(CS5020, Jul-Nov 2023) Nonlinear Optimisation: Theory and Algorithms
Worksheet - 2

(1) Let $\alpha_* = \max\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ (i.e., α_* is the maximum of m real numbers $\alpha_i, i = 1 \ldots, m$). Express this as a linear optimisation problem in one variable and m linear constraints (**Note**: α_i are given and are to be used as constants in the optimisation problem, please don't use them as variables. Also α_* is unknown and needs to be found).

(2) Sketch the feasible directions at $\begin{bmatrix}0\\1\end{bmatrix}$, $\begin{bmatrix}-1\\0\end{bmatrix}$ for the constraint sets below.

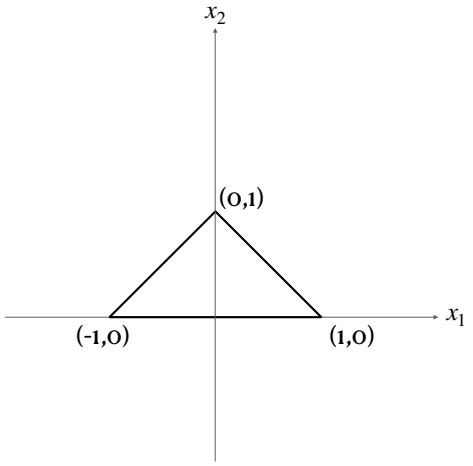

Fig (1)

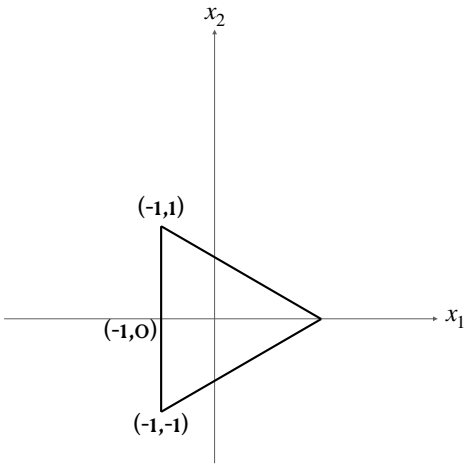

Fig (2)

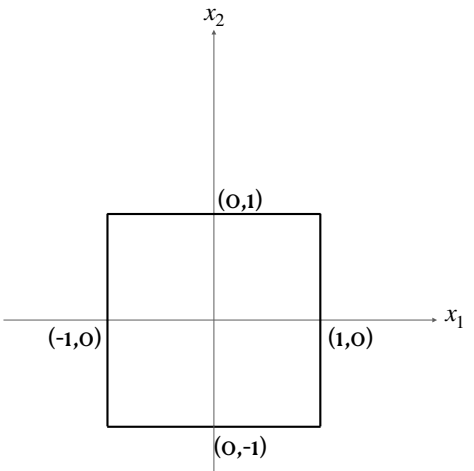

Fig (3)

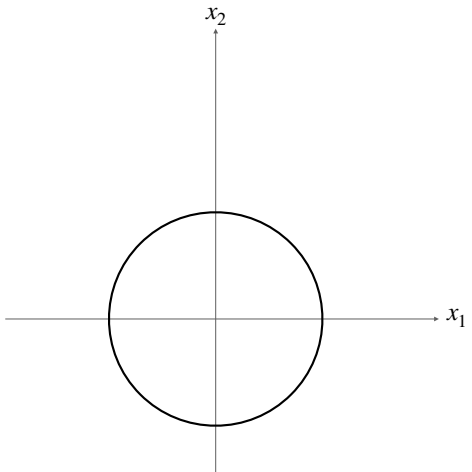

Fig (5)

In Fig(2) assume unit length for each segment. In Fig (5) assume circle of unit radius.

(3) Consider the problem of $\min_{x \in \mathbb{R}^2} f(x)$. For the following functions, sketch the improving directions at $\begin{bmatrix}0\\1\end{bmatrix}$, $\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}0\\-1\end{bmatrix}$, $\begin{bmatrix}-1\\0\end{bmatrix}$.

(1) $f(x) = x_1 + x_2$
(2) $f(x) = -x_1 + x_2$
(3) $f(x) = x_1 - x_2$
(4) $f(x) = -x_1 - x_2$

(5) $f(x) = 2x_1 + x_2$
(6) $f(x) = (x_1 - 1)^2 + (x_2 - 2)^2$
(7) $f(x) = (x_1 + 3)^2 + (x_2 + 2)^2$
(8) $f(x) = 5(x_1 - 1)^2 + 2(x_2 - 2)^2$
(9) $f(x) = x_1^2 + 2x_2^2 + x_1 - x_2 + 4$
(10) $f(x) = x_1^2 + x_2^2 + 2x_1x_2$

(4) For $n = 1, 2, \ldots$, find if the following sequences converge
(1) $\frac{1}{n}$
(2) $\frac{(-1)^n}{n}$
(3) $(-1)^n$
(4) $\sin((-1)^n)$
(5) $(\sin(-1))^n$
(6) n^2
(7) $\frac{n^2}{2^n}$
(8) $n^2(\sin(-1))^n$
(9) $n^2(\sin(-0.5))^n$
(10) $2^n(\sin(-0.5))^n$

(5) Find $f'(x), f''(x), f'''(x)$ for
(1) $f(x) = x$
(2) $f(x) = |x|$
(3) $f(x) = x^2$
(4) $f(x) = |x|^2$
(5) $f(x) = x^3$.
(6) $f(x) = |x|^3$.