Hints to homework 5:

1. One common trick here is to add something of the form $x - x$ (or if you prefer, $-x + x$) into the equation and that can help simplify things.

2. Your base case is $n = 3$, so you want to show that $a_3 = 3^3 - 2^3$. In your inductive step, you will need to assume that the result holds for $n = k - 1, k$. In other for both of these assumptions to work, you will need to do one more base case, namely $n = 3, 4$. [1]

3. Be careful with your inductive step. You want to show that $f_{k+2}f_k - {f_{k+1}}^2 = (-1)^{k+1}$. Also try expanding out some of the terms based on the sequence you are given. It is a good trick for induction involving sequences in general.

4. If you can do (1), you can do this question. It involves a similar trick.

5. For your inductive step, you want to show that $u_{k+1} = \cos((k+1)x)$ and you start with $u_{k+1} = 2u_1u_k - u_{k-1}$ since that is what u_{k+1} is defined as based on your sequence.

6. Try a bunch of examples. Pretty easy.

In general, just be careful what your inductive step is starting off with, and what you want to end up showing. Be very careful not to accidentally assume the conclusion. Sometimes working backwards can also be harder than working forwards for induction proofs.

[1] Your inductive step assumes that I can go from $k - 1$ to k and it will still work, so you need to show from the base case that to my next base case, and it will still work