Gamma Distribution - Intro

(Univariate) Normal with precision

$$X \sim \mathcal{N}(x; \mu, \tau^{-1}) = \sqrt{\frac{\tau}{2\pi}} \exp\left(-\frac{\tau}{2}(x-\mu)^2\right)$$

outside — inside exp

prior?

we want $p(\tau)$ as a prior

$\tau = \frac{1}{\sigma^2}$

$$\mathcal{N}(x; \mu, \tau^{-1}) = \frac{1}{\sqrt{2\pi}} \tau^{1/2} \exp\left(-\frac{1}{2}\tau(x-\mu)^2\right)$$

$$p(\tau) \propto \tau^{\alpha-1} \cdot e^{-\beta\tau} \sim \text{Gamma}(\tau; \alpha, \beta)$$

normalization constant

$$\text{Gamma}(\tau; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\beta\tau}$$

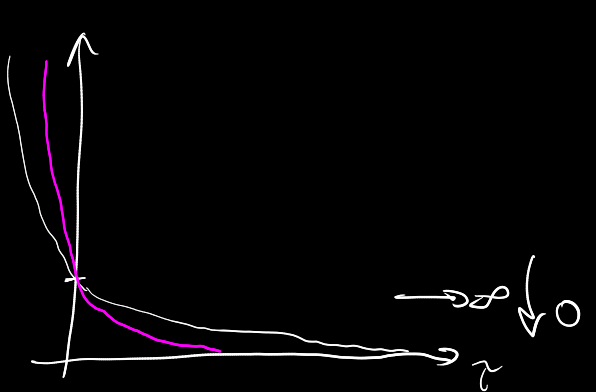
gamma function
"generalization of factorials"

Restrictions: $\alpha, \beta > 0$

$\tau > 0$

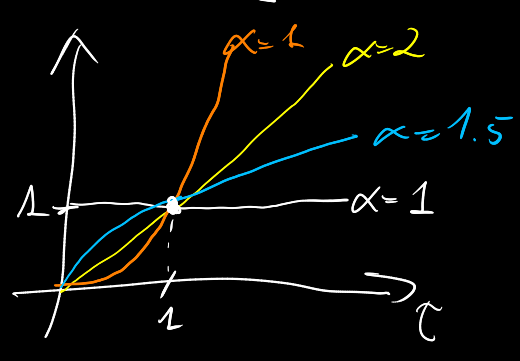
The two terms

$e^{-\beta\tau}$



$\tau^{\alpha-1}$

$\alpha \geq 1$



$\alpha < 1$
$\alpha > 0$