Motion :-

A body is said to be in motion if it is changing its position w.r.t. reference point.

Speed :-

Rate of change of Distance w.r.t. time but irrespective of the direction. It is scaler quantity. It's always positive.

Velocity :- ($v = \frac{x}{t}$)

Rate of change of displacement w.r.t. time in a particular direction. It is vector quantity.

Displacement :- (change in position of particle)

shortest distance betn initial & final position of a particle during motion. It may be positive or negative. It is denoted by 's' or 'x'.

Distance :-

Total path covered by a particle during given time interval. It is always positive. It is denoted by 's' or 'd'.

Distance $\geq$ Displacement.

* Average speed & Instantaneous speed :-

The avg. speed of a particle is the distance traveled in the particular time interval.

If particle travels the distance 's' in time t_1 to t_2, then, avg. speed

$$V_{avg} = \frac{s}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

Instantaneous speed is the speed of the particle at a particular instant when time interval approches to zero.

$\therefore$ Instantanious speed,

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

* Average velocity & Instantaneous Velocity :-

Avg. velocity is the ratio of displacement Δs (or Δx) to the time interval Δt.

$$\therefore \text{Avg. velocity} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity is the rate of change of displacement when time interval approches to zero.

$$\therefore \text{Instantaneous velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\therefore v = \frac{dx}{dt}$$

velocity may be positive or negative.

* Avg. Accelaration & Instantaneous Accl^n

Avg. accelaration is the rate of change of velocity of a particle for the time interval Δt.

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Instantaneous accl^n is the rate of change of velocity of a particle at a particular instant when time interval Δt approches to zero.

$$\therefore \text{Instantaneous Accl}^n = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

but $v = \frac{dx}{dt}$ $\quad \therefore a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$$\therefore a = \frac{d^2 x}{dt^2}$$

by chain rule

$$a = \frac{dv}{ds} \times \frac{ds}{dt}$$

$$\therefore a = \frac{dv}{ds} \times v \qquad \therefore a = v \cdot \frac{dv}{ds}$$

It may be positive or negative.

* Jerk

Rate of change of accl^n w.r.t. time.

$$J = \frac{da}{dt}$$

* Newtons first laws:-

Every body continues to be in its state of rest or of uniform motion, unless it is acted by some external agency.

* Newton's Second laws:-

Rate of change of momentum is directly proportional to the impressed force & it takes place in the direction of the impressed force.

$$F = m \cdot a.$$

* Newtons third law:-

For every action, there is equal & opposite reaction.

* Linear Motion (Rectilinear motion):-

(1) Uniform motion
- (Linear motion with <u>zero</u> Accl^n)

(2) Linear motion with uniform (constant) Accl^n.

(3) Linear motion with uniform (constant) Accl^n (under gravity).
(Motion under gravity)

(4) Linear motion with variable Accl^n.

Rectilinear Motion

Linear (Rectilinear) Motion:-

The motion of the particle along the straight line is called as Rectilinear motion.

* Uni-directional → particle moving only in one direction
* Un-directional → forward & reverse motion of particle.

Uniform motion (U.M.) :-
(motion with zero $Accl^n$).

If the particle is moving with constant velocity and zero acceleration, the motion of the particle is called as uniform motion.

Thus, Here,

$$v = \frac{dx}{dt} = \text{constant}.$$

taking integration,

$$\therefore \int v \cdot dt = \int dx.$$

$$\therefore v\int dt = \int dx.$$

$$\therefore \int dx = v\int dt$$

$$\therefore \boxed{x = v \times t}$$

$$\therefore \boxed{x = \text{velocity} \times \text{time}}$$

Examples on Uniform Motion – Velocity constant (Acceleration is zero)

① In a race of 100 m distance, A runner runs at constant velocity from 30 m to 100 m position at velocity of 12 m/sec. Find the time required for the runner to cover the distance from 30 m position to 70 m position.

⇒ $V = 12\,m/sec.$

$x_1 = S_1 = 30\,m$

$x_2 = S_2 = 70\,m$

distance covered / displacement = final position − initial position

$$x = S = S_2 - S_1 = x_2 - x_1$$

$$= 70 - 30$$

$$x = S = 40\,m.$$

for Uniform motion

$\therefore$ displacement/distance $= V \times t$

$$40 = 12 \times t$$

$$\therefore t = \frac{40}{12}$$

$$\boxed{t = 3.33\ sec.}$$

② A train accelerates Uniformly & gains the speed of 90 kmph. & runs at this speed for 17 minutes before getting Slowed down due to work in progress. Find the distance Covered by the train while running at constant speed.

⇒

$V = 90\ kmph = 90 \times \frac{5}{18} = 25\ m/sec.$

$t = 17$ minutes $= 17 \times 60 = 1020$ sec.

$\therefore$ distance covered / displacement of train while running at constant speed $= V \times t$

$$S = x = V \times t$$

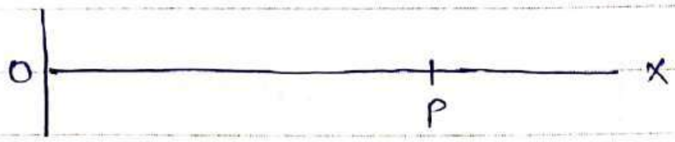
$$S = x = 25 \times 1020$$

$$= 25500\ m.$$

$$\boxed{S = x = 25.5\ Km}$$

Train covers 25.5 kms before getting slowed down due to work in progress.

* Rectilinear motion with uniform (constant) Accl^n :-
[U.A.M. — Uniform Acceleration Motion]



→ Consider linear motion of particle starting from O & moving along OX with uniform acceleration.

→ Let P is the position of particle after t sec.

→ Let u = initial velocity
v = final velocity
t = time taken by particle to change its velocity from u to v.
a = uniform positive acceleration
x or S = Displacement in 't' seconds.

Since, in 't' seconds, velocity of particle has increased steadily from (u) to (v) at the rate of Accl^n 'a',

∴ Total increase in velocity

$= at$

∴ final velocity, $\boxed{v = u + at}$ — (1)

avg. velocity $= \left(\frac{u+v}{2}\right)$

As we know that, Displacement by the Particle,

S = Avg. velocity × Time

$\therefore \quad s = \left(\frac{u+v}{2}\right) t \quad$ — (ii)

substituting value of v from eq^n (i)

$$\therefore \quad s = \left(\frac{u + u + at}{2}\right) t$$

$$= \frac{ut + ut + at^2}{2}$$

$$= \frac{ut}{2} + \frac{ut}{2} + \frac{at^2}{2}$$

$$= ut + \frac{at^2}{2}$$

$$\therefore \boxed{S = ut + \frac{1}{2} at^2} \quad \text{— eq}^n \text{ (iii)}$$

from eq^n (i), $\quad v = u + at$

$t = \frac{v-u}{a}$ — put this value in (ii)

$$\therefore \quad s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$= \frac{v^2 - u^2}{2a}$$

$$\therefore \quad 2as = v^2 - u^2$$

$$\therefore \boxed{v^2 = u^2 + 2as} \quad \text{— (iv)}$$

Thus the eq^{n}s of motion are

1) $V = u + at$
2) $S = ut + \frac{1}{2} at^2$
3) $V^2 = u^2 + 2as$

* Examples on- Motion with uniform Acceleration *
* UAM - Uniform Acceleration Motion. *
* (Acceleration is constant) *

(1) A car starting from rest is accelerated at the rate of $0.4\ m/s^2$. Find the distance covered by the car in 20 seconds.

⟹

Given :- Initial velocity $= u = 0$ ---- car is starting from rest.

Acceleration $= a = 0.4\ m/s^2$

time $= t = 20$ sec.

we know that, from the eqⁿ of motion for displacement,

$$S = ut + \frac{1}{2}at^2$$
$$= (0 \times 20) + \frac{1}{2} \times 0.4 \times 20^2$$

$$\boxed{S = 80\ m.}$$

car will cover 80 m distance in 20 seconds.

(2) A train travelling at 27 kmph is accelerated at the rate of $0.5\ m/sec^2$. What is the distance travelled by train in 12 seconds.

⟹

Given:- Initial velocity $= u = 27$ kmph $= 27 \times \frac{5}{18} = 7.5\ m/sec.$

Acceleration $= a = 0.5\ m/sec^2$

time $= t = 12$ sec.

∴ from eqⁿ of motion,

$$S = ut + \frac{1}{2}at^2$$
$$= (7.5 \times 12) + \left(\frac{1}{2} \times 0.5 \times 12^2\right)$$

$$S = 90 + 36$$

$\boxed{S = 126\ m.}$ ---- distance travelled by train.

(3) A scooter is starting from rest & moves with constant Accl^n of $1.2\ m/s^2$. Determine its velocity, after it has travelled for 60 meters.

⇒

<u>Given:</u> Initial velocity, $u = 0$ ------ start from rest.

Accl^n, $a = 1.2\ m/s^2$

distance travelled $= 60\ m = S$

$V = ?$

we know that,

$$v^2 = u^2 + 2as$$

$$= 0^2 + 2 \times 1.2 \times 60$$

$$V^2 = 144$$

Taking root,

$$\boxed{V = 12\ m/sec.}$$

(4) A motorist rushing at 20 m/sec, find a child on the road 50 m ahead. He instantly stops the engine & applies the brakes, so as to stop the car within 10 m of the child. Calculate retardation & time required to stop the car.

⇒

Initial velocity, $u = 20\ m/sec.$

final velocity, $v = 0$

distance travelled $= 50 - 10 = 40\ m.$

we know that, $v^2 = u^2 + 2as$

$$0 = 20^2 + 2 \times a \times 40$$

$$0 = 400 + 80a$$

$$\therefore 80a = -400$$

$$a = \frac{-400}{80}$$

$$a = -5\ m/sec^2$$ (– sign indicate Retardation)

Time req. to stop the car,

$$v = u + at$$

$$0 = 20 + (-5)t$$

$$0 = 20 - 5t$$

$$\therefore t = \frac{20}{5} = \underline{4\ sec.}$$

(5) A runner in a 100 m race accelerates uniformly for the first 30 m & then runs with constant velocity. If the runners time for the first 30 m is 5 sec, Determine the time required for the race.

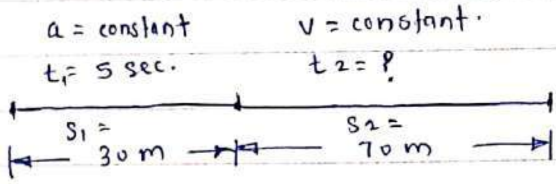
⇒ Given:-

Total distance covered during race = 100 m.

time req. for the first 30 m = $t_1 = 5$ sec.

let, time req. for the remaining race = t_2

total time taken for the entire race = t.



∴ $S = S_1 + S_2$

&

$\underline{\underline{S = 100 \text{ m}.}}$

Initial velocity, $u = 0$.

1) for first 30 m race, accl^n is uniform. Thus motion is Uniform Accelerated motion (UAM).

we know that, $S = ut + \frac{1}{2}at^2$

∴ Here, $S_1 = ut_1 + \frac{1}{2}at_1^2$

∴ $30 = (0 \times 5) + \frac{1}{2} a \times 5^2$

∴ $30 = 0 + 12.5a$

∴ $a = \frac{30}{12.5}$

∴ $\boxed{a = 02.4 \text{ m/sec}^2}$

we know that

$V = u + at$

$V = 0 + 2.4 \times 5$

$\underline{V = 12 \text{ m/sec}.}$

11) Motion for next 70 m - velocity is constant. Thus motion is Uniform motion.

∴ we know that,

$S_2 = 100 - S_1$

$= 100 - 30$

$\boxed{S_2 = 70 \text{ m}}$

For U.M.,

S_2 = velocity × time.

$70 = v \times t_2$ — ①

∴ $70 = 12 \times t_2$

∴ $\boxed{t_2 = 5.83 \text{ sec}}$

Time req. for the Race $t = t_1 + t_2 = 5 + 5.83 = \boxed{10.83 \text{ sec}.}$

(6) A burglar's car had a start with an acceleration $2.5\ m/s^2$. A police vigilant team came in a van to the spot at a velocity of 20 m/sec after 3.75 sec. & continued to chase burglar's car with uniform velocity. Find the time in which the police van will overtake the burglar's car.

$\Longrightarrow$

Given :- Acclⁿ of burglar's car, $a = 2.5\ m/s^2$
constant velocity of police van = 20 m/sec.
let, t = time taken by burglar's car.

1) Consider the motion of burglar's car (UAM).
Distance travelled by burglar's car, in t sec.

$$S = ut + \frac{1}{2}at^2$$

Here :- $S_1 = (0 \times t) + \frac{1}{2} \times 2.5 \times t^2$

$$S_1 = 0 + 1.25t^2$$

$$S_1 = 1.25\,t^2 \quad \text{------} \quad (1)$$

2) Distance travelled by police van
let S_2 = Distance travelled by police van.
As police van is running at constant velocity,

$$S_2 = \text{velocity} \times \text{time}.$$

To overtake the burglar's car, police van should also travel S_1 distance.
But time available for police van to cover that S_1 distance is $(t - 3.75)$ sec.

$$\therefore \quad S_2 = v \times (t - 3.75) = 20(t - 3.75)$$

But $S_2 = S_1$

$$1.25t^2 = 20(t - 3.75)$$

$$\therefore 1.25\,t^2 - 20t + 75 = 0$$

Solving the eqⁿ,

$t = 10$ sec. or $\boxed{t = 6\ \text{sec.}}$

As initially burglar's car is having less velocity than police van, thus $\boxed{t = 6\ \text{sec.}}$

Police will overtake the car in 6 seconds.

(7) A car comes to complete halt from an initial speed of 50 kmph in distance of 100 m. With same constant Retardation what would be stopping distance from an initial speed of 70 kmph.?

⇒

<u>Given :-</u> $u_1 = 50$ kmph $= 13.89$ m/sec.

$S_1 = 100$ m

$V_1 = 0$

when initial speed is 50 kmph,

we know that, $\therefore V_1^2 = u_1^2 + 2aS_1$

$\therefore 0 = 13.89^2 + 2a \times 100$

$\therefore 0 = 192.9 + 200a$

$$\therefore a = \frac{-192.9}{200}$$

$\therefore \boxed{a = -0.964 \ m/s^2}$ (-ve sign indicates retardation)

when initial speed is 70 kmph

i.e. $u = 70$ kmph $= 70 \times \frac{5}{18} = 19.44$ m/sec.

Then,

$\therefore V_2^2 = u_2^2 + 2aS_2$

$\therefore 0 = 19.44^2 + 2 \times (-0.964) \times S_2$

$\therefore 0 = 378.09 - 1.928 S_2$

$\therefore -1.928 S_2 = -378.09$

$$\therefore S_2 = \frac{378.09}{1.928}$$

$\therefore \boxed{S_2 = 196.10 \ m}$

If the initial speed of the car is 70 kmph, then it will stop after covering 196.1 m distance.

(8) Two trucks A & B travelling in the same direction on same adjecent lanes are stopped at traffic signal. As the signal turns green, Truck A accelerates at a constant rate of $2\ m/sec^2$. Three seconds later, truck B starts and accelerates at $3.6\ m/s^2$. Find ① when & where Truck B will overtake ② speed of each truck at that time.

Truck A ⇒ $u = 0\ m/s$

$a = 2\ m/s^2$

Time req. $= t_A$ (from signal to point p)

Signal —————————— P (overtaking point)

Truck B ⇒ $u = 0$

$a = 3.6\ m/s^2$

t_B = Time req $= (t_A - 3)$ (from signal to point p).

① For Truck A,

distance covered from signal to Point 'p' $= S_A$

$$\therefore\ S_A = ut_A + \frac{1}{2} a t_A^2$$
$$= 0 + \frac{1}{2} \times 2\, t_A^2$$
$$S_A = t_A^2 \quad \text{------ ①}$$

② for Truck B,

Distance covered from signal to point P $= S_B$

$$\therefore\ S_B = ut_B + \frac{1}{2} a t_B^2$$
$$= 0 + \frac{1}{2} \times 3.6 (t_A - 3)^2$$
$$S_B = 1.8 (t_A - 3)^2 \quad \text{------- (II)}$$

As the distance covered by both trucks from signal to Point P is same,

$$\therefore\ S_A = S_B$$
$$\therefore\ t_A^2 = 1.8\ (t_A - 3)^2$$
$$\therefore\ t_A^2 = 1.8\ (t_A^2 - 6t_A + 9)$$
$$\therefore\ t_A^2 - 1.8 t_A^2 + 10.8 t_A - 16.2 = 0$$
$$-0.8 t_A^2 + 10.8 t_A - 16.2 = 0$$

Solving above eqⁿ, $\boxed{t_A = 1.718\ sec\ \&\ t_A = 11.78\ sec}$

As $t_A = 1.718 < 3$ sec., it can't be considered.

$$\therefore\ \boxed{t_A = 11.78\ sec}$$

Truck B will overtake A at $t_A = 11.78$ sec.

Distance covered by truck A during 11.78 sec,

$$S_A = t_A^2 = 11.78^2 = \underline{\underline{138.76 \text{ m}}}.$$

Truck B will overtake truck A after 138.76 m from signal.

3) Speed of Each truck. after $t = 11.78$ sec & at $s = 138.76$ m.

$\therefore V_A = u + at$

$\therefore V_A = u_A + a_A t_A$

$\therefore V_A = 0 + 2 \times 11.78$

$V_A = 23.56$ m/sec.

Similarly,

$\therefore V_B = u_B + a_B t_B$

$\therefore V_B = 0 + 3.6\ (t_A - 3)$

$\therefore \quad = 0 + 3.6\ (11.78 - 3)$

$\therefore V_B = 31.608$ m/sec.

Ans :

Time req. to overtake = 11.78 sec.

Distance covered before overtaking = 138.76 m.

$V_A = 23.56$ m/s } while overtaking

$V_B = 31.608$ m/sec

3) A train which is at rest starts from station with constant acceleration of $5\ m/s^2$, 3 seconds later another train passes the same station on the same line & in same direction with constant velocity. Find the velocity of 2nd train to just avoid collision.

⇒

Let train A = a train starts from the station.
train B = a train passing the same station after 3 seconds.

1) Consider Train A - constant Accl^n (UAM).

$u = 0$ .... starts from rest

$a = 5\ m/s^2$

let t_A = time taken by train to cover S_A distance.

$\therefore \quad S = ut_A + \frac{1}{2} a t_A^2$

$\therefore \quad S_A = (0 \times t_A) + (\frac{1}{2} \times 5 \times t_A^2)$

$\therefore \quad \boxed{S_A = 2.5\, t_A^2} \quad$ - (I)

2) Consider Train B - constant velocity - (UM) -:-

let t_B = time taken by train B to cover S_B distance

$t_B = (t_A - 3)$ sec.

$\therefore$ For uniform motion,

S = velocity × time

$S_B = V_B \times (t_A - 3) \quad$ — (II)

Now To just avoid the collision distance travelled by both trains must be same.

$\therefore \quad S_A = S_B$

$\therefore \quad 2.5\, t_A^2 = V_B\, (t_A - 3)$

$\therefore \quad V_B = \dfrac{2.5\, t_A^2}{(t_A - 3)} \quad$ — (III)

But,

As velocity of train B is constant,

i.e. V_B = constant,

$\dfrac{dV_B}{dt} = a_B = 0$

Thus, $\quad \dfrac{dV_B}{dt} = \dfrac{d}{dt}\left(\dfrac{2.5\, t_A^2}{t_A - 3}\right)$

Using quotient Rule to take derivative.

$$\therefore \frac{dV_B}{dt} = a_B = \frac{(t_A-3)(5t_A) - (2.5t_A^2)(1)}{(t_A-3)^2}$$

$$\therefore \quad 0 = \frac{5t_A^2 - 15t_A - 2.5t_A^2}{(t_A-3)^2}$$

$$\therefore \quad 0 = 5t_A^2 - 15t_A - 2.5t_A^2$$

$$\therefore \quad 2.5t_A^2 - 15t_A = 0$$

$$\therefore \quad 2.5t_A^2 = 15t_A$$

$$\therefore \quad 2.5t_A = 15$$

$$\therefore t_A = \frac{15}{2.5}$$

$$\therefore \boxed{t_A = 6 \text{ sec.}}$$

$\therefore$ From eqn (I)

$$S = 2.5t_A^2$$

$$= 2.5 \times 6 \times 6$$

$$\underline{\underline{S_A = 90 \text{ m}}}$$

from eqn (III)

$$V_B = \frac{2.5t_A^2}{(t_A-3)}$$

$$= \frac{2.5 \times 6^2}{(6-3)}$$

$$\boxed{V_B = 30 \text{ m/sec}}$$

from eqn (II) $S_B = V_B(t_A - 3)$

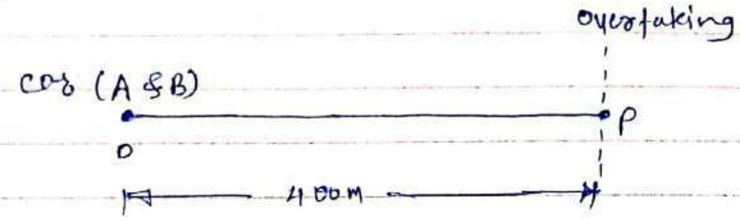
$$= 30(6-3)$$

$$\underline{\underline{S_B = 90 \text{ m}}}$$

Ans:-

Velocity of 2nd train to just avoid collisson should be $\boxed{V_B = 30 \text{ m/sec.}}$

(11) A car A starts from rest & accelerates uniformly on the straight road. Another car B starts from the same point after 6 sec; with zero initial velocity & accelerates at 5 m/s^2 uniformly. If car B overtakes car A at 400 m from starting point; determine accl^n of car A & velocity of each car while overtaking.



(1) for car A - (UAM) from O to P.

$u = 0$ $\quad\quad S = 400\ m$

$a_A = ?$ $\quad\quad$ Time of journey from O to P = t

$\therefore$

$$\therefore \quad S = ut + \frac{1}{2} a_A t^2$$

$$\therefore \quad 400 = 0 + 0.5\, a_A t^2$$

$$\therefore \quad a_A t^2 = 800 \quad\quad - (1)$$

(2) for Car B (Motion from O to P — UAM)

$u = 0$ $\quad\quad S = 400\ m$

$a_B = 5\ m/s^2$ $\quad\quad$ time of journey from O to P = $(t - 6)$

$$\therefore \quad S = ut + \frac{1}{2} a_B t^2$$

$$400 = 0 + 0.5 \times 5 \times (t-6)^2$$

$$\therefore \quad \frac{400}{2.5} = (t-6)^2$$

$$\therefore \quad (t-6)^2 = 160$$

$$\therefore \quad t - 6 = 12.649$$

$$\therefore \quad \boxed{t = 18.649\ sec.} \quad - \text{ put in eq}^n\ (1)$$

$$\therefore \quad a_A = \frac{800}{t^2} = \frac{800}{18.649^2} = 2.3\ m/s^2 \quad \therefore \boxed{a_A = 2.3\ m/sec^2}$$

$$\therefore V_{A(at\ P)} = u + at$$

$$= 0 + (2.3 \times 18.649) \quad \therefore \boxed{V_A = 42.9\ m/sec}$$

$$\therefore V_{B(at\ P)} = u + at$$

$$= 0 + 5 \times (18.649 - 6) \quad \therefore \boxed{V_B = 63.245\ m/sec.}$$

(10) Two cars A & B travelling at constant speed of 160 Kmph. Car A leads car B by 40 m. At $t = 0$, both car accelerate at constant rates, At $t = 8$ sec, car B passes car A & velocity of car A is 220 kmph. find accelerations of A & B.

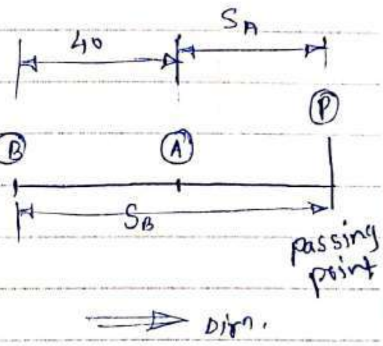
⟹

① Car A :-

$$u_A = 160 \text{ kmph} = 160 \times \frac{5}{18} = 44.45 \text{ m/sec.}$$

$$V_A = 220 \text{ Kmph} = 220 \times \frac{5}{18} = 61.11 \text{ m/sec.}$$

$$t = 8 \text{ sec.}$$



we know that,

$$\therefore \quad V = u + at$$

$$\therefore \quad V_A = u_A + a_A t$$

$$\therefore \quad 61.11 = 44.45 + a_A \times 8$$

$$\therefore \quad a_A = \frac{(61.11 - 44.45)}{8}$$

$$\therefore \boxed{a_A = 2.08 \text{ m/sec}^2}$$

As

$$S = ut + \frac{1}{2}at^2$$

$$S_A = u_A t + \frac{1}{2} a_A t^2$$

$$= 44.45 \times 8 + \frac{1}{2} \times 2.08 \times 8^2$$

$$S_A = 422.16 \text{ m.}$$

At $t = 8$ sec., car B passes car A. Thus car B has to travel the distance ($S_B = 40 + S_A$) as initially car A was leading by 40 m distance. Thus in the same time $t = 8$ sec, car B travels ($40 + S_A$) distance.

$\therefore$

$$S_B = 40 + S_A = 40 + 422.16 = \underline{462.16 \text{ m.}}$$

we know that,

$$S_B = u_B t + \frac{1}{2} a_B t^2$$

but initially Both cars are having same velocity.

$$\therefore \quad u_A = u_B = 44.45 \text{ m/sec.}$$

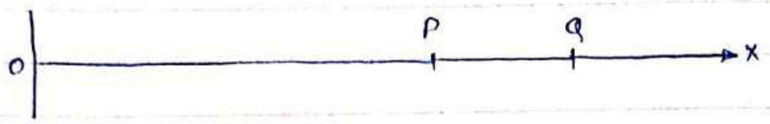
$$\therefore \quad 462.16 = 44.45 \times 8 + \frac{1}{2} \times a_B \times 8^2$$

$$\therefore \quad 462.16 = 355.6 + 32 a_B$$

$$\therefore \quad a_B = \frac{(462.16 - 355.6)}{32} \quad \therefore \boxed{a_B = 3.33 \text{ m/s}^2}$$

Displacement in n^{th} second :-

(with uniform or constant acceleration).



Consider the motion of particle moving along straight line with uniform / constant acceleration.

let the particle starts from O & moves along Ox.

let u = initial velocity of Particle
v = final velocity —"—
a = constant positive acceleration.

Let the Particle moves from O to Q in 'n' seconds &
– the particle moves from O to P in '(n-1)' seconds.

$S_n = x_n$ = Displacement in n seconds.
$S_{n-1} = x_{n-1}$ = Displacement in (n-1) seconds.
$\therefore$ $S = x = S_n - S_{n-1} = x_n - x_{n-1}$ = Displacement in n^{th} second.
n = no. of seconds.

We have eqn of motion for displacement,

$$S = ut + \frac{1}{2}at^2$$

for $t = n$,

$$S_n = u(n) + \frac{1}{2}an^2$$

for $t = n-1$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$\therefore$ Displacement by Particle in n^{th} second is given by,

$$S = S_n - S_{n-1} \quad \text{OR}$$
$$x = x_n - x_{n-1}$$

$$\therefore S = \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$\therefore S = un + \frac{1}{2}an^2 - \left[un - u + \frac{1}{2}a(n^2 - 2n + 1)\right]$$

$$\therefore S = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 + 1 - 2n)$$

$$\therefore S = \cancel{un} + \frac{1}{2}an^2 - \cancel{un} + u - \frac{1}{2}an^2 - \frac{1}{2}a + an$$

$$\therefore S = \cancel{\frac{1}{2}an^2} + u - \cancel{\frac{1}{2}an^2} - \frac{1}{2}a + an$$

$$\therefore S = u - \frac{1}{2}a + an$$

$$\therefore S = u + a\left(-\frac{1}{2} + n\right)$$

$$= u + a\left(n - \frac{1}{2}\right)$$

$$\therefore \boxed{S = u + \frac{a}{2}(2n-1)}$$ OR

$$\boxed{x = u + \frac{a}{2}(2n-1)}$$

* **Distance covered / Travelled :**

① For uni-directional motion along straight line Distance covered in t sec. is given by,

$$d = |x_t - x_0| = |S_t - S_0|.$$

② for un-directional motion, i.e. when particle reverses the direction in betn 0 & t sec;

Assume that particle velocity is zero at t_1, t_2, t_3 (where t_1, t_2, t_3 lie betn 0 & t). Then,

$$d = |x_{t_1} - x_0| + |x_{t_2} - x_{t_1}| + |x_t - x_{t_2}| \quad \text{OR}$$

$$d = |S_{t_1} - S_0| + |S_{t_2} - S_{t_1}| + |S_t - S_{t_2}|$$

Examples on Distance travelled in n^{th} second. (UAM).

1) A train starts from rest & moves with uniform accln along the straight line. It covers 120m distance in 8^{th} second. Find uniform acceln of train.

⇒

for Train, $u = 0$, $S^{8th} = 120m$.

Using distance travelled in n^{th} second,

$$S^{8th} = u + \frac{a}{2}(2n-1)$$

$$S^{8th} = u + \frac{a}{2}(2n-1) \quad \cdots \text{ Here } n = 8 \text{ sec.}$$

$$120 = 0 + \frac{a}{2}(2 \times 8 - 1)$$

$$120 = 7.5a$$

$$\boxed{a = 16 \ m/sec^2}$$

2) A Particle covers 90m in 5^{th} sec; & 140 m in 9^{th} sec. of its journey with uniform accln. Find distance travelled in 15^{th} sec.

⇒

When $n = 5$, $S = 90m$

$n = 9$ $S = 140m$

∴ Using 1^{st} condn

$$S^{nth} = u + \frac{a}{2}(2n-1)$$

$$090 = u + \frac{a}{2}(2 \times 5 - 1)$$

$$\therefore 4.5a + u = 090 \quad \text{----- (I)}$$

Using 2^{nd} condn,

$$S^{nth} = u + \frac{a}{2}(2n-1)$$

$$\therefore 140 = u + \frac{a}{2}(2 \times 9 - 1)$$

$$\therefore 140 = u + 8.5a$$

$$\therefore 8.5a + u = 140 \quad \text{—— (II)}$$

Solving (I) & (II), we get

$$\boxed{a = 12.5 m/s^2} \quad \& \quad \boxed{u \ 33.75 \ m/sec}$$

$$S^{15th} = 33.75 + \frac{12.5}{2}(2 \times 15 - 1) = \boxed{215 \ m}$$

* Motion under the Gravity *
(Linear motion with uniform gravitational Accl^n)

In this type of motion, the vertical motion of the particle under the influence of constant gravitational acceleration is considered.

$g = 9.81 \ m/s^2$ - gravitational acceleration.

The eq^{n}s of motions for motion under the gravity are as follows:-

A) when particle motion is towards earth surface
(Downward motion of particle)

$$v = u + gt$$
$$v^2 = u^2 + 2gs$$
$$s = ut + \frac{1}{2} g t^2$$

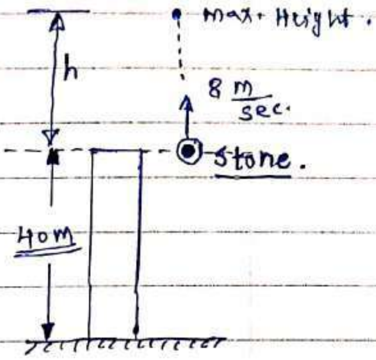
B] when particle motion is away from earth surface against the gravity force
(upward motion)

$$v = u - gt$$
$$v^2 = u^2 - 2gs$$
$$s = ut - \frac{1}{2} g t^2$$

Examples on Motion with Uniform ACCⁿ (UAM).
Motion Under the Gravity
(g = constant).

Q.① From the top of the tower which is 40 m high, a stone is thrown vertically upwards with a velocity of 8 m/sec. How long does the stone take to reach the ground? Also find out the velocity with which it strikes the ground.

⇒



$u = 8$ m/sec.

Tower height = 40m.

I] Consider upward motion of stone.

when stone is thrown upward,

let h = max. height reached by stone.

∴ we have $v = u + gt$

As stone will stop at max height, thus $v_1 = 0$

Also $g = -9.81$ m/s² for upward motion.

Thus, eqⁿ becomes,

$$V_1 = u_1 - gt_1$$

$$0 = 8 - 9.81 \times t_1$$

$$\therefore t_1 = \frac{8}{9.81}$$

$\boxed{t_1 = 0.81 \text{ sec}}$ --- Time req. to reach max. height.

Now,

let h = max. height reached by stone during upward motion.

∴ Thus we have, $V^2 = u^2 + 2as_1$.

Here for upward motion, $g = -9.81$ m/s²

∴

$$V_1^2 = u_1^2 - 2gh.$$

$$0 = 8^2 - 2 \times 9.81 \times h$$

$$0 = 64 - 19.62 \times h$$

$$\therefore 19.62h = 64$$

$$\therefore h = \frac{64}{19.62}$$

$\boxed{h = 3.26 \text{ m}}$ – Above the tower.

2) Now consider downward motion of the stone from the max. height (h).
Thus in this case, stone will cover $(h + 40)$ meter distance while coming down.

$\therefore$ $\therefore$ $S_2 = h + 40 = 40 + 3.26 = 43.26$ m.

while starting downward motion,
initial velocity $= u_2 = 0$
& final velocity $= V_2 = ?$

we have, $V^2 = u^2 + 2as$.
as $g = 9.81\ m/s^2$ for downward motion,

$$\therefore V_2^2 - u_2^2 = 2 g S_2.$$

$$\therefore V_2^2 - 0 = 2 \times 9.81 \times 43.26$$

$$\therefore V_2^2 = 848.76$$

$\therefore$ $\boxed{V_2 = 29.1 \ m/sec}$ with this velocity, stone will strike the ground.

Now, let t_2 = time req. for stone to reach ground level from the max height (h).
we have,

$$\therefore V_2 = u_2 + g \cdot t_2$$

$$\therefore 29.1 = 0 + 9.81 \times t_2$$

$$\therefore t_2 = \frac{29.1}{9.81}$$

$\therefore$ $\boxed{t_2 = 2.97 \ sec}$ - for downward motion.

$\therefore$ Total time required for stone to reach the ground = Time req. for upward motion + Time req. for downward motion

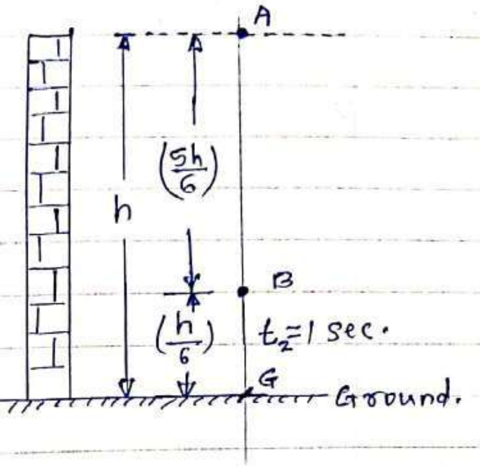
$$t = t_1 + t_2$$

$$t = 0.81 + 2.97$$

$$\boxed{t = 3.78 \ sec.}$$

Q. (2) A ball is released from a building (top of building) of height 'h' meter. It covers a vertical distance $\frac{h}{6}$ during its last second of descend. Find the height of building.

$\Rightarrow$ let h = total height of building



Let the Ball is released from the top of the Bldg. from point 'A'.

B - is the start point of last second of the travel of the ball.

G - is the ground level.

t_1 = time req. from A to B.

$\frac{h}{6}$ = distance covered during last second.

t_2 = time req. From B to G.

$\frac{5h}{6}$ = remaining distance covered.

t = Total time req = $t_1 + t_2 = (t_1 + 1)$ sec

1) Consider motion from A to B :-

$u_1 = 0$, $g = 9.81\ m/s^2$, $s_1 = \frac{5h}{6}$.

from eq^n of motion, $s = ut + \frac{1}{2}at^2$

$s_1 = u_1 t_1 + \frac{1}{2} g t_1^2$

Putting the values,

$\therefore \frac{5h}{6} = 0 + \frac{1}{2} \times 9.81 \times t_1^2$

$\therefore 5h = 29.43\ t_1^2$

$\therefore h = 5.886\ t_1^2$ ----- (I)

2) Consider motion from A to G.

$S = ut + \frac{1}{2} g t^2$

$h = 0 + \frac{1}{2} \times 9.81 \times (t_1 + 1)^2$

$h = 4.905\ (t_1 + 1)^2$ — (II)

Equating (I) & (II) as LHS are same,

$\therefore 5.886\ t_1^2 = 4.905\ (t_1 + 1)^2$

$\therefore = 4.905\ (t_1^2 + 2t_1 + 1)$

$5.886 t_1^2 = 4.905 t_1^2 + 9.81 t_1 + 4.905$

$0.981 t_1^2 - 9.81 t_1 - 4.905 = 0$ ---- solving this.

$\boxed{t_1 = 10.477 \text{ sec.}}$ $\therefore h = 5.886\ t_1^2 = \boxed{646.09\ m}$

$\therefore$ Height of Building = h = 646.09 m

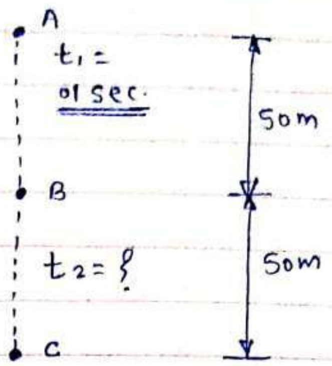
Q 3) A particle freely falls 50 m under gravity in a certain second. Calculate the time required to cover next 50 m.

Consider a partical freely falls from Point A to B (50m) in 1 sec.

$\therefore$ t_1 = time req. from A to B.

t_1 = 1 sec.

let partical covers next 50m i.e. From B to C in time $\underline{t_2}$ sec.



(1) Consider motion from A to B.

$u_1 = 0$, $t_1 = 01$ sec., $S_1 = 50$ m.

Using eqⁿ of motion,

$$\therefore \quad S_1 = u_1 t_1 + \frac{1}{2} g t_1^2$$

$$\therefore \quad 50 = u_1 + \frac{1}{2} \times 9.81 \times 1^2$$

$$\therefore \quad 50 = u_1 + 4.905$$

$$\therefore \quad \boxed{u_1 = 45.095 \text{ m/sec}}$$

V_1 = velocity at point B - is given by,

$$V_1 = u_1 + g t_1$$

$$= 45.095 + (9.81 \times 1)$$

$$\boxed{V_1 = 54.905 \text{ m/sec}}$$

(2)

Consider the motion from B to C :-

at the start point of motion i.e. at B,

$V_1 = 54.905 = u_2$

& $S_2 = 50$ m (from B to C).

Thus.

using eqⁿ of motion ;

$$\therefore \quad S_2 = u_2 t_2 + \frac{1}{2} g t_2^2$$

$$\therefore \quad 50 = 54.905\, t_2 + \frac{1}{2} \times 9.81 \times t_2^2$$

$$\therefore \quad 50 = 54.905\, t_2 + 4.905\, t_2^2$$

$$\therefore \quad 4.905\, t_2^2 + 54.905\, t_2^2 - 50 = 0$$

Solving this eqⁿ

$$\boxed{t_2 = 0.846 \text{ sec.}}$$

Q.4 A stone is thrown vertically downwards from the top of a 49 m high tower. A second later, one Ball is thrown vertically upwards from the ground with a velocity of 12.5 m/sec. At what distance above the ground, will both the stones cross each other?

Let, A - point from which stone is thrown

G - Ground leve (Ball is thrown up)

C - Crossing point of stone & Ball.

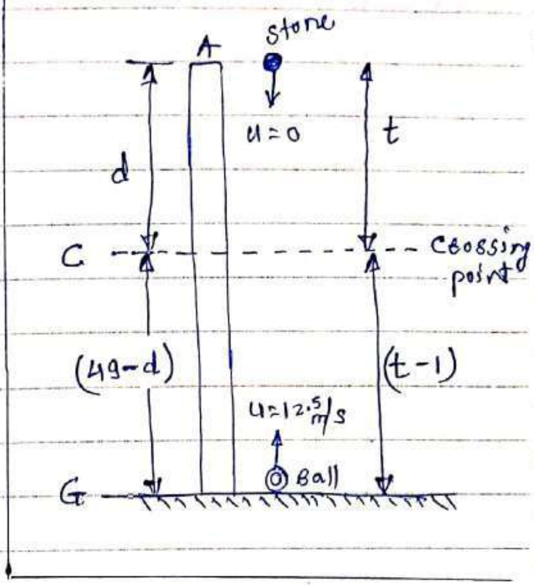
Let

d = distance covered by stone from A to C (crossing point)

(49-d) = distance covered by Ball from Ground to crossing point 'C'.

t = time taken by stone to reach from A to C

(t-1) = time taken by ball to reach from G to C.



1) consider motion from A to C (for stone) $\Rightarrow u = 0$.

$$S = ut + \frac{1}{2} g t^2$$

$$d = 0 \times t + \frac{1}{2} \times 9.81 \, t^2$$

$$\therefore d = 4.905 \, t^2 \quad \text{------} \quad (I)$$

2) Consider motion from G to C

$$S = ut - \frac{1}{2} g t^2$$

$$(49-d) = 12.5 \times (t-1) - \frac{1}{2} \times 9.81 \times (t-1)^2$$

$$= 12.5t - 12.5 - 4.905 \, (t^2 - 2t + 1)$$

$$49 - d = 12.5t - 12.5 - 4.905 \, t^2 + 9.81 t - 4.905$$

$$\therefore d = 4.905 t^2 - 22.31 t + 66.405 \quad -(II)$$

Equating (I) & (II)

$$\therefore \cancel{4.905 t^2} = \cancel{4.905 t^2} - 22.31 t + 66.405$$

$$\therefore 22.31 t = 66.405$$

$$\therefore \boxed{t = 2.97 \text{ sec.}} \approx 3 \text{ sec.}$$

$$\therefore d = 4.905 \times t^2 = 4.905 \times 3^2$$

$$\therefore \boxed{d = 44.14 \text{ m}}$$

$\therefore (49-d) = 4.85$ m. — Both objects cross each other at 4.85 m above ground.

Q5 Water drips from a tap at the rate of five drops per second. Determine the vertical separation betn two consecutive drops after the lower drop has attained a velocity of 7 m/sec.

⟹

Rate of droplets = 5 drops per second.

∴ Time difference betn two consecutive drops $= \frac{1}{5} = 0.2$ sec.

let us assume that, 1st or lower drop attains the velocity of 7 m/sec after 't_1' sec.

Then 2nd drop will take $(t_1 - 0.2)$ sec. $= t_2$

(1) consider first drop & its motion.

$$u_1 = 0 \quad , \quad v_1 = 7 \text{ m/sec}$$

$$\therefore \quad v_1 = u_1 + g t_1$$

$$7 = 0 + 9.81 \times t_1$$

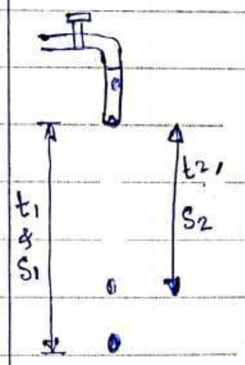
$$\therefore \boxed{t_1 = 0.714 \text{ sec.}}$$

During this time, distance travelled by 1st droplet will be,

$$S_1 = u_1 t_1 + \frac{1}{2} g t_1^2$$

$$= 0 + \frac{1}{2} \times 9.81 \times 0.714^2$$

$$\boxed{S_1 = 02.497 \text{ m.}}$$



(2) Consider 2nd drop & its motion.

∴ time for 2nd drop $= t_1 - 0.2$

$$= 0.714 - 0.2$$

$$t_2 = 0.514 \text{ sec.}$$

∴ distance travelled by 2nd drop in 0.514 sec.

$$S_2 = u_2 t_2 + \frac{1}{2} g t_2^2$$

$$= 0 + \frac{1}{2} \times 9.81 \times 0.514^2$$

$$\boxed{S_2 = 1.296 \text{ m}}.$$

The vertical separation betn two drops

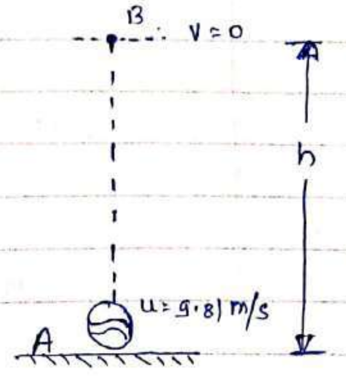
$$= S_1 - S_2$$

$$= 2.497 - 1.296$$

$$= 1.201 \text{ m.}$$

Q.6 A ball is projected vertically upward with a velocity of 9.81 m/s. determine maximum height travelled by the ball, velocity at which it strikes the ground & Total time of journey.

⇒



A) consider the motion of Ball from A to B :-

$u_1 = 9.8$ m/s

$V_1 = 0$ m/s

$S_1 = h$ = max. height travelled by ball.

t_1 = Time taken by ball for motion from A to B.

Using eqⁿ of motion $V^2 = u^2 + 2as$

$\therefore V_1^2 = u_1^2 - 2gS_1$

$\therefore 0 = 9.81^2 - 2 \times 9.81 \times h$

$\therefore h = \frac{9.81^2}{19.62}$

$\therefore \boxed{h = 4.905 \text{ m}}$ - max. height reached.

Now using

$S_1 = h = u_1 t_1 - \frac{1}{2} g \cdot t_1^2$

$\therefore 4.905 = 9.81 t_1 - 0.5 \times 9.81 \times t_1^2$

$\therefore 4.905 t_1^2 - 9.81 t_1 + 4.905 = 0$

Solving

$\boxed{t_1 = 01 \text{ sec.}}$ — for upward motion of Ball.

② Consider downward motion of ball,

$u_2 = 0$, $V_2 = ?$, $S_2 = h = 4.905$ m

$\therefore$ Using $V^2 - u^2 = 2as$

$V_2^2 - u_2^2 = 2gS_2$

$V_2^2 - 0 = 2 \times 9.81 \times 4.905$

$\therefore \boxed{V_2 = 9.81 \text{ m/s}}$ - with this velocity, Ball will strike the ground.

Now using

$\therefore V_2 = u_2 + g t_2$

$9.81 = 0 + 9.81 t_2$

$\therefore \boxed{t_2 = 01 \text{ sec.}}$

Total time of Journey = $t_1 + t_2 = 2$ sec.

* Rectilinear Motion with Variable Acceleration :-

We know, velocity of a body is

$$v = \frac{ds}{dt} \qquad \text{OR} \qquad v = \frac{dx}{dt}$$

&

Acceleration,

$$a = \frac{dv}{dt} \qquad\qquad a = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left(\frac{ds}{dt}\right) \qquad\qquad = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$a = \frac{d^2s}{dt^2} = v.\frac{dv}{ds} \qquad\qquad a = \frac{d^2x}{dt^2} = v.\frac{dv}{dx}.$$

- A body or particle sometimes moves along the straight line with variable acceleration.
- This variable acceleration may be the function of time or position or velocity.

① When acceleration is function of time (t) $[a = f(t)]$

$$a = \frac{dv}{dt} = f(t)$$

$$\therefore dv = f(t)\,dt$$

Integrating both sides, we get

$$\int dv = \int f(t)\,dt$$

This will give us eqn for velocity as a function of time

$$v = f(t)$$

velocity,

$$v = \frac{ds}{dt} = f(t).$$

$$\therefore ds = f(t)\,dt.$$

integrating above eqn we get s (displacement) in terms of 't'.

While solving the problems on variable acceleration, following cases will arise :-

① Given eqⁿ of motion is in terms of displacement (s) & time (t)

$$s = f(t) \quad \underline{\underline{or}} \quad x = f(t)$$

differentiating both sides will give, (w.r.t. time t)

$$v = \frac{ds}{dt} = f(t) \quad \underline{\underline{or}} \quad v = \frac{dx}{dt} = f(t)$$

Again differentiating above eqⁿ, w.r.t. (t),

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \cdot \frac{dv}{ds} = f(t) \quad \underline{\underline{OR}} \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{v \cdot dv}{dx} = f(t)$$

② Given eqⁿ is interms of acceleration (a) & time (t).

$$a = f(t)$$

Integrating once will give us velocity. &
Integrating again will give us the displacement.

③ Given eqⁿ is in terms of Accelⁿ (a) & displacement (x or s).

$$a = f(s) \quad OR \quad a = f(x).$$

Integrating once will give us velocity eqⁿ
Integrating twice will give the eqⁿ of motion for displacement

④ Given eqⁿ is interms of velocity (v) & time (t).

$$v = f(t).$$

- Integrate above eqn to get displacement.
- Differentiate above eqn to get acceleration.

(* In all above cases, use given conditions to find the constants of integration *.)

Examples Based on Variable acceleration:-
(eqn in terms of s & t OR x & t - given)

1) The position of a particle which moves along a straight line is defined by the relation $s = t^3 - 6t^2 - 15t + 40$, where s is in meters and t in sec;

Determine :- (a) time at which velocity will be zero.
(b) Position & distance travelled by particle at that time. (c) Accleration at that time.
(d) Distance travelled by particle from $t = 4$ sec to $t = 6$ sec.

$\Rightarrow$ $s = t^3 - 6t^2 - 15t + 40$

differentiating w.r.t. 'time 't'.

$$\frac{ds}{dt} = 3t^2 - 12t - 15$$

$$\therefore v = \frac{ds}{dt} = 3t^2 - 12t - 15$$

Again differntiating w.r.t. time 't'.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 12$$

(a) time at which velocity will be zero.

for, $v = 0$

$\therefore \quad v = 3t^2 - 12t - 15$

$\therefore \quad 0 = 3t^2 - 12t - 15$

solving the eqn

$\boxed{t = 5 \text{ sec.}}$

(b) $t = 5$ sec. at this time,

displacement will be,

$\therefore \quad S_5 = t^3 - 6t^2 - 15t + 40$

$\therefore S_5 = (5^3) - (6 \times 5^2) - (15 \times 5) + 40$

$\therefore S_5 = -60$ m.

at $t = 0$ sec; displacement will be,

$S_0 = t^3 - 6t^2 - 15t + 40$

$\therefore S_0 = 40$ m.

2) The motion of particle is defined by $x = t^3 - 6t^2 - 36t - 40$, in meters. Determine ① when the velocity is zero.
② Velocity, Acceleration & total distance travelled when $x = 0$.

⇒ Given,

$$x = t^3 - 6t^2 - 36t - 40$$

differentiating w.r.t. t, we get,

$$\therefore v = \frac{dx}{dt} = 3t^2 - 12t - 36$$

Again differentiating w.r.t. 't'

$$\therefore a = \frac{dv}{dt} = \frac{d^2x}{d^2t} = 6t - 12$$

① when the velocity is zero.

for, $v = 0$;

$$v = 3t^2 - 12t - 36$$

$$0 = 3t^2 - 12t - 36$$

Solving above eqn, we get

$$\boxed{t = 6 \text{ sec.}}$$

② Velocity, Accln & total distance travelled at $x = 0$.

for $x = 0$,

$$x = t^3 - 6t^2 - 36t - 40$$

$$0 = t^3 - 6t^2 - 36t - 40$$

Solving above eqn, we get

$$t = 10 \text{ sec.}$$

∴ Velocity (for $t = 10$)

$$V_{10} = 3t^2 - 12t - 36$$

$$= (3 \times 10^2) - (12 \times 10) - 36$$

$$\boxed{V_{10} = 144 \text{ m/sec.}}$$

∴ Accln (for $t = 10$)

$$a = 6t - 12$$

$$= (6 \times 10) - 12$$

$$\boxed{a = 48 \text{ m/s}^2}$$

Distance travelled $= |x_{10} - x_6| + |x_6 - x_0|$

$\therefore \quad x_{10} = 10^3 - (6 \times 10^2) - (36 \times 10) - 40$

$= 1000 - 600 - 360 - 40.$

$\underline{x_{10} = 0 \text{ m}}$

$\therefore \quad x_6 = 6^3 - (6 \times 6^2) - (36 \times 6) - 40$

$x_6 = \underline{-256 \text{ m}}$

$\therefore \quad x_0 = 0^3 - 6 \times 0^2 - 36 \times 0 - 40$

$x_0 = \underline{-40 \text{ m}}$

$\therefore$ Distance travelled. $= |0 - (-256)| + |-256 - (-40)|$

$= 256 + 216$

$= \underline{472 \text{ m.}}$

distance travelled $= |S_5 - S_0|$
$= |-60 - 40|$
$= 100 m.$

(c) Acceleration at $t = 5$ sec.

$a = 6t - 12$
$= (6 \times 5) - 12$
$\boxed{a = 18 \ m/s^2}$

(d) distance travelled from 4 to 6 sec.
As at $t = 5$ sec, $V = 0$,
Thus,
distance travelled = Distance travelled from 4 to 5 sec. + distance travelled from 5 to 6 sec.

$= |S_5 - S_4| + |S_6 - S_5|$

$\therefore$ at $t = 6$, $S_6 = 6^3 - (6 \times 6^2) - (15 \times 6) + 40$
$= -50$ m
at $t = 4$, $S_4 = 4^3 - (6 \times 4^2) - (15 \times 4) + 40$
$= -52$ m.

distance travelled $= |-60 - (-52)| + |-50 - (-60)|$
$= 8 + 10$
$= 18$ m.

3) A small part in the Mechanism travels on straight line Such that its position is $x = t^4 - 10t^2 + 24$ where x is in meters & t in sec. Determine

(a) when velocity is zero (b) when acceleration is zero

(c) minimum speed reached by partical

(d) distance travelled in 3 sec. (e) expression of x in terms of a.

$\Rightarrow$

$$x = t^4 - 10t^2 + 24$$

∴ differentiating w.r.t. 't'

$$v = \frac{dx}{dt} = 4t^3 - 20t$$

Again differentiating w.r.t. t,

$$a = \frac{dv}{dt} = 12t^2 - 20$$

(a) when velocity is zero. $(v=0)$

$\therefore \quad v = 4t^3 - 20t$

$\therefore \quad 0 = 4t^3 - 20t$

$\therefore \quad 0 = t(4t^2 - 20)$

$\therefore \quad \underline{t = 0}$ and $4t^2 - 20 = 0$ — Solving this eqn.

$\underline{t = 2.23 \text{ sec.}}$

∴ velocity is zero at

$\underline{t = 0 \ \& \ t = 2.23 \text{ sec.}}$

(b) when acceleration is zero. i.e. $(a=0)$

$\therefore \quad a = 12t^2 - 20$

$\therefore \quad 0 = 12t^2 - 20$

Solving this eqn

$t = 1.29$ sec.

∴ At $\underline{t = 1.29 \text{ sec.}}$, acceleration is zero.

(c) minimum speed reached by partical.

∴ Minimum speed will be reached when $\frac{dv}{dt} = 0$

i.e. $a = 0$. & for $a=0$, $t = 1.29$ sec.

Thus minimum speed will be reached at $t = 1.29$ sec.

$\therefore |V_{t=1.29}| = |4t^3 - 20t|$
$= |4 \times 1.29^3 - 20 \times 1.29|$

$$\boxed{V_{1.29} = 17.21 \text{ m/sec.}}$$

(d) distance travelled in 3 sec.

$$d = |x_3 - x_{2.23}| + |x_{2.23} - x_0|$$

$\therefore x_3 = 3^4 - 10 \times 3^2 + 24$
$\therefore x_3 = 15$ m. ....position at $t = 3$ sec.

$\therefore x_{2.23} = 2.23^4 - 10 \times 2.23^2 + 24$
$\therefore x_{2.23} = -0.99$ m ....position at $t = 2.23$ sec.

&

$x_0 = 0^4 - 10 \times 0^2 + 24$
$x_0 = 24$ m .... position at $t = 0$ sec.

$\therefore$ distance travelled $= |15 - (-0.99)| + |(-0.99) - 24|$
$= 15.99 + 24.99$
$= \underline{40.98 \text{ m.}}$

(e) Relation x in terms of a.
$\therefore$ we have, $a = 12t^2 - 20$ $\therefore t^2 = \frac{a+20}{12}$

Substituting this value in the eqn of x, we get,
$\therefore x = t^4 - 10t^2 + 24.$

$\therefore x = \left(\frac{a+20}{12}\right)^2 - 10\left(\frac{a+20}{12}\right) + 24$

$\therefore x = \frac{a^2 + 40a + 400}{144} - \frac{10}{12}(a+20) + 24$

$\therefore x = \frac{a^2 + 40a + 400}{144} - \frac{10(a+20)}{12} + 24$

$\therefore 144x = \frac{\cancel{144}(a^2 + 40a + 400)}{\cancel{144}} - \frac{{}^{12}\cancel{144} \times 10(a+20)}{\cancel{12}} + 24 \times 144$

$\therefore 144x = a^2 + 40a + 400 - 120a - 2400 + 3456.$

$$\therefore \boxed{144x = a^2 - 80a + 1456}$$

Examples on motion with variable Acceleration
(eqⁿ in terms of a & t i.e. $a = f(t)$ is given)

1) A Particle moves along straight line with an acceleration $a = (4t^2 - 2)$, where a is in m/s^2 & t is in sec. When $t = 0$, the particle is at 2m to the left of origin & when $t = 2$ sec, particle is at 20m to the left of origin. Determine the position of particle at $t = 4$ sec.

⟹ Given : $a = 4t^2 - 2$

when $t = 0$, $x = -2m$

$t = 2$, $x = -20$ m

$\therefore \quad a = \frac{dv}{dt} = 4t^2 - 2$

$\therefore dv = (4t^2 - 2)\,dt$

$\therefore \int dv = \int (4t^2 - 2)\,dt$

$\therefore \quad V = \frac{4t^3}{3} - 2t + C_1$

$\therefore \frac{dx}{dt} = \frac{4t^3}{3} - 2t + C_1$

$\therefore dx = \left(\frac{4t^3}{3} - 2t + C_1\right) dt$

$\therefore \int dx = \int \left(\frac{4t^3}{3} - 2t + C_1\right) dt$

$\therefore \quad x = \frac{t^4}{3} - t^2 + C_1 t + C_2$

Now, at $t = 0$, $x = -2$.

$\therefore -2 = 0 - 0 + 0 + C_2$

$\therefore \boxed{C_2 = -2}$

at $t = 2$, $x = -20$

$\therefore -20 = \frac{2^4}{3} - 2^2 + C_1 \times 2 - 2$

$\therefore -20 = \frac{16}{3} - 4 + 2C_1 - 2$

$\therefore \boxed{C_1 = -9.67}$

Thus, $x = \frac{t^4}{3} - t^2 - 9.67t - 2$

$\therefore$ at $t = 4$, $\therefore x = \frac{4^4}{3} - 4^2 - 9.67 \times 4 - 2$

$\therefore x = \frac{256}{3} - 16 - 38.68 - 2$

$\boxed{x = 28.65 \text{ m}}$

2) The acceleration of a point moving along a straight line is given by the equation $a = 12t - 20$. It is known that its displacement $s = -10m$ at time $t = 0$ & $s = +10m$ at time $t = 5$ sec. Derive eqn of motion.

$\Longrightarrow$

$a = 12t - 20.$

Given condition : 1) $s = -10m$ at $t = 0$ sec.

2) $s = 10m$ at $t = 5$ sec

$\therefore$ we know that,

$\therefore \quad a = \frac{dv}{dt} = 12t - 20$

$\therefore dv = (12t - 20)\, dt$

Taking integration

$\therefore \int dv = \int (12t - 20)\, dt$

$\therefore \quad v = 6t^2 - 20t + C_1$

$\therefore \quad \frac{ds}{dt} = 6t^2 - 20t + C_1$

$\therefore \quad ds = (6t^2 - 20t + C_1)\, dt$

Integrating, again,

$\therefore \int ds = \int (6t^2 - 20t + C_1)\, dt$

$\therefore \quad s = 2t^3 - 10t^2 + C_1 t + C_2$

Now, using given conditions, 1) $s = -10$ & $t = 0$

2) $s = 10$ & $t = 5$

$\therefore \quad -10 = 2 \times 0^3 - 10 \times 0^2 + C_1 \times 0 + C_2$

$\therefore \boxed{C_2 = -10}$ &

$\therefore \quad 10 = 2 \times 5^3 - 10 \times 5^2 + C_1 \times 5 - 10$

$\therefore \quad 10 = 250 - 250 + 5C_1 - 10$

$\therefore \quad 10 + 10 = 5C_1$

$\therefore \quad C_1 = \frac{20}{5}$

$\boxed{C_1 = 4}$

Thus, eqn of motion is $\boxed{s = 2t^3 - 10t^2 + 4t - 10}$

Examples Based on Motion with Variable Acceleration
(eqn in terms of a & x is given. i.e. $a = F(x)$)

1) A particle oscillates betn the points $x = 40$ mm & $x = 160$ mm with an acceleration $a = k(100 - x)$ where k is constant. The velocity of particle is 18 mm/sec when $x = 100$ mm & is zero at both $x = 40$ mm & 160 mm. Determine ① value of k ② The velocity when $x = 120$ mm.

⇒ $a = k(100 - x)$ – Given relation.

Given conditions are –
1) $v = 18$ mm/s at $x = 100$ mm
2) $v = 0$ mm/sec at $x = 40$ mm
3) $v = 0$ mm/s at $x = 160$ mm

$\therefore$

$$a = \frac{v \cdot dv}{dx} = k(100 - x)$$

$\therefore\ v \cdot dv = k(100 - x)\,dx$ --- Integrating this

$\therefore\ \int v \cdot dv = \int k(100 - x)\,dx$.

$$\therefore\ \frac{v^2}{2} = k\left(100x - \frac{x^2}{2}\right) + C_1$$

Now using conditions (I) & (II)

$$\therefore\ \frac{18^2}{2} = k\left(100 \times 100 - \frac{100^2}{2}\right) + C_1$$

$$0 = k\left(100 \times 40 - \frac{40^2}{2}\right) + C_1$$

$0 = 3200k + C_1$

$\therefore\ C_1 = -3200k$

$\therefore\ 162 = 5000k + C_1$

$\therefore\ 162 = 5000k - 3200k$

$\therefore\ 162 = 1800k$

$\therefore\ \boxed{k = 0.09}$ & $C_1 = -3200 \times 0.09$

$\therefore\ \boxed{C_1 = -288}$

The eqn of velocity becomes,

$$\therefore\ \frac{v^2}{2} = 0.09\left(100x - \frac{x^2}{2}\right) - 288$$

$$\therefore\ v^2 = 0.18\left(100x - \frac{x^2}{2}\right) - 576.$$

Now for $x = 120$ mm.

$$v^2 = 0.18\left(100 \times 120 - \frac{120 \times 120}{2}\right) - 576.$$

$$\boxed{v = \pm 16.97 \text{ mm/sec.}}$$

2) A particle travels in a straight line with accelerated motion such that $a = -Kx$, where x is the distance from starting point. Find constant K if for $x = 2\,m$, velocity is 4 m/sec. and for $x = 3.5\,m$, the velocity is 10 m/sec. Also find x when the velocity is zero.

⟹ Given data:

$a = -Kx.$

Conditions given:

1) $x = 2\,m$, $v = 4\,m/sec.$
2) $x = 3.5\,m$, $v = 10\,m/sec.$

Find ① K & ② find x for $v = 0$.

$$\therefore \quad a = v \cdot \frac{dv}{dx} = -K \cdot x$$

$$\therefore \quad v \cdot dv = -Kx\,dx.$$

$$\therefore \int v \cdot dv = -K \int x \cdot dx$$

$$\frac{v^2}{2} = -K\frac{x^2}{2} + C_1$$

Using 1st condition, $x = 2$, $v = 4\,m/sec.$

$$\therefore \quad \frac{4^2}{2} = -K \times \frac{2^2}{2} + C_1$$

$$\therefore \quad 8 = -2K + C_1 \quad \text{---- ①}$$

Using 2nd condition, $x = 3.5$, $v = 10$

$$\frac{10^2}{2} = -K \times \frac{3.5^2}{2} + C_1$$

$$50 = -6.125K + C_1 \quad \text{---- (ii)}$$

Solving eqn ① & (ii) we get

$$\boxed{C_1 = -12.36} \quad \& \quad \boxed{K = -10.18}$$

$\therefore$ eqn of velocity is $\frac{V^2}{2} = 10.18\frac{x^2}{2} - 12.36$

$$\therefore \quad V^2 = 10.18x^2 - 24.72$$

for, $V = 0$, $\quad 0 = 10.18x^2 - 24.72$

$\therefore$ Solving above eqn for x.

$$\boxed{x = 1.558\,m.}$$

Example on Motion with variable Acceleration
(eqⁿ is given interms of v & t i.e. v = F(t))

① A particle moves along a straight line with velocity $v = 3t^2 - 6t$ (in m/sec.). If it is initially at origin. Determine the average velocity, average speed & distance travelled during interval $0 \le t \le 3.5$ sec.

⇒ Given, $v = 3t^2 - 6t$

if $t = 0$, then $v = 0$. & Also $x = 0$.

$\therefore \quad v = 3t^2 - 6t$

$\therefore \quad v = \frac{dx}{dt} = 3t^2 - 6t$

$\therefore \quad dx = (3t^2 - 6t)\,dt$.

Integrating above eqⁿ ;

$$\int dx = \int (3t^2 - 6t)\,dt$$

$$x = t^3 - 3t^2 + C_1$$

from the condition, $t = 0$, $x = 0$

$\therefore \quad 0 = 0^3 - 3 \times 0 + C_1$

$\therefore \quad \boxed{C_1 = 0}$

$\therefore \quad x = t^3 - 3t^2$.

① Average velocity, $= \frac{\text{Change in position}}{\text{Time interval}}$

$$V_{avg} = \frac{X_{3.5} - X_0}{3.5 - 0}$$

$\therefore \quad X_{3.5} = 3.5^3 - 3 \times 3.5^2 = 6.125$ m.

$X_0 = 0^3 - 3 \times 0 = 0$ m.

$\therefore \quad V_{avg} = \frac{6.125 - 0}{3.5 - 0} = 1.75$ m/sec.

② Average speed.

To find avg. speed, always check where the velocity is zero. & Then find distance travelled.

$\therefore \quad v = 3t^2 - 6t$

$\therefore \quad 0 = 3t^2 - 6t$

solving the eqn

$t = 0$ & $t = 2$ sec.

distance travelled $= |x_{3.5} - x_2| + |x_2 - x_0|$

$\therefore \quad x_{3.5} = 6.125$ m.

$\therefore \quad x_0 = 0$ m.

$x_2 = 2^3 - 3 \times 2^2$

$= 8 - 12$

$\underline{x_2 = -4 \text{ m}}$

$\therefore$ Thus distance travelled is,

$d = |6.125 - (-4)| + |-4 - 0|$

$= 10.125 + 4$

$= 14.125$ m.

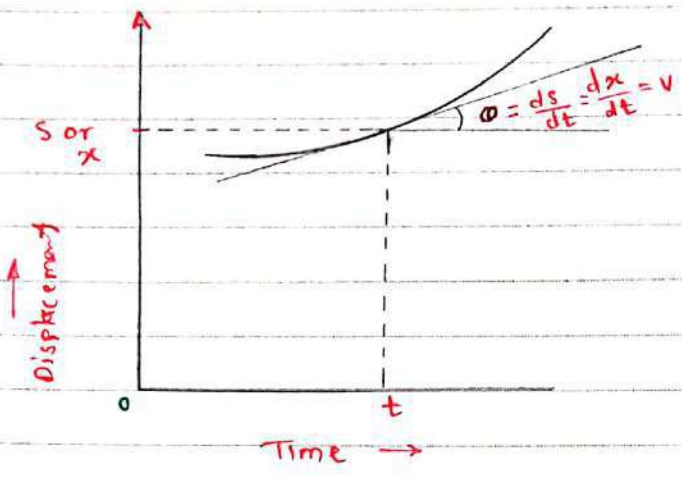
$\therefore$ Avg. speed $= \dfrac{\text{distance travelled}}{3.5} = \dfrac{14.125}{3.5}$

$\therefore$ Avg. speed $= 4.08$ m/sec.

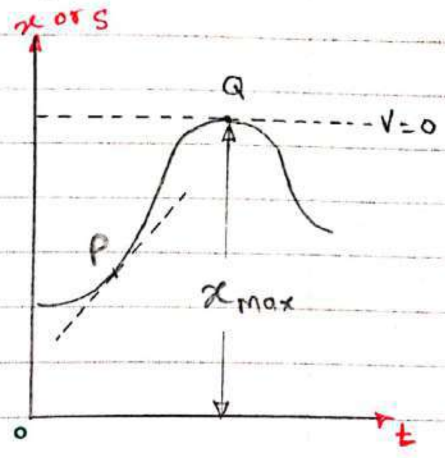
** Motion Diagrams (motion curves):-

It is a graphical representation of displacement, velocity Accl^n with time.

(1) Displacement - time curve: ($x - t$ diagram)



(i)



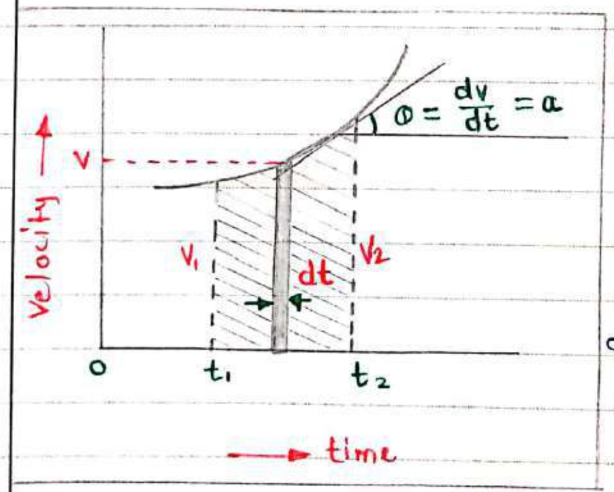
(ii)

- This diagram represents position of particle w.r.t. to time. Here time is taken on x axis & displacement (s or x) is taken on y-axis.
- Slope of $x-t$ diagram at any point represents the velocity at that instant.

$\therefore$ At any instant t (time),

velocity is given by $V = \frac{ds}{dt} = \frac{dx}{dt}$

(2) Velocity - time curve: [($v-t$ diagram)] —



This diagram represents velocity of partical w.r.t. time. Here velocity is taken on y axis & time on-x axis.

a) The slop of the $v-t$ diagram represents accl^n at that instant.

$\therefore a = \frac{dv}{dt}$

b) Area under the curve ($v-t$ curve) represents change in displacement (x or s) bet^n two instants of time.

$\therefore$ let us select elementary strip bet^n t_1 & t_2.

$\therefore$ The area of strip, $dA = V \times dt$

Area bounded betn t_1 & t_2 can be find out by integrating Area of of elementary strip.

$$\therefore \quad dA = v \cdot dt$$

$$\therefore \quad \int dA = \int v \cdot dt$$

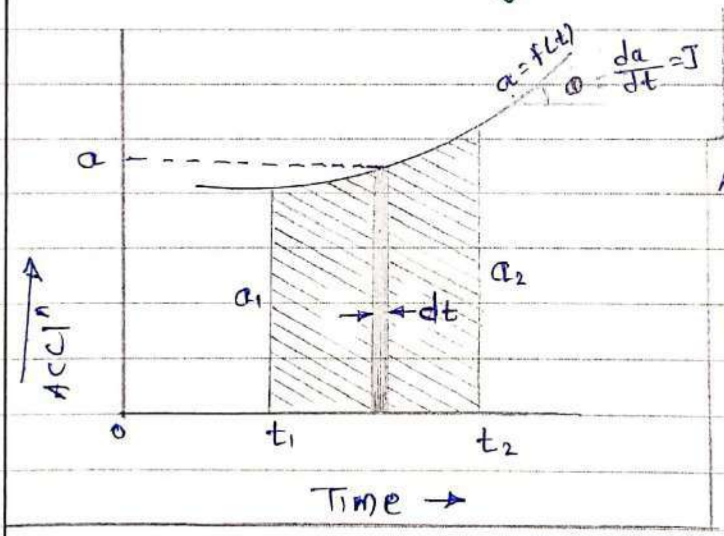
$$\therefore \quad A = \int_{t_1}^{t_2} v \cdot dt$$

But $V = \frac{dx}{dt}$ $\quad \therefore \; v \cdot dt = dx.$

$$\therefore \quad A = \int_{x_1}^{x_2} dx = [x]_{x_1}^{x_2}$$

$$\therefore \quad A = \text{Area bet}^n (t_1 \,\&\, t_2) = x_2 - x_1$$

<3> Accln – time diagram (a – t curve) :-



This represents accln of particle w.r.t. time 't'. Accln is plotted on y axis & time is plotted on x axis.

The slop of the curve represents Jerk $J = \frac{da}{dt}$

The area under a-t diagram betn two instants of time (t_1 & t_2) represents change in velocity.

let us consider an elementary strip.

$\therefore$ area of strip ; $\quad dA = a \cdot dt$

$\therefore$ Total Area ; $\quad A = \int_{t_1}^{t_2} a\,dt \qquad$ but $a = \frac{dv}{dt}$

$\therefore \; a \cdot dt = dv,$

$$\therefore \quad A = \int_{v_1}^{v_2} dv$$

$$\therefore \text{Area} = A = V_2 - V_1 .$$

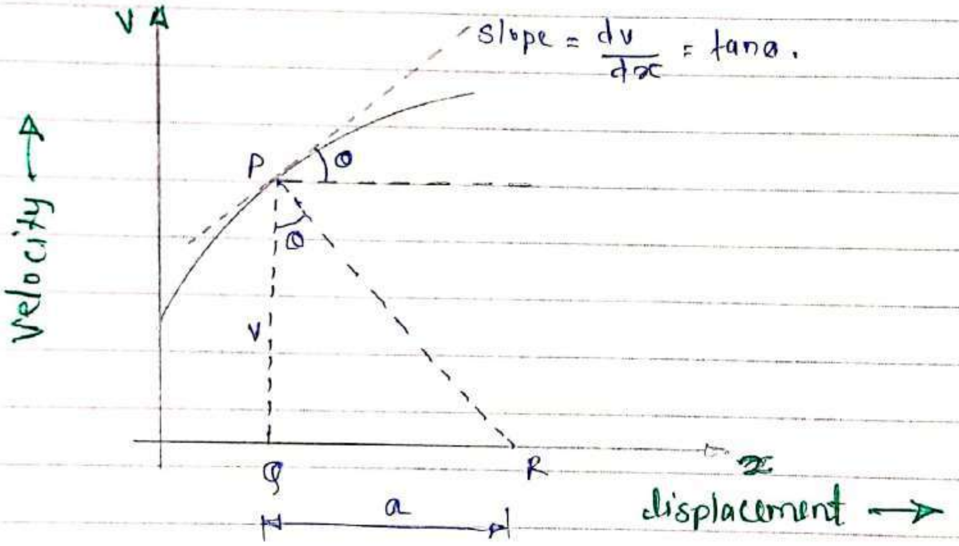
The position co-ordinate (x) is directly find out from the following moment eqn (from a-t diagram).

$$\boxed{x_t = x_0 + V_0 t + M_t}$$

where

x_t = Position of particle at time 't'.
x_0 = initial position of particle
V_0 = initial velocity of particle.
M_t = Moment of area under a-t diagram about the instant t.

4] Velocity - Displacement diagram (v-x)



Here plot or graph of velocity (on y axis) & displacement on (x axis) is drawn.

If a normal is drawn to the tangent on curve, the subnormal on x axis represents the accln.

let PR is the normal drawn at P.

The subnormal $QR = PQ \cdot \tan\theta$

$$= v \cdot \tan\theta$$

$$= v \cdot \frac{dv}{dx}$$

$$= a.$$

$$\therefore \quad a = v\frac{dv}{dx}$$

$\therefore$

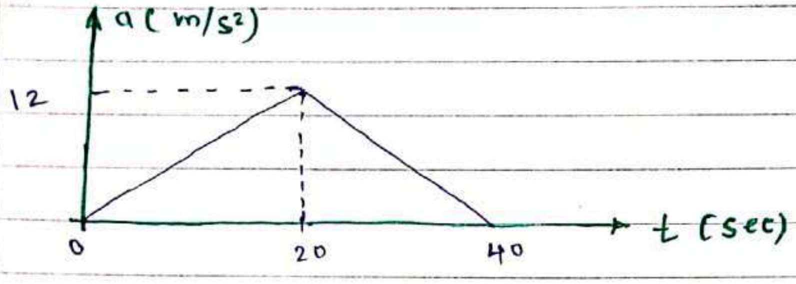
Accln = velocity × slope of v-x diagram.

Numericals on Motion Diagrams.

① The accelaration versus time for a particle moving along x-axis is given in the figure given below. The time interval is 0 to 40 sec. For the same time interval, plot –

1) v – t diagram 2) x – t diagram

3) Also find max speed attained & max. distance covered.



⇒ We know that,

Change in velocity = Area under a – t diagram.

∴ from the give a – t diagram

at $t = 20$ sec.

$$V_{20} - V_0 = \frac{1}{2} \times 20 \times 12$$

$$V_{20} - V_0 = 120 \text{ m/s}$$

But at $t = 0$, $V_0 = 0$

$$\therefore \boxed{V_{20} = 120 \text{ m/sec.}}$$

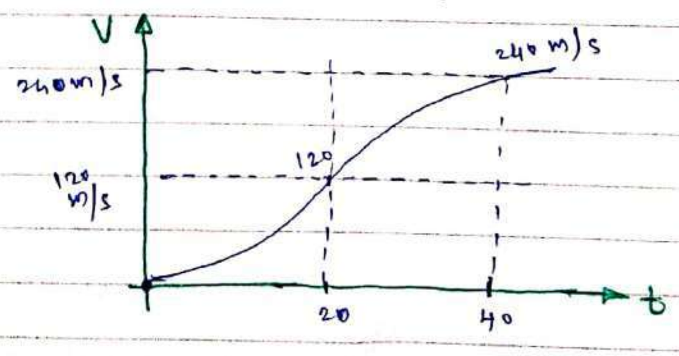
Now at $t = 40$ sec.

$$\therefore V_{40} - V_{20} = \frac{1}{2} \times 20 \times 12 = 120$$

$$\therefore V_{40} = 120 + V_{20}$$

$$\therefore \boxed{V_{40} = 240 \text{ m/sec}}$$

Thus v – t diagram will be as follows



Now from above v-t diagram,
Change in displacement = Area under v-t diagram.

At $t = 20$ sec.

$$X_{20} - X_0 = \frac{1}{3} \times 20 \times 120 = 800 \text{ m.}$$

as $(X_0 = 0$ at $t = 0.)$

$$\therefore \boxed{X_{20} = 800\text{m.}}$$

At $t = 40$ sec.

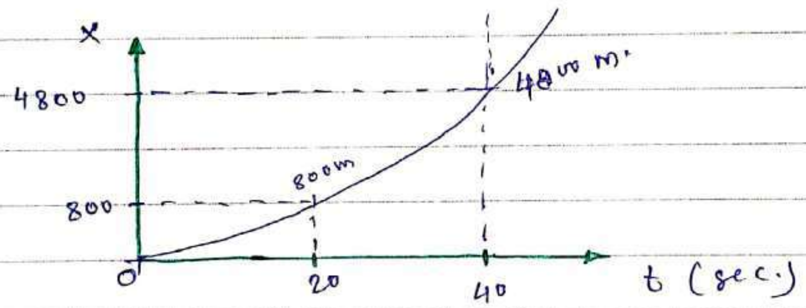
$$X_{40} - X_{20} = (20 \times 120) + \left(\frac{2}{3} \times 20 \times 120\right)$$

$$X_{40} = 2400 + 1600 + X_{20}$$

$$X_{40} = 2400 + 1600 + 800$$

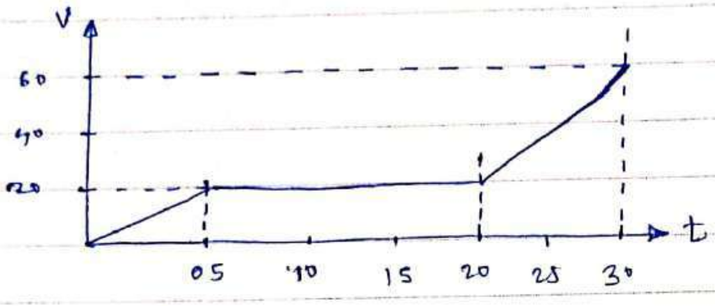
$$\boxed{X_{40} = 4800\text{m.}}$$

$\therefore$ X - t diagram will be as follows.



Max. speed attained = 240m/sec.
max. distance travelled = 4800 m.

(2) The figure below shows v-t graph of jet plane travelling along runway. Construct x-t & a-t graph for the motion of jet plane. The plane starts from rest.



⇒ ① x-t diagram :

∴ Change in displacement = Area under v-t diagram.

∴ At $t = 0$, $x_0 = 0$

At $t = 5$ sec.

$$X_5 - X_0 = \frac{1}{2} \times 5 \times 20 = 50$$

$$\therefore X_5 = 50 - X_0$$

$$X_5 = 50 m$$

At $t = 20$ sec.

$$\therefore X_{20} - X_5 = 15 \times 20 = 300$$

$$\therefore X_{20} = 300 + X_5 = 300 + 50 = 350 \text{ m.}$$

At $t = 30$ sec.

$$\therefore X_{30} - X_{20} = (10 \times 20) + (\tfrac{1}{2} \times 10 \times 40)$$

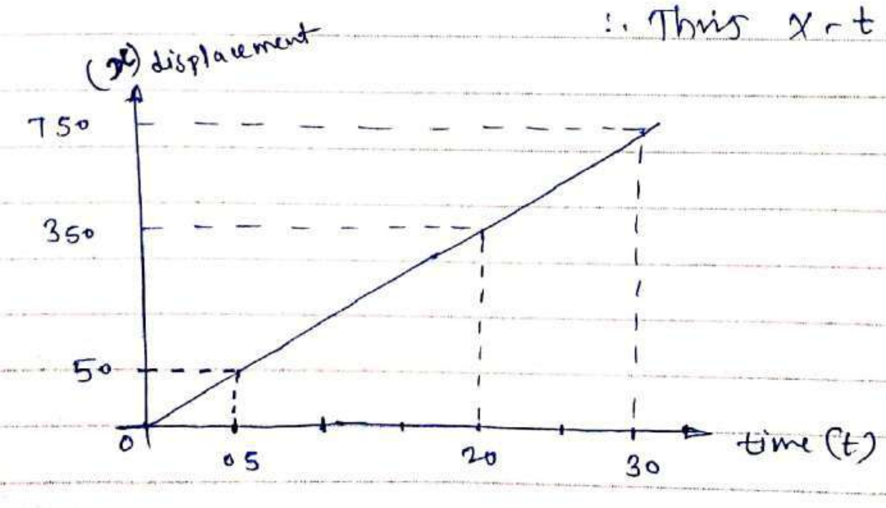
$$\therefore = 200 + 200$$

$$X_{30} = 400 + X_{20}$$

$$X_{30} = 400 + 350$$

$$X_{30} = 750 \text{ m.}$$

∴ This x-t diagram is as below-



Now;
from given v-t diagram,

Slope of v-t diagram = acceleration.

At $t = 0$, slope $= a = \frac{20}{5} = 4 \, m/s^2$

Before $t = 5$, slope $= a = \frac{20}{5} = 4 \, m/s^2$

After $t = 5$ slope $= a = 0$ - as v is constant.

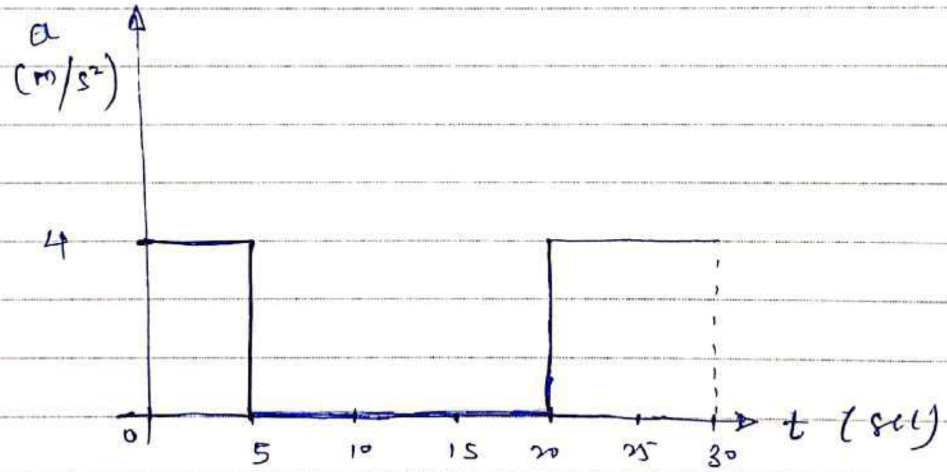
Before $t = 20$, Slope $= a = 0$

After $t = 20$, Slope $a = \frac{40}{10} = 4 \, m/s^2$

At $t = 30$,

slope $= a = \frac{40}{10} = 4 \, m/s^2$

$\therefore$ a-t diagram is drawn below.



Relative Velocity / motion.

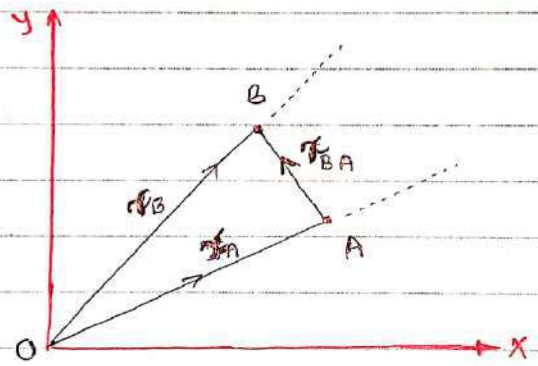
Absolute motion :-
The motion of particle with respect to fixed frame of reference.
Relative motion :-
The motion of particle with respect to set of axes which are moving.

OR

The motion of particle with respect to another particle when both particles are moving.

* Relative motion betⁿ two Particles.

Consider two particles A & B moving on different paths. let A is moving along OA & B is moving along OB.



Now as both A & B are moving, Thus, motion of Partical B w.r.t. A is called as Relative motion B w.r.t A.

$\therefore \quad \bar{r}_A = OA =$ absolute position of A = position vector of A.

$\bar{r}_B = OB =$ absolute position of B = position vector of B.

Then,

$r_{BA} =$ position vector of B w.r.t. A

from triangle law of vectors,

$$\bar{r}_B = \bar{r}_A + \bar{r}_{BA}$$

$$\therefore \quad \bar{r}_{BA} = \bar{r}_B - \bar{r}_A.$$

Differentiating above eqⁿ.

$$V_{BA} = V_B - V_A$$

again differntiating

$$a_{BA} = a_B - a_A .$$

So, $x_{BA} = x_B - x_A$

$V_{BA} = V_B - V_A$

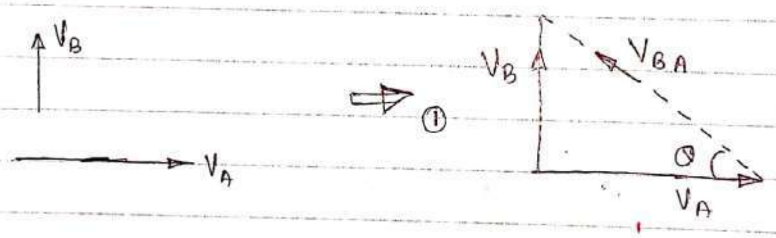
$a_{BA} = a_B - a_A$

Graphical Representation

To find the relative velocity, place the two vectors in such a way that their tails are joined at common Point.

① Two velocities are perpendicular to each other.

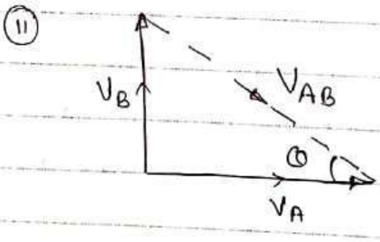
let V_A & V_B are the two velocity vectors, acting perpendicular to each other.



$$V_{BA} = \sqrt{V_A^2 + V_B^2}$$

$$\tan\theta = \frac{V_B}{V_A}$$

(ii)

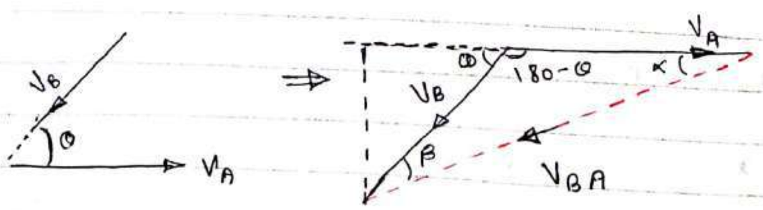


$$V_{AB} = \sqrt{V_A^2 + V_B^2}$$

$$\tan\theta = \frac{V_B}{V_A}$$

② when two velocities are inclined to each other.

let V_A & V_B are the two velocities making angle θ with each other.



in Above triangle by using cosine rule

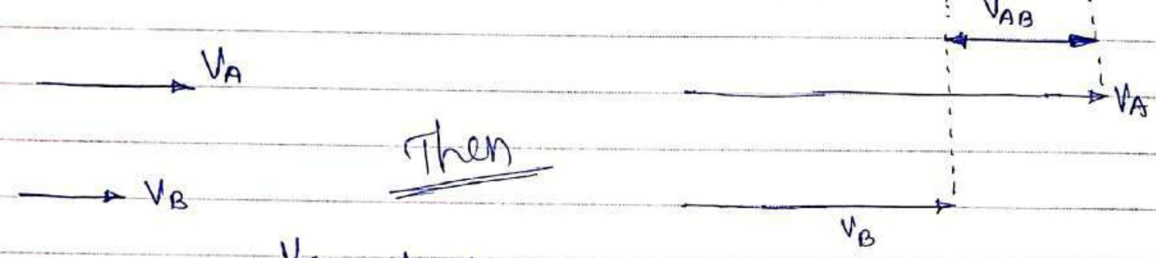
we have, magnitude, $V_{BA}^2 = V_A^2 + V_B^2 - 2V_AV_B\cdot\cos(180-\theta)$

$$\therefore\ V_{BA} = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cdot\cos(180-\theta)}$$

Direction,

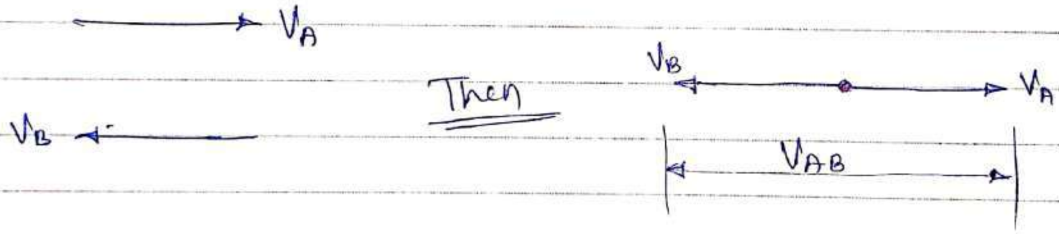
$$\sin\alpha = \frac{V_B\cdot\sin\theta}{V_{BA}}$$

③ when velocities are parallel & unequal.



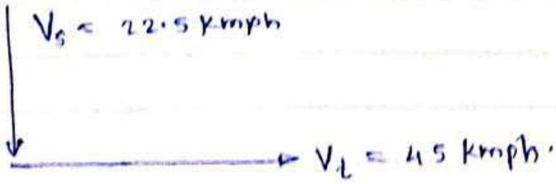
$V_{AB} = V_A - V_B$

④ when velocities are equal & opposite.



$V_{AB} = V_A - V_B$.

A train is moving at 45 kmph is hit by a stone thrown at right angle to it with a velocity of 22.5 kmph. Find the velocity & direction with which the stone appears to hit a person travelling in the train.



$V_t = 45$ kmph $= 45 \times \frac{5}{18} =$.

$\therefore \; V_t = 12.5$ m/sec

$V_s = 22.5$ kmph $= 22 \times \frac{5}{18} = 6.25$ m/sec.

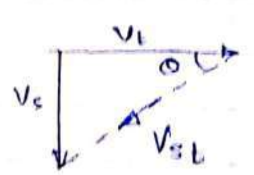
velocity vectors

$$\bar{V_t} = 12.5\,i$$
$$V_s = -6.25\,j$$

$\therefore$ velocity of stone relative to train will be

$$\therefore \bar{V_{st}} = \bar{V_s} - \bar{V_t}$$
$$\bar{V_{st}} = -6.25\,j - 12.5\,i$$
$$\bar{V_{st}} = (-12.5)i + (-6.25)j$$

magnitude of velocity



$$V_{st} = \sqrt{12.5^2 + 6.25^2}$$
$$V_{st} = 13.975 \text{ m/sec.}$$

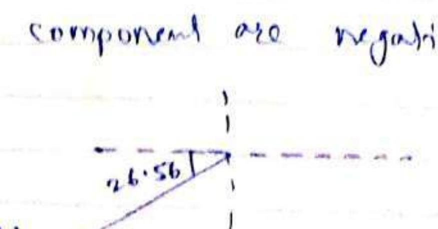
Direction

$$\tan\theta = \frac{j \text{ coefficient}}{i \text{ coefficient}}$$

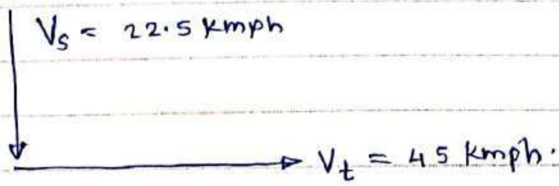
$$\tan\theta = \frac{6.25}{12.5}$$

$$\therefore \theta = 26.56^\circ$$

As both i & j component are negative, so V_{st} is in third quadrant.



A train is moving at 45 kmph is hit by a stone thrown at right angle to it with a velocity of 22.5 kmph. Find the velocity & direction with which the stone appears to hit a person travelling in the train.



$V_t = 45 \text{ kmph} = \frac{45 \times 5}{18} =$.

$\therefore \quad V_t = 12.5 \text{ m/sec}$

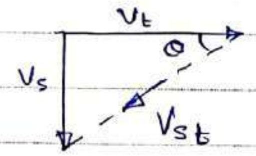
$V_s = 22.5 \text{ kmph} = \frac{22 \times 5}{18} = 6.25 \text{ m/sec.}$

velocity vectors

$$\bar{V_t} = 12.5\, i$$
$$\bar{V_s} = -6.25\, j$$

$\therefore$ velocity of stone relative to train will be

$$\therefore \bar{V}_{st} = \bar{V_s} - \bar{V_t}$$
$$\bar{V}_{st} = -6.25\, j - 12.5\, i$$
$$\bar{V}_{st} = (-12.5)i + (-6.25)j$$



magnitude of velocity

$$V_{st} = \sqrt{12.5^2 + 6.25^2}$$
$$V_{st} = 13.975 \text{ m/sec.}$$

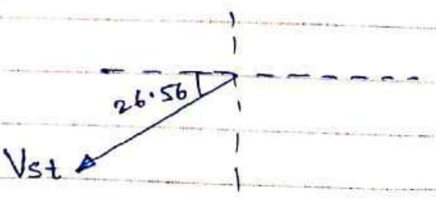
Direction

$$\tan \theta = \frac{j \text{ coefficient}}{i \text{ coefficient}}$$

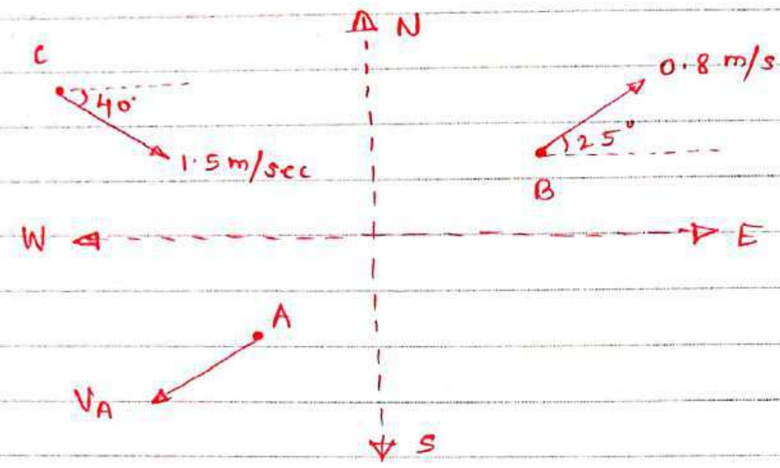
$$\tan\theta = \frac{6.25}{12.5}$$

$$\therefore \theta = 26.56^\circ$$

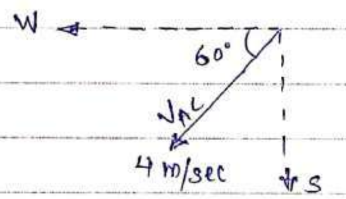
As both i & j component are negative, so V_{st} is in third quadrant.



Following figure shows 3 ships sailing in differnt directions. If the sailer of ship C observes the ship A to sail with a velocity of 4 m/s at 60° in the direction South of west. Find - 1) velocity of ship A 2) velocity of ship B observed by A 3) velocity of ship C observed by B.



Given - V_{AC} = 4m/sec. at 60° in south of west.



∴ Velocity vector = $\bar{V}_{AC}$

$$\therefore \bar{V}_{AC} = (-4\cos 60)\,i - (4\sin 60)\,j$$

$$V_{AC} = -2i - 3.464j \qquad - (1)$$

From the given diagram, $V_c = 1.5$ m/sec.

$$\bar{V}_c = (1.5\cos 40)\,i + [-(1.5\sin 40)\,j]$$

$$\bar{V}_c = 1.149i + (-0.964)j \qquad -(ii)$$

$$\bar{V}_c = 1.149i - 0.964j$$

$V_B = 0.8$ m/sec.

$$\therefore \bar{V}_B = (0.8\cos 25)i + (0.8\sin 25)j$$

$$\bar{V}_B = 0.725i + 0.338j \qquad -(iii)$$

Now, $\bar{V}_{AC} = \bar{V}_A - \bar{V}_C$

from eqn ① & ⑪

$\therefore -2i - 3.464j = \bar{V}_A - [1.149i - 0.964j]$

$\therefore -2i - 3.464j = \bar{V}_A - 1.149i + 0.964j$

$\therefore -2i + 1.149i - 3.464j - 0.964j = \bar{V}_A$

$\therefore \bar{V}_A = -0.851i - 4.428j$

$\therefore$ magnitude of $V_A = \sqrt{0.851^2 + 4.428^2}$

$V_A = 4.509$ m/sec

Direction,

$$\theta_A = \tan^{-1}\left(\frac{(j\text{-coefficient})}{(i\text{-coefficient})}\right)$$

$$= \tan^{-1}\frac{4.428}{0.851}$$

$$\boxed{\therefore \theta_A = 79.12^\circ}$$

velocity of ship B as observed by A :-

$\bar{V}_{BA} = \bar{V}_B - \bar{V}_A$

$= 0.725i + 0.338j - [-0.851i - 4.428j]$

$= 0.725i + 0.851i + 0.338j + 4.428j$

$\bar{V}_{BA} = 1.576i + 4.766j$

Magnitude

$V_{BA} = \sqrt{1.576^2 + 4.766^2} = \underline{5.02 \text{ m/sec.}}$

Direction, $\theta_{BA} = \tan^{-1}\left(\frac{4.766}{1.576}\right) = \underline{71.70^\circ}$.

Velocity of ship C observed by B.

$\bar{V}_{CB} = \bar{V}_C - \bar{V}_B$

$= 1.149i - 0.964j - (0.725i + 0.338j)$

$\bar{V}_{CB} = 0.424i - 1.302j$

Magnitude,

$V_{CB} = \sqrt{0.424^2 + 1.302^2} = \underline{1.369 \text{ m/sec}}$

Direction, $\theta_{CB} = \tan^{-1}\left(\frac{1.302}{0.424}\right)$

$\underline{\theta_{CB} = 71.96^\circ}$

Two trains P & Q are 190 m & 160 m long moving in opposite directions on parallel tracks. The velocity of shorter train is 3 times that of larger train. If train takes 5 sec. to pass each other, find velocity of each train.

$\Rightarrow$ Length of train P $= L_P = 190\text{m}$

— " — Q $= L_Q = 160\text{m}$

$\therefore \quad V_Q = 3V_P$ --- Given.

As trains are moving in opposite directions & parallel to each other.

$\therefore$ Relative speed

$$V_{PQ} = V_{QP} = V_P + V_Q$$
$$= V_P + 3V_P$$
$$= 4V_P$$

$$\text{Time taken to cross each other} = \frac{\text{total length of trains} = (L_P + L_Q)}{\text{Relative speed}}$$

$$5 = \frac{190 + 160}{4V_P}$$

$\therefore \quad V_P = 17.5 \text{ m/s}$ &

$V_Q = 3V_P = 3 \times 17.5 = 52.5 \text{ m/sec.}$

Dependent Motion

When motion of one particle depends upon the motion of other particle or several particles, then that kind of motion is called as Dependent motion.

* String Law :-

When only two particles are connected by a continuous, inextensible string passing over smooth pulleys, then following relation holds good.

$$N_1 S_1 = N_2 S_2$$

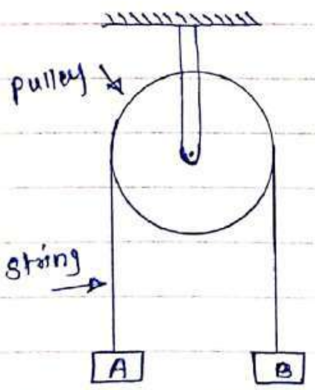
$$N_1 V_1 = N_2 V_2$$

$$N_1 a_1 = N_2 a_2$$

---- where,

N_1 = No. of strings parts connected to particle 1.
N_2 = No. of string parts connected to particle 2.

① Consider the figure shown below.



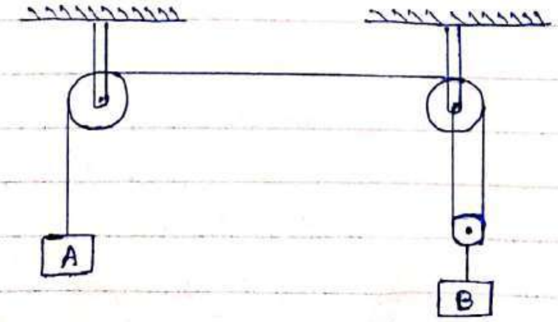
Here, $N_A = 1$ & $N_B = 1$

Also, $S_A = S_B$

$V_A = V_B$

$a_A = a_B$

② consider following Diagram,



Here $N_A = 1$ & $N_B = 2$.

$\therefore N_A S_A = N_B S_B$

$S_A = 2 S_B$

Simillarly,

$V_A = 2 V_B$

$a_A = 2 a_B$

Consistency of string :-

The length of the string during the motion of the particles remains constant as string is inextensible.

* Analysis of dependent motion :-

Following are the steps to be followed while solving the numericals of dependent motion.

Step - 1 :-

Select the datum passing through a fixed point or fixed points or fixed surface.

a) If string parts are horizontal in the given system, then select vertical datum.

b) If string parts are vertical in the given system, then select Horizontal datum.

c) If string parts are both vertical & horizontal in the given ~~string~~ system; then select two datums i.e. vertical & Horizontal.

Step - 2 :-

a) Select proper sign conventions w.r.t. datum.

b) +ve sign is selected in the Direction of string parts.

Step - 3 :-

a) Decide the variables & mark their position co-ordinate (x) w.r.t. datum.

b) Mark 'x_0' bet^n datum & any other fixed point/surface.

Step - 4 :-

a) Measure the length of strings in terms of variables using selected sign convention. (x_A, x_B, x_C -------)

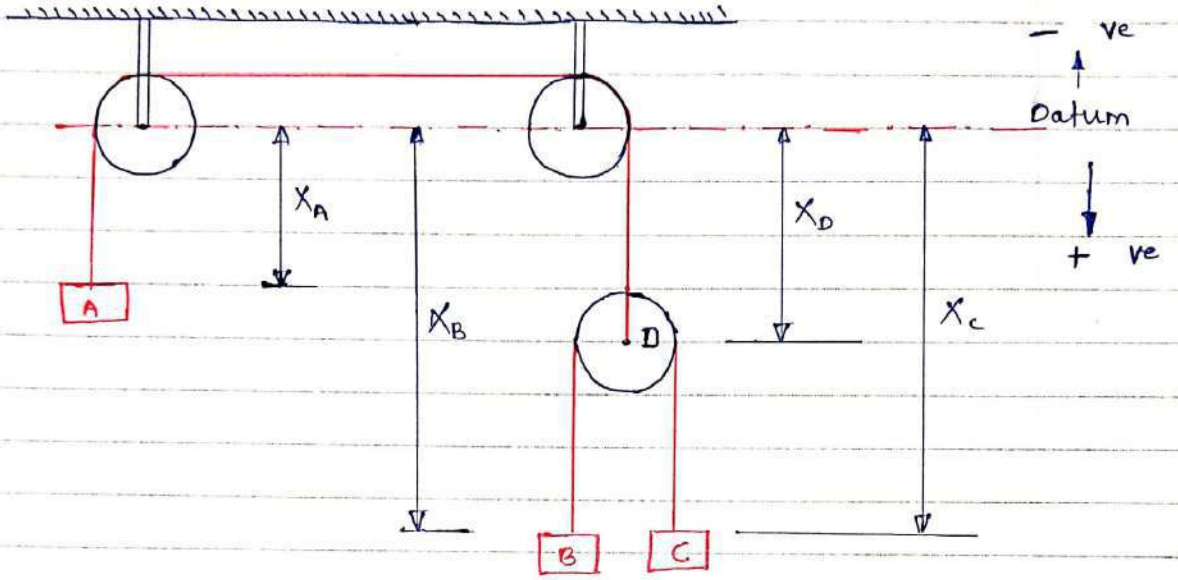
b) Neglect overlaps with pulley & constant length if any.

Step 5 :- Measure the length of each string separately if system consists of more than one string.

Step 6 :- Differentiate once w.r.t. t to get velocity & Differentiate twice w.r.t. t to get acceleration.

Numericals on Dependent Motion

Three blocks move with constant velocities as shown in figure. Find the velocity of each block. The relative velocity of A w.r.t. C is 120 mm/sec. (↑) & Relative velocity of B w.r.t. A is 40 mm/s (↑).



Mark positions of the variables w.r.t. datum as shown above. Let X_A, X_B, X_C, X_D be the distances of the blocks A, B, C, D respectively.

No. of strings in the given system are two.

9 ∴

Let L_1 = length of the string connecting block A & pulley D.

$\therefore L_1 = X_A + X_D$

Differntiate w.r.t. 't' (L_1 = constant)

$$0 = \frac{d}{dt}(X_A) + \frac{d}{dt}(X_D)$$

$$0 = V_A + V_D$$

$$\therefore V_D = -V_A \quad -①$$

Let L_2 = length of the string connecting block B & C

$$\therefore L_2 = (X_B - X_D) + (X_C - X_D)$$

$$L_2 = X_B + X_C - 2X_D$$

Differntiating w.r.t. 't' (L_2 = constant)

$$0 = \frac{d}{dt}(X_B) + \frac{d}{dt}(X_C) - 2 \cdot \frac{d}{dt}(X_D)$$

$\therefore \quad 0 = V_B + V_c - 2V_D$

But $V_D = -V_A$ ------ from ①

$\therefore \quad 0 = V_B + V_c - 2 \times (-V_A)$

$0 = V_B + V_c + 2V_A \quad$ — ⑪

$\therefore V_{AC} = 120$ mm/sec. ↑ --- given.

$\therefore V_{AC} = -120$ mm/sec (upward is Negative as per datum). & sign conventions.

$\therefore V_A - V_c = -120$

$\therefore \quad -V_c = -120 - V_A$

$\therefore \quad V_c = V_A + 120 \quad$ ------ ③

Now $V_{BA} = 40$ mm/sec (↑)

$\therefore V_{BA} = -40$ mm/sec

$\therefore V_B - V_A = -40$

$\therefore V_B = -40 + V_A \quad$ ---- ④

put eqn ③ & ④ in eqn ②

$\therefore (-40 + V_A) + (V_A + 120) + 2V_A = 0$

$4V_A + 120 - 40 = 0$

$\therefore 4V_A = -120 + 40$

$4V_A = -80$

$\therefore V_A = \frac{-80}{4} = -20$ mm/sec.

$\therefore \boxed{V_A = 20 \text{ mm/sec}}$ – (upward)

$\therefore V_B = -40 + V_A \quad \therefore V_B = -40 + (-20) = -60$ m/sec.

$\boxed{V_B = 60 \text{ mm/sec}}$ – (upward)

$\therefore V_c = V_A + 120$

$= -20 + 120$

$\therefore V_c = 100$ mm/sec. (downward)

$\therefore \boxed{V_c = 100 \text{ mm/sec (downward)}}$

from eqⁿ ②, $V_W = -V_E$

$V_W = -5$ m/sec

$\boxed{V_W = 5 \text{ m/sec}}$ upward.

putting above values in eqⁿ ①,

$V_C = -2V_E$

$= -2 \times (+5) = -10$ m/sec.

$\boxed{V_C = 10 \text{ m/sec.}}$ upward.

Now, Relative velocity of cable w.r.t. elevator,

$V_{CE} = V_C - V_E$

$V_{CE} = -10 - (+5) = -15$ m/sec.

$\boxed{V_{CE} = 15 \text{ m/sec.}}$ upward.

Relative velocity of counter weight w.r.t. elevator

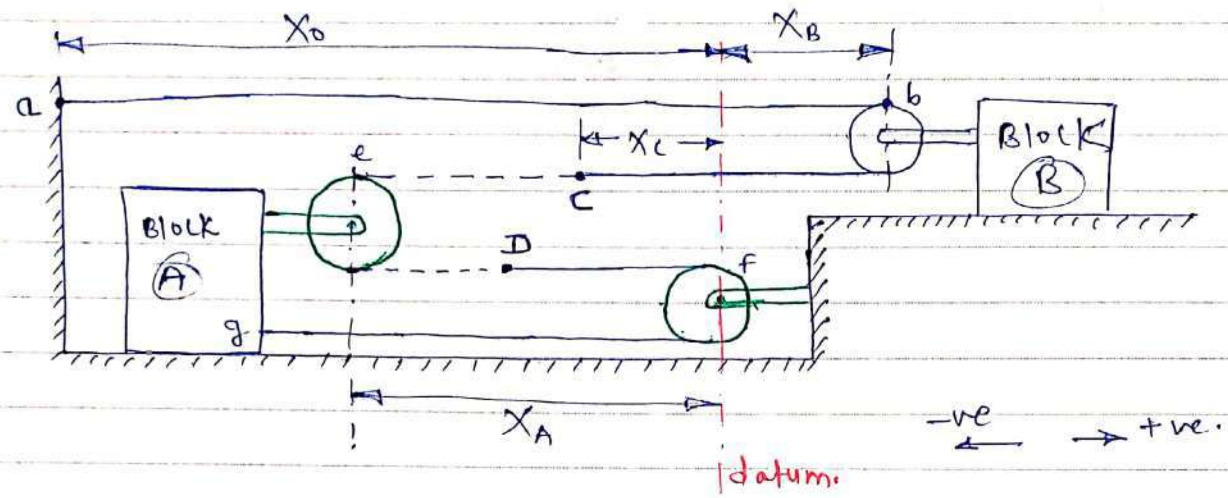
$V_{W/E} = V_W - V_E$

$V_{WE} = -5 - (+5) = -10$ m/sec.

$\boxed{V_{WE} = 10 \text{ m/sec.}}$ upward.

Slider Block B moves to the right with constant velocity of 300 mm/sec. Determine:

1) velocity of the slider block A
2) velocity of portion c of the cable
3) velocity of portion D of cable.
4) Relative velocity of portion c of cable w.r.t. A.



Total length of String connecting slider Block A & B

$$L_1 = 2X_B - 3X_A - X_0$$

neglecting X_0

$$L_1 = 2X_B - 3X_A$$

As L = constant

differentiating w.r.t. t

$$0 = 2V_B - 3V_A$$

$$\therefore 3V_A = 2V_B$$

as $V_B = 0.3$ m/sec ($\rightarrow$) ----- given

$$\therefore V_A = \frac{2 \times 0.3}{3} = 0.2 \text{ m/sec. } (\rightarrow)$$

Now to find velocity of portion C of cable,
Consider the part of the cable starting from any one end upto point C.
length of the part a-b-c will be,

$$L_2 = -X_0 + X_B + X_B - X_C$$

Neglecting X_0,

$\therefore\ L_2 = 2X_B - X_C$

differentiating w.r.t. 't'

$\therefore\ 0 = 2V_B - V_C$

$\therefore\ V_C = 2V_B$

$\therefore\ V_C = 2 \times 0.3$

$\boxed{V_C = 0.6\ m/sec}\ (\rightarrow)$

To find velocity of portion D of the cable,
consider the part of cable starting from any end point upto D.
Thus
for the portion a-b-c-D,

$L_3 = -X_0 + X_B + X_B - X_C - (X_A - X_C) - (X_A - X_D)$

$= 2X_B - \cancel{X_C} - X_A + \cancel{X_C} - X_A + X_D$

$L_3 = 2X_B - 2X_A + X_D$

differentiating w.r.t. 't'

$\therefore\ 0 = 2V_B - 2V_A + V_D.$

$\therefore\ V_D = 2V_A - 2V_B$

$= 2\times 0.2 - 2\times 0.3$

$= 0.4 - 0.6$

$V_D = -0.2\ m/sec.$

$\therefore\ \boxed{V_D = 0.2\ m/sec}\ (\leftarrow)$ toward left

OR

Consider portion gf-D.

$L_3 = -X_A - X_D$

Differentiating w.r.t. 't'

$0 = -V_A - V_D$

$\therefore\ -V_D = V_A$

$\therefore\ V_D = -V_A$

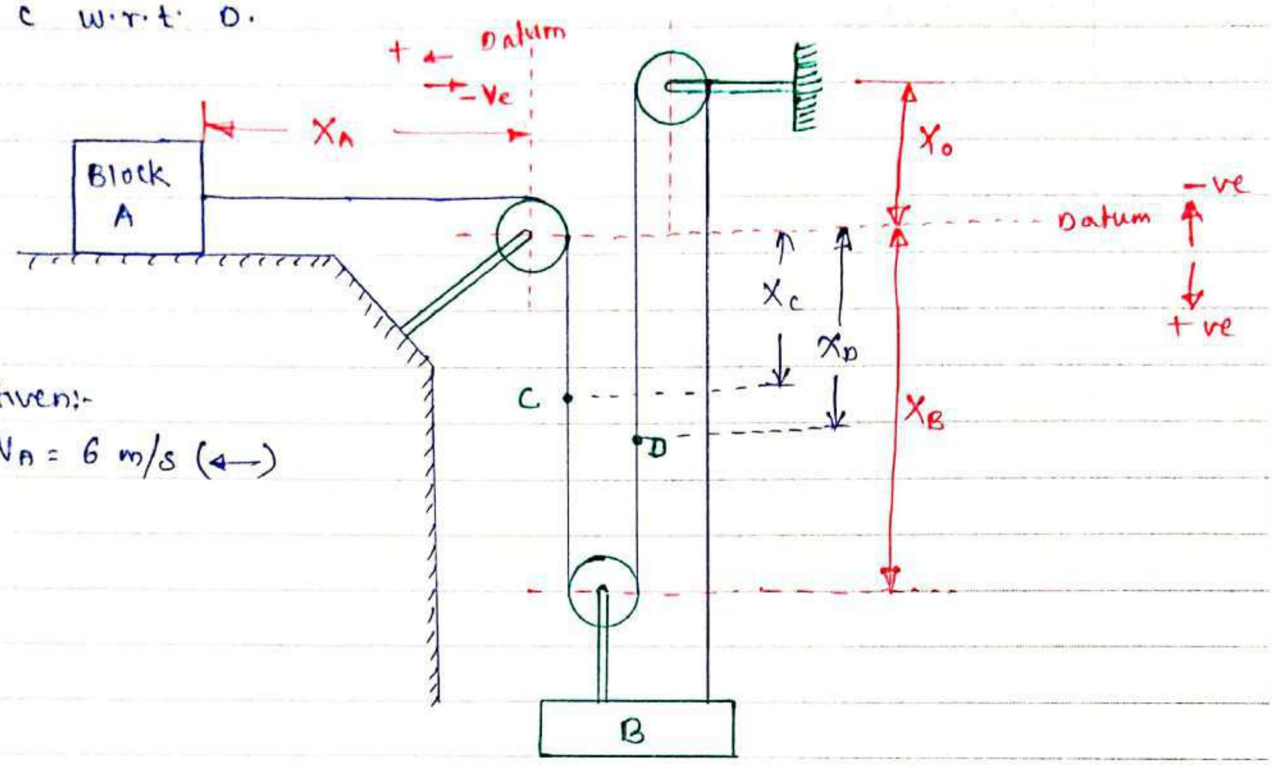
$\therefore\ V_D = -0.2\ m/sec.\ (\leftarrow)$

$\therefore\ \boxed{V_D = 0.2\ m/sec}$ towards left

8

$V_{CA} = V_C - V_A = 0.6 - (+0.2) = \cancel{0.8\ m/sec}\ 0.4\ m/sec$

Slider Block A moves to the left with constant velocity of 06 m/sec. Determine the velocity of portion C & D of cable. Also find velocity of Block B & relative velocity of C w.r.t. D.



Given:-

$V_A = 6$ m/s (←)

Consider length of the string connecting Block A & B

$$L = X_A + 3X_B - 2X_o$$

neglecting X_o

$$L = X_A + 3X_B$$

∴ differentiating w.r.t. t

$$V_A + 3V_B = 0$$

$$\therefore V_B = -\frac{V_A}{3}$$

$$\therefore V_B = -\frac{6}{3} = -2 \text{ m/sec.}$$

$$\boxed{V_B = 02 \text{ m/sec}} \uparrow$$

Now,

Consider the portion of string from end A to point C.

$$\therefore L = X_A + X_C$$

∴ differentiating w.r.t. t

$$V_A + V_C = 0$$

$$\therefore V_C = -V_A.$$

$\therefore \; V_c = -6 \text{ m/sec.}$

$\boxed{V_c = 6 \text{ m/sec} \uparrow}$ upward.

Now consider the cable upto point D starting from Block B.

$\therefore \quad L = x_B - x_0 - x_0 + x_D$

$L = x_B - 2x_0 + x_D$

Neglecting x_0 as it is constant.

$L = x_B + x_D$

$\therefore$ differentiating w.r.t. t

$V_B + V_D = 0$

$V_B = -V_D$

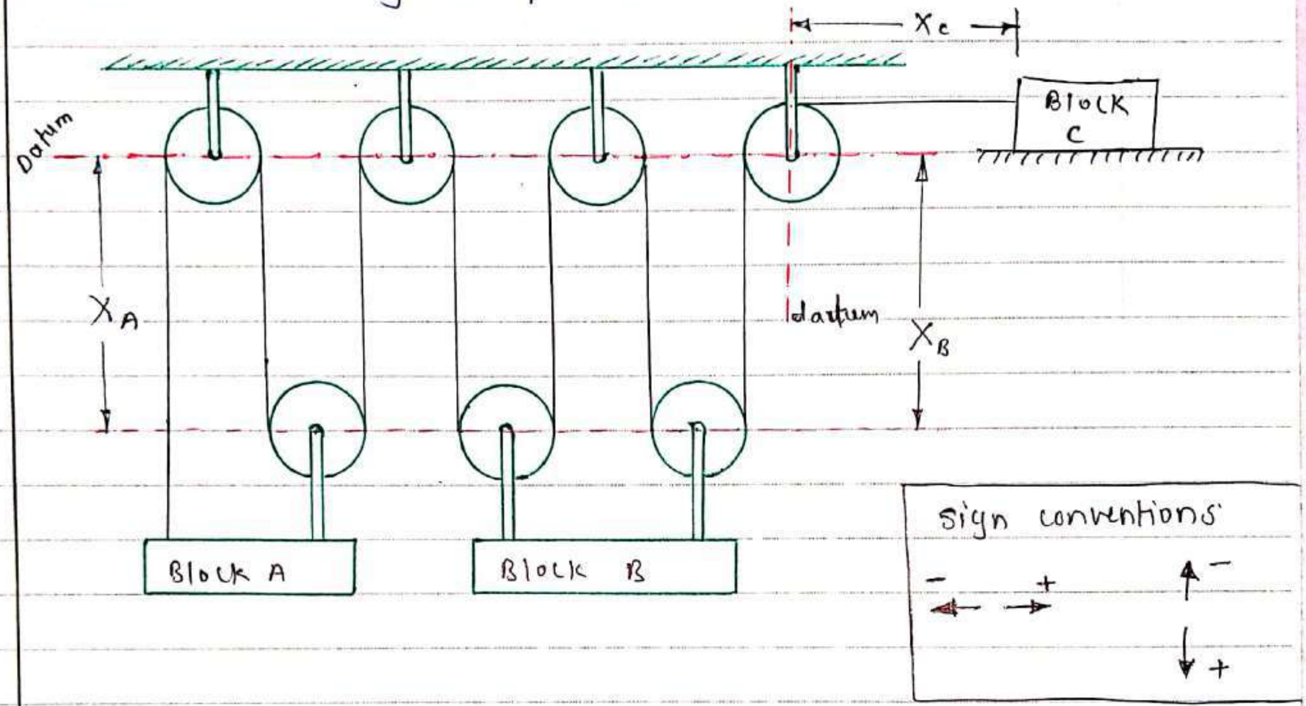
or $V_D = -V_B$

$= -(-2)$

$V_D = 2 \text{ m/sec.}$

$\therefore \boxed{V_D = 2 \text{ m/sec}} \downarrow$

Block A moves with constant acceleration. Block B starts from rest and slider block 'C' moves to right with constant acceleration of 75 mm/sec^2. At $t = 2$ sec, velocity of B & C are 0.45 m/s & 0.25 m/s to the right respectively.
Determine 1) The acc^n of A & B
2) Initial velocities of A & C
3) change in position of C after 3 sec.



Consider the length of the string connecting A, B, C;

$L = 3X_A + 4X_B + X_C$

Differentiating w.r.t. 't'

$\therefore 0 = 3V_A + 4V_B + V_C$ Similarly. --- (i)

$0 = 3U_A + 4U_B + U_C$ --- (ii)

Again differentiating w.r.t. t

$\therefore 0 = 3a_A + 4a_B + a_C$ --- (iii)

Given: a_A = constant

$U_B = 0$

$a_C = 75 \; mm/s^2 = 0.075 \; m/s^2 \rightarrow$

At $t = 2$, $V_B = 0.45$ m/s ($\rightarrow$) & $V_C = 0.25$ m/s ($\rightarrow$)

Now for Block A & B (UAM)

$$V_B = u_B + a_B t$$

$$0.45 = 0 + a_B \times 2$$

$$\therefore a_B = \frac{0.45}{2} = 0.225 \text{ m/s}^2$$

$$\therefore \boxed{a_B = 0.225 \text{ m/s}^2 \downarrow}$$

$$\therefore 3a_A + 4a_B + a_c = 0 \quad \text{.....from eq}^n \text{ (iii)}$$

$$3a_A + 4 \times 0.225 + 0.075 = 0$$

$$\therefore \cancel{a_A = \frac{-1.975}{3}}$$

$$\therefore a_A = \frac{-0.975}{3} = -0.325 \text{ m/s}^2$$

$$\therefore \boxed{a_A = -0.325 \text{ m/s}^2} \uparrow$$

for Block A, $\therefore 3V_A + 4V_B + V_c = 0$

$$\therefore 3 \cdot V_A + 4 \times 0.45 + 0.25 = 0$$

$$\therefore 3V_A = -2.05$$

$$V_A = \frac{-2.05}{3} = -0.6833 \text{ m/s}$$

$$\boxed{V_A = 0.6833 \text{ m/s} \uparrow}$$

using $V_A = U_A + a_A t$

$$-0.6833 = U_A + (-0.325 \times 2)$$

$$\therefore U_A = -0.0333 \text{ m/sec}$$

$$\boxed{U_A = 0.0333 \text{ m/sec} \uparrow}$$

for Block C,

$$V_c = U_c + a_c t$$

$$0.25 = U_c + (0.075 \times 2)$$

$$\therefore \boxed{U_c = 0.1 \text{ m/sec}} \rightarrow$$

Now at $t = 3$,

$$V_c^2 = U_c^2 + 2a_c S_c$$

$$0.25^2 = 0.1^2 + (2 \times 0.075) S_c$$

$$0.0625 - 0.01 = 0.15 S_c$$

$$\therefore 0.15 S_c = 0.0525$$

$$\therefore S_c = \frac{0.0525}{0.15}$$

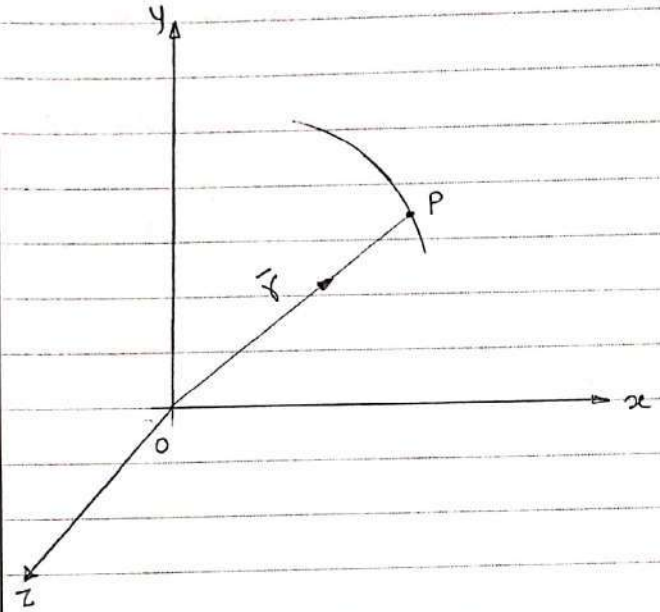
Change in position $\Rightarrow \boxed{S_c = 0.35 \text{ m}} \rightarrow$

* Curvilinear Motion :-

When a particle moves along a curved path, then motion of the particle is said to be curvilinear.

* Basic Terminology used to describe curvilinear motion :-

① Position vector :- ($\bar{r}$)



- consider that particle is moving along the curve as shown in figure.
- Let 'P' is the position of Particle at any time instant 't'
- let we have fixed reference axes x, y, z as shown.

- The line 'OP' represents the position of Particle & it is known as position vector of particle at time 't'.

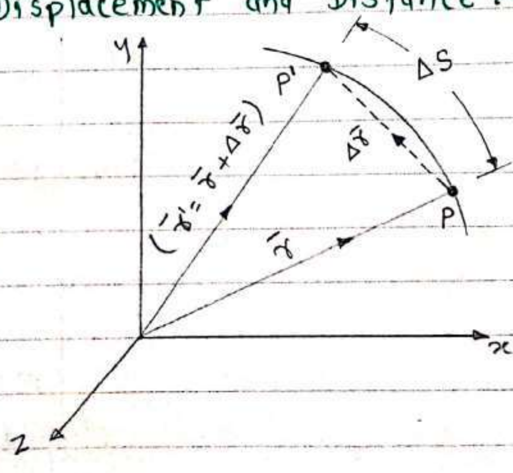
$\therefore$ Position vector $\overline{OP} = \bar{r}$

In cartesian co-ordinates,

$$\bar{r} = xi + yj + zk \quad \text{and}$$

$$r = OP = \sqrt{x^2 + y^2 + z^2} \quad - \text{magnitude of position.}$$

② Displacement and Distance :-



- consider that particle is moving along the plane curve P - P' as shown in figure.
- Let Particle is located at Point P, at time instant 't'.

- Position of Particle at point p is given by vector $\bar{r}$. Now after time $(t + \Delta t)$, let particle is moved to a new position p'. This position of particle is given by the vector $(\bar{r}' = \bar{r} + \Delta\bar{r})$.

- The vector joining p & p' is $\Delta\bar{r}$. (dashed line).
  $\Delta\bar{r}$ = change of position of particle during the time interval 'Δt'.
  $\therefore \Delta\bar{r}$ = Displacement of particle.

- The distance travelled by the particle along the curve from Point p to p' is Δs. This is measured along the curved path & is scalar quantity.

③ velocity :-

In the above figure, $\Delta\bar{r}$ = Displacement vector,
Δr = Displacement of particle (magnitude)
Δt = time taken by particle to move from p to p'.

$$\text{Avg. Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta r}{\Delta t} = V_{avg}.$$

- when the time interval approches to zero, $\Delta t \to 0$
Instantanious velocity at p will be,

$$V = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

$$V = \frac{dr}{dt} \qquad \& \qquad \bar{v} = \frac{d\bar{r}}{dt}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

④ Acceleration :-

$$\text{Avg. acc}^n \quad a = \frac{\Delta\bar{v}}{\Delta t}$$

for very small interval of time, $\Delta t \to 0$

The acc^n at point p (Instantanious acc^n) $= \lim_{\Delta t \to 0} \frac{\Delta\bar{v}}{\Delta t} = \frac{dv}{dt}$.

Thus,

$$a = \frac{dv}{dt} \quad \&$$

$$\bar{a} = \frac{d\bar{v}}{dt}$$

* Co-ordinate systems in curvilinear motion :-

There are 3 different co-ordinate systems that are used to describe the motion of particle along the curved path.

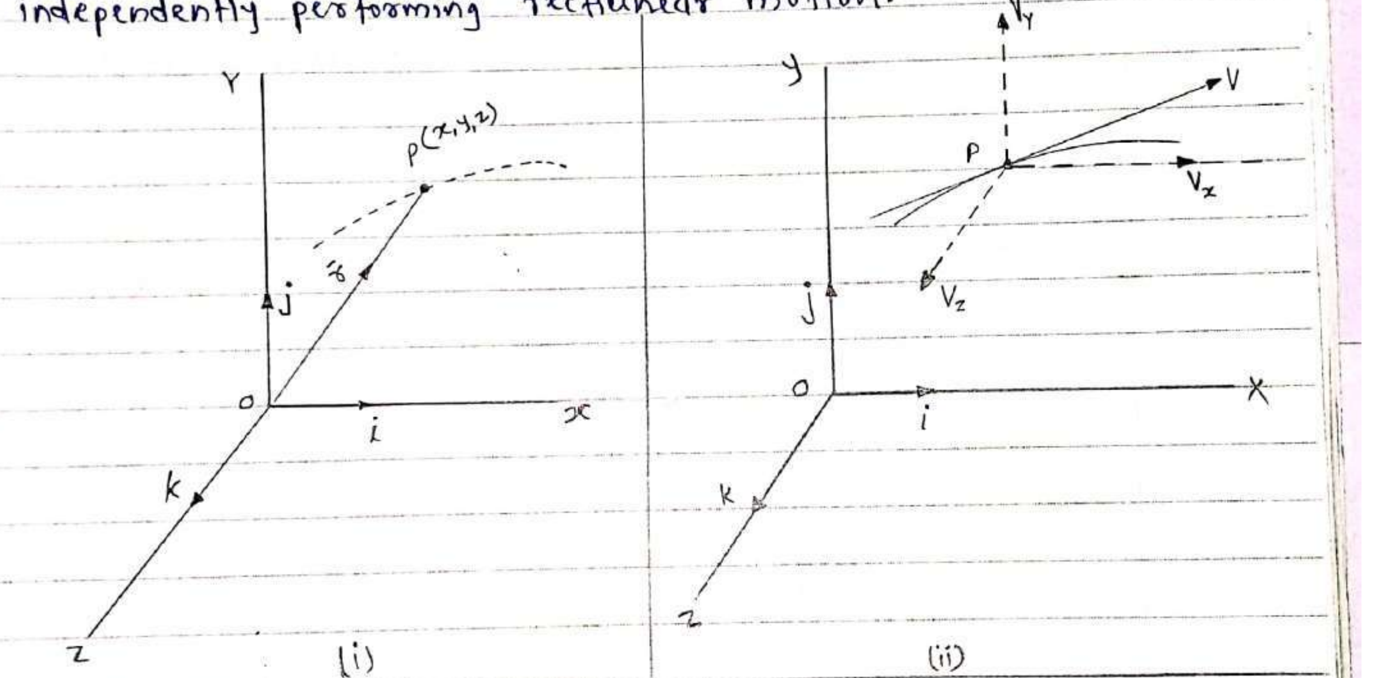
1) Rectangular co-ordinates system.
Also known as Cartesian co-ordinate sys.

2) Normal & tangential co-ordinate system.
Also known as path variables or path co-ordinates.

3) Radial & Transverse co-ordinate system.
Also known as polar co-ordinate sys.

* ① Rectangular / Cartesian co-ordinate system.

If particle is moving along curved path, we can split its motion into x, y, z directions as an independently performing rectilinear motion.



- In the figure (i) above,
consider the particle moving along the curve & at any instant 't', it is at point $P(x,y,z)$. Thus position vector for particle at P is

$\bar{OP} = \bar{r} = xi + yj + zk$ ---- Position vector.

where x, y, z are the functions of time (t).

To find velocity & Accⁿ of particle, let us differentiate position vector $\bar{r}$ w.r.t. 't' (time)

$$\bar{r} = xi + yj + zk$$

Taking derivative,

$$\frac{d\bar{r}}{dt} = \bar{V} = \left(\frac{dx}{dt}\right)i + \left(\frac{dy}{dt}\right)j + \left(\frac{dz}{dt}\right)k$$

$$\bar{V} = V_x i + V_y j + V_z k$$ ---- Velocity vector.

where,

$$V_x = \frac{dx}{dt}, \quad V_y = \frac{dy}{dt} \ \& \ V_z = \frac{dz}{dt}.$$

Thus V_x, V_y, V_z = x, y, z component of velocity respectively.

Magnitude of velocity = $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$ - for space curve

$V = \sqrt{V_x^2 + V_y^2}$ - for plane curve

Direction of velocity = $\tan\alpha = \frac{V_y}{V_x}$ ---- for plane curve.

Now differntiating velocity w.r.t. time,

$$\frac{d\bar{V}}{dt} = \bar{a} = \left(\frac{dV_x}{dt}\right)i + \left(\frac{dV_y}{dt}\right)j + \left(\frac{dV_z}{dt}\right)k$$

$$\bar{a} = a_x i + a_y j + a_z k$$ ---- Accl^n vector.

where

$a_x = \frac{dV_x}{dt}$ ---- x component of Acceleration

$a_y = \frac{dV_y}{dt}$ ---- y component of Accl^n

$a_z = \frac{dV_z}{dt}$ ---- z component of Accl^n

Magnitude of Accl^n

$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$ - for space curve

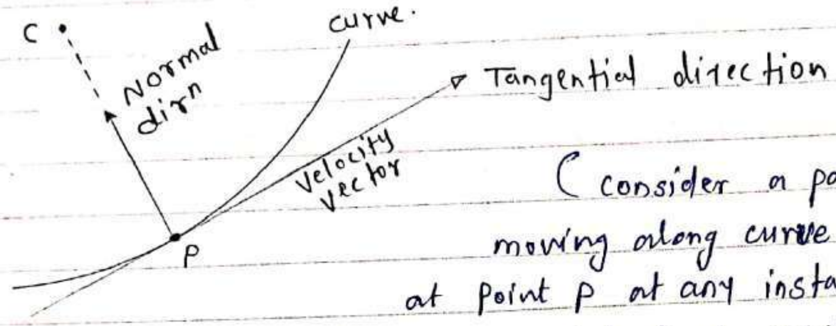
$a = \sqrt{a_x^2 + a_y^2}$ - for plane curve.

Direction of Accl^n

$\tan\beta = \frac{a_y}{a_x}$ ---- for plane curve.

② Normal & Tangential co-ordinate system :-
[Path variables]

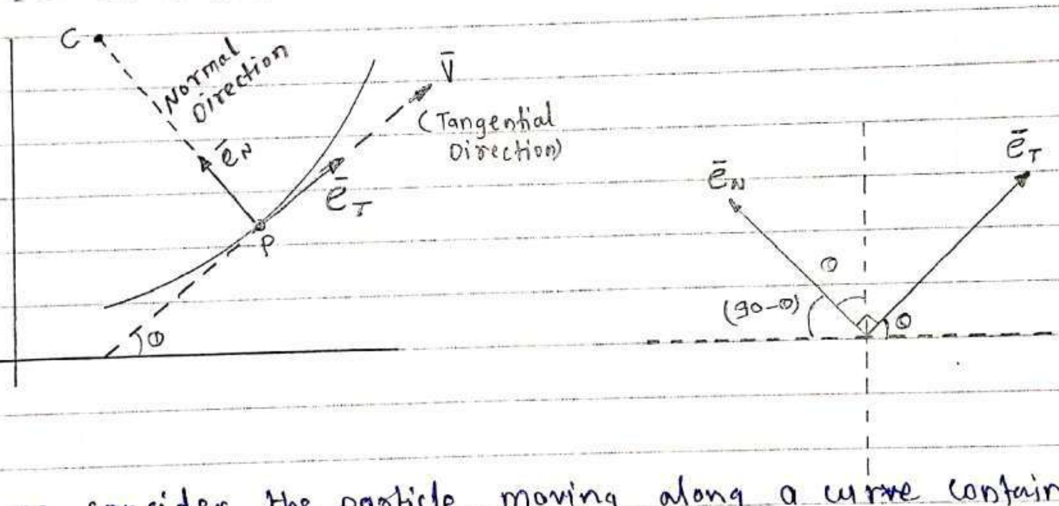
- We know that, velocity of particle is always tangential to the path along which it is moving.



(consider a particle moving along curve & is located at point P at any instant 't'.)

So, the tangent drawn at that point (at point P) represents the tangential direction. &

The direction which is perpendicular to the velocity vector (or tangent at P) & passing through centre of curvature is known as Normal direction.



- Let us consider the particle moving along a curve contained in the plane as shown in figure above

Let P is the position of the particle at a given instant.

Now, let us attach unit vector $\bar{e}_T$ & $\bar{e}_N$ at point P. $\bar{e}_T$ is along tangential direction & $\bar{e}_N$ is along Normal direction.

$\bar{e}_T$ = unit vector along tangential direction

$\bar{e}_N$ = Unit vector along Normal direction.

- Unit vectors can be written as

$$\bar{e}_T = \cos\theta\, i + \sin\theta\, j \quad \& \quad \bar{e}_N = -\sin\theta\, i + \cos\theta\, j$$

Differentiating above eqns w.r.t. θ, we get,

$$\therefore \frac{d\bar{e}_T}{d\theta} = -\sin\theta\, i + \cos\theta\, j \quad \& \quad \therefore \frac{d\bar{e}_N}{d\theta} = -\cos\theta\, i - \sin\theta\, j$$

$$\therefore \frac{d\bar{e}_T}{d\theta} = \bar{e}_N \quad \text{------ (I)} \qquad \text{And} \qquad \frac{d\bar{e}_N}{d\theta} = -\bar{e}_T \quad \text{------ (II)}$$

- Velocity components.

As the velocity is always tangential to the path along which partical moves, Then Tangential component of velocity is equal to velocity itself. and Normal component will be zero.

$$\therefore V_T = V$$
$$V_N = 0$$

The velocity vector of particle is tangent to path. It will be

$\bar{v}$ = magnitude of velocity $\times$ unit vector along tangential Dirn.

$$\boxed{\bar{v} = v \times \bar{e}_T}$$

- Accln of Particle:

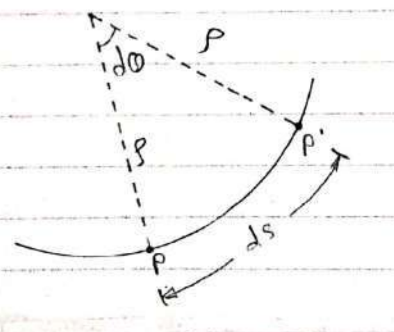
$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(v\bar{e}_T)$$

$$\bar{a} = \frac{dv}{dt}\bar{e}_T + \frac{d\bar{e}_T}{dt} \cdot v \quad \text{— (III)}$$

$$\text{But } \frac{d\bar{e}_T}{dt} = \frac{d\bar{e}_T}{d\theta} \cdot \frac{d\theta}{ds} \cdot \frac{ds}{dt} \quad \text{---- by double chain rule.}$$

$$\frac{d\bar{e}_T}{dt} = \bar{e}_N \cdot \frac{d\theta}{ds} \cdot v \quad \text{— (IV)}$$

- Now let us find the value of $\frac{d\theta}{ds}$.



consider particle moving along curve is at P at time t. Then after small time interval Δt, Particle attains new position at P'. During this it covers 'ds' distance along the curve.

let C = centre of Radius of curvature
ρ = Radius of curvature of curved path.

- Then, Arc length = Radius × Angle subtended by arc

$$ds = \rho \times d\theta$$

$$\therefore \frac{d\theta}{ds} = \frac{1}{\rho}.$$

putting above value in eqⁿ (iv), we get,

$$\frac{d\bar{e}_T}{dt} = \bar{e}_N \cdot \frac{1}{\rho} \cdot v$$

putting this value in eqⁿ (iii), we get,

$$\bar{a} = \left(\frac{dv}{dt}\right)\bar{e}_T + \bar{e}_N \cdot \frac{1}{\rho} \cdot v \cdot v$$

$$\bar{a} = \left(\frac{dv}{dt}\right)\bar{e}_T + \left(\frac{v^2}{\rho}\right)\bar{e}_N$$

$$\bar{a} = (a_T)\bar{e}_T + (a_N)\bar{e}_N$$ --------- Acceleration vector.

where

$$a_T = \frac{dv}{dt}$$ & ---- Tangential component of Accⁿ

$$a_N = \frac{v^2}{\rho}$$ ---- Normal component of Accⁿ.

Magnitude of Acceleration, (Total)

$$\boxed{a = \sqrt{(a_T)^2 + (a_N)^2}}$$

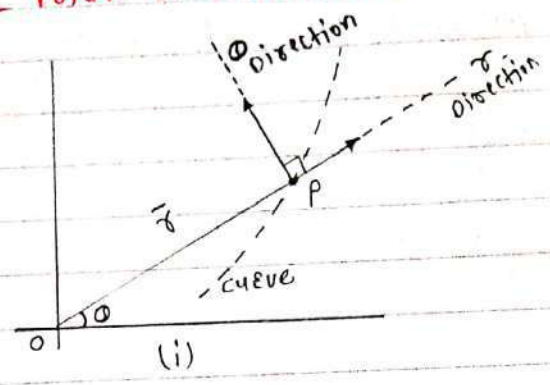
Direction of Total Accⁿ is,

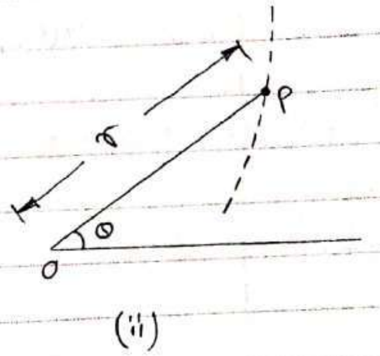
$$\tan\alpha = \frac{a_T}{a_N}$$ where

α = angle made by Accⁿ with normal direction.

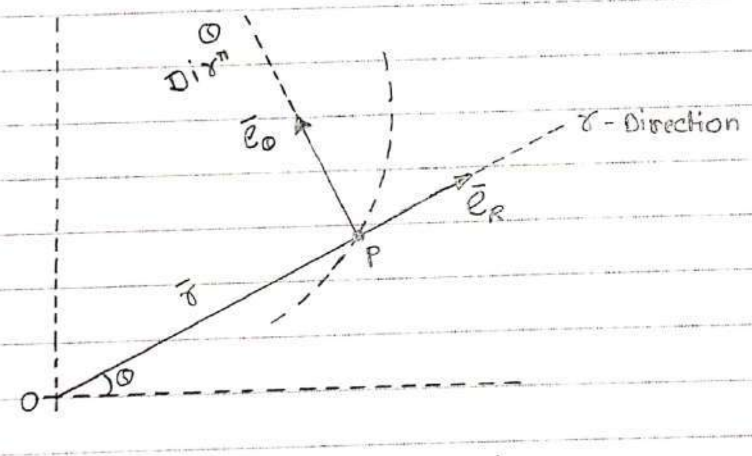
- a_N is always directed towards centre of curvature.
- a_T reflects the change in speed of particle
- a_N reflects the change in dirⁿ of motion.
- when ρ becomes infinity at inflection point then $\underline{a_N = 0}$.

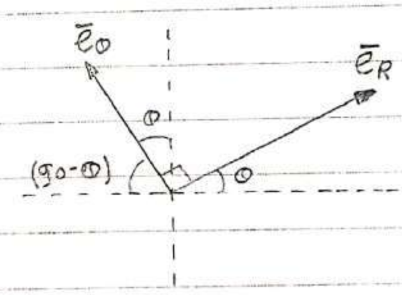
* ③ Radial & Transverse Co-ordinates system
[Polar Co-ordinates]





- consider a particle moving along the curve & located at Point P at any instant 't'.
- To define the position of the particle, we can say that P is located at a radial distance r from origin & at an angular measurement θ to the radial line. (Fig-ii)
- Thus, the direction along the position vector $\bar{r}$ is called as radial direction (r) and & the direction which is perpendicular to the position vector $\bar{r}$ is called as transverse direction (θ). (Fig-i)





- let particle is moving along a curve & is located at point P at any time instant 't'. let r is radial distance & θ is angle made by radial dirⁿ.
- Let us attach unit vector $\bar{e}_R$ at point P along Radial direction & $\bar{e}_\theta$ at point P along transverse direction.
- Unit vector $\bar{e}_R$ defines Radial direction & Unit vector $\bar{e}_\theta$ defines Transverse direction.

- Unit vectors can be written as,

$$\bar{e}_R = \cos\theta\, i + \sin\theta\, j \quad \text{And} \quad \bar{e}_\theta = -\sin\theta\, i + \cos\theta\, j$$

Differentiating above vectors with 'θ'.

$$\therefore \frac{d\bar{e}_R}{d\theta} = -\sin\theta\, i + \cos\theta\, j \quad \text{And} \quad \frac{d\bar{e}_\theta}{d\theta} = -\cos\theta\, i - \sin\theta\, j$$

$$\therefore \frac{d\bar{e}_R}{d\theta} = \bar{e}_\theta \quad ---- (i) \qquad \therefore \frac{d\bar{e}_\theta}{d\theta} = -(\cos\theta\, i + \sin\theta\, j)$$

$$\therefore \frac{d\bar{e}_\theta}{d\theta} = -\bar{e}_R \quad --- (ii)$$

Now by using chain rule,

$$\therefore \frac{d\bar{e}_R}{dt} = \frac{d\bar{e}_R}{d\theta} \cdot \frac{d\theta}{dt} \quad \text{And} \quad \therefore \frac{d\bar{e}_\theta}{dt} = \frac{d\bar{e}_\theta}{d\theta} \cdot \frac{d\theta}{dt}$$

let $\dot{\theta} = \frac{d\theta}{dt}$ = time derivative of θ, then,

$$\therefore \frac{d\bar{e}_R}{dt} = \frac{d\bar{e}_R}{d\theta} \cdot \dot{\theta} \quad \& \quad \frac{d\bar{e}_\theta}{dt} = \frac{d\bar{e}_\theta}{d\theta} \cdot \dot{\theta}$$

$$\therefore \frac{d\bar{e}_R}{dt} = \bar{e}_\theta \cdot \dot{\theta} \quad \& \quad \frac{d\bar{e}_\theta}{dt} = -\bar{e}_R \cdot \dot{\theta} \quad \cdots\cdots (3)$$

- **velocity components -:**

we know that,

position vector = magnitude × unit vector along the dir^n.

$$\bar{r} = r \times \bar{e}_R$$

Differentiating this w.r.t. time,

$$\bar{V} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(r \cdot \bar{e}_R)$$

$$\bar{V} = r \cdot \frac{d\bar{e}_R}{dt} + \bar{e}_R \cdot \frac{dr}{dt}$$

$$\bar{V} = r \cdot (\bar{e}_\theta \cdot \dot{\theta}) + \bar{e}_R \cdot \dot{r} \quad \text{- by using eq}^n\ (3)$$

$$\bar{V} = (\dot{r})\bar{e}_R + (r \cdot \dot{\theta})\bar{e}_\theta \quad \left(\text{where } \dot{r} = \frac{dr}{dt}\right)$$

$$\bar{V} = (V_r)\bar{e}_R + (V_\theta)\bar{e}_\theta \quad \cdots\cdots \text{where}$$

Radial component of velocity $= V_r = \dot{r} = \frac{dr}{dt}$

Transverse component of velocity $= V_\theta = r\dot{\theta}$

Resultant velocity or total velocity,

$$V = \sqrt{V_r^2 + V_\theta^2} \quad \text{------ magnitude}$$

Direction of velocity, $\tan\alpha = \frac{V_\theta}{V_r}$ where

α = angle made by velocity with Radial direction.

- **Accl^n components**

we know that,

Accl^n vector,

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(\dot{r}\bar{e}_R + r\dot{\theta}\bar{e}_\theta)$$

$$\therefore \bar{a} = \frac{d}{dt}(\dot{r}\bar{e}_R) + \frac{d}{dt}(r\dot{\theta}\bar{e}_\theta)$$

$$\therefore \bar{a} = \dot{r}\frac{d\bar{e}_R}{dt} + \bar{e}_R\frac{d\dot{r}}{dt} + r\dot{\theta}\frac{d\bar{e}_\theta}{dt} + r\bar{e}_\theta\frac{d\dot{\theta}}{dt} + \dot{\theta}\bar{e}_\theta\frac{dr}{dt}$$

$$= \dot{r}\cdot\dot{\theta}\bar{e}_\theta + \ddot{r}\bar{e}_R + r\dot{\theta}(-\bar{e}_R\cdot\dot{\theta}) + r\bar{e}_\theta\ddot{\theta} + \dot{r}\dot{\theta}\bar{e}_\theta$$

$$= \dot{r}\dot{\theta}\bar{e}_\theta + \ddot{r}\bar{e}_R - r(\dot{\theta})^2\bar{e}_R + r\ddot{\theta}\bar{e}_\theta + \dot{r}\dot{\theta}\bar{e}_\theta$$

$$= \ddot{r}\bar{e}_R - r(\dot{\theta})^2\bar{e}_R + 2\dot{r}\dot{\theta}\bar{e}_\theta + r\ddot{\theta}\bar{e}_\theta$$

$$= [\ddot{r} - r(\dot{\theta})^2]\bar{e}_R + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\bar{e}_\theta$$

$$\bar{a} = (a_r)\bar{e}_R + (a_\theta)\bar{e}_\theta \quad \text{---- Accl}^n \text{ vector.}$$

where,

a_r = Radial component of Accl^n = $[\ddot{r} - r(\dot{\theta})^2]$

a_θ = Transverse component of Accl^n = $(2\dot{r}\dot{\theta} + r\ddot{\theta})$

Resultant or Total Accl^n = $\sqrt{a_r^2 + a_\theta^2}$ ---- magnitude.

$$\tan\beta = \frac{a_\theta}{a_R} \quad \text{------ Direction of Accl}^n$$

where β = Angle made by Accl^n with radial dir^n.

* Special case of curvilinear Motion (Circular Motion)

For the motion in circular path -: (r is constant)

- Radial component of velocity = $V_r = 0$
- Transverse component of velocity = $V = V_\theta = r\dot{\theta}$
- Radial component of Acceleration = $a_r = -r(\dot{\theta})^2$
- Transverse component of Acceleration = $a_\theta = r\ddot{\theta}$
- $a_T = a_\theta$ & $a_r = -a_N$

* Radius of curvature *

① when Rectangular components of velocity & Accl^n are known,

- Radius of curvature $= \rho = \dfrac{[V_x^2 + V_y^2]^{3/2}}{[V_x \cdot a_y - a_x V_y]} = \left[\dfrac{V^3}{V_x \cdot a_y - a_x V_y}\right]$

② when $\bar{V}$ & $\bar{a}$ are known, Then

ⓐ magnitude of $a_T = \dfrac{\bar{V} \cdot \bar{a}}{|V|}$

b) Magnitude of $a_N = \sqrt{a^2 - a_T^2}$

c) Radius of curvature $= \rho = \dfrac{V^2}{a_N}$

③ when path eq^n $y = f(x)$ is given,

$$|\rho| = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Numericals on Rectangular co-ordinates / Eqⁿ of motion in cartesian co-ordinates

* A particle moves along a curved path given by the relation $y = 4x^2 + 8x + 10$ starting with initial velocity $\bar{V}_0 = 5i + 3j$. If V_x = constant, determine V_y & a_y at $x = 3m$. Also determine the magnitude of velocity and acceleration.

$\Longrightarrow$

Given: $\vec{V}_0 = 5i + 3j$ so, $V_{x_0} = 5$ m/sec. & $V_{y_0} = 3$ m/sec.

$y = 4x^2 + 8x + 10.$

$y = 4x^2 + 8x + 10$

Differentiating w.r.t. 't'

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$V_y = (8x + 8)\frac{dx}{dt}$$
$$= (8x + 8) V_x$$
$$\therefore V_y = 8x \cdot V_x + 8V_x$$

Again differentiating w.r.t. 't'

$$\frac{dV_y}{dt} = \frac{dV_y}{dx} \cdot \frac{dx}{dt}$$

$$a_y = \left[8x \cdot \frac{dV_x}{dx} + V_x \cdot \frac{d(8x)}{dx} + 8 \cdot \frac{dV_x}{dx}\right] V_x$$

$$\therefore a_y = [0 + 8V_x + 0] V_x$$

$$\therefore a_y = 8V_x^2$$

Now, at $x = 3m$,

$$V_y = 8 \cdot x \cdot V_x + 8V_x$$
$$= (8 \times 3 \times 5) + (8 \times 5)$$
$$= 120 + 40$$
$$= 160 \text{ m/sec.}$$

[(As V_x = constant) ($V_{x_0} = V_x = 5$ m/sec.)]

$$a_y = 8 \cdot V_x^2$$
$$= 8 \times 5^2$$
$$a_y = 200 \text{ m/s}^2$$

$a_x = 0$

$\therefore$ magnitude of velocity

$$V = \sqrt{V_x^2 + V_y^2}$$
$$V = \sqrt{5^2 + 160^2}$$
$$V = 160.07 \text{ m/s}$$

magnitude of Accⁿ

$$a = \sqrt{a_x^2 + a_y^2}$$
$$a = \sqrt{0 + 200^2}$$
$$\boxed{a = 200 \text{ m/s}^2}$$

* A Particle moves along the path $\bar{r} = (8t^2)i + (t^3+5)j$, where 't' is in seconds. Determine the magnitude of particle velocity and acc$|^n$ when $t = 4$ sec. Find the eq^n of path, $y = f(x)$.

⇒

<u>Given</u> : $\bar{r} = (8t^2)i + (t^3+5)j$ – position vector.

$\therefore x = 8t^2$ & $y = t^3+5$

differentiation w.r.t. t

$V_x = \frac{dx}{dt} = 16t$ & $V_y = \frac{dy}{dt} = 3t^2$

Again differntiation w.r.t. t

$a_x = \frac{dV_x}{dt} = 16$ m/s^2 & $a_y = \frac{dV_y}{dt} = 6t$

Now, at $t = 4$ sec.

$V_x = 16t = 16 \times 4 = 64$ m/s

$V_y = 3t^2 = 3 \times 4^2 = 48$ m/sec.

magnitude of velocity,

$V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{64^2 + 48^2}$

$\boxed{V = 80 \text{ m/sec.}}$

$a_x = 16$ m/sec^2

$a_y = 6t = 6 \times 4 = 24$ m/s^2

$\therefore$ magnitude of Acc$|^n$

$a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{16^2 + 24^2}$

$\boxed{a = 28.84 \text{ m/s}^2}$

Now as,

$x = 8t^2$

$\therefore t^2 = \frac{x}{8}$

& $\therefore t = \sqrt{\frac{x}{8}} = \left(\frac{x}{8}\right)^{\frac{1}{2}}$

But $y = t^3 + 5$

put the value of t in this eq^n.

$\therefore y = \left[\left(\frac{x}{8}\right)^{\frac{1}{2}}\right]^3 + 5$

$\boxed{y = \left(\frac{x}{8}\right)^{\frac{3}{2}} + 5}$

--- path eq^n.

* The Y co-ordinate of the particle is given by $y = 4t^3 - 3t$. If $a_x = 12t$ m/s^2 & $V_x = 8$ m/s at $t=0$, calculate the magnitude of velocity & acceleration of particle at time $t = 2$ seconds.

$\Rightarrow$

Given :- at $t=0$, $a_x = 12t$ & $V_x = 8$ m/sec.
at $t=2$, $V = ?$ $a = ?$

$y = 4t^3 - 3t$ ----- Given

Differentiating w.r.t. time 't'.

$$\frac{dy}{dt} = 12t^2 - 3$$

$$V_y = 12t^2 - 3 \quad \text{--- (i)}$$

Differentiating again

$$a_y = 24t \quad \text{--- (ii)}$$

$$a_x = 12t \quad \text{--- (iii)} \quad \text{— Given.}$$

Now,

$$a_x = \frac{dV_x}{dt} = 12t$$

$$\therefore dV_x = 12t\, dt$$

Taking integeation

$$\int dV_x = \int 12t\, dt$$

$$V_x = 12\frac{t^2}{2} + C_1$$

$$V_x = 6t^2 + C_1$$

Now using the given conditions.

$t=0$, $V_x = 8$ m/s

$$8 = 6 \times 0 + C_1$$

$$\therefore \boxed{C_1 = 8}$$

$$\therefore V_x = 6t^2 + 8 \quad \text{----- (4)}$$

At time $t = 2$ sec;

$$V_x = 6t^2 + 8$$
$$= 6 \times 2^2 + 8$$
$$V_x = 32 \text{ m/sec.}$$

$$V_y = 12t^2 - 3$$
$$= 12 \times 2^2 - 3$$
$$V_y = 45 \text{ m/sec.}$$

$$a_x = 12t$$
$$= 12 \times 2$$
$$a_x = 24 \text{ m/s}^2$$

$$a_y = 24t = 24 \times 2 = 48 \text{ m/s}^2$$

$\therefore$ Magnitude of velocity at $t=2$,

$$V = \sqrt{V_x^2 + V_y^2}$$
$$= \sqrt{32^2 + 45^2}$$
$$\boxed{V = 55.21 \text{ m/sec.}}$$

Magnitude of Accl^n at $t=2$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{24^2 + 48^2}$$

$$\boxed{a = 53.66 \text{ m/s}^2}$$

* A Particle is moving along a curve, $y = x - \frac{x^2}{600}$. If $V_x = 4$ m/s and is constant. Determine the magnitude of velocity & Acc^n when $x = 30$ m.

⇒

Given: $V_x = 4$ m/sec. (constant)

$$y = x - \frac{x^2}{600}$$

Given eq^n of curve is

$$y = x - \frac{x^2}{600}$$

Differentiating this w.r.t 't'

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \quad \text{-- by chain rule}$$

$$\therefore V_y = \frac{d}{dx}\left(x - \frac{x^2}{600}\right) \cdot \frac{dx}{dt}$$

$$= \left(1 - \frac{2x}{600}\right) \cdot V_x$$

$$= \left(1 - \frac{x}{300}\right) \cdot V_x$$

$$V_y = V_x - \frac{x}{300} \cdot V_x$$

$$\therefore V_y = V_x\left[1 - \frac{x}{300}\right] \quad \text{---- (1)}$$

Again differentiating w.r.t. 't'

$$\frac{dV_y}{dt} = \frac{dV_y}{dx} \cdot \frac{dx}{dt} \quad \text{- using chain rule.}$$

$$\therefore a_y = \frac{d}{dx}\left[V_x\left(1 - \frac{x}{300}\right)\right] \cdot \frac{dx}{dt}$$

$$= \left[V_x \frac{d}{dx}\left(1 - \frac{x}{300}\right) + \left(1 - \frac{x}{300}\right)\frac{dV_x}{dx}\right]\frac{dx}{dt}$$

$$\therefore a_y = \left[V_x\left[0 - \frac{1}{300}\right] + 0\right] V_x$$

$$= \frac{-V_x}{300} \cdot V_x$$

$$a_y = \frac{-V_x^2}{300} \quad \text{----- (2)}$$

Now, at $x = 30$ m,

$$\therefore \boxed{V_x = 4 \text{ m/sec.}}$$

$$\therefore V_y = V_x\left(1 - \frac{x}{300}\right)$$

$$\therefore V_y = 4\left(1 - \frac{30}{300}\right)$$

$$\therefore \boxed{V_y = 3.6 \text{ m/sec.}}$$

Now as

V_x = constant

$$\therefore \boxed{a_x = 0} \quad \&$$

$$a_y = \frac{-V_x^2}{300}$$

$$a_y = -\frac{4^2}{300}$$

$$\boxed{a_y = -0.053 \text{ m/s}^2}$$

$$\therefore a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{0^2 + (-0.053)^2}$$

$$\boxed{a = 0.053 \text{ m/s}^2} \downarrow$$

as $a_x = 0$, a must be downward because a_y is negative.

Numericals on Tangential & Normal Components (Eqⁿ of motion in Path co-ordinates)

* A Car is travelling on a curved portion of Highway of radius 350 m at a speed of 72 kmph. The brakes are suddenly applied, causing the speed to decrease at a constant rate of $1.25\ m/s^2$. Determine the magnitude of total accⁿ of car (a) immediately after the brakes have been applied (b) 4 sec. later.

⇒

Given : $\rho = 350\ m$

$V = 72\ kmph = 20\ m/s$

a_T = Tangential Accⁿ represents change in speed.

$\therefore a_T = -1.25\ m/s^2$

At the instant when brakes are applied;

speed of car $= V = 20\ m/s$

$\therefore a_N = \frac{V^2}{\rho} = \frac{20^2}{350} = 1.143\ m/s^2$

∴ Magnitude of Total Accⁿ is

$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(1.25)^2 + 1.143^2} = 1.69$

$\therefore \boxed{a = 1.69\ m/s^2}$

After $t = 4$ sec,

velocity of car will be decreased due to application of brake.

∴ final velocity after 4 sec $= V$

initial velocity before 4 sec $= u = 20\ m/sec.$

∴ using $V = u + a_T t = 20 + (-1.25)\times 4 = 20 - 5$

$\therefore V = 15\ m/sec.$

$\therefore a_N = \frac{V^2}{\rho} = \frac{15^2}{350} = 0.64\ m/s^2$

∴ magnitude of Total Accⁿ is,

$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(1.25)^2 + 0.64^2}$

$\therefore \boxed{a = 1.4\ m/s^2}$

* A rocket follows a path such that its accl^n is given by $\bar{a} = (4i + tj)$ m/s^2 at $x = 0$ it start from rest. At $t = 20$ sec., determine, 1) speed of rocket, 2) Radius of curvature of its path 3) Magnitude of Tangential & Normal components of acceleration.

⇒

$$\bar{a} = 4i + tj$$

$$\therefore \bar{a} = \frac{d\bar{V}}{dt} = 4i + tj$$

Integrating Both sides,

$$\int d\bar{V} = \int (4i + tj)\,dt$$

$$\therefore \bar{V} = \int (4\,dt)\,i + \int (t\,dt)\,j$$

$$\bar{V} = (4t)\,i + \left(\frac{t^2}{2}\right) j + C_1.$$

Now, at $t = 0$, $V = 0$.

$$\therefore \boxed{C_1 = 0}$$

Now At $t = 20$ sec.

$$\bar{a} = 4i + tj$$

$$\bar{a} = 4i + 20j$$

$$\therefore a_x = 4 \text{ m/s}^2 \quad \& \quad a_y = 20 \text{ m/s}^2$$

Also, $\bar{V} = (4t)\,i + \left(\frac{t^2}{2}\right) j$

$$= (4 \times 20)\,i + \left(\frac{20^2}{2}\right) j$$

$$\bar{V} = 80i + 200j$$

$$\therefore V_x = 80 \text{ m/s} \quad \& \quad V_y = 200 \text{ m/sec}$$

$\therefore$ magnitude of velocity

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{80^2 + 200^2} = \underline{215.40 \text{ m/s}}$$

Magnitude of Acceleration.

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 20^2}$$

$$\therefore \boxed{a = 20.39 \text{ m/s}^2}$$

Radius of curvature (ρ)

$$|\rho| = \frac{[V_x^2 + V_y^2]^{3/2}}{V_x \cdot a_y - V_y \cdot a_x}$$

$$|\rho| = \frac{[80^2 + 200^2]^{3/2}}{(80 \times 20) - (200 \times 4)}$$

$$\therefore |\rho| = 12492.5 \text{ m}.$$

Normal Accl^n,

$$a_N = \frac{V^2}{\rho} = \frac{215.40^2}{12492.5}$$

$$\therefore \boxed{a_N = 3.71 \text{ m/s}^2}$$

Tangential Accl^n

$$a_T = \sqrt{a^2 - a_N^2}$$

$$\therefore a_T = \sqrt{20.39^2 - 3.71^2}$$

$$\therefore \boxed{a_T = 20.04 \text{ m/s}^2}$$

* Determine the distance travelled & time taken by a car starting from rest, moving on a circular curve having a radius of 275 m, and accelerates at constant rate of tangential acc^n of 1 m/s^2 & Total acc^n of 1.4 m/s^2.

Given :
$\rho = 275 m$
$a_T = 1 m/s^2$ - constant
$a = 1.4 m/s^2$
$u = 0 m/sec$ - car starting from rest.

Normal component of Acc^n,

$$a_N = \sqrt{a^2 - a_T^2}$$

$$= \sqrt{1.4^2 - 1^2}$$

$$\boxed{a_N = 0.979 m/s^2}$$

But $a_N = \frac{v^2}{\rho}$ $\therefore$ $v^2 = a_N \cdot \rho$

$\therefore v^2 = 0.979 \times 275$

$\therefore \boxed{v = 16.41 m/s}$

As a_T = constant, motion is UAM.

Using eq^n of motion,

$$v^2 = u^2 + 2 a_T s.$$

$\therefore (16.41)^2 = 0 + 2 \times 1 \times s.$

$\therefore \boxed{s = 134.64 m.}$ - distance travelled.

Using $v = u + a_T t$

$\therefore 16.41 = 0 + 1 \times t$

$\therefore \boxed{t = 16.41 sec.}$ - time taken.

* A Particle travels a curved path of radius 600m with a speed of 108 kmph & a tangential acceleration of $4 m/s^2$. Determine the total Acceleration of the particle.

⇒ Given: $\rho = 600m$

$V = 108$ kmph $= 108 \times \frac{5}{18} = 30$ m/sec.

$a_T = 4 m/sec^2$.

∴ we know that

Normal component of Accl^n is given by

$$a_N = \frac{V^2}{\rho}$$

$$a_N = \frac{30^2}{600} = \frac{900}{600} = 1.5 \; m/s^2.$$

magnitude of total accl^n of a particle is,

$$a = \sqrt{a_T^2 + a_N^2}$$

$$a = \sqrt{4^2 + 1.5^2}$$

$$\boxed{a = 4.27 \; m/s^2}$$

* A particle is moving along a curve $y = 1 + \cos x$ with constant speed of 4m/s. Find Tangential & Normal components of velocity & Accl^n.

⇒ $V = 4 m/s$ – constant

$\therefore a_T = 0$

Tangential component of velocity $= V_T = V = 4 m/s$

Normal component of velocity $= V_N = 0$.

$\therefore y = 1 + \cos x$

$\therefore \frac{dy}{dx} = -\sin x$

$\therefore \frac{d^2y}{dx^2} = -\cos x$

$$|\rho| = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

putting all values in above eq^n of 'ρ'.

$$|\rho| = \frac{\left[1 + (-\sin x)^2\right]^{3/2}}{-\cos x} = \frac{(1 + \sin^2 x)^{3/2}}{\cos x}$$

$$\therefore a_N = \frac{V^2}{\rho} = \frac{4^2}{\frac{(1 + \sin^2 x)^{3/2}}{\cos x}}$$

$$\therefore a_N = \frac{16 \cos x}{(1 + \sin^2 x)^{3/2}}$$

Numericals on Radial & Transverse co-ordinates.
(Eqn of motion in polar co-ordinates.)

* A particle moves in a circular path of radius 0.4m. Calculate magnitude of Accln of the particle if its speed is 0.6 m/sec but it is increasing at a rate of 1.2m/sec each second.

⇒ Given :

$$\rho = 0.4 \text{ m.}$$
$$V = 0.6 \text{ m/s}$$
$$a_\theta = a_T = 1.2 \text{ m/s}^2$$

In case of circular motion θ & tangential direction coincides. $\therefore a_\theta = a_T = 1.2 \text{ m/s}^2$

Also, positive r dirn is in negative Normal dirn.

$$a_r = -a_N,$$

$\therefore$

$$a_N = \frac{V^2}{\rho} = \frac{0.6^2}{0.4}$$

$$\therefore a_N = 0.9 \text{ m/s}^2$$

$$\therefore a_r = -a_N = -0.9 \text{ m/s}^2$$

$$\therefore \text{Total Accl}^n = a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{0.9^2 + 1.2^2} = 1.5 \text{ m/s}^2.$$

* A particle moves on a curved path defined by polar co-ordinates. At certain instant, $\theta = \frac{5\pi}{4}$, $a_\theta = 16 \frac{mm}{s^2}$, $r = 5mm$, $\ddot{r} = 6 mm/s^2$ & $\dot{\theta} = 0.5$ rad/s & increasing at the rate of 4 rad/s each second. Calculate magnitude of velocity of particle.

⇒ Given :-

$$r = 5mm, \quad \theta = \frac{5\pi}{4}, \quad \dot{\theta} = 0.5 \text{ rad/s}$$
$$\ddot{r} = 6 \text{ mm/s}^2, \quad \ddot{\theta} = 4 \text{ rad/s}^2, \quad a_\theta = 16 \text{ mm/s}^2$$

$$\therefore a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$$
$$\therefore 16 = (2\dot{r} \times 0.5 + 5 \times 4)$$
$$\therefore \boxed{\dot{r} = -4 \text{ mm/s}}$$

$$\bar{V} = \dot{r}.\bar{e}_R + (r\dot{\theta})\bar{e}_\theta = (-4)\bar{e}_R + (5 \times 0.5)\bar{e}_\theta$$

$$\therefore \bar{V} = -4\bar{e}_R + 2.5\bar{e}_\theta \qquad \therefore V_r = -4 \text{ mm/s} \ \& \ V_\theta = 2.5 \text{ mm/sec}$$

$$\therefore V = \sqrt{V_r^2 + V_\theta^2} = \sqrt{4^2 + 2.5^2} = \boxed{4.716 \text{ mm/s}}$$

* A particle position is described by the co-ordinates $r = 2\sin 2\theta$ meters & $\theta = (4t)$ rad; where t is in seconds. Determine the radial & transverse component of its velocity & Accl^n at t = 1 sec.

$r = 2\sin 2\theta$ & $\theta = 4t$

Differentiating w.r.t. t,

$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$ & $\therefore \frac{d\theta}{dt} = \dot{\theta} = 4$

$\therefore \dot{r} = 2 \times 2\cos 2\theta \cdot \dot{\theta}$

$= 4\cos 2\theta \cdot \dot{\theta}$

$\dot{r} = 4\dot{\theta} \cdot \cos 2\theta$

Again differentiating w.r.t t

$\frac{d\dot{r}}{dt} = \frac{d\dot{r}}{d\theta} \cdot \frac{d\theta}{dt}$ & $\frac{d\dot{\theta}}{dt} = \ddot{\theta} = 0$

$\ddot{r} = 4\dot{\theta} \times (-2\sin 2\theta) \cdot \dot{\theta}$

$= 4(\dot{\theta})^2 \times (-2\sin 2\theta)$

$\ddot{r} = -8(\dot{\theta})^2 \sin 2\theta$

at $t = 1$, $\theta = 4t = 4$ radians

$\therefore t = 1$, $\theta = 4$ rad.

$\therefore r = 2\sin 2\theta = 2 \times \sin(2 \times 4) = 1.978$ m

$\therefore \dot{r} = 4\dot{\theta}\cos 2\theta = 4 \times 4 \cdot \cos(2 \times 4) = -2.33$ m/s

$\therefore \ddot{r} = -8 \times (\dot{\theta})^2 \sin 2\theta = -8 \times 4^2 \times \sin(2 \times 4) = -126.637$ m/s²

$\therefore \dot{\theta} = 4$ rad/s

$\ddot{\theta} = 0$

$V_r = \dot{r} = -2.33$ m/s. $\therefore \boxed{V_r = -2.33 \text{ m/s}}$

$V_\theta = r\dot{\theta} = 1.978 \times 4 = 7.192$ m/s $\therefore \boxed{V_\theta = 7.192 \text{ m/s}}$

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.637 - 1.978 \times 4^2$

$\boxed{a_r = -158.28 \text{ m/s}^2}$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 2 \times (-2.33) \times 4 + (1.978 \times 0)$

$= -18.64$ m/s²

$\therefore \boxed{a_\theta = -18.64 \text{ m/s}^2}$

* Projectile motion *

- When a particle is freely thrown in the air along any direction other than vertical, it follows the parabolic path. The motion of a particle along this parabolic path is called as projectile motion.
- i.e. when we project the particle in the space, its motion is a combination of horizontal & vertical motion. This motion is called as projectile motion.
- Wind Resistance, curvature & rotation of the earth affects the actual path.
  But these parameters are neglected.
- $\boxed{\text{The path Traced by projectile is called as "Trajectory".}}$
- The motion of projectile in Horizontal direction is Uniform motion.
  a_x = Horizontal component of acceleration $= 0$.
- The acceleration in vertical direction is affected by gravity. Thus motion in y direction is considered as "Motion under gravity."

  $\therefore \quad a_y = -g$.

* Basic Terms involved in the projectile motion *

1) Time of flight :- (t)

- The time taken by the projectile to move from point of projection to the point of target is called as "Time of Flight".
- It is the total time during which projectile remains in space.

2) "Horizontal Range" :- (R)

It is Horizontal distance from point of projection to the point of target. OR

It is Horizontal distance betn point of projection & point of landing.

3) Maximum Height :- (H) or (H_{max}).

It is the vertical distance betn the point of projection and the point (c) where the vertical component of velocity is zero.

4] Angle of Projection: (α)

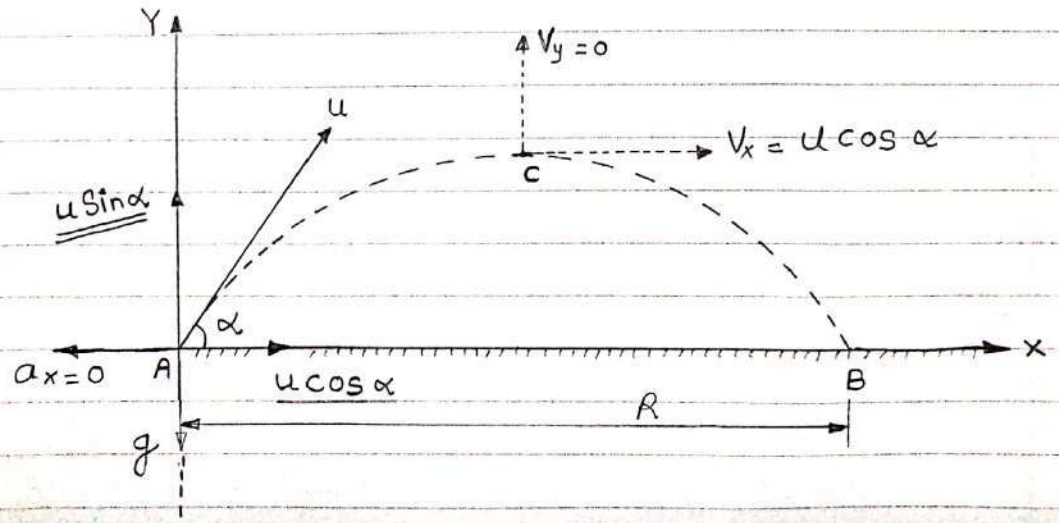
- It is the angle made by velocity with the Horizontal.

- If velocity is directed up the horizontal, then it is called as angle of elevation.
- If the velocity is directed down the Horizontal, then it is called as angle of depression.

5) Trajectory :- It is the path traced by a projectile during its motion. It is parabolic in nature.

* Projectile on Horizontal plane *

Consider a projectile projected from point A with
u = initial velocity of projection &
α = angle of projection.
let t = total time of flight.
Thus projectile will land at point B after time 't'.
Both Point A & B are on H.P.



As the air resistance is neglected, the motion in x-direction is uniform motion & y dirⁿ motion is "motion under Gravity".

a) Time of Flight (t).

$$\boxed{t = \frac{2u \sin\alpha}{g}}$$

b) Horizontal Range (R)

$$\boxed{R = \frac{u^2 \cdot \sin 2\alpha}{g}}$$

c) Maximum Range (R_{max})

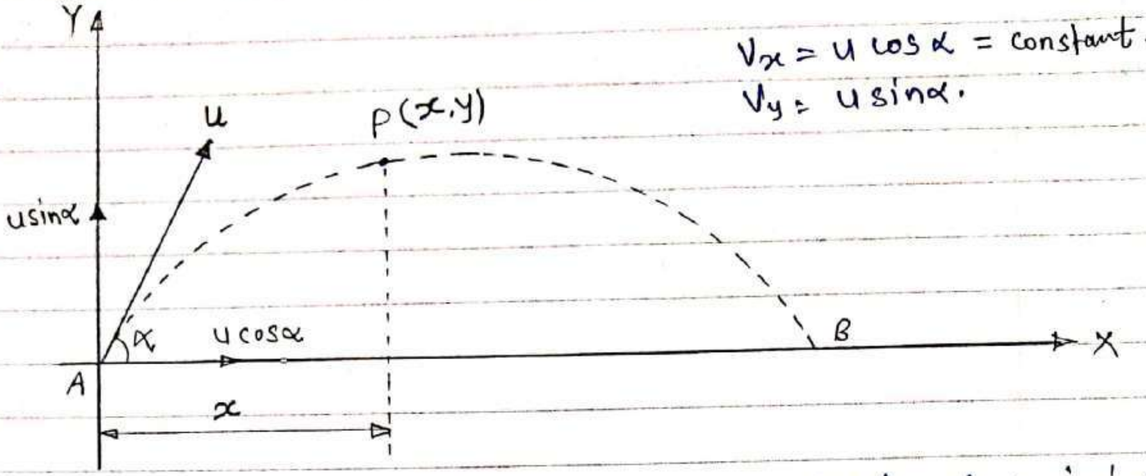
for maximum Range angle of projection must be $45°$.

$$\boxed{R_{max} = \frac{u^2}{g}}$$

d) Maximum Height

$$\boxed{H = \frac{u^2 \cdot \sin^2\alpha}{2g}}$$

* Derivation of Path Equation *
[Eqn of Trajectory]



$V_x = u\cos\alpha$ = constant.
$V_y = u\sin\alpha$.

Consider a particle projected from A with initial velocity 'u' & angle of projection 'α'.
let after time 't_1', the particle has reached at point P (x,y).

Consider the motion of projectile in x dirn (UM) :- [A → P]

$$S = \text{velocity} \times \text{time}$$

$$x = u\cos\alpha \cdot t_1$$

$$\therefore t_1 = \frac{x}{u\cos\alpha} \quad \text{-----} \quad (1)$$

consider the motion of projectile in y dirn (M.U.G.) [A → P]

$$\therefore S_y = U_y t_1 - \frac{1}{2} g t_1^2$$

$$y = u\sin\alpha . t_1 - \frac{1}{2} g t_1^2$$

from eqn (1), put the value of time t_1.

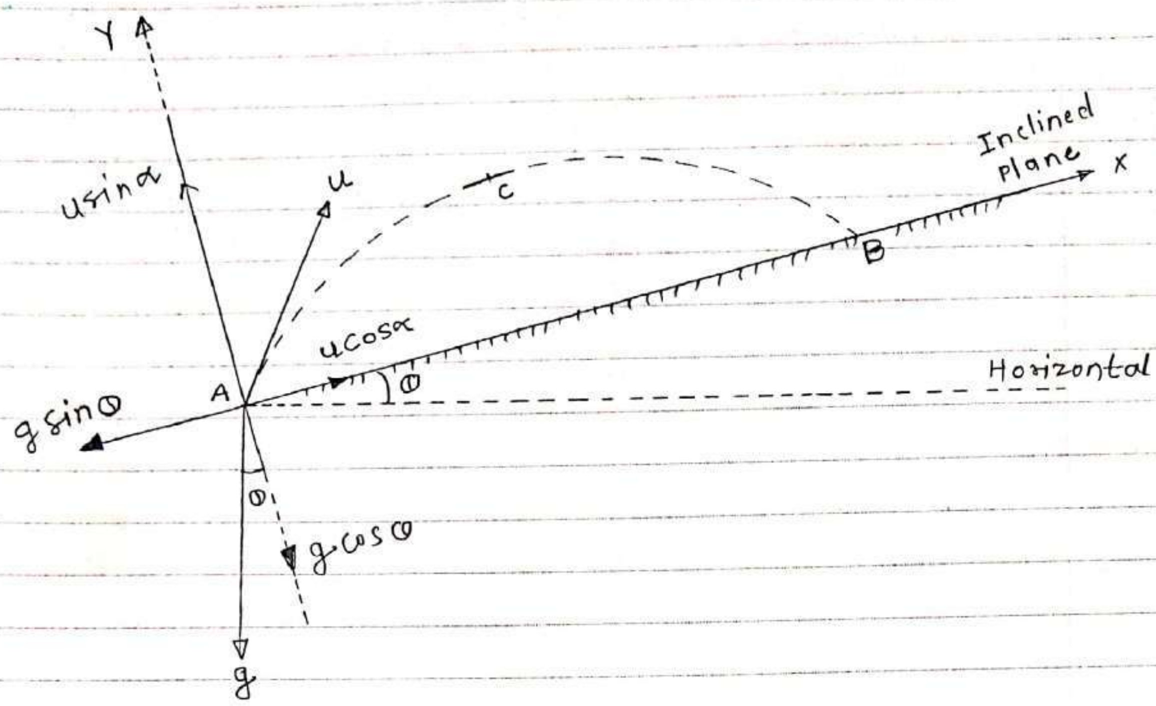
$$\therefore y = u\sin\alpha . \left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2} g \left(\frac{x}{u\cos\alpha}\right)^2$$

$$\therefore y = \frac{\cancel{u} . x . \sin\alpha}{\cancel{u}\cos\alpha} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\alpha}$$

$$\therefore \boxed{y = x . \tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}}$$

Eqn of Trajectory.

* Projectile on Inclined Plane *



Let projectile is projected from point A.

let α = angle of projection with (inclined) plane.

θ = Angle of inclined plane with Horizontal.

Now let us select x axis along the inclined plane and y-axis perpendicular to the inclined plane.

$\therefore$ x component of velocity = $u \cos\alpha$

$\therefore$ y —"— —"— —"— —"— = $u \sin\alpha$

Similarly for gravitational acc^n 'g',

x component = $g \sin\theta$

y component = $g \cos\theta$

[a] Time of flight (t)

$$\boxed{t = \frac{2u \cdot \sin\alpha}{g \cos\theta}}$$

[b] Range along the plane (R)

$$\boxed{R = \frac{2u^2 \sin\alpha}{g \cos^2\theta} \cdot \cos(\alpha+\theta)}$$

[c] Maximum Range (Rmax)

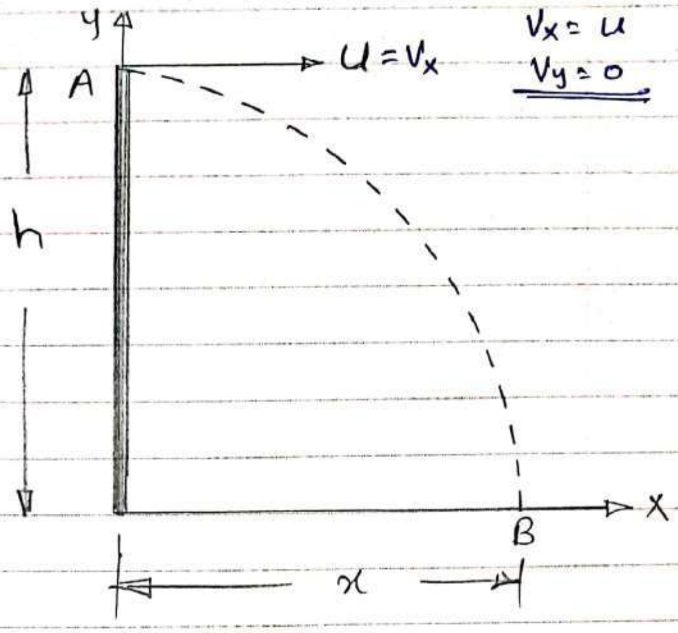
$$\boxed{R_{max} = \frac{u^2}{g(1+\sin\theta)}}$$

[d] Max. Height ($\perp^{ar}$ to plane)

$$\boxed{H = \frac{u^2 \sin^2\alpha}{2g \cdot \cos\theta}}$$

* Special cases of projectile *

* Projectile Projected with Horizontal velocity :-



$V_x = u$
$V_y = 0$

x motion
consider motion $A \to B$
(U.M.)

$$\boxed{x = u \times t}$$

consider motion (y-motion)
from $A \to B$ (M.U.G)

$$s = ut + \frac{1}{2}gt^2$$
$$h = 0 + \frac{1}{2}gt^2$$
$$\therefore \boxed{t = \sqrt{\frac{2h}{g}}}$$

$\therefore$ Horizontal distance, $= \boxed{x = u\sqrt{\frac{2h}{g}}}$

$$y = x\tan\alpha - \frac{gx^2}{u^2\cos^2\alpha} \quad \text{— eq}^n \text{ of Trajectory}$$

But $\alpha = 0$ At point A.

$$\therefore -h = -y = 0 - \frac{gx^2}{2u^2}$$
$$\therefore -h = 0 - \frac{gx^2}{2u^2}$$
$$\therefore \boxed{h = \frac{gx^2}{2u^2}}$$

* For given value of u, two angle gives us the same Range.

$$\boxed{\begin{aligned} \alpha_1 &= \alpha \\ \alpha_2 &= \frac{\pi}{2} - \alpha \end{aligned}}$$

Numericals on Projectile Motion.
(Projectile on Horizontal plane)

* A projectile is fired with a velocity of 60 m/s on Horizontal plane. Find its time of flight in the following 3 cases.

a) Its Range is 4 times the max. Height
b) Its max height is 4 times Horizontal range
c) Its max. Height & Horizontal range are equal.

⇒ $u = 60$ m/s

a) when $R = 4H$

$$\frac{u^2 \sin 2\alpha}{g} = 4\left[\frac{u^2 \cdot \sin^2\alpha}{2g}\right]$$

$$\therefore \frac{u^2}{g} 2\sin\alpha\cos\alpha = \frac{2u^2}{g}\sin^2\alpha$$

$$\therefore \cos\alpha = \sin\alpha.$$

$$\therefore \cos\alpha - \sin\alpha = 0$$

$$\therefore \boxed{\alpha = 45^\circ}$$

Time of flight $t = \frac{2u\sin\alpha}{g} = \frac{2\times 60\times \sin 45}{9.81} = 8.65$ sec.

b) when $H = 4R$

$$\therefore \frac{u^2\sin^2\alpha}{2g} = 4\left[\frac{u^2\sin 2\alpha}{g}\right]$$

$$\therefore \sin^2\alpha = 8\sin 2\alpha$$

$$\therefore \sin^2\alpha = (8\times 2)\sin\alpha\cos\alpha$$

$$\therefore \sin\alpha = (2\times 8)\cos\alpha$$

$$\therefore \sin\alpha = 16\cos\alpha$$

$$\therefore \tan\alpha = 16 \quad \& \quad \boxed{\alpha = 86.42^\circ}$$

$$t = \frac{2u\sin\alpha}{g} = \frac{2\times 60\times \sin 86.42}{9.81} = 12.21 \text{ sec}$$

c) when $H = R$

$$\frac{u^2\sin^2\alpha}{2g} = \frac{u^2\sin 2\alpha}{g}$$

$$\therefore \frac{\sin^2\alpha}{2} = \sin 2\alpha$$

$$\therefore \sin^2\alpha = 2\times 2\sin\alpha\cos\alpha$$

$$\therefore \sin\alpha = 4\cos\alpha.$$

$$\therefore \tan\alpha = 4 \quad \therefore \boxed{\alpha = 75.96^\circ}$$

$$t = \frac{2u\sin\alpha}{g}$$

$$= \frac{2\times 60\times \sin 75.96}{9.81}$$

$$t = 11.87 \text{ sec.}$$

A projectile is aimed at an object on a H.P. through the point of projection and falls 8 m short when the angle of projection is 15°; while it overshoots the object by 18 m when the angle of projection is 45°. Determine the angle of projection to hit the object exactly.

⇒ let R = actual Range required to hit the object.

⇒ Case . I — $\alpha = 15°$

$$\text{Range} = R - 8$$

$$\therefore \frac{u^2 \sin 2\alpha}{g} = R - 8$$

$$\therefore \frac{u^2 \times \sin(2\times 15)}{g} = (R-8)$$

$$\therefore 0.5\frac{u^2}{g} = (R-8)$$

multiply both sides by 2

$$\therefore \frac{u^2}{g} = 2R - 16 \text{ ---- (I)}$$

Case : (II) $\alpha = 45°$

$$\text{Range} = R + 18$$

$$\frac{u^2 \sin 2\alpha}{g} = R + 18$$

$$\frac{u^2}{g} \cdot \sin 90 = R + 18$$

$$\therefore \frac{u^2}{g} = R + 18 \text{ ---- (II)}$$

from (I) & (II),

$$R + 18 = 2R - 16$$

$$\therefore 2R - R = 18 + 16 = 34.$$

$$\therefore \boxed{R = 34\,m}$$ --- Actual Range to hit the object

Actual Range

$$R = \left(\frac{u^2}{g}\right)\sin 2\alpha.$$

$$34 = (R+18)\sin 2\alpha.$$

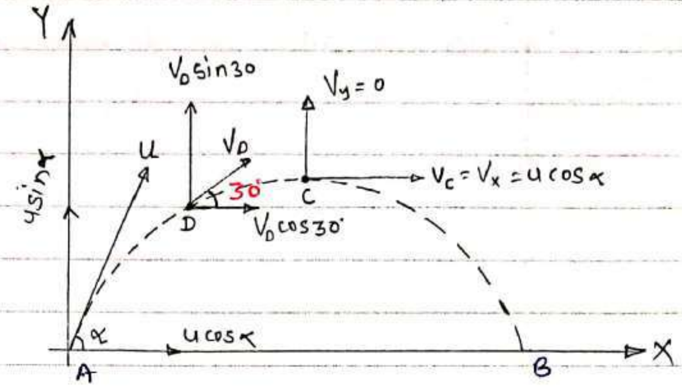
$$34 = (34+18)\sin 2\alpha$$

$$34 = 52\sin 2\alpha.$$

$$\therefore \sin 2\alpha = 0.653$$

$$\boxed{\alpha = 20.38°}$$ — Angle of projection to hit the object.

A shot is fired from the gun. After 2 sec. the velocity of shot is inclined at 30° up the horizontal. After 1 more second, it attains max. height. Determine the initial velocity and angle of projection.



let, u = initial velocity

α = angle of Projection.

let, after 2 seconds, the shot fired from gun reaches at point D. Here V_D makes 30° angle with Horizontal.

&

let after one more second, shot attains max. Height at point C as shown in figure.

At point D, V_D makes 30° with Horizontal.

∴ x component of velocity at 'D' = $V_D \cos 30$

But we know that velocity in x dir^n is constant (U.M.)

∴ $V_D \cos 30 = u\cos\alpha$

∴ $0.87 V_D = u\cos\alpha$ ---- ①

Consider y-motion from A to D. This is motion under gravity.

∴

$V = u + at$

∴ $V_D \sin 30 = u\sin\alpha - g t_{AD}$

∴ $0.5 V_D = u\sin\alpha - 9.81 \times 2$

∴ $0.5 V_D = u\sin\alpha - 19.62$ — ②

Now,

consider Y motion from D–C, (M.U.G)

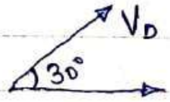
$V = u + at$

$0 = V_D \sin 30 - g \times t_{DC}$

$0 = 0.5 V_D - 9.81 \times 1$

∴ $0.5 V_D = 9.81$

∴ $\boxed{V_D = 19.62 \text{ m/s}}$



By substituting the value of V_D in eq^n ① & ⑪

$0.87 V_D = u\cos\alpha.$

∴ $u\cos\alpha = 0.87 \times 19.62$

∴ $u\cos\alpha = 17.069$ m/s → — ③

Also,

$u\sin\alpha = 0.5 V_D + 19.62$

$= 0.5 \times 19.62 + 19.62$

∴ $u\sin\alpha = 29.43$ m/s ↑ — ④

from eq^n ③ & ④

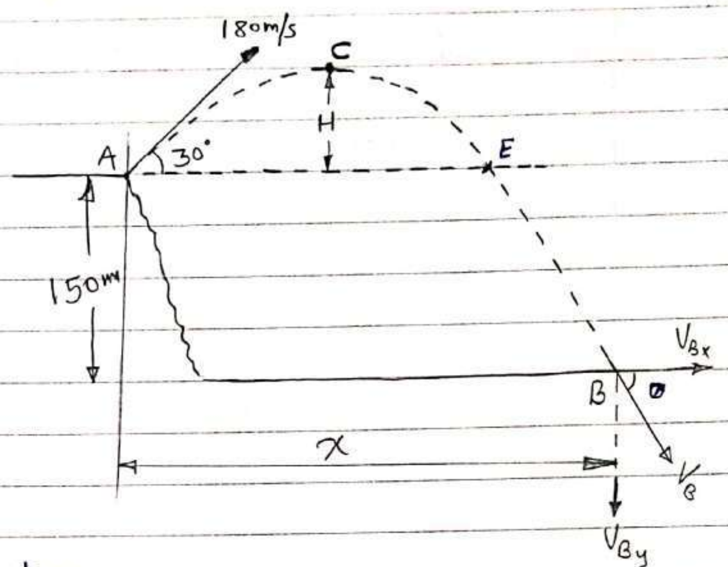
$$\frac{u\sin\alpha}{u\cos\alpha} = \frac{29.43}{17.069}$$

∴ $\tan\alpha = 1.724$

∴ $\boxed{\alpha = 59.886°}$

& $u = \frac{29.43}{\sin 59.886} = \boxed{34.02}$ m/s

A projectile is fired from the edge of a 150m cliff with an initial velocity of 180 m/s at 30° angle with Horizontal. Find ① The Horizontal distance from the gun to the point where the projectile strikes the ground 2) The greatest elevation above the ground reached by projectile ③ striking velocity. Refer the given figure. (PU - may - 15)



let

x = Horizontal distance betn A & B

A = point of projection

B = point of striking.

we can see from the fig. that A & B are Not on same level.

t_{AB} = time Req. $= t_{AC} + t_{CB}$

consider the Horizontal motion from A to B (U.M.)

$\therefore$ Distance = velocity × time

$$x = 180 \cos 30^\circ \times t_{AB}$$

$$x = 155.88\, t_{AB} \quad - ①$$

consider vertical motion from A to C,

$\therefore V = u + at$

$$V_{cy} = 180 \sin 30 - g \times t_{AC}$$

$$0 = 90 - 9.81\, t_{AC}$$

$$\therefore t_{AC} = \frac{90}{9.81}$$

$$\therefore \boxed{t_{AC} = 9.17 \text{ sec.}}$$

consider vertical motion from A to E

$$H = \frac{u^2 \sin^2\alpha}{2g} = \frac{180^2 \times \sin^2 30}{2 \times 9.81}$$

$$H = 412.84 \text{ m.}$$

$\therefore$ Now using, eqn of motion

$$S = ut + \frac{1}{2} g t^2$$

$$H + 150 = 0 + \frac{1}{2} \times 9.81 \times t_{CB}^2$$

$$412.84 + 150 = 4.905\, t_{CB}^2$$

$$\therefore t_{CB}^2 = \frac{562.84}{4.905}$$

$$\therefore t_{CB}^2 = 114.748$$

$$\therefore \boxed{t_{CB} = 10.71 \text{ sec}}$$

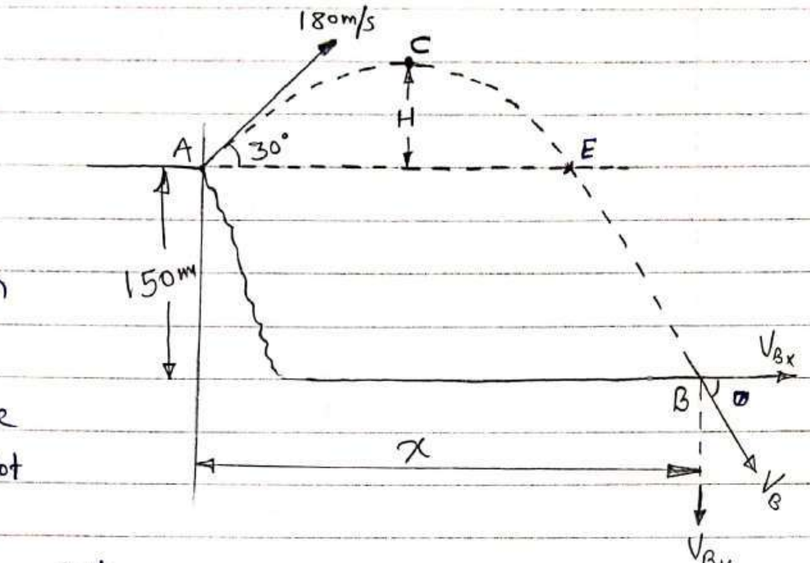
$$t_{AB} = 9.17 + 10.71 = 19.88 \text{ sec.}$$

from eqn ①

$$x = 155.88\, t_{AB} = 155.88 \times 19.88$$

$$\boxed{x = 3098.9 \text{ m}}$$

A projectile is fired from the edge of a 150 m cliff with an initial velocity of 180 m/s at 30° angle with Horizontal. Find ① The Horizontal distance from the gun to the point where the projectile strikes the ground 2) The greatest elevation above the ground reached by projectile ③ striking velocity. Refer the given figure. (PU - may - 15)



Let
x = Horizontal distance betn A & B

A = point of projection
B = point of striking.

we can see from the fig. that A & B are Not on same level.

t_{AB} = time Req. = $t_{AC} + t_{CB}$

consider the Horizontal motion from A to B (U.M.)

$\therefore$ Distance = velocity × time

$$x = 180 \cos 30 \times t_{AB}$$

$$x = 155.88\, t_{AB} \quad \text{— ①}$$

consider vertical motion from A to C,

$\therefore V = u + at$

$$V_{cy} = 180 \sin 30 - g \times t_{AC}$$

$$0 = 90 - 9.81\, t_{AC}$$

$$\therefore t_{AC} = \frac{90}{9.81}$$

$$\therefore \boxed{t_{AC} = 9.17 \text{ sec.}}$$

consider vertical motion from A to E

$$H = \frac{u^2 \sin^2\alpha}{2g} = \frac{180^2 \times \sin^2 30}{2 \times 9.81}$$

$$H = 412.84 \text{ m.}$$

$\therefore$ Now using, eqn of motion

$$S = ut + \frac{1}{2} g t^2$$

$$H + 150 = 0 + \frac{1}{2} \times 9.81 \times t_{CB}^2$$

$$412.84 + 150 = 4.905\, t_{CB}$$

$$\therefore t_{CB}^2 = \frac{562.84}{4.905}$$

$$\therefore t_{CB}^2 = 114.748$$

$$\therefore \boxed{t_{CB} = 10.71 \text{ sec}}$$

$$t_{AB} = 9.17 + 10.71 = 19.88 \text{ sec.}$$

from eqn ①

$$x = 155.88\, t_{AB} = 155.88 \times 19.88$$

$$\boxed{x = 3098.9 \text{ m}}$$

Horizontal distance from the to the point of striking is

$$\boxed{x = 3098.9 \text{ m}}$$

Time req. from A to B = $t_{AB} = 19.88$ sec.

greatest Height reached by projectile above the ground is

$$H_{max} = H + 150 = 412.84 + 150$$

$$\boxed{H_{max} = 562.84 \text{ m}}$$

Now,

consider that V_B = striking velocity. &

θ = angle made by striking velocity with Horizontal. as shown.

let V_{Bx} = x component of V_B.

V_{By} = y component of V_B.

But we know that, in x dirn, motion is uniform.

Thus $V_{Bx} = u\cos\alpha = 180\cos 30 = 155.88$ m/sec.

To find V_{By}, consider the motion from C to B.

$\therefore \quad V = u + gt$

$V_{By} = V_{cy} + g \times t_{CB}$

$V_{By} = 0 + 9.81 \times 10.71$

$$\boxed{V_{By} = 105.06 \text{ m/sec.}}$$

$\therefore \; V_B = \sqrt{V_{Bx}^2 + V_{By}^2} = \sqrt{155.88^2 + 105.06^2}$

$$\boxed{V_B = 187.9 \text{ m/sec}}$$

$\tan\theta = \dfrac{V_{By}}{V_{Bx}} = \dfrac{105.06}{155.88}$

$$\therefore \boxed{\theta = 33.97^\circ}$$