We will prove that:

$$\int f(x) + g(x)\, dx = \int f(x) dx + \int g(x) dx$$

Using the fundamental theorem of calculus:

$$\frac{d}{dx}\left(\int f(x) + g(x)\, dx\right) = f(x) + g(x)$$

$$\frac{d}{dx}\left(\int f(x)\, dx + \int g(x)\, dx\right) = f(x) + g(x)$$

$$\therefore \int f(x) + g(x)\, dx = \int f(x) dx + \int g(x)\, dx$$

We will prove that:

$$\int a\,dx = ax$$

We know that:

$$\frac{d}{dx}ax = a$$

So, the antiderivative of a is ax.

We will prove that:

$$\int af(x)\,dx = a\int f(x)\,dx$$

$$\frac{d}{dx}\left(\int af(x)\,dx\right) = af(x)$$

$$\frac{d}{dx}\left(a\int f(x)\,dx\right) = af(x)$$

$$\therefore \int af(x)\,dx = a\int f(x)\,dx$$

We will prove that:

$$\int x^a \, dx = \frac{x^{a+1}}{a+1}$$

We derive $\frac{x^{a+1}}{a+1}$ using the quotient rule:

$$\frac{d}{dx}\left(\frac{x^{a+1}}{a+1}\right) = \frac{(a+1)(a+1)x^a}{(a+1)^2} = x^a$$

So, the antiderivative of x^a is $\frac{x^{a+1}}{a+1}$.