MEA (Means - End Analysis)

State Space Tree
- Tree which shows all possible states from current state.

Imp

MEA Algorithm:
1. Define initial state, goal state and calculate goal difference (Δ).
2. Choose action / procedure with least goal difference that will ultimately reach to goal.
3. When goal difference (Δ) = 0, goal is reached.

Production Rule System:

Q) Using  and  jugs find out 2l water.

4l 3l

Solution, [production-rule system (if-then)]

Possible actions:
1. Fill 4l Jug
2. Fill 3l Jug
3. Empty 4l Jug
4. Empty 3l Jug
5. Pour 4l to 3l → 3l full / → 3l not full
6. Pour 3l to 4l → 4l Jug full / → 4l not full.

let, x = Quantity of water in 4l Jug
y = " " " " 3l "

1> Fill 4l Jug (F4)
$x<4 \rightarrow (4,y)$

2> Fill 3l Jug (F3)
$y<3 \rightarrow (x,3)$

3> Empty 4l Jug (E4)
$x>0 \rightarrow (0,y)$

4.> Empty 3l Jug (E3)
$y>0 \rightarrow (x,0)$

5> Pour water from 4l to 3l to fill it. (P_{4-3} fill)
$x>0$ and $x+y \geq 3 \rightarrow (x-(3-y), 3)$

6.> Pour water from 4l to 3l to not fill (P_{4-3} not fill)
$x>0$ and $x+y<3 \rightarrow (0, (x+y))$

7> Pour 3l to 4l to fill it (P_{3-4} (fill))

$y>0$ and $x+y \geq 4 \rightarrow (4, y-(4-x))$

8> Pour water from 3l to 4l to not fill (P_{3-4} not fill)

$y>0$ and $x+y<4 \rightarrow (x+y, 0)$

Note: Path cost = number of steps.

goal: $\boxed{\begin{matrix} x=2 \\ y=n \end{matrix}}$ or $\boxed{\begin{matrix} x=n \\ y=2 \end{matrix}}$

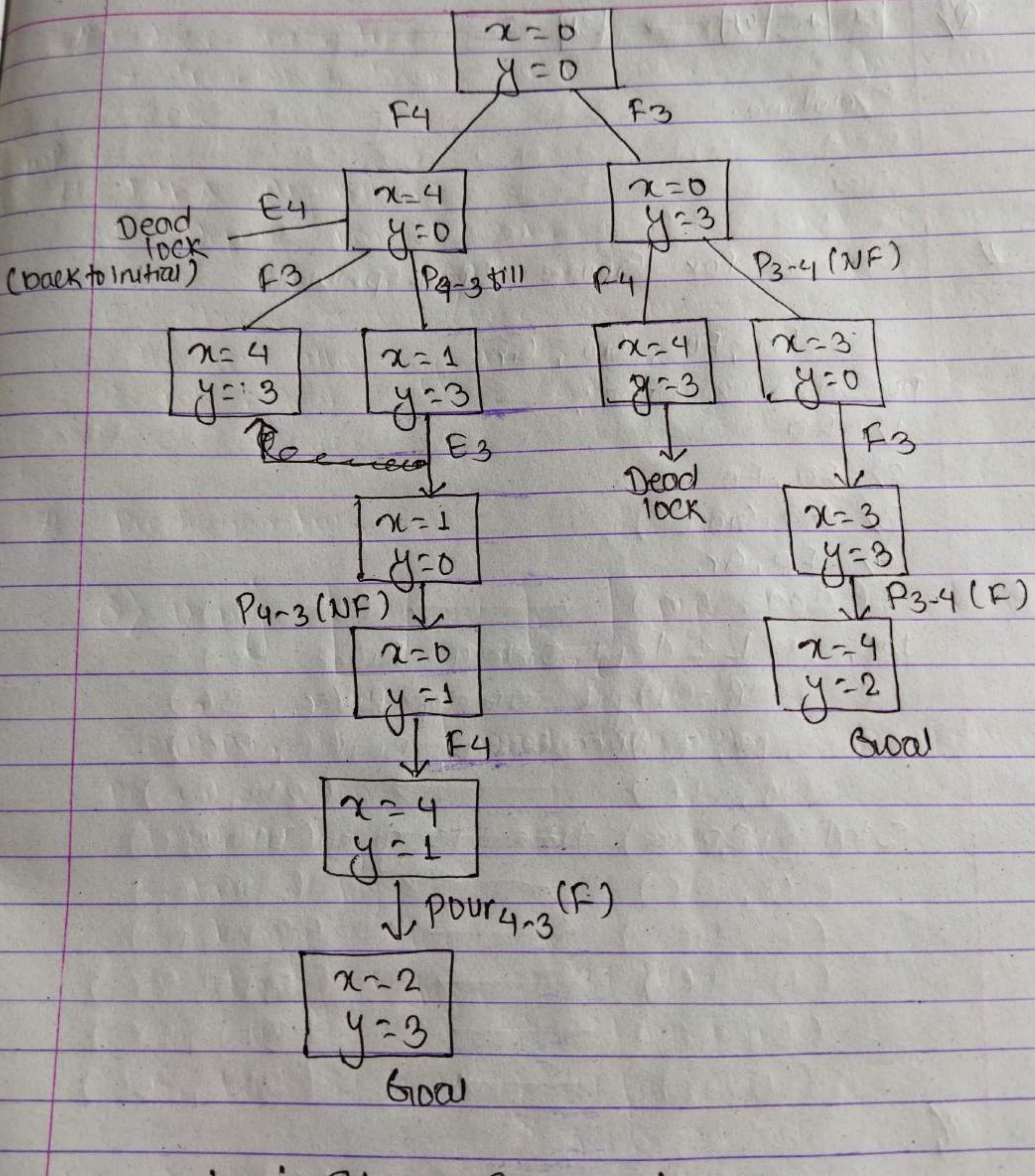


fig: State Space tree

Water-Jug problem

Q) $\boxed{5l}$ + $\boxed{4l}$ → $\boxed{3l}$

Solution.

Farmer Fox Goose Grain problem

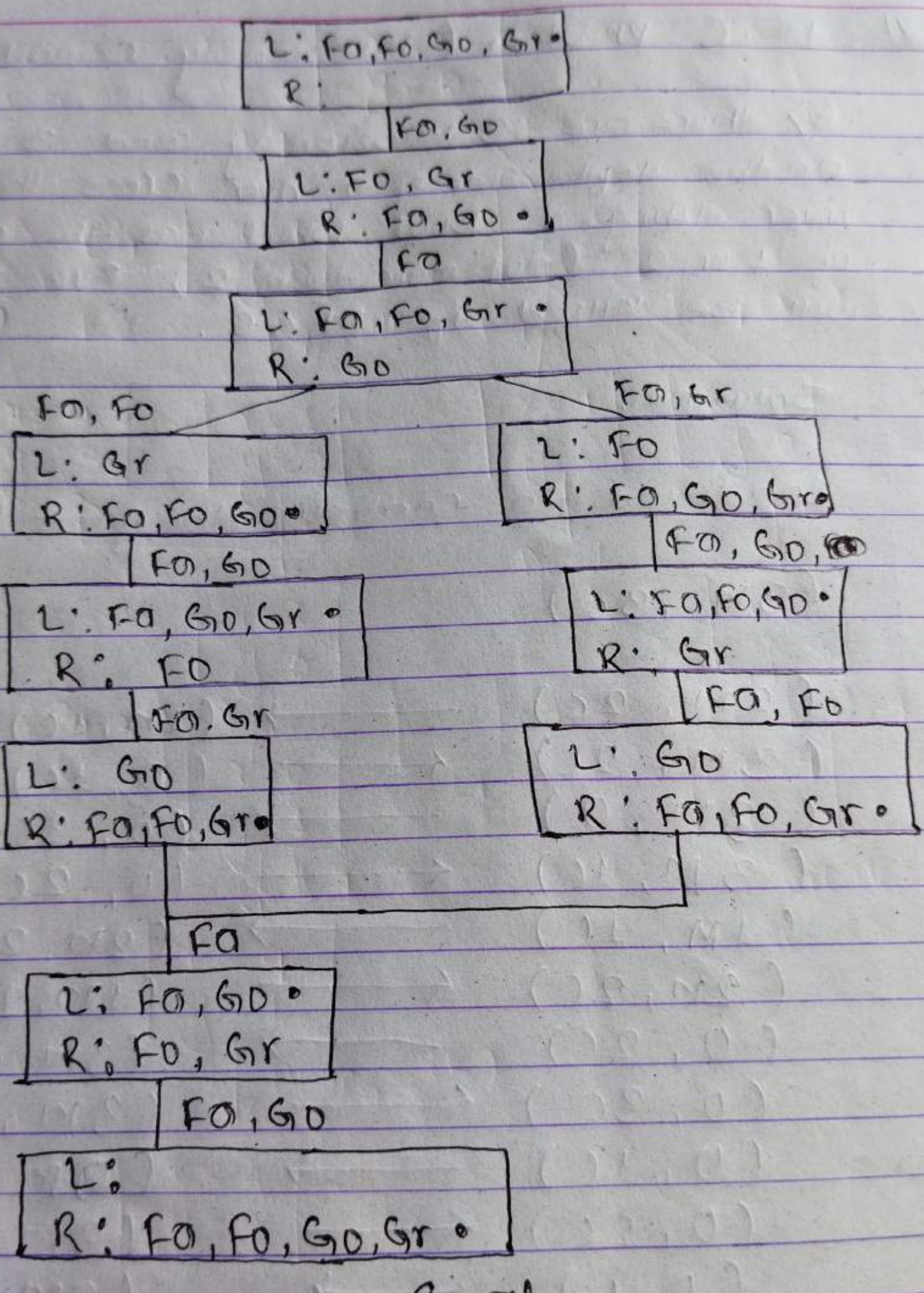
Fa, Fo, Go, Gr

Possible actions:

1) Fa
2) Fa, Fo
3) Fa, Go
4) Fa, Gr.

let, L = left side of River
R = Right side of River.
• = Location of Boat

State space tree is given below:



∴ Total number of States = 8
Total number of Steps = 7
Total ~~number~~ of pathcost = 7

M-C problem (Missionary cannibal problem)

Q) There are 3 Missionary and 3 cannibal on the left side of a river. cross them the river using a boat which has the capacity of 1 or 2 (not more than 2). But missionary will be killed if $M < C$.

Solution.

Condition to continue the game is $M \geq C$.

(3M, 3C)

(2M, 2C) ⟶ (1M, 1C)
(3M, 2C) ⟵ (0, 1C)
(3M, 0) ⟶ (0, 3C)
(3M, 1C) ⟵ (0, 2C)
(1M, 1C) ⟶ (2M, 2C)
(2M, 2C) ⟵ (1M, 1C)
(0, 2C) ⟶ (3M, 1C)
(0, 3C) ⟵ (3M, 0)
(0, 1C) ⟶ (3M, 2C)
(0, 2C) ⟵ (3M, 1C)
(0, 0) ⟶ (3M, 3C)

or,

(0, 1C) ⟶ (3M, 2C)
(1M, 1C) ⟵ (2M, 2C)

M-C problem (Missionary Cannibal problem)

Q) There are 3 missionary and 3 cannibal on the left side of a river. Cross them the river using a boat which has the capacity of 1 or 2 (not more than 2). But missionary will be killed if M < C.

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(2M, 2C) ⟶ (1M, 1C)
(3M, 2C) ⟵ (0, 1C)
(3M, 0) ⟶ (0, 3C)
(3M, 1C) ⟵ (0, 2C)
(1M, 1C) ⟶ (2M, 2C)
(2M, 2C) ⟵ (1M, 1C)
(0, 2C) ⟶ (3M, 1C)
(0, 3C) ⟵ (3M, 0)
(0, 1C) ⟶ (3M, 2C)
(0, 2C) ⟵ (3M, 1C)
(0, 0) ⟶ (3M, 3C)

or,

(0, 1C) ⟶ (3M, 2C)
(1M, 1C) ⟵ (2M, 2C)
(0, 0) ⟶ (3M, 3C)

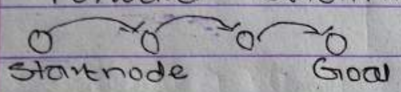
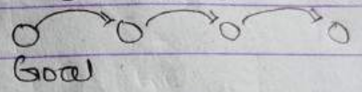
Imp

Learning by Analogy (Analogical reasoning / Learning)

- Comparison of problems and applying solution from known problem to the new problem.

Imp

Forward Chaining vs Backward Chaining

Forward Chaining	Backward Chaining
	
- Starts from initial state and aims for conclusion.	- Starts from goal and backward search to necessary conclusion.
- Data Driven inference technique.	- Goal driven inference Technique.
- Bottom-up approch.	- Top-Down approch.
- Breadth First Search.	- Depth First Search.
eg: Diagnosis Recommendation in Disease prediction bots.	eg: Navigation System in Driverless car.
- The number of final Solution is infinite (Multiple)	- The number of final Solution is limited.

Imp #

Mycin
- Expert system to identify bacterial infections and recommend antibiotics.
- 500 production rules.

eg: $\boxed{A \rightarrow B}$. if A, then B.
If Symptoms → Disease
If Disease → Antibiotics

(i) linear and nonlinear
(ii) Forward and backward
(iii) MEA

Chapt

Imp #
HI V

Hum
C

1) Operational Ability -

2) Speed -

3) accuracy -

4) Storage Cap

5) Dicison / Reason

6) Perception

Chapter-2

Intelligence

HI vs MI

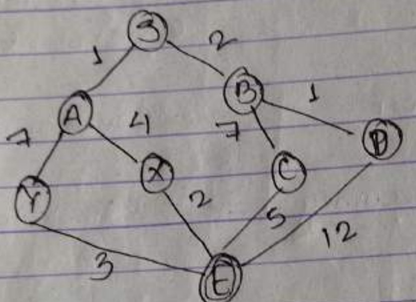
	Human Intelligence (Natural)	Machine Intelligence (Artifical)
1) Operational Ability	- Performance declines over time.	- Multitasking with consistent performance
2) Speed	- Relatively Slower	- Comparatively faster.
3) accuracy	- Inconsistent accuracy Comparatively.	- Consistent accuracy.
4) Storage capacity	- Unlimited	- limited.
5) Dicison making / Reasoning.	- Might be based on emotions.	- Unbiased and Rational.
6) Perception	- Perceives by pattern.	- Perceives in terms of Datasets.

~~Chapter~~ Chapter- 4
Inference and Reasoning.

Search Techniques:

1) Blind search or Uniformed Search — BFS, DFS
2) Heuristic Search or Informed search. — Greedy, A*

Q)



Values of h' (heuristic)

A	5
B	6
C	4
D	15
X	5
Y	8
E	0

- Greedy search:

$f(n) = h(n)$

where $f(n)$ = evaluation function.
$h(n)$ = heuristic value of node 'n'.

Start node S:

$S \to A$: $f(n) = h(n)$
$f(A) = h(A) = 5$
$S \to B$: $f(B) = h(B) = 6$

Expanding $S \to A$:
$S \to A \to X$: $f(X) = h(X) = 5$
$S \to A \to Y$: $f(Y) = h(Y) = 8$

Expanding $S \to A \to X$:
$S \to A \to X \to E$: $f(E) = h(E) = 0$

$\therefore$ $S \to A \to X \to E$ is the best path (path cost $= 7$)

- A* Search :

$f(n) = g(n) + h(n)$
where, $g(n)$ = path cost to reach node 'n'.

Start node S.
$S \to A$: $f(A) = g(A) + h(A)$
$= 1 + 5 = 6$
$S \to B$: $f(B) = g(B) + h(B)$
$= 2 + 6 = 8$.

Expanding $S \to A$
$S \to A \to X$: $f(X) = g(X) + h(X)$
$= (1+4) + 5 = 10$
$S \to A \to Y$: $f(Y) = g(Y) + h(Y)$
$= (1+7) + 8 = 16$

Expanding $S \to B$.
$S \to B \to C$: $f(C) = g(C) + h(C)$
$= (2+7) + 4 = 13$
$S \to B \to D$: $f(D) = g(D) + h(B)$
$= (2+1) + 15 = 18$

Expanding $S \to A \to X$:

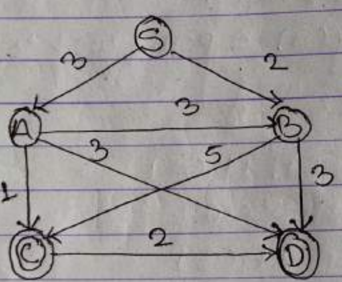
$S \to A \to X \to E$: $f(E) = g(E) + h(E)$

$= (1+4+2)+0$

$= 7.$

$\therefore$ $S \to A \to X \to E$ is the shortest path.

Q)



$h(S) = 1$
$h(A) = 3$
$h(B) = 3$
$h(C) = 0$
$h(D) = 0.$

Solution,

Using Greedy Search method:

$f(n) = h(n)$

Start node S:

$S \to A$: $f(A) = h(A) = 3$

$S \to B$: $f(B) = h(B) = 3$

Extending $S \to B$:

$S \to B \to C$: $f(C) = h(C) = 0$

$S \to B \to D$: $f(D) = h(D) = 0$

Extending

$S \to A$

$S \to A$

$S \to A$

Hence,

path for

Using A

Start n

$S \to$

S

Expand

$S \to$

S

$\therefore$

Extending $S \to A$:

$S \to A \to C$: $f(C) = h(C) = 0$

$S \to A \to D$: $f(D) = h(D) = 0$.

$S \to A \to B$: $f(B) = h(B) = 3$

Hence, $S \to A \to C$ is the shortest path for Goal C. ($\because$ path cost = 4.) and $S \to B \to D$ is the shortest path for Goal D (cost = 5).

Using A* Search method:

$$f(n) = h(n) + g(n)$$

Start node S:

$S \to A$: $f(A) = g(A) + h(A)$
$= 3 + 3 = 6$.

$S \to B$: $f(B) = g(B) + h(B)$
$= 2 + 3 = 5$.

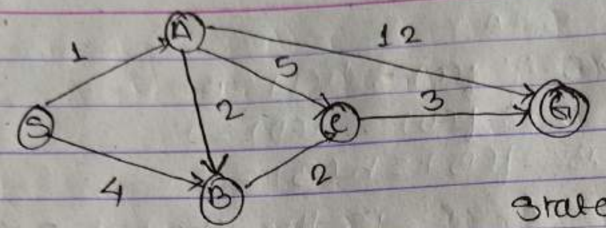
Expanding $S \to B$:

$S \to B \to C$: $f(C) = g(C) + h(C)$
$= (2+5) + 0 = 7$.

$S \to B \to D$: $f(D) = g(D) + h(D)$
$= (2+3) + 0 = 5$

$\therefore$ $S \to B \to D$ is the shortest path.

Q)



State	h
S	7
A	6
B	2
C	1
G	0.

Solution,

Using Greedy search method:

$$f(n) = h(n).$$

Here,

Start node S:

$S \to A$: $f(A) = h(A) = 6$

$S \to B$: $f(B) = h(B) = 2$.

Expanding $S \to B$:

$S \to B \to C$: $f(C) = h(C) = 1$.

Expanding $S \to B \to C$

$S \to B \to C \to G$: $f(G) = h(G) = 0$

$\therefore$ path cost $= 9$

Using A* Search method:

$$f(n) = g(n) + h(n).$$

Here,

Start node S:

$S \to A$: $f(A) = g(A) + h(A)$
$= 1 + 6 = 7$

$S \to B$: $f(B) = g(B) + h(B)$
$= 4 + 2 = 6.$

Expanding $S \to B$:

$S \to B \to C$: $f(C) = g(C) + h(C)$
$= (4+2) + 1 = 7.$

Here,

Expanding $S \to A$:

$S \to A \to B$: $f(B) = g(B) + h(B)$
$= (1+2) + 2 = 5.$

$S \to A \to C$: $f(C) = g(C) + h(C)$
$= (1+5) + 1 = 7.$

$S \to A \to G$: $f(G) = g(G) + h(G)$
$= (1+12) + 0 = 13.$

Expanding $S \to A \to B$:

$S \to A \to B \to C$: $f(C) = g(C) + h(C)$
$= (1+2+2) + 1 = 6$

Expanding $S \to A \to B \to C$

$S \to A \to B \to C \to G$: $f(G) = g(G) + h(G)$
$= (1+2+2+3) + 0 = 8.$

Expanding $S \to A \to C$:

$S \to A \to C \to G$: $f(G) = g(G) + h(G)$

$= (1+5+3)+0 = 9$

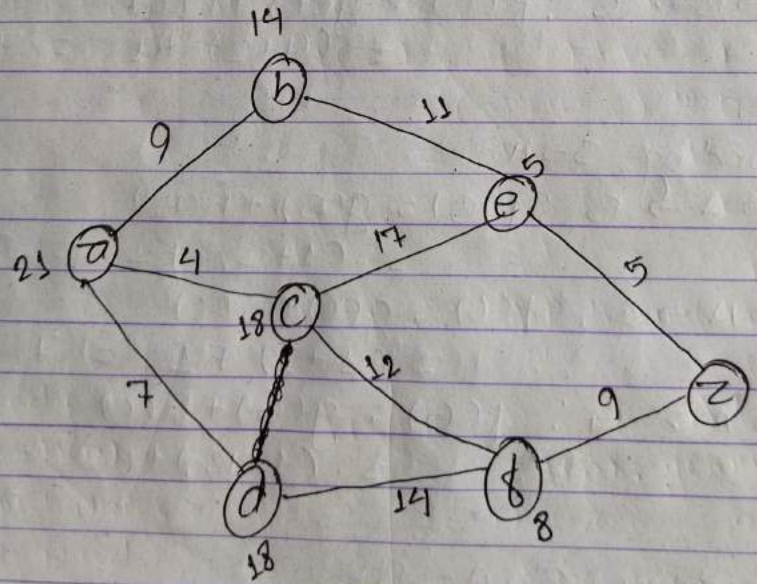
Expanding $S \to B \to C$

$S \to B \to C \to G$: $f(G) = g(G) + h(G)$

$= (4+2+3)+0 = 9$

Hence, $S \to A \to B \to C \to G$ is the shortest path.

1)



Solution,

- Using Greedy Search method:

$$f(n) = h(n).$$

Start node a:

a → b : $f(b) = h(b) = 14$

a → c : $f(c) = h(c) = 18$

a → d : $f(d) = h(d) = 18$

Extending a → b.

a → b → e : $f(e) = h(e) = 5$

Extending a → b → e:

a → b → e → z : $f(z) = h(z) = 0$.

∴ a → b → e → z is the shortest path. path cost = 25

- Using A* search method:

$$f(n) = g(n) + h(n)$$

Here,

Start node a:

a → b : $f(b) = g(b) + h(b)$
$= 9 + 14 = 23$

a → c : $f(c) = g(c) + h(c)$
$= 4 + 18 = 22$

a → d : $f(d) = g(d) + h(d)$
$= 7 + 18 = 25.$

Extending a → c:

a → c → e: $f(e) = g(e) + h(e)$
$= (4 + 17) + 5 = 26$

$a \to c \to f$: $f(f) = g(f) + h(f)$
$= (4+12) + 8 = 24$

Extending $a \to b$:
$a \to b \to e$: $f(e) = g(e) + h(e)$
$= (9+11) + 5 = 25$

Extending $a \to c \to f$:
$a \to c \to f \to z$: $f(z) = g(z) + h(z)$
$= (4+12+9) + 0$
$= 25.$
$a \to c \to f \to d$: $f(d) = 48$

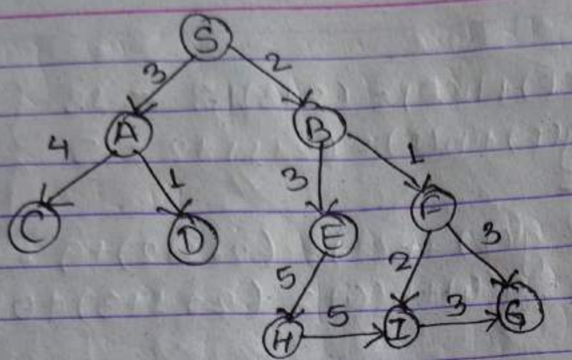
Extending $a \to b \to e$:
$a \to b \to e \to z$: $f(z) = g(z) + h(z)$
$= (9+11+5) + 0$
$= 25$

Hence, $a \to c \to f \to z$ and $a \to b \to e \to z$ are the shortest path ∴ Path cost = 25.

Greedy	A*
- $f(n) = h(n)$	- $f(n) = g(n) + h(n)$
- Heuristic value	- Also considers path cost.
- Not required	- Backtracing
- Simple and faster	- Comparatively slower
- Comparatively lower	- Time complexity and Space complexity higher.

Q)

Q)



node	h(n)
A	12
B	4
C	7
D	3
E	8
F	2
G	0
H	4
I	9
S	13

Solution,

- Using Greedy Search method:

$f(n) = h(n)$

Here, start node S:

$S \to A$: $f(A) = h(A) \Rightarrow 12$

$S \to B$: $f(B) = h(B) \Rightarrow 4$

Expanding $S \to B$:

$S \to B \to E$: $f(E) = h(E) \Rightarrow 8$

$S \to B \to F$: $f(F) = h(F) \Rightarrow 2$

Expanding $S \to B \to F$:

$S \to B \to F \to G$: $f(G) = h(G) \Rightarrow 0$

$\therefore S \to B \to F \to G$ is the shortest path. $\therefore$ path cost = 6.

- Using A* Search method:

$f(n) = g(n) + h(n)$

Here, start node S:

$S \to A$: $f(A) = g(A) + h(A) \Rightarrow 3 + 12 \Rightarrow 15$

$S \to B$: $f(B) = g(B) + h(B) \Rightarrow 2 + 4 \Rightarrow 6$

Expanding S→B:

S→B→E: $f(E) = g(E) + h(E) = (2+3)+8 = 13$

S→B→F: $f(F) = g(F) + h(F) = (2+1)+2 = 5$

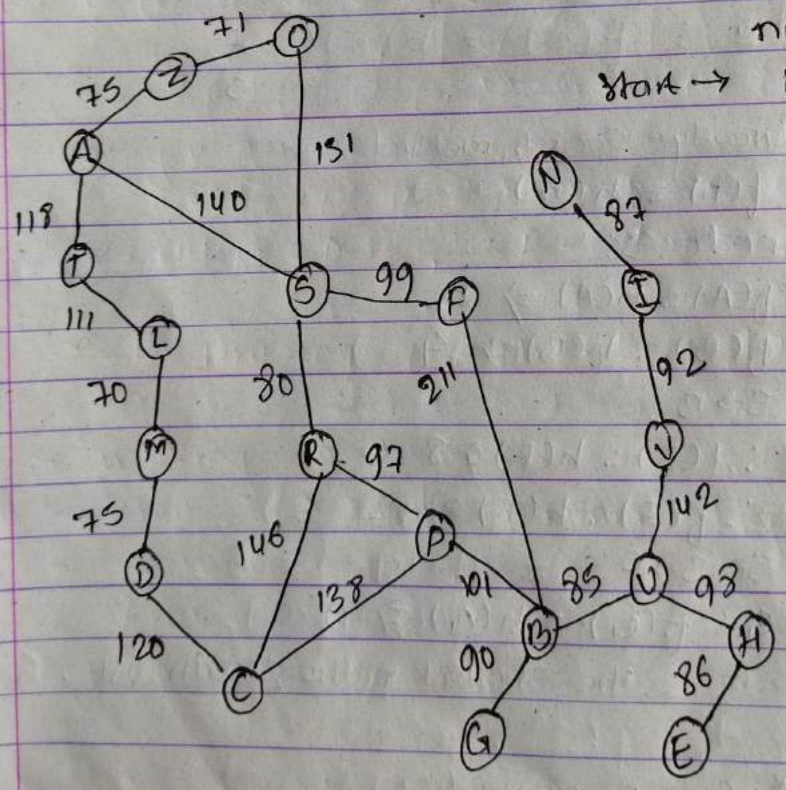
Expanding S→B→F

S→B→F→I: $f(I) = g(I) + h(I) = (2+1+2)+9 = 14$

S→B→F→G: $f(G) = g(G) + h(G) = (2+1+3)+0 = 6$

∴ S→B→F→G is the shortest path. ∴ ~~shortest path~~

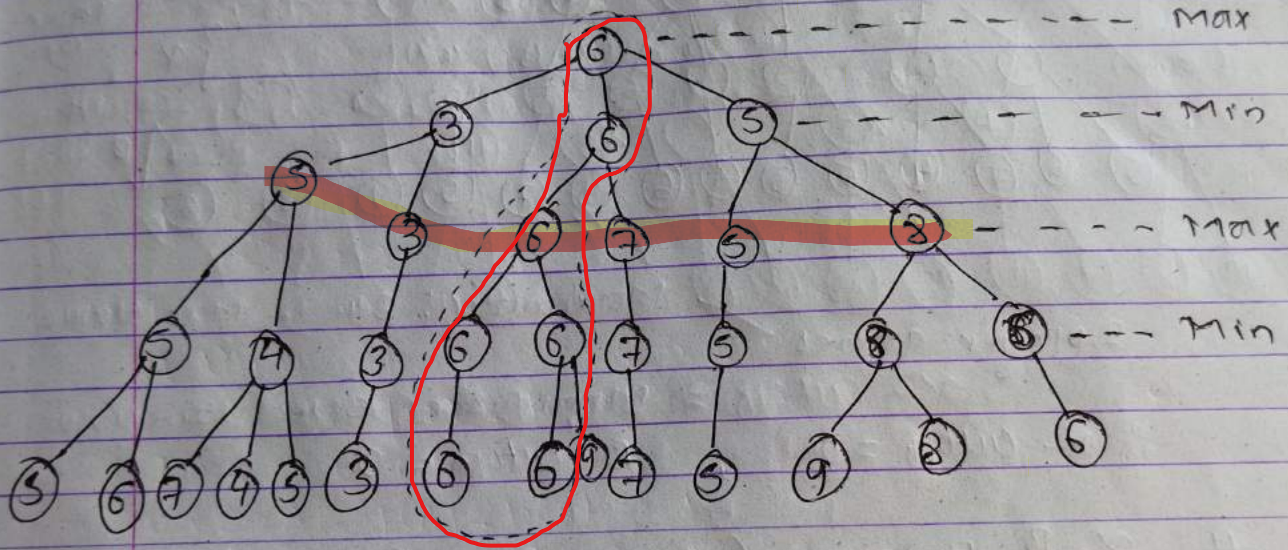
Q)



	node	h(n)
Start →	A	366
	B	0
	C	160
	D	242
	E	161
	F	178
	G	77
	H	151
	I	226
	L	244
	M	241
	N	234
	O	380
	P	98
	R	193
	S	253
	T	329
	U	80
	V	199
	Z	374

Minmax Algorithm

Maximizer : Selects max value
Minimizer : Selects Min value.



- Minmax algorithm is a game playing algorithm used in 2 player games. (Tic-Tac-Toe, Chess)
- Two players : Maximizer : Selects max value
  : Minimizer : Selects min value.

Limitations :

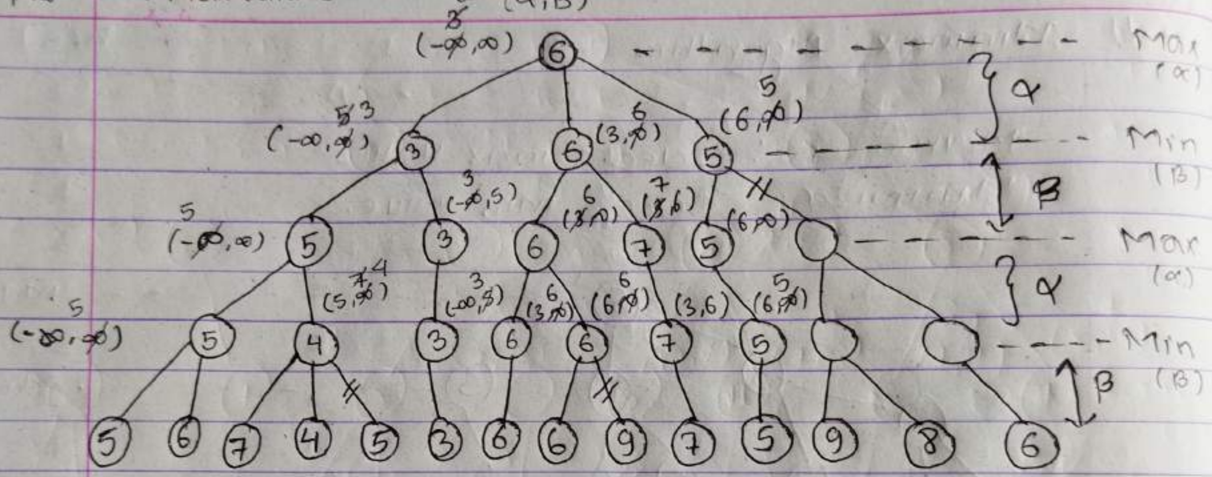
- Time Complexity and Space complexity higher.
- Alpha-beta pruning overcomes these limitations.

1) check min (B) or Max (α) level

2) update α-B values

3) push min Max values

let $(\alpha, B) = (-\infty, \infty)$



2 player games
- Maximizer : Selects Max Value
- Minimizer : Select Min value.

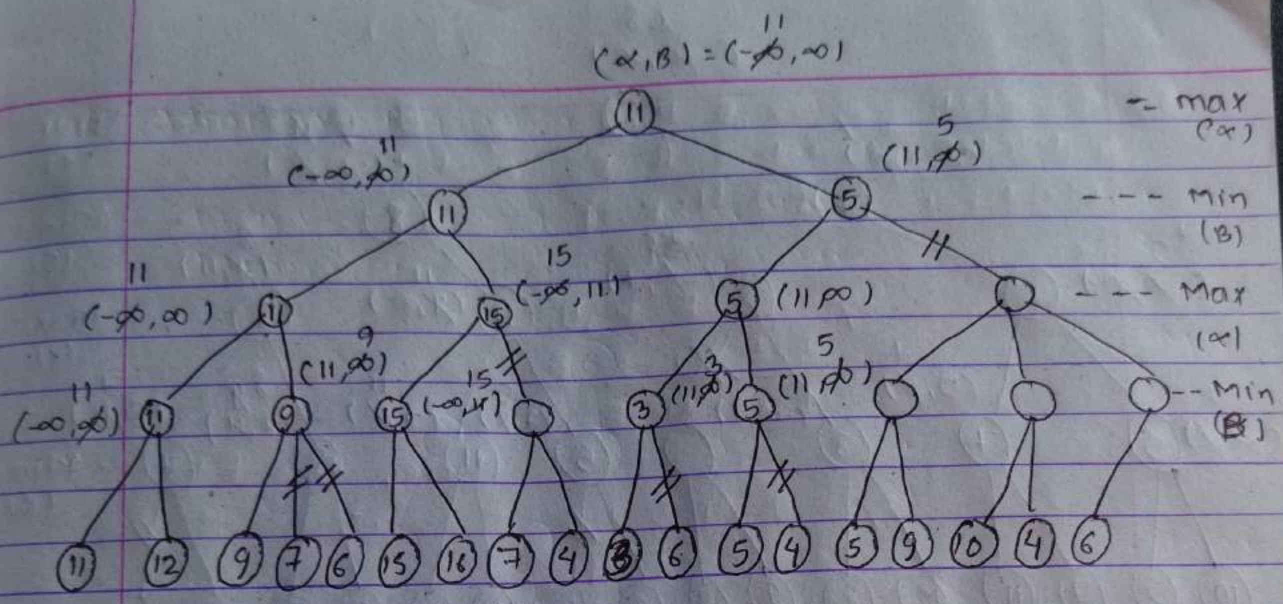
$\therefore$ α - cutoff = 0 , B - cutoff = 3

α-B Pruning (Alpha - Beta Pruning)

- working condition :

$\boxed{\alpha \geq B}$, ~~PU~~ prune the brach (Ignore the node).

$\alpha \geq \beta$ Max



$\therefore$ α - cutoff = 1

β - cutoff = 5

→ α-β pruning

- Extension of min-max

α = Maximizer ⇒ Selects Max value

β = Minimizer ⇒ Selects Min value

Initial values:

$\alpha = -\infty$

$\beta = \infty$

Working Condition: $\alpha \geq \beta$ ⇒ Prune the branch.

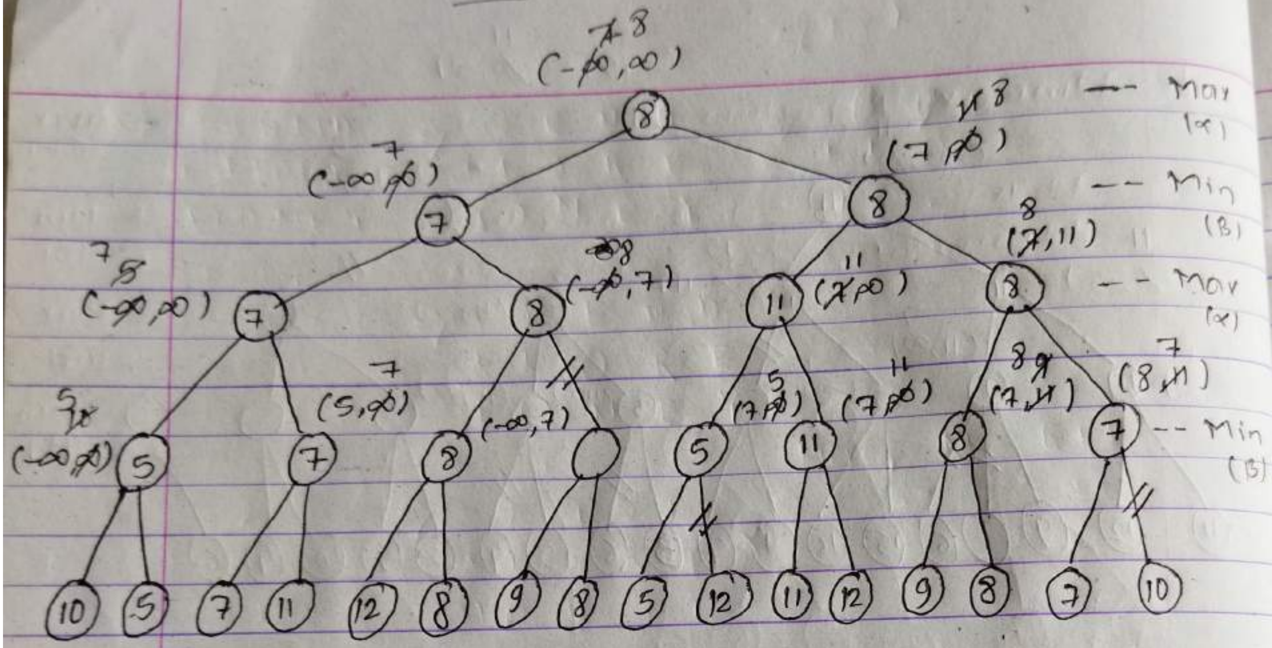
Algo:

1) Check min (β) or Max (α) level

2) Update α value if in α-level else update β level.

3) Push Max value if in α-level else push min value.

$\alpha \geq \beta$ let, $(\alpha, \beta) = (-\infty, \infty)$



α - cutoff = 1 , β - cutoff = 2

CSP (Constraint Satisfaction Problem)

-> Rules / condition to satisfy.
-> Solving problem by satisfying some conditions / rules.

e.g: Sudoku / Crypto arithmetic problem.

Constraints:

1) Each alphabet's value must be 0-9.
2) Each alphabet's value must be unique.
3) Starting alphabet can't be '0'.
4) The final solution should be arithmetically correct.

L O G I C
+ L O G I C
P R O L O G

and O = zero.

Here, P = 1 then, (L, L) = (5,5), (6,6), (7,7), (8,8) (9,9)

$\boxed{6}$ $\boxed{0}$ $\boxed{G}$ $\boxed{I}$ $\boxed{C}$ ×
+ $\boxed{6}$ $\boxed{0}$ $\boxed{G}$ $\boxed{I}$ $\boxed{C}$
1 2 0 6 0 G

7 0 G ×
+ 7 0 G
1 4 0

8 0 G I ×
+ 8 0 G I
1 6 0 8 0

9 0 G I C
+ 9 0 G I C
1 8 0 9 0 G

⇒ 9 0 4 I C
+ 9 0 4 I C
1 8 0 9 0 4

⇒ 9 0 4 5 2
+ 9 0 4 5 2
1 8 0 9 0 4

S(9,8)

```
  SEND      =>   S E N D       =>   8 9
+ MORE         + 1 O R E          + 1 0
 MONEY         1 O N E Y          1 0
```

(1,---7) ~~(5,5)~~ (6,6 (5,7) (5,8), (5,9) (6,6), (6,7) (6,8), (6,9)

```
=>   9 E N D
   + 1 O R E
   1 O N E Y
```

Here, N = E + 1
- 5 = 4 + 1
6 + 5+1
7 + 6

```
=>   9 4 5 D
   + 1 0 R 4     X
   1 0 5 4 Y
```

```
=>   9 5 6 D
   + 1 0 R 5
   1 0 6 5 Y
```

=> R = 8.

```
=>   9 5 6 7
   + 1 0 8 5
   1 0 6 5 2
```

Unc
- U

So
1>

2)
3)

->

#

Uncertainty With Baye's Rule:

Uncertainty
- unable to predicted outcome / conclusion.

Sources of Uncertainty
1> Incomplete data / Unknown data (insufficient) (Underfitting)
2> Vague data / Redundent data (over fitting) / (weak Implication)
3> Impercise language
   - Use of ambiguous words (hardly ever, rarely, often, sometimes)
4> Combining / Multiple views of experts.

→ Baye's rule / Bayesian Network / BBN (Bayes' Belief Network) is used to overcome uncertainty.

Baye's Rule
- handles uncertainty / uncertain data.
- deals with conditional probability, probability of an event 'A' given that 'B' has occured prior.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A/B) \cdot P(B) \qquad \Rightarrow P(B \cap A) = P(B/A) \cdot P(A)$$

$$\therefore P(A|B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$

Q) While watching a game of Champions league football in a cafe, you observe a person who is clearly supporting Man Utd. What is the prob. that they were actually born within 25 miles of Manchester. Assume that:

i) The prob. that is randomly selected person in that bar is born within 25 miles of Manchester is 1/20.

ii) The chance that a person born within 25 miles of manchester actually supports Man Utd is 7/10.

iii) The prob. that a person not born within 25 miles of machester supports man utd with prob. 1/10.

Solution,

let, S = Supports Man Utd.

B = Born within 25 miles of Manchester.

Given, $\bar{B}$ = not Born

$P(B) = 1/20$. $\quad P(\bar{B}) = 19/20$

$P(S/B) = 7/10$

$P(S/\bar{B}) = 1/10$

$P(B/S) = ?$

Here,

$$P(B/S) = \frac{P(S/B) \cdot P(B)}{P(S)} \quad \text{--- (1)}$$

$$P(S) = P(B) \cdot P(S/B) + P(\bar{B}) \cdot P(S/\bar{B})$$

$$= 1/20 \cdot 7/10 + 19/20 \cdot 1/10$$

$$= \frac{7}{200} + \frac{19}{200}$$

$$= \frac{26}{200} = \frac{13}{100}$$

$\therefore$ From eqn (1).

$$P(B/S) = \frac{P(S/B) \cdot P(B)}{P(S)}$$

$$= \frac{7/10 \cdot 1/20}{13/100}$$

$$= \frac{7}{26}$$

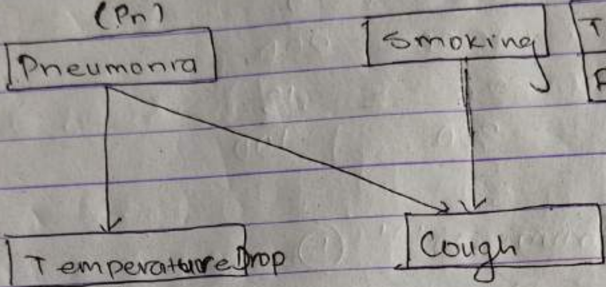
#

Bayesian Network

Pneumonia

T	0.1
F	0.9

(Pn) (S)



Smoking

T	0.2
F	0.8

Temperature

Pneumonia	Y	N
Y	0.9	0.1
N	0.2	0.8

Cough

Pneumonia	Smoking	T	F
T	Y	0.95	0.05
T	N	0.8	0.2
F	Y	0.6	0.4
F	N	0.05	0.95

Find:

1) $P(C / S \cap P_n)$
2) $P(C)$
3) $P(C/S)$

Solution,

i) $P(C / S \cap P_n) = 0.95$

ii) $P(C)$ = Prob. of Cough

$$P(C) = P(P_n).P(S)\,P(C / P_n \cap S) + P(P_n).P(\bar{S}).P(C/P_n \cap \bar{S}) + P(\bar{P}_n).P(S).P(C/\bar{P}_n \cap S) + P(\bar{P}_n).P(\bar{S}).P(C/\bar{P}_n \cap \bar{S})$$

$$= (0.1 \times 0.2 \times 0.95) + (0.1 \times 0.8 \times 0.8) + (0.9 \times 0.2 \times 0.6) + (0.9 \times 0.8 \times 0.05)$$

iii) $P(C/S) = P(P_n) \cdot P(S) \cdot P(C/P_n \cap S) +$
$P(\bar{P}_n) \cdot P(S) \cdot P(C/\bar{P}_n \cap S)$

$= (0.1 \times 0.2 \times 0.95) + (0.9 \times 0.2 \times 0.6)$

$=$

Chapter-3

Knowledge Representation. (KR)

- The branch of AI that deals with storing or representing knowledge / datasets in Database / Knowledge Base for inference or reasoning.

Imp
KR Schemes:
i) production Rule
ii) logic
iii) Semantic Network
iv) frames

* logic
a) Propositional logic
b) Predicate logic

a) Propositional logic
- Declarative sentence that can be either True or False

⇒ Imperative Sentence:
"Get out"
"Close the door"

⇒ Equations:
$2x+y=4$; depends on x and y.

=> Interogative / Question.
Can you close the door ?

=> Paradox.
"The second sentence is false. The first sentence is True".

Logical Operators:
1> Unary operator ($\neg$)
2> Binary operator ($\wedge, \vee, \rightarrow, \leftrightarrow$)

Truth table:

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	T	F	T	F
F	F	T	T	F	F	T	T

$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$	$P \leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Imp
Normal Forms:
a) CNF (Conjunctive Normal Form)
b) DNF (Disjunctive Normal Form)

e.g: Disjunction, conjunction

$(P \vee Q \vee R) \wedge (R \vee S) \wedge S$ → CNF

$(P \wedge Q) \vee (R \wedge S) \vee (P \wedge S)$ → DNF

CNF → Conjunction of Disjunction

DNF → Disjunction of conjunction.

Imp

CNF Conversion Steps:

- Step 1: Eliminate conditional and Biconditional / Double Implication (if present)

$P \rightarrow Q \equiv \neg P \vee Q$

P = "It is cold"

Q = "I will wear a sweater".

$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

$= (\neg P \vee Q) \wedge (\neg Q \vee P)$

- Step 2: De-Morgan's Law

$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

$\neg(P \wedge Q) = \neg P \vee \neg Q$

- Step 3: Apply Distributive law

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$
$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

eg: Convert : $P \to Q \to R$ into CNF.

Step 1: $(P \to Q) \to R$

$\equiv (\neg P \vee Q) \to R$

$\equiv \neg(\neg P \vee Q) \vee R$

Step 2: De-Morgan's Law

$\neg(\neg P \vee Q) \vee R$

$\equiv (P \wedge \neg Q) \vee R$

Step 3: Distributive law

$(P \wedge \neg Q) \vee R$

$\equiv (P \vee R) \wedge (\neg Q \vee R)$

Required CNF

P	Q	R	$(P \to Q)$	$(P \to Q) \to R$	$\neg Q$	$P \vee R$	$\neg Q \vee R$	$(P \vee R) \wedge (\neg Q \vee R)$
1	1	1	1	1	0	1	1	1
1	1	0	1	0	0	1	0	0
1	0	1	0	1	~~0~~1	1	1	1
1	0	0	0	1	1	1	1	1
0	1	1	1	1	0	1	1	1
0	1	0	1	0	0	0	0	0
0	0	1	1	1	1	1	1	1
0	0	0	1	0	1	0	1	0

proved

Proof by Resolution / Resolution Refutation.

⇒ If it is hot (P), then it is humid (Q). If it is humid (Q), then it will rain (R). It is hot. prove that it will rain (R).

Steps:

1) Convert all statement into CNF.
2) Negate the desired conclusion.
3) Apply resolution rule.

$$\begin{array}{l} P \vee Q \\ \underline{\neg Q \vee R} \\ \therefore P \vee R \end{array}$$

When we derive contradiction, our assumption was false and given statement is proved.

Solution,

Given,

P = "It is hot".

Q = "It is humid".

R = "It will rain".

Given;

(i) $P \rightarrow Q$

(ii) $Q \rightarrow R$

(iii) P

Prove that: R

Step 1 : 1) $P \rightarrow Q$

$\equiv \neg P \vee Q$

2) $Q \rightarrow R$

$\equiv \neg Q \vee R$

3) P

4) $\neg R$ (assumption).

Steps	Formula	Derivation.
1	$\neg P \vee Q$	Given
2	$\neg Q \vee R$	Given
3.	P	Given
4.	$\neg R$	Given.
5)	$\neg P \vee R$	Using resolution in (i) & (ii)
6)	R	Using resolution in (iii) & (v)

We reach to a contradiction hence "it will rain".

Q) Either Heather attended the meeting or Heather was not invited. If the boss wanted Heather at the meeting, then she was invited. Heather didn't attend the meeting. If the boss didn't want Heather there and the boss didn't invite her there, then she is going to be fired.
prove: Heather is going to be fired.

Solution, let,

P = Heather attended the meeting
Q = Heather was invited
R = Boss wanted Heather at meeting.
S = Heather is fired.

Here,

(i) $(P \vee \neg Q)$
(ii) $R \rightarrow Q$
(iii) $\neg P$
(iv) $(\neg R \wedge \neg Q) \rightarrow S$
(v) $\neg S$ (assumption)

To prove: S.

Steps: $(\neg R \wedge \neg Q) \rightarrow S$
$\equiv \neg(\neg R \wedge \neg Q) \vee S$ (∵
$\equiv (R \vee Q) \vee S$ (∵ De-Morgan's law)

$R \rightarrow Q \equiv \neg R \vee Q$

Steps	Formula	Derivations.
1.	$P \vee \neg Q$	Given
2.	$R \vee Q$	Given
3.	$\neg P$	Given
4.	$(R \vee Q) \vee S$	Given
5.	$\neg S$	Given
6	$P \vee \neg R$	Using resolution in 1 & 2.
7.	$\neg R$	Using resolution in 3 & 6
8.	$\neg Q$	Using resolution in 1 & 3
9.	S	Using resolution in 4, 7 & 8

<u>Proved</u>