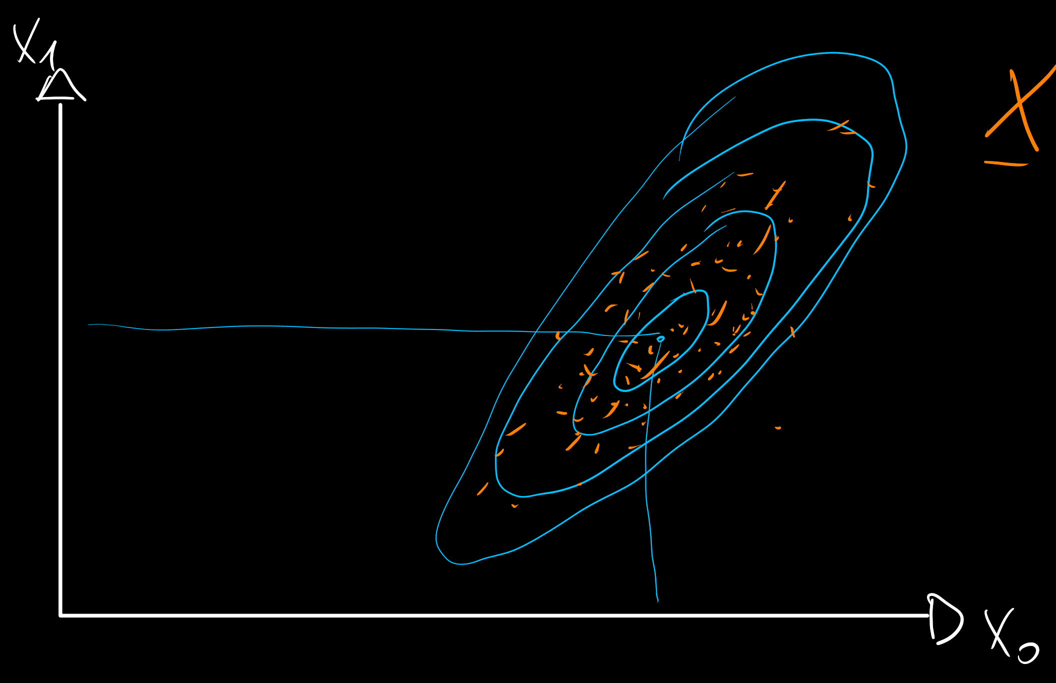
Sampling the Multivariate Normal



$\underline{X} \sim \mathcal{N}(\underline{X}; \underline{\mu}, \underline{\underline{\Sigma}})$

Similar to the univariate case, start with $\mathcal{N}(\underline{0}, \underline{\underline{I}})$

and then transform $\mathcal{N}(\underline{\mu}, \underline{\underline{\Sigma}})$

note: $\mathcal{N}(\underline{0}, \underline{\underline{I}})$ is a standard univariate

Normal in each of the K

dimensions

example: $K = 3$

$$\mathcal{N}(\underline{0}, \underline{\underline{I}}) \sim \underline{X} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \sim \begin{bmatrix} \mathcal{N}(0,1) \\ \mathcal{N}(0,1) \\ \mathcal{N}(0,1) \end{bmatrix}$$

to get one $\underline{X} \sim \mathcal{N}(\underline{0}, \underline{\underline{I}})$ sample

we draw K samples from $\mathcal{N}(0,1)$

↳ e.g. Box-Müller transform

[univariate : $Y = \mu + \sigma X$, $Y \sim \mathcal{N}(\mu, \sigma^2)$]

then $\underline{Y} = \underline{\mu} + \underline{\underline{L}}\, \underline{X}$

[$\underline{\underline{\Sigma}} = \underline{\underline{L}}\, \underline{\underline{L}}^T$ cholesky factor]

$$\underline{Y} \sim \mathcal{N}(\underline{\mu}, \underline{\underline{\Sigma}})$$