We will prove that:

$$\int f(x) + g(x)\,dx = \int f(x)dx + \int g(x)dx$$

Using the fundamental theorem of calculus:

$$\frac{d}{dx}\left(\int f(x) + g(x)\,dx\right) = f(x) + g(x)$$

$$\frac{d}{dx}\left(\int f(x)\,dx + \int g(x)\,dx\right) = f(x) + g(x)$$

$$\therefore \int f(x) + g(x)\,dx = \int f(x)dx + \int g(x)\,dx$$