

Roll No: CS23E001

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Collaborators (if any):

References/sources (if any):

- Use \LaTeX to write-up your solutions (in the solution blocks of the source \LaTeX file of this assignment), and submit the resulting pdf files (one per question) at Crowdmark by the due date. (Note: **No late submissions** will be allowed, other than one-day late submission with 10% penalty or four-day late submission with 30% penalty! Instructions to join Crowdmark and submit your solution to each question within Crowdmark **TBA** later).
- For the programming question, please submit your code (rollno.ipynb file and rollno.py file in rollno.zip) directly in moodle, but provide your results/answers (including Jupyter notebook **with output**) in the pdf file you upload to Crowdmark.
- Collaboration is encouraged, but all write-ups must be done individually and independently, and mention your collaborator(s) if any. Same rules apply for codes written for any programming assignments (i.e., write your own code; we will run plagiarism checks on codes).
- If you have referred a book or any other online material or LLMs (Large Language Models like ChatGPT) for obtaining a solution, please cite the source. Again don't copy the source *as is* - you may use the source to understand the solution, but write-up the solution in your own words (this also means that you cannot copy-paste the solution from LLMs!). Please be advised that *the lesser your reliance on online materials or LLMs for answering the questions, the more your understanding of the concepts will be and the more prepared you will be for the course exams.*
- Points will be awarded based on how clear, concise and rigorous your solutions are, and how correct your answer is. The weightage of this assignment is 12% towards the overall course grade.

The solution of question (b)

The decision criterion that minimizes the expected loss is to choose the class C_j that minimizes the expected risk:

$$\text{Decision: } j = \arg \min_j \sum_k P(C_k|\text{input}) L_{kj} + P(\text{reject}|\text{input})\psi$$

Here, $P(C_k|\text{input})$ is the posterior probability of the true class C_k given the input data, and $P(\text{reject}|\text{input})$ is the probability of selecting the reject option given the input data.

Now, let's simplify this decision criterion for the case of 0-1 loss, which means L_{kj} is 0 if $k = j$ (correct classification) and 1 if $k \neq j$ (incorrect classification), and ψ is a constant representing the loss for rejecting.

In this case, the decision criterion becomes:

$$\text{Decision: } j = \arg \min_j \sum_k P(C_k|\text{input}) \cdot \mathbb{I}(k \neq j) + P(\text{reject}|\text{input})\psi \quad (1)$$

Here, $\mathbb{I}(k \neq j)$ is an indicator function that equals 1 when $k \neq j$ (incorrect classification) and 0 when $k = j$ (correct classification).

So, for the case of 0-1 loss, the decision criterion becomes select the class with the highest posterior probability, and if the probability of all classes is below a certain threshold (determined by ψ), then choose the reject option.