

Roll No: CS23E001

Name: Shuvrajeet Das

Collaborators (if any):

References/sources (if any):

- Use  $\text{\LaTeX}$  to write-up your solutions (in the solution blocks of the source  $\text{\LaTeX}$  file of this assignment), submit the resulting rollno.asst2.answers.pdf file at Crowdmark by the due date, and properly drag that pdf's answer pages to the corresponding question in Crowdmark (do this properly, otherwise we won't be able to grade!). (Note: **No late submissions** will be allowed, other than one-day late submission with 10% penalty or four-day late submission with 30% penalty.)
- Please upload to moodle a rollno.zip file containing three files: rollno.asst2.answers.pdf file mentioned above, and two code files for the programming question (rollno.ipynb file and rollno.py file). Do not forget to upload to Crowdmark your results/answers (including Jupyter notebook **with output**) for the programming question.
- Collaboration is encouraged, but all write-ups must be done individually and independently, and mention your collaborator(s) if any. Same rules apply for codes written for any programming assignments (i.e., write your own code; we will run plagiarism checks on codes).
- If you have referred a book or any other online material or LLMs (Large Language Models like ChatGPT) for obtaining a solution, please cite the source. Again don't copy the source *as is* - you may use the source to understand the solution, but write-up the solution in your own words (this also means that you cannot copy-paste the solution from LLMs!). Please be advised that *the lesser your reliance on online materials or LLMs for answering the questions, the more your understanding of the concepts will be and the more prepared you will be for the course exams*.
- Points will be awarded based on how clear, concise and rigorous your solutions are, and how correct your answer is. The weightage of this assignment is 12% towards the overall course grade.

1. (8 points) [SPECTRAL CLUSTERING - LAPLACIAN EIGENMAP] Consider a simple undirected graph  $G = (V, E)$  with  $|V| = n$  nodes and  $|E| = m$  edges. Let  $A$  be the binary adjacency matrix of the graph (i.e., the symmetric 0-1 matrix where 1 indicates the presence of the corresponding edge; diagonal entries of  $A$  are zero). Let  $x \in \mathbb{R}^n$  denote the node scores.

Let the graph Laplacian matrix be  $L = D - A$  seen in class, with  $D$  being the diagonal matrix of node degrees. Let  $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$  denote the eigenvalues of the graph Laplacian  $L$  (sometimes also referred to as  $L_G$  to explicitly mention the graph).

- (a) (2 points) Let's add one edge to  $G$  to obtain a new graph  $G'$ . What can you say about the multiplicity of eigen value 0 of  $L_{G'}$  relative to that of  $L_G$ ? Will the sum of eigen values of  $L_{G'}$  change compared to that of  $L_G$ ; and if so, by what amount?

**Solution:** The multiplicity of eigenvalue 0 of  $L_{G'}$  will be the same as the multiplicity of eigenvalue 0 of  $L_G$ . The sum of eigenvalues of  $L_{G'}$  will be greater than the sum of eigenvalues of  $L_G$  by 2.