Generative Adversarial Networks (GANs) & Game Theory

IMPORTANT NOTES:
Study lecture materials at least 1 hour and prepare the questions prior to the tutorial session. The questions will be discussed in the tutorial session.

1. Two friends, Alice and Bob, want to meet up but have two possible locations: Cafe or Library. They both prefer to meet at the same location rather than going to different places.

$(Alice, Bob)$	Cafe	Library
Cafe	(2,2)	(0,0)
Library	(0,0)	(1,1)

Is there a nash equilibrium?

If Alice chooses Cafe and Bob chooses Cafe, both get (2,2). Neither can do better by changing alone $\rightarrow$ Nash Equilibrium. If Alice chooses Library and Bob chooses Library, both get (1,1). Again, neither can improve by deviating $\rightarrow$ Nash Equilibrium. If they choose different places $\rightarrow$ payoff (0,0). Here, either player has an incentive to switch to match the other. So these are not equilibria.

Thus, there are two pure strategy Nash equilibria $\rightarrow$ (Cafe, Cafe) and (Library, Library).

2. Hawk–Dove Game: Two agents, A and B, compete for a single resource. Each agent chooses one of two strategies: They can either act as a Hawk (aggressive) or a Dove (peaceful).

The payoffs are as follows:

- If both act as Hawks , they fight, and each gets a payoff of -1 (due to injury).
- If one acts as a Hawk and the other as a Dove, the Hawk gets the resource (payoff of 3), and the Dove gets nothing (payoff of 0).
- If both act as Doves, they share the resource equally (payoff of 1 each).

Based on the payoff structure given, complete the following tasks:

(a) Construct the payoff matrix.
(b) What is the Pure Strategy Nash Equilibrium ?
(c) What is the Mixed Strategy Nash Equilibrium?
(d) What is the optimal outcome?

(a) Payoff matrix

(A, B)	Hawk	Dove
Hawk	(-1,-1)	(3,0)
Dove	(0,3)	(1,1)

(b) Pure Strategy Nash Equilibria (PSNE) Evaluate unilateral deviations for each profile:

- (Hawk, Hawk) = $(-1, -1)$
  If A unilaterally switches to Dove, A gets $0 > -1$. A has incentive to deviate. Not a NE.
- (Hawk, Dove) = (3, 0)
  A (Hawk) cannot improve by switching to Dove (would get $1 < 3$. B (Dove) cannot improve by switching to Hawk (would get $-1 < 0$).
- (Dove, Hawk) = (0, 3)
  Symmetric argument (Dove, Hawk) is a Nash equilibrium. So, (Hawk, Dove) is a Nash equilibrium.

- (Dove, Dove) = (1, 1)
  If A switches to Hawk (while B stays Dove) A would get 3 ¿ 1. A has incentive to deviate. Not a NE.

There are two pure-strategy Nash equilibria: (Hawk, Dove) and (Dove, Hawk) (i.e., one agent plays Hawk and the other plays Dove).

(c) Mixed Strategy Nash Equilibrium (MSNE):
Compute expected payoffs (call opponent's Hawk probability p):

- Expected payoff for playing Hawk: $E(Hawk) = p \cdot (-1) + (1-p) \cdot 3$
  $E(Hawk) = -p + 3 - 3p = 3 - 4p$
- Expected payoff for playing Dove: $E(Dove) = p \cdot 0 + (1-p) \cdot 1 = 1 - p$

Indifference condition $E(Hawk) = E(Dove) : 3 - 4p = 1 - p$
Solve for p:
$3 - 1 = 4p - p$
$2 = 3p$
$p = \frac{2}{3}$
So, in the symmetric MSNE each player plays Hawk with probability $\frac{2}{3}$ and Dove with probability $\frac{1}{3}$.

(d) The outcome that gives the highest total payoff is when one plays Hawk and the other plays Dove $\rightarrow$ total payoff = 3. The outcome that is fairest for both players is when both play Dove $\rightarrow$ payoff = (1,1). So the socially optimal choice (fair and stable for both) is **(Dove, Dove)**, even though it is not a Nash equilibrium.

3. Partnership Game: Two players A and B can choose between Work Hard or Shirk.

   Below are the payoff matrix

(A, B)	Work Hard	Shirk
Work Hard	(2,2)	(-1,1)
Shirk	(1,-1)	(0,0)

   Identify the Nash equilibrium for this game.

   Two Nash equilibria: (Work Hard, Work Hard) and (Shirk, Shirk).

   - (work hard, work hard): neither A nor B can improve their outcome by changing strategies, so this could be an equilibrium.
   - (shirk, shirk): both players have no incentive to change the strategies

4. Stag Hunt Game: Two hunters A and B can hunt either Stag (requires cooperation) or Hare (individually sufficient).

   Two hunters, A and B, must each choose one type of hunting equipment.

   - Stag option: If both choose to hunt the stag, they must cooperate, and together they can capture it, yielding a total of 6 units of meat (3 units each).
   - Hare option: If a hunter chooses to hunt hare, he can do so alone and will obtain 1 unit of meat for himself, regardless of the other's choice.

   Below are the payoff matrix

(A, B)	Stag	Hare
Stag	(3,3)	(0,2)
Hare	(2,0)	(1,1)

Identify the Nash equilibrium for this game.

Two Nash equilibria: (Stag, Stag) (efficient but risky) and (Hare, Hare) (safe but lower payoff).

- (Stag, Stag): Both gain higher payoff (3,3). Neither has incentive to deviate.
- (Hare, Hare): Both get (1,1). If one deviates to Stag alone, payoff drops to 0.

5. Traffic Light Game: Two drivers, A and B, approach an intersection from perpendicular directions. At any given moment, the traffic light is green for one driver and red for the other. There is no enforcement (no police, no traffic fines). Each driver faces a dilemma: Should they obey the light and stop, or break the rule and go?

   Below are the payoff matrix

(A, B)	Go	Stop
Go	(-5,-5)	(1,0)
Stop	(0,1)	(-1,-1)

   Identify the Nash equilibrium for this game.

   Two asymmetric Nash equilibria: (Go, Stop) and (Stop, Go).

   - If both Go, both crash $(-5, -5)$, so each would want to change. Not equilibrium.
   - If A Go, B Stop; payoff (1,0). B switching to Go reduces payoff, so equilibrium.
   - If A Stop, B Go; payoff (0,1). Symmetrical equilibrium.
   - If both Stop, $(-1,-1)$. Either one can improve by deviating to Go. Not equilibrium.

6. What does Nash equilibrium mean, and how does this concept apply to the training dynamics of Generative Adversarial Networks (GANs)?

   In game theory, a Nash equilibrium is a situation where no player can improve their outcome by changing strategy alone.

   In a GAN (Generative Adversarial Network), the generator (G) tries to create realistic samples, while the discriminator (D) tries to tell real from fake. Training is a two-player zero-sum game: G minimizes the loss, D maximizes the same loss.

   A Nash equilibrium in GANs occurs when: G produces samples that look exactly like real data ($p_{\text{model}} = p_{\text{real}}$). D outputs 0.5 for all inputs (cannot distinguish real from fake).

   At this point, neither G nor D can improve unilaterally. This is the ideal balance in GAN training, although in practice training often oscillates around this point rather than fully converging.

7. What key challenges arise when training Generative Adversarial Networks (GANs), and why do they make the training process unstable or difficult to converge?

   Training Generative Adversarial Networks (GANs) is difficult because the generator and discriminator must learn in balance. Common challenges include:

   - Mode Collapse – The generator produces only a narrow range of outputs instead of diverse samples.
   - Discriminator Saturation – If the discriminator learns too quickly, the generator receives little useful feedback.
   - Gradient Instability – Training can suffer from vanishing or exploding gradients, causing unstable updates.
   - Oscillations / Freezing – One network can dominate, leading to cycles or stagnation in training.

- Evaluation Difficulty – It is hard to measure sample quality objectively; metrics like FID and Inception Score are used but imperfect.
- Slow Convergence & High Cost – GANs often require long training times and significant compute resources.
- Hyperparameter Sensitivity – GANs are highly sensitive to learning rates, batch sizes, and architecture choices.
- Data Demands – GANs typically need large datasets; performance drops with limited data.

GAN training is unstable and resource-intensive because both networks co-evolve. Careful design choices (e.g., Wasserstein loss, regularization, learning rate tuning) are often required for stable convergence.

8. How can the challenges in training Generative Adversarial Networks (GANs) be effectively addressed?

Several strategies can help stabilize and improve GAN training:

- Transfer Learning – Start from pre-trained GANs and fine-tune on target datasets to reduce data and compute requirements.
- Regularization – Apply techniques like weight decay, dropout, and batch normalization to improve gradient flow and reduce mode collapse.
- Advanced Architectures & Loss Functions – Use GAN variants (e.g., Wasserstein GAN, StyleGAN) designed for more stable and high-quality training.
- Ensemble Methods – Combine multiple GANs to enhance sample diversity and robustness.
- High-Quality Datasets – Ensure training data is diverse and representative to support stable learning.
- Consistent Evaluation – Use standardized metrics (e.g., FID, precision-recall curves) to objectively assess sample quality.
- Optimized Computing Resources – Employ GPUs, TPUs, or distributed training to accelerate convergence and handle large models.

9. How can Generative Adversarial Networks (GANs) be applied to support privacy-preserving data release?

GANs can generate synthetic data that protects sensitive information while remaining useful for analysis. Generator's role: Acts as a perturbation function, producing data that hides or removes private attributes. Discriminator's role: Ensures the generated data is realistic and statistically similar to the original, so that it remains usable but difficult for attackers to distinguish from real data. Applications: GANs have been applied to preserve privacy across many domains, including images, videos, text, speech, spatio-temporal data, and graphs. GANs enable the creation of synthetic datasets that balance two goals: protecting privacy and maintaining utility for downstream machine learning or analytics tasks.