# MATH 2 HANDBOOK

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Unit-I

# chapter-1. Differential Equation.

## * Method to solve diff equation:

### 1] variable seprable form.

### 2] Diff Equation Reducible to variable seperable form.

a) if in the form $\frac{dy}{dx}$, substitute $\frac{y}{x} = u$, $y = ux$

b) if in the form $\frac{dx}{dy}$, substitute $\frac{x}{y} = u$, $x = uy$.

### 3] Homogeneous differential equation:

Substitute $y = ux$.

### 4] Non homogeneous diff equation

a] if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ $\quad lx + my = u$ (substitution).

b) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, Substitute $X = x + h$, $Y = y + k$.

$$\therefore \frac{dy}{dx} = \frac{dY}{dX}$$

### 5] Exact diff. Equation :-

Consider equation $M(x,y) \cdot dx + N(x,y) \cdot dy = 0$

$$\therefore M dx + N . dy = 0$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then

$$\int M . dx + \int N . dy = c$$



N = not contains terms having $x$.

## 6] Non Exact diff equation

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Rules for finding integrating factors.

a] if D.E is homogeneous.

$$I.F = \frac{1}{xM + y \cdot N}$$

b] if D.E is in the form $y f_1(xy).dx + x f_2(xy)\,dy = 0$ then

$$I.F = \frac{1}{Mx - Ny}$$

c] if $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then.

$$I.F = e^{\int f(x).dx}.$$

d] if $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = f(y)$ then.

$$I.F = e^{\int f(y).dy}$$

## 7] Linear diff equation :-

form $\frac{dy}{dx} + Py = Q$

$$\therefore I.F = e^{\int P.dx}$$

Solution method $= y.e^{\int P.dx} = \int Q.I.F \cdot dx + c.$

## 8] Reducible to linear form:-

a) Bernoulli's Diff. Equation :- $\frac{dy}{dx} + Py = Q \cdot y^n$.

Sol$^n$ method = $\frac{du}{dx} + (1-n)P \cdot u = (1-n)Q$ where $y^{1-n} = u$

b] Equation of form $f'(y) \cdot \frac{dy}{dx} + P f(y) = Q$

Substitute $f(y) = u$

$\therefore f' \frac{dy}{dx} = \frac{du}{dx}$

# chapter 2 - Applications of Diff Equation.

## 1] Newton's law of cooling:-

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

$\theta_0$ = room tempreture K = constant

## 2] Rectilinear Motion:-

m = mass of body  v = velocity
F = force  x = displacement.
t = time  a = accelaration.

velocity (v) $= \frac{dx}{dt}$

Accelaration (a) $= \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{dv}{dx} \cdot v$

Newton's / second law of motion $= F = ma = m \cdot \frac{dv}{dt} = m \cdot v \frac{dv}{dx}$

## 3] Simple Electric circuits:-

a) $i = \frac{dq}{dt}$ or $q = \int i \, dt$.

b) voltage drop across resistance $= Ri$

c] — " — — " — — " — inductance $= L \cdot \frac{di}{dt}$

d] — " — — " — — " — capacitance $= q/c$

e] Kirchoff's law $= L \cdot di/dt + Ri = E$

A] Circuit involving

a] L and R with voltage source and E in series

$$\boxed{i = \frac{E}{R} + c e^{-Rt/L}}$$

b] R and C " " " " " $i = \left[\frac{E}{R} - \frac{q_0}{RC}\right] e^{-t/RC}$

## 4] Heat flow :-

$$q = -K \cdot A \cdot \frac{dT}{dx} \quad \text{where } A = 2\pi x \text{ (area)}.$$

# chapter 3 - Reduction formulae, Beta & Gamma functions.

1] for $\int \sin^n x \cdot dx$

$$I_n = \frac{n-1}{n} I_{n-2}$$

2] for $n =$ positive integer.

$$\int_0^{\pi/2} \sin^n x \cdot dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \quad n = \text{even.}$$

$$= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1 \cdot \quad n = \text{odd}$$

$$\int_0^{\pi/2} \sin^n x \cdot dx = \int_0^{\pi/2} \cos^n x \cdot dx$$

Additional Results :-

$$\int_0^{\pi} \sin^n x \cdot dx = 2 \int_0^{\pi/2} \sin^n x \cdot dx.$$

$$\int_0^{\pi} \cos^n x \cdot dx = 2 \int_0^{\pi/2} \cos^n x \cdot dx \quad n = \text{even.}$$

$$= 0 \quad n = \text{odd}$$

$$\int_0^{2\pi} \sin^n x \cdot dx = 4 \int_0^{\pi/2} \sin^n x \cdot dx \quad n = \text{even}$$

$$= 0 \quad n = \text{odd}$$

5] $\int_0^{\pi/2} \sin^m x \cdot \cos^n x \cdot dx$

$$= \frac{\{(m-1)(m-3)\cdots 2 \text{ or } 1\}\{(n-1)(n-3)\cdots 2 \text{ or } 1\}}{(m+n)(m+n-2)(m+n-4)\cdots 2 \text{ or } 1} \times P$$

$P = \frac{\pi}{2}$ · m and n are even

$= 1$ · for all values of 'm' and 'n'

## B] Gamma funcutions.

$$\Gamma n = \int_0^\infty e^{-x} \cdot x^{n-1} \cdot dx \quad (n>0).$$

*** properties**

1] $\Gamma n = 2\int_0^\infty e^{-x^2} \cdot x^{2n-1} \cdot dx$

2] $\Gamma 1 = 1$

3] $\Gamma n+1 = n\Gamma n$ if $n =$ fraction.

4] $\Gamma \frac{1}{2} = \sqrt{\pi}$

5] $\Gamma 0 = \infty$

6] $\Gamma n+1 = n!$ if $n =$ integer

7] $\int_0^\infty e^{-ky} \cdot y^{n-1} dy = \frac{\Gamma n}{k^n}$

5] $\int_0^{\pi/2} \sin^m x \cdot \cos^n x \cdot dx$

$$= \frac{\{(m-1)(m-3)\cdots 2 \text{ or } 1\}\{(n-1)(n-3)\cdots 2 \text{ or } 1\}}{(m+n)(m+n-2)(m+n-4)\cdots 2 \text{ or } 1} \times P$$

$P = \frac{\pi}{2}$ · m and n are even

$= 1$ · for all values of 'm' and 'n'

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5] $\Gamma 0 = \infty$

6] $\Gamma n+1 = n!$ if $n =$ integer.

7] $\int_0^{\infty} e^{-ky} \cdot y^{n-1} \cdot dy = \frac{\Gamma n}{k^n}$

## c] Beta funcutions.

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} \cdot dx$$

* properties :-

1) $B(m,n) = B(n,m)$

2] $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \cdot d\theta.$

3] $B\left[\frac{p+1}{2}, \frac{q+1}{2}\right] = 2\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \cdot d\theta.$

4] $B(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$

5] $\Gamma p \, \Gamma 1+p = \frac{\pi}{\sin p\pi}$

*a) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left[\frac{x}{a}\right] + C$

b) $\int \sqrt{x^2+a^2} \cdot dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log_e\left(x+\sqrt{x^2+a^2}\right) + C$

c) $\int \sqrt{x^2-a^2} \cdot dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log_e\left(x+\sqrt{x^2-a^2}\right) + C.$

* Reduction of $\int \tan^n x \cdot dx = \int \tan^n x \cdot dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \cdot dx.$

Reduction of $\int \sec^n\theta \cdot d\theta = \frac{\sec^{n-2}\theta \cdot \tan\theta}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}\theta \cdot d\theta.$

Unit-3

# chapter-4 - Differentiation under the integral sign and Error function.

## Rule -I-

$$I(\alpha) = \int_a^b f(x,\alpha)\cdot dx \text{ then } \frac{dI}{d\alpha} = \int_a^b \frac{df}{d\alpha}(x,\alpha)\cdot dx.$$

## Rule-II-

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\cdot dx = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial\alpha}(f x,\alpha)\cdot dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\cdot\frac{da}{d\alpha}$$

# chapter 4 - Differential Under Integral Sign and Error function.

## Error function!

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \cdot du$$

## 1] Complementary Error function:-

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2} \cdot du.$$

## 2] Alternative defination:

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{t} \cdot t^{-1/2} \cdot dt$$

## properties of Error function:

1) $erf(\infty) = 1$
2) $erf(0) = 0$
3) $erf(x) + erf(x) = 1$
4) $erf(-x) = -erf(x)$
5] $erf(x) = \frac{2}{\sqrt{\pi}} \left[ x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots \right]$

## Diffⁿ of Error function:-

$$\frac{d}{dx} erf(ax) = \frac{2a\, e^{-a^2 x^2}}{\sqrt{\pi}}$$

## Integration of Error function

$$\int_0^t erf(ax) \cdot dx = t\,(erf(at)) + \frac{1}{\sqrt{\pi}} e^{-a^2 t^2} - \frac{1}{a\sqrt{\pi}}$$

Unit-4.

## chapter-5 - Curve tracing and Rectification of Curves.

### 1] Tracing of cartesian curves:

a) Symmetry about X-axis :- powers of 'y' are even everywhere

b) Symmetry about Y-axis :- power of 'x' are even everywhere

c) Symmetry about both axis : power of x and y are even

d) Symmetry about opposite quadrant : Equation remains unchanged if x and y are changed to −x and −y

e) Symmetry about Y-X line : Equation remains unchanged if 'x' is replaced with 'y' and y is replaced with 'x'.

### 2] point of intersection

a) with X-axis :- put y=0 and obtain values of 'x'
(a,0), (a, 0) are point of intersection.

b) with Y-axis :- put x=0 and obtain values of 'y'
(0,a) $(0,a_2)$ are point of intersection.

c) with origin :- put x=0 and y=0

### 3] Tangent at origin.

if curve passes through origin, then find tangent at origin [illegible] lowest degrees term.

## 4] Sepecial point.

$\frac{dy}{dx} = \pm\infty$ . – parallel to y-axis

$\frac{dy}{dx} > 0$ curve increasing

$\frac{dy}{dx} < 0$ curve decreasing

## 5] Asymptote

a) parallel to X-axis : Equate co-efficient of highest power of $x$ to zero, we get asymptote parallel to X-axis.

b) parallel to Y-axis :- Equate co-efficient of highest power of 'y' to zero, we get asymptote parallel to Y-axes.

## 6] Region of Absence!

a) for $y^2 = f(x)$ : Suppose $y^2 < 0$ and find values of 'x'

b) for $x^2 = f(y)$ ; Suppose $x^2 < 0$ and find values of 'y'.

## II] Tracing of polar form.

### A] Symmetry

i) initial line $\theta = 0$ : Replace $\theta$ by $-\theta$, if equation remains unchanged then curve is Symmetric.

ii) About pole : Replace $r$ by $-r$, if given equation remains unchanged then curve is symmetric about pole.

iii) About $\theta = \frac{\pi}{2}$ : Replace $\theta$ by $(\pi - \theta)$. if equation remains unchanged curve is symmetrical about $\theta = \pi/2$

iv) if equation remains unchanged by changing $\theta$ to $-\theta$. and $r$ to $-r$, then curve is symmetric about. line $\theta = \pi/2$, through pole perpendicular to initial line.

### B] Rose curves

$r = a\sin n\theta$ or $r = a\cos n\theta$.

1] For $r = a\sin n\theta$, first loop drawn along $\theta = \frac{\pi}{2n}$

2] For $r = a\cos n\theta$, first loop is drawn along $\theta = 0$

3] If 'n' is odd there are 'n' number of loops.

4] If 'n' is even there are '2n' number of loops.

Unit-V

chapter-6 - Coordinate System, plane, straight line.

## I] Relations Between three Co-ordinate System:-

a) Relation Between cartesian and spherical polar system of coordinates

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$.

b] Relation between cartesian and cylindrical system of co-ordinates

$x = \rho\cos\theta$

$y = \rho\sin\phi$

$z = z$

c] for spherical polar coordinates $(r, \theta, \phi)$

$r = \sqrt{x^2+y^2+z^2}$

$\cos\theta = \frac{z}{r}$

$\tan\phi = y/x$

d] for cylindrical coordinates $(\rho, \phi, z)$

$\rho = \sqrt{x^2+y^2}$

$\tan\phi = y/x$

$z = z$

## II] Distance formula:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

## III] Division of Join of Given points

Internally in ratio
m: n.

$$x = \frac{mx_2+nx_1}{m+n} \qquad y = \frac{my_2+my_1}{m+n} \qquad z = \frac{mz_2+nz_1}{m+n}.$$

Externally in ratio m:n.

$$x = \frac{mx_2-nx_1}{m-n} \qquad y = \frac{my_2-ny_1}{m-n} \qquad z = \frac{mz_2-mz_1}{m-n}.$$

## IV] Direction cosines (Relaution)

$$l^2+m^2+n^2=1$$

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

## V] Direction Ratios:

If a, b, c are dr's in $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$. then dc axes

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad m = \frac{b}{\sqrt{a^2+b^2+c^2}} \qquad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

## v] Angle

$(l_1, m_1, n_1)\ (l_2, m_2, n_2)$ = direction cosines

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

## SPHERE

### I] Centre Radius from:

centre $= (a, b, c)$

radius $= r$

$\therefore$ Equation : $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

Note:- if centres lies at origin equation becomes:

$x^2 + y^2 + z^2 = r^2$ which is the standard form.

### II] General form:

$$x^2 + y^2 + z^2 + 2ux + 2wz + d = 0$$

where centre $= (-u, -v, -w)$.

radius $= \sqrt{u^2 + v^2 + w^2 - d}$

### III] Intercept form:-

if sphere cuts x axis at $x = a$

y axis at $y = b$

z axis at $z = c$

$u = -a/2 \quad v = -\frac{b}{2} \quad w = -c/2$

Intercept form eqn : $x^2 + y^2 + z^2 - ax - by - cz = 0$

### IV] Diameter form:-

$$(x-x_1)(x-x_2) + (y-y_1) + (z-z_1)(z-z_2) = 0$$

## Touching spheres:

if spheres touch

a) externally then $d = r_1 + r_2$

b) internally then $d = r_1 - r_2$

## INTERSECTION OF PLANE AND SPHERE.

$$S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

$$\gamma = lx + my + nz + p = 0$$

Equation : $S + \lambda \gamma = 0$

Length of perpendicular $= \left| \dfrac{lu + mv + nw + p}{\sqrt{l^2 + m^2 + n^2}} \right|$

where $u, v, w$ are centers of sphere

$l, m, n$ are points of constact on plane.

## ORTHOGONAL SPHERES

$$\left[ 2u_1 u_2 + 2v_1 v_2 + 2w_1 w_2 = d_1 + d_2 \right]$$

## Cone :-

fixed point = vertex $(\alpha, \beta, \gamma)$

Given curve = guiding curve or base

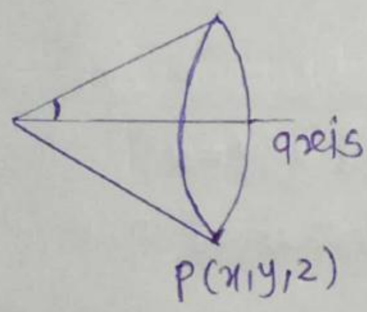
Any straight line = generator

Direction Ratios $(l, m, n)$

$\therefore$ Equation of generator passing through $(\alpha, \beta, \gamma)$ with direction ratios $l, m, n$ is given by

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

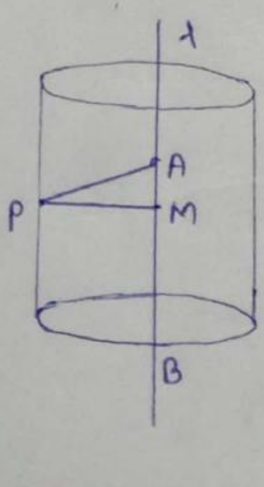
## RIGHT CIRCULAR CONE




Angle between generator AP and axis is given as

$$\cos\alpha = \frac{l(x-\alpha)+(m)(y-\beta)+n(z-\gamma)}{\sqrt{(l^2+m^2+n^2)}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$$

## RIGHT CIRCULAR CYLINDER




$P = (x, y, z)$ = any point on cylinder

$A(\alpha, \beta, \gamma)$ = fixed point on axis AB

PM = Radius of cylinder $= r$

$$AP = \sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}$$

AM = projection of AP on axis AB

$$= \frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}$$

Unit - 6

# chapter-7 - Multiple Integrals:

## I] Double Integration

a] when $x_1$ and $x_2$ are function of $y$ and $y_1$ and $y_2$ are constant, then we integrate first w.r.t 'x' keeping 'y' constant and then integrate 'y' between $y_1$ and $y_2$

$$\int_{y_1=a}^{y_2=d} \left[ \int_{x_1=f(y)}^{x_2=f(y)} f(x,y)\,dx \right] . dy$$

b] when $y_1$ and $y_2$ are functions of 'x' and $x_1$ and $x_2$ are constants, then we integrate first w.r.t 'y' keeping 'x' constant and then integrate 'x' between $x_1$ and $x_2$

$$\int_{x_1=a}^{x_2=d} \left[ \int_{y_1=f(x)}^{y_2=f(x)} f(y,x).dy \right] . dx.$$

c] when $y_1$ and $y_2$ are functions of $x$ and $x_1$ and $x_2$ in this
$x_1$ $x_2$ and $y_1$ $y_2$ are constant limits then region of integration is rectangle. so we can use the given order of integration

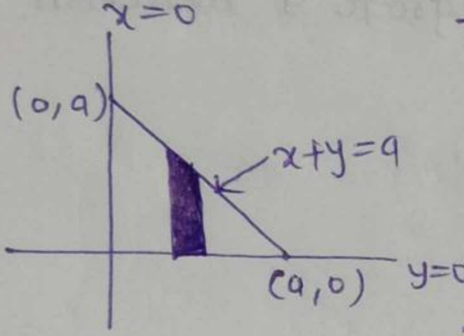
d] if $f(x,y) = h(x). g(y)$ and if both limits are constant the

$$\int_a^b \int_c^d f(x,y).dx.dy = \int_a^b h(x).dx \cdot \int_c^d g(y).dy.$$

## II] Evaluation of Double integrals without limits.

### a] Integrating first w.r.t 'y' then 'x'.

- Draw region 'r' where $x=0$ $y=0$ and $x+y=a$.
- for this integration consider vertical strip.




- for $x=0$ and $y=a$ for $y=0$ $x=a$

∴ points of intersection are $(0,a)$ & $(a,0)$.

- Limits for integration of vertical strip is given as

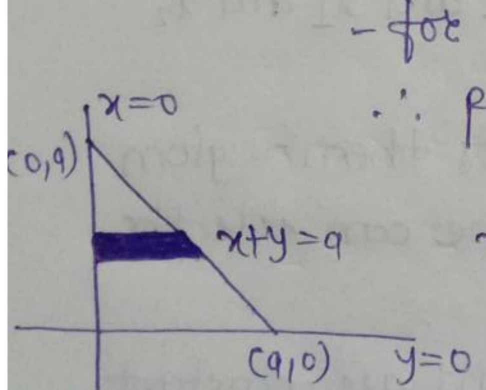
$x=0$ to $a$

$y=0$ to $a-x$.

$$I = \int_0^a \int_0^{a-x}$$

### b] Integrating first w.r.t 'x' then 'y'

- Draw region 'r' where $x=0$ $y=0$ and $x+y=a$
- for this integrating consider horizontal strip.




- for $x=0$ $y=a$ and for $y=0$ $x=a$

∴ points of intersection of horizontal strip is

$x=0$ to $a-y$

$y=0$ to $a$.

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c] Double integration in polar co-ordinate form.

i] Function is always integrated first w.r.t $r$ and then w.r.t $\theta$.

ii] The strip is always radials and it is taken from pole.

iii] Rotate strip in anticlockwise direction.

iv] $x = r\cos\theta \qquad y = r\sin\theta$

$$x^2 + y^2 = r^2$$
$$dx\,dy = r \cdot dr \cdot d\theta$$

v] For first Quadrant $= \theta = 0$ to $\pi/2$

first, Second " $= \theta = 0$ to $\pi$

first, second, third $= \theta = 0$ to $3\pi/2$

1st, 2nd, 3rd, 4th $= \theta = 0$ to $2\pi$

## TRIPLE INTEGRATION

- If function $f(x,y)$, then integrate 1st w.r.t 'z' and then w.r.t 'y' and lastly with respect $x$

- Limits of z are in terms of $x$ and $y$
- Limits of y are in terms of 'x'
- Limits of 'x' are always constants.

$$\int_a^b \int_{f_1(x)}^{f_2(x)} \int_{\theta_1(x,y)}^{\theta_2(x,y)} f(x,y,z)\, dz\, dy\, dx =$$

$$\int_a^b \left\{ \int_{f_1(x)}^{f_2(x)} \left[ \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x,y,z)\, dz \right] dy \right\} \cdot dx.$$

## TRIPLE INTEGRATION IN SPHERICAL POLAR COORDINATES

$$x^2+y^2+z^2=r^2$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$dx\,dy\,dz = r^2\sin\theta\, dr\, d\theta \cdot d\phi.$$

# AREA

1) $\int y \cdot dx \longrightarrow$ Area bounded on $x$-axis
2) $\int x \cdot dy \longrightarrow$ Area bounded on $y$-axis
3) $\iint dx \cdot dy \longrightarrow$ Area is in cartesian form.
4) $\iint r \cdot dr \cdot d\theta \rightarrow$ Area is in polar form.

— shaikh Altaf Tayyab.