CSI 4900: Honours Project Report

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1 Introduction

An Erdös-Selfridge-Spencer game consists of an attacker A, a defender D, and a game state S. We use p_i to represent a piece that is i levels away from the top of the board. In other words, a piece p_0, if not destroyed by the defender this turn, will attain tenure and provide a point for the attacker.

Definition 1.1. Let v be a value function, which maps a level i to a real value representing the value of a p_i piece. A value function is represented by an array $(w_0, w_1, w_2, ..., w_k)$ where:

$$v(i) = w_i.$$

Definition 1.2. Let S be a game state. At any point, the state can be described by a K dimensional vector $(n_0, n_1, n_2, ..., n_k)$ representing a multiset of pieces consisting of a_i pieces on the ith level. We define the value function applied to S as:

$$v(S) = \sum_{i=0}^{k} a_i v(i).$$

A defender would apply its value function to the two sets partitioned by the attackers, and it would destroy the set it deems to be more valuable.

Theorem 1.1. *The optimal value function for the defender is:*

$$v_*(i) = \frac{1}{2^{i+1}}. \tag{1}$$

1.1 Project Summary

We intend to investigate using reinforcement learning to train attackers to exploit the sub-optimality of biased defenders (nearsighted and farsighted).

2 Biased Defenders

We notice that the optimal value function satisfies this property that $v(i) = 2v(i+1)$. By deviating from this equality, we create sub-optimal defenders with biases that either favour pieces that are closer to the top of the board, or those that are at the bottom, depending on the direction of the inequality.

Definition 2.1. A nearsighted (myopic) defender is a defender whose value function satisfies the property, for all defined values of i:

$$v(i) > 2v(i+1). \tag{2}$$

A nearsighted defender disproportionally values pieces that are close to the top of the board.

Definition 2.2. A farsighted defender is a defender whose values function satisfies the property, for all defined values of i:

$$v(i) < 2v(i+1). \tag{3}$$

A farsighted defender disproportionally value pieces that are close to the bottom of the board.

Theorem 2.1. *When playing against an optimal defender, the optimal approach for the attacker is to try to make the value according to v_* of the two partitioned sets as close as possible.*

In order to maximize the number of pieces tenured, it is sufficient to maximize. at each turn, the value according to v_* of the surviving pieces. This idea can be proven in two steps:

1. show that against an optimal defender, the best score that can be achieved for starting position S is $\lfloor v_*(S) \rfloor$.
2. playing according to *Theorem 2.1* achieves that best score.

2.1 Nearsighted (Myopic) Defender

Theorem 2.2. *To maximize to real value of the surviving pieces on any round against a nearsighted defender, we follow the following algorithm:*

Algorithm 1: `Maximizing Real Value Playing Nearsighted Defender`

Input: a board position S, and the value function of a nearsighted defender v
Result: (S_1, S_2), such that $a + b = c + d$

if $|b| > |a|$ **then**
  exchange a and b ;
end
$c \leftarrow a + b$;
$z \leftarrow c - a$;
$d \leftarrow b - z$;
return (c, d) ;

Proof. □

2.2 Farsighted Defender

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Algorithm 2: `Maximizing Real Value Playing Nearsighted Defender`

Input: a board position P, and the value function of a nearsighted defender v
Result: (c, d), such that $a + b = c + d$

if $|b| > |a|$ **then**
  exchange a and b ;
end
$c \leftarrow a + b$;
$z \leftarrow c - a$;
$d \leftarrow b - z$;
return (c, d) ;

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