Hints to homework 4:

1. You know from Bézout's lemma that there exists $x, y \in \mathbb{Z}$ such that $ax + by = 1$, so you can fix these variables. Try playing around with expressions by fixing x and y.

3. Usually, multiply things out and simplifying expressions like $|\frac{2x^2}{2x^2+1} - 2|$ will make things less confusing.

4. You should think about what special properties sine have. It would be really nice if I can just somehow "delete" sine, or just not have to deal with it at all.

5. Since x_n converges to 0, you can fix any arbitrary $\varepsilon_N > 0$ and you will get that for all $n > N$, $|x_n - 0| \leq \varepsilon_N$. Notice $\{x_1, x_2, x_3, \ldots, x_N\}$ is a finite set.

6. What special properties do $(-1)^n$ have? What happens when n is very big?

(2) is pretty straight forward, just be careful.