Assignment

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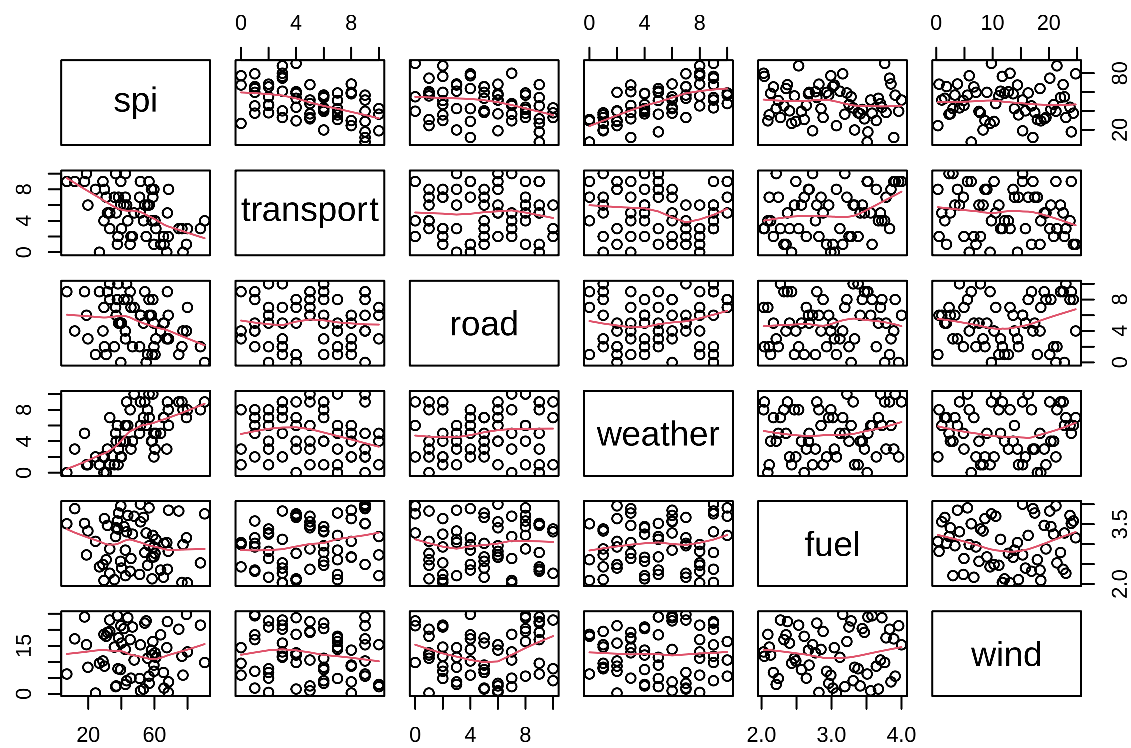
QUESTION 1:

a) Correlation plot and matrix, comments on relationships of predictors and response:

```r
traffic = read.csv('data/traffic.csv', header = TRUE)
pairs(traffic, panel = panel.smooth)
```




```r
cor(traffic)
```

```
##                  spi    transport         road     weather         fuel
## spi       1.00000000 -0.472909967 -0.303836850  0.66672345 -0.138153417
## transport -0.47290997  1.000000000 -0.005714728 -0.16971072  0.240947972
## road      -0.30383685 -0.005714728  1.000000000  0.12495993  0.043675635
```

```
## weather    0.66672345 -0.169710717  0.124959926  1.00000000  0.110531767
## fuel      -0.13815342  0.240947972  0.043675635  0.11053177  1.000000000
## wind      -0.03466263 -0.131014749  0.080481857  0.00751783  0.006532832
##                  wind
## spi       -0.034662632
## transport -0.131014749
## road       0.080481857
## weather    0.007517830
## fuel       0.006532832
## wind       1.000000000
```

- The response variable ***spi*** has a strong negative relationship with the predictor ***transport***; a weak negative relationship with the predictor ***road***; a strong positive relationship with the predictor ***weather***; and no obvious relationship with both the predictors ***fuel*** and ***wind***.
- There does not seem to be a relationship between the predictors themselves.

b) Fit full model and estimate the impact of *weather* on *spi* with 95% CI:

```r
M1 = lm(spi ~ ., data = traffic)
summary(M1)
```

```
##
## Call:
## lm(formula = spi ~ ., data = traffic)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.1596  -4.9415   0.1278   5.1686  21.7415
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  62.8071     7.4080   8.478 1.27e-11 ***
## transport    -2.1750     0.4611  -4.717 1.63e-05 ***
## road         -2.4097     0.4365  -5.520 9.04e-07 ***
## weather       4.2456     0.4473   9.492 2.92e-13 ***
## fuel         -3.6145     2.2759  -1.588    0.118
## wind         -0.1358     0.1764  -0.769    0.445
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.913 on 56 degrees of freedom
## Multiple R-squared:  0.7405, Adjusted R-squared:  0.7174
## F-statistic: 31.96 on 5 and 56 DF,  p-value: 3.039e-15
```

```r
summary.M1 = summary(M1)
sqrt(diag(summary.M1$cov.unscaled * summary.M1$sigma^2))[4]
```

```
##   weather
## 0.4472731
```

```r
qt(1 - 0.05/2,56)
```

```
## [1] 2.003241
```

The require CI is:

$$\hat{\beta}_{\text{weather}} \pm t_{n-p,1-\frac{\alpha}{2}} \text{se}(\hat{\beta}_{\text{weather}}) = \hat{\beta}_{\text{weather}} \pm t_{56,0.975} \text{se}(\hat{\beta}_{\text{weather}}) = 4.2456 \pm 2.003241 \times 0.4472731 = (3.349604, 5.141596)$$

That is, we are 95% confident that for every percentage increase in relative ***weather***, the ***spi*** concentration will increase between **3.349604** and **5.141596** on average.

c) Conduct F-test for overall regression and examine relationship between predictors and response:

Theorical Model is:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_5 Xi5 + \epsilon_i, \quad i = 1, 2, \ldots, n$$

- Y_i is the response variable ***spi***
- X_{ij} is th are the predictors variables for the i-th observation:
  - X_{i1} = annual mean ***transport*** of test locations
  - X_{i2} = annual mean ***road*** of test locations
  - X_{i3} = annual mean ***weather*** of test locations
  - X_{i4} = annual mean ***fuel*** of test locations
  - X_{i5} = annual mean ***wind*** of test locations
- $\epsilon \sim \mathcal{N}(0, \sigma^2)$ denotes the random variation with constant variance

Now we conduct the F-test:

- Hypotheses: $H_0 : \beta_1 = \ldots = \beta_5 = 0$ vs $H_1 :$ not all $\beta_i = 0,$ for $i = 1, 2, \ldots, 5$
- Standard R output ANOVA table:

```r
anova(M1)
```

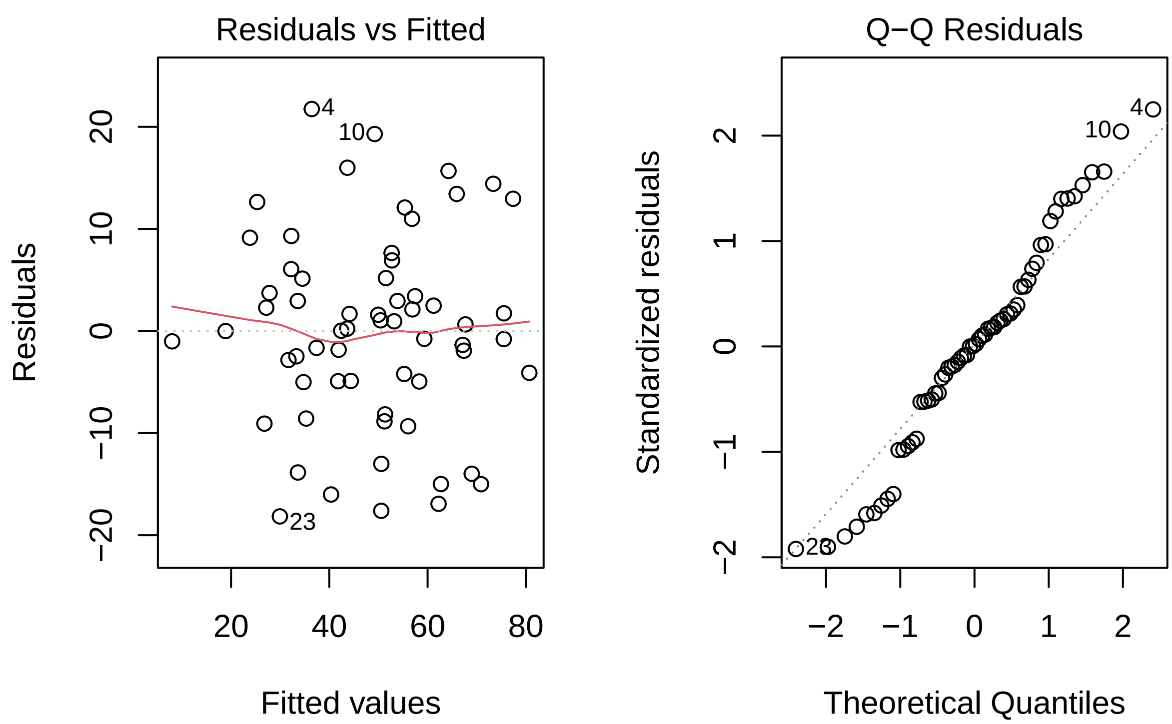
```
## Analysis of Variance Table
##
## Response: spi
##           Df Sum Sq Mean Sq F value    Pr(>F)
## transport  1 4742.6  4742.6 48.2656 4.228e-09 ***
## road       1 1992.7  1992.7 20.2800 3.441e-05 ***
## weather    1 8651.9  8651.9 88.0507 4.355e-13 ***
## fuel       1  258.1   258.1  2.6264    0.1107
## wind       1   58.2    58.2  0.5921    0.4449
## Residuals 56 5502.6    98.3
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Note the $RegressionSS = 4742.6 + 1992.7 + 8651.9 + 258.1 + 58.2 = 15703.5$

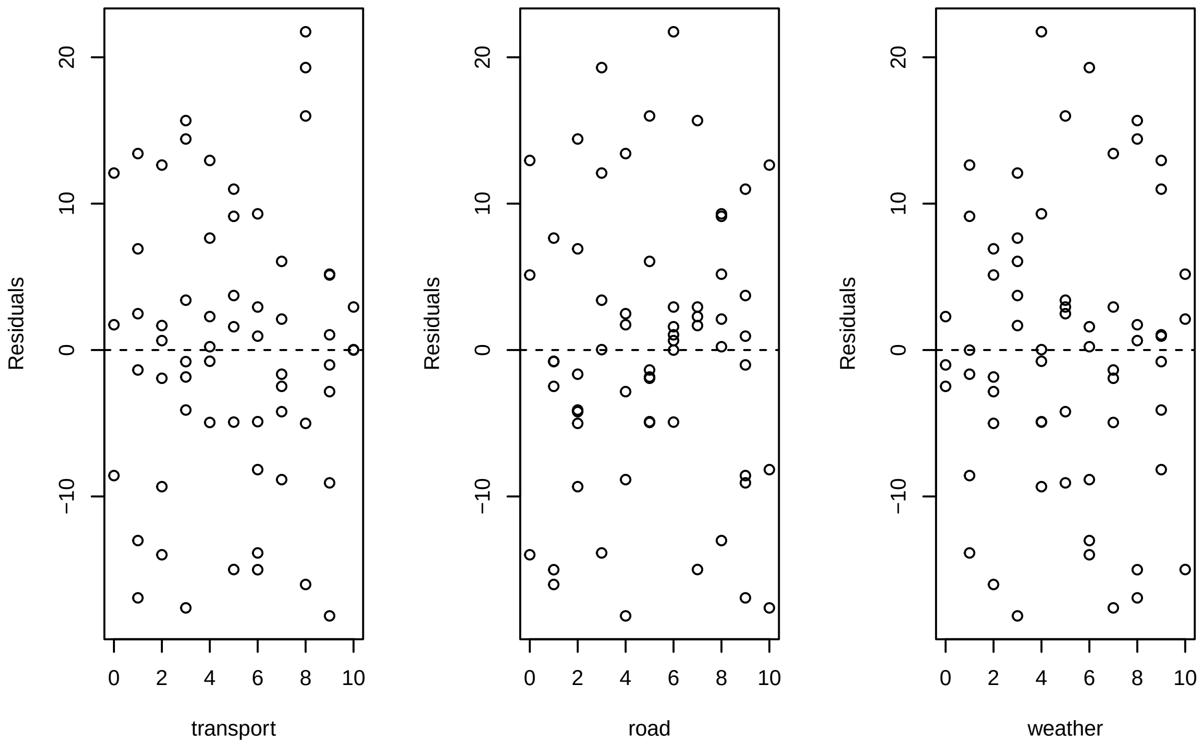
- Therefore the Mean Squared Reg = Reg SS / Reg df = 15703.5/5 = 3140.7
- Test statistics: $F_{obs} = MS_{Reg}/MS_{Res} = 3140.7/98.26094 = 31.96285$
- The null distribution for the test statistics is: $F_{5,56}$
- P-value: $P(F_{5,56} \geq 31.96285) = 0 = 3.0386681751212669e - 15 < 0.05$
- As the P-value is small:
  - (Statistical) There is enough evidence to reject H_0
  - (Contextual) There is significant linear relationship between ***spi*** and at least one of the 4 predictor variables.

d) For the diagnostics:

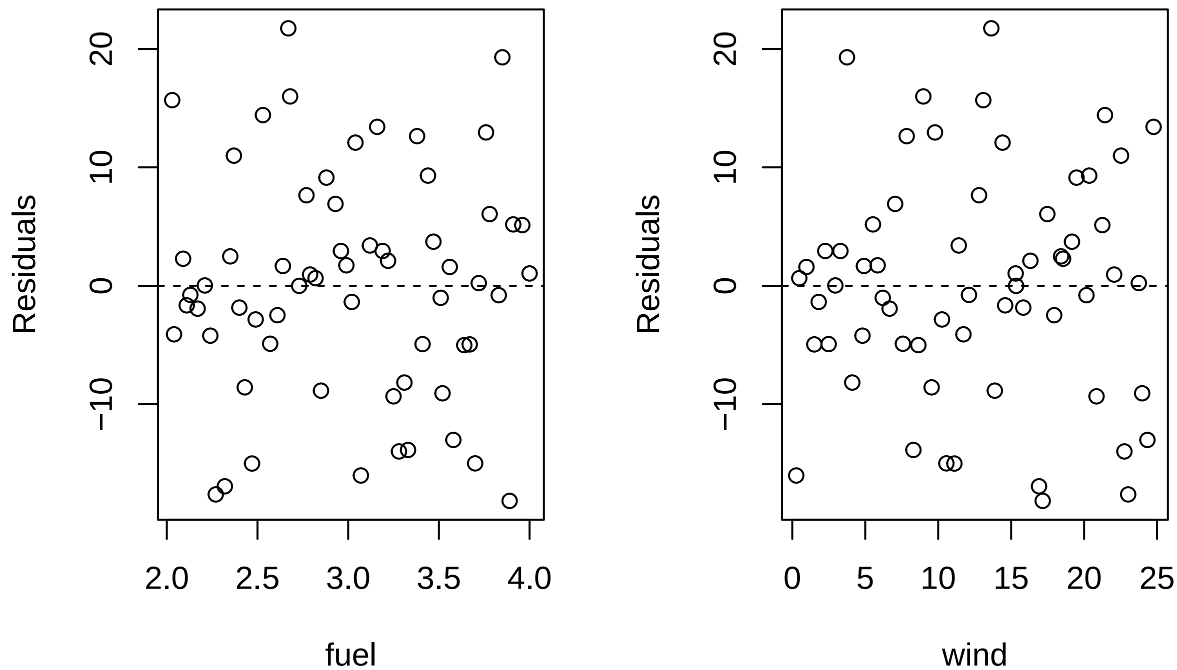
```r
par(mfrow = c(1,2))
plot(M1, which = 1:2)
```




```r
par(mfrow = c(1,3))
plot(resid(M1) ~ transport, data = traffic, xlab = "transport", ylab = "Residuals")
abline(h = 0, lty = 2)
plot(resid(M1) ~ road, data = traffic, xlab = "road", ylab = "Residuals")
abline(h = 0, lty = 2)
plot(resid(M1) ~ weather, data = traffic, xlab = "weather", ylab = "Residuals")
abline(h = 0, lty = 2)
```




```r
par(mfrow = c(1,2))
plot(resid(M1) ~ fuel, data = traffic, xlab = "fuel", ylab = "Residuals")
abline(h = 0, lty = 2)
plot(resid(M1) ~ wind, data = traffic, xlab = "wind", ylab = "Residuals")
abline(h = 0, lty = 2)
```



- The quantile plot of residuals look approximately linear, so the normality assumption for residuals is appropriate
- There is no obvious pattern in any of the residual plots so it appears the linearity and constant variance assumptions of the multiple linear model are

e) Find R2:

- Here $R^2 = 0.741 = 74.1\%$, which is a goodness of fit metric. It means 74,1% of the variation in ***spi*** is explained by the full linear regression model.

f) Find best regression model:

```
summary(M1)
```

```
##
## Call:
## lm(formula = spi ~ ., data = traffic)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.1596  -4.9415   0.1278   5.1686  21.7415
##
## Coefficients:
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  62.8071     7.4080   8.478 1.27e-11 ***
## transport    -2.1750     0.4611  -4.717 1.63e-05 ***
## road         -2.4097     0.4365  -5.520 9.04e-07 ***
## weather       4.2456     0.4473   9.492 2.92e-13 ***
## fuel         -3.6145     2.2759  -1.588    0.118
## wind         -0.1358     0.1764  -0.769    0.445
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.913 on 56 degrees of freedom
## Multiple R-squared:  0.7405, Adjusted R-squared:  0.7174
## F-statistic: 31.96 on 5 and 56 DF,  p-value: 3.039e-15
```

- ***wind*** has the highest P-value so remove it first

```r
M2 = update(M1, . ~ . - wind)
summary(M2)
```

```
##
## Call:
## lm(formula = spi ~ transport + road + weather + fuel, data = traffic)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.9347  -4.2440   0.0528   5.0544  21.4515
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  61.1610     7.0669   8.655 5.69e-12 ***
## transport    -2.1257     0.4550  -4.672 1.86e-05 ***
## road         -2.4372     0.4335  -5.622 5.92e-07 ***
## weather       4.2565     0.4454   9.555 1.94e-13 ***
## fuel         -3.6853     2.2659  -1.626    0.109
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.877 on 57 degrees of freedom
## Multiple R-squared:  0.7378, Adjusted R-squared:  0.7194
## F-statistic: 40.09 on 4 and 57 DF,  p-value: 5.959e-16
```

- ***fuel*** still has large P-value so remove it

```r
M3 = update(M2, . ~ . - fuel)
summary(M3)
```

```
##
## Call:
## lm(formula = spi ~ transport + road + weather, data = traffic)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -21.672  -5.643   1.067   4.656  23.164
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  51.7370     4.1027  12.611  < 2e-16 ***
## transport    -2.3216     0.4449  -5.218 2.54e-06 ***
## road         -2.4563     0.4394  -5.590 6.40e-07 ***
## weather       4.1450     0.4463   9.286 4.48e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.02 on 58 degrees of freedom
## Multiple R-squared:  0.7256, Adjusted R-squared:  0.7114
## F-statistic: 51.12 on 3 and 58 DF,  p-value: 2.724e-16
```

- In model M3 although all predictors are significant, the value of R^2 adjust decrease, which indicates that the recently removed predictor ***fuel*** has contribution to the model. Therefore, we will stop at the final model M2.

$$\hat{Y} = 61.16096 - 2.12565X_1 - 2.43721X_2 + 4.25645X_3 - 3.68533X_4$$

g) Explain R2 and R2 adjust:

- The R2 goodness of fit metric always decreases/increases when a predictor is removed/added from/into the model. The adjusted R2 has a penalty for the number of predictors in the model. So it will sometimes increase when a predictor is removed. In this case, from the full to final model, the R2 decreases from 74.1% to 73.8% but the adjusted R2 increases from 71.7% to 71.9%.

QUESTION 2:

a) Balanced study checking and explain:

- A study is balanced if there are equal number of replicates across all the levels factors in the study.
- We can check replicates by:

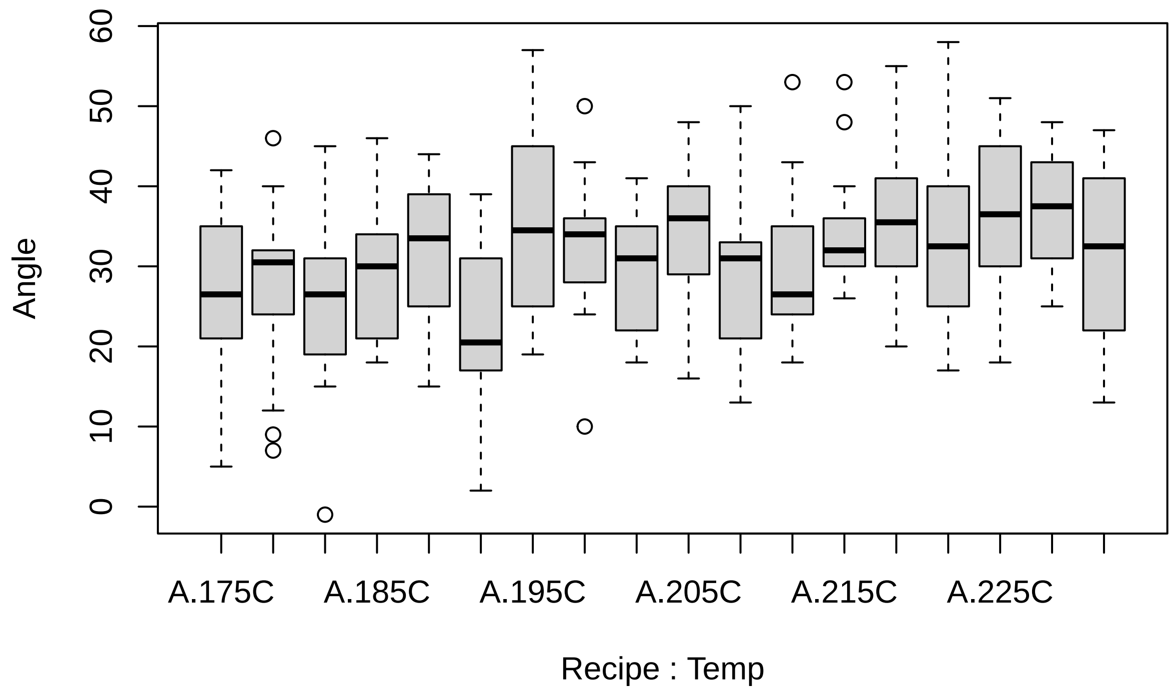
```r
cake  = read.csv('data/cake.csv', header = TRUE, stringsAsFactors = TRUE)
table(cake[, c("Recipe", "Temp")])
```

```
##       Temp
## Recipe 175C 185C 195C 205C 215C 225C
##      A   14   14   14   14   14   14
##      B   14   14   14   14   14   14
##      C   14   14   14   14   14   14
```

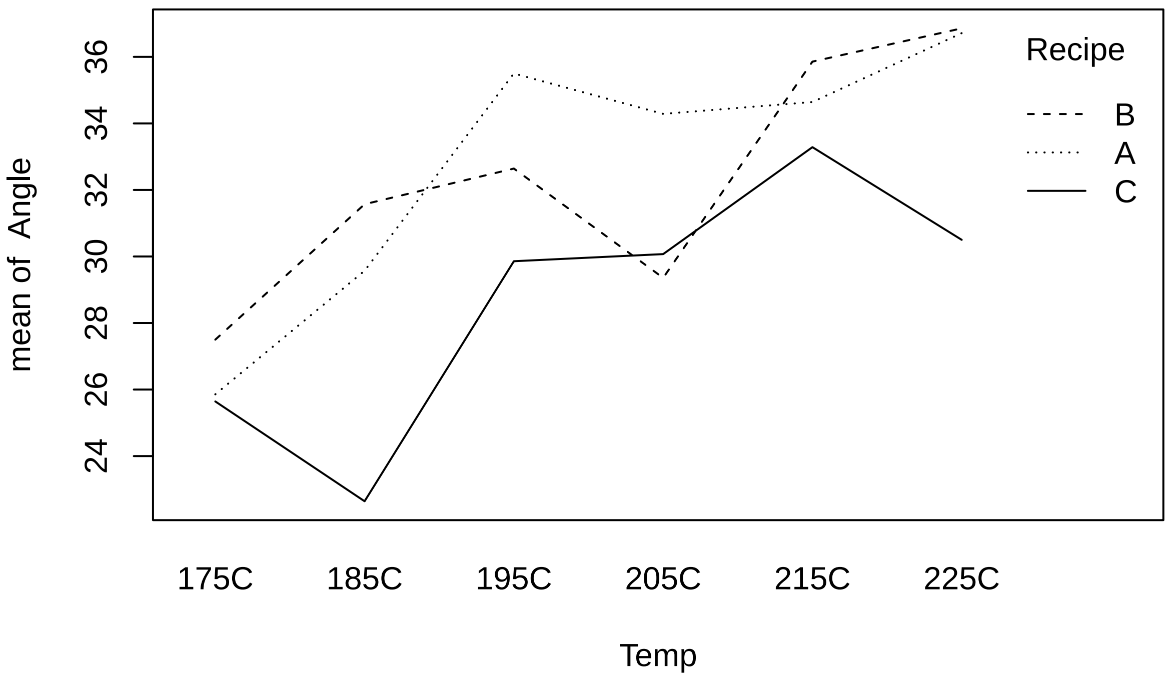
- From the above we can see that the design is balanced with an equal number of replicates for each combination of levels of the two factors.

b) Preliminary graphs that investigate different features of the data:

```r
boxplot(Angle ~ Recipe + Temp, data = cake)
```




```r
with(cake, interaction.plot(Temp, Recipe, Angle))
```



- From the boxplot, we can see that the assumption of equal variance among levels seems approximately valid due to the similar box sizes.
- From the interaction plot we can see there is non-parallel lines for the means of each group at different levels of the independent variables, this indicates a significant interaction effect between the two independent variables.

c) Interaction model:

- The Two-Way ANOVA model with interaction is:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$

- The parameters are:
  - Y_{ijk}: the reponse breaking angle of the cake
  - α_i: the ***Recipe*** effect, there are 3 levels: A, B, C
  - β_j: the ***Temp*** effect, there are 6 levels: 175C, 185C, 195C, 205C, 215C, 225C
  - γ_{ij}: the interaction effect between ***Recipe*** and ***Temp***
  - $\epsilon_{ijk} \sim \mathcal{N}(0, \sigma^2)$: the unexplained variation

d) Study the effect of *Recipe* and *Temp* on *Angle*:

- We will conduct a hypotheses test:

$$H_0 : \gamma_{ij} = 0 \text{ for all } i, j H_1 : \text{at least one } \gamma_{ij} \neq 0$$

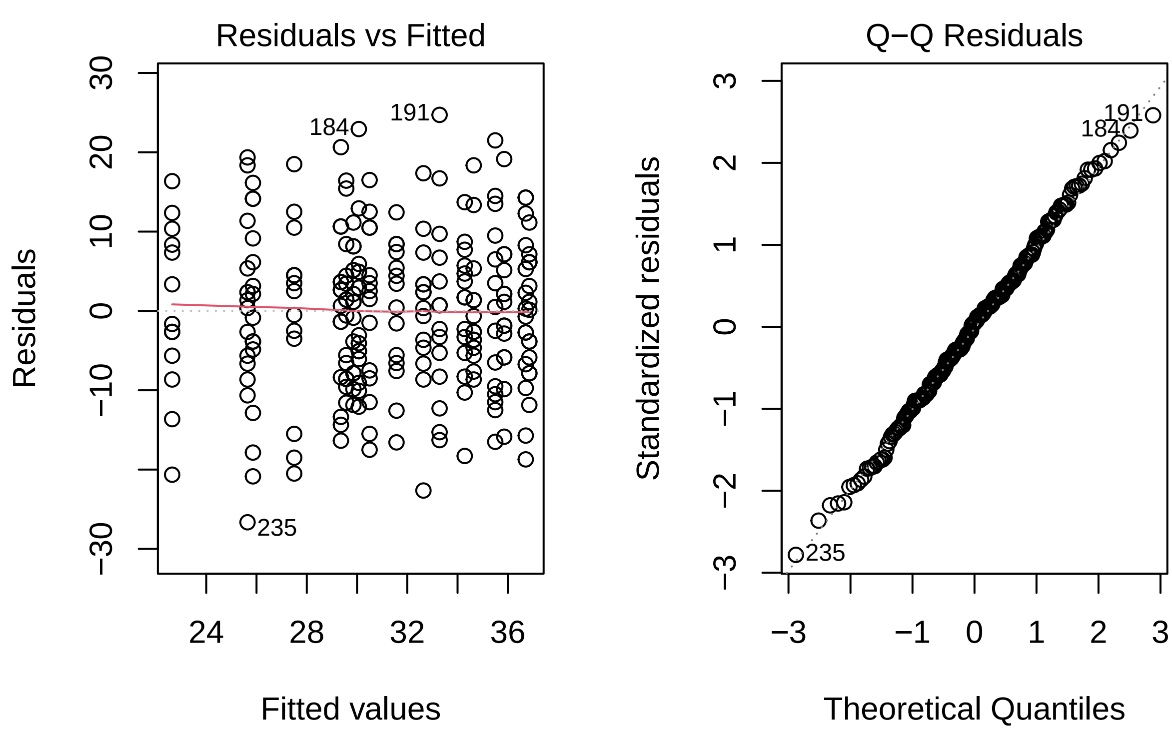
- Now we fit the interaction model:

```
cake.int = lm(Angle ~ Recipe * Temp, data = cake)
anova(cake.int)
```

```
## Analysis of Variance Table
##
## Response: Angle
##               Df  Sum Sq Mean Sq F value   Pr(>F)
## Recipe         2   844.8  422.38  4.2762 0.014998 *
## Temp           5  2530.1  506.01  5.1228 0.000177 ***
## Recipe:Temp   10   635.6   63.56  0.6435 0.775632
## Residuals    234 23113.8   98.78
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Since the P-value is $0.775632 > 0.05$. we do not have enough evidence to reject H_0. We can see that the interaction term is insignificant. Therefore, this is not the final model yet.
- We should validate the interaction model with diagnostic plots:

```
par(mfrow = c(1:2))
plot(cake.int, which = 1:2)
```



- The residuals are close to linear in the QQ-plot, and so the normal assumption should be valid. The residual plot seems to show equal spread around the fitted values and so the constant variance assumption is also appropriate.

e) Repeat test analysis for the main effects:

- We will conduct hypotheses tests for main effects:

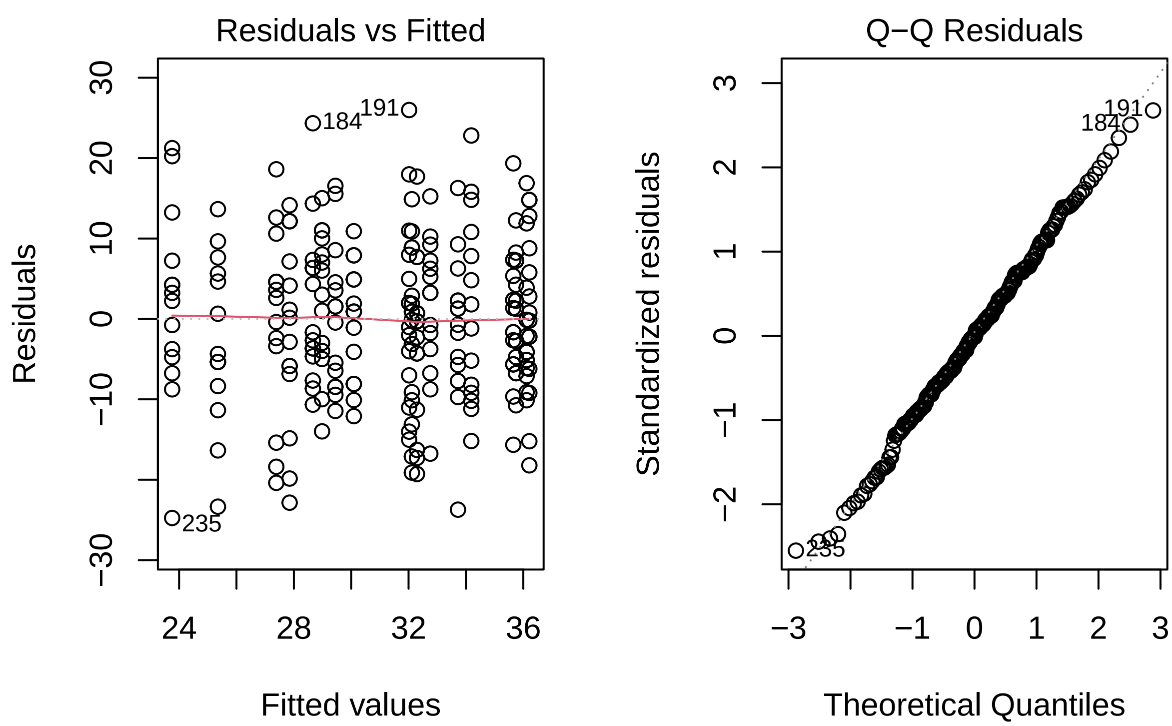
$$H_0 : \alpha_i = 0 \text{ for all } i \qquad \text{against} \qquad H_1 : \alpha_i \neq 0 H_0 : \beta_j = 0 \text{ for all } j \qquad \text{against} \qquad H_1 : \beta_j \neq 0$$

- Now we refit the model without interaction term:

```
cake.main = lm(Angle ~ Recipe + Temp, data = cake)
anova(cake.main)
```

```
## Analysis of Variance Table
##
## Response: Angle
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## Recipe      2   844.8  422.38  4.3396 0.0140636 *
## Temp        5  2530.1  506.01  5.1988 0.0001489 ***
## Residuals 244 23749.4   97.33
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
par(mfrow = c(1,2))
plot(cake.main, which = 1:2)
```



- The result of ANOVA table shows that the P-values for both ***Recipe*** and ***Temp*** are smaller than 0.05, which help reject all H_0 above and indicates that the main effects are significant.
- Again, the residuals are close to linear in the QQ-plot, and so the normal assumption should be valid, and the residual plot seems to show equal spread around the fitted values and so the constant variance assumption is also appropriate.

f) Conclusion on effect:

- Overall, the effect of ***Recipe*** and ***Temp*** on the quality of cakes ***Angle*** are not depend on each other since the interaction term is insignificant. However, these effects separately have significant impact on the response ***Angle***, that is, ***Angle*** seems to be higher with ***Recipe*** A and B, within the ***Temp*** ranges of 195C, 215C, 225C.
- We can interpret the main effects separately since there is no significant interaction effect them.