Input

$$\int 4x^2 \, \mathrm{d}x$$

Output

Step 1

We start by using the Constant multiple rule:

$$\int af(x) \, \mathrm{d}x = a \int f(x) \, \mathrm{d}x$$

So:

$$\int 4x^2 \, \mathrm{d}x = 4 \int x^2 \, \mathrm{d}x$$

Step 2

Next, we find $\int x^2 \, \mathrm{d}x$ using the Power rule:

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1}$$

$$\int x^2 \, \mathrm{d}x = \frac{x^3}{3}$$

Final step

$$\int 4x^2 \, \mathrm{d}x = 4 \int x^2 \, \mathrm{d}x = \frac{4x^3}{3}$$

A Constant multiple rule

We can prove that $\int af(x)\,\mathrm{d}x = a\int f(x)\,\mathrm{d}x$.

Proof. Let $F(x) = \int f(x)\,\mathrm{d}x$.
We know that:

$$\big(aF(x)\big)' = aF'(x) = af(x)$$

From here we can prove the constant multiple rule:

$$\begin{aligned}\big(aF(x)\big)' &= af(x)\\ aF(x) &= \int af(x)\,\mathrm{d}x\\ a\int f(x)\,\mathrm{d}x &= \int af(x)\,\mathrm{d}x\end{aligned}$$

□

B Power rule

We can prove that $\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1}$.

Proof. We will do this by finding $\left(\frac{x^{a+1}}{a+1}\right)$:

$$\left(\frac{x^{a+1}}{a+1}\right)' = \frac{(a+1)(a+1)(x^a)}{(a+1)^2} = x^a$$

□