Summary Report

THE MODEL:

Considering the problem of the projectile, isolating the two components X and Y from the XY plane makes the problem much easier, since the change in the Y axis is independent of the change in X axis, and they are both a function of the time, T. Next, I worked out the patterns of change of X and Y in respect to T by plotting each of them against T. X had a linear relationship with T while Y formed the equation of a parabola. The problem of fitting a curve given a few points using linear regression has a trivial solution. But I realized that I needed to pick and train my hypothesis in such a way, that given only the first two points in time-index 0 and 1 respectively, I could predict the third point and so on. The problem, thus, reduces to the following: "If two points that are a particular time interval apart are given in a parabola, I am to predict the points in the consecutive time intervals."

While the laws of motion can be easily used to solve the simple problem of finding the co-ordinates of object thrown with an initial velocity of 10 ms^{-1} with an angle of 45 degrees at any given time, the point of this exercise is to use a machine learning algorithm to learn the motion of the projectile, and as such I refrained from using prior knowledge of Newton's laws of motion to derive my model for prediction.

Considering for a projectile P, at any time-index t_n, the coordinate of the object of unit mass is (x_n, y_n). Initially, I used a neural network and input t_n, x_{n-1} as features to train x_n and likewise, t_n, y_{n-1} as features to train y_n. While the NN would give moderately good result, a better model can be easily formulated. The graph for X,T gives a straight line passing through the origin. Since it is sufficient to find a third point given two previous points in fixed time intervals, and since the T intercept for the X-T graph is zero, I could rely on the simple hypothesis:

$$h(x_n) = ax_{n-1} + bx_{n-2}$$

The Y-T graph is polynomial, but displacement over a time period can be considered linear. The following expresses the hypothesis for Y:

$$h(y_n) = ay_{n-1} + by_{n-2} + c$$

This model is simple and gives accurate results without making too many assumptions. Moreover, a simple multivariate linear regression can be used to implement it.

EVALUATION OF THE MODEL

The model can be evaluated by using K-Fold Cross Validation. In this particular problem, I did not need to use cross-validation data to test the parameters of my model because I had worked out the mathematics involved on paper and ensured that these are all the parameters needed. In a more complex problem, or a problem where we do not know for sure what the order of the parameters are, cross-validation data can be used to address problem in the model such as overfitting. For my purposes, I can simple divide the dataset into train and test data and train my hypothesis and then check it against the test data to derive accuracy.

ASSUMPTIONS (AND CONSIDERATIONS FOR MOTION OTHER THAN PROJECTILE)

The model can predict the coordinates of projectiles at given time intervals launched at arbitrary angles and velocities equally well since it is only trained to predict the position of X and Y given two prior points and a fixed time interval and makes no assumptions on the velocity or angle of launching with the horizon. The model does make some assumptions, such as- the object in question is a projectile and follows a consistent path, there are no external forces acting on the body aside from g and there is no air pressure or resistance and lastly, gravity g is constant. The model also assumes that the data is free of noise or error.

If I was not told the object in question is a projectile, I would have to change my approach somewhat. Initially, I would start with a model that includes more parameters and include higher order variables as well as add a regularization term. I would need to check them against a cross-validation set to determine the order and parameter set that would give the best results. That would be an ideal model for the motion of an object we do not know the behavior of.

LITERATURE

I consulted the following books on Machine Learning: *Artificial Intelligence: A Modern Approach Third Edition* by Stuart Ruseel and Peter Norvig, and *Artificial Intelligence: A Guide to Intelligent Systems Second Edition* by Michael Negnevitsky. While my first instinct upon receiving the assignment was to consult existing work on the problem, or even relevant literature, I failed to find much that was relevant to my specific problem on the internet. There were a number of research papers that I had downloaded and started reading but realized they do not address the problem at hand. Examples of such papers are *Machine learning methods for predicting the outcome of hypervelocity impact events* (Article in Expert Systems with Applications 45:23-39 · March 2016), *Input Generalization in Delayed Reinforcement Learning: An Algorithm and Performance Comparisons.* (IJCAI. Vol. 91. 1991)

FUTURE WORK

Given more time, I would try to modify the model so it can work for a projectile without the assumptions that I had made for my model e.g. no air resistance and variable gravity. I am also curious on how to solve the problem for all paths rather than only projectiles. Out of personal interest, I would also like to implement the model using different ML algorithms, particularly Markov models and Reinforcement Learning.