In[8]:=
```
u[x_] = a + b * x + c * x ^ 2 + d * x ^ 3
```

Out[8]=
$$a + b\,x + c\,x^2 + d\,x^3$$

In[3]:=
```
Solve[{u[0] == u1, u[1 / 3] == u2, u[2 / 3] == u3, u[1] == u4}, {a, b, c, d}]
```

Out[3]=
$$\left\{\left\{a \to u1,\ b \to \frac{1}{2}\,(-11\,u1 + 18\,u2 - 9\,u3 + 2\,u4),\right.\right.$$
$$\left.\left.c \to \frac{9}{2}\,(2\,u1 - 5\,u2 + 4\,u3 - u4),\ d \to -\frac{9}{2}\,(u1 - 3\,u2 + 3\,u3 - u4)\right\}\right\}$$

In[38]:=
```
eqn1 := u1 + ((1 / 2) * (-11 u1 + 18 u2 - 9 u3 + 2 u4)) * x +
  ((9 / 2) * (2 u1 - 5 u2 + 4 u3 - u4)) * x ^ 2 + (-9 / 2) * (u1 - 3 u2 + 3 u3 - u4) * x ^ 3
```

In[40]:=
```
eqn2 := N1 * u1 + N2 * u2 + N3 * u3 + N4 * u4
```

In[41]:=
```
Thread[Equal[CoefficientList[eqn1, u1], CoefficientList[eqn2, u1]]]
```

Out[41]=
$$\left\{9\,u2\,x - \frac{9\,u3\,x}{2} + u4\,x - \frac{45\,u2\,x^2}{2} + 18\,u3\,x^2 - \frac{9\,u4\,x^2}{2} + \frac{27\,u2\,x^3}{2} - \frac{27\,u3\,x^3}{2} + \frac{9\,u4\,x^3}{2} ==\right.$$
$$\left.N2\,u2 + N3\,u3 + N4\,u4,\ 1 - \frac{11\,x}{2} + 9\,x^2 - \frac{9\,x^3}{2} == N1\right\}$$

In[42]:=
```
Thread[Equal[CoefficientList[eqn1, u2], CoefficientList[eqn2, u2]]]
```

Out[42]=
$$\left\{u1 - \frac{11\,u1\,x}{2} - \frac{9\,u3\,x}{2} + u4\,x + 9\,u1\,x^2 + 18\,u3\,x^2 - \frac{9\,u4\,x^2}{2} - \frac{9\,u1\,x^3}{2} - \frac{27\,u3\,x^3}{2} + \frac{9\,u4\,x^3}{2} ==\right.$$
$$\left.N1\,u1 + N3\,u3 + N4\,u4,\ 9\,x - \frac{45\,x^2}{2} + \frac{27\,x^3}{2} == N2\right\}$$

In[43]:=
```
Thread[Equal[CoefficientList[eqn1, u3], CoefficientList[eqn2, u3]]]
```

Out[43]=
$$\left\{u1 - \frac{11\,u1\,x}{2} + 9\,u2\,x + u4\,x + 9\,u1\,x^2 - \frac{45\,u2\,x^2}{2} - \frac{9\,u4\,x^2}{2} - \frac{9\,u1\,x^3}{2} + \frac{27\,u2\,x^3}{2} + \frac{9\,u4\,x^3}{2} ==\right.$$
$$\left.N1\,u1 + N2\,u2 + N4\,u4,\ -\frac{9\,x}{2} + 18\,x^2 - \frac{27\,x^3}{2} == N3\right\}$$

In[44]:=
```
Thread[Equal[CoefficientList[eqn1, u4], CoefficientList[eqn2, u4]]]
```

Out[44]=
$$\left\{u1 - \frac{11\,u1\,x}{2} + 9\,u2\,x - \frac{9\,u3\,x}{2} + 9\,u1\,x^2 - \frac{45\,u2\,x^2}{2} + 18\,u3\,x^2 - \frac{9\,u1\,x^3}{2} + \frac{27\,u2\,x^3}{2} - \frac{27\,u3\,x^3}{2} ==\right.$$
$$\left.N1\,u1 + N2\,u2 + N3\,u3,\ x - \frac{9\,x^2}{2} + \frac{9\,x^3}{2} == N4\right\}$$

In[6]:=
```
N1[x_] := 1 - 11 x / 2 + 9 x ^ 2 - 9 x ^ 3 / 2
```

In[7]:=
```
N2[x_] := 9 x - 45 x ^ 2 / 2 + 27 x ^ 3 / 2
```

In[18]:=
```
N3[x_] := -9 x / 2 + 18 x ^ 2 - 27 x ^ 3 / 2
```

In[10]:=
```
N4[x_] := x - 9 x ^ 2 / 2 + 9 x ^ 3 / 2
```

In[27]:= `N[Integrate[(N1'[x]*N1'[x]), {x, 0, 1}]]`

Out[27]= 3.7

In[28]:= `N[Integrate[(N1'[x]*N2'[x]), {x, 0, 1}]]`

Out[28]= -4.725

In[29]:= `N[Integrate[(N1'[x]*N3'[x]), {x, 0, 1}]]`

Out[29]= 1.35

In[36]:= `N[Integrate[(N1'[x]*N4'[x]), {x, 0, 1}]]`

Out[36]= -0.325

In[30]:= `N[Integrate[(N2'[x]*N2'[x]), {x, 0, 1}]]`

Out[30]= 10.8

In[33]:= `N[Integrate[(N2'[x]*N3'[x]), {x, 0, 1}]]`

Out[33]= -7.425

In[31]:= `N[Integrate[(N2'[x]*N4'[x]), {x, 0, 1}]]`

Out[31]= 1.35

In[32]:= `N[Integrate[(N3'[x]*N3'[x]), {x, 0, 1}]]`

Out[32]= 10.8

In[34]:= `N[Integrate[(N3'[x]*N4'[x]), {x, 0, 1}]]`

Out[34]= -4.725

In[35]:= `N[Integrate[(N4'[x]*N4'[x]), {x, 0, 1}]]`

Out[35]= 3.7