2D heat equation

Aditya Bhandari

May 2024

The following equation is the heat equation in two dimensions

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$u(t,x,y) \quad t \in [0,1]$$

$$u(t,x,0) = u(t,0,y) = 1$$

$$u(0,x,y) = \delta_{x0} + \delta_{x1}$$

$$x \in [0,1] \qquad y \in [0,1]$$

$$\implies \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \alpha \{ \frac{u_{i+1,j}^{n+1} + u_{i+1,j}^{n} - 2(u_{i,j}^{n+1} + u_{i,j}^{n}) + u_{i-1,j}^{n+1} + u_{i-1,j}^{n}}{2(\Delta x)^2}$$
$$+ \frac{u_{i,j+1}^{n+1} + u_{i,j+1}^{n} - 2(u_{i,j}^{n+1} + u_{i,j}^{n}) + u_{i,j-1}^{n+1} + u_{i,j-1}^{n}}{2(\Delta y)^2} \}$$

$$r_x = \frac{\alpha \Delta t}{(\Delta x)^2} \ , \ r_y = \frac{\alpha \Delta t}{(\Delta y)^2}$$

$$\implies u_{i,j}^{n+1} - u_{i,j}^{n} = \frac{r_x}{2} \{ u_{i+1,j}^{n+1} + u_{i+1,j}^{n} - 2(u_{i,j}^{n+1} + u_{i,j}^{n}) + u_{i-1,j}^{n+1} + u_{i-1,j}^{n} \}$$
$$+ \frac{r_y}{2} \{ u_{i,j+1}^{n+1} + u_{i,j+1}^{n} - 2(u_{i,j}^{n+1} + u_{i,j}^{n}) + u_{i,j-1}^{n+1} + u_{i,j-1}^{n} \}$$

$$\implies u_{i-1,j}^{n+1} \{ \frac{-r_x}{2} \} + u_{i,j-1}^{n+1} \{ \frac{-r_y}{2} \} + u_{i,j}^{n+1} \{ 1 + r_x + r_y \} + u_{i,j+1}^{n+1} \{ \frac{-r_y}{2} \} + u_{i+1,j}^{n+1} \{ \frac{-r_x}{2} \} = d_{i,j}^{n}$$

$$where, \quad d_{i,j}^{n} = u_{i-1,j}^{n} \{ \frac{r_x}{2} \} + u_{i,j-1}^{n} \{ \frac{r_y}{2} \} + u_{i,j}^{n} \{ 1 - r_x - r_y \} + u_{i,j+1}^{n} \{ \frac{r_y}{2} \} + u_{i+1,j}^{n} \{ \frac{r_x}{2} \}$$

$$\implies a u_{i-1,j}^{n+1} + b u_{i,j-1}^{n+1} + c u_{i,j}^{n+1} + b u_{i,j+1}^{n+1} + a u_{i+1,j}^{n+1} = d_{i,j}^{n}$$

$$where, a = \frac{-r_x}{2}$$
$$b = \frac{-r_y}{2}$$
$$c = 1 + r_x + r_y$$

When the x-axis, y-axis, and time t are divided into p,q, and td divisions respectively. Note that for boundary points aka for values of form $u_{i,q}^{n+1}$ the value after c is not b but replaced by 0 instead. Similarly, for boundary points aka for values of form $u_{k,0}^{n+1}$ the value before c is not b but replaced by 0 instead.

$$
\begin{bmatrix}
c & b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 \\
b & c & b & 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 \\
0 & b & c & b & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & . & . & 0 & b & c & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 \\
a & 0 & 0 & . & . & 0 & 0 & c & b & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & . & . & 0 & 0 \\
0 & a & 0 & 0 & . & . & 0 & b & c & b & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & a & 0 & 0 & 0 & . & . & 0 & 0 \\
0 & 0 & a & 0 & 0 & . & . & 0 & b & c & b & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & a & 0 & 0 & . & . & 0 & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & a & 0 & 0 & 0 & 0 & . & . & 0 & b & c & 0 & 0 & 0 & 0 & 0 & . & 0 & a \\
0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & . & . & 0 & 0 & c & b & 0 & 0 & 0 & 0 & . & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & a & 0 & . & 0 & b & c & b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & 0 & a & 0 & . & 0 & b & c & b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & 0 & 0 & a & 0 & . & 0 & b & c
\end{bmatrix}
\begin{bmatrix}
u_{0,0}^{n+1} \\ u_{0,1}^{n+1} \\ u_{0,2}^{n+1} \\ . \\ . \\ u_{0,q}^{n+1} \\ u_{1,0}^{n+1} \\ u_{1,1}^{n+1} \\ u_{1,2}^{n+1} \\ . \\ . \\ u_{p-1,q}^{n+1} \\ u_{p,0}^{n+1} \\ . \\ . \\ u_{p,q-2}^{n+1} \\ u_{p,q-1}^{n+1} \\ u_{p,q}^{n+1}
\end{bmatrix}
=
\begin{bmatrix}
d_{0,0}^{n} \\ d_{0,1}^{n} \\ d_{0,2}^{n} \\ . \\ . \\ d_{0,q}^{n} \\ d_{1,0}^{n} \\ d_{1,1}^{n} \\ d_{1,2}^{n} \\ . \\ . \\ d_{p-1,q}^{n} \\ d_{p,0}^{n} \\ . \\ . \\ d_{p,q-2}^{n} \\ d_{p,q-1}^{n} \\ d_{p,q}^{n}
\end{bmatrix}
$$

Suppose we acquire a matrix where p=5 and q=5 then the system of linear equations would be as such

$$
\begin{bmatrix}
c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & b & c & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a & 0 & 0 & 0 & 0 & 0 & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 & 0 & a & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & 0 & 0 & 0 & 0 & 0 & a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & c & b & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c & b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & b & c
\end{bmatrix}
\begin{bmatrix}
u_{0,0}^{n+1} \\ u_{0,1}^{n+1} \\ u_{0,2}^{n+1} \\ u_{0,3}^{n+1} \\ u_{0,4}^{n+1} \\ u_{0,5}^{n+1} \\
u_{1,0}^{n+1} \\ u_{1,1}^{n+1} \\ u_{1,2}^{n+1} \\ u_{1,3}^{n+1} \\ u_{1,4}^{n+1} \\ u_{1,5}^{n+1} \\
u_{2,0}^{n+1} \\ u_{2,1}^{n+1} \\ u_{2,2}^{n+1} \\ u_{2,3}^{n+1} \\ u_{2,4}^{n+1} \\ u_{2,5}^{n+1} \\
u_{3,0}^{n+1} \\ u_{3,1}^{n+1} \\ u_{3,2}^{n+1} \\ u_{3,3}^{n+1} \\ u_{3,4}^{n+1} \\ u_{3,5}^{n+1} \\
u_{4,0}^{n+1} \\ u_{4,1}^{n+1} \\ u_{4,2}^{n+1} \\ u_{4,3}^{n+1} \\ u_{4,4}^{n+1} \\ u_{4,5}^{n+1} \\
u_{5,0}^{n+1} \\ u_{5,1}^{n+1} \\ u_{5,2}^{n+1} \\ u_{5,3}^{n+1} \\ u_{5,4}^{n+1} \\ u_{5,5}^{n+1}
\end{bmatrix}
=
\begin{bmatrix}
d_{0,0}^{n} \\ d_{0,1}^{n} \\ d_{0,2}^{n} \\ d_{0,3}^{n} \\ d_{0,4}^{n} \\ d_{0,5}^{n} \\
d_{1,0}^{n} \\ d_{1,1}^{n} \\ d_{1,2}^{n} \\ d_{1,3}^{n} \\ d_{1,4}^{n} \\ d_{1,5}^{n} \\
d_{2,0}^{n} \\ d_{2,1}^{n} \\ d_{2,2}^{n} \\ d_{2,3}^{n} \\ d_{2,4}^{n} \\ d_{2,5}^{n} \\
d_{3,0}^{n} \\ d_{3,1}^{n} \\ d_{3,2}^{n} \\ d_{3,3}^{n} \\ d_{3,4}^{n} \\ d_{3,5}^{n} \\
d_{4,0}^{n} \\ d_{4,1}^{n} \\ d_{4,2}^{n} \\ d_{4,3}^{n} \\ d_{4,4}^{n} \\ d_{4,5}^{n} \\
d_{5,0}^{n} \\ d_{5,1}^{n} \\ d_{5,2}^{n} \\ d_{5,3}^{n} \\ d_{5,4}^{n} \\ d_{5,5}^{n}
\end{bmatrix}
$$

ALGORITHM

1) Start

2) Define p,q, and td divisions of x,y, and t respectively.

3) Define α which is thermal diffusivity.

4) Define ti,tf, xi, xf, yi, and yf where i stands for initial and f stands for final.

5) Set dx=$\frac{xf-xi}{p}$ and x(n)=xi+n.dx

6) Set dy=$\frac{yf-yi}{q}$ and y(n)=yi+n.dy

7) Set dt=$\frac{tf-ti}{td}$ and t(n)=ti+n.dt

8) Set rx = $\frac{\alpha dt}{(dx)^2}$, ry = $\frac{\alpha dt}{(dy)^2}$

9) set al=$\frac{-rx}{2}$, bl=$\frac{-ry}{2}$, cl= 1+rx+ry

10) Define a zero matrix of dimensions p+1 x q+1 x td+1 of name fs.

11) Set i=2 and repeat steps 12-15 till i<=q+1

12) Set j=2 and repeat steps 13,14 till j<=p+1

13) set fs(i,j,1)=sin(πx(j-1)).sin(πy(i-1))

14) j=j+1

15) i=i+1

16) Set ptr=q+2

17) Define zero matrix ls of dimensions (p+1.q+1) x (p+1.q+1)

18) Set ls(1,1)=cl , ls(1,2)=bl and ls(1,q+2)=al

19) Set r=2 and repeat steps 20-24 till r<= (p+1).(q+1)-1

20) Set ls(r,r-1)=bl , ls(r,r) = cl , ls(r,r+1) = bl

21) if ptr<= (p+1).(q+1) then set ls(r,ptr)=al

22) if r-q-1>= 1 then set ls(r,r-q-1) = al

23) if mod(r,q+1)=0 then set ls(r,r-1) = 0 and if mod(r,q+1)=q then set ls(r,r+1)=0

24) r=r+1 and ptr=ptr+1

25) set ls((p+1).(q+1),(p+1).(q+1))=cl , ls((p+1).(q+1),(p+1).(q+1)-1)=bl and ls((p+1).(q+1),(p+1).(q+1)-q-1)=al and zero matrix rs of dimensions (p+1).(q+1) x 1

26) Set tt=1 and repeat steps 27-47 till tt<=td+1

27) Set row = 1

28) Set i=1 and repeat steps 29-37 till i<=p+1

29) Set j=1 and repeat steps 30-36 till j<=q+1

30) Set xc=i and yc = j

31) Set

$$d^n_{i,j} = u^n_{i-1,j}\{\frac{r_x}{2}\} + u^n_{i,j-1}\{\frac{r_y}{2}\} \tag{1}$$

$$+ u^n_{i,j}\{1 - r_x - r_y\} + u^n_{i,j+1}\{\frac{r_y}{2}\} + u^n_{i+1,j}\{\frac{r_x}{2}\} \tag{2}$$

Where $u^n_{i,j}$ = fs(yc,xc,tt-1) , $u^n_{i-1,j}$ = fs(yc,xc-1,tt-1) and so on

32) If j=q+1 then remove term $u^n_{i,j+1}\{\frac{r_y}{2}\}$ from $d^n_{i,j}$ and if j=0 then remove term $u^n_{i,j-1}\{\frac{r_y}{2}\}$

33) If i=p+1 then remove term $u^n_{i+1,j}\{\frac{r_x}{2}\}$ from $d^n_{i,j}$ and if i=0 then remove term $u^n_{i-1,j}\{\frac{r_x}{2}\}$

34) Set rs(row,1) = $d^n_{i,j}$

35) set row = row+1

36) j=j+1

37) i=i+1

38) Set matrix soln=$ls^{-1}.rs$

39) set row=1

40) set i=1 and repeat steps 41-46 till i<=p+1

41) set j=1 and repeat steps 42-45 till j<=q+1

42) Set xc=i and yc=j

43) Set fs(yc,xc,tt)=soln(row,1)

44) set row=row+1

45) set j=j+1

46) set i=i+1

47) tt=tt+1 and set boundary conditions

48) The solution is acquired in fs where right along a row denotes increasing x, down a column denotes increasing y and outwards along z axis denotes increasing t

49) The solution is acquired in fs

50) End