CS5020: Nonlinear Optimisation: Theory and Algorithms
Worksheet - 7

For all the questions below:
(a) Sketch the constraints and shade the constraint set.
(b) Find the set of global minima.
(c) Find the set of active constraints $\mathcal{A}(x_*)$ at $x_* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.
(d) At at $x_* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, find all possible ways in which

$$-\nabla f(x_*) = \sum_{i \in \mathcal{A}(x_*)} \lambda_i \nabla g_i(x_*), \quad \text{where} \quad \lambda_i \geq 0, i \in \mathcal{A}(x_*)$$

Note: Here 'all possible ways' means all possible values for λ_i that satisfy our needs.
(e) Assume that a budget of $\epsilon \geq 0$ is given. At $x_* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, we are allowed to relax an active constraint $i \in \mathcal{A}(x_*)$ by $\epsilon_i \geq 0$, such that the total budget $\sum_{i \in \mathcal{A}(x_*)} \epsilon_i = \epsilon$. Under this assumption, find the least possible objective value achieved after relaxing the constraints.
(f) Assume that a budget of $\epsilon \geq 0$ is given. At $x_* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, find **all possible ways** to relax the constraints, wherein, an active constraint $i \in \mathcal{A}(x_*)$ is relaxed by $\epsilon_i \geq 0$, such that the total budget $\sum_{i \in \mathcal{A}(x_*)} \epsilon_i = \epsilon$, so that the least possible objective value achieved after relaxing the constraints.
Note: Here 'all possible ways' means all possible values for ϵ_i that satisfy our needs.

(1)

$$\begin{array}{rrcl} & \min & & f(x) = -(x(1) + x(2)) \\ \text{such that} & g_1 & : & x(1) \leq 0 \\ & g_2 & : & x(2) \leq 0 \end{array}$$

(2)

$$\begin{array}{rrcl} & \min & & f(x) = -x(1) \\ \text{such that} & g_1 & : & x(1) \leq 0 \\ & g_2 & : & x(2) \leq 0 \end{array}$$

(3)

$$\begin{array}{rrcl} & \min & & f(x) = -x(2) \\ \text{such that} & g_1 & : & x(1) \leq 0 \\ & g_2 & : & x(2) \leq 0 \end{array}$$

(4)

$$\begin{array}{rrcl} & \min & & f(x) = -(x(1) + x(2)) \\ \text{such that} & g_1 & : & 2x(1) + x(2) \leq 0 \\ & g_2 & : & x(1) + 2x(2) \leq 0 \end{array}$$

(5)

$$\begin{array}{rlcl} & \min & & f(x) = -x(1) \\ \text{such that} & g_1 & : & 2x(1) + x(2) \leq 0 \\ & g_2 & : & x(1) + 2x(2) \leq 0 \end{array}$$

(6)

$$\begin{array}{rlcl} & \min & & f(x) = -x(2) \\ \text{such that} & g_1 & : & 2x(1) + x(2) \leq 0 \\ & g_2 & : & x(1) + 2x(2) \leq 0 \end{array}$$

(7)

$$\begin{array}{rlcl} & \min & & f(x) = -x(2) \\ \text{such that} & g_1 & : & -x(1) + 0.1x(2) \leq 0 \\ & g_2 & : & x(1) \leq 0 \end{array}$$

(8)

$$\begin{array}{rlcl} & \min & & f(x) = -x(2) \\ \text{such that} & g_1 & : & -0.1x(1) + x(2) \leq 0 \\ & g_2 & : & x(1) \leq 0 \end{array}$$

(9)

$$\begin{array}{rlcl} & \min & & f(x) = -x(2) \\ \text{such that} & g_1 & : & x(1) \leq 0 \\ & g_2 & : & x(2) \leq 0 \\ & g_3 & : & -x(1) + x(2) \leq 0 \end{array}$$

(10)

$$\begin{array}{rlcl} & \min & & f(x) = -(x(1) + x(2)) \\ \text{such that} & g_1 & : & x(1) \leq 0 \\ & g_2 & : & x(2) \leq 0 \\ & g_3 & : & 2x(1) + x(2) \leq 0 \\ & g_4 & : & x(1) + 2x(2) \leq 0 \end{array}$$