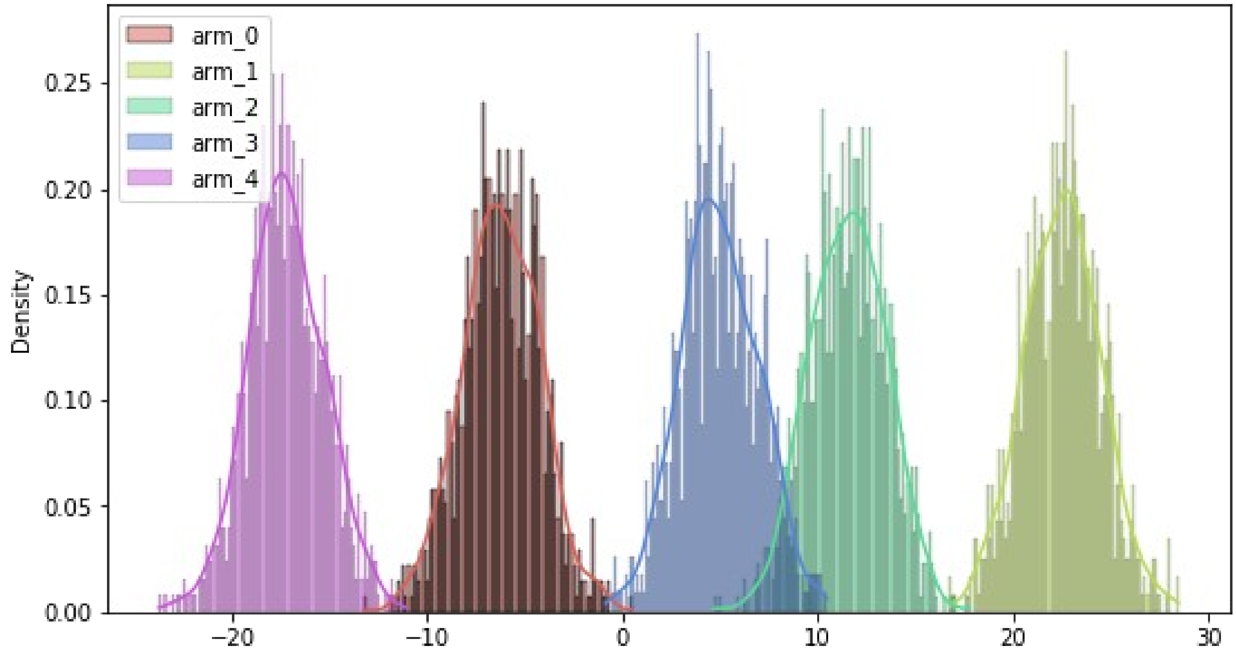
CS6700 : Tutorial 1 - Multi-Arm Bandits




```
    Goal: Analysis 3 types of sampling strategy in a MAB
```

Import dependencies

```python
# !pip install seaborn

import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from typing import NamedTuple, List
```

Gaussian Bandit Environment

```python
class GaussianArm(NamedTuple):
    mean: float
    std: float


class Env:
    def __init__(self, num_arms: int, mean_reward_range: tuple, std: float):
        """
        num_arms: number of bandit arms
        mean_reward_range: mean reward of an arm should lie between
```

```
the given range
        std: standard deviation of the reward for each arm
        """
        self.num_arms = num_arms
        self.arms = self.create_arms(num_arms, mean_reward_range, std)

    def create_arms(self, n: int, mean_reward_range: tuple, std:
float) -> dict:
        low_rwd, high_rwd = mean_reward_range
        # creates "n" number of mean reward for each arm
        means = np.random.uniform(low=low_rwd, high=high_rwd,
size=(n,))
        arms = {id: GaussianArm(mu, std) for id, mu in
enumerate(means)}
        return arms

    @property
    def arm_ids(self):
        return list(self.arms.keys())

    def step(self, arm_id: int) -> float:
        arm = self.arms[arm_id]
        return np.random.normal(arm.mean, arm.std)   # Reward

    def get_best_arm_and_expected_reward(self):
        best_arm_id = max(self.arms, key=lambda x: self.arms[x].mean)
        return best_arm_id, self.arms[best_arm_id].mean

    def get_avg_arm_reward(self):
        arm_mean_rewards = [v.mean for v in self.arms.values()]
        return np.mean(arm_mean_rewards)

    def plot_arms_reward_distribution(self, num_samples=1000):
        """
        This function is only used to visualize the arm's distrbution.
        """
        fig, ax = plt.subplots(
            1, 1, sharex=False, sharey=False, figsize=(9, 5))
        colors = sns.color_palette("hls", self.num_arms)
        for i, arm_id in enumerate(self.arm_ids):
            reward_samples = [self.step(arm_id) for _ in
range(num_samples)]
            sns.histplot(reward_samples, ax=ax, stat="density",
kde=True,
                             bins=100, color=colors[i],
label=f'arm_{arm_id}')
        ax.legend()
        plt.show()
```

Policy

```
class BasePolicy:
    @property
    def name(self):
        return 'base_policy'

    def reset(self):
        """
        This function resets the internal variable.
        """
        pass

    def update_arm(self, *args):
        """
        This function keep track of the estimates
        that we may want to update during training.
        """
        pass

    def select_arm(self) -> int:
        """
        It returns arm_id
        """
        raise Exception("Not Implemented")
```

Random Policy

```
class RandomPolicy(BasePolicy):
    def __init__(self, arm_ids: List[int]):
        self.arm_ids = arm_ids

    @property
    def name(self):
        return 'random'

    def reset(self) -> None:
        """No use."""
        pass

    def update_arm(self, *args) -> None:
        """No use."""
        pass

    def select_arm(self) -> int:
        return np.random.choice(self.arm_ids)

class EpGreedyPolicy(BasePolicy):
    def __init__(self, epsilon: float, arm_ids: List[int]):
        self.epsilon = epsilon
        self.arm_ids = arm_ids
```

```python
        self.Q = {id: 0 for id in self.arm_ids}
        self.num_pulls_per_arm = {id: 0 for id in self.arm_ids}

    @property
    def name(self):
        return f'ep-greedy ep:{self.epsilon}'

    def reset(self) -> None:
        self.Q = {id: 0 for id in self.arm_ids}
        self.num_pulls_per_arm = {id: 0 for id in self.arm_ids}

    def update_arm(self, arm_id: int, arm_reward: float) -> None:
        # your code for updating the Q values of each arm
        self.num_pulls_per_arm[arm_id] += 1
        self.Q[arm_id] += (1/self.num_pulls_per_arm[arm_id]) * \
            (arm_reward-self.Q[arm_id])

    def select_arm(self) -> int:
        # your code for selecting arm based on epsilon greedy policy
        random = np.random.uniform(0, 1)
        if random < self.epsilon:
            action = np.random.randint(0, len(self.Q.keys()))
        else:
            action = np.argmax([self.Q[key] for key in self.Q.keys()])
        return action

class SoftmaxPolicy(BasePolicy):
    def __init__(self, tau, arm_ids):
        self.tau = tau
        self.arm_ids = arm_ids
        self.Q = {id: 0 for id in self.arm_ids}
        self.num_pulls_per_arm = {id: 0 for id in self.arm_ids}

    @property
    def name(self):
        return f'softmax tau:{self.tau}'

    def reset(self):
        self.Q = {id: 0 for id in self.arm_ids}
        self.num_pulls_per_arm = {id: 0 for id in self.arm_ids}

    def update_arm(self, arm_id: int, arm_reward: float) -> None:
        # your code for updating the Q values of each arm
        self.num_pulls_per_arm[arm_id] += 1
        self.Q[arm_id] += (1/self.num_pulls_per_arm[arm_id]) * \
            (arm_reward-self.Q[arm_id])

    def select_arm(self) -> int:
        # your code for selecting arm based on softmax policy
        Q_vector = np.array([self.Q[key] for key in self.Q.keys()])
```

```python
        def softmax(x, t):
            exp_x = np.exp(x/t - np.max(x/t))
            sum_exp_x = np.sum(exp_x)
            return exp_x / sum_exp_x

        probs = softmax(Q_vector, self.tau)
        action = np.random.choice(len(self.Q.keys()), 1, p=probs)[0]
        return action

class UCB(BasePolicy):
    # your code here
    def __init__(self, arm_ids, c=4) -> None:
        self.c = c
        self.arm_ids = arm_ids
        self.Q = {id: 0 for id in self.arm_ids}
        self.num_pulls_per_arm = {id: 0 for id in self.arm_ids}
        self.counter = 0

    @property
    def name(self):
        return f"Upper Confidene Bound algorithm {self.c}"

    def reset(self):
        self.Q = {id: 0 for id in self.arm_ids}
        self.num_pulls_per_arm = {id: 0 for id in self.arm_ids}
        self.counter = 0

    def update_arm(self, arm_id: int, arm_reward: float) -> None:
        # your code for updating the Q values of each arm
        self.num_pulls_per_arm[arm_id] += 1
        self.Q[arm_id] += (1/self.num_pulls_per_arm[arm_id]) * \
            (arm_reward-self.Q[arm_id])

    def select_arm(self) -> int:
        # your code for selecting arm based on softmax policy
        self.counter += 1
        EPSILON = 1e-8
        Q_vector = np.array([self.Q[key] for key in self.Q.keys()])
        N_vector = np.array([self.num_pulls_per_arm[key]
                            for key in self.num_pulls_per_arm.keys()])
        action = np.argmax(Q_vector + self.c *
                           np.sqrt(self.counter/(N_vector + EPSILON)))
        return action
```

Trainer

```python
def train(env, policy: BasePolicy, timesteps):
    policy_reward = np.zeros((timesteps,))
    for t in range(timesteps):
```

```python
        arm_id = policy.select_arm()
        reward = env.step(arm_id)
        policy.update_arm(arm_id, reward)
        policy_reward[t] = reward
    return policy_reward


def avg_over_runs(env, policy: BasePolicy, timesteps, num_runs):
    _, expected_max_reward = env.get_best_arm_and_expected_reward()
    policy_reward_each_run = np.zeros((num_runs, timesteps))
    for run in range(num_runs):
        policy.reset()
        policy_reward = train(env, policy, timesteps)
        policy_reward_each_run[run, :] = policy_reward

    # calculate avg policy reward from policy_reward_each_run
    # your code here (type: nd.array, shape: (timesteps,))
    avg_policy_rewards = np.mean(policy_reward_each_run, axis=0)
    # your code here (type: float)
    total_policy_regret = expected_max_reward * \
        timesteps - np.sum(avg_policy_rewards)

    return avg_policy_rewards, total_policy_regret

def plot_reward_curve_and_print_regret(env, policies, timesteps=200,
num_runs=500):
    fig, ax = plt.subplots(1, 1, sharex=False, sharey=False,
figsize=(10, 6))
    for policy in policies:
        avg_policy_rewards, total_policy_regret = avg_over_runs(
            env, policy, timesteps, num_runs)
        print('regret for {}: {:.3f}'.format(policy.name,
total_policy_regret))
        ax.plot(np.arange(timesteps), avg_policy_rewards,
                '-', label=policy.name)

    _, expected_max_reward = env.get_best_arm_and_expected_reward()
    ax.plot(np.arange(timesteps), [expected_max_reward]*timesteps,
'g-')

    avg_arm_reward = env.get_avg_arm_reward()
    ax.plot(np.arange(timesteps), [avg_arm_reward]*timesteps, 'r-')

    plt.legend(loc='lower right')
    plt.show()
```

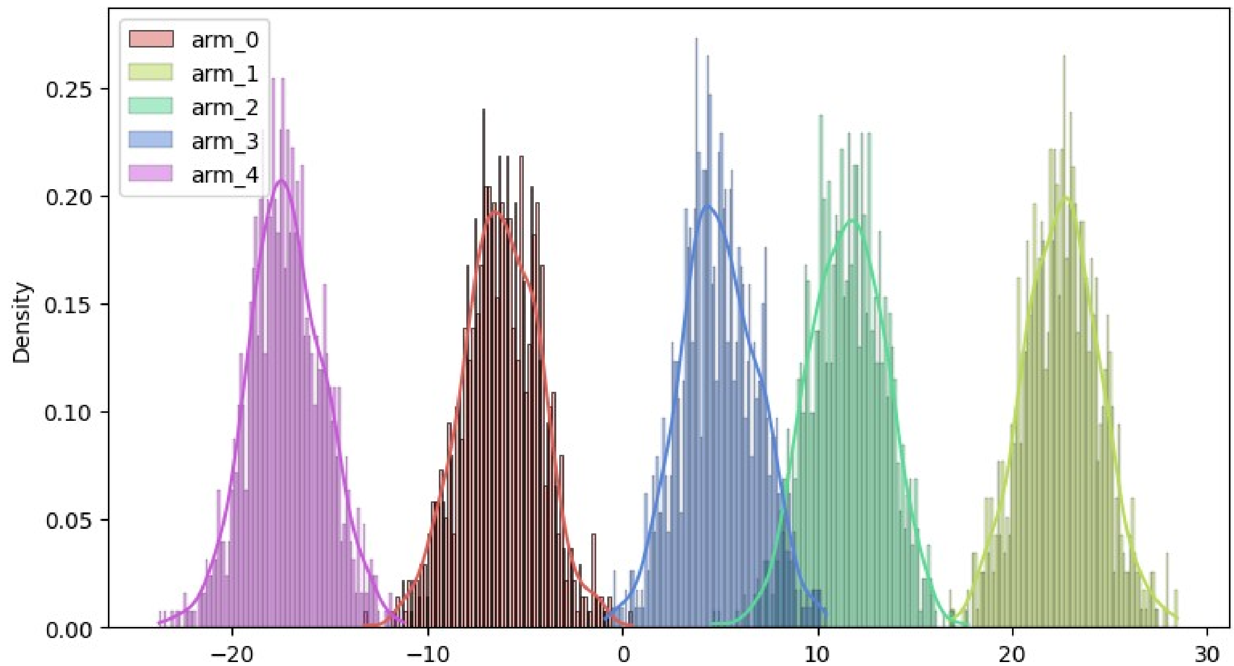
Experiments

```python
seed = 42
np.random.seed(seed)
```

```
num_arms = 5
mean_reward_range = (-25, 25)
std = 2.0

env = Env(num_arms, mean_reward_range, std)

env.plot_arms_reward_distribution()
```




```
best_arm, max_mean_reward = env.get_best_arm_and_expected_reward()
print(best_arm, max_mean_reward)
```

```
1 22.53571532049581
```

```
print(env.get_avg_arm_reward())
```

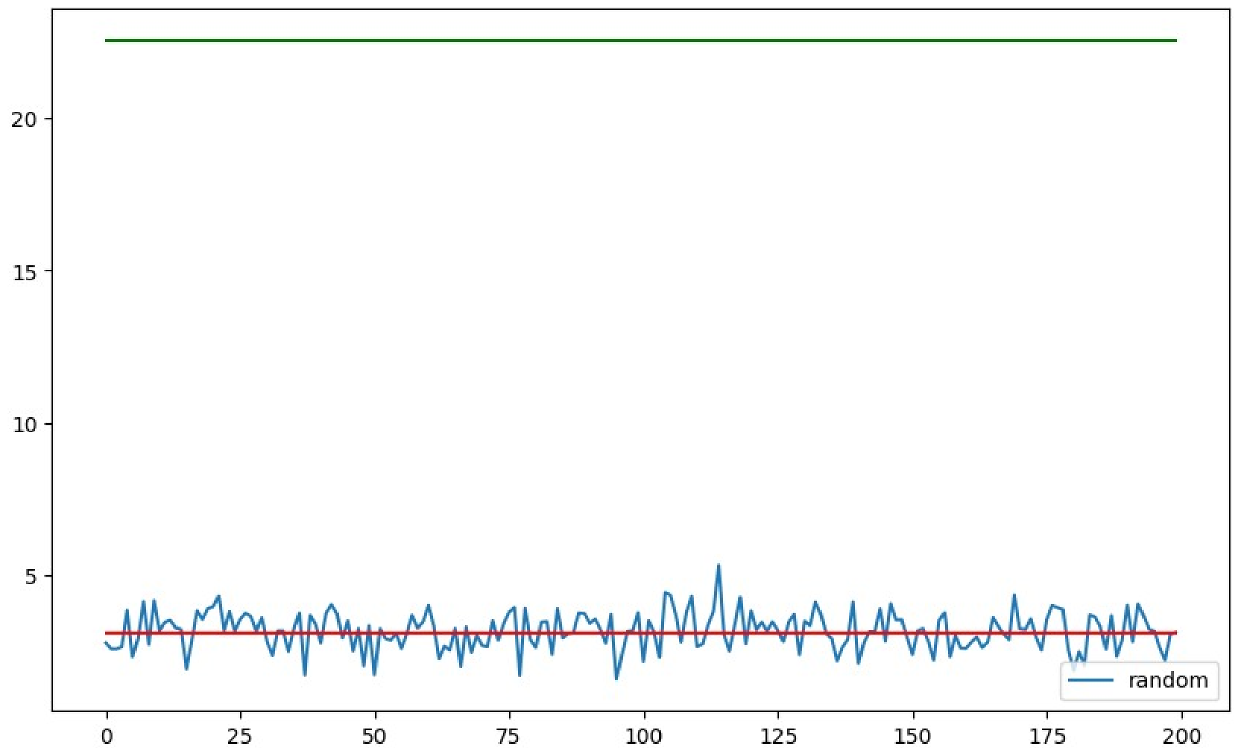
```
3.119254917081568
```

Please explore following values:

- Epsilon greedy: [0.001, 0.01, 0.5, 0.9]
- Softmax: [0.001, 1.0, 5.0, 50.0]

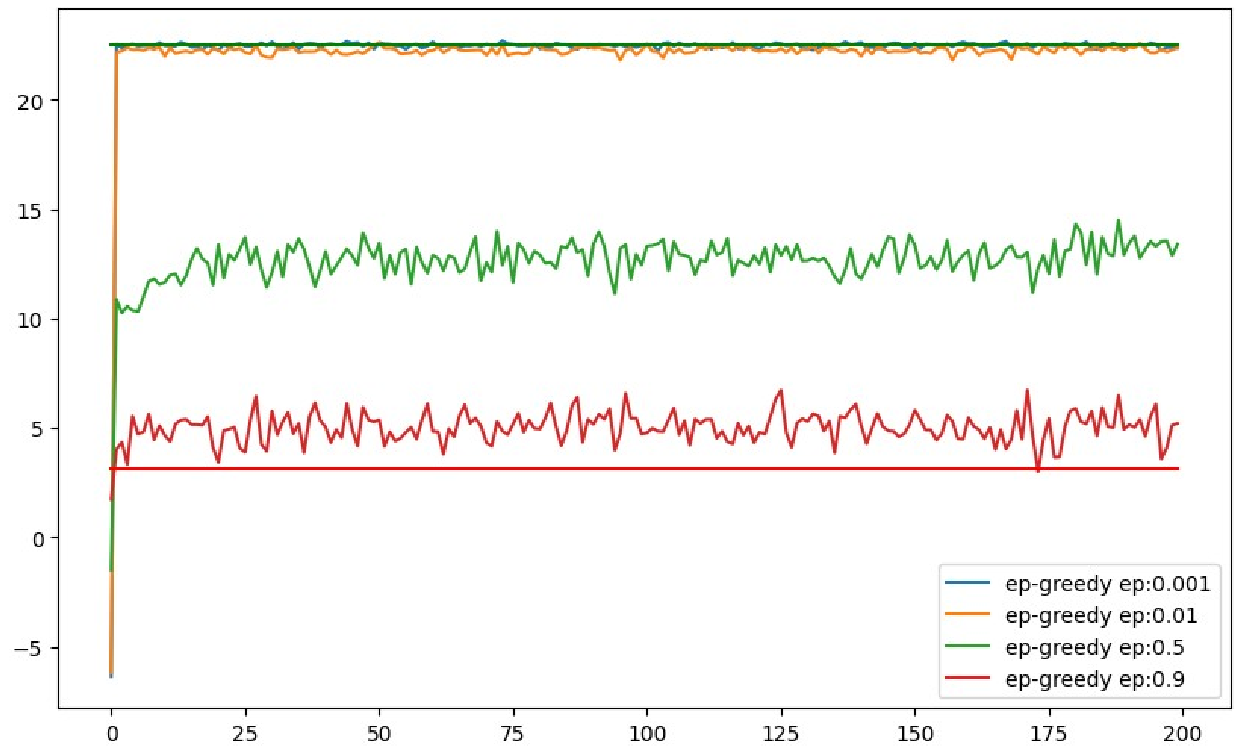
```
random_policy = RandomPolicy(env.arm_ids)
plot_reward_curve_and_print_regret(
    env, [random_policy], timesteps=200, num_runs=500)
```

```
regret for random: 3871.625
```



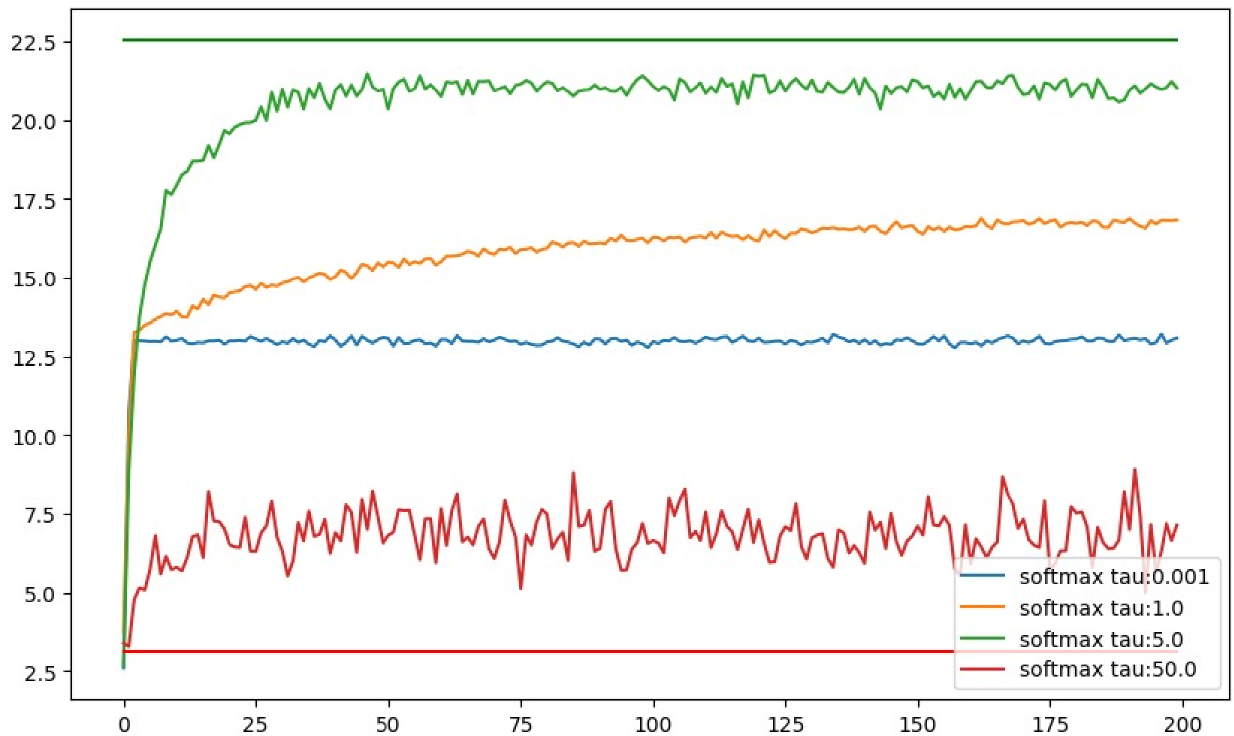

```
explore_epgreedy_epsilons = [0.001, 0.01, 0.5, 0.9]
epgreedy_policies = [EpGreedyPolicy(ep, env.arm_ids)
                     for ep in explore_epgreedy_epsilons]
plot_reward_curve_and_print_regret(
    env, epgreedy_policies, timesteps=200, num_runs=500)
```

```
regret for ep-greedy ep:0.001: 39.590
regret for ep-greedy ep:0.01: 83.511
regret for ep-greedy ep:0.5: 1980.353
regret for ep-greedy ep:0.9: 3505.350
```



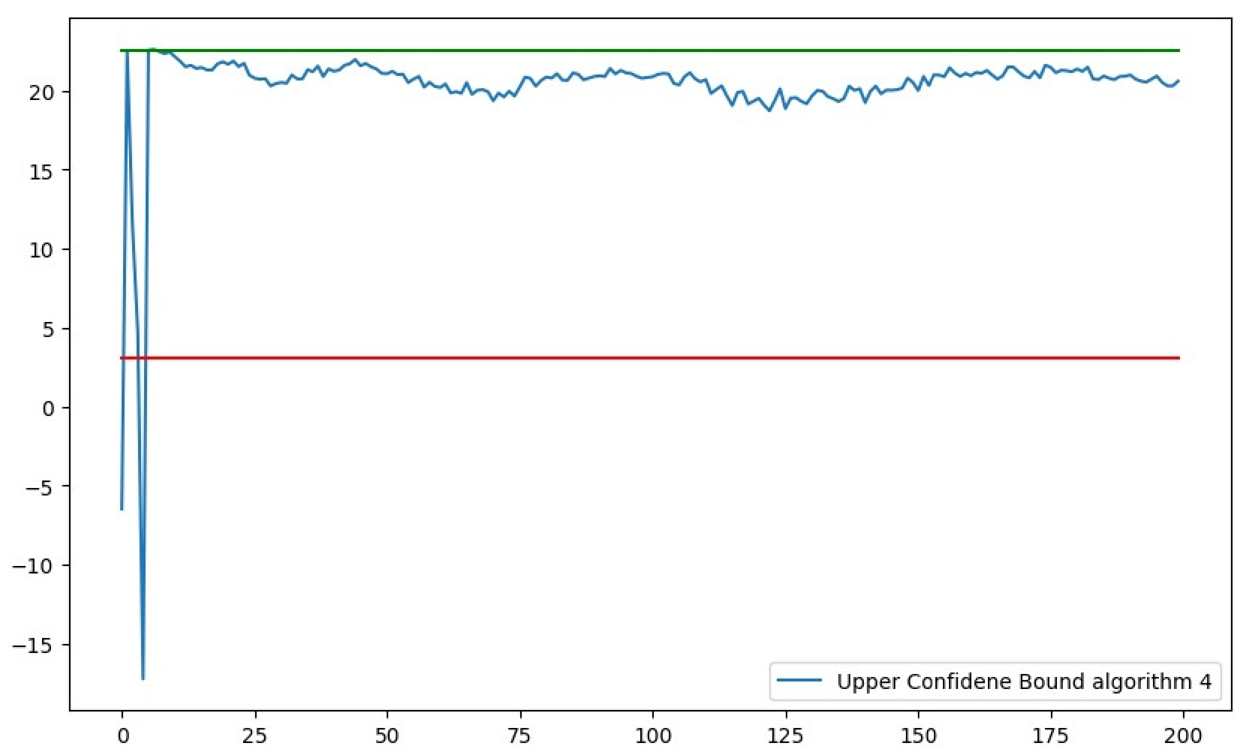

```
explore_softmax_taus = [0.001, 1.0, 5.0, 50.0]
softmax_polices = [SoftmaxPolicy(tau, env.arm_ids)
                   for tau in explore_softmax_taus]
plot_reward_curve_and_print_regret(
    env, softmax_polices, timesteps=200, num_runs=500)
```

```
regret for softmax tau:0.001: 1922.557
regret for softmax tau:1.0: 1344.711
regret for softmax tau:5.0: 411.401
regret for softmax tau:50.0: 3150.510
```




```
plot_reward_curve_and_print_regret(
    env, [UCB(env.arm_ids, c=4)], timesteps=200, num_runs=500)
```

```
regret for Upper Confidene Bound algorithm 4: 455.986
```



Optional: Please explore different values of epsilon, tau and verify how does the behaviour changes.

```
plot_reward_curve_and_print_regret(
    env, [UCB(env.arm_ids, c=c) for c in [1,2,3,4]], timesteps=200,
num_runs=500)
```

```
regret for Upper Confidene Bound algorithm 1: 108.248
regret for Upper Confidene Bound algorithm 2: 175.628
regret for Upper Confidene Bound algorithm 3: 304.709
regret for Upper Confidene Bound algorithm 4: 457.232
```