PARAMETER ESTIMATION USING CONTINUOUS KERNEL HOUGH TRANSFORM

J.Turán, D.Šiškovičová, P.Farkaš, P.Filo
Department of Electronics and Multimedia Telecommunications
University of Technology Košice
Park Komenského 13, 041 20 Košice, Slovakia

Abstract – The paper suggests a new approach to estimating parameters based on using Continuous Kernel Hough Transform. Definition of Continuous Kernel Hough Transform and basics of parameter estimation are given. This technique has been applied to the problem of parameter estimation for linear models. The experimental results confirm the efficiency of the proposed estimation algorithm also its comparison with the known technique of estimation – the Least Square Method.

Keywords: Hough transform, parameter estimation

INTRODUCTION

Automatization of many processes becomes moving force of powerful progress of industry and economy of the 20-th century. Modelling of reality using the computer algorithms become to priority with growth of the artificial intelligence. Machines with support of computational techniques and improved sensing systems are able to manage and carry out more complex and sophisticated processes, which can be managed only by people to present time. One of many fields, where these machines can be used, is estimation of parameters of various systems. In practice appear events, when the internal structure of system isn't known, but its performance is. By means of these information can be found out information about structure of the system and after, there is a possibility to describe it by any model. Process of parameter estimation is useful in approximation of this model's parameters [1].

Classical least squares techniques [1,2] have been widely used, despite the problems encountered when applied to noisy or occluded data sets. This paper presents new robust parameter estimation method. The Hough transform [3] was originally developed and applied to the problem of curve detection. More recently, the researchers have started to examine how the HT can be applied to parameter estimation and non-images data [4,5,6]. A new method – the parameter estimation method using Continuous Kernel Hough Transform (CTKHT) is described in proposed paper.

As first a definition of CTKHT is presented. This transform has been applied to the problem of parameter estimation of linear models. A comparison between the more traditional Least Square technique and method using CTKHT was made and result are presented.

CONTINUOUS KERNEL HOUGH TRANSFORM

The Hough transform [3,7,8] is used in image processing to extract geometric primitives from digital images. A comprehensive review of the HT has given by Illingworth and Kittler [7], who have placed heavy emphasis on the extraction of line segments from edge data in images.

A special transformation is applied to all the points and then a two-dimensional search is accomplished to find the maxima in the transform plane. The line parameters are r, the normal distance of the desired line from the origin, and the θ, the angle between x – axis and normal. Each data point (x,y) is mapped under the Hough Transform using Duda-Hart parameterisation [8] (Fig.1.) $r = x\cos\theta + y\sin\theta$ in the $r - \theta$ plane.

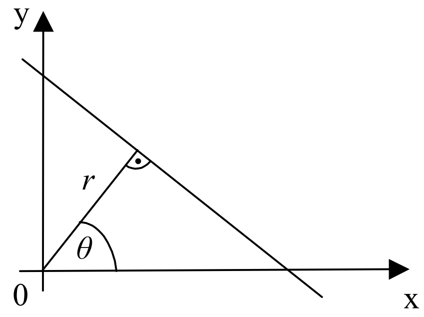

Fig.1 The normal parameterization of the Hough transform

To fit a straight line to a set of data points, both r and θ axes have to be quantized and hence a two-dimensional accumulator array must be constructed in the $r - \theta$ plane. The Hough Transform equation is applied to each point in the data set and the contents of all the cells in the transform plane that the corresponding curve passes through are incremented. Then, a search is made to locate the point with maxima values in the $r - \theta$ plane.

However, when curves in image are corrupted by noise the previous Hough Transform is not able to detect significant peaks. To solving this problem, a new modification of HT - Continuous Kernel Hough Transform (CKHT) [9,10] was defined:

$$H(r,\theta) = \sum_{m=1}^{M}\sum_{n=1}^{N} f(x_m, y_m)\frac{T}{T + (x_m\cos\theta + y_m\sin\theta - r)^2} \tag{1}$$

where $f(x_m,y_m)$ is an MxN binary digital image, $H(r, \theta)$ is corresponding parameter space and T is a constant, which determines sensitivity of HT. There is possibility to set together “wideness” of kernel selectivity and determine smoothness of approximation of the HT parameter space [9,10] (Fig.2).

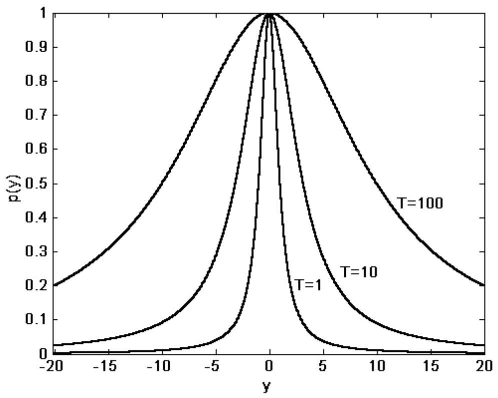

Fig.2 Influence of selectivity constant on continuous kernel of Hough transform

PARAMETER ESTIMATION

Hough transform has been generally applied to the problem of fitting curves to image data, but can be applied to non-image data as system's parameter estimation [4,5,6], too.

If system can be presented by following equation

$$y(t)+a_1 y(t-1)+\ldots+a_n y(t-n)=b_1 u(t-1)+\ldots+b_m u(t-m) \tag{2}$$

where $a_1, \ldots, a_n$ and $b_1, \ldots, b_m$ are real constant with a_n, $b_m \neq 0$, then input and output at the sample interval k is $u(k)$ and $y(k)$. Model (2) can be rewritten into

$$y(k)=-a_1 y(k-1)-\ldots-a_n y(k-n)+b_1 u(k-1)+\ldots+b_n u(k-n) \tag{3}$$

For convenience we use substitution $k = k+n$:

$$y(k+n)=-a_1 y(k+n-1)-\ldots-a_n y(k)+b_1 u(k+n-1)+\ldots+b_n u(k) \tag{4}$$

For N input-output pairs $\{[u(k), y(k); k=0,1,2,3,\ldots,N]\}$ can be created equation system:

$$\begin{aligned}
y(n+1)&=-a_1 y(n)-\ldots-a_n y(1)+b_1 u(n)+\ldots+b_n u(1)\\
y(n+2)&=-a_1 y(n+1)-\ldots-a_n y(2)+b_1 u(n+1)+\ldots+b_n u(2)\\
&\vdots\\
y(N)&=-a_1 y(N-1)-\ldots-a_n y(N-n)+b_1 u(N-1)+\ldots+b_n u(N-n)
\end{aligned} \tag{5}$$

If linear system is considered, a matrix formulation is also possible

$$\mathbf{Y}=\mathbf{\Phi}\boldsymbol{\beta} \tag{6}$$

where $\mathbf{Y}=\begin{bmatrix} y(n+1)\\ \vdots \\ y(N)\end{bmatrix}$, $\boldsymbol{\beta}=[a_1,\ldots,a_n,b_1,\ldots,b_n]^T$

and $\mathbf{\Phi}=\begin{bmatrix} -y(n) & \cdots & -y(1) & u(n) & \cdots & u(1)\\ \vdots & & \vdots & \vdots & & \vdots \\ -y(N-1) & \cdots & -y(N-n) & u(N-1) & \cdots & u(N-n)\end{bmatrix}$

Process of estimation can be summarized into following steps (Fig.3.):

For known input-output pairs has been created equation system (5) or (6) from which is selected P equation, where P is number of estimated parameters. Using Gauss elimination method is determined solution of created system. Above steps must be repeated until is obtained solution for whole input-output pairs. Result is set of $(N-(P-1))$ approximations of vector of parameters β, which present points in Hough transform parameter space. Using these results in equation of CTKHT (7) with input-output values can be obtained unambiguous value which subsistent to one point in parameter space. Each estimation model is described by differential equation, for which is necessary to create continuous kernel of Hough transform:

$$H_T(\beta)=\sum_{i=1}^{N}\frac{T}{T+\left(y_i(k)+a_1 y_i(k-1)+\ldots+a_n y_i(k-n)-b_0 u_i(k)-\ldots-b_n u_i(k-n)\right)^2} \tag{7}$$

Maximal value point presents starting point for searching in accumulators field. In neighbourhood of starting point is setting up the field of accumulators for covering desirable interval of values with adequate accurateness. By progressive substitution of centres of the cell of the accumulator field into CTKHT has been obtained unambiguous values of points from parameter space. Next, the maximum value point was seek out and checking of accumulators array start again, but with refined accuracy.

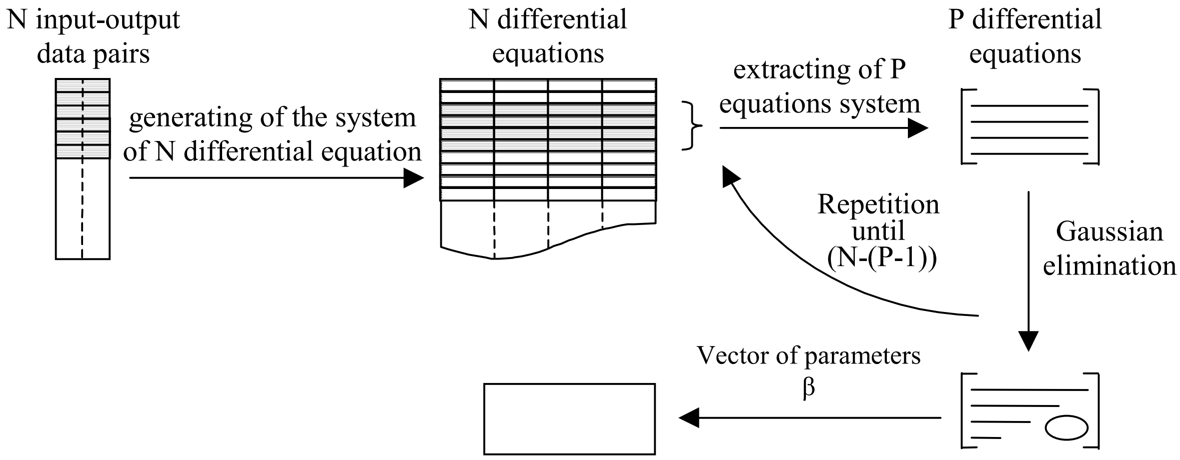

Fig.3. Preprocessing steps for obtain the vector of parameter β used to create HT parameter space

Result of the cell-by-cell searching process is the peak, which indicate the estimated value of the parameter. Quality evaluating is gone using simulation with verification data and computing of square of the output signals deflection

$$\hat{\boldsymbol{\beta}}_N = \left[\sum_{t=1}^{N} \boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right]^{-1} \sum_{t=1}^{N} \boldsymbol{\varphi}(t) y(t) \tag{8}$$

where $\boldsymbol{\beta}_N$ is vector of estimated parameters, *y(t)* output at given time and $\boldsymbol{\varphi}(t)=[-y(t-1) \ldots -y(t-n)u(t-1) \ldots u(t-m)]^T$. The value of (8) presents the ability of the system to express input measured data.

Advantage of using the CTHT in process of parameter estimation is possibility to set sensitivity of searching the accumulators field by choosing right parameters and possibility to set the kernel selectivity and determine smoothness of approximation of the HT parameter space.

EXPERIMENT AND RESULTS

Performance of proposed method of parameter estimation was tested in experiment on linear models with two parameters. Because of two estimated parameter, the following equation was created for build the Hough Transform parameter space:

$$H_T(a_1, b_0) = \sum_{i=1}^{N} \frac{T}{T + \left(y_i(k) + a_1 y_i(k-1) - b_0 u_i(k)\right)^2} \tag{9}$$

To test the resistance of method towards the influence of noise were used different kinds of noise. A comparison with least square technique was made.

For experiments was created set of input-output data, which output part *y(t)* was distorted by noise:

$$y_{\breve{S}}(t) = y(t) + \varepsilon(t) \tag{10}$$

In experiments were used three kinds of noise:

- Gauss white noise with zero mean value and variation σ

$$\varepsilon(t) = g(0,\sigma) \tag{11}$$

- Gauss white noise with unipolar impulse noise defined by

$$\varepsilon_\sigma(t) = \begin{cases} g(0,\boldsymbol{\sigma}) \\ 100.|g(0,\boldsymbol{\sigma})| & \text{if} \quad |g(0,\boldsymbol{\sigma})| \geq 2\boldsymbol{\sigma} \end{cases} \tag{12}$$

- Gauss white noise with bipolar impulse noise defined by following

$$\varepsilon_\sigma(t) = \begin{cases} g(0,\boldsymbol{\sigma}) \\ 100.g(0,\boldsymbol{\sigma}) & \text{if} \quad |g(0,\boldsymbol{\sigma})| \geq 2\boldsymbol{\sigma} \end{cases} \tag{13}$$

The linear model used for tests can be presented by differential equation:

$$y(t) = a_1.y(t-1) + b_0.u(t) + n_1.\varepsilon(k) + n_2.\varepsilon(k-1) + n_3.\varepsilon(k-2) \tag{14}$$

where n_1, n_2 a n_3 are constants.

In partial experiments was tested resistance presented methods of estimation for different value of noise variation (0,01; 0,05; 0,1 and 0,2) for every kind of above mentioned noise and for 500 and 2000 values of input-output data.

Results are presented in Tables No1. and No2.

Table 1: Experiment results – used 500 values of input-output data, original values of parameters a_1 = -1, b_1 = 1, HT is method using CTKHT, LMS is Least Square Method

			Noise variation			
			0,01	0,05	0,1	0,2
			Estimated values			
Gauss white noise	HT	a_1	-1,00007	-1,00035	-1,00071	-1,00141
		b_1	1,00021	1,00104	1,00208	1,00416
		e	1,613E-03	4,369E-02	1,951E-01	1,008E+00
	LSM	a_1	-1,01319	-1,01227	-1,01114	-1,00886
		b_1	1,94762	1,93860	1,92741	1,90536
		e	7,121E+05	3,112E+05	1,099E+05	1,311E+04
Gauss white noise with unipolar impulse noise	HT	a_1	-1,00027	-1,00035	-1,00071	-1,00141
		b_1	0,99989	1,00104	1,00208	1,00416
		e	2,453E-02	4,369E-02	1,951E-01	1,008E+00
	LSM	a_1	-0,99897	-0,94580	-0,88415	-0,76623
		b_1	1,72735	1,16377	0,79964	0,45295
		e	1,767E+01	2,748E+01	3,022E+01	3,218E+01
Gauss white noise with bipolar impulse noise	HT	a_1	-1,00027	-1,00035	-1,00071	-1,00141
		b_1	0,99989	1,00104	1,00208	1,00416
		e	2,453E-02	4,369E-02	1,951E-01	1,008E+00
	LSM	a_1	-0,99897	-0,94580	-0,88415	-0,76623
		b_1	1,72735	1,16377	0,79964	0,45295
		e	1,767E+01	2,748E+01	3,022E+01	3,218E+01

Table 2: Experiment results – used 2000 values of input-output data, original values of parameters $a_1 = -1$, $b_1 = 1$, HT is method using CTKHT, LMS is Least Square Method

			Noise variation			
			0,01	0,05	0,1	0,2
			Estimated values			
Gauss white noise	HT	a_1	-1,00000	-1,00001	-1,00001	-1,00002
		b_1	0,99977	0,99885	0,99770	0,99540
		e	8,085E-05	1,890E-03	7,578E-03	3,101E-02
	LSM	a_1	-1,00036	-1,00027	-1,00016	-0,99994
		b_1	0,53630	0,53632	0,53633	0,53637
		e	1,521E+02	1,705E+02	1,937E+02	2,395E+02
Gauss white noise with unipolar impulse noise	HT	a_1	-1,00007	-1,00001	-1,00001	-1,00002
		b_1	0,99781	0,99885	0,99770	0,99540
		e	7,516E-01	1,890E-03	7,578E-03	3,101E-02
	LSM	a_1	-0,99918	-0,99437	-0,98835	-0,97631
		b_1	0,53600	0,53480	0,53329	0,53023
		e	3,813E+02	7,904E+02	9,125E+02	9,812E+02
Gauss white noise with bipolar impulse noise	HT	a_1	-1,00007	-1,00001	-1,00001	-1,00002
		b_1	0,99781	0,99885	0,99770	0,99540
		e	7,516E-01	1,890E-03	7,578E-03	3,101E-02
	LSM	a_1	-0,99918	-0,99437	-0,98835	-0,97631
		b_1	0,53600	0,53480	0,53329	0,53023
		e	3,813E+02	7,904E+02	9,125E+02	9,812E+02

CONCLUSION

We have presented a new approach to parameter estimation using Continuous kernel Hough transform. This estimation method has been successfully applied to the problem of parameter estimation of linear systems. Comparing with the Least square technique, results that the method using CTKHT is more efficient. The Hough transform was found to be robust in the presence of each from used three kind of noise.

Currently the authors are work on experiments with higher number of parameters and testing the performance of the proposed method.

ACKNOWLEDGMENTS

This work was supported from the grant to the project COST 276 and VEGA grant No.1/7444/20.

References

[1] A.C. O. Mutambara: *Design and Analysis of Control systems*. CRC Press, New York, 1999.

[2] L. Ljung: *System identification; Theory for the user.* Prentice Hall, Englewood Cliffs, N.J., 1987.

[3] P.V.C. Hough: *Method and means for recognizing complex patterns.* US pattent 3069654, 1962.

[4] T. Flint, M. Ingleby, D. Morton: "A New Generalisation of the Hough Transform in Trend Analysis", in *Proceedings of IEEE Conference on Intelligent Control'92*, Glasgow, 1995, 261 – 267.

[5] V.S. Donchenko, V.M. Shpak: „Estimation of Autoregression Parameters on the Basis of the Hough Transform." *Izv. VUZ. Radioelektronika,* Vol.37, No.8, 1994, 38-44.

[6] M. Herdenson, M. Ingleby, D. Ford: „Robust System Identification Using the Hough Transform." *Proceedings of the I MECH E Part I Journal of Systems & Control Engineering*, Vol.211, 1997, 135-144.

[7] J. Illingworth, J. Kittler: "A survey of the Hough Transform." *Comput. Vision, Graphics, Image Process. 44(1)*, 1988, 87-116.

[8] R.D. Duda, P.E. Hart: *Pattern Recognition and Scene Analysis.* John Wiley, New York, 1973.

[9] J. Turán, P. Farkaš: „Line Fitting Using Hough-Like Procedure." *Radioengineering,* Vol.10, No.1, 2001, 25-30.

[10] P. Farkaš, J. Turán, D. Šiškovičová, K. Fazekaš, M. Benča: "New Invariant Feature Extractor Based on Hybrid Hough-Rapid Transform", in *Proceedings of DSP-MCOM 2001*, Košice, 2001, 94-97.

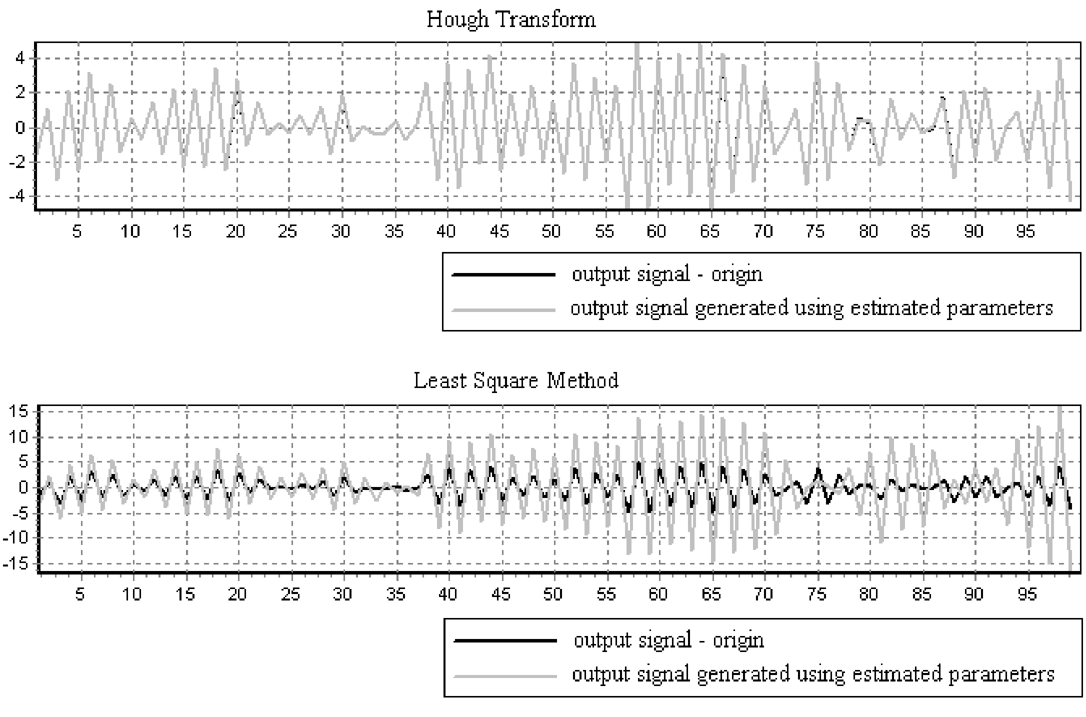

Fig.4 Example of estimation process outputs using CTKHT and Least Square Method (σ = 0.01, number of samples 500, used noise: Gauss white noise).