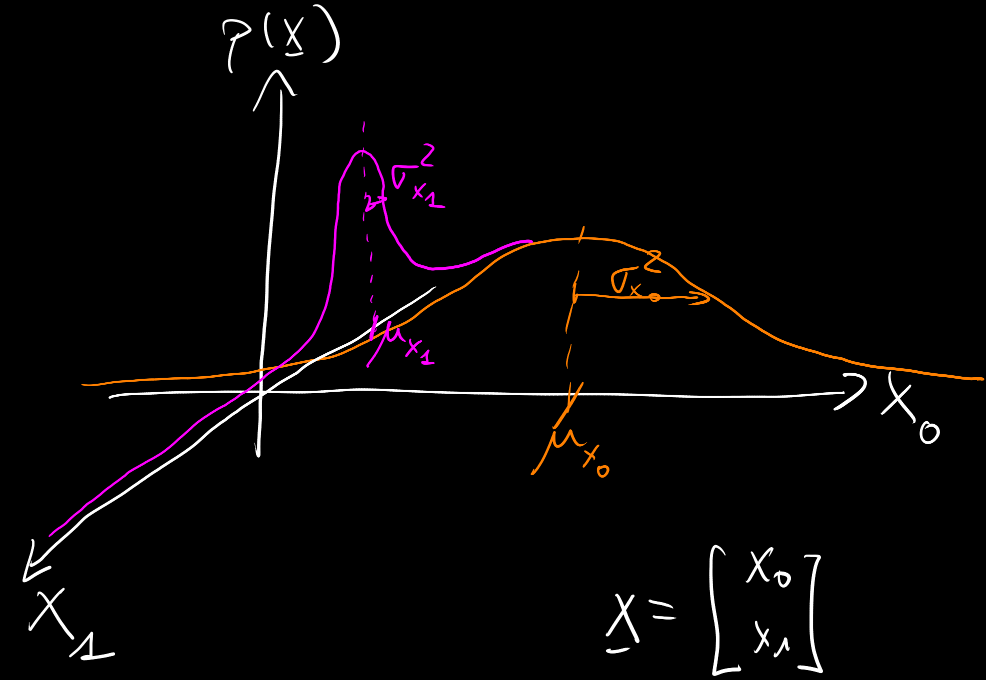
Multivariate Normal - Intro



$\underline{X} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$

Two Variables are both Normally Distributed

→ can they interact?

What to save:

RV : X_0	RV : X_1
mean μ_{x_0}	mean : μ_{x_1}
variance $\sigma^2_{x_0}$	variance : $\sigma^2_{x_1}$

"interaction" : covariance $\sigma_{x_0 x_1}$

("rotation around the third axis")

intro : random vector

$$\underline{X} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \in \mathbb{R}^2$$

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pdf

[univariate: $p(x) = \mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{\sigma^2 2\pi}} \exp(-\frac{1}{2\sigma^2}(x-\mu)^2)$]

$$p(\underline{X}) \sim \exp(-\frac{1}{2}(\underline{X}-\underline{\mu})^T \underline{\underline{\Sigma}}^{-1}(\underline{X}-\underline{\mu}))$$

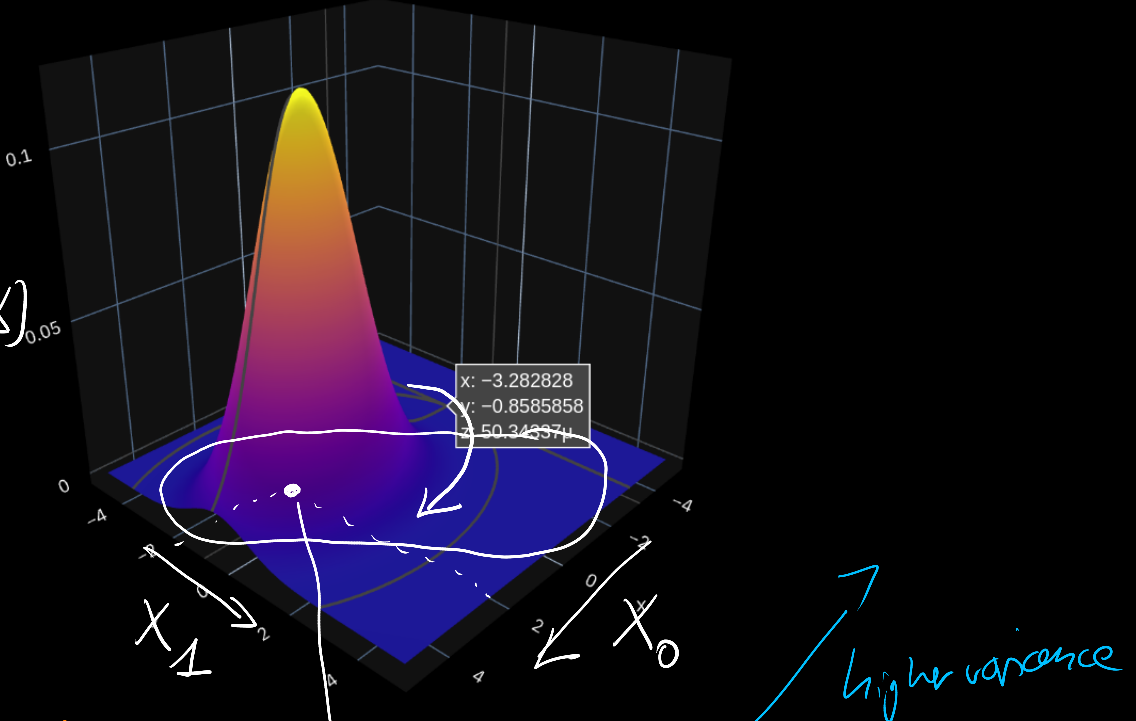
mean/mu vector

$$\underline{\mu} = \begin{bmatrix} \mu_{x_0} \\ \mu_{x_1} \end{bmatrix}$$

Covariance/Sigma matrix

$$\underline{\underline{\Sigma}} = \begin{bmatrix} \sigma^2_{x_0} & \sigma_{x_0 x_1} \\ \sigma_{x_0 x_1} & \sigma^2_{x_1} \end{bmatrix}$$

covariances



still axis-aligned

→ covariances = 0

$\underline{\mu} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

higher variance

Generalizes to D = 3, 4, 100, 50'000, ... dimensions

Normalization Constant

$$p(\underline{X}) = \frac{1}{\sqrt{\det(\underline{\underline{\Sigma}})(2\pi)^D}} \exp(-\frac{1}{2}(\underline{X}-\underline{\mu})^T \underline{\underline{\Sigma}}^{-1}(\underline{X}-\underline{\mu}))$$

number of dimensions

$$=: \mathcal{N}(\underline{X}; \underline{\mu}, \underline{\underline{\Sigma}})$$

Requirements:

$\underline{\mu} \in \mathbb{R}^D$

[univariate, $\sigma, \sigma^2 > 0$]

Multivariate $\underline{\underline{\Sigma}} > 0$

symmetric positive definite

i.f.f. there is a Cholesky decomposition

$$\underline{\underline{\Sigma}} = \underline{\underline{L}}\,\underline{\underline{L}}^T$$

lower triangular matrix

→ it is also relevant for "the inverse"

What is about the precision

[univariate case: $\mathcal{N}(X; \mu, \tau^{-1}) = \sqrt{\frac{\tau}{2\pi}} \exp(-\frac{\tau}{2}(x-\mu)^2)$

$\tau := \frac{1}{\sigma^2}$

Multivariate Case: $\underline{\underline{\Lambda}} = \underline{\underline{\Sigma}}^{-1}$ (useful for derivations)

↳ Bayesian Analyses

$$p(\underline{X}) = \mathcal{N}(\underline{X}; \underline{\mu}, \underline{\underline{\Lambda}}^{-1}) = \sqrt{\frac{\det(\underline{\underline{\Lambda}})}{(2\pi)^D}} \exp(-\frac{1}{2}(\underline{X}-\underline{\mu})^T \underline{\underline{\Lambda}}(\underline{X}-\underline{\mu}))$$