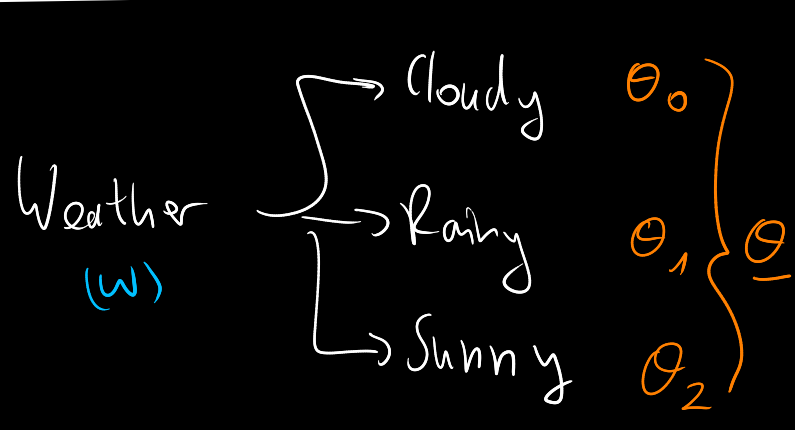


# Multinomial Distribution - Intro



observe the weather

$$D = \{C, S, S, S, R, C, S\}$$

$N = 7$  days

$$w \in \{C, R, S\}$$

what is the probability for that?

2 out of 7 cloudy  
1 out of 7 rainy  
4 out of 7 sunny

just use a Categorical

$$w \sim \text{Cat}(\theta)$$

$$\text{e.g. } \theta = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}$$

likelihood

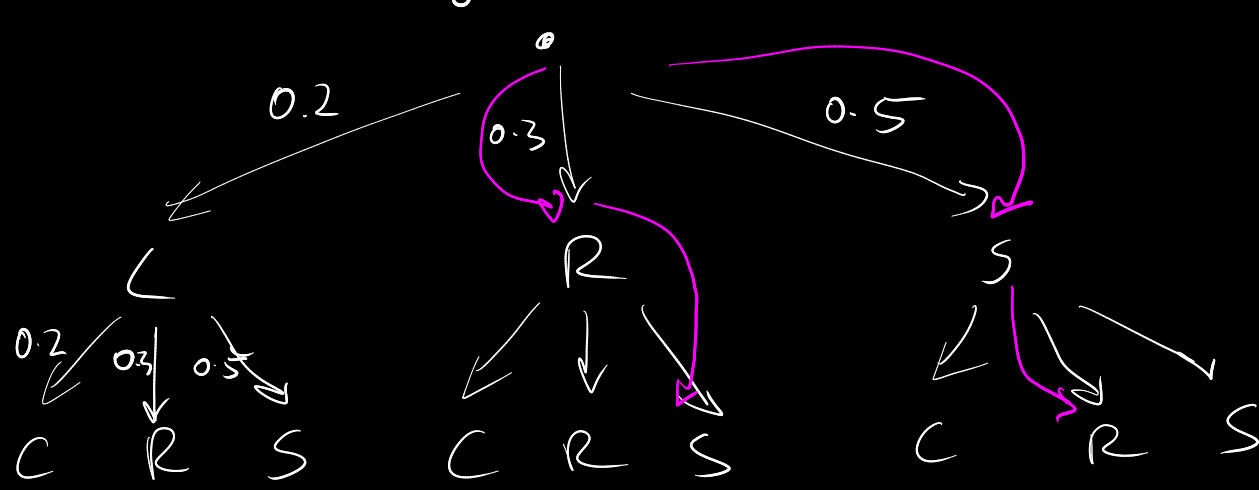
$$p(D) = 0.2^2 \cdot 0.3^1 \cdot 0.5^4$$

$$= 0.0075$$

→ wrong (it's too low)

⇒ there are multiple paths (similar to Binomial)

e.g.  $n=2$  days



e.g. prob 1 out of 2 rains  
1 out of 2 sunny

⇒ 2 paths

$$p(\dots) = 0.3 \cdot 0.5 + 0.5 \cdot 0.3$$

$$= 2 \cdot 0.3 \cdot 0.5$$

# of paths prob for any of paths

Multinomial coefficients

similar to the Categorical

(why the Categorical is sometimes called Multinomial)

↳ has to be a composition of one-hot Categoricals

$$k = [0, 1, 1]^T = \underbrace{[0, 1, 0]^T}_{\text{raining}} + \underbrace{[0, 0, 1]^T}_{\text{sunny}}$$

(but also  $k = [2, 0, 2]^T$   
 $n=3$ )

$$p(k) = \frac{n!}{\prod_{d=0}^{D-1} k_d!} \prod_{d=0}^{D-1} \theta_d^{k_d} = \text{Multinomial}(\theta, n)$$

Multinomial coefficient      prob of one path

To save

•  $\theta$  ... prob for each state (for each weather-type)

•  $n$  ... Number of observations (number of days)

restrictions

$$\theta \in [0, 1]^D, \quad \sum_{d=0}^{D-1} \theta_d = 1, \quad \sum_{d=0}^{D-1} k_d = n$$

Dataset (e.g.  $n=7$  days)

$$D = \{ [2, 3, 2], [1, 1, 5], [2, 1, 4], \dots \}$$