

MATH 220 Practice Midterm — September, 2024, Duration: 50 minutes*This test has **5 questions** on **8 pages**, for a total of 50 points.*

First Name:	Last Name:
Student Number:	Section:
Signature:	

Question:	1	2	3	4	5
Points:					
Total:	/50				

10 Marks

1. Negate each of the following and prove or disprove the original statement:

(a) For all $x \in \mathbb{R}$, there exists $q \in \mathbb{Q}$ such that for all $r < q \in \mathbb{Q}$, $r + q < x$.

- (b) For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that for all $z \in \mathbb{R}$, if $xy < z$ then $x < 0$ or $x^2 + y^2 < z$

10 Marks

2. Let $p \in \mathbb{N}$ and assume $p > 1$. Prove that if there exists $x \in \mathbb{Z}$ with $x \not\equiv 0 \pmod{p}$ such that for all $y \in \mathbb{Z}$, $xy \not\equiv 1 \pmod{p}$, then p is not prime.

10 Marks

3. Let $a, b \in \mathbb{Z}$ where $a, b \neq 0$ and $S = \{ax + by \mid x, y \in \mathbb{Z}, ax + by > 0\}$. You may not use Bézout's lemma for this section.

(a) Prove that S is non-empty.

- (b) Prove that the minimal element $d = as + bt \in S$ for some $s, t \in \mathbb{Z}$ divides both a and b . (Hint: Euclidean division of a by d and b by d)

(c) Prove that if $c \mid a$ and $c \mid b$, then $c \leq d$.

10 Marks

4. Let $A = \{a_1, a_2, \dots, a_n\} \subseteq \mathbb{Q}$ and $S = \{c_1a_1 + c_2a_2 + \dots + c_na_n \mid c_i \in \mathbb{Z}\}$. Prove by induction that if A has $n \in \mathbb{N}$ elements, then there exists $q \in \mathbb{Q}$ such that $S = \{cq \mid c \in \mathbb{Z}\}$. (Hint: Show that $S \subseteq \{cq \mid c \in \mathbb{Z}\}$ and $S \supseteq \{cq \mid c \in \mathbb{Z}\}$ and find q in your inductive step)

10 Marks

5. Prove or disprove that the sequence $(x_n)_{n \in \mathbb{N}} = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}$ converges. (Hint: Consider an inequality between this sequence and $\frac{1}{\sqrt{3n+1}}$)