IITM-CS5691 : Pattern Recognition and Machine Learning — Release Date: August 30, 2023
Assignment 1 — Due Date : Wednesday, 27 September 2023, 11:59 PM

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Collaborators (if any):
References/sources (if any):

- Use LaTeX to write-up your solutions (in the solution blocks of the source LaTeX file of this assignment), and submit the resulting pdf files (one per question) at Crowdmark by the due date. (Note: **No late submissions** will be allowed, other than one-day late submission with 10% penalty or four-day late submission with 30% penalty! Instructions to join Crowdmark and submit your solution to each question within Crowdmark **TBA** later).
- For the programming question, please submit your code (rollno.ipynb file and rollno.py file in rollno.zip) directly in moodle, but provide your results/answers (including Jupyter notebook **with output**) in the pdf file you upload to Crowdmark.
- Collaboration is encouraged, but all write-ups must be done individually and independently, and mention your collaborator(s) if any. Same rules apply for codes written for any programming assignments (i.e., write your own code; we will run plagiarism checks on codes).
- If you have referred a book or any other online material or LLMs (Large Language Models like ChatGPT) for obtaining a solution, please cite the source. Again don't copy the source *as is* - you may use the source to understand the solution, but write-up the solution in your own words (this also means that you cannot copy-paste the solution from LLMs!). Please be advised that *the lesser your reliance on online materials or LLMs* for answering the questions, *the more your understanding* of the concepts will be and *the more prepared you will be for the course exams.*
- Points will be awarded based on how clear, concise and rigorous your solutions are, and how correct your answer is. The weightage of this assignment is 12% towards the overall course grade.

The solution of question (a)

In a two-class optimal Bayes classifier, the decision regions are typically separated by a single cutoff point, resulting in a single interval for each region. This is often the case when the class-conditional probability density functions (pdfs) are well-behaved and exhibit a clear separation between the two classes.

However, there can be scenarios where the decision region R1 may be composed of more than one discontiguous interval if the class-conditional pdfs overlap in such a way that it's not possible to define a single cutoff point to separate the classes. This situation is more common when the class-conditional pdfs overlap significantly.

Let's illustrate this with an example by plotting the pdfs $p(x, C_1)$ and $p(x, C_2)$ against x:

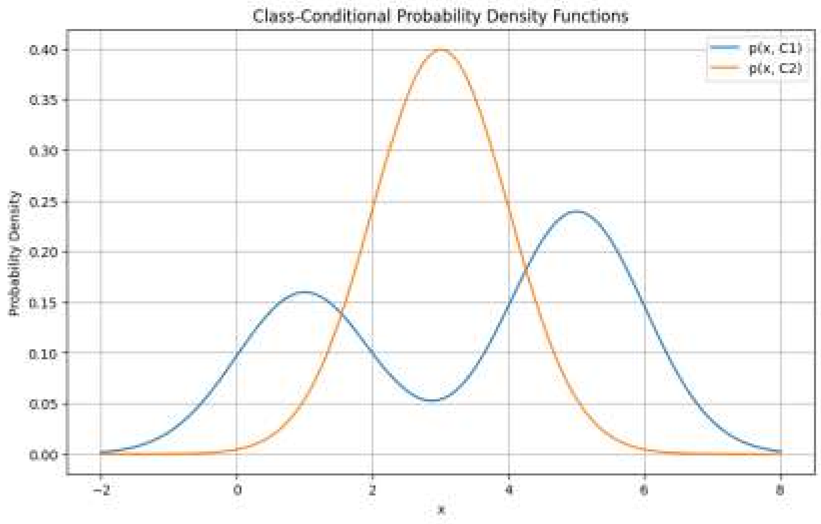
Suppose we have two classes, C1 and C2, and their class-conditional pdfs are given as follows:

For Class C1: $p(x, C1) = 0.4 * N(1, 1) + 0.6 * N(5, 1)$

For Class C2: $p(x, C2) = N(3, 1)$

Here, $N(\mu, \sigma^2)$ represents the normal distribution. In this example, the pdf for Class C_1 is a mixture of two Gaussian distributions with different means and weights.

Let's plot these pdfs:



In this plot, you can see that the two class-conditional pdfs overlap significantly, and there isn't a single cutoff point that cleanly separates the two classes. Therefore, in such cases, the decision region R1 for Class C1 would consist of multiple discontiguous intervals.