

Unit 5: Magnetism and Superconductivity

Syllabus

Magnetism

- Origin of magnetism
- Classification of magnetism on the basis of permeability (qualitative)
- Applications of magnetic devices: transformer cores, magnetic storage, magneto-optical recording

Superconductivity

- Introduction to superconductivity; Properties of superconductors: zero electrical resistance, critical magnetic field, persistent current, Meissner effect
- Type I and Type II superconductors
- Low and high temperature superconductors (introduction and qualitative)
- AC/DC Josephson effect; SQUID: basic construction and principle of working; Applications of SQUID
- Applications of superconductors

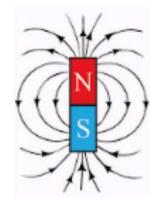
Introduction

- Magnetic materials play a prominent role in modern technology. They are widely used in industry, electronics and storage technologies.
- Magnetic materials can broadly be classified into soft and hard materials. Soft magnetic materials are easily magnetized and demagnetized and. They are used in AC applications. Hard magnetic materials retain magnetism on a permanent basis.
- Further, depending on their response to the external magnetic field, magnetic materials can be broadly classified into three groups: diamagnetic, paramagnetic and ferromagnetic materials.

5.1.1 Magnetism: basic terms and definitions

1. Magnetic dipole

- Magnetic dipole is generally a tiny magnet of microscopic to subatomic dimensions generated due to current loop.
- Electrons circulating around atomic nuclei, electrons spinning around their axes, and rotating positively charged atomic nuclei all are magnetic dipoles.
- Magnetic dipole is referred to smallest magnetic unit having two poles (north and south) that produces magnetic field and are inseparable from each other.
- A single monopole has never been observed.



2. Magnetic dipole moment

- The strength of a magnetic dipole is represented by magnetic dipole moment. It is a measure of a dipole's ability to align itself according to direction of external magnetic field.
- The magnetic moment can be considered to be a vector quantity with direction perpendicular to the current loop in the right-hand-rule direction.
- Magnetic dipole moment is defined as the maximum amount of torque caused by magnetic force on a dipole that arises per unit value of surrounding magnetic field in vacuum. This torque is given by

$$\mu = iA$$

i (current) *A* = area

$$\tau = \mu \times B$$

3. Magnetic field strength (**H**)

- The strength (or intensity) of magnetic field at any point in magnetic field is force experienced by a unit north pole placed at that point. It is denoted by **H**.
- The unit of **H** is ampere-turns per meter (A/m) in SI system.

4. Magnetization (**M**)

- Magnetization (or intensity of magnetization) is the measure of magnetism of magnetic materials. It is defined as magnetic moment per unit volume and denoted by **M**.
- The unit of magnetization in SI system is amperes per meter (A/m).
- As magnetization is induced by magnetic field, **M** is proportional to **H**.
Thus, $M \propto H$ or $M = \chi H$
Where, χ is called as magnetic susceptibility.

5. Magnetic Susceptibility (χ)

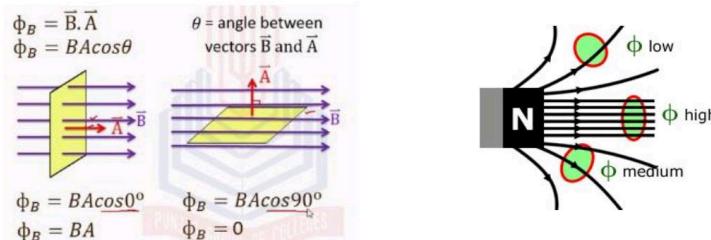
- The magnetic susceptibility of a material is a measure of the ease with which the material can be magnetized. It is defined as magnetization (**M**) produced in the material per unit applied magnetic field (**H**).
- Hence, $\chi = M/H$
- Materials having high susceptibility are easily magnetized.

6. Magnetic flux (ϕ)

- Magnetic flux is a measurement of the total magnetic field which passes through a given area. The lines of induction are collectively called as flux.
- The SI unit of magnetic flux is the Weber (Wb).

7. Magnetic Induction or magnetic flux density (**B**)

- A magnetic field is schematically represented by lines of magnetic induction or magnetic flux density).
- The magnetic flux per unit area is defined as magnetic flux density.
- Magnetic induction is the number of lines of force through a unit area of cross section perpendicularly.
- Thus, $B = \phi/A$.
- The SI unit of **B** is Weber per square meter (Wb/m^2) and CGS unit is Tesla (T)
- 1 Gauss = 10^{-4} Tesla



8. Relation between **B** and **H**

When a material is kept in a magnetic field, two types of induction arise – one due to magnetizing field **H** and other due to magnetization **M** of the material itself. The magnetic induction **B** produced inside the material is given by

$$B = \mu_0(H + M)$$

Where, μ_0 is known as permeability of the free space. It is equal to $4\pi \times 10^{-7}$ henry per meter (H/m)

$$B = \mu_0 \left(1 + \frac{M}{H}\right) H$$

Or

$$B = \mu_0(1 + \chi)H$$

Or

$$B = \mu H$$

Where μ is called as absolute permeability of the medium.

In free space, $M=0$, and $B = \mu_0 H$

9. Absolute permeability (μ)

- When a magnetic material is placed in a magnetic field, the magnetic field lines are redistributed and tend to pass through the material. The absolute permeability of the material is a measure of the degree of which the field lines penetrate (or permeate) the material.
- It is defined as the ratio of magnetic induction \mathbf{B} in the medium to the magnetizing field \mathbf{H} .
- Thus, $\mu = \frac{B}{H}$. The unit of absolute permeability is henry per meter (H/m).

10. Relative Permeability (μ_r)

- The relative permeability of a material is defined as the ratio of absolute permeability of the material to the permeability of free space.
- Thus, $\mu_r = \frac{\mu}{\mu_0}$.
- μ_r is only a number and has no units.
- Its value for air or vacuum is one.

11. Relation between μ_r and χ

$$\begin{aligned} \text{The magnetic induction} & B = \mu_0(1 + \chi)H \\ \text{Or} & B = \mu H \\ \text{Thus,} & \mu = \mu_0(1 + \chi) \\ \text{As} & \mu_r = \frac{\mu}{\mu_0} \quad \mu_r = (1 + \chi) \end{aligned}$$

12. Bohr Magneton (μ_B)

- Bohr magneton is the elementary electron magnetic moment and no electron can have a magnetic moment below it.
- It is the natural unit for the measurement of atomic magnetic moments.
- It has value

$$\mu_B = \frac{e\hbar}{4\pi m} = 9.28 \times 10^{-24} A.m^2$$

Example: A magnetic material has a magnetization of 2300 A/m and produces a flux density of 0.00314 Wb/m². Calculate magnetizing force and relative permeability of the material.

Solution: $B = \mu_0(H + M)$

$$\text{Thus, Magnetizing force, } H = \frac{B}{\mu_0} - M = \frac{0.00314}{12.57 \times 10^{-7}} - 2300 = 198 \text{ A/m}$$

$$\text{Relative permeability } \mu_r = \frac{\mu}{\mu_0} = \frac{1}{\mu_0} \times \mu = \frac{1}{\mu_0} \times \frac{B}{H} = \frac{0.00314}{12.57 \times 10^{-7} \times 198} = 12.56$$

Example: Diamagnetic Al_2O_3 is subjected to an external magnetic field of 10^5 A/m. Evaluate magnetization and magnetic flux density. [Susceptibility of Al_2O_3 is -5×10^{-5}].

Solution: Magnetization $M = \chi H = 5 \times 10^{-5} \times 10^5 = 5 \text{ A/m}$

$$\text{Magnetic flux density } B = \mu_0(H + M)$$

$$B = 12.57 \times 10^{-7}(10^5 - 5) = 0.126 \text{ wb/m}^2$$

Example: Find the relative permeability of the ferromagnetic material if a magnetic field of strength 220 A/m produces magnetization of 3300 A/m in it.

Solution: $\mu_r = (1 + \chi) = \left(1 + \frac{M}{H}\right) = \left(1 + \frac{3300}{220}\right) = 16$

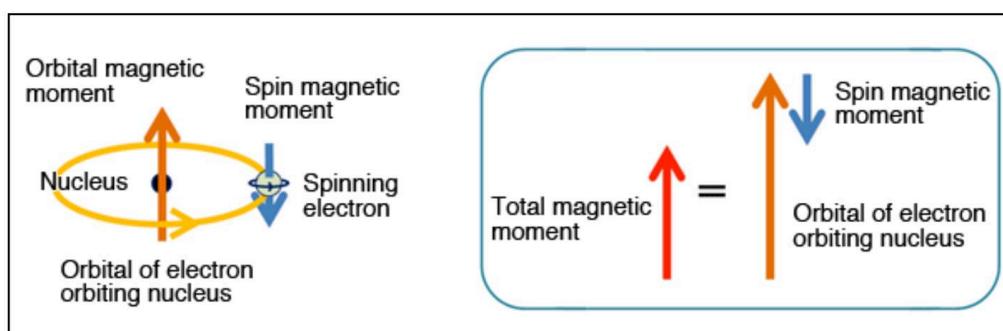
5.1.2 Origin of magnetization

- Magnetic properties of solids arise due to electrons undergoing different motions in atom. These motions give rise to magnetic dipole moments.
- In general, the magnetic dipole moment of the atom arises because of three sources:

(a) Orbital motion of electrons

- Inside an atom, electrons move around a nucleus in specific orbits. Each electron orbit is equivalent to a tiny current loop. It behaves as an elementary magnet having a magnetic dipole moment. The sum of orbital magnetic moments of individual electrons generates the total orbital magnetic moment of an atom.
- It can be shown that elementary electron magnetic moment is given by Bohr Magneton

$$\mu_B = \frac{e\hbar}{4\pi m_e} = 9.28 \times 10^{-24} A.m^2$$



(b) The electron spin

Each electron is spinning about itself and this gives rise to a magnetic dipole moment. It can be shown that spin magnetic moment is given by

$$\mu_s = g \frac{e}{2m} S = \gamma S$$

Where

$\gamma = g \frac{e}{2m}$ is called gyromagnetic ratio (g-factor) [For electron $g=-2.0023$]

$S = m_s \hbar$ is angular momentum ($m_s = \text{spin quantum number} = \pm \frac{1}{2}$)

(c) The nuclear spin

- The nucleus spins around itself and it also contributes to magnetic moment of atoms due to magnetic field produced by protons.
- It can be shown that elementary nuclear magnetic moment is given by

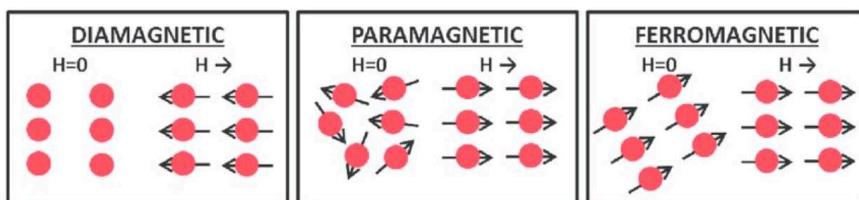
$$\mu_N = \frac{e\hbar}{4\pi m_p} = 5.05 \times 10^{-27} A.m^2$$

- The magnetic moment of the nucleus is about 1/1837 of the magnetic moment of the electron. Therefore, in studying magnetic properties of solids, the magnetic moment due to nuclear spin is neglected.

Magnetization of atom and materials

- For a solid, the resultant magnetic moment of an atom is sum of the orbital and spin magnetic moments of its electrons.
- The major contribution to atomic magnetic moment comes from the spin of unpaired valence electrons. Number of such magnetic moments aligns in different directions to and it results into a net non-zero magnetic moment.

- (iii) When material is placed in a magnetic field, the atomic dipoles respond to the external magnetic field.
- (iv) In **diamagnetic materials**, atomic moments are weakly aligned along opposite direction of external magnetic field.
- (v) In **paramagnetic materials**, atomic moments are weakly aligned along same direction of external magnetic field.
- (vi) In **ferromagnetic materials**, atomic moments are strongly aligned along same direction of external magnetic field.



Example: Calculate the values of Bohr magneton and nuclear magneton. Hence find their ratio.

Solution:

$$\text{Bohr magneton: } \mu_B = \frac{eh}{4\pi m_e} = \frac{1.6 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}} = 9.28 \times 10^{-24} \text{ A.m}^2$$

$$\text{Nuclear magneton: } \mu_B = \frac{eh}{4\pi m_p} = \frac{1.6 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 1.673 \times 10^{-27}} = 5.05 \times 10^{-27} \text{ A.m}^2$$

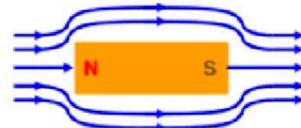
$$\text{Thus, } \frac{\mu_B}{\mu_N} = \frac{9.28 \times 10^{-24}}{5.05 \times 10^{-27}} = 1837.62$$

5.1.3 Classification of magnetism on the basis of permeability

In accordance with the value of relative permeability the materials are classified as:

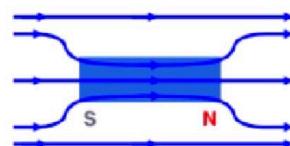
1. Diamagnetic Materials:

- Diamagnetic materials are substances which when placed in an external magnetic field develop a weak magnetism in opposite direction of the external magnetic field.
- The atoms have very small magnetic moment
- They slightly repel the magnetic lines of forces.
- The examples are bismuth, silver, copper and hydrogen.
- The relative permeability of these materials is slightly less than one i.e. $\mu_r < 1$
- For example the relative permeability of bismuth is 0.00083, copper is 0.000005 and wood is 0.9999995
- The diamagnetic susceptibility is very small and negative i.e. $\chi < 0$. It is due to the repulsion experienced by diamagnetic materials when placed in a magnetic field.



2. Paramagnetic Materials:

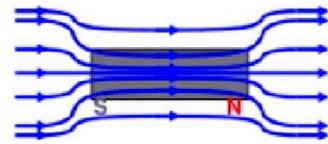
- Paramagnetic materials are substances which when placed in a magnetic field acquire a feeble (small) magnetism in the direction of the magnetic field.
- The atoms are slightly oriented along the direction of the external magnetic field.
- They attract the lines of force slightly.
- The examples are aluminum, tin magnesium etc.



- The relative permeability of these materials is slightly above than one i.e. $\mu_r > 1$
- For example the relative permeability of aluminum is: 1.00000065
- The paramagnetic susceptibility is less than one but positive i.e. $\chi < 1$. It is due to the slight attraction of magnetic field when placed in a magnetic field.

3. Ferromagnetic Materials:

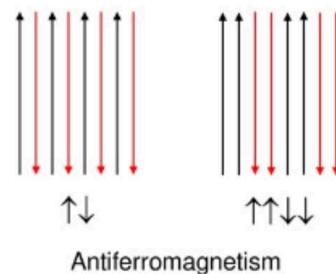
- Ferromagnetic materials are metallic crystals which when placed in external magnetic field becomes strongly magnetized in the direction of the field. These materials are strongly attracted by a magnetic field.
- These materials are strongly oriented along the direction of the external magnetic field.
- They attract the lines of force strongly.
- The examples are iron, steel, nickel, cobalt etc.
- The relative permeabilities of these materials are much greater than one and dependent on the field strengths i.e. $\mu_r > 1$
- For example, the purified iron and many magnetic alloys have relative permeabilities of iron and many magnetic alloys is 100,000 or more.
- The ferromagnetic susceptibility is greater than one and positive i.e. $\chi \gg 1$. It is due to the large attraction of magnetic field when placed in a magnetic field.
- Above the Curie temperature (T_c), ferro-magnetic materials behave as para-magnetic materials and their susceptibility is given by the Curie-Weiss law, defined as
- $\chi = \frac{C}{T - T_c}$, where C is material constant, T and T_c are Temperature and Curie temperatures



| Sr. | Property | Diamagnetics | Paramagnetics | Ferromagnetics |
|-----|---|--|--|--|
| 1 | Behavior in external magnetic field | Develop a weak magnetism in opposite direction | Develop a weak magnetism in the same direction | Develop a strong magnetism in the same direction |
| 2 | Alignment of magnetic dipoles in external magnetic field | Opposite and weak | Same direction and weak | Same direction and strong |
| 3 | Response to external magnetic field | Slightly repelled by external magnetic field | Slightly attracted by external magnetic field | Strongly attracted by external magnetic field |
| 4 | Relative permeability (μ_r) | $\mu_r < 1$ (Bi: 0.00083) | $\mu_r > 1$ (Al: 1.00000065) | $\mu_r \gg 1$ (Fe: 1,00,000 or more) |
| 5 | Susceptibility (χ) | Very small and negative i.e. $\chi < 0$ | Less than one but positive i.e. $\chi < 1$ | Very high $\chi \gg 1$ |
| 6 | Effect of temperature on susceptibility | χ is independent of temperature | Obeys Curie law $\chi \propto \frac{1}{T}$ | Susceptibility χ decrease with temperature in complex way |
| 7 | Curie Point (above which become paramagnetic) | No Curie point | No Curie point | Have definite Curie point |
| 8 | Hysteresis (Lag in change on magnetization) | Not exhibited | Not exhibited | Exhibited |
| 9 | Rententivity (Residual magnetism) | Not exhibited | Not exhibited | Exhibited |
| 10 | Examples | Bismuth, silver, copper, hydrogen, etc | Aluminum, tin magnesium etc | Iron, steel, nickel, cobalt etc |

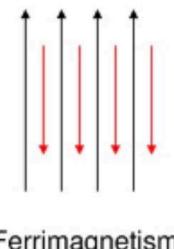
4. Antiferromagnetism

- Antiferromagnetic materials are crystalline materials.
- In the presence of the strong magnetic field, antiferromagnetic materials are weakly magnetized in the direction the field. This property of the materials is called antiferromagnetism.
- The magnetic moments of atoms are aligned in opposite directions and are equal in magnitude. Thus, when unmagnetized, their net magnetisation is zero.
- Examples of antiferromagnetic substances: MnO, FeO, CoO, NiO, Cr, Mn
- This is the case below a particular temperature, called as Néel temperature (T_N) above which the material behaves as a paramagnet.
- Antiferromagnetic materials exhibit a small positive susceptibility χ of order of 10^{-3} and 10^{-5} from 0 K to T_N and show antiferromagnetic behavior. Above T_N , the susceptibility show paramagnetic behavior.



5. Ferrits and Ferrimagnetism

- Some ceramic materials exhibit net magnetization.
- For example: Fe_3O_4 , NiFe_2O_4 ,
- In a magnetic field, the dipoles of some cation may line up with the field, while dipoles of other cation may not. These ceramics are called ferrites, and the effect is known as ferri-magnetism.
- Ferri-magnetism is similar to anti-ferro-magnetism in that the spins of different atoms or ions line up anti-parallel. However, the spins do not cancel each other out, and a net spin moment exists.
- Below the Neel temperature (T_N), ferrimagnetic materials behave very much like ferromagnetic materials and are paramagnetic above the Neel temperature.
- These materials exhibit a large but field dependent magnetic susceptibility similar to ferromagnets.
- They also show Curie-Weiss behavior. As these ceramics are good insulators, electrical losses are minimal, and hence ferrites have lot of applications in devices such as high frequency transformers.



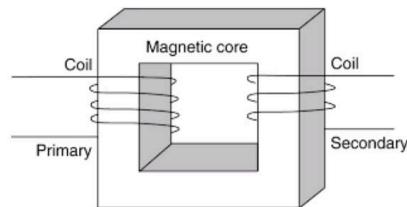
5.1.4 Applications of magnetic materials

Soft magnetic materials: Magnetic materials, which are easily magnetized and demagnetized are known as soft magnetic materials. They have high initial permeability due to which they reach saturation magnetization with a relatively low applied magnetic field. They are particularly used in alternating current applications. Susceptibility and permeability are high.

Hard magnetic materials: Hard magnetic materials retain their magnetism and are difficult to demagnetize. They have high permeability and a high resistance to demagnetization. They retain their magnetism even after the removal of the applied magnetic field. Susceptibility and permeability are low.

(a) Transformer core

- A magnetic core is a piece of magnetic material usually made up of ferro-magnetic materials which have high permeability.
- The magnetic core confines and increases the magnetic field of a coil by a factor of several thousands.



Following are some of the types of materials used for producing magnetic cores for the transformers.

| Sr. | Material | Manufacturing | Advantages | Applications |
|-----|---------------------------------|---|--|---|
| 1 | Solid Iron Core | A block of solid iron | Retain high magnetic fields without iron saturation. Large eddy currents produced by magnetic fields. | Not recommended for transformers operating at AC applications because large eddy currents are produced by the magnetic field. These eddy currents produce heat at high frequencies. |
| 2 | Carbonyl iron cores | Highly pure powder comprises of micrometer-sized iron spheres coated with a thin insulating layer | Provide stability across wide range of magnetic flux level and temperatures | They are efficient at high temperature and reduce effects of eddy currents. These ceramic materials serve as efficient insulators, and help decrease eddy currents |
| 3 | Amorphous Steel | Made of many layers of paper-thin metallic tapes | Reduce flow of eddy currents and have fewer losses | Can be operated at high temperatures. Used in high efficiency transformers operated at medium frequencies |
| 4 | Silicon steel | Silicon steel cores | High electrical resistivity and offers high saturation flux density. It also has high permeability and low losses. | Mostly in low frequency transformers to flow through narrow loops between every lamination layer |
| 5 | Amorphous metals | Amorphous or vitreous metals are glassy and non-crystalline | They have low conductivity to reduce eddy losses. Highly responsive to magnetic fields. | High-efficiency and high-performance transformers efficient insulators to prevent eddy currents. |
| 6 | Ferrite ceramics | Made from iron oxide and one or multiple metallic elements | Ceramic materials are produced in different specifications to meet diverse electrical requirements | These cores are used in high frequency applications |
| 7 | Laminated magnetic cores | Made of stacks of thin iron sheets coated with an insulated layer | Prevent eddy currents and confine them to narrow loops within each single lamination layer | General domestic applications |

(b) Magnetic storage

Magnetic storage is one of the most widely used digital data storage using a magnetized medium. Several types of magnetized media are used to store data such as **magnetic tape**, **floppy disks** and **hard disk drives**. The basic approach to magnetic data storage is almost similar for the different types of media.

Storage medium

The medium used in magnetic storage devices is coated with iron oxide, which is a ferromagnetic material. The storage media contains magnetic surface and it is divided into very small regions of mostly uniform magnetization.

Basic Principle

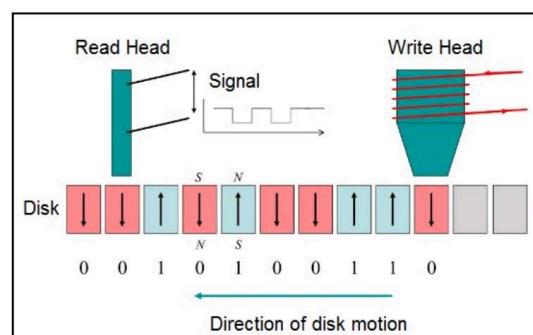
There are two types of magnetic polarities i.e. N-S and S-N each one used to represent either zero or one. Computer systems need to store data in digital format consists of binary information i.e. data in the form of zero and ones. The region where data is to be stored is magnetized (read as 1). The unmagnetized region is read as 0.

Read and write heads

The drive uses a motor to rotate the media at a high speed. The data is written and read using a small device called heads. Each head has a tiny electromagnetic, which consists of an iron core wrapped with wire. A read-write head moves very close (few nanometer) to the magnetic surface.

Writing the data:

Data signal is sent through the coil of wire which generates a magnetic flux. At the gap, the magnetic flux forms a fringe pattern. This flux magnetizes small region of the oxide on the media and magnetization of that area change. The information is stored on the disk in the form of zero's (unmagnetized region) and ones (magnetized regions).

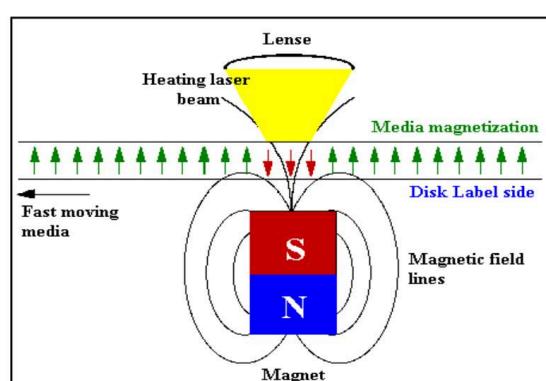


Reading the data: The read head is moved along the surface of the storage media. The information stored on the magnetic devices i.e. the regions of magnetization and demagnetizations are detected by the read head by detecting a varying magnetic field across the gap.

(c) Magneto-optical recording

Basic arrangement

- Magneto-optical recording is a method of storing and retrieving data using a laser and a magnet.
- A thin film of magnetic material (e.g. amorphous Terbium iron cobalt, TbFeCo) magnetic film is coated on the substrate over which the data is written.



Recording the data

- The disk is coated with film that initially is uniformly magnetized. The magnetic layer side faces towards the laser beam.
- A laser beam is used to demagnetize a small spot on the film by heating it above a critical temperature (the Curie point) by disordering the magnetic domains. A local magnetic field determines the direction in which the spot is magnetized when it cools.
- A relatively weak magnetic field is applied to this region to modify its magnetization. The tiny region that is magnetized is treated as 1 while the un-magnetized region is treated as 0.

Reading the data

- To read the information, the disk is scanned by polarized light from a low-power laser. The laser light is reflected from the surface of the media. Its plane of polarization is rotated slightly according to the direction of the magnetic field (Kerr effect) of the media.
- When the laser beam is reflected from the regions that are magnetized it has certain polarization. When it is reflected from the regions that are un-magnetized, its polarization is reversed due to Kerr effect.
- Thus, the change in direction of magnetization could be associated with numbers 0 or 1. Thus these techniques are useful for storage of binary data.

Erasing the data

To erase the data, magnetic field is applied to the material in the opposite direction. The focused laser pulse heats the disk above Curie temperature and causes local heating to demagnetize the surface. The disk is again de-magnetized and data written on in the form of magnetization is erased.

Questions on Magnetism

1 or 2 marks each

Explain the terms in brief with unit wherever required

1. Magnetic dipole moment
2. Magnetic field strength (H) [Dec 19, 2m]
3. Magnetization (M) [Dec 19, 2m]
4. Magnetic Susceptibility (χ)
5. Magnetic Induction (B)
6. Absolute permeability (μ) [Dec 19, 2m]
7. Relative Permeability (μ_r) [Dec 19, 2m]
8. Bohr magneton

6 marks

1. Explain the origin of magnetism.
2. Explain how materials are classified on the basis of permeability.
3. Differentiate between paramagnetism, diamagnetism and ferromagnetism.
4. Explain how the information is recorded and retrieved in magneto-optical recording devices. [Dec 19, 6m]
5. Explain the process of recording and retrieving the information in magnetic storage devices.

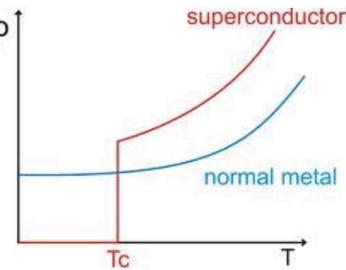
3/4 marks

1. What are the different types of transformer core? Discuss in brief any two of them and state their applications.
2. Explain in brief what are paramagnetic materials?
3. Explain in brief what are diamagnetic materials?
4. Explain in brief what are ferromagnetic materials?
5. Explain in brief what are anti-ferromagnetic materials?
6. Explain in brief what are ferrimagnetic materials?
7. Differentiate between paramagnetism, diamagnetism and ferromagnetism. [Dec 19, 4m]

5.2 Superconductivity

- Superconductivity is complete disappearance of electrical resistance (zero electrical resistance) in some materials when they are cooled below a characteristic temperature. This temperature is called as transition temperature (T_c). The zero resistivity means almost infinite conductivity.
- A superconductor is a material that exhibits superconductivity. Above T_c , the superconducting material behaves as a normal conductor.
- Thus the observed phenomenon is called as superconductivity.

Different material shows superconductivity below their characteristic critical temperature. For example: Tin (3.72 K), Lead (7.2 K), Niobium (9.2 K), Aluminium (1.1 K), etc.



However, copper, silver and gold which are very good conductor at room temperature do not show superconductivity.

5.2.1 BCS theory of superconductivity [For Information]

In the year 1957 three American Scientists John Bardeen, Leon Cooper and John Schrieffer jointly developed the theory to explain superconductivity which is known as BCS theory.

1. Electron-lattice-electron interaction

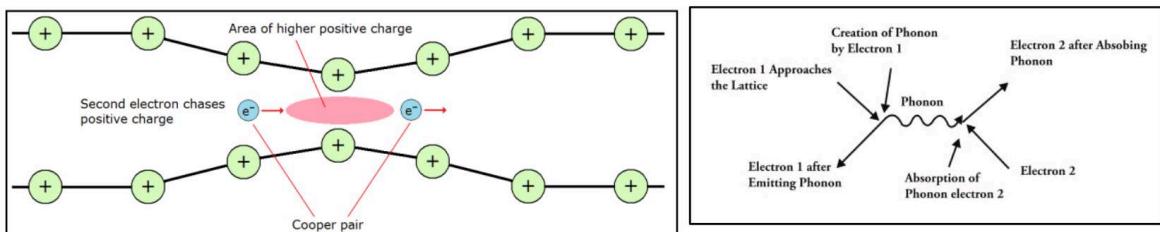
The BCS theory is based on electron – lattice – electron interaction and pairing of electrons close to the Fermi Level into cooper pairs through lattice interaction.

2. Lattice distortion

When an electron (say electron 1) approaches an ion in the lattice, there is a coulomb attraction between the electron and the lattice ion. This produces a distortion in the lattice and there is increase density of positive ions surrounding to electron 1. Due to the lattice vibration, the energy is emitted in the form of a quanta i.e. phonon.

3. Formation of Cooper pair

The distorted positive region attracts another electron of “opposite spin” (say electron 2). Thus, a free electron (electron 1) exerts a small attractive force on another electron (electron 2) through phonons. A pair of free electrons thus coupled through a phonon is called a Cooper pair.



4. Zero resistance below critical temperature

At $T < T_c$: Thermal vibrations of lattice are minimum which leads to the formation of Cooper pairs. Cooper pairs move almost without any resistance.

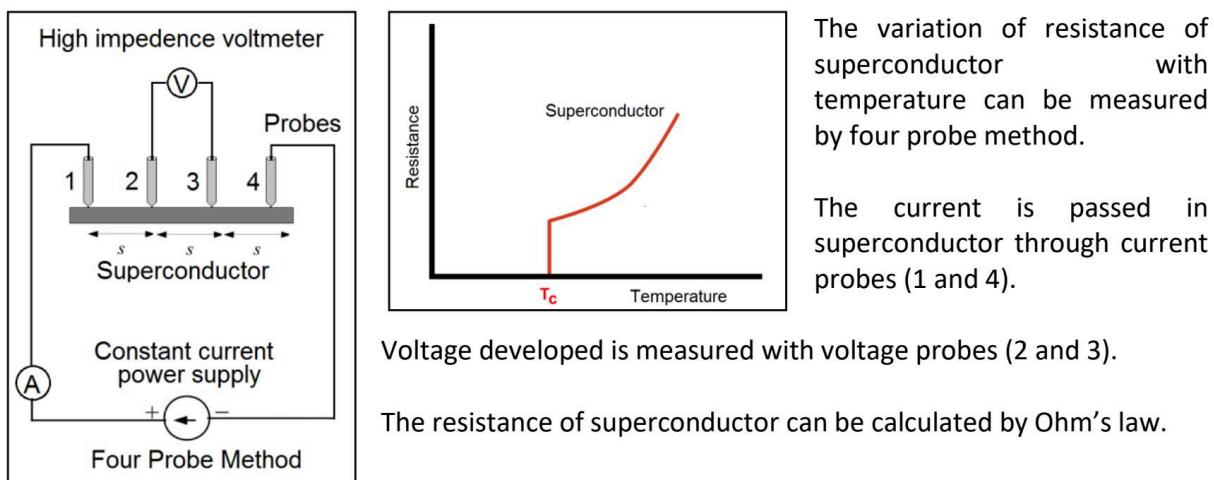
At $T > T_c$: The thermal vibrations of ions increases and provides sufficient energy to break Cooper pairs.

5. Superconducting current

A large number of Cooper pairs drift cooperatively through the superconductor in an ordered state forming a highly collective "condensate".

5.2.2 Properties of superconductors

Zero electrical resistance

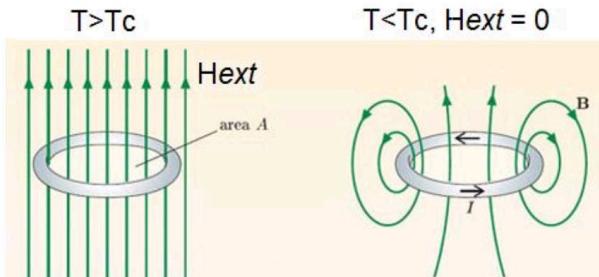


Normal state ($T > T_c$): A voltage is developed across voltage probes and Ohm's law $V = IR$ is obeyed.

Superconducting state ($T > T_c$): The resistance of superconductor vanishes ($R = 0$) and hence the voltage across voltage probes disappear i.e. ($V = 0$) while current continues to flow.

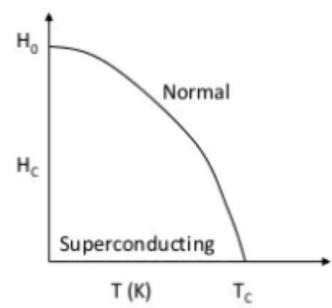
Persistent Currents

- Once a current is started in a closed loop of superconducting material, it will continue to keep flowing, of its own accord, around the loop as long as the loop is held below $T < T_c$.
- Such a steady current, which flows without diminishing in strength, is called persistent current.
- The persistent current does not require external power to maintain. In superconducting state resistance is zero and thus there are no I^2R losses. Once the current flow is initiated, it is estimated it persists for more than 10^5 years.
- Persistent current is one of the most important properties of superconductor. Superconductor coils with persistent current flowing through them produce magnetic fields and can therefore act as magnets.



Critical magnetic field (H_c)

- The critical magnetic field refers to the maximum magnetic field strength below which a material remains superconducting.
- It depends on temperature. Superconductivity is characterized both by temperature and magnetic field. Changes in either temperature or magnetic field can cause the phase transition between normal and superconducting states.
- The highest temperature under which the superconducting state is seen is known as the critical temperature. At that temperature even the weakest



external magnetic field will destroy the superconducting state, so the strength of the critical field is zero. As temperature decreases, the critical field increases generally to a maximum at absolute zero.

- The critical magnetic field at temperature T is given by $H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$
Where, $H_c(0)$ is the critical magnetic field at 0 K, T_c is critical temperature.
- Critical field at 0K, $H_c(0)$, for few materials Nb: 198 mT, Pb: 80.3 mT, Sn: 30.9 mT

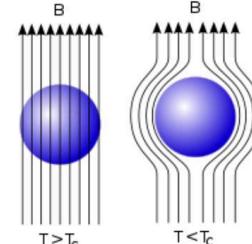
Critical current and critical current density

- An electric current flowing through the superconducting material produces a magnetic field surrounding to it. As current increases, the associated magnetic field also increases.
- Thus, it is important to have current flowing through the superconductor below a certain critical value (I_c) which is associated with critical magnetic field (H_c).
- If the current in superconductor is increased beyond I_c , the associated magnetic field will exceed critical magnetic field H_c and superconductor may destroy its own superconductivity.
- A superconducting ring or radius R ceases to be a superconductor if the critical current exceeds: $I_c = 2\pi R H_c$
- Current density is defined as current per unit area. The current density associated with critical current (I_c) is called critical current density (J_c). It is the maximum current density at which the superconductivity disappears.

5.2.3 Meissner Effect and perfect diamagnetism

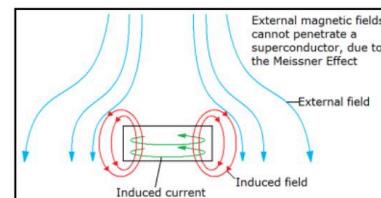
Meissner effect

When certain superconductors (type-I) are cooled below critical temperature $T < T_c$ in presence of external magnetic field, magnetic flux is expelled from the interior of the superconductor and it becomes perfect diamagnetic. This phenomenon is called Meissner effect.



Reason for magnetic flux expulsion

In presence of magnetic field, induced circulating currents are generated on the surface of the material. These circulating currents creates magnetic field opposite to the externally applied magnetic field in accordance with Lenz's law. Thus the external magnetic flux is expelled by the superconductor.



At $T > T_c$: The material is in normal conductor state and not a superconductor. When a magnetic field is applied to it, the it penetrates through the material.

If, H is external magnetic field and M is magnetization produced within the sample, then magnetic induction inside the specimen (B) is given by

$$B = \mu_0(H + M)$$

At $T < T_c$: The material turns a superconductor and as magnetic field is expelled out of superconductor. The magnetic induction inside the material $B=0$

Thus

$$0 = \mu_0(H + M)$$

And

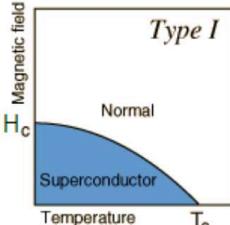
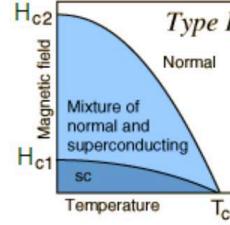
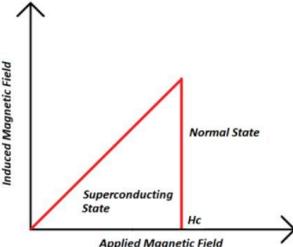
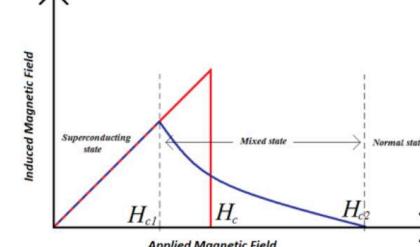
$$M = -H$$

Susceptibility of the material, $\chi = \frac{M}{H} = -1$

Perfectly diamagnetism: Magnetic susceptibility is the degree of magnetization of a material in response to an applied magnetic field. If magnetic susceptibility is negative the material is diamagnetic. For superconductors, magnetic susceptibility, $\chi = \frac{M}{H} = -1$. This indicates that superconductors are characterized by perfect diamagnetism.

5.2.4 Types of superconductors (Type I and Type II)

In Type I superconductors, the transition from superconducting state to normal state in presence of magnetic field occurs sharply at critical field H_c . However certain alloys show different behavior. The transition from superconducting to normal state is gradual. They have two critical magnetic fields. At lower critical magnetic field (H_{c1}) external magnetic field enters into superconductor. It does not lose its superconductivity and exists in mixed/vortex state. If the value of external magnetic field is increased above upper magnetic field (H_{c2}), they lose their superconductivity.

| Sr. | Property | Type-I Superconductor | Type-II Superconductor |
|-----|--|--|---|
| 1 | <i>Meissner effect</i> | They exhibit complete Meissner effect. | They do not exhibit Meissner effect completely. |
| 2 | <i>Diamagnetic Behavior</i> | They show perfect diamagnetic behavior | They do not show perfect diamagnetic behavior |
| 3 | <i>Critical magnetic field</i> | They have only one critical magnetic field H_c . | They have two critical magnetic fields. At lower critical magnetic field H_{c1} flux starts penetrating the superconductor. At upper critical magnetic field H_{c2} flux enters into superconductor and it loses superconductivity. |
| 4 | <i>Variation of magnetic field with temperature</i> |  |  |
| 5 | <i>Variation of Magnetization (M) of superconductor with applied Magnetic Field (H)</i> |  |  |
| 6 | <i>States of the material</i> | Material exists in two states for $T < T_c$ $H < H_c$ – superconductor $H > H_c$ – conductor | Material exists in three states for $T < T_c$ $H < H_{c1}$ – superconductor $H_{c1} > H < H_{c2}$ – mixed/vortex state $H > H_{c2}$ – conductor |
| 7 | <i>Change in magnetization</i> | The materials loses magnetization abruptly | The materials loses magnetization gradually |
| 8 | <i>Critical magnetic field</i> | Highest value for H_c is about 0.01 to 0.2 Wb/m ² | Highest value for H_{c2} is about 30 Wb/m ² |
| 9 | <i>Type</i> | They are known as soft superconductors | They are known as hard superconductors |
| 10 | <i>Applications</i> | Not much useful due to low H_c | Useful due to high H_{c2} |
| 11 | <i>Examples</i> | Aluminum, lead, mercury, etc | Nb-Sn, Nb-Ti, Nb-Zr, V-Ga, etc |

5.2.5 Low and high temperature superconductors

Superconductors are classified into low T_c and high T_c superconductors based on their transition temperatures. Broadly, the superconductors having transition temperature (T_c) below 24 K are low temperature superconductors. The superconductors having transition temperature above 27 K are high temperature superconductors.

| Sr. | Property | Low Temperature Superconductors | High Temperature Superconductors |
|-----|------------------------|--|---|
| 1 | Transition temperature | Below 24 K | Above 27 K upto 138 K so far |
| 2 | Coolant required | Liquid Helium (4 K) | Liquid Nitrogen (77 K) |
| 3 | Examples | They are typically elements | They are generally Nb based alloys, mixed oxide materials containing lanthanum and yttrium. |
| 4 | Applications | They are generally not useful for commercial applications. | Useful for commercial applications such as motors, generators, cables, transformers, magnets, MRI, NMR, etc |

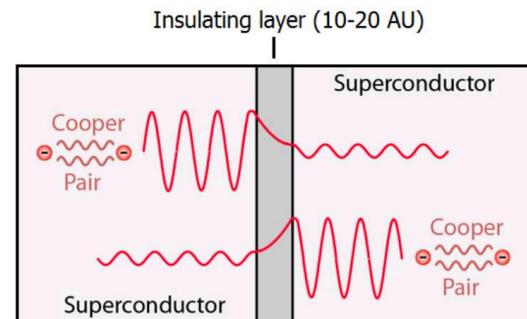
5.2.6 Josephson Effect

The dc Josephson effect

- When two superconductors are separated by a thick insulating layer ($> 10\text{nm}$) behaves as two independent superconductors. When the insulating layer is thin ($\sim 1\text{nm}$), it becomes a system of coupled conductors.
- The cooper pairs tunnel easily through the barrier (insulating layer) as a single unit.
- The cooper pairs can be represented by a wave function, which is the same for all pairs. The insulating layer introduces a phase difference between the wave function of cooper pairs on opposite sides. Due to this, a super-current appears across the junction even though the applied voltage is zero. This effect is known as **dc Josephson effect**.
- The super-current through the junction is given by

$$I_s = I_c \sin \phi$$

Where, ϕ is the phase difference between the wave functions describing Cooper pairs, and I_c is critical current at zero voltage condition.



The ac Josephson effect

- If a dc voltage is applied across Josephson junction, it introduces an additional phase on the Cooper pairs during tunneling. The dc voltage generates an alternating current I , given by

$$I = I_c \sin(\phi + \Delta\phi)$$

- Because of the dc voltage, the energies of Cooper pairs on both sides of the barrier differ in energy by $2eV$. Using quantum mechanical calculations, it can be shown that

$$\Delta\phi = 2\pi t \left(\frac{2eV}{h} \right)$$

Thus, the alternating current across the junction is given by

$$I = I_c \sin\left(\phi + 2\pi t \left(\frac{2eV}{h}\right)\right)$$

This current can be represented by an alternating current of frequency $\nu = \frac{2eV}{h}$. This frequency only depends on the applied voltage. This effect is known as **ac Josephson Effect**.

- The frequency of alternating currents does not depend on the dimensions of the superconductors. Also this frequency does not depend on properties of superconductors such as critical temperature, chemical composition.
- At $V=1 \mu V$, ac current of frequency 483.6 MHz is produced.

5.2.7 Superconducting Quantum Interference Devices (SQUID)

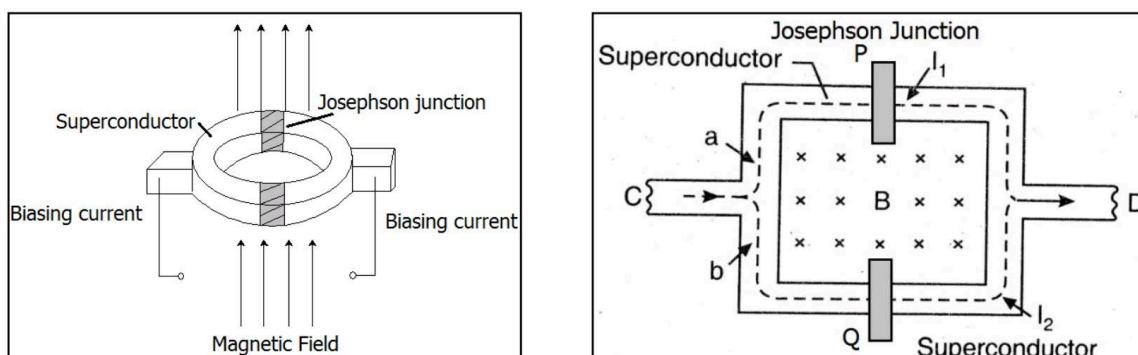
A SQUID (Superconducting Quantum Interference Device) is a very sensitive magnetometer used to measure extremely subtle (small) magnetic flux of the order of 10^{-18} Tesla. Their working is based on superconducting loops containing Josephson junctions.

Construction of SQUID

There are two main types of SQUID: direct current (DC) and radio frequency (RF). A radio frequency (RF) SQUID is made up of one Josephson junction, which is mounted on a superconducting ring. A direct current (DC) SQUID consists of two Josephson junctions in parallel, which is more sensitive.

Construction

SQUIDs are usually fabricated from lead or pure niobium. The tunnel barrier is oxidized onto lead or niobium surface. The entire device is cooled to within a few degrees of absolute zero with liquid helium. A schematic of a two-junction dc SQUID is shown in figure. It consists of two Josephson junctions arranged in parallel.



Working

A constant dc supercurrent is applied to the SQUID. This current is known as bias current which enters into the SQUID through arm C. It is divided along the paths a and b and again merge into one and leaves through the arm D. I_1 and I_2 are currents tunneling through Josephson junctions.

In absence of magnetic field

- In a superconductor, a single wave function describes all the Cooper pairs. The wave function experiences a phase shift at the Josephson junctions P and Q.
- In absence of magnetic field, Phase difference across P and Q = 0

In presence of magnetic field

- When a magnetic field is applied, it changes the quantum mechanical phase difference across each of the two junctions. This phase difference between reuniting currents (δ_0) is directly proportional to the magnetic flux Φ through the ring.

- The voltage across the Josephson junction oscillates with the changes in phase at the two junctions. This voltage depends upon the change in the magnetic flux. Thus, by noting down the voltage across the junction, the change in flux and corresponding magnetic field can be measured.
- Total current through parallel Josephson junctions is $I_T = 2(I_0 \sin \delta_0) \cos \frac{e\phi}{\hbar c}$
- Thus, a progressive increase or decrease of the magnetic flux causes the current to oscillate between a maximum and minimum when the magnetic flux increases by one flux quantum.
- One flux quantum $\phi_0 = \frac{\hbar}{2e} = 2.06 \times 10^{-15}$ webers.

Thus, a SQUID can detect extremely small magnetic fields of the order of 10^{-15} Wb (10^{-11} T). This sensitivity can further be increased using a flux transformer.

Applications of SQUID

SQUIDs are sensitive magnetometers that detect very small magnetic fields of the order of 10^{-14} T. SQUIDs have many applications. Few are as below:

- Magnetoencephalography (MEG)** is a technique for mapping brain activities. It can detect magnetic fields produced naturally in the brain which is of the order of 10^{-14} T. The human heart also produces a feeble magnetic field of the order of 10^{-14} T which can also be detected by SQUIDs.
- MR Scan:** Magnetic resonance imaging (MRI), also known as nuclear magnetic resonance imaging, is a scanning technique for creating detailed images of the human body. The scan uses a strong magnetic field and radio waves to generate images of parts of the body that can't be seen with X-rays, CT scans or ultrasound. Water molecules in human body contain hydrogen nuclei (protons). An MRI scanner applies a very strong magnetic field (about 0.2 to 3 teslas), which aligns the proton "spins". This alignment of spin can be detected by SQUIDs.
- Non-destructive corrosion testing: The magnetism of the material changes due to the corrosion, which can be detected using SQUID.
- Oil prospecting: Presence of oil field changes the magnetism of the region, which can be detected using SQUID.
- Earthquake prediction can be done by detecting change in earth's magnetism
- Mineral exploration by detecting variation in magnetic fields inside earth crust
- Geothermal energy survey, etc.

5.2.8 Applications of Superconductors

Superconducting Transmission Lines and electricity

Conventionally the electricity is transported through copper cables. Generally 10% to 15% of generated electricity is lost in overcoming resistance of cables. It also requires a huge setup. The use of superconducting transmission lines would have following advantages:

- Minimum (zero) heat losses**
When superconductors will be used as cables, resistive and heat losses are avoided and electrical power transmission can be done more efficiently. A network of superconducting power cables could be carrying 1000 times more electric current than copper cables.
- Carrying a large power**
Using superconducting transmission lines a large power could be transmitted at a fairly low voltage. Large infrastructure such as huge transformers banks and multiple high voltage AC transmission lines on towers could be minimized.
- High current carrying density**
Using superconductors such as BSCCO in tape forms and YBCO in thin film forms, current densities above 10,000 amperes per square centimeter could be transmitted.

- **For storing current in superconducting coils**

Superconductors are used in Superconducting Magnetic Energy Storage (SMES) that store electric energy in the form of electric current in a closed superconducting coil. The current remains trapped forever in the coil as there is absolutely no energy loss. This current can be recovered in a very short amount of time.

- **Transformers**

Superconducting coils in transformers and electrical machines generate much stronger magnetic fields. It will also eliminate eddy current losses and hysteresis losses. Therefore, the size of motors and generators will be drastically reduced.

Superconducting magnets

Conventional electromagnets are very much bigger in size. They consume large electrical power to maintain the magnetic field and also require continuous cooling. The use of superconductors in designing of magnets would have following advantages:

- **Intense magnetic field**

Superconducting magnet has ability to support a very high current density with almost no resistance. Due to this electromagnets can be constructed that generate intense magnetic fields with little electrical power input. Type-II superconductors such as niobium-titanium (NbTi) alloys, niobium-tin (Nb₃Sn) alloys can produce high magnetic fields of around 9-10 Tesla.

- **Compact and more efficient setup**

Superconducting magnet systems are quite compact and occupy a small space.

- **Can be operated for longer duration**

In the persistent mode of operation, the L/R time constant is extremely long and the magnet can be operated for days or even months at a nearly constant field.

Friction-less bearings

A bearing is a machine element that bears the load and minimizes the friction between moving parts. Most of the energy of bearing is consumed in overcoming friction hence it results into reduction of efficiency. The use of superconductors in designing of bearings uses principle of Meissner effect and magnetic field repulsion. Due to this, it is kind of a magnetic cushion and it would have following advantages:

- **No lubrication and maintenance**

Superconducting bearings provide the highest efficiency of all bearing technologies and prevent contact, friction and wear. They need no lubrication or maintenance and can be used under extreme conditions: vacuum, cryogenic environments.

- **More efficient bearings**

Superconducting magnetic bearings are virtually frictionless, dust-free, wear & tear less.

- **High driving speed**

Rotational bearings for very high driving speeds are possible.

- **Frictionless motors**

In frictionless motors the principle of magnetic air cushion between stator and rotator is used.

More efficient electronic components

Variety of devices such as SQUIDS, transistors, ICs, etc can be designed using superconductors. Main advantage of using superconductors in designing of electronic components is reduction in heat losses and flow of current in the electronic circuit without any resistance. Some of the applications of superconductors in electronics are listed as below:

- **Josephson junctions**

Josephson junctions are used in fast electronic switches or sensitive magnetometers. A magnetometer is able to detect very small magnetic fields of the order of 10^{-15}T .

- **Supercomputers**

The semiconductor logic elements have a speed limit. They operate at a speed in orders of few nanoseconds. Logic elements based on Josephson junction can operate at the speed of few picoseconds. Use of superconductors in logic gates will drastically increase the speed of computers.

- **SQUID (superconducting quantum interference device)**

SQUIDs are used for Non-destructive corrosion testing, Magnetoencephalography (MEG), observing neural activities inside the brain (MRI scan), study of magnetic properties of material, oil prospecting, mineral exploration, earthquake prediction, geothermal energy survey, etc.

- **Transistors**

Superconducting transistors based on Josephson junctions could be used to switch voltages very quickly. They will significantly speed up the processing of signals or data in microprocessors.

- **Circuitry connections**

Circuit connections can be made through superconductive films. This would have advantage that information can be transmitted more quickly without losses.

- **IC fabrication**

At present processing power of ICs is limited due to I^2R losses of components. Use of superconductors will make ICs more efficient.

Medical applications of superconductors

Two of the properties of superconductors are extremely useful in medical field – production of extensive magnetic field in devices such as MRI (upto 1.5 Tesla) and detection of smaller magnetic fields using SQUIDs (of the order of 10^{-14} Tesla)

Magnetoencephalography (MEG)

- Magnetoencephalography (MEG) is a technique for investigating human brain activities on a millisecond basis. It can detect where in the brain activity is produced.
- Brain generates neuromagnetic signals that are extremely small (of the order of 10^{-12} Tesla). MEG scanners use SQUIDs to detect this extremely small magnetic field.

Magnetic Resonance Imagery (MRI)

- Magnetic resonance imaging (MRI), also known as nuclear magnetic resonance imaging, is a scanning technique for creating detailed images of the human body. The scan uses a strong magnetic field (of the order of 1.5 Tesla or higher) and radio waves to generate images of parts of the body that can't be seen with X-rays, CT scans or ultrasound. If conventional metal wires are used to produce such a strong magnetic field, they would heat to the extent of melting.
- Water molecules in human body contain hydrogen nuclei (protons). An MRI scanner applies a very strong magnetic field (about 0.2 to 3 teslas), which aligns the proton "spins". This alignment of spin can be detected by SQUIDs.
- MRI diagnosis is used for investigation of brain, including detection of tumors. MRI is also used for investigation of sports injuries, muscle injuries, skeletal problems, spinal injuries, etc.

Transport - MagLev Trains

In conventional transportation system, friction between the wheels and ground or rail is one of the crucial elements. Due to friction there is a limitation of speed of the vehicle also it also increases wear and tear.

Magnetic Levitation Trains (Maglev Trains) is a floating vehicle for land transportation. MagLev trains do not slide over the rails but float on an air cushion over a strongly magnetized track. As there is no mechanical friction, speeds upto 500 km/h can be easily achieved. The advantages of superconductors are:

- Permanent currents as high as about 700000 amperes can be passed through the superconducting coils.
- This produces a strong magnetic field of order of almost 5 Tesla enough to levitate the train.

Numericals: Superconductivity

Example: In a superconductor ring of radius 0.02 m, the critical magnetic field is 2×10^3 A/m at 5K. Find the value of critical current.

Solution:

$$I_c = 2\pi R H_c = 2 \times 3.143 \times 0.02 \times 2 \times 10^3 = 251.4 \text{ A}$$

Example: Calculate the critical magnetic field for lead at 4.2 K. The critical temperature for lead is 7.18 K and $H_c(0)=6.5 \times 10^4$ A/m.

Solution:

$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right] = 6.5 \times 10^4 \left[1 - \left(\frac{4.2}{7.18} \right)^2 \right] = 4.28 \times 10^4 \text{ A/m}$$

Example: The transition temperature for lead is 7.2K. However, at 5K it loses the superconducting property if subjected to magnetic field of 3.3×10^4 A/m. Find the maximum value of H which will allow the metal to retain its superconductivity at 0K. [Dec 19, 4m]

Solution:

$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$H_c(0) = \frac{H_c(T)}{1 - (T^2/T_c^2)} = \frac{3.3 \times 10^4}{\left(1 - \frac{25}{51.28} \right)} = 6.37 \times 10^4 \text{ A/m}$$

Example: The critical field of niobium is 1×10^5 A/m at 8 K and 2×10^5 at 0 K. Calculate the transition temperature of the element.

Solution: $H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$

$$T_c = \frac{T}{\left[1 - \frac{H_c(T)}{H_c(0)} \right]^{1/2}} = \frac{8}{\left[1 - \frac{1 \times 10^5}{2 \times 10^5} \right]^{1/2}} = 11.3 \text{ K}$$

Example: The transition temperature for lead is 7.26 K. The maximum critical field for the material is 8×10^5 A/m. Lead has to be used as a superconductor subjected to a magnetic field of 4×10^4 A/m. At what maximum temperature it can be operated.

Solution: $H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$

$$T = T_c \left[1 - \frac{H_c(T)}{H_c(0)} \right]^{1/2} = 7.261 - \left[1 - \frac{4 \times 10^4}{8 \times 10^5} \right]^{1/2} = 7.08 \text{ K}$$

Therefore, the temperature of the materials should be kept below 7.08 K

Example: A Josephson junction with a voltage difference of $650 \mu\text{V}$ radiates electromagnetic radiations. Calculate its frequency.

Solution: $\nu = \frac{2eV}{h} = \frac{2 \times 1.6 \times 10^{-19} \times 650 \times 10^{-6}}{6.63 \times 10^{-34}} = 3.13 \times 10^{11} \text{ Hz}$

Example: Calculate the voltage required to produce a frequency of $2 \times 10^{11} \text{ Hz}$ across the Josephson junction.

Solution: $\nu = \frac{2eV}{h}$

Thus, $V = \frac{\nu h}{2e} = \frac{2 \times 10^{11} \times 6.63 \times 10^{-34}}{2 \times 1.6 \times 10^{-19}} = 414.38 \mu\text{V}$

Questions on superconductivity

6 marks

1. Define superconductivity with resistance vs temperature graph and example. Explain zero electrical resistance in superconductivity. [Dec 19, 6m]
2. Explain Meissner effect and its cause. Show that superconductors exhibit perfect diamagnetism.
3. Differentiate between type I and type II superconductors.
4. What are SQUIDs? Explain principle, construction and working of SQUID.

3/4 marks

1. Explain following terms in superconductivity. Draw the diagram and write equation wherever necessary:
 - (i) Critical transition temperature (ii) Zero electrical resistance (iii) Persistent currents
 - (iv) Critical magnetic field (v) Critical current and critical current density
2. Differentiate between type I and type II superconductors.
3. Discuss in brief low and high temperature superconductors. Where are they used?
4. Explain DC and AC Josephson effect with diagram. [Dec 19, 4m]
5. What are SQUIDs? Explain their any two applications in brief. [Dec 19, 4m]
6. Explain applications of superconductors in the field of (any one of below):
 - (i) Transmission lines and electricity (ii) Superconducting magnets (iii) Frictionless bearings
 - (iv) Electronics (v) Medical field (vi) MagLev trains.