Digital Logic Design

Lecture - 4

Lecture

Last?

- ✓ Boolean Analysis
- ✓ Boolean Expressions SOP and POS
- ✓ Converting into Standard Form
- ✓ Min- & Max-terms
- ✓ Extracting SOP/POS from Truth table

Today!

- K-Maps
- Quiz
- Assignment
- Project Proposal

PAF KIET

Boolean Algebra and Logic Simplification

- Do you have some difficulty to apply Boolean algebra laws, rules, and theorems?
- In 1953, an American mathematician **Maurice Karnaugh** introduced a graphical method to simplify logic expressions. That is known as **K-Map**
  - K-map provides a systematic method with an array of cells
  - It is similar to a truth table, presents all of the possible values of input variables and the resulting output for each value.
  - Used for simplifying **2, 3, 4** and **5 variable** expressions
  - It produces, simple SOP & POS expressions

Boolean Algebra and Logic Simplification

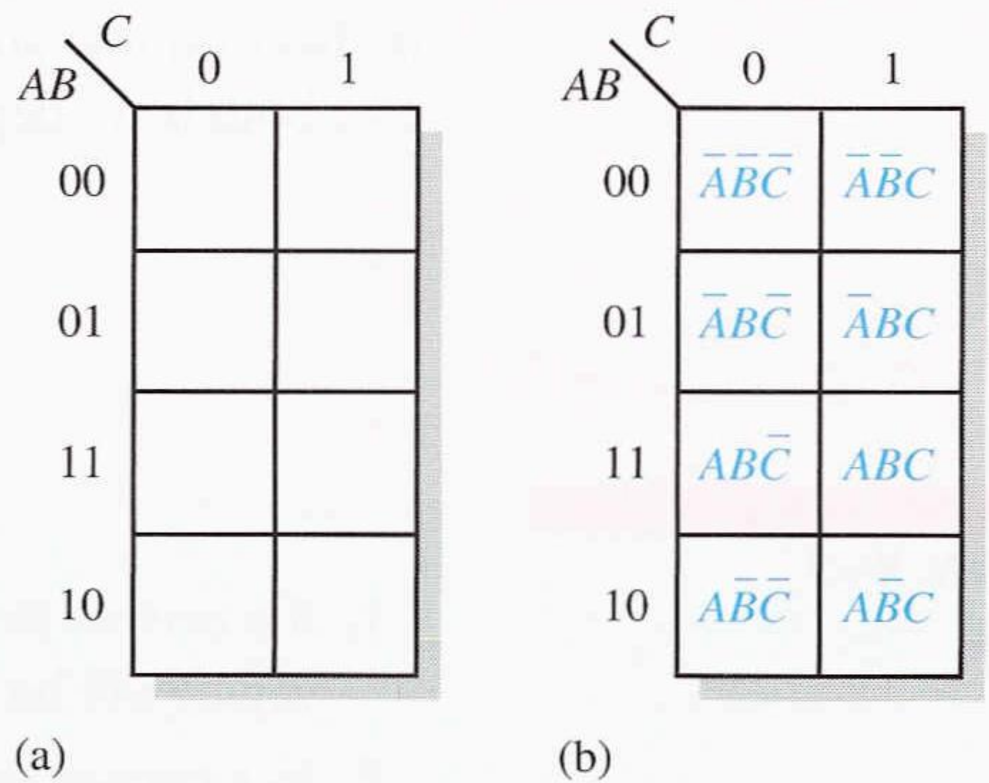
- K-Map is similar to truth table;
  - instead of i/p and o/p into columns and rows, the K-map is an array of cells in which each cell represents a binary value of the input variables.
- The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells.

Two main things to learn:
- **Mapping from expression**
- **Grouping / Minimization of expression**

K-Maps

FIGURE 4–21

A 3-variable Karnaugh map showing product terms.



K-Maps

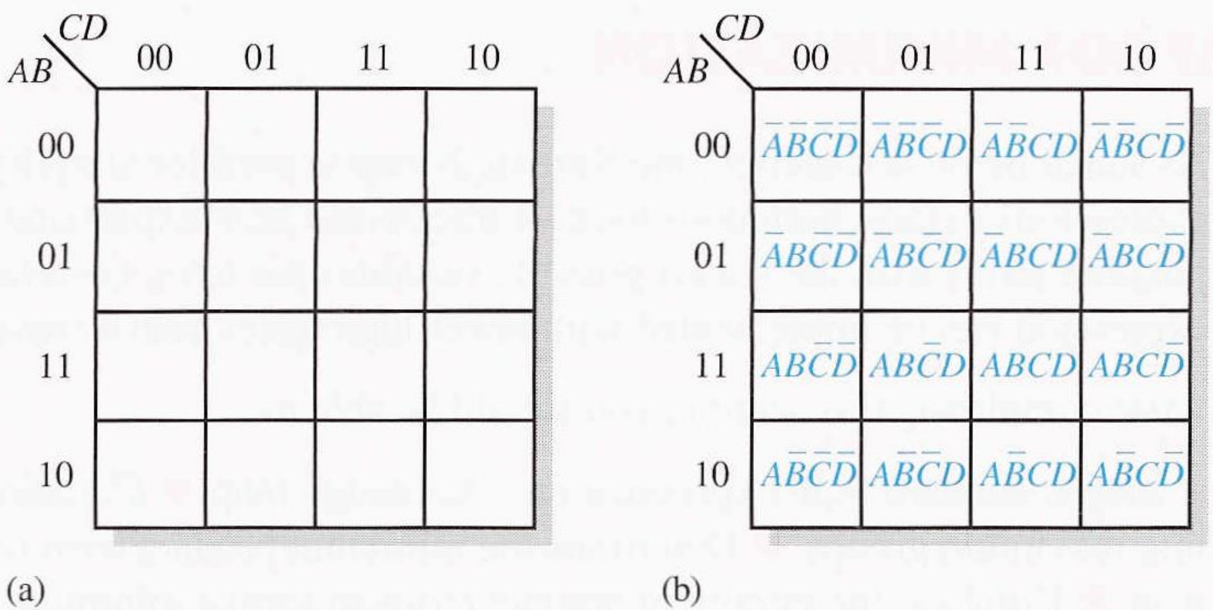


FIGURE 4–22

A 4-variable Karnaugh map.

Cell Adjacency

The cells in a Karnaugh map are arranged so that there is only a single-variable change between adjacent cells. **Adjacency** is defined by a single-variable change. In the 3-variable map the 010 cell is adjacent to the 000 cell, the 011 cell, and the 110 cell. The 010 cell is not adjacent to the 001 cell, the 111 cell, the 100 cell, or the 101 cell.

Physically, each cell is adjacent to the cells that are immediately next to it on any of its four sides. A cell is not adjacent to the cells that diagonally touch any of its corners. Also, the cells in the top row are adjacent to the corresponding cells in the bottom row and the cells in the outer left column are adjacent to the corresponding cells in the outer right column. This is called "wrap-around" adjacency because you can think of the map as wrapping around from top to bottom to form a cylinder or from left to right to form a cylinder. Figure 4–23 illustrates the cell adjacencies with a 4-variable map, although the same rules for adjacency apply to Karnaugh maps with any number of cells.

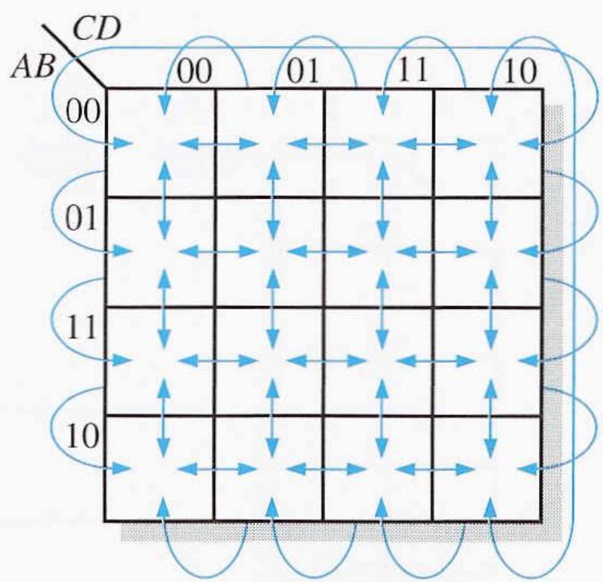


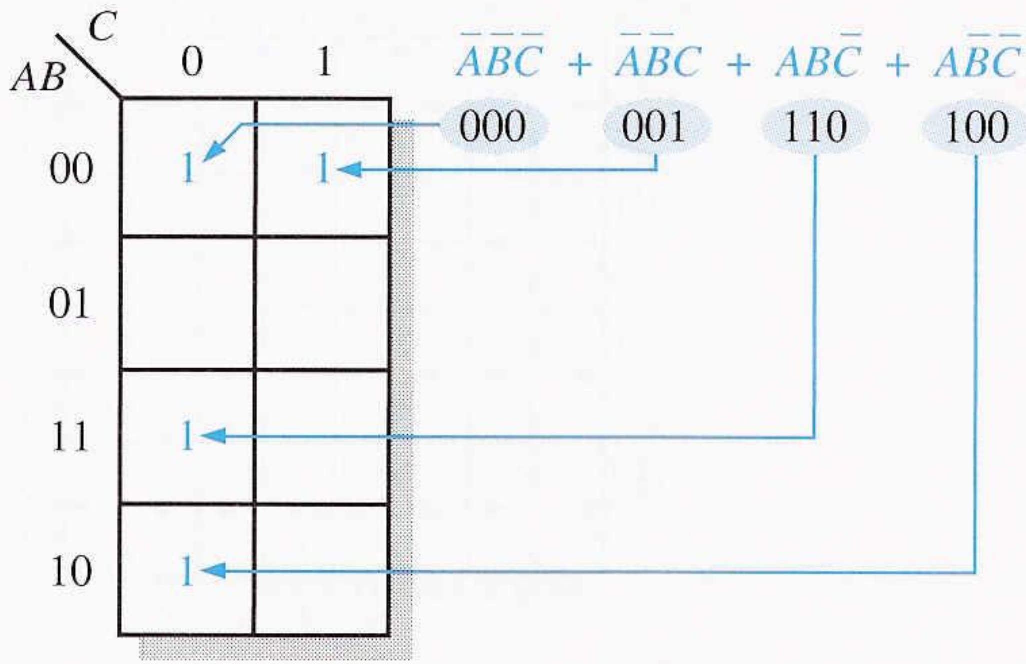
FIGURE 4–23

Adjacent cells on a Karnaugh map are those that differ by only one variable. Arrows point between adjacent cells.

K-Maps

FIGURE 4–24

Example of mapping a standard SOP expression.



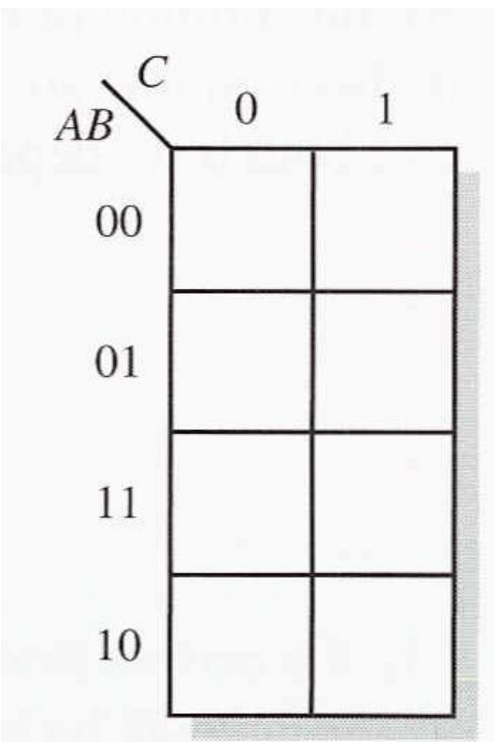
K-Maps

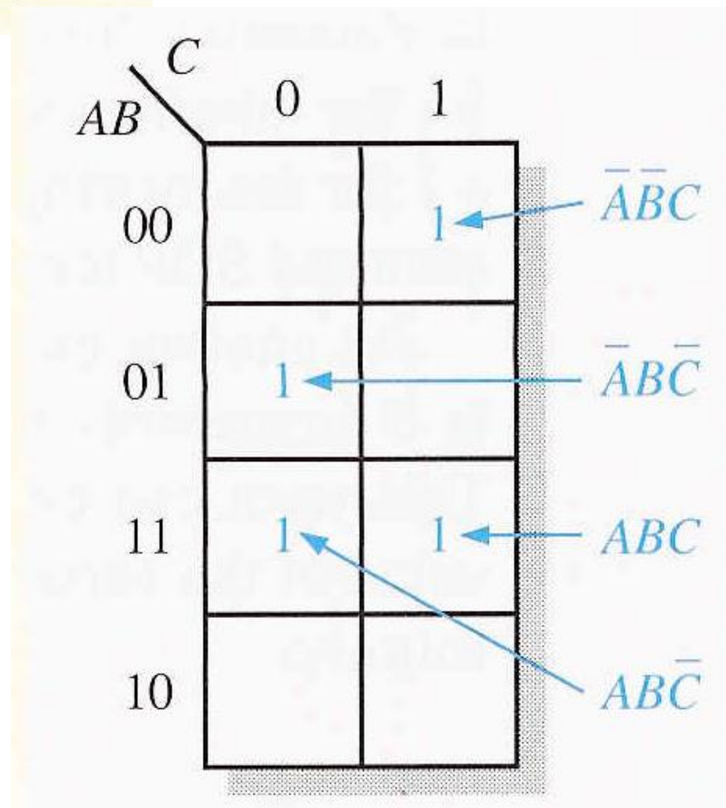
Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

$$\begin{array}{cccc} 001 & 010 & 110 & 111 \end{array}$$

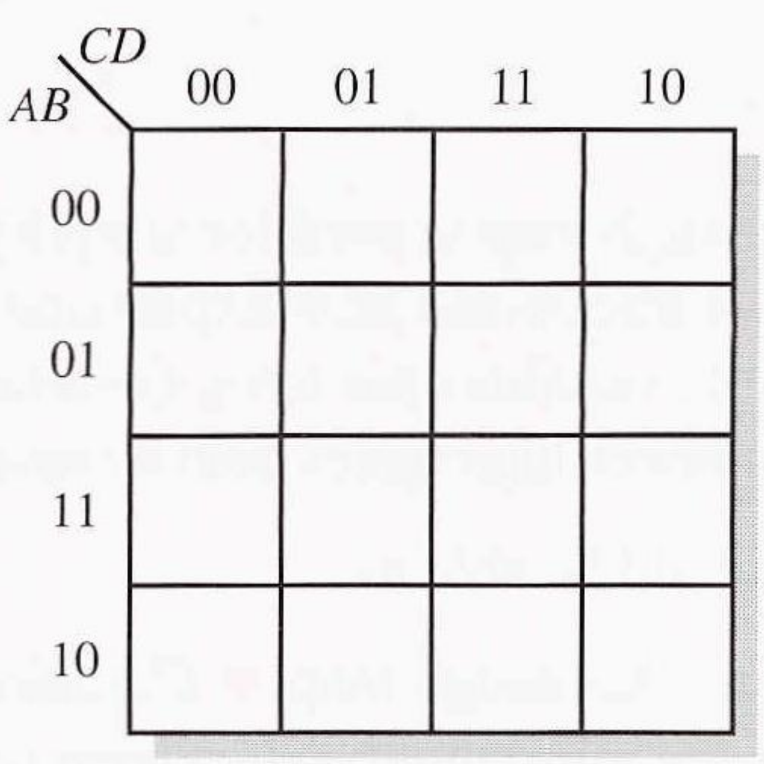


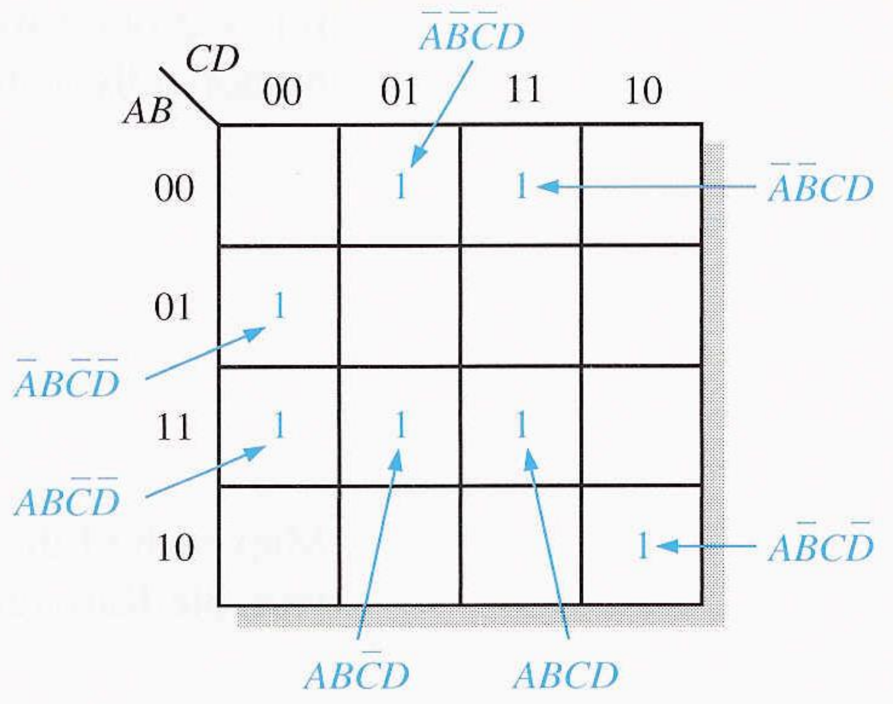


K-Maps

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$





K-Maps

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}BC\overline{D} + ABC\overline{D} + AB\overline{C}\,\overline{D} + ABCD$$

$AB \backslash CD$	00	01	11	10
00				
01				
11				
10				

K-Maps

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

Related Problem Map the SOP expression $BC + \overline{A}\,\overline{C}$ on a Karnaugh map.

$AB \backslash C$	0	1
00		
01		
11		
10		

$AB \backslash C$	0	1
00	1	1
01	1	1
11	1	
10	1	1

$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		

EXAMPLE 4–24

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

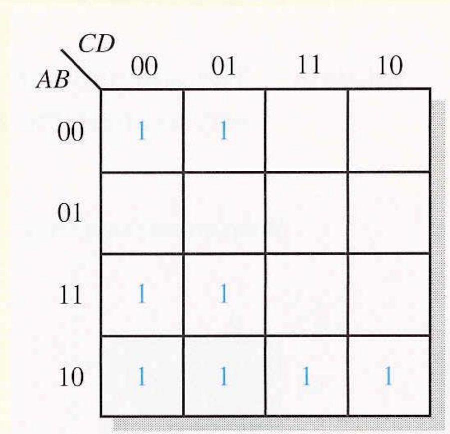
Solution

The SOP expression is obviously not in standard form because each product term does not have four variables. The first and second terms are both missing two variables, the third term is missing one variable, and the rest of the terms are standard. First expand the terms by including all combinations of the missing variables numerically as follows:

$\overline{B}\,\overline{C}$	$A\overline{B}$	$+\ AB\overline{C}$	$+\ A\overline{B}C\overline{D}$	$+\ \overline{A}\,\overline{B}\,\overline{C}D$	$+\ A\overline{B}CD$
0000	1000	1100	1010	0001	1011
0001	1001	1101			
1000	1010				
1001	1011				

Map each of the resulting binary values by placing a 1 in the appropriate cell of the 4-variable Karnaugh map in Figure 4–28. Notice that some of the values in the expanded expression are redundant.

▶ **FIGURE 4–28**



Related Problem

Map the expression $A + \overline{C}D + AC\overline{D} + \overline{A}BC\overline{D}$ on a Karnaugh map.

Cell Adjacency

The cells in a Karnaugh map are arranged so that there is only a single-variable change between adjacent cells. **Adjacency** is defined by a single-variable change. In the 3-variable map the 010 cell is adjacent to the 000 cell, the 011 cell, and the 110 cell. The 010 cell is not adjacent to the 001 cell, the 111 cell, the 100 cell, or the 101 cell.

Physically, each cell is adjacent to the cells that are immediately next to it on any of its four sides. A cell is not adjacent to the cells that diagonally touch any of its corners. Also, the cells in the top row are adjacent to the corresponding cells in the bottom row and the cells in the outer left column are adjacent to the corresponding cells in the outer right column. This is called "wrap-around" adjacency because you can think of the map as wrapping around from top to bottom to form a cylinder or from left to right to form a cylinder. Figure 4–23 illustrates the cell adjacencies with a 4-variable map, although the same rules for adjacency apply to Karnaugh maps with any number of cells.

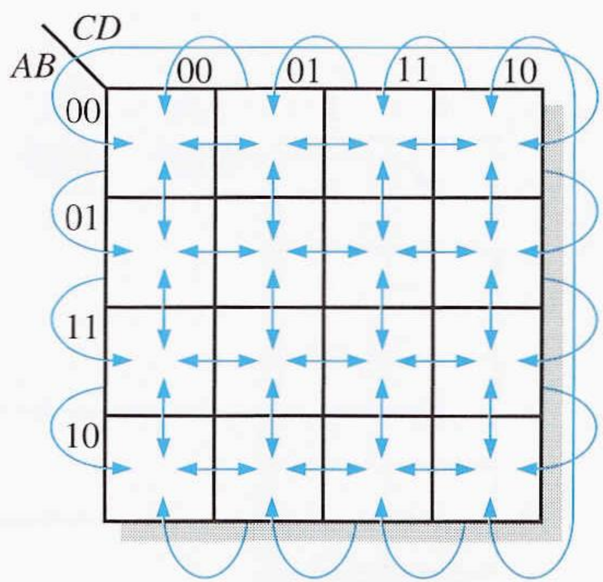
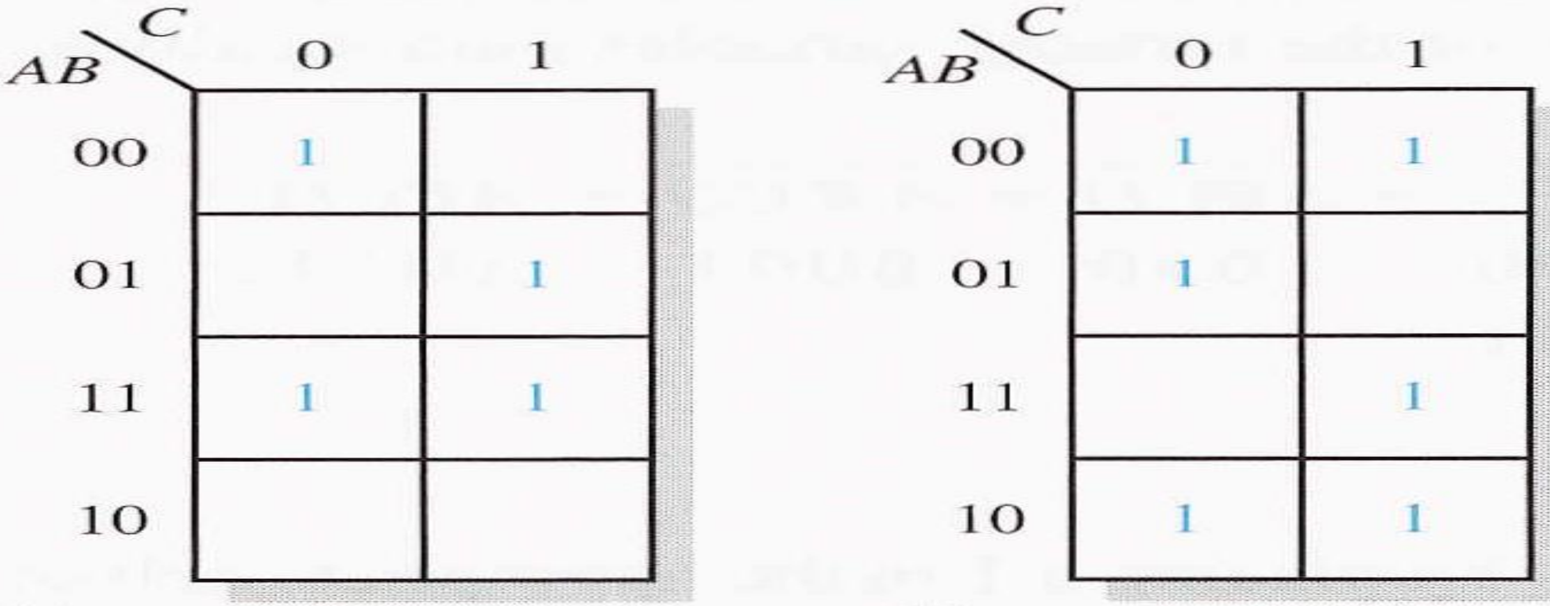


FIGURE 4–23

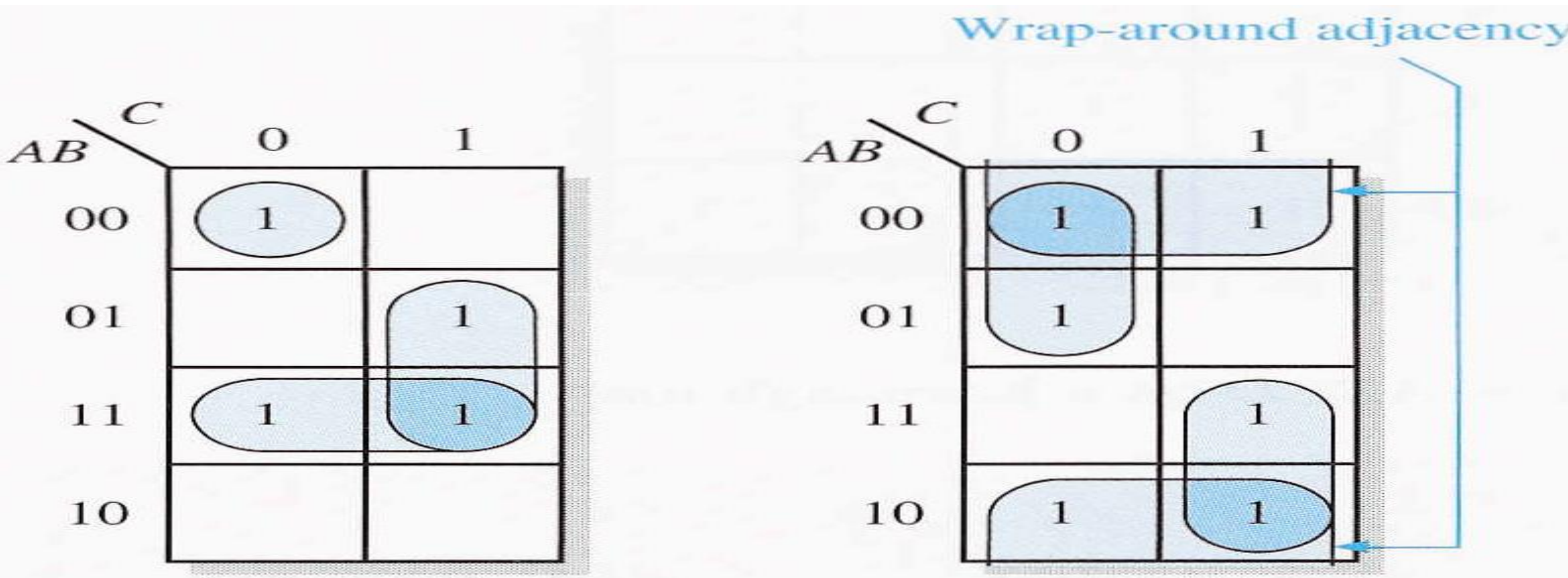
Adjacent cells on a Karnaugh map are those that differ by only one variable. Arrows point between adjacent cells.

Grouping the 1s You can group 1s on the Karnaugh map according to the following rules by enclosing those adjacent cells containing 1s. The goal is to maximize the size of the groups and to minimize the number of groups.

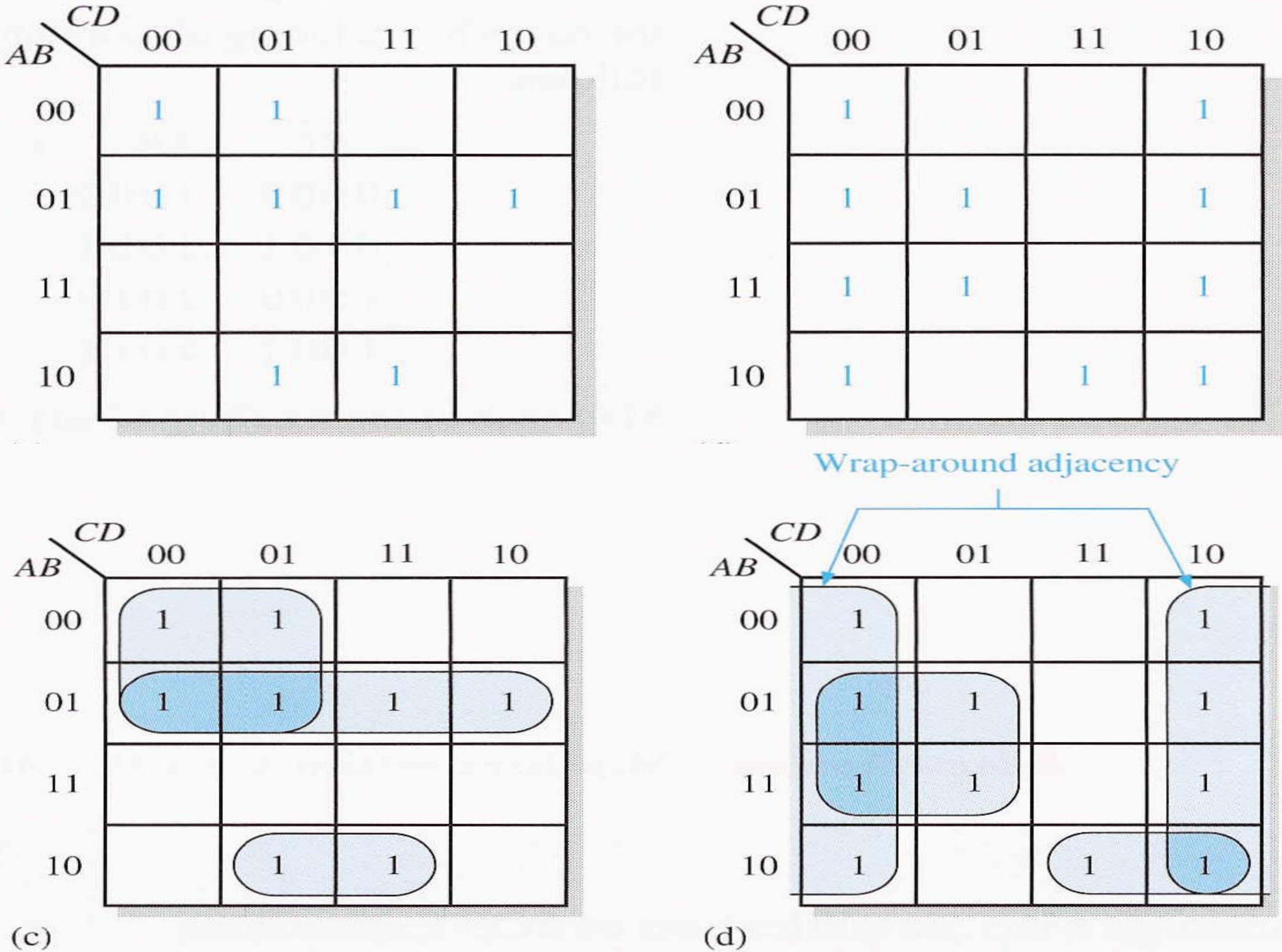
1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.


C
AB
0
1
00
01
11
10
1
1
1
1
C
AB
0
1
00
01
11
10
1
1
1
1
1
1

(a) (b)


Wrap-around adjacency
C
AB
0
1
00
01
11
10
1
1
1
1
C
AB
0
1
00
01
11
10
1
1
1
1
1
1

(a) (b)


CD
AB
00 01 11 10
Wrap-around adjacency
(c)
(d)

EXAMPLE 4–24

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

Solution The SOP expression is obviously not in standard form because each product term does not have four variables. The first and second terms are both missing two variables, the third term is missing one variable, and the rest of the terms are standard. First expand the terms by including all combinations of the missing variables numerically as follows:

$\overline{B}\,\overline{C}$	$+ A\overline{B}$	$+ AB\overline{C}$	$+ A\overline{B}C\overline{D}$	$+ \overline{A}\,\overline{B}\,\overline{C}D$	$+ A\overline{B}CD$
0000	1000	1100	1010	0001	1011
0001	1001	1101			
1000	1010				
1001	1011				

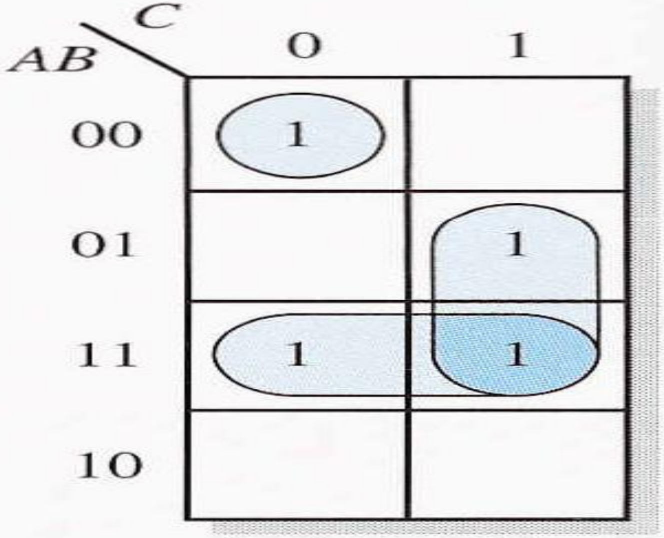
Lec # 04, Ch # 04, RECAP

Map each of the resulting binary values by placing a 1 in the appropriate cell of the 4-variable Karnaugh map in Figure 4–28. Notice that some of the values in the expanded expression are redundant.

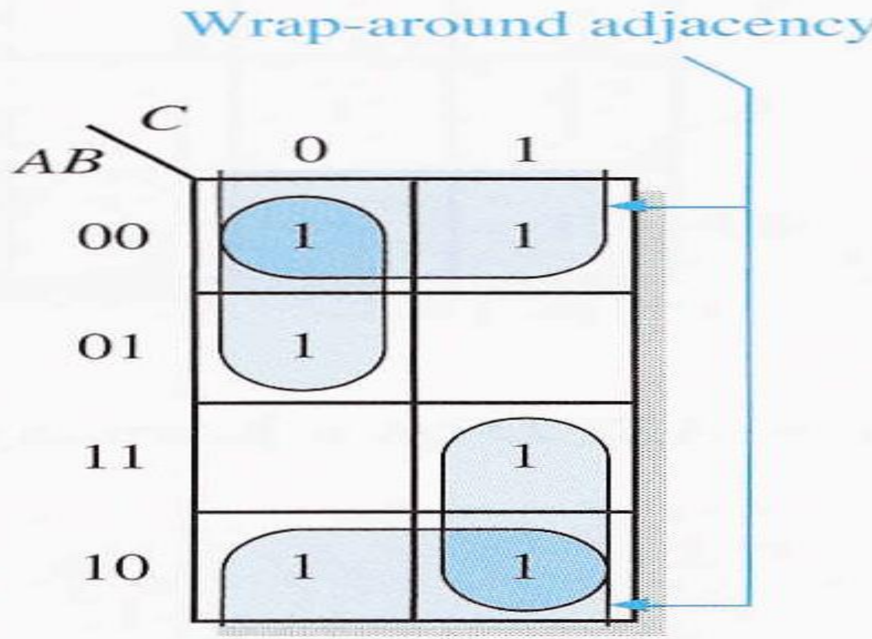
FIGURE 4–28

AB \ CD	00	01	11	10
00	1	1		
01				
11	1	1		
10	1	1	1	1

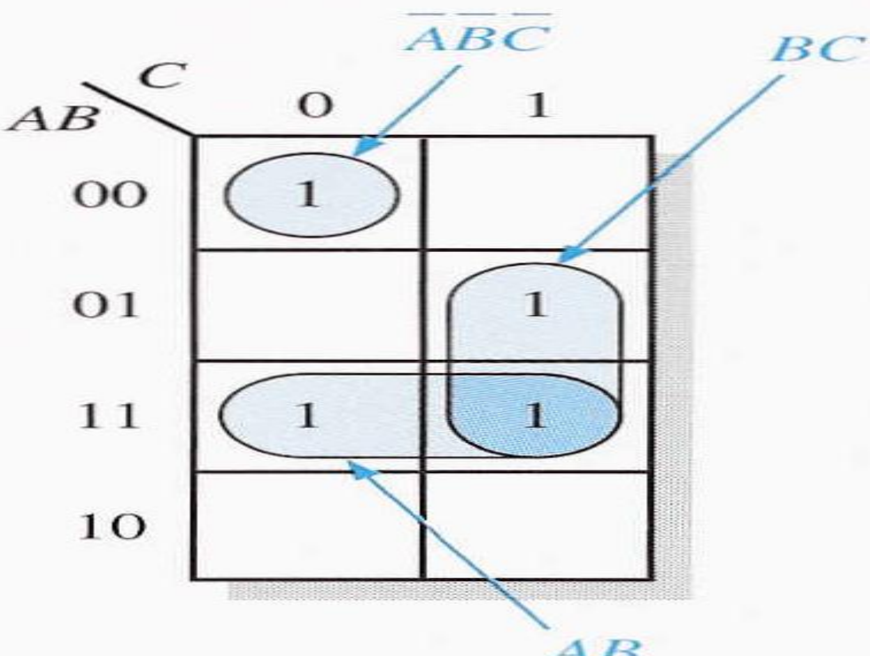
Related Problem Map the expression $A + \overline{C}D + AC\overline{D} + \overline{A}BC\overline{D}$ on a Karnaugh map.


C
AB
0
1
00
01
11
10
1
1
1
1

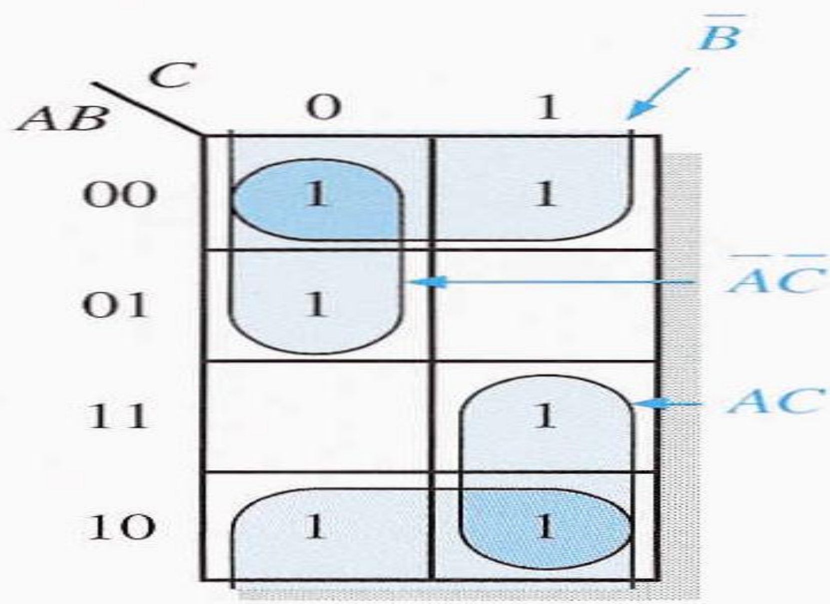
(a)


Wrap-around adjacency
C
AB
0
1
00
01
11
10
1
1
1
1
1
1

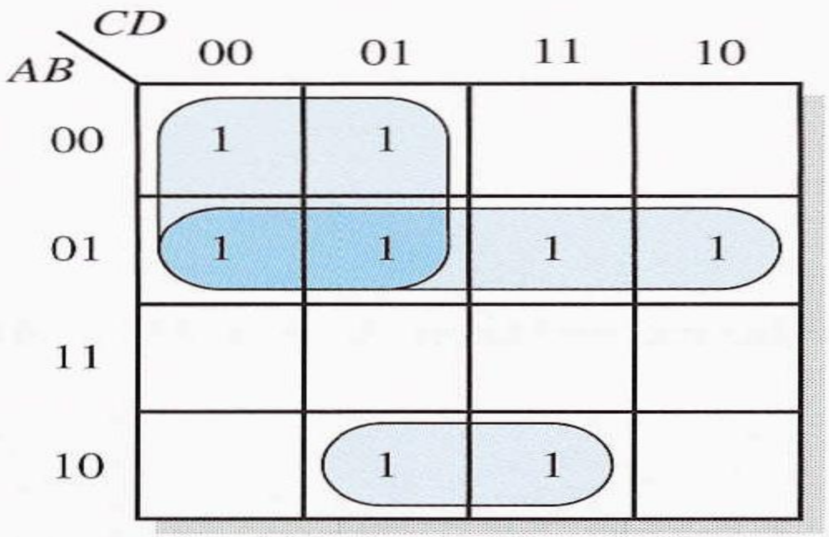
(b)


$\overline{A}\overline{B}\overline{C}$
BC
C
AB
0
1
00
01
11
10
1
1
1
1
AB

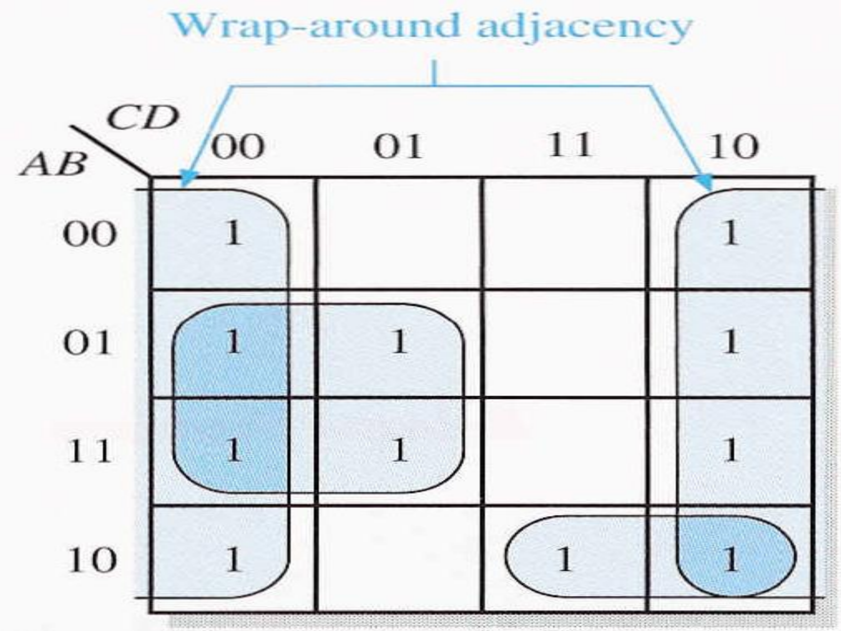
(a)


$\overline{B}$
C
AB
0
1
00
01
11
10
1
1
1
1
1
1
$\overline{A}\overline{C}$
AC

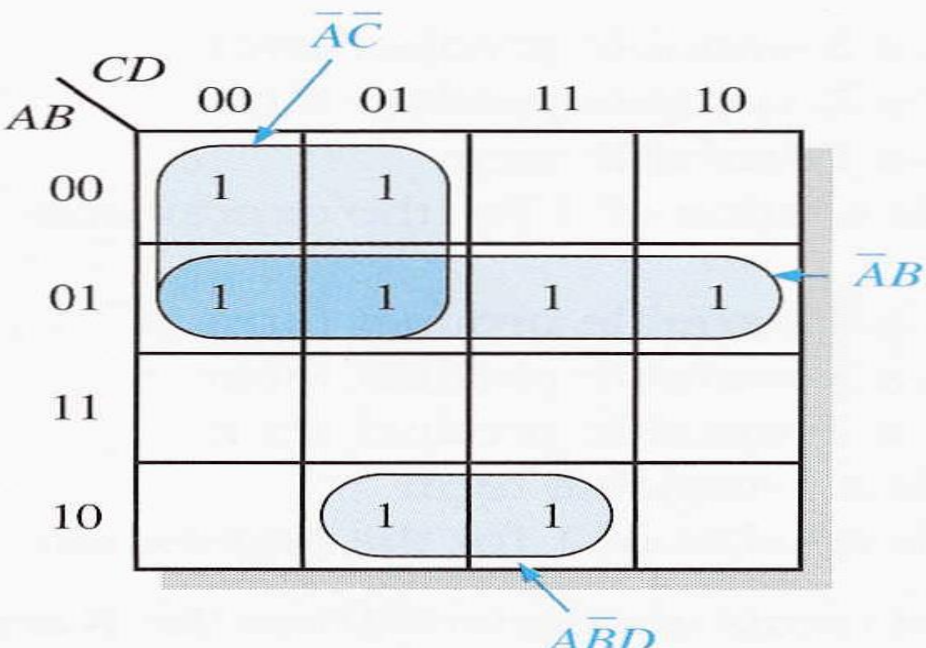
(b)


CD
AB
00
01
11
10

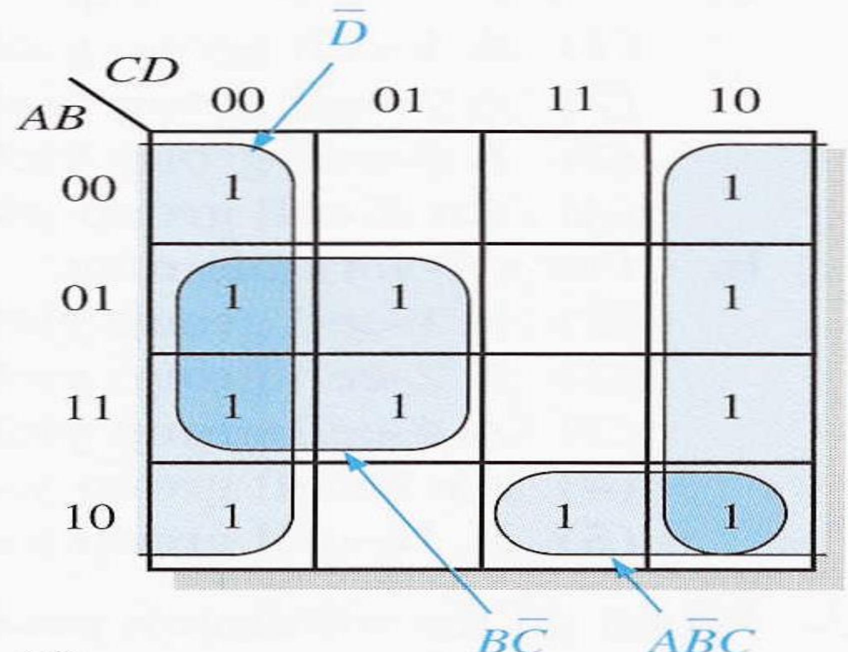
(c)


Wrap-around adjacency
CD
AB
00
01
11
10

(d)


$\overline{A}\overline{C}$
CD
AB
00
01
11
10
$\overline{A}B$
$A\overline{B}D$

(c)


$\overline{D}$
CD
AB
00
01
11
10
$B\overline{C}$
$A\overline{B}C$

(d)

Determining the Minimum SOP Expression from the Map When all the 1s representing the standard product terms in an expression are properly mapped and grouped, the process of determining the resulting minimum SOP expression begins. The following rules are applied to find the minimum product terms and the minimum SOP expression:

1. Group the cells that have 1s. Each group of cells containing 1s creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group. Variables that occur both uncomplemented and complemented within the group are eliminated. These are called *contradictory variables.*

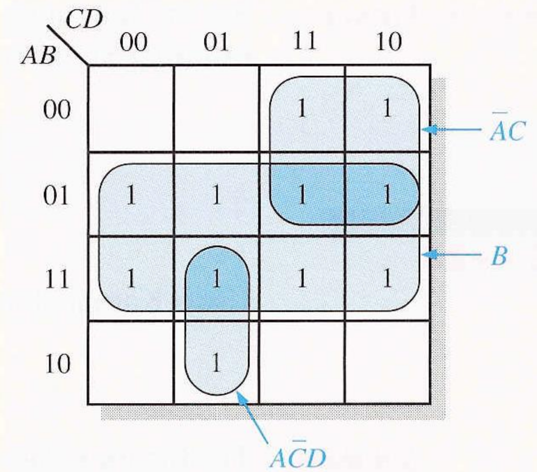
2. Determine the minimum product term for each group.
   - **a.** For a 3-variable map:
     - (1) A 1-cell group yields a 3-variable product term
     - (2) A 2-cell group yields a 2-variable product term
     - (3) A 4-cell group yields a 1-variable term
     - (4) An 8-cell group yields a value of 1 for the expression
   - **b.** For a 4-variable map:
     - (1) A 1-cell group yields a 4-variable product term
     - (2) A 2-cell group yields a 3-variable product term
     - (3) A 4-cell group yields a 2-variable product term
     - (4) An 8-cell group yields a 1-variable term
     - (5) A 16-cell group yields a value of 1 for the expression

3. When all the minimum product terms are derived from the Karnaugh map, they are summed to form the minimum SOP expression.

EXAMPLE 4–26

Determine the product terms for the Karnaugh map in Figure 4–31 and write the resulting minimum SOP expression.

▶ **FIGURE 4–31**



Solution Eliminate variables that are in a grouping in both complemented and uncomplemented forms. In Figure 4–31, the product term for the 8-cell group is B because the cells within that group contain both A and $\overline{A}$, C and $\overline{C}$, and D and $\overline{D}$, which are eliminated. The 4-cell group contains B, $\overline{B}$, D, and $\overline{D}$, leaving the variables $\overline{A}$ and C, which form the product term $\overline{A}C$. The 2-cell group contains B and $\overline{B}$, leaving variables A, $\overline{C}$, and D which form the product term $A\overline{C}D$. Notice how overlapping is used to maximize the size of the groups. The resulting minimum SOP expression is the sum of these product terms:

$$B + \overline{A}C + A\overline{C}D$$

Related Problem For the Karnaugh map in Figure 4–31, add a 1 in the lower right cell (1010) and determine the resulting SOP expression.

4–28

Use a Karnaugh map to minimize the following standard SOP expression:

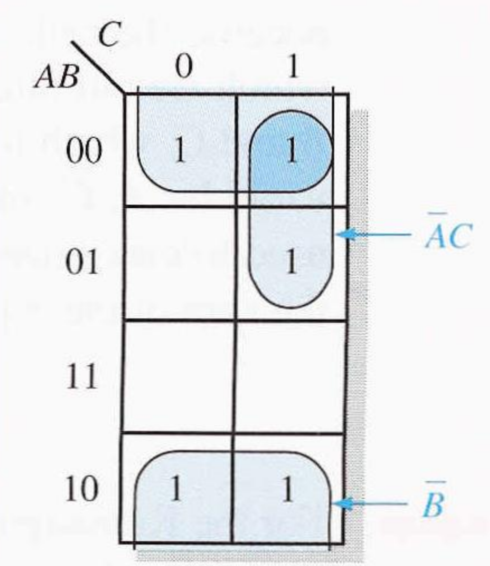
$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

Solution The binary values of the expression are

$$101 + 011 + 011 + 000 + 100$$

Map the standard SOP expression and group the cells as shown in Figure 4–33.

FIGURE 4–33



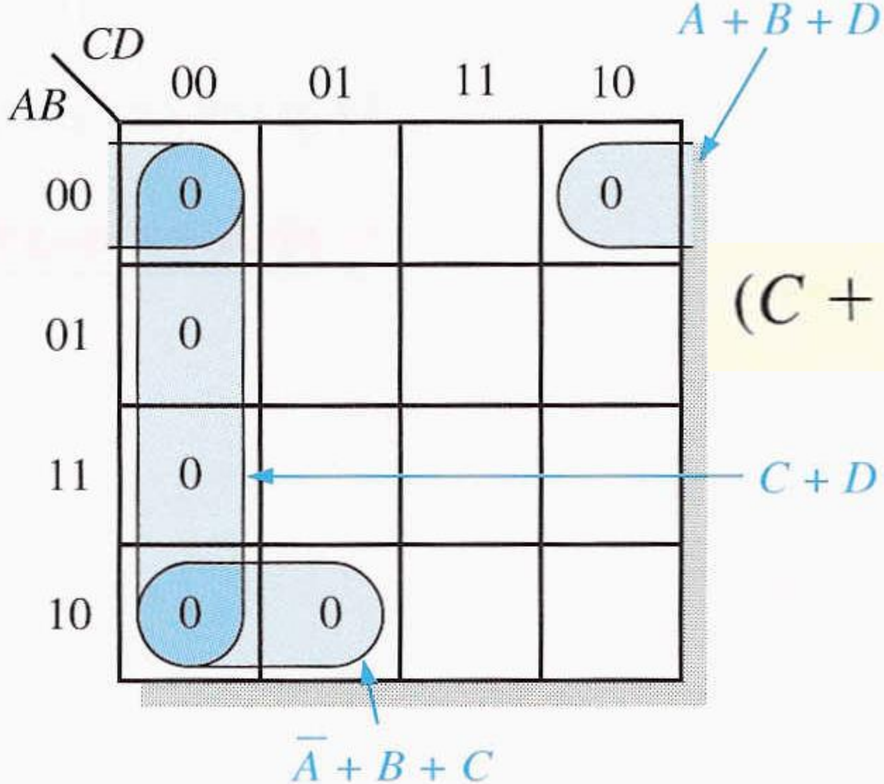
K-Map POS Minimization

EXAMPLE 4–32

Use a Karnaugh map to minimize the following POS expression:

$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$

Solution The first term must be expanded into $\overline{A} + B + C + D$ and $A + B + C + D$ to get a standard POS expression, which is then mapped; and the cells are grouped as shown in

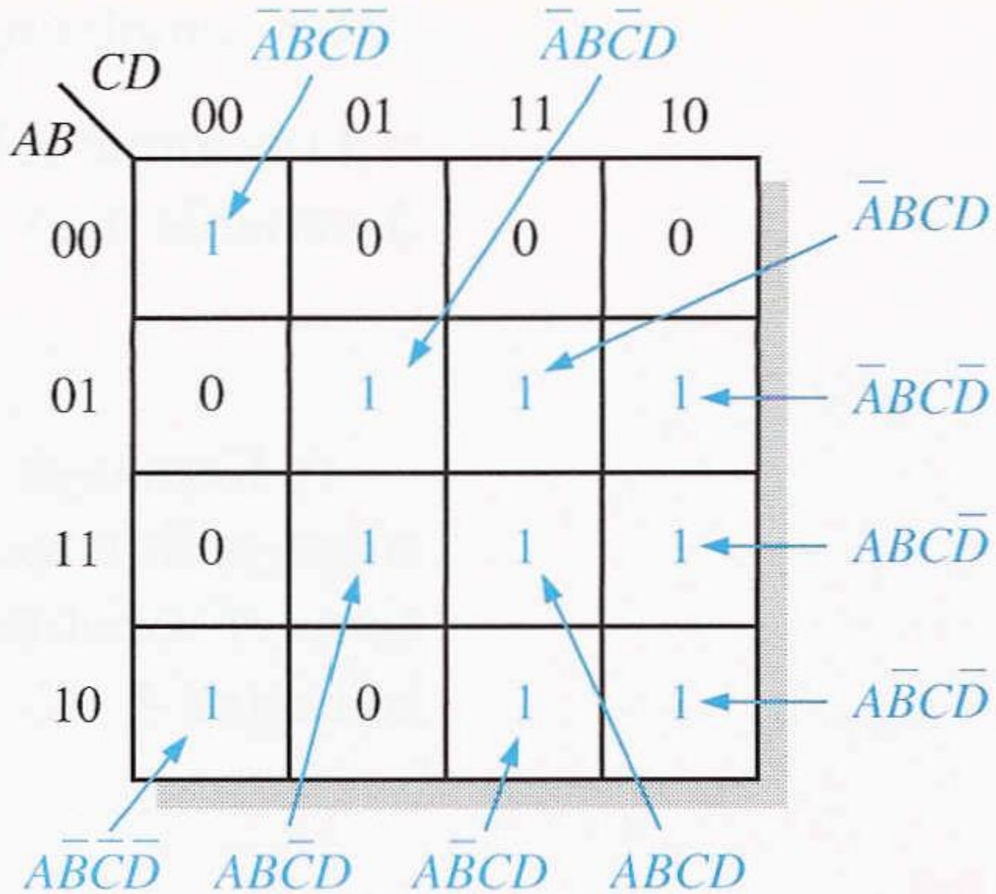


$$(C + D)(A + B + D)(\overline{A} + B + C)$$

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})$$
$$(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$

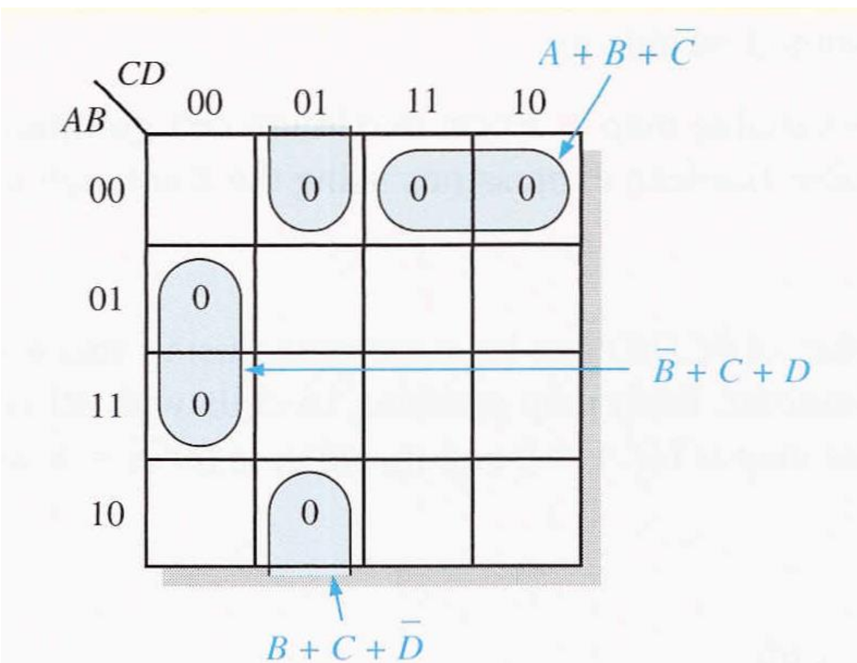
$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})$$
$$(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



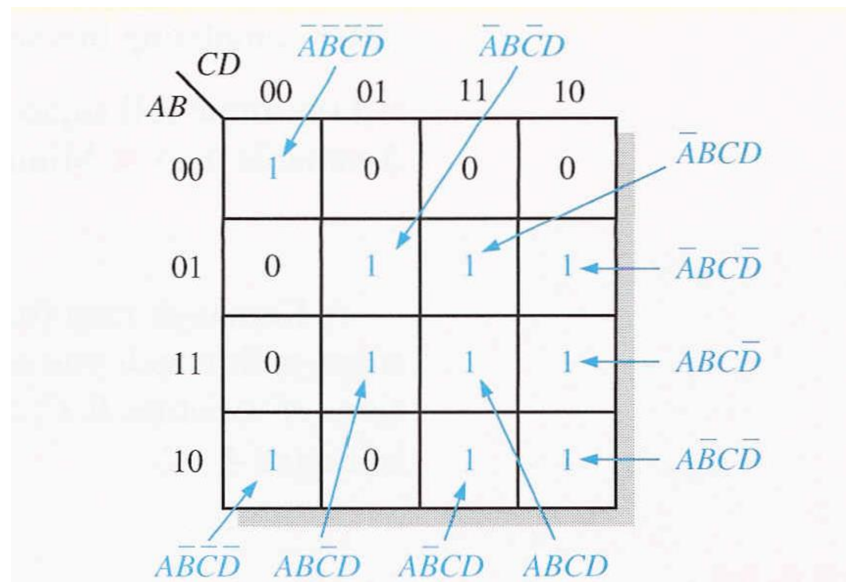
(b) Standard SOP:
$\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} +$
$A\overline{B}\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}CD + ABCD$

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



(a) Minimum POS: $(A + B + C)(\overline{B} + C + D)(B + C + \overline{D})$

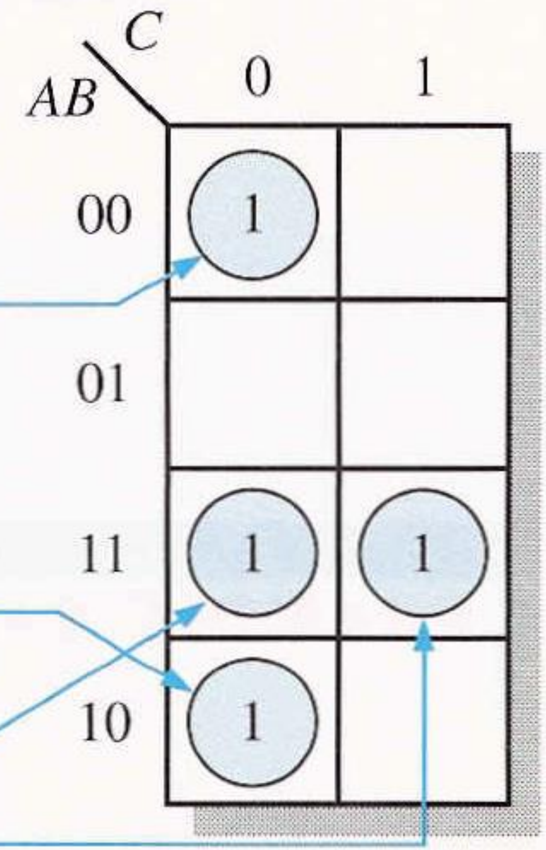


(b) Standard SOP:
$\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}CD + ABCD$

Mapping Directly from a Truth Table

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

Inputs A	B	C	Output X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Related Problem Use a Karnaugh map to convert the following expression to minimum SOP form:

$$(W + \overline{X} + Y + \overline{Z})(\overline{W} + X + \overline{Y} + \overline{Z})(\overline{W} + \overline{X} + \overline{Y} + Z)(\overline{W} + \overline{X} + \overline{Z})$$

SECTION 4–10 REVIEW

1. What is the difference in mapping a POS expression and an SOP expression?
2. What is the standard sum term expressed with variables A, B, C, and D for a 0 in cell 1011 of the Karnaugh map?
3. What is the standard product term expressed with variables A, B, C, and D for a 1 in cell 0010 of the Karnaugh map?

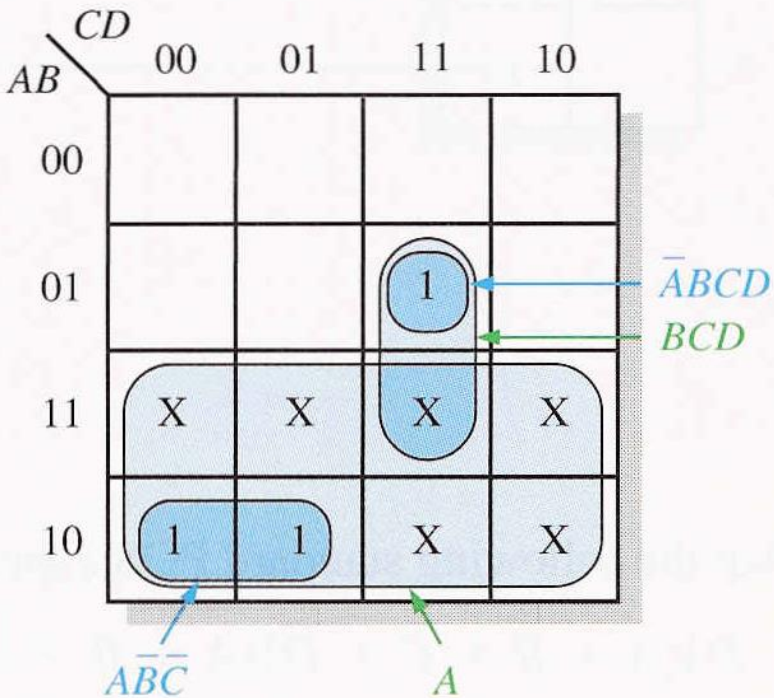
"Don't Care" Conditions

Sometimes a situation arises in which some input variable combinations are not allowed. For example, recall that in the BCD code covered in Chapter 2, there are six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111. Since these unallowed states will never occur in an application involving the BCD code, they can be treated as **"don't care"** terms with respect to their effect on the output. That is, for these "don't care" terms either a 1 or a 0 may be assigned to the output; it really does not matter since they will never occur.

Inputs $ABCD$	Output Y
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With "don't cares" $Y = A + BCD$

4–11 FIVE-VARIABLE KARNAUGH MAPS

Cell Adjacencies

BC \ DE	00	01	11	10
00				
01				
11				
10				

$A = 0$

BC \ DE	00	01	11	10
00				
01				
11				
10				

$A = 1$

Cell Adjacencies

$D\overline{E}$

$\overline{B}CE$

$\overline{A}B\overline{D}$

$B\overline{C}\,\overline{D}E$

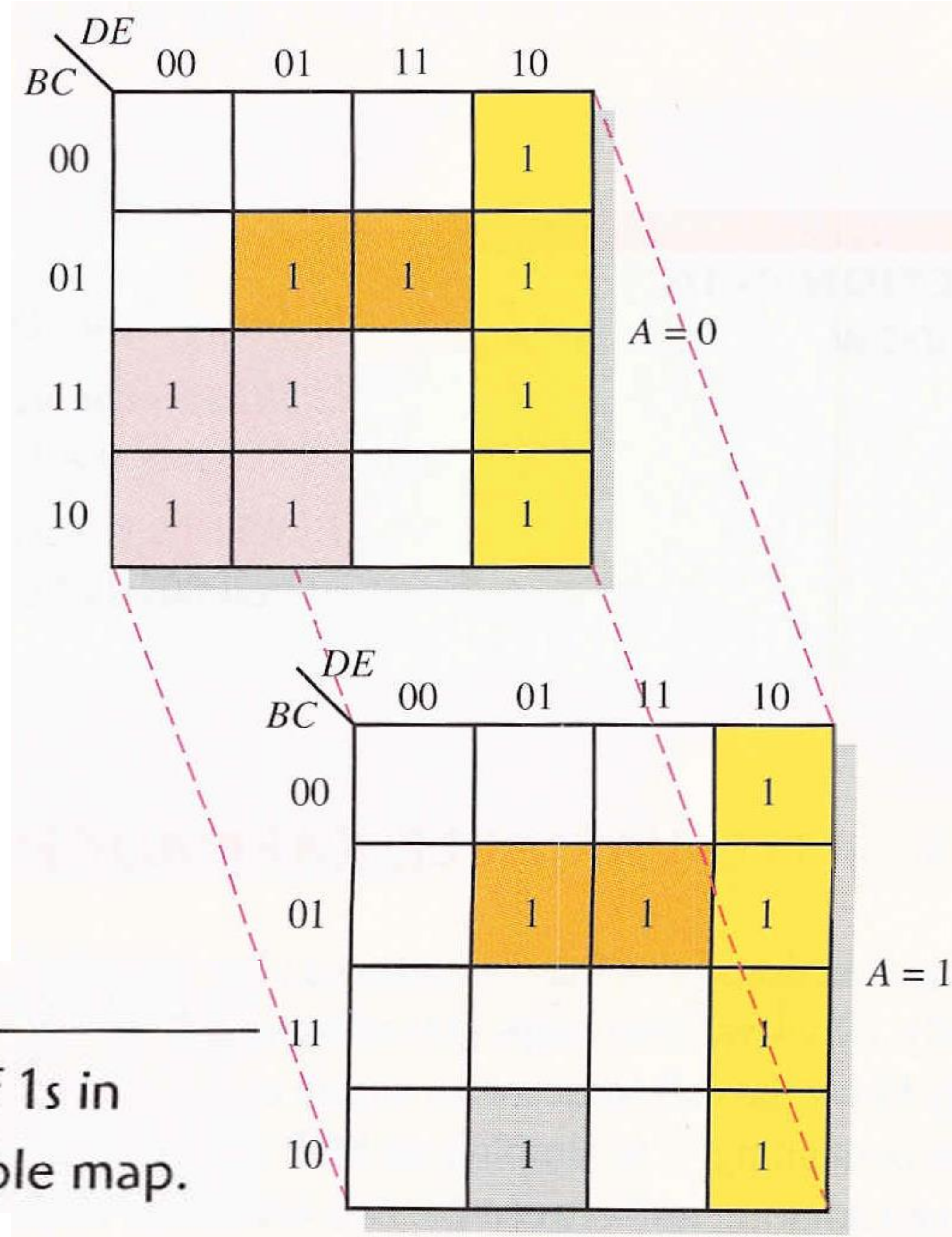


FIGURE 4–43

Illustration of groupings of 1s in adjacent cells of a 5-variable map.

- The term for the yellow group is $D\overline{E}$.
- The term for the orange group is $\overline{B}CE$.
- The term for the light red group is $\overline{A}B\overline{D}$.
- The term for the gray cell grouped with the red cell is $B\overline{C}\,\overline{D}E$.

Combining these terms into the simplified SOP expression yields

$$X = D\overline{E} + \overline{B}CE + \overline{A}B\overline{D} + B\overline{C}\,\overline{D}E$$

EXAMPLE 4–34

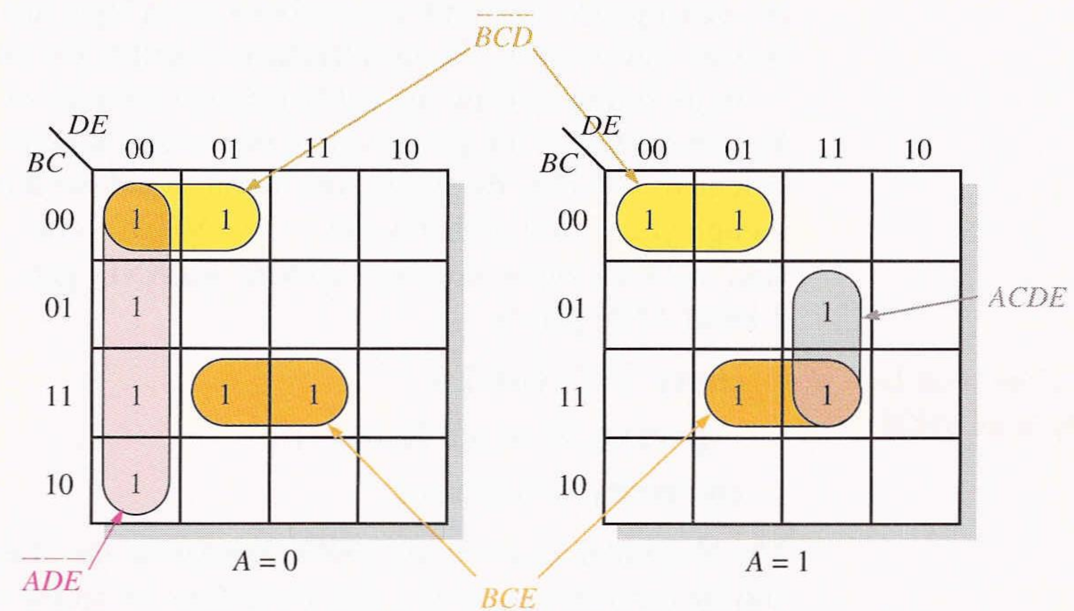
Use a Karnaugh map to minimize the following standard SOP 5-variable expression:

$$X = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + \overline{A}\,\overline{B}C\overline{D}\,\overline{E} + \overline{A}BC\overline{D}\,\overline{E} + \overline{A}B\overline{C}\,\overline{D}\,\overline{E} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}E + \overline{A}BC\overline{D}E$$
$$+ \overline{A}BCDE + A\overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + A\overline{B}\,\overline{C}\,\overline{D}E + ABC\overline{D}E + ABCDE + A\overline{B}CDE$$

Solution Map the SOP expression. Figure 4–44 shows the groupings and their corresponding terms. Combining the terms yields the following minimized SOP expression:

$$X + \overline{A}\,\overline{D}\,\overline{E} + \overline{B}\,\overline{C}\,\overline{D} + BCE + ACDE$$

FIGURE 4–44



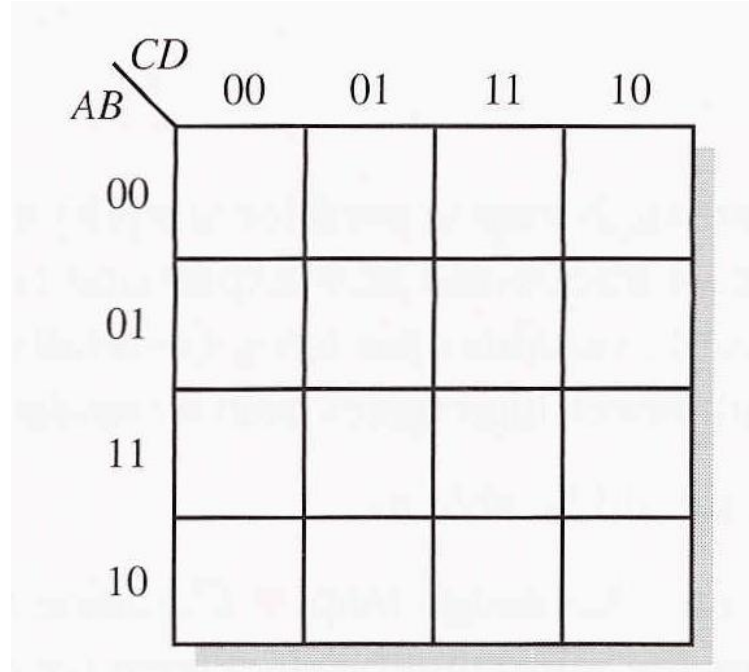
Related Problem Minimize the following expression:

$$Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + \overline{A}\,\overline{B}C\overline{D}\,\overline{E} + \overline{A}BC\overline{D}\,\overline{E} + \overline{A}B\overline{C}\,\overline{D}\,\overline{E} + A\overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + A\overline{B}C\overline{D}\,\overline{E} + ABC\overline{D}\,\overline{E} + AB\overline{C}\,\overline{D}\,\overline{E}$$
$$+ \overline{A}\,\overline{B}\,\overline{C}D\overline{E} + \overline{A}\,\overline{B}CD\overline{E} + \overline{A}BCD\overline{E} + \overline{A}B\overline{C}D\overline{E} + A\overline{B}\,\overline{C}D\overline{E} + A\overline{B}CD\overline{E} + ABCD\overline{E} + AB\overline{C}D\overline{E}$$

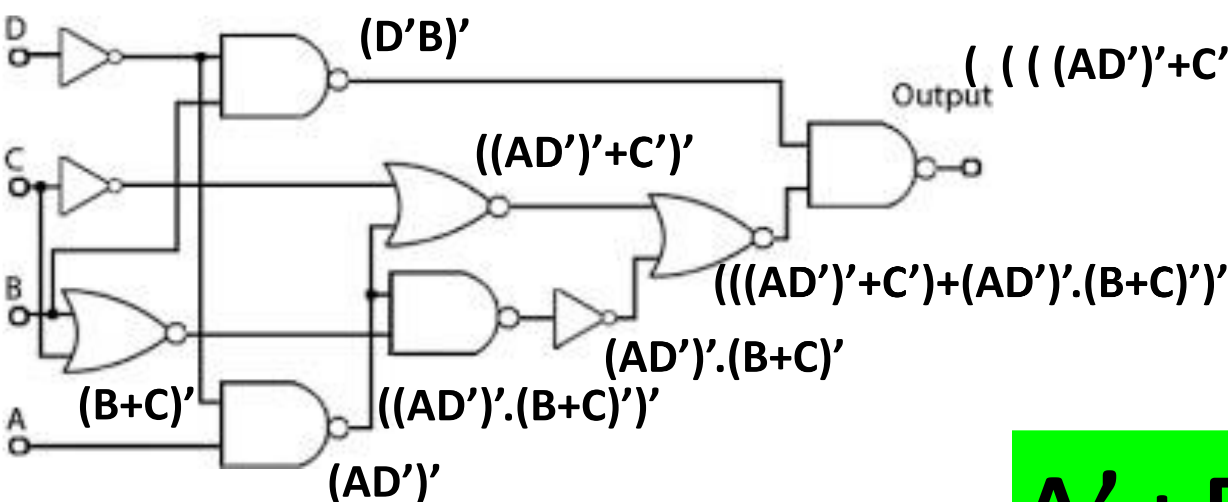
KARNAUGH MAPS

Exercise?

- Find the
  - SOP Expression,
  - Draw K-Map
  - Find the Minimized SOP Expression



Also Implement the Simplified Logic



((((AD')'+C')' + (AD')'.(B+C)')'. (D'B)')'

A'+B'+C+D'