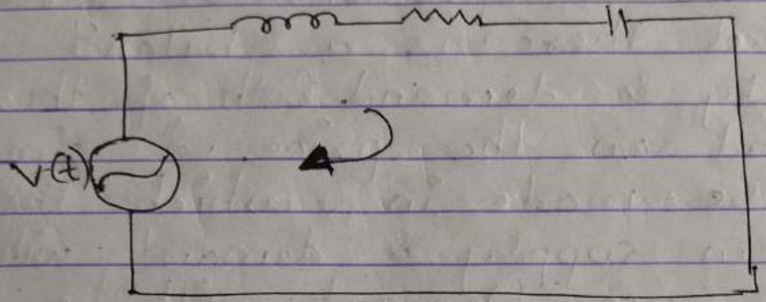
Static physical model

As discussed above, the static physical model uses analogy between systems to represent a system in terms of another and the system itself is in equillibrium. One well known example of static physical model is the scale models. Scale models mean the mineature model of a large system such as the DNA molecule. For eg: to make a static model of a molecule, we can use spheres to represent the atoms and rods or metal plates to

represent the bond. This technique was used and had great success in deciphering the characteristics of DNA molecule.
Another example, if we want to measure the heat distribution in a complex shape. Then we can make the shape and flow charge through it. We can measure the value of charges at different points to give the value of heat as heat & charge distribute in analog manner.

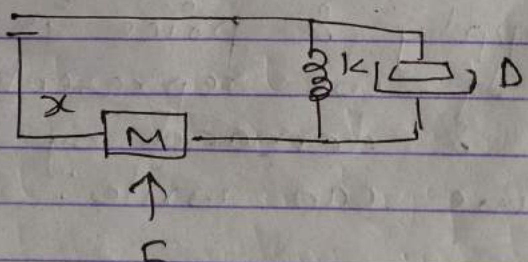
Dynamic physical model
-In a dynamic model, the state is constantly changing. Thus, in dynamic physical model, analogy between the system being studied and some other system is used, the analogy depends upon an underlined similarity in the forces governing the behaviour of the system.
Example: To study the mechanical system in which the force 'F' is applied to the mass 'm' with spring constant 'k' and damping constant 'D' is equivalent to electrical circuit. In electrical circuit, voltage gives the force applied. Inductance is equal to mass, Resistance is equal to D and Capacitance is equal to 'K'.

The value of charge 'q' will give the distance moved by mass 'm'



Electrical System

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + q \times \frac{1}{C} = v(t)$$



Mechanical System

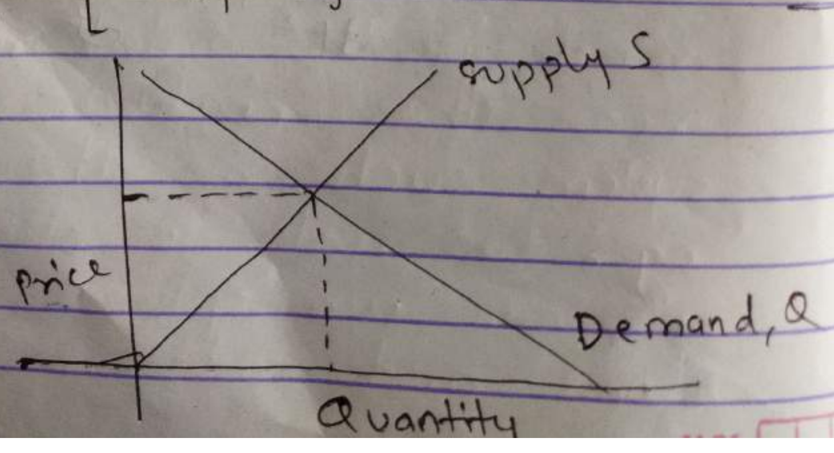
$$M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + kx = k\,F(t)$$

Static Mathematical Model

- This model gives the values of the system variable when the system is in equillibrium in terms of mathematical equations & expressions. For eg: in the market there is a balance between supply & demand. Both of these factors depend on the price. A simple model can be made in which the balance between supply & demand will occur at right price. Demand will be low when the price is high and it will increase as the price drops. The relationship betⁿ demand ~~queue~~ Q and price 'p' is called demand curve. The supply increase as the price increases. And decreases the price decreases. The relationship betⁿ the supply & price is called supply curve. If we represent both the supply & demand by straight line. Then, we can write:

$Q = a - bp$ [Extracted from mathematical expⁿ $y = mx + c$

$S = c + dP$



For balance or equillibrium condition,
$Q = S$.
If we put $a = 600$, $b = 3{,}000$

$c = -100$ and $d = 2{,}000$
Then,

$$p = \frac{(a-c)}{b+d}$$

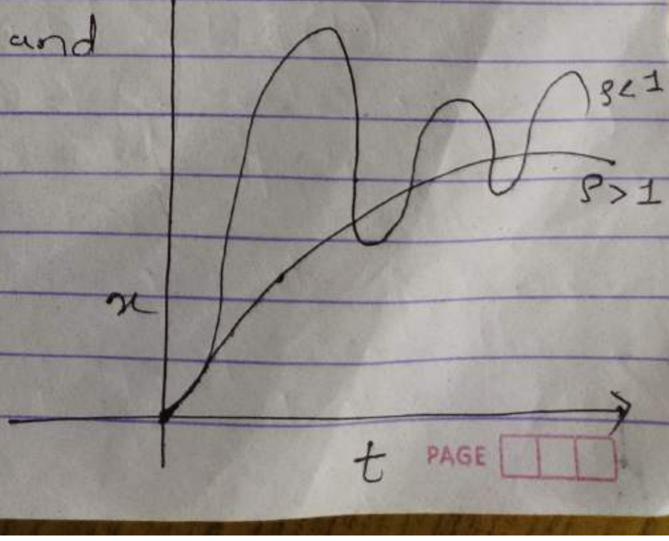
$$= 0.14$$

Dynamic Mathematical Model
- This model allows changes of a system attributes to be derived as a function of time. The equation that is used to describe the behaviour of a car wheel is an example of a dynamic mathematical model. The eqⁿ is given as:

$$\frac{d^2x}{dt^2} + 2\rho\omega\frac{dx}{dt} + \omega^2 x = \omega^2 F(t)$$

where $\frac{D}{M} = 2\rho\omega$ and

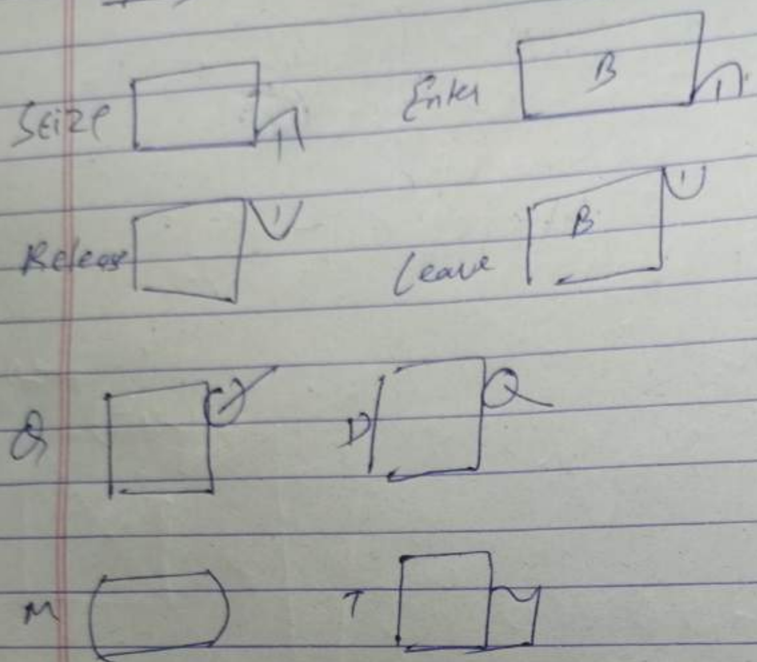
$\frac{K}{M} = \omega^2$



For $\zeta < 1$, the motion is oscillatory,
This can be seen as how x varies
when we increase time 't'. Since, it
can be seen that the x varies with
time 't', we know that the system
is dynamic.

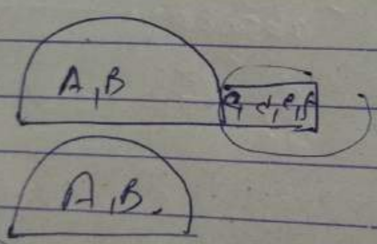
Flow chart

GPSS

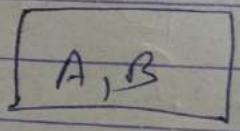


Start

Generate



Advance



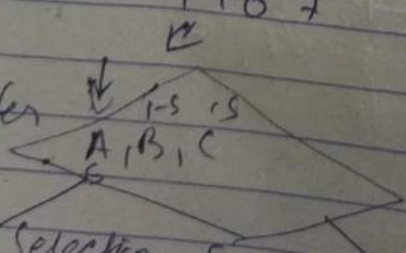
mean
modifie

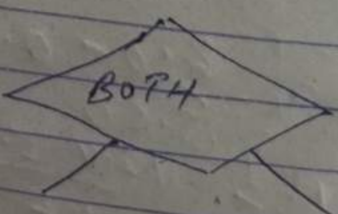
4±3

A => mean 4 minutes
B => modifies
time => 4-3 to 4+3
1 to 7

5±2

Transfer





Selection = S
B = next block
C = next

0.8 /.

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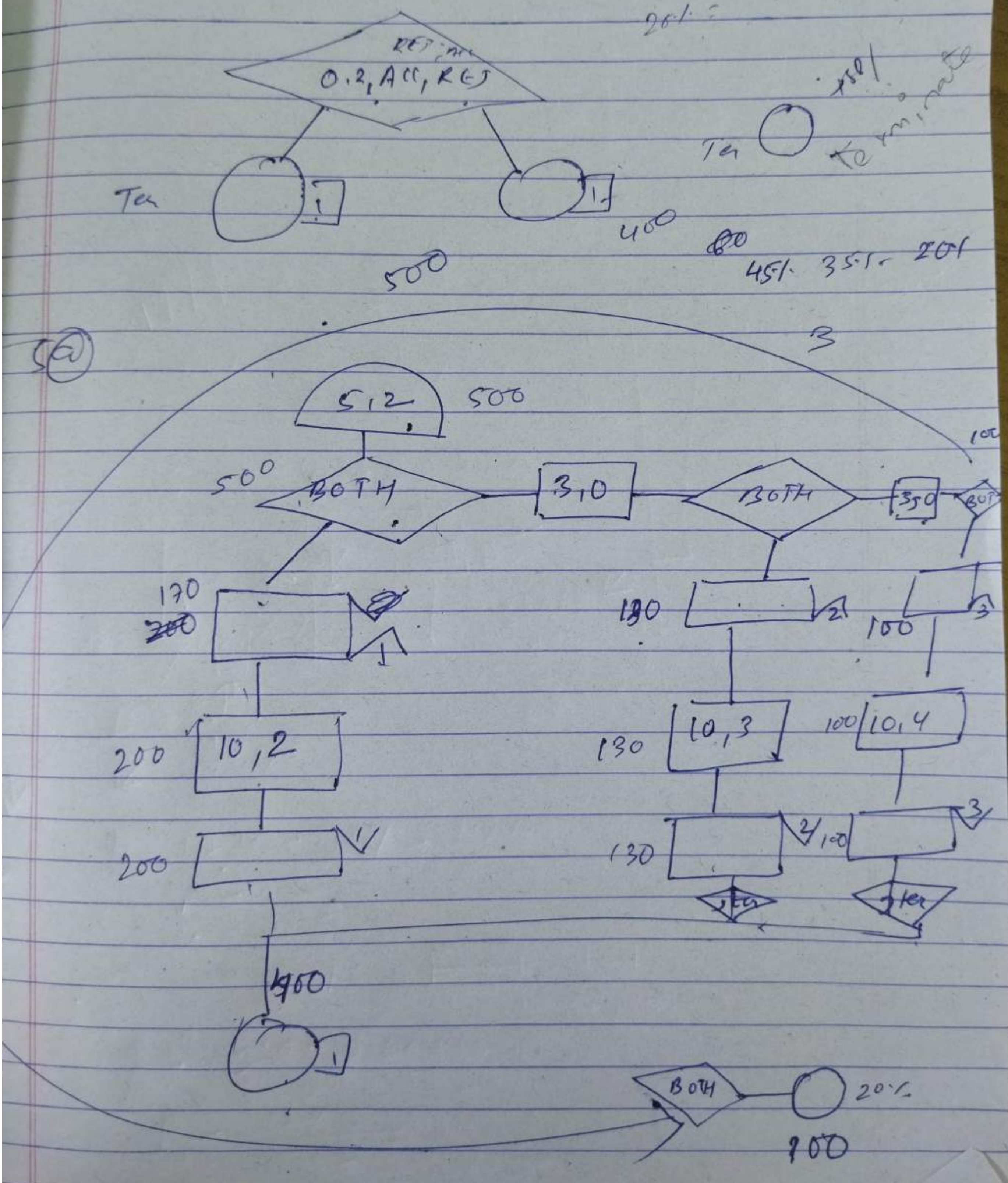
5(a)



0.2, ACC, REJ
Ten
1
400
500
20%
Ten
terminate
45% 35% 20%
3
5,12
500
500
BOTH
3,0
BOTH
350
170
~~200~~
1
180
2
100
3
10,2
200
10,3
130
100
10,4
200
1
130
2/100
3
Ter
Ter
400
1
BOTH
20%
900

PAGE