MATH 220 Practice Midterm 1 — Duration: 50 minutes

This test has ***5 questions*** *on* ***8 pages****, for a total of 50 points.*

Disclaimer: This test is definitely harder than actual 220 midterm exams. Treat it more like extra homework and take time to think through the problems, the duration is technically 2.5 hours but you will likely not be able to finish most of the test so give yourself more time if needed, your performance in this practice test is not a good indicator of success nor failure in the course.

First Name:	Last Name:
Student Number:	Section:
Signature:	

Question:	1	2	3	4	5
Points:					
Total:					/50

10 Marks

1. For each of the following statements, write down its negation and prove or disprove the statement.

   (a) For all $x \in \mathbb{R}$, there exists $q \in \mathbb{Q}$ such that for all $r < q \in \mathbb{Q}$, $r + q < x$.

(b) For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that for all $z \in \mathbb{R}$, if $xy < z$ then $x < 0$ or $x^2 + y^2 < z$

$\boxed{\text{10 Marks}}$ 2. Let $p \in \mathbb{N}$ and assume $p > 1$. Prove that if there exists $x \in \mathbb{Z}$ with $x \not\equiv 0 \mod p$ such that for all $y \in \mathbb{Z}$, $xy \not\equiv 1 \mod p$, then p is not prime.

10 Marks 3. Let $a, b \in \mathbb{Z}$ where $a, b \neq 0$ and $S = \{ax + by | x, y \in \mathbb{Z}, ax + by > 0\}$. You may not use Bézout's lemma for this section.

(a) Prove that S is non-empty.

(b) Prove that the minimal element $d = as + bt \in S$ for some $s, t \in \mathbb{Z}$ divides both a and b. (Hint: Euclidean division of a by d and b by d)

(c) Prove that if $c \mid a$ and $c \mid b$, then $c \leq d$.

10 Marks

4. Let p be a prime. Prove by induction that for all $n \in \mathbb{Z}$, $p \mid n^p - n$. (Hint: You will need to split into different cases for induction, and use the binomial theorem)

$\boxed{\text{10 Marks}}$ 5. Prove or disprove that the sequence $(x_n)_{n\in\mathbb{N}} = \frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{2\cdot 4\cdot 6\cdot\ldots(2n)}$ converges. (Hint: First prove an inequality between this sequence and $\frac{1}{\sqrt{3n+1}}$ holds for all $n \in \mathbb{N}$)