Time Travelling to Avoid Trump
A Mathematical Analysis

Vincent Macri David White Aviv Haber

November 22, 2017

Contents

Part I

The Twin Paradox

Chapter 2

Resolving the Twin Paradox

2.1 The relativistic Doppler effect

The simplest way to solve the twin paradox is to use the relativistic Doppler effect in order to analyze what Donald sees and what Alice sees.

We will have both Alice and Donald flash a light at the other once per second, according to their own proper time. Their lights are infinitely powerful, and can be seen from light years away (once the light has travelled there of course). We will assume the path of the lights are entirely through a perfect vacuum.

2.1.1 More notation

First, let's define some notation specific to this section:

f_s The frequency the emitter (source) flashes their light at.

f_o The frequency the other person (observer) sees the light flashing at.

λ The distance to the next wave front of the approaching light wave. λ is calculated as:

$$\lambda = \frac{c_0 \, \mathrm{m/s}}{f_s \, \mathrm{s}^{-1}} = \frac{c_0}{f_s} \, \mathrm{m}$$

2.1.2 Relativistic Doppler effect derivation

For now, we will make Alice the source of the flashes, and Donald the observer. We could do it the other way around, but we will soon see that it does not have any effect on the fully derived formula, as both Δt_A and Δt_D will cancel out.

We start by relating Δt_D to λ and v when Alice is moving away from Donald:

$$\begin{aligned}
\Delta t_D &= \frac{\lambda}{c_0} + \frac{v \times \Delta t_D}{c_0} \\
c_0 \Delta t_D &= \lambda + v \Delta t_D \\
c_0 \Delta t_D - v \Delta t_D &= \lambda \\
\Delta t_D (c_0 - v) &= \lambda \\
\Delta t_D &= \frac{\lambda}{c_0 - v} \\
\Delta t_D &= \frac{1}{c_0 - v} \times \lambda
\end{aligned}$$

Now substitute in $\lambda = \frac{c_0}{f_s}$:

$$\begin{aligned}
\Delta t_D &= \frac{1}{c_0 - v} \times \frac{c_0}{f_s} \\
\Delta t_D &= \frac{c_0}{c_0 - v} \times \frac{1}{f_s} \\
\Delta t_D &= \frac{1}{1 - \frac{v}{c_0}} \times \frac{1}{f_s} \\
\Delta t_D &= \frac{1}{f_s (1 - \beta)} \qquad (2.1)
\end{aligned}$$

Next, we will perform a unit analysis to verify that (2.1) gives us a value in seconds:

$$\begin{aligned}
\Delta t_D &= \frac{1}{\mathrm{s}^{-1}} \\
\Delta t_D &= \mathrm{s}
\end{aligned}$$

So, we have derived (2.1) and verified that it gives us a value in seconds. Now we need to use this formula.

The next step is to develop the actual Doppler effect formula. We will work off of the special relativity time dilation formula (Bruni, Dick, Speijer, & Stewart, 2012, p. 593):

$$\Delta t_D = \frac{\Delta t_A}{\sqrt{1 - \beta^2}} \qquad (2.2)$$

We will now substitute (2.1) into (2.2) to combine our two equations in order to develop a third formula:

$$\begin{aligned} \Delta t_D &= \frac{\Delta t_A}{\sqrt{1-\beta^2}} \\ \frac{1}{f_s(1-\beta)} &= \frac{\Delta t_A}{\sqrt{1-\beta^2}} \\ \Delta t_A &= \frac{\sqrt{1-\beta^2}}{f_s(1-\beta)} \qquad (2.3) \end{aligned}$$

Next, we will finish developing the relativistic Doppler shift formula.

Note that, by definition:

$$f_o = \frac{1}{\Delta t_D} \qquad (2.4)$$

Bibliography

BIPM. (2006). *SI brochure: the international system of units* (Eighth). Bureau international des poids et mesures. Retrieved from http://www.bipm.org/en/si/si_brochure/

Bouttell, D. (2017).

Bruni, D., Dick, G., Speijer, J., & Stewart, C. (2012). *Physics 12*. Nelson Education.

Einstein, A. (1916). Relativity: the special and general theory.

Gibbs, P. (1996). Can special relativity handle acceleration? In D. Koks (Ed.), *Physics FAQ*. (n.p.)

Kogut, J. (2012). *Introduction to relativity: for physicists and astronomers*. Complementary Science. Elsevier Science. Retrieved from https://books.google.ca/books?id=9AKPpSxiN4IC

Weiss, M. (1992–2016). The twin paradox. In D. Koks (Ed.), *Physics FAQ*. (n.p.)