

# Worksheet on “Probability”

PRML – CS5691 (Jul–Nov 2023)

August 4, 2023

1. For three events A, B, and C, we know that

- A and C are independent,
- B and C are independent,
- A and B are disjoint,

Given  $P(A \cup C) = \frac{2}{3}$ ,  $P(B \cup C) = \frac{3}{4}$ , and  $P(A \cup B \cup C) = \frac{11}{12}$ . Find  $P(A)$ ,  $P(B)$ , and  $P(C)$ .

2. There are three bags that contain 100 marbles each:

- Bag 1 contains 75 red and 25 blue marble;
- Bag 2 contains 60 red and 40 blue marble;
- Bag 3 contains 45 red and 55 blue marble;

A bag is chosen uniformly at random and a marble is also chosen uniformly at random. (a) What is the probability that the chosen marble is red? (b) If the chosen marble is red, what is the probability that Bag 1 was chosen?

3. You toss a fair coin three times. Given that you have observed at least one tails, what is the probability that you observe at least two tails?

4. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

5. A fair coin is tossed twice, and  $\mathbf{X}$  is defined as the number of heads that are observed. Find the range of  $\mathbf{X}$  ( $R_X$ ) and the probability mass function ( $P_X$ ).

6. (RECOMMENDED FOR NEXT WEEK CLASSES) Consider a continuous random variable  $X$  and a discrete random variable  $Y$ . Let

- $P_Y(Y = 1) = 0.5$  and  $P_Y(Y = -1) = 0.5$ , and
- $(X|Y = 1) \sim \text{Unif}(1, 2)$  and  $(X|Y = -1) \sim \text{Unif}(-2, -1)$ .

- a. What is the marginal distribution of  $X$ ? Specifically, plot the pdf of  $X$  denoted  $f_X(x)$ .
- b. Write down the pdf  $f_X(x)$  of  $X$ .
- c. Let  $Z = X^2$ . What is the pdf of  $Z$ ? Use it to compute  $E[Z]$ . (Hint: To obtain pdf of  $Z$ , you could simply derive the cdf of  $Z$  and differentiate it (or you could also use the change-of-variables formula).)
- d. Now, use the pdf of  $X$  directly to compute  $E[X^2]$  (using the law of the unconscious statistician). Does this give the same answer as the previous question?