A. Algorithm

A.1. Conditions

- Complete knowledge of the structure of the labeled petri net
- No information about the initial marking

A.2. Input

- Observed sequence of labels (generated byt transition activity in the net)

A.3. Output : Minimum initial marking

A starting marking that allows :

1. Allows (at least one possible sequence) to fire the observed labels
2. Minimum total number of tokens

B. Notations & Definitions

Silent transitions

> Transitions whose firing cannot be observed

Petri net $N = (P, T, A, W)$

> Directed, Weighted, Bipartite Graph

Bipartite Graph

> In the mathematical field of graph theory, a bipartite graph (or bigraph) is a graph whose vertices can be divided into two disjoint and independent sets U and V, that is, every edge connects a vertex in U to one in V. Equivalently, a bipartite graph is a graph that does not contain any odd-length cycles.

$$N = (P, T, A, W)$$

- $P = \{p_1, p_2, \ldots, p_n\}$ → set of places (circles)
- $T = \{t_1, t_2, \ldots, t_m\}$ → set of transitions (bars)
- $A \subseteq (P \times T) \cup (T \times P)$ → set of arcs (from places to transitions and from transitions to places)
- $W : A \to \{w_1, w_2, \ldots, w_{\dim A}\}$ → weight function on the arcs (entry / exit price of a transition)

Marking $M : P \to \mathbb{N}$ → maps each *place* to a number of *tokens* (black dots)

Notation of the sum total of *tokens* $|M|$:

$$|M| = \sum_{i=1}^{n} M(p_i)$$

Enabled transition t

> if for all input places p, $M(p) \geqslant B^{-}(p, t) = B_{p \to t}$
> enabled transition in marking M written $M[t\rangle$

Notation	Meaning	Definition
$N = (P, T, A, W)$	Petri net	
$p = p_i$	Place	«Buffer» in the wrokflow, contains tokens
P	Finite set of places	
$t = t_i$	Transition	
T	Finite set of transitions	
A	Set of arcs	An arc links a place to a transition or a transition to a place
$M(p)$	Marking of a place	Number of tokens in the place
$\|M\|$	Marking of a net	Total number of tokens in the net
$B_{p \to t}$	Weight of the arc from a place to a transition	Number of tokens consumed from p to enter t
$B_{t \to p}$	Weight of the arc from a transition to a place	Number of tokens given to p when exiting t
$M\ [t\rangle$	Enabled transition	
$M\ [t\rangle M'$	Reachability	Marking M' is reachable from marking M via the firing of transition t

All transitions are observable (λ-free labeled petri net)

C. Problem

$$\underbrace{\min |M|}_{\text{minimum}}, \left(\underbrace{M[\sigma\rangle}_{\text{initial marking}} \quad \& \quad \underbrace{L(\sigma) = \omega}_{\text{that produces the given label sequence}} \right)$$

D. Complexity study

Polynomial time complexity in length of the observed label sequence.

Recursive