CS6700: Tutorial 3 - Policy Iteration

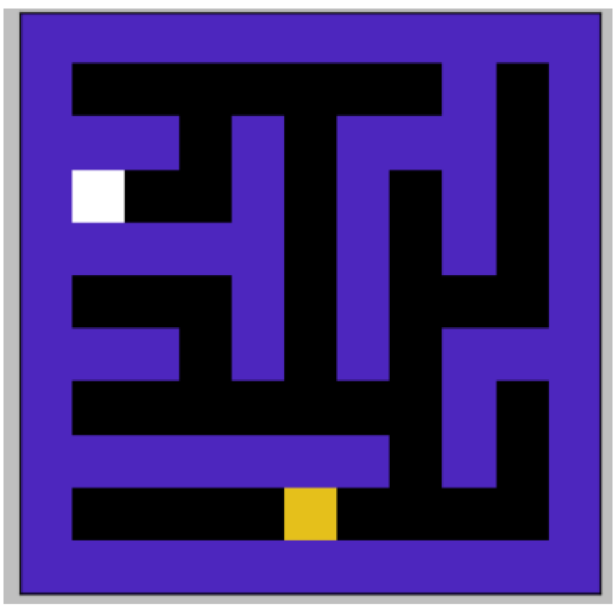
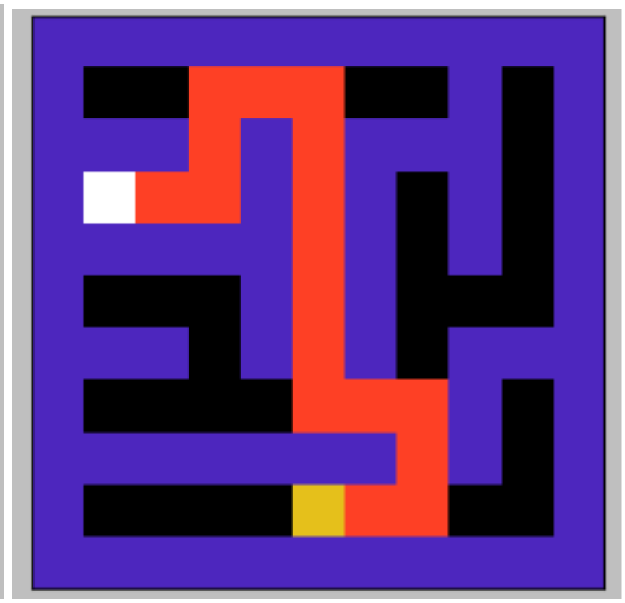
```python
import numpy as np
from enum import Enum
import copy
```

Consider a standard grid world, where only 4 (up, down, left, right) actions are allowed and the agent deterministically moves accordingly, represented as below. Here yellow is the start state and white is the goal state.

Say, we define our MDP as:

- S: 121 (11 x 11) cells
- A: 4 actions (up, down, left, right)
- P: Deterministic transition probability
- R: -1 at every step
- gamma: 0.9

Our goal is to find an optimal policy (shown in right).

```python
# Above grid is defined as below:
#   - 0 denotes an navigable tile
#   - 1 denotes an obstruction/wall
#   - 2 denotes the start state
#   - 3 denotes an goal state

# Note: Here the upper left corner is defined as (0, 0)
#       and lower right corner as (m-1, n-1)
```

```
# Optimal Path: RIGHT RIGHT UP UP LEFT LEFT UP UP UP UP UP UP LEFT
LEFT DOWN DOWN LEFT LEFT


GRID_WORLD = np.array([
    [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],
    [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1],
    [1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1],
    [1, 3, 0, 0, 1, 0, 1, 0, 1, 0, 1],
    [1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1],
    [1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1],
    [1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1],
    [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1],
    [1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1],
    [1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1],
    [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
])
```

Actions

```
class Actions(Enum):
    UP = (0, (-1, 0))  # index = 0, (xaxis_move = -1 and yaxis_move =
0)
    DOWN = (1, (1, 0))   # index = 1, (xaxis_move = 1 and yaxis_move =
0)
    LEFT = (2, (0, -1))  # index = 2, (xaxis_move = 0 and yaxis_move =
-1)
    RIGHT = (3, (0, 1))   # index = 3, (xaxis_move = 0 and yaxis_move
= -1)

    def get_action_dir(self):
        _, direction = self.value
        return direction

    @property
    def index(self):
        indx, _ = self.value
        return indx

    @classmethod
    def from_index(cls, index):
        action_index_map = {a.index: a for a in cls}
        return action_index_map[index]

# How to use Action enum
for a in Actions:
    print(
        f"name: {a.name}, action_id: {a.index}, direction_to_move:
{a.get_action_dir()}")
```

```
print("\n------------------------------------\n")

# find action enum from index 0
a = Actions.from_index(0)
print(f"0 index action is: {a.name}")
```

```
name: UP, action_id: 0, direction_to_move: (-1, 0)
name: DOWN, action_id: 1, direction_to_move: (1, 0)
name: LEFT, action_id: 2, direction_to_move: (0, -1)
name: RIGHT, action_id: 3, direction_to_move: (0, 1)

------------------------------------

0 index action is: UP
```

Policy

```
class BasePolicy:
    def update(self, *args):
        pass

    def select_action(self, state_id: int) -> int:
        raise NotImplemented


class DeterministicPolicy(BasePolicy):
    def __init__(self, actions: np.ndarray):
        # actions: its a 1d array (|S| size) which contains action for
each state
        self.actions = actions

    def update(self, state_id, action_id):
        assert state_id < len(self.actions), f"Invalid state_id
{state_id}"
        assert action_id < len(Actions), f"Invalid action_id
{action_id}"
        self.actions[state_id] = action_id

    def select_action(self, state_id: int) -> int:
        assert state_id < len(self.actions), f"Invalid state_id
{state_id}"
        return self.actions[state_id]
```

Environment

```
class Environment:
    def __init__(self, grid):
        self.grid = grid
        m, n = grid.shape
```

```
        self.num_states = m*n

    def xy_to_posid(self, x: int, y: int):
        _, n = self.grid.shape
        return x*n + y

    def posid_to_xy(self, posid: int):
        _, n = self.grid.shape
        return (posid // n, posid % n)

    def isvalid_move(self, x: int, y: int):
        m, n = self.grid.shape
        return (x >= 0) and (y >= 0) and (x < m) and (y < n) and
(self.grid[x, y] != 1)

    def find_start_xy(self) -> int:
        m, n = self.grid.shape
        for x in range(m):
            for y in range(n):
                if self.grid[x, y] == 2:
                    return (x, y)
        raise Exception("Start position not found.")

    def find_path(self, policy: BasePolicy) -> str:
        max_steps = 50
        steps = 0

        P, R = self.get_transition_prob_and_expected_reward()
        num_actions, num_states = R.shape
        all_possible_state_posids = np.arange(num_states)

        path = ""
        curr_x, curr_y = self.find_start_xy()
        while (self.grid[curr_x, curr_y] != 3) and (steps <
max_steps):
            curr_posid = self.xy_to_posid(curr_x, curr_y)
            action_id = policy.select_action(curr_posid)
            next_posid = np.random.choice(
                all_possible_state_posids, p=P[action_id, curr_posid])
            action = Actions.from_index(action_id)
            path += f" {action.name}"
            curr_x, curr_y = self.posid_to_xy(next_posid)
            steps += 1
        return path

    # P(s_next | s, a), R(s, a)
    def get_transition_prob_and_expected_reward(self):
        m, n = self.grid.shape
        num_states = m*n
        num_actions = len(Actions)
```

```
        P = np.zeros((num_actions, num_states, num_states))
        R = np.zeros((num_actions, num_states))
        for a in Actions:
            for x in range(m):
                for y in range(n):
                    xmove_dir, ymove_dir = a.get_action_dir()
                    # find the new co-ordinate after the action a
                    xnew, ynew = x + xmove_dir, y + ymove_dir

                    posid = self.xy_to_posid(x, y)
                    new_posid = self.xy_to_posid(xnew, ynew)

                    if self.grid[x, y] == 3:
                        # the current state is a goal state
                        P[a.index, posid, posid] = 1
                        R[a.index, posid] = 0
                    elif (self.grid[x, y] == 1) or (not
self.isvalid_move(xnew, ynew)):
                        # the current state is a block state or the
next state is invalid
                        P[a.index, posid, posid] = 1
                        R[a.index, posid] = -1
                    else:
                        # action a is valid and goes to a new position
                        P[a.index, posid, new_posid] = 1
                        R[a.index, posid] = -1
        return P, R
```

Policy Iteration

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$; $V(\mathit{terminal}) \doteq 0$

2. Policy Evaluation
   Loop:
   $\quad \Delta \leftarrow 0$
   $\quad$ Loop for each $s \in \mathcal{S}$:
   $\quad\quad v \leftarrow V(s)$
   $\quad\quad V(s) \leftarrow \sum_{s',r} p(s', r \mid s, \pi(s))\big[r + \gamma V(s')\big]$
   $\quad\quad \Delta \leftarrow \max(\Delta, |v - V(s)|)$
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   $\mathit{policy\text{-}stable} \leftarrow \mathit{true}$
   For each $s \in \mathcal{S}$:
   $\quad \mathit{old\text{-}action} \leftarrow \pi(s)$
   $\quad \pi(s) \leftarrow \operatorname{argmax}_a \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma V(s')\big]$
   $\quad$ If $\mathit{old\text{-}action} \neq \pi(s)$, then $\mathit{policy\text{-}stable} \leftarrow \mathit{false}$
   If $\mathit{policy\text{-}stable}$, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

```python
def policy_evaluation(P: np.ndarray, R: np.ndarray, gamma: float,
                      policy: BasePolicy, theta: float,
                      init_V: np.ndarray = None):
    _, num_states = R.shape

    # Please try different starting point for V you will find it will
always
    # converge to the same V_pi value.

    if init_V is None:
        init_V = np.zeros(num_states)

    V = copy.deepcopy(init_V)

    delta = 100.0

    while delta > theta:
        delta = 0.0

        for state_id in range(num_states):
            action_id = policy.select_action(state_id)
```

```
            # Following equation is a different way of writing the
same equation given in the slide.
            # Note here R is an expected reward term.
            V_old = V[state_id]
            V[state_id] = R[action_id, state_id] + \
                gamma * np.dot(P[action_id, state_id], V)

            # YOUR CODE HERE
            # Calculate delta which determines when to terminate the
evaluation step
            delta = max(delta, abs(V_old - V[state_id]))

    return V


def policy_improvement(P: np.ndarray, R: np.ndarray, gamma: float,
                       policy: BasePolicy, V: np.ndarray):
    _, num_states = R.shape
    policy_stable = True

    for state_id in range(num_states):

        old_action_id = policy.select_action(state_id)

        # YOUR CODE HERE
        values = [R[action_id, state_id] + gamma *
                  np.dot(P[action_id, state_id], V) for action_id in
range(len(Actions))]

        # Update policy to choose the action with the highest Q-value
        new_action_id = np.argmax(values)

        policy.update(state_id, new_action_id)
        if old_action_id != new_action_id:
            policy_stable = False

    return policy_stable


def policy_iteration(P: np.ndarray, R: np.ndarray, gamma: float,
                     theta: float = 1e-3, init_policy: BasePolicy =
None):
    _, num_states = R.shape

    # Please try exploring different policies you will find it will
always
    # converge to the same optimal policy for valid states.
    if init_policy is None:
        # Say initial policy = all up actions.
        init_policy = DeterministicPolicy(
```

```
            actions=np.zeros(num_states, dtype=int))

    # creating a copy of a initial policy
    policy = copy.deepcopy(init_policy)
    policy_stable = False

    while not policy_stable:
        V = policy_evaluation(P, R, gamma, policy, theta)
        policy_stable = policy_improvement(P, R, gamma, policy, V)

    return policy, V
```

Experiments

```
def is_same_optimal_value(V1, V2, diff_theta=1e-3):
    diff = np.abs(V1 - V2)
    return np.all(diff < diff_theta)

seed = 0
np.random.seed(seed)

gamma = 0.9
theta = 1e-5

env = Environment(GRID_WORLD)
P, R = env.get_transition_prob_and_expected_reward()
```

Exercise 1: Using Policy iteration algorithm find the optimal path from start to goal position

```
# # Start with random choice of init_policy.
# One such choice could be: init_policy = np.ones(env.num_states,
dtype=int)
init_policy = DeterministicPolicy(actions=np.ones(env.num_states,
dtype=int))

pitr_policy, pitr_V_star = policy_iteration(
    P, R, gamma, theta=theta, init_policy=init_policy)
pitr_path = env.find_path(pitr_policy)
print(pitr_path)
```

```
 RIGHT RIGHT UP UP LEFT LEFT UP UP UP UP UP UP LEFT LEFT DOWN DOWN
LEFT LEFT
```

Exercise 2: Using initial guess for V as random values, find the optimal value function using policy evaluation and compare it with the optimal value function

```
# Start with random choice of init_V.
# One such choice could be: init_V = np.random.randn(env.num_states)
# Another choice could be: init_V = 10*np.ones(env.num_states)
init_V = 10*np.ones(env.num_states)
```

```
V_star = policy_evaluation(P, R, gamma, pitr_policy, theta, init_V)
is_same_optimal_value(pitr_V_star, V_star)
```

```
True
```

To-do: Repeat Exercise 1 with a random Deterministic policy

```
init_policy =
DeterministicPolicy(actions=np.random.randint(low=0,high=len(Actions),
size= env.num_states, dtype=int))

pitr_policy, pitr_V_star = policy_iteration(
    P, R, gamma, theta=theta, init_policy=init_policy)
pitr_path = env.find_path(pitr_policy)
print(pitr_path)
```

```
 RIGHT RIGHT UP UP LEFT LEFT UP UP UP UP UP UP LEFT LEFT DOWN DOWN
LEFT LEFT
```