Understanding Analysis (Stephen Abbott) Exercises

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1 The Real Numbers

1.1 Discussion: The Irrationality of $\sqrt{2}$

1.2 Some Preliminaries

> **Exercise 1.2.1.**
>
> (a) Prove that $\sqrt{3}$ is irrational. Does a similar argument work to show $\sqrt{6}$ is irrational?
> (b) Where does the proof of Theorem 1.1.1 break down if we try to use it to prove $\sqrt{4}$ is irrational?

Proof of (a). Suppose by way of contradiction that $\sqrt{3}$ is rational. Then there exist $p, q \in \mathbb{Z}$ coprime such that $\sqrt{3} = p/q$. Square both sides to get

$$\left(\frac{p}{q}\right)^2 = 3.$$

Rewriting the above equality yields

$$p^2 = 3q^2.$$

$\square$