(CS5020, Jul-Nov 2023) Nonlinear Optimisation: Theory and Algorithms
Worksheet - 3

(1) Give linear approximations to the following functions from $\mathbb{R}$ to $\mathbb{R}$ around $x_0 = 0$:
   (a) $\sin(x)$
   (b) $3x^4 + 4x^3 + 8$
   (c) $e^x + 2x$
   (d) $\ln(2 + x)$, where $x \in (-2, \infty)$
   (e) $2x^2$
   (f) $\sin(x^2)$
   (i) Repeat problem, by giving linear approximations around $x_0 = 1$.
   (ii) Repeat problem, but give quadratic approximations around $x_0 = 0$ instead.
   (iii) Repeat problem, but give quadratic approximations around $x_0 = 1$ instead.

(2) Let $f(x) = \ln(2 + x)$ for $x \in (-2, \infty)$. Approximate f around $x_0 = 0$ using a fourth degree polynomial.

(3) Let $f(x) = \ln(2 + x)$ for $x \in (-2, \infty)$. Let $g(x) = a + bx + ce^x$. Find values of a, b, c such that g approximates f around $x_0 = 0$. (Match zeroth, first and second order derivatives).

(4) For the function $f(x) = x^4 - x^2$. Find the
   (a) stationary points
   (b) local/global minima/maxima
   (c) first order approximation at $x = 1$
   (d) second order approximation at $x = -1$

(5) Find a polynomial that has exactly 2 global minima.

(6) Write down gradient descent update rule and find the range of valid learning rate/step-size for minimising the function (a) $f(x) = 5x^2$, (b) $f(x) = \frac{x^2}{100}$, and (c) $f(x) = (x-5)^2$.