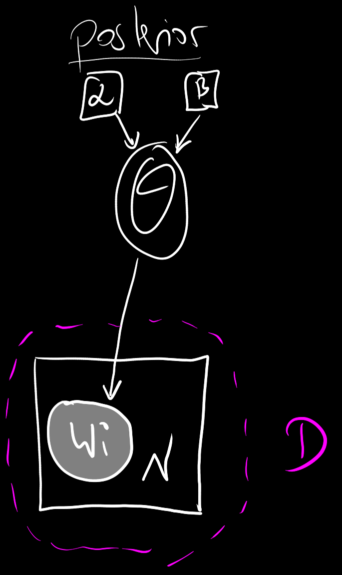
Predictive Posterior for Bernoulli Distribution



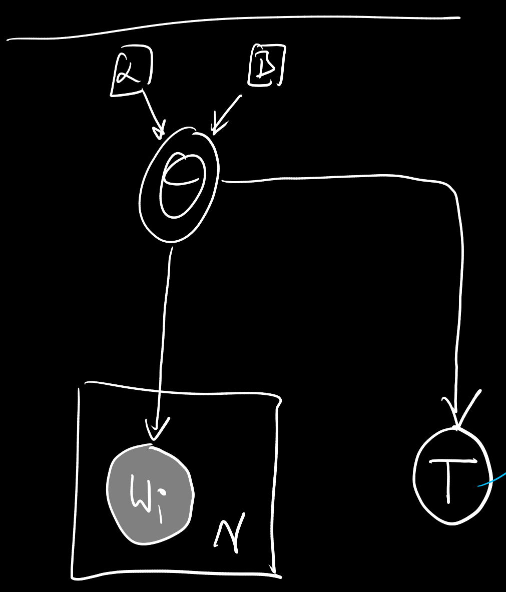
$\Theta \sim Beta(\alpha, \beta)$

$w_i \sim Bernoulli(\Theta)$

$$p(\Theta|D) = Beta(\alpha', \beta') = \frac{1}{B(\alpha', \beta')} \Theta^{\alpha'-1} \cdot (1-\Theta)^{\beta'-1}$$

$$\alpha' = \alpha + \sum_{i=0}^{N-1} w^{[i]}$$

$$\beta' = \beta + N - \sum_{i=0}^{N-1} w^{[i]}$$



T ... tomorrow's weather

$T \sim Brn(\theta)$

We want $p(T|D)$

joint:

$$p(\theta, D, T) = p(\theta) \cdot p(D|\theta) \cdot p(T|\theta)$$

$$p(\theta, T|D) p(D) = p(\theta) \cdot p(D|\theta) \cdot p(T|\theta) \quad | : p(D)$$

$$p(\theta, T|D) = \underbrace{\frac{p(\theta) \cdot p(D|\theta)}{p(D)}}_{\substack{\text{Bayes' Rule} \\ p(\theta|D) \\ \text{posterior}}} \cdot p(T|\theta)$$

$$p(\theta, T|D) = p(\theta|D) \cdot p(T|\theta)$$

marginalize over θ (θ continuous, $\theta \in [0,1]$)

$$p(T|D) = \int_\theta p(\theta, T|D) \, d\theta$$

$$= \boxed{\int_\theta p(\theta|D) \cdot p(T|\theta) \, d\theta} = \mathbb{E}_{\theta \sim p(\theta|D)}[p(T|\theta)]$$

(can be evaluated by sampling if integral is intractable)

$$= \int_{\theta=0}^{\theta=1} Beta(\alpha', \beta') \cdot Brn(\theta) \, d\theta$$

$$= \int_0^1 \frac{1}{B(\alpha', \beta')} \cdot \theta^{\alpha'-1} (1-\theta)^{\beta'-1} \cdot \theta^T \cdot (1-\theta)^{1-T} \, d\theta$$

(Integral is difficult)

$$= \frac{(\alpha')^T \cdot (\beta')^{1-T}}{\alpha' + \beta'}$$