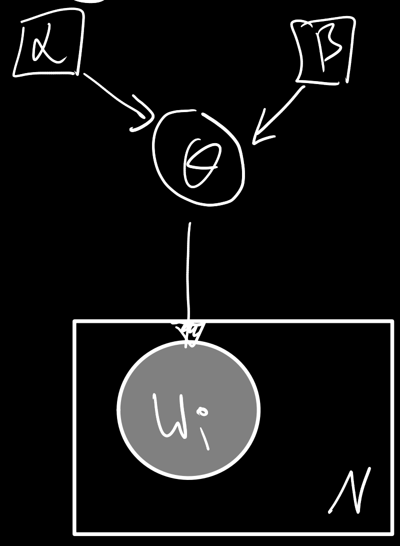
Marginal for the Beta-Bernoulli



$\theta \sim \underbrace{\mathrm{Beta}(\alpha,\beta)}$

$w_i \sim \mathrm{Bernoulli}(\theta)$

Bayes' Rule

posterior $p(\theta|D) = \dfrac{p(D|\theta)\,p(\theta)}{p(D)}$

difficult!

what is difficult?

joint: $p(\theta, D) = p(\theta)\,p(D|\theta)$

$$= \frac{1}{B(\alpha,\beta)} \cdot \theta^{\alpha-1}(1-\theta)^{\beta-1} \cdot \prod_{i=0}^{N-1} \theta^{w^{[i]}} \cdot (1-\theta)^{1-w^{[i]}}$$

$$= \frac{1}{B(\alpha,\beta)}\, \theta^{\alpha+\sum_{i=0}^{N-1} w^{[i]} - 1} \cdot (1-\theta)^{\beta+N-\sum_{i=0}^{N-1} w^{[i]} - 1}$$

---

find $\underbrace{p(D)}_{\text{marginal}}$ by marginalizing $\underbrace{p(\theta, D)}_{\text{joint}}$ over θ

$$p(D) = \int_\theta p(\theta, D)\, d\theta \quad \left(= \int_\theta p(\theta)\,p(D|\theta)\,d\theta = \mathbb{E}_{\theta\sim p(\theta)}\left[p(D|\theta)\right] \right)$$

evaluated by sampling

$$= \int_{\theta=0}^{1} \frac{1}{B(\alpha,\beta)} \cdot \theta^{\alpha+\sum_{i=0}^{N-1} w^{[i]} - 1} \cdot (1-\theta)^{\beta+N-\sum_{i=0}^{N-1} w^{[i]} - 1}\, d\theta$$

hmm.... hard to find anti-derivative

(sometimes impossible to find exact solution)

→ intractable

$$= \boxed{\frac{B(\alpha',\beta')}{B(\alpha,\beta)}}$$

$\alpha' = \alpha + \sum_{i=0}^{N-1} w^{[i]}$

$\beta' = \beta + N - \sum_{i=0}^{N-1} w^{[i]}$

→ no longer a distribution, just a probability

because $p(D)$ actually means $p(\underline{W} = D)$