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Traffic Cellular Automata Model:

An analysis of traffic behaviour in single and multi-road systems

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March 27, 2019

Contents

1 Abstract

In this report, using a traffic cellular automata model, we run various simulations of both a single and multi-road environment. Our data highlights the over simplistic nature of some of the earlier traffic models whilst successfully recreating real world data. An analysis of traffic jam propagation suggests an exponential relationship with density. Finally we demonstrate how our model can be built upon to create complex multi-road systems.

2 Introduction

Traffic has a notorious reputation for being difficult to precisely model, most likely due to its high level of variability and the vast number of extraneous factors. Broadly speaking, there are three scales at which the problem can be addressed: microscopic, macroscopic and kinetic. Microscopic models follow the behaviour of individual cars, macroscopic are based on fluid dynamics and finally kinetic sit somewhere in between, based on Boltzmann type kinetic equations. This report focuses on a traffic cellular automata (TCA) model, a type of microscopic model that is particularly computationally efficient.

3 Traffic Cellular Automata Model

The general aim of TCA models, which are born from statistical mechanics, is to produce accurate macroscopic phenomena based on very simple microscopic principles. These microscopic rules (Section 3.2) are not designed to be realistic necessarily but are instead designed so that the entire system reproduces the correct macroscopic behaviour (i.e. how traffic behaves on a large scale).

3.1 Lattice Road

A basic road can be thought of as a one-dimensional lattice, consisting of L sites. A site may either be empty or contain a car with a "quantised" velocity, v, which may take integer values $0, 1, 2, \ldots, V_{max}$ (where V_{max} is the imposed speed limit on the road). Time, t, also advances in integer steps and we adopt the units of $\Delta t = 1s, \Delta x = 7.5$ m, meaning that v takes units of 27 km/hr, per integer value. To begin with the road is considered a closed loop so that cars at the end of the lattice simply return to the start thus conserving the total number of cars on the road, N. In our simulations this lattice is represented by an array, with each index in the array representing a site and the value at that index representing a car's velocity. For the basic road, at time $t = 0$, cars are randomly distributed around the entire array, each with an initial speed $v = 0$.

3.2 Car Movement

The following rules are used to determine the state of the cars at time t + 1:

- *Acceleration* — For each car, if $v < V_{max}$ then its velocity is increased to $v + 1$.
- *Avoiding Collisions* — Drivers will slow down to avoid colliding with the car in front. If a car is at site i and there is a car at site i + d, then if $v \geq d, v$ is set to d–1. This overrides the first rule.
- *Randomly slowing down* — Drivers will react to unexpected events by reducing their speed. For each car, if $v > 0$, there is a probability p that v is set to v–1. [1] This occurs after rules 1 and 2. This random component is designed to emulate external factors (for example an accident by the side of the road or careless driving).
- *Cars Move* — Each car is moved forward v places, taking the updated value of v.

[1] For the rest of this report this process will simply be referred to as an *event*.

These rules highlight the efficiency of TCA models, consisting of very basic instructions for each car to follow but on a larger scale providing us with the macroscopic behaviour we are looking for.

3.3 Parameters

The following section is here to define the parameters we will be measuring.

It should be noted that there is a distinction between local and global measurements. Local measurements consist of a "detector" of finite length, whereas global measurements are conducted over the entire array (and thus only reasonable for a closed loop).

Global density, k, is simply the total number of cars, N, per sites in the road:

$$k = \frac{N}{L} \tag{1}$$

For a finite detecting length of L_d, local density, $k_l(t)$, is therefore:

$$k_l(t) = \frac{n(t)}{L_d} \tag{2}$$

where $n(t)$ cars are in detecting length at time t. Note that local density equals global density when $L_d = L$ as we would expect.

Global mean velocity, $V(t)$, is simply the average velocity of all the cars on the road:

$$V = \frac{1}{N} \sum_{i=1}^{N} v_i \tag{3}$$

where v_1 is velocity of 1st car, v_2 the 2nd etc. As with density we may then define a local mean velocity, V_l, as:

$$V_l = \frac{1}{n(t)} \sum_{i=1}^{n(t)} v_i \tag{4}$$

Both velocities have units site/second and therefore need converting into physical units.

If we define global traffic flow, q, over time t, as cars passing a site per second and use N_d detectors of length $L_d = V_{max}$, spread around the entire road, then:

$$q \;=\; \frac{1}{N_d\, V_{max}\, t} \sum_{t=1}^{t} \sum_{i=1}^{n_m(t)} n_m(t) \tag{5}$$

where $n_m(t)$ is the number of cars moving (i.e $v > 0$) in all detecting lengths at time t. Note that q is in units of moving cars/site/second, any stationary cars in the detecting lengths are not counted.

The local variant of flow is also found using equation (5) however the locations of the detectors are simply moved to only enclose the area of interest.

4 Macroscopic Behaviour

There have been many proposed models for describing the relationship between traffic parameters. The simplest of them all is Greenshield's macroscopic stream model [1] which proposes a linear relationship between velocity and density [2]:

$$V = V_{max} - \frac{V_{max}}{k_{jam}}\, k \tag{6}$$

where k_{jam} is the jam density (i.e the density at which traffic is unable to move, due to how we have defined k this is equal to 1). This relationship is shown below in Figure (1)

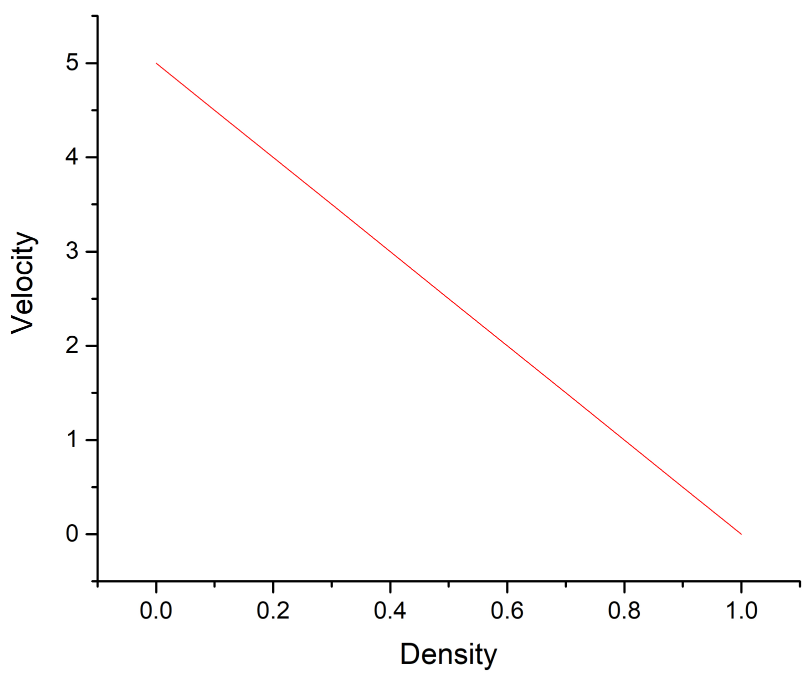


Figure 1: Greenshield's relation between velocity and density.

The model also provides us with a velocity-flow and flow-density relationships shown in Figures (2)

[2]Unless stated otherwise will be referring to global measurements of parameters and velocity, V, refers to average global mean velocity.

and (3) respectively:

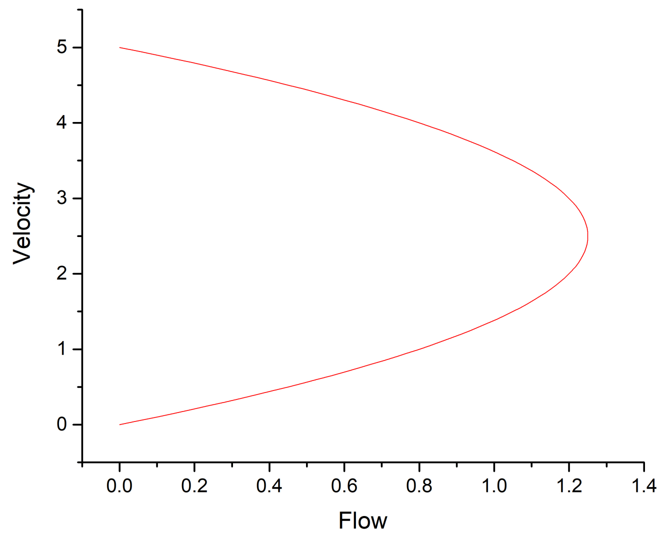


Figure 2: Greenshield's relation between velocity and flow.

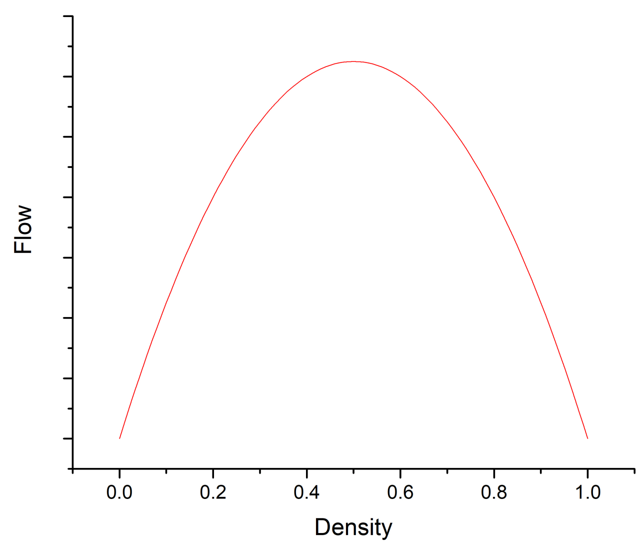


Figure 3: Greenshield's relation between flow and density.

We will see shortly that these models are too simplistic to accurately describe these parameters (especially Figures (1) and (2)) however they provide a useful starting point.

5 Results

In this section we will compare the data produced by our TCA model and compare it to some of the theoretical predictions laid out by existing models.

5.1 Velocity and Density

The first result we will discuss is the relationship between velocity and density. Recall that Figure (1) predicts that as density increases, velocity will linearly decrease. Our results are shown below:

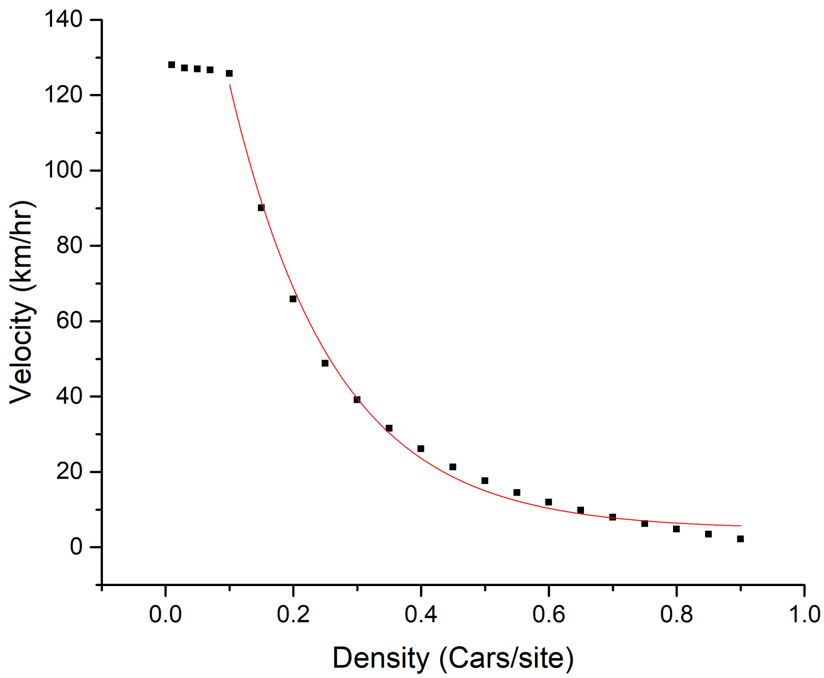


Figure 4: Relation between velocity and density (L = 200, p = 0.25, t = 600s).

It is clear to see that Greenshield's prediction of a linear relationship does not hold up under scrutiny. Aware of the limitations, Underwood put forward the following exponential model:

$$V = V_{max}\ \exp\left(\frac{-k}{k_0}\right) \tag{7}$$

where k_0 is the optimum density (i.e. the density at which maximum flow occurs). This exponential relationship has been plotted against the curved portion of our graph and acts as a reasonable predictor. However the model does break down at extreme low and high densities. The linear portion of the graph (up to around a density of 0.17) arises as a result of our $V_{max} = 5$. Once a car is at top speed it will move forward by 5 sites, therefore occupying 6 sites in the array for the duration of one time interval. The maximum velocity density, $k_{V_{max}}$, at which every car in the array may travel at maximum velocity is therefore $L/(1 + V_{max})$, i.e. $0.167L$ for $V_{max} = 5$.

At any density less than this, all cars are free to travel at V_{max} (unless experiencing an event and once given enough time to spead out), any higher and it becomes physically impossible for every car to do so simultaneously. This phenomena will be demonstrated later in (5.4) when addressing velocity-time graphs.

5.2 Velocity and Flow

Figure (5) below shows our data for velocity against flow and we see a significant departure from the simple quadratic relationship predicted in Figure (2). So much so that it was simply not possible to fit a 2nd order quadratic plot to this data. There are clearly three very distinct sections to this graph and despite its lack of concurrence with Greenshield's predictions, we will see in Section 6.2 that this plot shows striking similarity to some real world data, encouraging its validity.

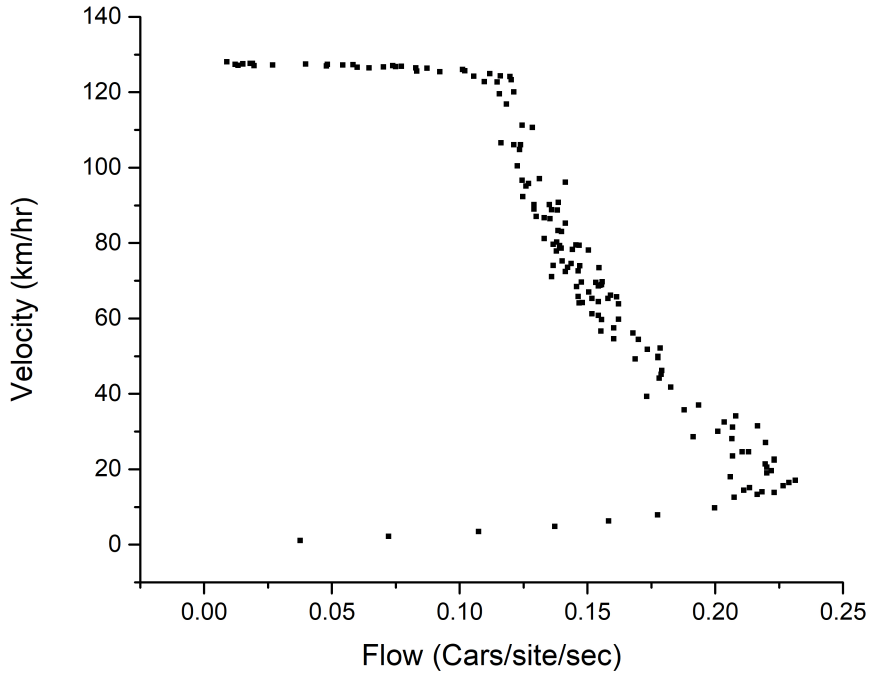


Figure 5: Relation between velocity and flow (L = 200, p = 0.25, t = 600s)

5.3 Flow and Density

Finally we get to the relationship between flow and density; Greenshield's model predicts the following:

$$q = V_{max} - \left(\frac{V_{max}}{k_{jam}}\right) k^2 \tag{8}$$

A reference back to Figure (3) visually confirms that $q \; \alpha \; k^2$ and below, in Figure (6), we can see our results.

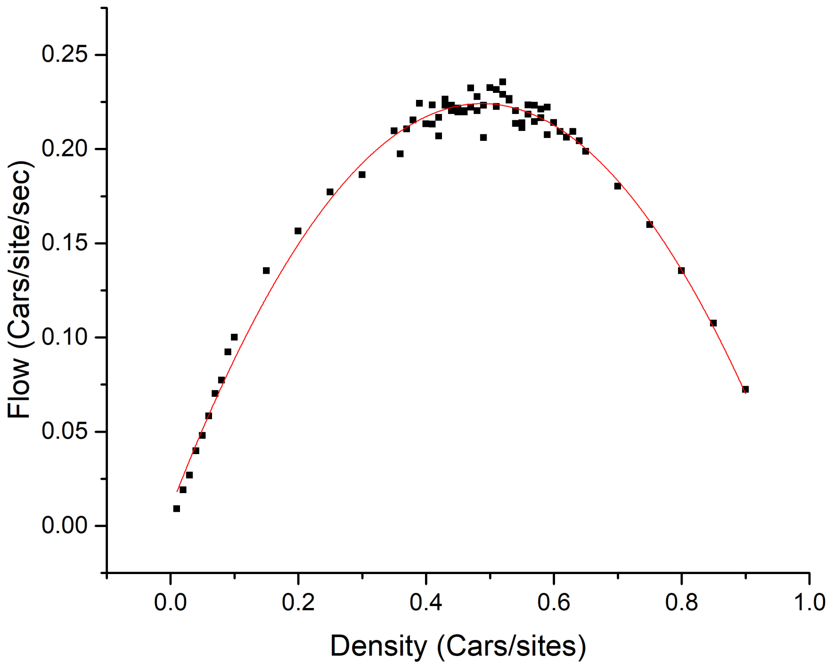


Figure 6: Relation between flow and density (L = 200, p = 0.25, t = 600s)

Unlike the previous two relationships, our results for here are very well predicted by Greenshield's model and a quadratic fit can be seen in red. As well as this, if we differentiate Equation (8) with respect to k and set that to zero we can achieve a theoretical value for optimum density:

$$\frac{dq}{dk} = V_{max} - \left(\frac{V_{max}}{k_j}\right) 2k = 0$$

$$k_0 = \frac{k_j}{2} \tag{9}$$

Recall that for our units, $k_j = 1$, therefore according to this model, optimal flow should occur at $k = 0.5$. Figure (6) shows that this is an extremely accurate prediction for the peak of our data.

5.4 Distance Time Graphs

So far we have only discussed how the traffic parameters interact with one another and are yet to address the formation and propagation of traffic within the road. An effective way to visualize this is with a distance time graph plotting the displacement of every car, at every time interval, and then colour coding the velocity. An exmaple of such a plot is shown below:

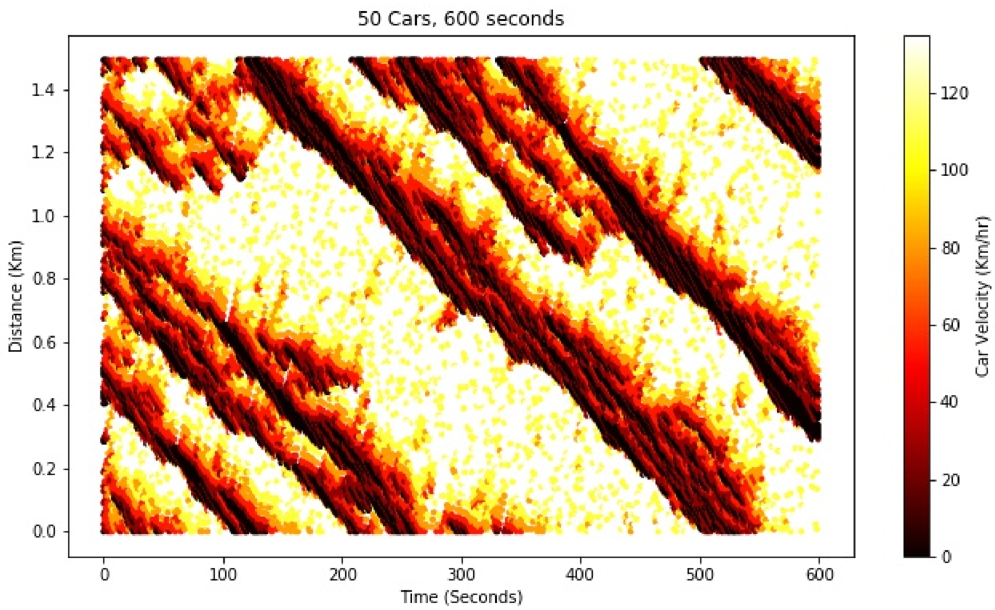


Figure 7: Distance Time Graph, for all cars on a simple looped lattice road (L = 200, p = 0.25)

Figure (7) clearly demonstrates the large scale macroscopic behaviour we aimed to produce from our simple microscopic rules. Individual cars travelling at V_{max} that experience an event can be seen as the individual yellow marks in the white region of the graph. The majority of these events have very little effect and the traffic flow adjusts to the minor fluctuation. The dominating phenomena however is the wave of traffic (represented by the black and red regions) that can be seen travelling in the opposite direction to the cars as time advances.

By setting the event probability to zero ($p = 0$), Figure (8) demonstrates the phenomena briefly mentioned in Section 5.1:

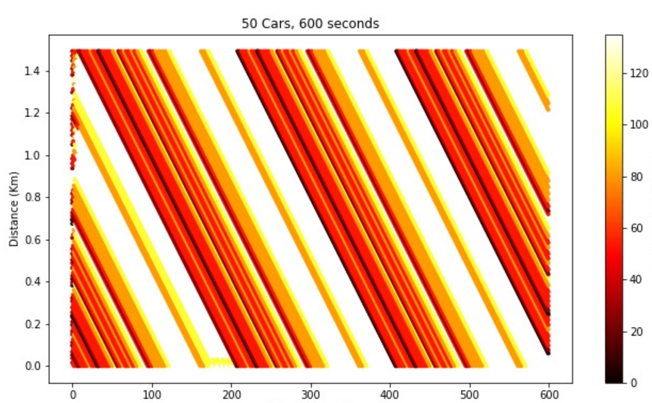


Figure 8: Distance Time Graph, for all cars on a simple looped lattice road (L = 200, p = 0)

With 50 cars on a road with 200 sites (i.e. $k = 0.25$) we exceed the maximum velocity density,

$k_{V_{max}}$, (not to be confused with optimal density, k_0, which is the density at which maximum flow can occur). This results in periodic traffic jams that propagate faster and contain a higher average car velocity within them, than the jams shown in Figure (7).

This result leads to the hypothesis that traffic jam propagation speed, V_{jam} (i.e. the speed at which the traffic wave travels backwards down the road), is proportional to events, p, (for a fixed density, k). We will investigate this hypothesis in the next section.

5.5 Traffic Jam Propagation

An important point to note is that whilst the location of the jams is visually easily to identify via the distance-time graphs, quantifying their characteristics from within the array is fairly complex, especially due to their high level of variability. It was therefore deemed that the most efficient way of determining V_{jam} was from the distance-time graphs themselves as follows:

$$V_{jam} = \frac{L}{t_{jam}} \tag{10}$$

where t_{jam} is simply the time it takes for the traffic jam to travel from the end of the array to the start (i.e. top to bottom on the distance time graph). Then, for a fixed k, we varied p, Figure (9) below, shows this (V_{jam} converted to km/hr from L/s):

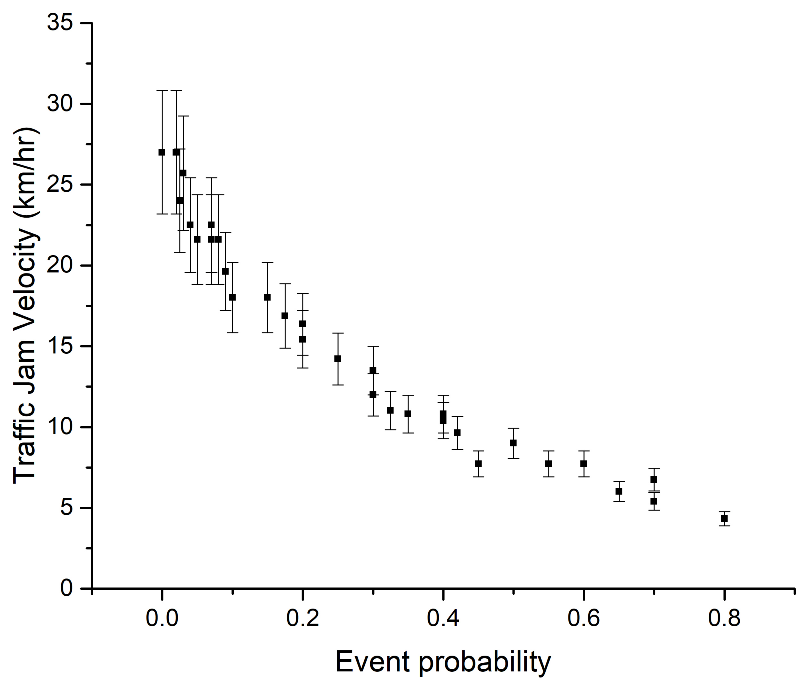
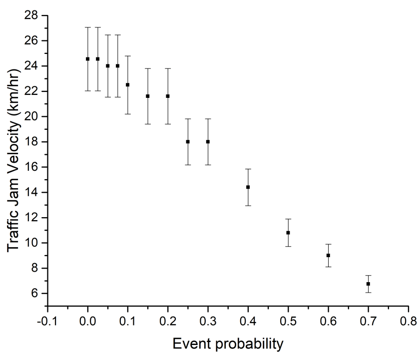


Figure 9: Traffic Jam Velocity against Event Probability (L = 200, k = 0.25)

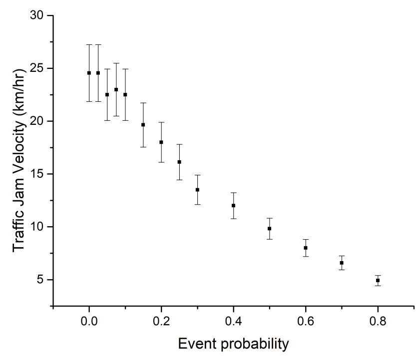
Our results seemed to suggest an exponential relationship between the two and so a trial fit of the following form was applied to our data:

$$V_{jam}(p) \;=\; A\,\exp(-p/B) \;+\; C \tag{11}$$

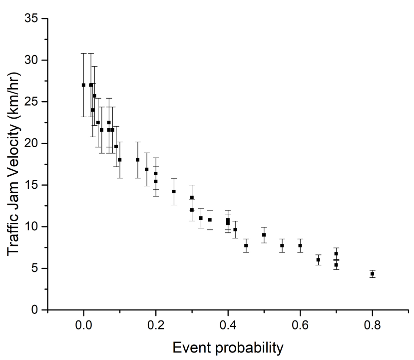
where A,B,C are all constants. This returned an R-squared value of 0.98. The next step was to attempt to define the constants in terms of model parameters. Recall that we hypothesised that V_{jam} is a function of both p and k, because of this we repeated the data collection of V_{jam} against p, for $k = 0.17, 0.5$ and 0.75:



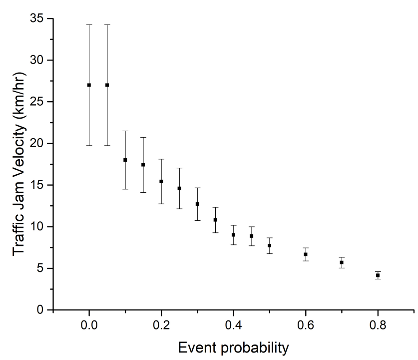
(a) $k = 0.75$



(b) $k = 0.5$



(c) $k = 0.25$



(d) $k = 0.17$

Figure 10: Traffic Jam Velocity against Event Probability for multiple values of Density

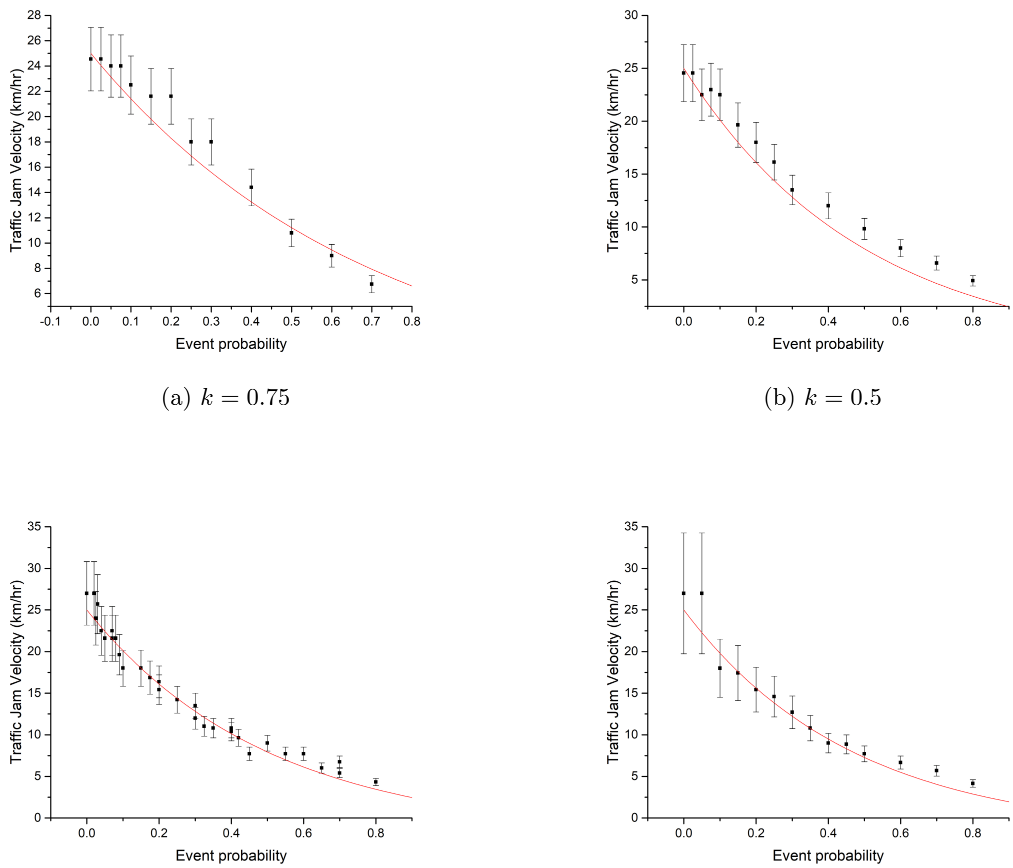
By visual analysis of the graphs in Figure (10) one can see that as k is increased, the exponential relationship becomes more linear - suggesting that B in Equation (11) is a function of k. After varying $B(k)$ for our four values of k, by trial and error we obtained this following relation:

$$B(k) \;=\; 0.4\,k \;+\; 0.4 \tag{12}$$

Substituting Equation (12) into (11) and setting $A = V_1$ and $C = -2$, we reach the following:

$$V_{jam}(p,k) \ = \ V_1 \ \exp\left(\frac{-p}{B(k)}\right) \ - 2 \tag{13}$$

where $V_1 = 27km/hr$ (i.e. the speed of a car when travelling at speed of 1 site/sec). V_{jam} also has units km/hr as k is dimensionless in this context. So taking Equation (13) and plotting it against our 4 graphs in Figure (10) we get the following:



(a) $k = 0.75$

(b) $k = 0.5$

(c) $k = 0.25$

(d) $k = 0.17$

Figure 11: V_{jam} against p for multiple values of k with Equation (13) plotted.

Following Figure (11) it would be reasonable to describe Equation (13) as a fairly accurate expression for determining the speed of propagation of a traffic jam, for a given p and k.

6 Real World Data

A further check of validity for our model is to see how it compares to real world traffic data. The following data is from TRL Insight Report INS003 [2].

6.1 Velocity Density Data

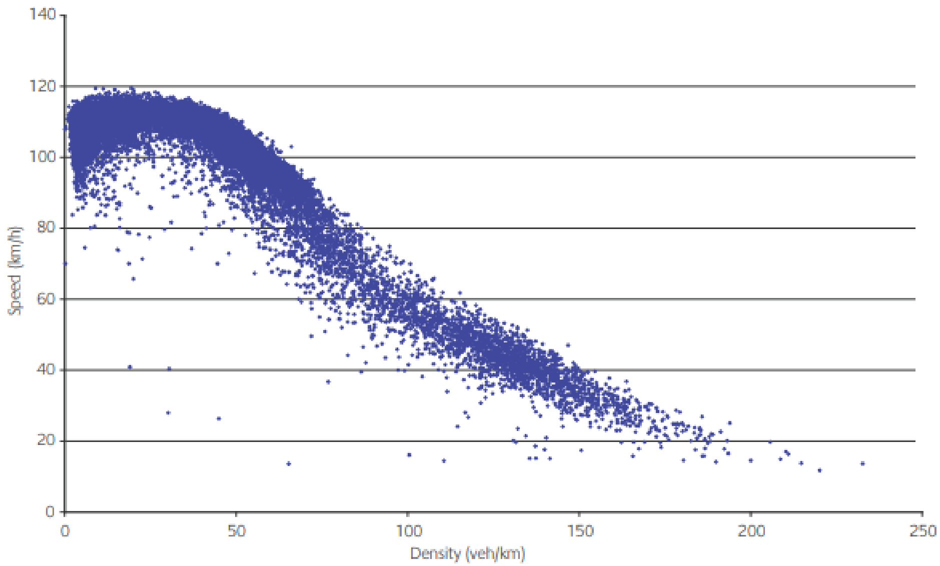


Figure 12: Speed-Density plot for M25 Motorway link LM299

Figure (12) compares favourably to our Velocity Density graph in Figure (4): a low density region of roughly constant velocity and then above a certain $k_{V_{max}}$, velocity starts to decrease in a roughly exponential manner.

6.2 Velocity Flow Data

Recall how our Velocity Flow plot shown in Figure (5) varied fairly significantly to the predictions presented by Greenshield's model. Compared instead to Figure (13), the two graphs look much more similar. The upper and lower regions (circled in red) are much more concentrated, whereas the high flow gap in between shows a much larger variance. The circled regions represent the two *phases* of traffic, often described as free-flowing and congested (upper and lower on the graph, respectively).

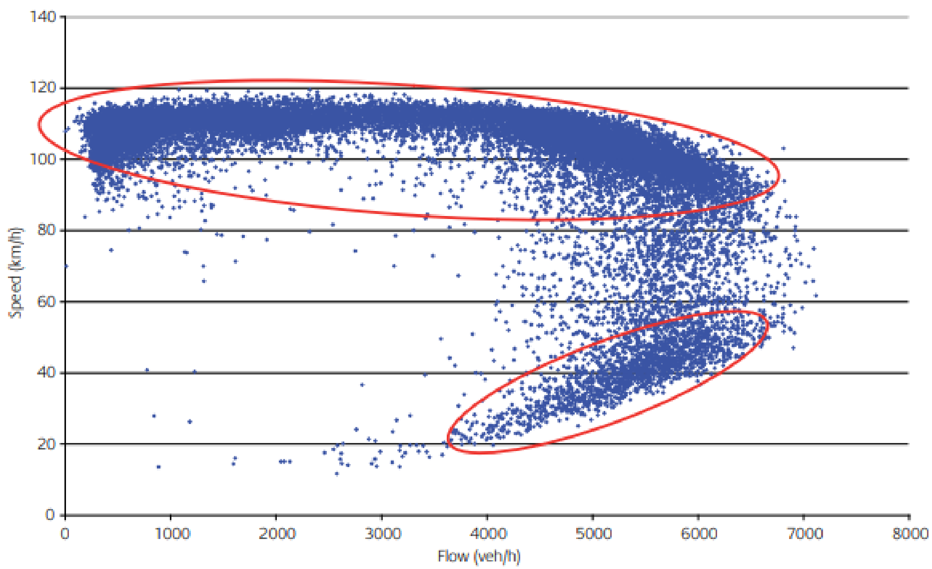


Figure 13: Speed-Flow plot for M25 Motorway link LM299

The similarity between our data and Figures (12) and (13) are a strong indicator that our model acts as a more than robust representation of traffic phenomena.

7 Multi-Road Model

We have established that our model exhibits macroscopic behaviour that agree with both theoretical predictions and real world data. Due to the simplistic nature of the TCA model at the microscopic level, it is possible to model multiple roads simultaneously without significantly increasing computational time. The additions of extra roads has several potential applications; For this report we have decided to create multiple slip roads - all connected to our main road [3]. This allows cars to join and leave the main road, therefore constantly changing the global density of the main road. The system consists of three pairs of slip roads, one joining and one leaving the main road per pair:

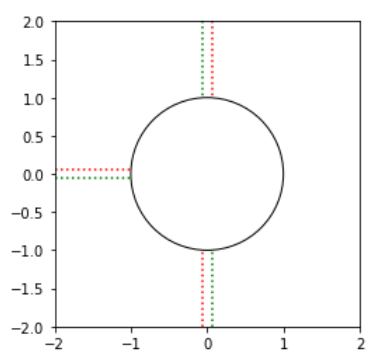


Figure 14: Layout of the multi-road model

[3]From this point we will be referring to the closed loop road, set up in Section 3.1, as the **main** road and the extra roads as **slip** roads. The point at which two roads are connected is a **junction**.

7.1 Car Movement and Road Interactions

There are many ways of defining rules for how these roads should interact with one another. We adopted the following:

Joining main road:

- *Cars slow down as they approach the main road* — Cars on a slip road in will gradually reduce their velocity as they approach the main road, so that at the junction, their highest possible velocity = 1.
- *Cars "give way" at slip road junction* — A car may only join the main road if there is a space free. For each time interval, the main road moves first, then the slip roads.

Leaving main road:

- *Cars leave randomly* — A car at a leaving junction has a probability, p_l, of driving onto the exit slip road.
- *Cars stuck in a jam leave* — If for a car at a leaving junction, $v = 0$ and there is a car directly in front, $p_l = 1$ (this increases the chance of cars leaving if there is a traffic jam present at the junction).

7.2 Distance Time Graphs for Multi-Road Model

The addition of multiple roads allows for different initial car distributions in our model. As the main road functions identically to the single road system everywhere but the junctions, the largest changes in the distance time graphs will be located at these junctions.

Take the following initial distribution:

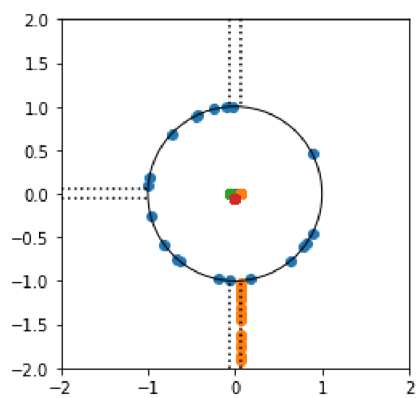


Figure 15: Main road - 20 cars, South slip road - 20 cars

The corresponding distance time graph for this inital distribution is shown below in Figure (16):

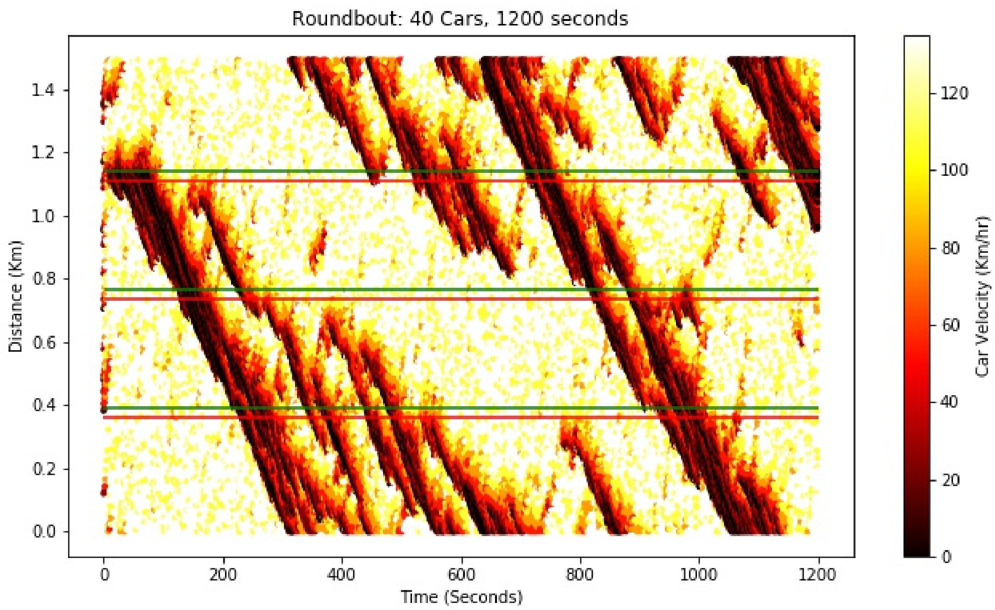


Figure 16: Distance Time Graph ($L = 200, p = 0.25, p_l = 0$)

The green and red horizontal lines represent slip roads in and out respectively. A distance of zero is point $(0, 1)$ in Figure (15) and cars travel counter-clockwise around the main road. The south slip roads are therefore located at distance ≈ 1.15km and in Figure (16) can be clearly seen as the source of the largest traffic jam the main road experiences. This is an expected result, as cars join the main road from the slip road, local density in that area exceeds k_{Vmax} and a jam forms behind. This effect is particularly noticeable in our next example:

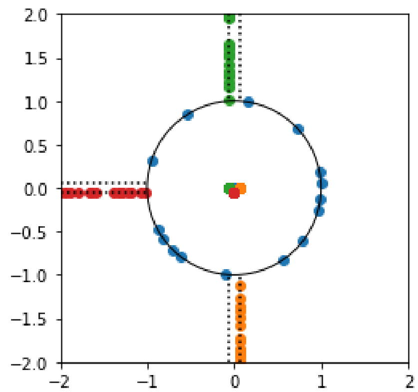


Figure 17: Main road - 15 cars, Each slip road - 15 cars

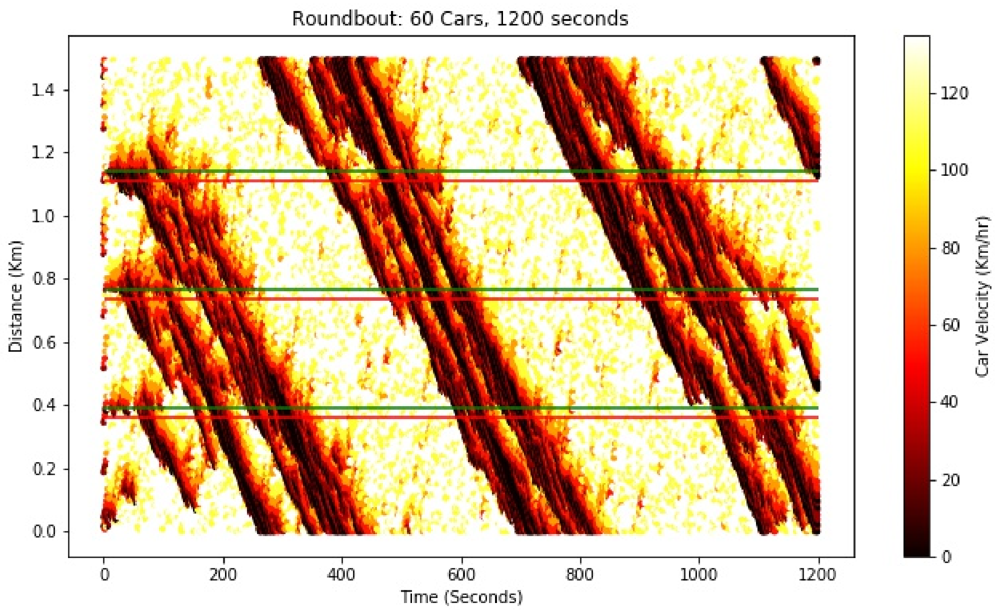


Figure 18: Distance Time Graph ($L = 200, p = 0.25, p_l = 0$)

Figure (18) nicely shows that with a final global density of 0.3, and 75% of those cars fed into the main road from the slip roads, the locations of the slip roads become the dominating factor in determining where the traffic jams originate. This is an exaggerated example as $p_l = 0$ and a road is unlikely to experience such a high influx of cars without losing any, but it does highlight the knock on effect junctions have on a road.

There are countless more variations on initial distributions, but even the little shown here once again demonstrate the strengths of a TCA model.

8 Conclusions

In conclusion, we have established the legitimacy of our traffic model with comparisons to both theoretical predictions and real life data. Many of the theoretical predictions turned out to be too simplistic an approach, with our data provided a more accurate representation.

Greenshield's macroscopic stream model, shown in Equation (6), which suggests a linear relationship between velocity and density, was contradicted by both our model and the real world data provided by TRL - at densities greater than $k_{V_{max}}$ an exponential relationship seems far more likely. Our relationship between velocity and flow also contradicts the simplistic nature of the predictions whilst supporting the real world data. Both our data in Figure (5) and the TRL data in Figure (13) exhibit characteristic "free-flowing" and "congested" regions.

Our analysis of traffic jam propagation resulted in Equation (13). Theoretically this would allow one to determine the speed at which a traffic jam would propagate backwards down a road, given p and k. Whilst density is easily adapted from real world data, p is rather non-physical and would be more accurately described as a mechanism by which the model functions, rather than a measurable quantity. It would require some statistical analysis in order to estimate an accurate value, but given that this equation may be able to provide meaningful estimates.

Finally we demonstrated how our model could be adapted into a more complex, interconnected road system. The addition of slip roads and the ability to fully control a multitude of variables such as initial distributions and p_l values, allow for a huge range possible simulations. The following variations have also been successfully tested within this model, but as of yet they have not had enough data collected to be analysed. These include: varying V_max, allowing p to be a function of v or k, removing the slip road speed reduction and more.

References

[1] Transportation Systems Engineering. "[ONLINE] Available at: "$https://nptel.ac.in/courses/105101008/downloads/cete_03.pdf$" *Traffic Stream Models*. Date accessed:27/03/2019.

[2] S O Notley, N Bourne and N B Taylor. "TRL Insight Report INS003" *Speed, flow and density of motorway traffic*. Date accessed:27/03/2019.