Kinetics :-

Motion of the bodies by considering the force causing motion.

OR

motion of the bodies under the action of unbalanced force acting on it.

Newton's 1st law :-

Every body continues in its state of rest or state of uniform motion unless it is acted upon by some external force.

Newton's 2nd law :-

The rate of change of momentum is directly proportional to the impressed force & takes place in the dirn of force.

$$F = m \cdot a.$$

Newton's Third law :-

To every action, there is equal & opposite reaction.

Application of Newton's 2nd law.

1) For Rectangular co-ordinate system

When unbalanced force system is acting on particle having accln 'a', then, by newton's 2nd law, we can write,

$$\Sigma \left[F_x i + F_y j + F_z k \right] = m \left[a_x i + a_y j + a_z k \right]$$

OR

$$\Sigma F_x = m a_x$$
$$\Sigma F_y = m a_y$$
$$\Sigma F_z = m a_z$$

F_x, F_y, F_z = are the forces acting along i, j, k directions respectively.

i, j, k = directions along x, y, z ~~dirn~~ axis respectively.

a_x, a_y, a_z = Component of accln 'a' acting in x, y, z directions or acting in i, j, k dirn.

(2) Appl^n of 2nd law for curvilinear motion of particle
(Using Tangential & Normal component).

let, a = accl^n of particle.

a_T = Tangential component of Accl^n

a_N = Normal component of Accl^n

Then,

according to newton's 2nd law,

$\therefore \Sigma F_T = m \times a_T$ & $\Sigma F_N = m \cdot a_N$

$\therefore \Sigma F_T = m \cdot \left(\frac{dv}{dt}\right)$ & $\Sigma F_N = \left(\frac{v^2}{\rho}\right) \times m.$

(3) Appl^n of 2nd law for Radial & Transverse coordinates,

when particle moves along curve & No. of forces are acting on it, then, according to 2nd law,

$\Sigma F_r = m \cdot a_r$ & $\Sigma F_\theta = m \cdot a_\theta$

The force of equal magnitude but acting in the opposite direction of motion is called as inertia force.

Inertia force can keep the particle in equillibrium.

* D'-Alembert's Principle *

It states that the sum of external force acting on a body and inertia force is always equal to zero.

$$\Sigma F + (-ma) = 0.$$

1) for Rectilinear motion,

$\Sigma F_x + (-m a_x) = 0$ & $\Sigma F_y + (-m a_y) = 0$

2) for curvilinear motion

$\Sigma F_T + (-m a_T) = 0$ &

$\Sigma F_N + (-m a_N) = 0$.

* D'Alembert's principle for curvilinear motion *

(A) D'Alembert's principle for curvilinear motion in Cartesian co-ordinates

$$\Sigma F = ma$$

$\therefore \Sigma F_x = ma_x$

$\therefore \Sigma F_y = ma_y$

$\Sigma F_z = ma_z$

(B) D'Alembert's principle for curvilinear motion in path variables (Tangential & Normal co-ordinates)

$\Sigma F_T = ma_T$ & $\Sigma F_N = ma_N$

$\Sigma F_T = m.\left(\frac{dv}{dt}\right)$ $\Sigma F_N = m\left(\frac{v^2}{\rho}\right)$.

(C) For Radial & Transvers co-ordinates.

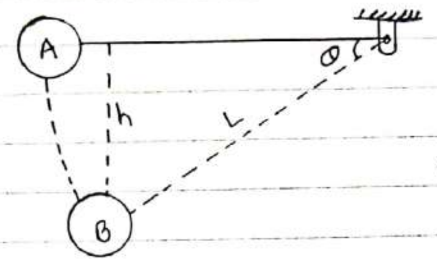
$\Sigma F_r = ma_r$ & $\Sigma F_\theta = ma_\theta$

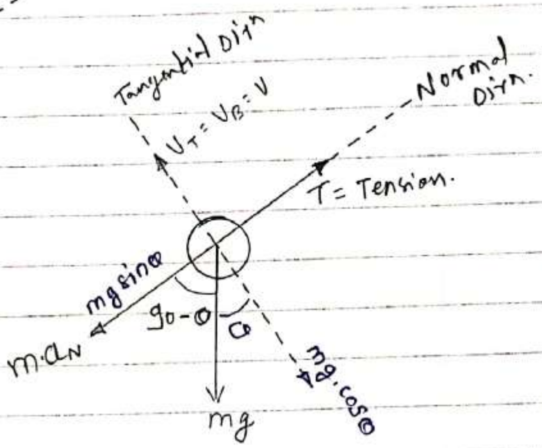
$\therefore \Sigma F_r = m.(\ddot{r} - r(\dot{\theta})^2)$ $\Sigma F_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$

Numericals on D'Alembert's principle for curvilinear Motion

* A pendulum bob has a mass of 10 kg & is realeased from rest when $\theta = 0^\circ$ as shown in figure. Determine the tesion in the cord at $\theta = 30^\circ$. Neglect the size of Bob. (Dec-15)

$h = L\sin\theta$





consider the pendulum is initially at A & when it is released, it occupy new position at B.

θ = angle displaced.

L = length of string of pendulum = Radius of curvature $= \rho$.

$\therefore \quad L = \rho$.

Now, at point B, Apply D'Alembert principle in Normal dirⁿ.

$$\Sigma F_N = m \cdot a_N$$

$$T - m \cdot g \cdot \sin\theta = m \cdot \left(\frac{V^2}{\rho}\right)$$

$$\therefore \quad T = mg\sin\theta + \frac{m \cdot V^2}{\rho} \quad \text{— (1)}$$

Here V = velocity at B = V_B

$\therefore$ using eqⁿ of motion: $\therefore V_B^2 - U_A^2 = 2gs$

$$\therefore \quad V_B^2 - 0 = 2gh$$

$$\therefore \quad V_B = \sqrt{2gh} = \sqrt{2gL\sin\theta}$$

$\therefore \quad V = V_B = \sqrt{2gL\sin\theta}$

$\therefore \quad V^2 = V_B^2 = 2gL\sin\theta$ — put in eqⁿ (1), we get

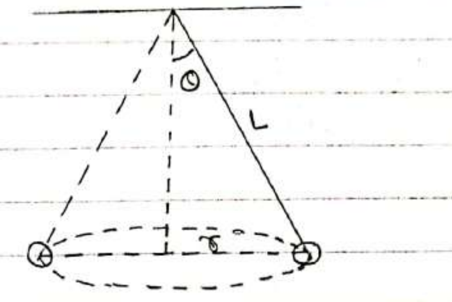
$$\therefore \quad T = mg\sin\theta + \frac{m \cdot 2g\cancel{L}\sin\theta}{\cancel{L}}$$

$$\therefore \quad T = mg\sin\theta + 2mg\sin\theta = 3mg\sin\theta$$

at $\theta = 30^\circ$, $\quad T = 3mg\sin\theta = 3mg\sin 30 = \boxed{0.5 \times 3mg}$ N.

A 0.5 kg ball revolves in a Horizontal circle as shown in figure. If L = 1m, & maximum allowable Tension in the string is 100N, Determine max. allowable speed & Corresponding angle θ.

consider free body diagram of ball as shown in 2nd figure.



Apply D'Alembert's principle, in x dirn

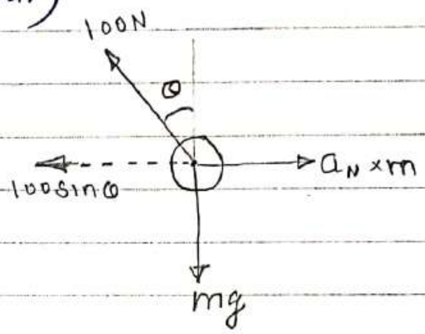
$\Sigma F_N = -(m \cdot a_N)$

$-100 \sin\theta = -(m \cdot a_N)$

$-100 \sin\theta = -\left(0.5 \times \frac{V^2}{\rho}\right)$

Here $\rho = L\sin\theta = r$

(positive radial dirn is negative Normal dirn)



$\therefore 100 \sin\theta = 0.5 \frac{V^2}{\rho}$

$\therefore \frac{100 \sin\theta \times \rho}{0.5} = V^2$

$\therefore V^2 = 200 \sin\theta \times \rho$

$V^2 = 200 \rho \sin\theta$ — (1)

Apply dealemberts principle in y dirn.

$\Sigma F_y = m \cdot a_y$

but as there is no velocity in y dirn

Thus $a_y = 0$.

$\therefore 100\cos\theta - mg = m \times 0 = 0$

$100 \cos\theta = 0.5 \times 9.81$

$\cos\theta = \frac{4.905}{100}$

$\boxed{\theta = 87.18°}$ — put in eqn (1)

$V^2 = 200 \times \rho \sin\theta = 200 \times L \sin\theta \times \sin\theta$

$V^2 = 200 \times 1 \sin 87.18 \times \sin 87.18$

$V^2 = 199.52$

$\therefore \boxed{V = 14.12\ m/s}$ — max, allowable speed.

Work Energy Principle & Impulse-Momentum Theorem.

Unit -: 4 [A] Work Energy principle *

* Work and Workdone: *

The work is said to be done when a force acts on a body & body moves through certain distance.

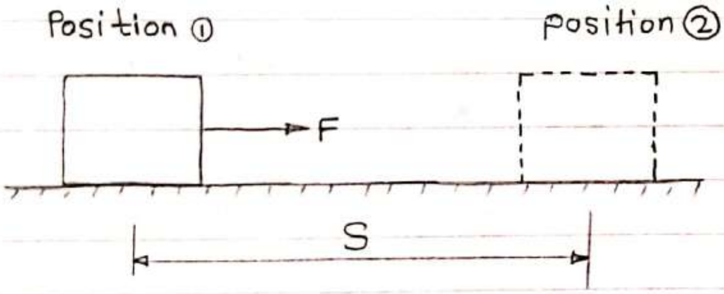
W.D. = Force x distance
$U = F \times S$

Unit of W.D.
N.m or Joule (J)
1 N.m = 1 J

If body does not move in the direction of force, then
W.D. = Component of force in the direction of motion x Displacement or distance.

① Workdone by an External force :-

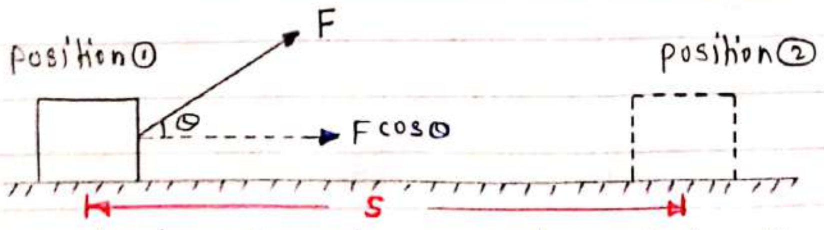
a]



consider an object at position ① & subjected to force 'F'. Due to force F, an object will be displaced from the position ① to new position ② at a distance 'S'. Then workdone is given as,

W.D = Force x displacement
$U = F \times S$ .... (+ve if W.D. is in the dirn of force).

b]

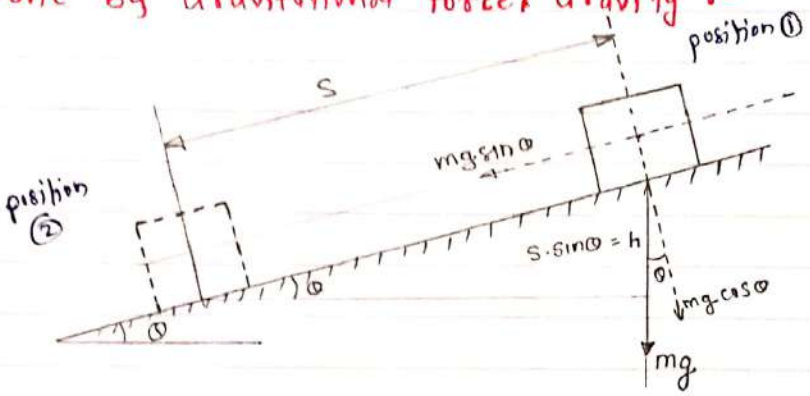


If an object is subjected to a force 'F' acting at an angle θ with Horizontal, then object is displaced from position ① to position ② at a distance 'S'. Here the work is done due to component of force in dirn of motion (displacement)

W.D. = U = component of force in dirn of displacement x displacement.

$$\boxed{U = F\cos\theta \times S}$$

② Workdone by Gravitational Force / Gravity :-

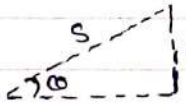


As shown in figure, if object of mass 'm' is displaced from position ① to position ②, then w.D. is given by,

w.D. = weight × displacement in the dirⁿ of weight.

$U = m.g. \times h$

$U = m.g.S.\sin\theta$ $\therefore h = S.\sin\theta$



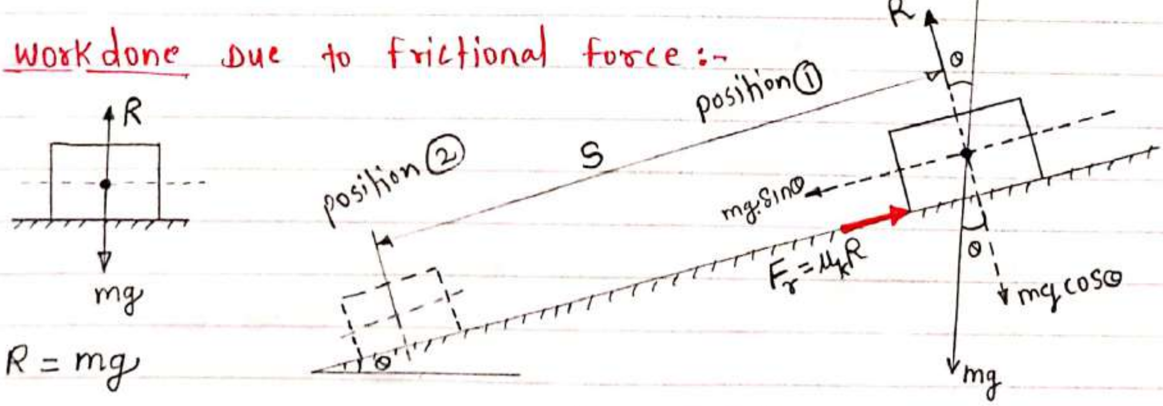
$\therefore \boxed{U = mg.\sin\theta \cdot S}$

If Block is moving from ① to ② i.e. in the down dirⁿ (in the dirⁿ of gravity) then w.D. is +ve.

If block is moving from position ② to ① then it against the gravity, so w.D. is negative (−ve).

$\boxed{U = -mg\sin\theta.S}$

③ Workdone Due to frictional force :-



$R = mg$

R = Normal Reaction

R = mg - for Horizontal plane

R = mg cosθ - for inclined plane.

$\therefore$ Frictional force = $\boxed{F_r = \mu_k R}$

$\therefore$ W.D. by Frictional force = Frictional force × displacement

$\therefore$ $U = -F_r \times S$

$\boxed{U = -\mu_k R \times S}$

μ_k = Coefficient of kinetic friction

* Frictional Force (F_r) always acts in the opposite Direction of displacement. Thus w.D. is −ve.

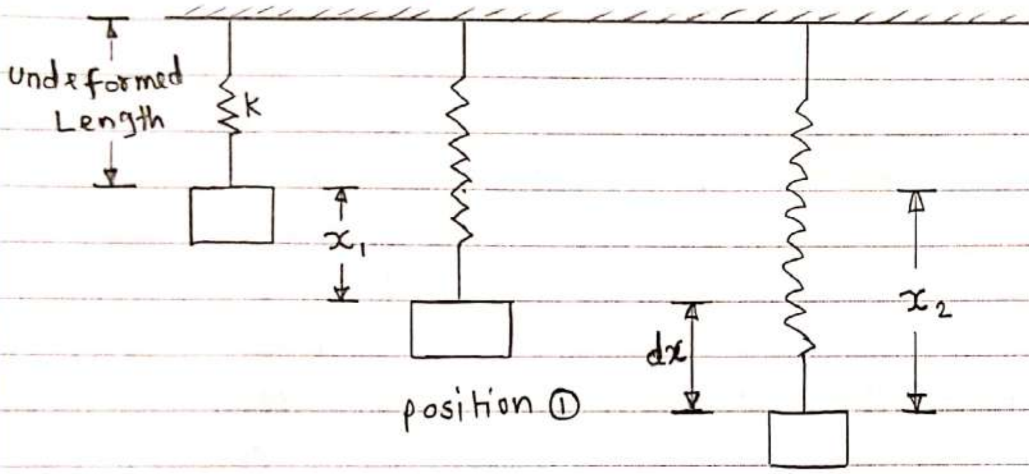
- W.D. by frictional force is always negative.
- Use coefficient of kinetic friction μ_K
- when object is moving on the Horizontal plane, then

$$R = mg$$

- when object is moving on the inclined plane, then

$$R = mg \cdot \cos\theta.$$

(4) Workdone by Spring force:-



Consider spring with stiffness K as shown in figure with undeformed length.

let x_1 = deformation of spring at position (1) of an object.
x_2 = deformation of spring at position (2) of an object.

Then Spring Force = $F = K \cdot x$

$\therefore$ W.D. = U = Spring force x displacement betn (1) & (2)

$$\therefore \quad U = -K \cdot x \cdot dx$$

$$= \int_{x_1}^{x_2} -K x \cdot dx$$

$$= -\frac{1}{2} K \left(x_2^2 - x_1^2 \right)$$

$$\boxed{U = \frac{1}{2} K \left(x_1^2 - x_2^2 \right)}$$

K = spring constant or stiffness ⇒ Unit (N/m)

–ve sign indicates that spring force acts in the dirn opposite to displacement i.e. toward original position.

* Energy * (Energy of a particle)

The capacity to do work is called as Energy.
It is Scaler quantity.
Unit - Joule

Types of Energy: 1] Mechanical Energy - potential Energy
- kinetic Energy.

2] Thermal Energy.
3] Electrical Energy.

Potential Energy :-

The energy possesed by a particle by virtue of its position is known as potential Energy.

$$P.E. = m.g.h.$$

Kinetic Energy :-

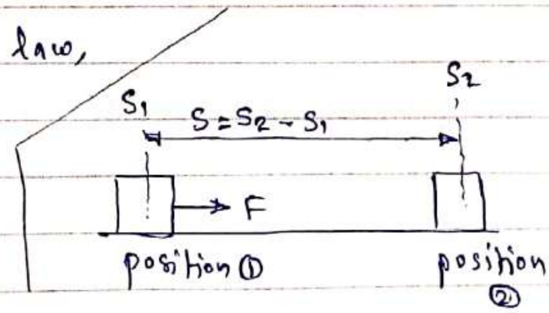
The energy possesed by a particle due to its motion is called as kinetic Energy.

$$K.E. = \frac{1}{2} m.V^2$$

* Work-Energy principle *

It states that "The total workdone by all the forces acting on a particle during some displacement is equal to the change in kinetic energy during that displacement".

From Newton's 2nd law,



$$\therefore \quad F = m.a.$$

$$\therefore F = m.v.\frac{dv}{ds}$$

$$\therefore F.ds = m.v.dv$$

Integrate both sides,

$$\therefore \int_{S_1}^{S_2} F ds = \int_u^v mv dv$$

$$\therefore F \int_{S_1}^{S_2} ds = m \int_u^v v.dv$$

$$\therefore F[S]_{S_1}^{S_2} = m.\left[\frac{V^2}{2}\right]_u^V$$

$\therefore F(S_2 - S_1) = m \cdot \left[\frac{V^2}{2} - \frac{u^2}{2}\right]$

$\therefore$ But $S_2 - S_1 = S$

$F \times S = \frac{mV^2}{2} - \frac{mu^2}{2}$

& $F \times S$ = workdone by particle from position 1 to 2.

$\therefore U_{1-2} = \frac{mV^2}{2} - \frac{mu^2}{2}$

$\therefore U_{1-2}$ = Final K.E. − Initial K.E.

$\therefore \boxed{U_{1-2} = K.E._2 - K.E_1}$

$\boxed{U_{1-2} = \text{change in kinetic Energy}}$

$K.E_1$ = kinetic Energy of particle at position ①
$K.E_2$ = — " —— of particle at position ②
U_{1-2} = Algebric sum of W.D. by all forces acting on the particle.

* Conservative Force :-

If workdone by a force is independent of the path followed by particle, then a force is known as conservative force.

Here, W.D. = change in potential energy of particle.

Ex. Spring force, Gravity force etc.

$\therefore W.D. = (P.E_2 - P.E_1)$

* Non Conservative Force :-

If W.D. by a force is dependent upon the path followed by particle, then force is called as - - Non-conservative force.

Ex. Frictional Force, viscous force etc.

* principle of conservation of Energy *

It states that " The total energy of a particle (P.E. + K.E.) remains constant during the displacement of a particle from position ① to position ② under the action of Conservative forces.

From work Energy principle :-

$$U_{1-2} = K.E._2 - K.E._1 \qquad -①$$

But,

under the action of conservative forces,

$$U_{1-2} = -(P.E._2 - P.E._1) \qquad ⑪$$

Equating ① & ⑪

$$K.E._2 - K.E._1 = -P.E._2 + P.E._1$$

$$K.E._2 + P.E._2 = K.E._1 + P.E._1$$

$$\therefore E_2 = E_1$$

$$\therefore \boxed{E_1 = E_2} \quad \text{where,}$$

$E_1 = P.E_1 + K.E._1$ = Total Energy at position ①

$E_2 = P.E._2 + K.E._2$ = Total Energy at position ②

when frictional forces are involved, then, above principle can not be used.

* Power :-

It is rate of doing work.

$$\therefore \text{power} = \frac{\text{Work}}{\text{Time}}$$

$$\therefore \text{power} = \frac{\text{Force} \times \text{displacement}}{\text{Time}}$$

$$\therefore \text{power} = \text{force} \times \frac{\text{Displacement}}{\text{Time}}$$

$$\therefore \text{Power} = \text{Force} \times \text{velocity}$$

$$\boxed{\text{Power} = F \times V}$$

Unit of power :

S.I. Unit

$$\frac{N.m}{sec} = \frac{\text{Joule}}{\text{sec}} = \text{watt}$$

M.K.S.

1 H.P = 746 watt

1 HP = 0.746 kw.

* Efficiency :-

The ratio of workdone by machine to the workdone on the machine during same time interval is called as efficiency.

$$\boxed{\eta = \frac{\text{Output power}}{\text{Input power}}}$$

Unit - 4 :- B] Impulse Momentum Theorem & Impact.

Impulse momentum theorem directly relates Force, velocity and time. The Theorem is useful where forces acts for very small intervals.

Important Terms :-

① Linear Momentum :-

It is the total quantity of motion possed by a body or Particle or object.

∴ Linear Momentum = mass x velocity

$= m \times v$

S.I. unit :- $\frac{kg \cdot m}{sec}$

② Angular Momentum :-

It is the product of mass moment of inertia of the body (I) and angular velocity of the Body.

∴ Angular Momentum = mass moment of inertia x Angular speed

∴ Angular Momentum = $I\omega$

S.I. unit is :- $\frac{kg \cdot m^2}{sec.}$ or $\frac{kg \cdot m^2 \cdot rad}{sec.}$

③ Impulsive force :-

A very large force acting on a particle during short time interval.

④ Impulsive Torque :-

A large torque acting on a particle during short time interval.

⑤ Linear Impulse :- (vector quantity)

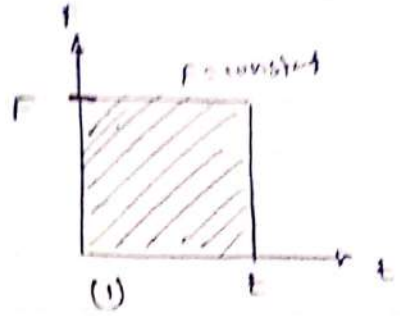
It is impulsive force acting on particle x time.

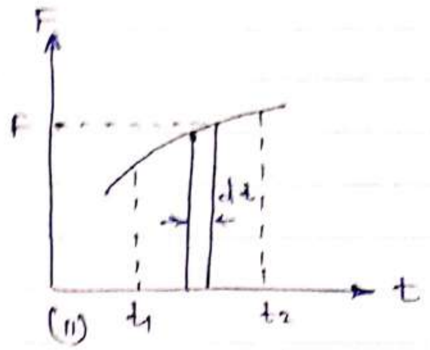
If large force (F) is acting on body for short time interval betn t_1 & t_2, then

∴ Linear Impulse$_{1 \to 2}$ $= \int_{t_1}^{t_2} F \cdot dt$

∴ Linear Impulse $= F \cdot (t_2 - t_1) = F \Delta t$

S.I unit → N. Sec.





Graphically, Impulsive force is Area under F-t diagram.

Impulse: $F \times t$

Impulse = Area under F-t diagram betn t_1 & t_2

$$= \int_{t_1}^{t_2} F \cdot dt$$

⑥ Angular Impulse

It is Large Torque acting on a body x time duration.

$$\therefore \text{Angular impulse} = \int_{t_1}^{t_2} T \cdot dt$$

$$= T(t_2 - t_1)$$
$$= T \Delta t$$

where
T = Torque
Δt = time duration.

S.I. Unit:
N·m·sec

⑦ Impulsive motion:-

The resulting motion induced due to or created due to Impulsive force or torque is called as Impulsive Motion.

✱ Impulse - momentum Theorem:-

It states that, when an Unbalanced system of force is acting on the particle for short time interval, The impulse produced by all impulsive forces is equal to change in momentum of particle.

from 2nd law of motion, we have

$$F = m \cdot a.$$

$$F = m \cdot \frac{dv}{dt}$$

$$\therefore F \cdot dt = m \cdot dv$$

Integrating both sides betn the position ① & ②,

$$\therefore \int_{t_1}^{t_2} F\,dt = \int_{u}^{v} m \cdot dv$$

if force and & mass are constant.

$$\therefore F\int_{t_1}^{t_2} dt = m\int_{u}^{v} dv$$

$$\therefore F\,(t)_{t_1}^{t_2} = m \cdot [v]_{u}^{v}$$

$$\therefore F\,[t_2 - t_1] = m \cdot [v - u]$$

$$\therefore F \cdot \Delta t = mv - mu$$

But $F \cdot \Delta t$ = impulse & mass x velocity = momentum.

$\therefore$ $\text{Impulse}_{1-2} = M_1 - M_2$

$$\boxed{\begin{array}{l}\therefore \text{Impulse} = \text{Final momentum} - \text{Initial momentum.} \\ \therefore \text{Impulse} = \text{Change in momentum.}\end{array}}$$

where,

Impulse = $F \cdot \Delta t$ = $F \times (t_2 - t_1)$,

Final momentum = mV = M_1

Initial momentum = mu = M_2

$\therefore$ Impulse in x direction = $m\,(V_x - u_x)$

$\therefore$ Impulse in y dirn = $m\,(V_y - u_y)$

—"— z dirn = $m\,(V_z - u_z)$

Non-Impulsive force :-

Any Force which does not produces any kind of impulse, is called as non-impulsive force.

Ex. 1) force exerted by spring
2) Forces of action & Reaction.
3) Internal forces. etc.

Conservation of momentum - principle :-

we have, Impulse momentum theorem,

Impulse = Final momentum - Initial momentum,

But when, 1) Resultant of forces is zero,
2) time interval Δt is very very small,
3) All external forces are non-impulsive,

Then
Impulse = 0.

∴ Impulse momentum Theorem can be written as,

0 = final Momentum - Initial Momentum.

∴ $\boxed{\text{Final Momentum} = \text{Initial momentum}}$

$$\Sigma mv = \Sigma mu.$$

The Total momentum remains constant.
i.e. Total momentum of particle is conservative.

* Total momentum is conserved only in one dirn.

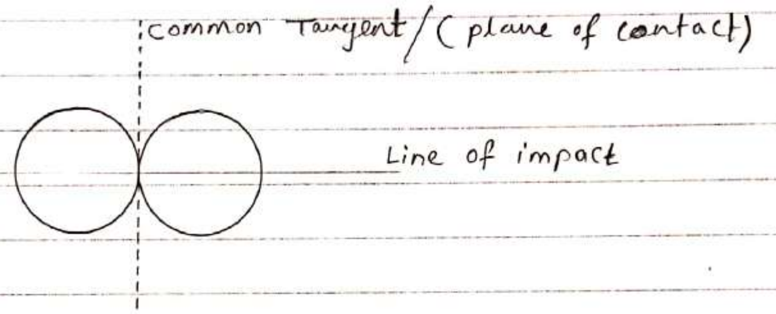
Impact :-

It is the collision betn two bodies, which occurs for a very small time interval. During this small time interval both bodies will exert large amount of force on each other.

① Line of impact :-

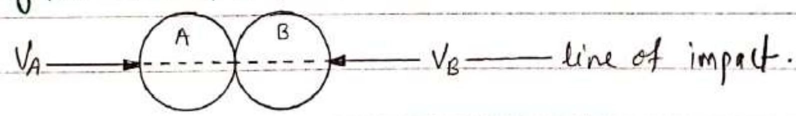
It is the common normal to plane of contact. OR It is the common normal to the surface of two bodies in contact during impact.

It is perpendicular to common tangent.



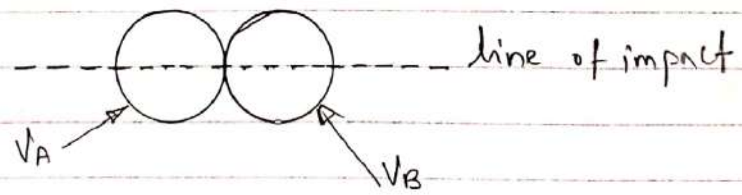
② Direct impact

When the velocities of the two bodies are along the line of impact then impact is said to be direct impact. Both particles are moving along the line of impact.



③ Oblique impact :-

When both particles does not move along the line of impact, i.e. when velocities are not along the line of impact then it is called as oblique impact.



④ Central Impact :-

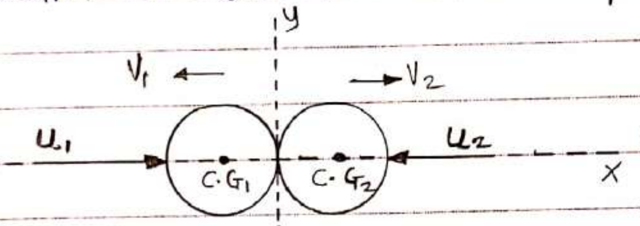
When mass centres of two colliding bodies lie on the line of impact, the impact is called as central impact.

(5) Non central impact

when mass centres (C.G.) of the two colliding bodies do not lie on the line of impact, it is then known to be noncentral impact.

(6) Direct-central Impact :-

when the velocities of the two colliding bodies and their mass centre lies on the line of impact, then it is called as Direct central impact.

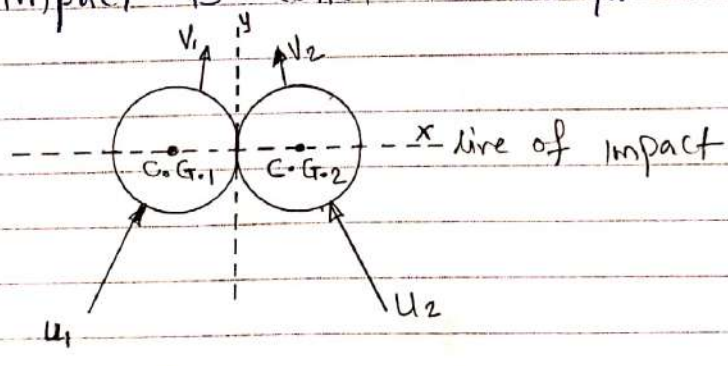


Acc to law of conservation of momentum } - Total momentum before impact = Total momentum after impact

$$\therefore \boxed{m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2.}$$

(7) oblique central impact

when the mass centres (C.G.) of the two colliding bodies ~~are~~ lies on the same line of impact But their velocities do not lie on the line of impact, then the impact is called as oblique central impact.



As per the law of conservation,

component of total momentum of two bodies along the line of impact is given by -

$$m_1u_{1x} + m_2u_{2x} = m_1V_{1x} + m_2V_{2x}$$

component of momentum along the common tangent is

$$m_1u_{1y} + m_2u_{2y} = m_1V_{1y} + m_2V_{2y}$$

* Restitution (Restoration)

After the impact, both objects tends to regain their original shape and size. This process is known as Restitution or Restoration. This process contunes for small time duration which is called as period of Restitution.

- Coefficient of Restitution (e)

It is the ratio of impulses during the restoration period and deformation period. It is denoted by e.
This is always positive and lies betn $0 \le e \le 1$.

$$e = \frac{\text{Impulse during restoration}}{\text{Impulse during deformation}}$$

when the impact takes place, the two objects initially goes under the deformation for small duration, & then, they tries to regain their original shape, size.

* Types of IMPACT *

① Perfectly Elastic Impact :-

In this impact, both bodies regain their shape & size completely. i.e. There is complete restoration. Here,

a] Momentum is conserved along the line of impact.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

b] K.E. is also conserved. No loss of K.E during impact.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

c] Coeff. of restitution (e) = 1

$$e = 1 = \left(\frac{v_2 - v_1}{u_1 - u_2}\right)$$

$$\therefore u_1 - u_2 = v_2 - v_1$$

$$\therefore u_1 + v_1 = u_2 + v_2$$

d] Both bodies separates after impact.

② Perfectly plastic Impact:-

In this impact, both bodies are coupled together after the impact and they move together with same velocity. Here permanent deformation takes place. So there is no Restitution.

a) Here, momentum is conserved:-

$$m_1u_1 + m_2u_2 = (m_1+m_2)V$$, where, V = common velocity of both bodies after impact.

b) K.E is not conserved. There is loss of K.E. during impact.

Loss of K.E. = Total K.E. before Impact − Total K.E. After Impact

$$\therefore \text{Loss of K.E} = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2\right)$$

c) Both bodies move together after impact.

d) Coefficient of Restitution (e) = 0.

③ Partially Elastic (semi elastic) Impact.

In this impact, both bodies do not regain their original shape & size completely. But there is a partial restoration.

a) Here, momentum is conserved.

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

b) K.E. is not conserved. There is some loss in K.E.

Initial K.E. > K.E. After impact.

K.E. before impact > K.E After impact.

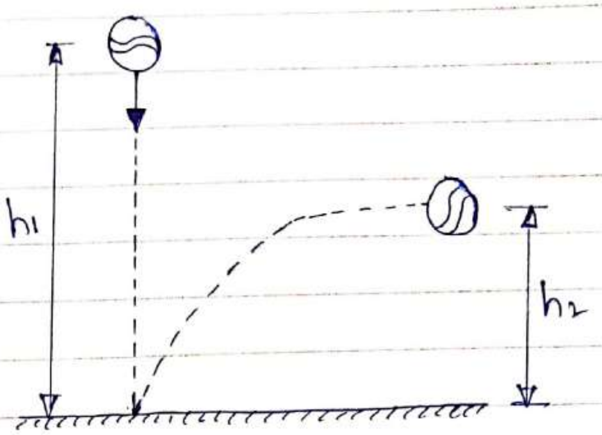
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 > \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

c) Coeff. of Restitution is more than zero & less than 1.

$$0 < e < 1.$$

(4) Impact with very large mass (infinite mass)

When a body of small mass collides with a body of very large mass as compared to first body, then the impact is considered as <u>impact with infinite mass.</u>



Ex. A ball is dropped on the floor.

a) In this type of Impact, law of conservation of momentum can not be applied.

b) There is no loss in K.E. due to impact.

c) coeff. of Restitution is.

$$e = \frac{V_2 - V_1}{U_1 - U_2}$$

But velocity of floor before & After impact is zero.

$$U_2 = V_2 = 0$$

$$\therefore \quad e = \frac{-V_1}{U_1}$$

velocity of ball just before the impact when it is dropped from height h_1 is,

$$U_1 = \sqrt{2gh_1} \ (\downarrow) \quad \therefore U_1 = -\sqrt{2gh_1} \quad \text{downward.}$$

After the impact ball rises to the height h_2, then its velocity after the impact is $V_1 = \sqrt{2gh_2} \ (\uparrow)$

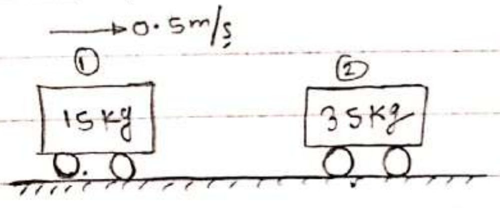
$$\therefore \quad e = \frac{-V_1}{U_1} = \frac{-\sqrt{2gh_2}}{-\sqrt{2gh_1}} = \frac{\sqrt{h_2}}{\sqrt{h_1}}$$

$$\therefore \boxed{e = \sqrt{\frac{h_2}{h_1}}}$$

where, h_1 = height just before impact

h_2 = height after impact.

A 15 kg wagon moving with a speed of 0.5 m/s towards right collides with 35 kg wagon which is at rest. If after collision the 35 kg wagon is observed to move towards right at a speed of 0.3 m/s, determine the coefficient of restitution between the two wagons. Also comment on the impact.



Given data :-

wagon-1

$m_1 = 15\ kg$

$u_1 = 0.5\ m/s$

wagon-2

$m_2 = 35\ kg$

$u_2 = 0$

$V_2 = 0.3\ m/s$

Using law of conservation of momentum,

Initial momentum = final momentum

$\therefore\ m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$\therefore\ (15 \times 0.5) + (35 \times 0) = (15 \times V_1) + (35 \times 0.3)$

$\therefore\ 7.5 = 15V_1 + 10.5$

$V_1 = \frac{7.5 - 10.5}{15} = -0.2\ m/s\ (\leftarrow)$

Coefficient of Restitution is given by

$$e = \frac{V_2 - V_1}{u_1 - u_2}$$

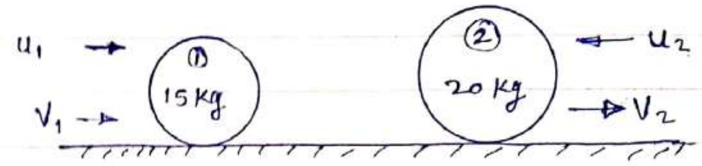
$$e = \frac{0.3 - (-0.2)}{0.5 - 0}$$

$$\boxed{e = 1}$$

As $e = 1$, the impact between two wagons is perfectly elastic impact.

Two masses of 15 kg & 20 kg are moving along a straight line towards each other at velocities of 4 m/s & 2 m/s respectively. If e = 0.6, find the velocities of the masses immidiately after the collision. Also find % loss in K.E. due to impact.

$m_1 = 15\,kg$
$u_1 = 4\,m/s\ (\rightarrow)$



$m_2 = 20\,kg$
$u_2 = -2\,m/s$
$(\leftarrow)$

let V_1 & V_2 are the velocities after collision, assume that both are towards Right. ($\rightarrow$ +ve, $\leftarrow$ -ve).

∴ Using law of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$(15\times4) + (20\times-2) = 15\times V_1 + 20V_2$$

$$20 = 15V_1 + 20V_2 \quad -①$$

using the coeff. of Restitution,

$$e = \frac{V_2 - V_1}{u_1 - u_2}$$

$$0.6 = \frac{V_2 - V_1}{4-(-2)}$$

$$V_2 - V_1 = 0.6\times6$$

$$V_2 - V_1 = 3.6$$

$$\therefore -V_1 + V_2 = 3.6 \quad -②$$

Solving ① & ②

$$15V_1 + 20V_2 = 20$$

$$-V_1 + V_2 = 3.6$$

we get, $V_2 = 2.11\,m/s\ (\rightarrow)$ And $V_1 = -1.49\,m/s\ (\leftarrow)$

Loss in K.E. = Initial K.E. − Final K.E.

$$= \left(\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2\right) - \left(\tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2\right)$$

$$= \left(\tfrac{1}{2}\times15\times4^2 + \tfrac{1}{2}\times20\times2^2\right) - \left(\tfrac{1}{2}\times15\times1.49^2 + \tfrac{1}{2}\times20\times2.11^2\right)$$

$$= (120 + 40) - (16.65 + 44.52)$$

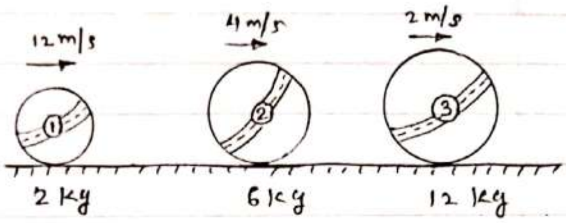
$$= 160 - 60.97$$

Loss in K.E. $= 99.029$ Joules.

$$\%\ \text{Loss in KE} = \frac{\text{Loss in K.E.}}{\text{Initial K.E.}}\times100 = \frac{99.029}{160}\times100$$

∴ % loss in K.E. = 61.89 %

Three perfectly elastic balls 1, 2, 3 of masses 2 kg, 6 kg, 12 kg respectively move along a line in same direction with velocities 12 m/s, 4 m/s & 2 m/s respectively. If the ball 1 strikes the ball 2, which in turn strikes ball 3, determine velocity of each ball after impact.



Given data :

Ball - 1

$m_1 = 2\,kg,\ u_1 = 12\,m/s$

Ball - 2

$m_2 = 6\,kg,\ u_2 = 4\,m/s$

Ball - 3

$m_3 = 12\,kg,\ u_3 = 2\,m/s$

a) Consider the impact betn ball 1 & ball 2 :-

for law of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

$$2\times12 + 6\times4 = 2V_1 + 6V_2$$

$$48 = 2V_1 + 6V_2$$

$$\therefore\ 2V_1 + 6V_2 = 48 \quad \text{-----} \ (1)$$

as the balls are perfectly elastic,

$$e = \frac{V_2 - V_1}{u_1 - u_2} = 1$$

$$\therefore\ V_2 - V_1 = u_1 - u_2 = 12 - 4 = 8$$

$$\therefore\ V_2 - V_1 = 8 \quad \text{-----} \ (2)$$

Solving eqn (1) & (2), $\boxed{V_1 = 0}$ & $\boxed{V_2 = 8\,m/s \rightarrow}$

As the ball strikes each other, ball 1 comes to rest & ball 2 moves forward ball 3 with new velocity of 8 m/s.

$$u_{2_{New}} = 8\,m/s$$

b) Consider the impact betn ball 2 & ball 3 .

$$m_2u_{2_{New}} + m_3u_3 = m_2V_{2_{New}} + m_3V_3$$

$$6\times8 + 12\times2 = 6V_{2_{New}} + 12V_3$$

$$\therefore\ 6V_{2_{New}} + 12V_3 = 72 \quad \text{----} \ (3)$$

for perfectly elastic balls, $e = 1$

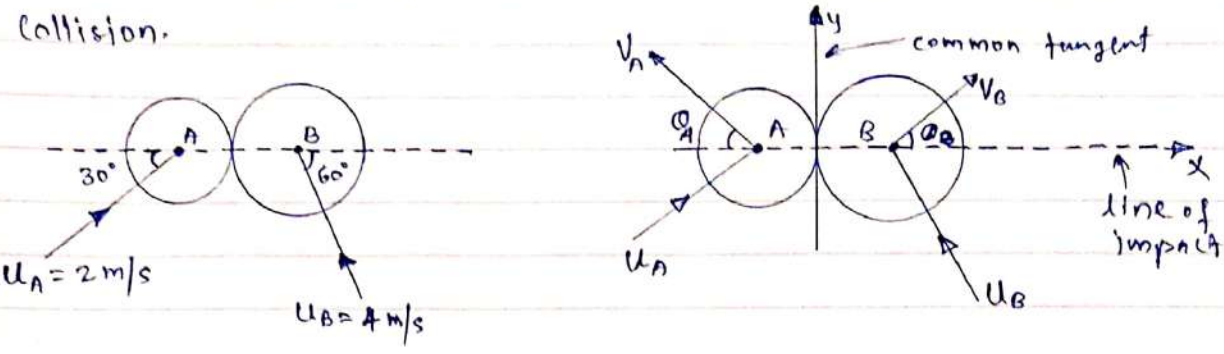
$$e = 1 = \frac{V_3 - V_{2_{New}}}{u_{2_{New}} - u_3} = \frac{V_3 - V_{2_{New}}}{8 - 2}$$

$$\therefore\ -V_{2_{New}} + V_3 = 6 \quad \text{----} \ (4)$$

Solving eqn (3) & (4) we get,

$\boxed{V_{2_{New}} = 0}$ And $\boxed{V_3 = 6\,m/s \rightarrow}$

Two smooth balls A & B having a mass of 4 kg and 8 kg resp. collide with initial velocities as shown in figure. If coeff. of restitution is $e = 0.8$, find the velocities of each ball after the collision.



Ball-A

$m_A = 4$ kg, $u_A = 2$ m/s

Ball-B

$m_B = 8$ kg, $u_B = 4$ m/s

Using law of conservation of momentum along x dirn,

$$m_A u_{A_x} + m_B u_{B_x} = m_A V_{A_x} + m_B V_{B_x}$$

$$\therefore [(4 \times 2\cos 30) + (8 \times (-4\cos 60))] = [(4 \times (-V_{Ax})) + (8 \times V_{Bx})]$$

$$\therefore 6.928 + (-16) = -4V_{Ax} + 8V_{Bx}$$

$$\therefore -4V_{Ax} + 8V_{Bx} = -9.072 \quad \text{----- (1)}$$

Coeff. of Restitution, $e = 0.8 = \dfrac{V_{Bx} - (-V_{Ax})}{u_{Ax} - u_{Bx}}$

$$\therefore 0.8 = \frac{V_{Bx} + V_{Ax}}{2\cos 30 - (-4\cos 60)}$$

$$V_{Ax} + V_{Bx} = 2.986 \quad \text{----- (2)}$$

Solving eqn (1) & (2) we get $V_{Ax} = 2.746$ m/s $\rightarrow$

$V_{Bx} = 0.239$ m/s

similarly., Now component of velocity before and after the impact is conserved i.e. Remains constant along the common tangent.

$$\therefore V_{Ay} = u_{Ay} = 2\sin 30 = 1 \text{ m/s } (\uparrow)$$

$$V_{By} = u_{By} = 4\sin 60 = 3.46 \text{ m/s } (\uparrow)$$

$$\therefore V_A = \sqrt{V_{Ax}^2 + V_{Ay}^2} = \sqrt{2.746^2 + 1^2} = 2.92 \text{ m/s}$$

$$\therefore V_B = \sqrt{V_{Bx}^2 + V_{By}^2} = \sqrt{0.239^2 + 3.46^2} = 3.468 \text{ m/s}$$

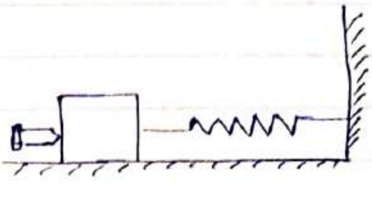
$$\theta_A = \tan^{-1}\left(\frac{V_{Ay}}{V_{Ax}}\right) = \tan^{-1}\left(\frac{1}{2.746}\right) = 20°$$

$$\theta_B = \tan^{-1}\left(\frac{V_{By}}{V_{Bx}}\right) = \tan^{-1}\left(\frac{3.46}{0.239}\right) = 086.05°$$

A 20 gm bullet is fired Horizontally into 300 gm block, which rests on smoth surface. After impact the bullet penetrates into block. the block moves to the right through 300 mm before momentarily coming to rest. Determine speed of bullet as it strikes the block. The block is resting on smooth surface against the spring which is originally unstreched & has a constant of 200 N/m.

⇒

Given data:



For Bullet,

$m_1 = 0.02\ kg$, $u_1 = ?$

For block,

$m_2 = 0.3\ kg$, $u_2 = 0$.

As the bullet penetrates into the block after the impact, The impact is perfectly plastic.

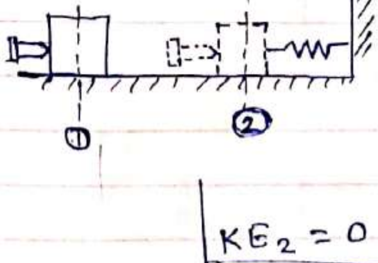
∴ After impact velocity of block & bullet will be equal.

∴ $V = V_2 = V_1 =$ common velocity.

By Applying law of conservation of momentum, For impact;

$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$$
$$0.02 u_1 + 0.3 \times 0 = m_1 V + m_2 V$$
$$0.02 u_1 = (m_1 + m_2) V$$
$$0.02 u_1 = (0.02 + 0.3) V$$
$$0.02 u_1 = 0.32 V \quad - ①$$

As the impact takes place, the block along with bullet will move towards right against spring force before coming to rest. Here spring will be compressed by 300 mm.



Appling work Energy principle,
Just After the impact;

Total W.D = K.E$_2$ - K.E.$_1$

W.D. by spring force = K.E.$_2$ - K.E$_1$ | $KE_2 = 0$.

$$\frac{1}{2}k(x_1^2 - x_2^2) = 0 - \left(\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2\right)$$

$$\frac{1}{2} \times 200 (0 - 0.3^2) = 0 - \left[\frac{V^2}{2}(0.02 + 0.3)\right]$$

$$-9 = -0.16 V^2$$

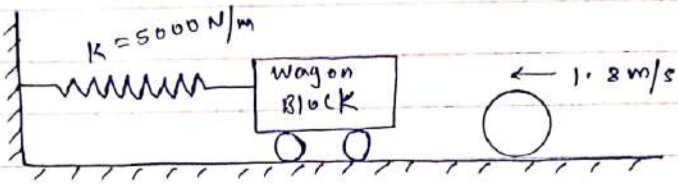
$$\therefore \boxed{V = 7.5\ m/s} \rightarrow$$ put this in eqn ①

$$\therefore 0.02 u_1 = 0.32 V_1 = 0.32 \times 7.5$$

$$\therefore \boxed{u_1 = 120\ m/s}$$ → speed of bullet as it strikes the block.

A 2.5 kg sphere is moving with a velocity of 1.8 m/s when it strikes the vertical face of 4kg block which is at rest. Block is supported on rollers and is attached to a spring of constant K = 5200 N/m as shown in figure. If e = 0.8 betn the block & sphere, determine max. compression of spring due to impact.

$\Rightarrow$



For Block:

$m_1 = 4\,kg$

$u_1 = rest = 0$

$v_1 = ?$

for sphere :- $m_2 = 2.5\,kg$, $u_2 = -1.8\,m/s\ (\leftarrow)$, $v_2 = ?$

Applying law of conservation, we have,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$4\times 0 + (-2.5\times 1.8) = 4v_1 + 2.5v_2$$

$$\therefore\ 4v_1 + 2.5v_2 = -4.5 \quad ----- \ (1)$$

from the coeff. of restitution,

$$e = 0.8 = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{0-(-1.8)}$$

$$\therefore\ 0.8\times 1.8 = v_2 - v_1$$

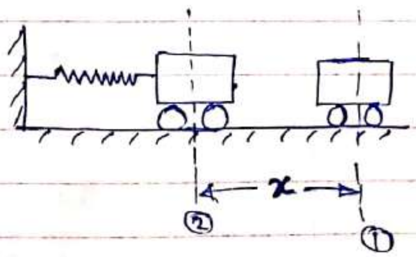
$$\therefore\ -v_1 + v_2 = 1.44 \quad ---- \ (2)$$

Solving eqn (1) and eqn (2) $v_1 = -1.246\,m/s\ (\leftarrow)$

$v_2 = 0.194\,m/s\ (\rightarrow)$

Because of the impact betn sphere & block, block will move towards left from position (1) to position (2) as shown.

At the max. compression of spring at 2nd position, Block will stop; Velocity of block at 2nd position is zero.



Applying W-E principle,

Total WD = $KE_2 - K.E_1$

$$\tfrac{1}{2}K(x_1^2 - x_2^2) = 0 - \tfrac{1}{2}mu^2$$

$$\tfrac{1}{2}\times 5000\times(0 - x^2) = -\tfrac{1}{2}\times 4\times(1.246^2)$$

$$\therefore\ -2500\,x^2 = -3105$$

$$\boxed{x = 0.0352\,m}$$

max. compression of spring is 0.0352m.

Here

$x_1 = 0$

$x_2 = x$

deflection $= x_1 - x_2$

$= 0 - x$

$= -x$

A sphere of mass 2.5 kg is released from rest when $\theta = 60°$. It strikes Block B of 3 kg mass which is at rest. Sphere velocity is zero after the impact & block moves through a distance of 1.5 m before coming to rest. Determine ① coeff. of friction betn the block and surface, ② coeff. of restitution betn the block & sphere.

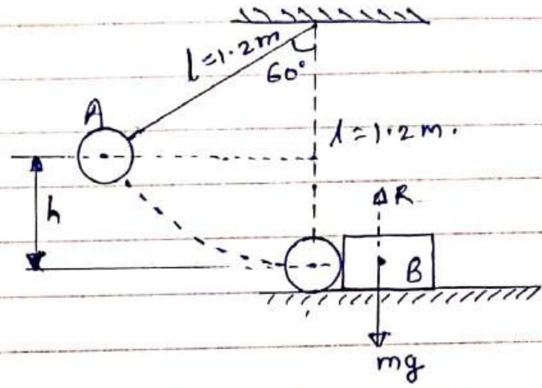
→ Given data:

$m_1 = 2.5\ kg$, $l = 1.2\ m.$

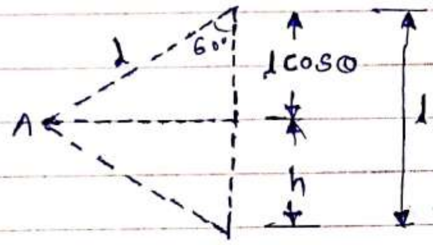
u_1 = velocity of sphere before impact

v_1 = velocity of sphere after impact.

$\therefore \underline{v = 0}$



To find h, consider the geometry of the diagram as shown below



$\therefore\ h = l - l\cos\theta$

consider the vertical displacement of ball.

$\therefore\ v^2 - u^2 = 2gs$

$v^2 - u^2 = 2gh$

$v^2 - 0 = 2gh$

$\therefore\ v = \sqrt{2gh}$

$v = \sqrt{2 \times 9.81 \times (1.2 - 1.2\cos 60)}$

$v = 3.43\ m/s$

This is the velocity of ball when it reaches the Block. i.e. it is velocity of ball before impact.

$\therefore\ v = u_1 = 3.43\ m/s.$

$v_1 = 0$ given.

v_2 = velocity of block after impact.(comes to rest) = 0

Applying law of conservation of momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2.5 \times 3.43 + 3 \times 0 = 2.5 \times 0 + 3 \times v_2$

$8.575 = 3v_2$

$\therefore\ \boxed{v_2 = 2.858\ m/s}$ →

This is velocity of block just after impact.

$\therefore$ coeff. of Restitution $= e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$e = \dfrac{2.858 - 0}{3.43 - 0} = \underline{\underline{0.833}}$

W.D. due to friction for block.

$= -\mu_k \cdot R \cdot S = -\mu_k \cdot 3 \times 9.81 \times 1.5$

$\therefore$ W.D $= -44.145\ \mu_k$

Applying work Energy principle,

Total WD = $KE_2 - KE_1$

$-44.145\mu_k = 0 - \frac{1}{2} \times 3 \times 2.85^2$

$\therefore\ \boxed{\mu_k = 0.277}$

A 100 gm ball is dropped from a height of 600 mm on a small plate. It rebounds to a height of 400 mm when plate is directly resting on ground floor and it rebounds to a height of 250 mm when foam rubber matt is placed betn plate and ground. Determine the coefficient of restitution betn the plate and ground.

case - I $h_1 = 600$ mm

$h_2 = 400$ mm

$$e = \sqrt{\frac{h_2}{h_1}}$$

$$e = \sqrt{\frac{400}{600}}$$

$$\boxed{e = 0.816}$$

case - II

$h_1 = 600$ mm

$h_2 = 250$ mm

$$e = \sqrt{\frac{250}{600}}$$

$$\boxed{e = 0.645}$$