IITM-CS5691 : Pattern Recognition and Machine Learning — Release Date: August 30, 2023

Assignment 1 — Due Date : Wednesday, 27 September 2023, 11:59 PM

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Collaborators (if any):

References/sources (if any):

- Use LaTeX to write-up your solutions (in the solution blocks of the source LaTeX file of this assignment), and submit the resulting pdf files (one per question) at Crowdmark by the due date. (Note: **No late submissions** will be allowed, other than one-day late submission with 10% penalty or four-day late submission with 30% penalty! Instructions to join Crowdmark and submit your solution to each question within Crowdmark **TBA** later).
- For the programming question, please submit your code (rollno.ipynb file and rollno.py file in rollno.zip) directly in moodle, but provide your results/answers (including Jupyter notebook **with output**) in the pdf file you upload to Crowdmark.
- Collaboration is encouraged, but all write-ups must be done individually and independently, and mention your collaborator(s) if any. Same rules apply for codes written for any programming assignments (i.e., write your own code; we will run plagiarism checks on codes).
- If you have referred a book or any other online material or LLMs (Large Language Models like ChatGPT) for obtaining a solution, please cite the source. Again don't copy the source *as is* - you may use the source to understand the solution, but write-up the solution in your own words (this also means that you cannot copy-paste the solution from LLMs!). Please be advised that *the lesser your reliance on online materials or LLMs* for answering the questions, *the more your understanding* of the concepts will be and *the more prepared you will be for the course exams.*
- Points will be awarded based on how clear, concise and rigorous your solutions are, and how correct your answer is. The weightage of this assignment is 12% towards the overall course grade.

The solution of question (b)

we know from the taylor series expnasion,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

The linear approximation of a function at a given point is its first-order Taylor series expansion around that point. To find the linear approximation of the function $f(x_1, x_2) = x_1^2 + x_2^2 + x_1x_2$, we need to choose a point (a, b) around which we want to approximate the function. Let's choose the point (a, b).

The linear approximation of $f(x_1, x_2)$ at the point (a, b) is given by:

$$L(x_1, x2) = f(a, b) + \frac{\partial f(a, b)}{\partial x_1}(x_1 - a) + \frac{\partial f(a, b)}{\partial x_2}(x_2 - b)$$

To find the linear approximation, we need to calculate the partial derivatives of $f(x_1, x_2)$ with respect to x and y. Here are the derivatives:

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 2x_1 + x_2 \quad \frac{\partial f(x_1, x_2)}{\partial x_2} = 2x_2 + x_1$$

Now, we can plug these derivatives into the linear approximation formula:

$$L(x_1, x_2) = (a^2 + b^2 + ab) + (2a + b)(x_1 - a) + (2b + a)(x_2 - b)$$

So, the linear approximation of $f(x_1, x_2) = x_1^2 + x_2^2 + x_1x_2$ around the point (a, b) is:

$$L(x_1, x_2) = a^2 + b^2 + ab + (2a + b)(x_1 - a) + (2b + a)(x_2 - b)$$

Filling up all the necessary values we get

$$L(x_1, x_2) = 49 + 11(x - 3) + 13(y - 5)$$

3D Plot of 49 + 11(x - 3) + 13(y - 5) and $x^2 + y^2 + xy$