CS6046 Problem Set 2

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1. (8+2 marks) Let X_1 and X_2 be σ_1 and σ_2-sub-Gaussian random variables respectively. Then, show that $X_1 + X_2$ is $\sigma_1 + \sigma_2-$sub-Gaussian. Additionally, if X_1 and X_2 are independent, then show that $X_1 + X_2$ is $\sqrt{\sigma_1^2 + \sigma_2^2}-$sub-Gaussian.

2. (5+5 marks) If $X_1, X_2, ..., X_n$ be n independent non-identical random variables such that $X_i \sim$ Bern(p_i). Let $S_n = \sum_{i=1}^n X_i$ and $\mu = \mathbb{E}[S_n]$. Then, prove

$$\mathbb{P}[S_n \geq (1+\epsilon)\mu] \leq \exp\big(-\frac{\epsilon^2\mu}{3}\big),\ 0 \leq \epsilon \leq 1$$
$$\mathbb{P}[S_n \leq (1-\epsilon)\mu] \leq \exp\big(-\frac{\epsilon^2\mu}{2}\big),\ 0 \leq \epsilon \leq 1$$

3. (10 marks) Consider a K-armed stochastic bandit with $1-$sub-Gaussian rewards. The Explore-Then-Commit (ETC) strategy (ETC) is played for n rounds. If each arm is played for $m = (\frac{n}{K})^{\frac{2}{3}}(\log n)^{\frac{1}{3}}$ times, then show that the regret $R_n \leq c_1 n^{\frac{2}{3}}(K \log n)^{\frac{1}{3}}$ for some constant c_1.

4. (**Computer Assignment**- 20 marks) In this exercise you will investigate the empirical behavior of ETC on a two-armed Gaussian bandit with means $\mu_1 = 0$ and $\mu_2 = -\Delta$. The horizon is set to $n = 1000$, and the sub-optimality gap Δ is varied between 0 and 1 as follows: $\Delta \in \{0, 0.1, 0.2, ..., 1\}$. Plot the performance of Explore-Then-Commit with $m = \max\{1, \frac{4}{\Delta^2}\log(\frac{n\Delta^2}{4})\}$ where m is the input to the ETC algorithm. Compare this with the theoretical upper bound,

$$R_n \leq \min\left\{n\Delta, \Delta + \frac{4}{\Delta}\left(\max\left\{0, 1 + \log\left(\frac{n\Delta^2}{4}\right)\right\}\right)\right\}$$

Note: In the above assignment, repeat the experiment 100 times for each value of Δ, and take the average value to get the average regret.

5. (Practice) For some $0 < \alpha < 1/2$, let p, q and r correspond to the pmfs of Bernoulli random variables with parameters $\frac{1}{2}$, $\frac{1+\alpha}{2}$ and $\frac{1-\alpha}{2}$ respectively. Then, prove

$$D(p,q) \leq \alpha^2, D(q,p) \leq 2\alpha^2, D(p,r) \leq \alpha^2 \text{ and } D(r,q) \leq 4\alpha^2.$$

Hint: Use the following type of inequalities on the logarithmic functions:

$\log(1-x) \geq -2x$ for $0 \leq x \leq 1/2$.