Thomas algorithm

Aditya Bhandari

May 2024

1 Thomas Algorithm

i) For a given set of linear equations of the form

$$\begin{bmatrix} b_1 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_4 & b_4 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & b_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_6 & b_6 & c_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_7 & b_7 & c_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_8 & b_8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{bmatrix}$$

ii) The following set of linear equations is to be obtained is to be obtained

$$\begin{bmatrix} 1 & \gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \gamma_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \gamma_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \gamma_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \gamma_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \gamma_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \rho_7 \\ \rho_8 \end{bmatrix}$$

iii) The value of γ_n is computed according to the formula

$$\gamma_n = \frac{c_n}{b_n - a_n\gamma_{n-1}}$$

iv) Where the value of γ_1 is calculated as

$$\gamma_1 = \frac{c_1}{b_1}$$

v) The value of ρ_n is computed according to the formula

$$\rho_n = \frac{r_n - a_n\rho_{n-1}}{b_n - a_n\gamma_{n-1}}$$

vi) Where the value of ρ_1 is calculated as

$$\rho_1 = \frac{r_1}{b_1}$$

vii) To obtain the solutions of x_n as required, start from the bottom row. x_8=ρ_8

viii) Now to obtain the solution for x_7. We know that x_7+$\gamma_7 x_8 = \rho_7$. Therefore we compute x_7 as $x_7 = \rho_7 - \gamma_7 x_8$

ix) Following the same pattern we computer x_n as $x_n = \rho_n - \gamma_n x_{n+1}$

x) Thus we acquire the required solutions

xi) For the problem involving the 1D heat equation, the set of linear equations are such

$$\begin{bmatrix} 1+2r & -r & 0 & 0 & 0 & 0 & 0 & 0 \\ -r & 1+2r & -r & 0 & 0 & 0 & 0 & 0 \\ 0 & -r & 1+2r & -r & 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 1+2r & -r & 0 & 0 & 0 \\ 0 & 0 & 0 & -r & 1+2r & -r & 0 & 0 \\ 0 & 0 & 0 & 0 & -r & 1+2r & -r & 0 \\ 0 & 0 & 0 & 0 & 0 & -r & 1+2r & -r \\ 0 & 0 & 0 & 0 & 0 & 0 & -r & 1+2r \end{bmatrix} \begin{bmatrix} u_1^i \\ u_2^i \\ u_3^i \\ u_4^i \\ u_5^i \\ u_6^i \\ u_7^i \\ u_8^i \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{bmatrix}$$