

# CSC4140 Assignment 6

Computer Graphics

April 21, 2022

## Ray Tracing

This assignment is 10% of the total mark.

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This assignment represents my own work in accordance with University regulations.

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# 1 Overview

This project implement various methods of ray tracing, including generating rays, creating volum hierarchy (BVH) data structures, achieving direct and indirect illumination from multiple ray bounces to adaptive sampling. Rendering speed and effect have greatly improved through each step.

## 2 Implement and Results

### 2.1 Part 1: Ray Generation and Scene Intersection

Ray generation requires to build an image space and a camera space first. Their coordinates can easily be transformed with given formula. Then I use Samplers to sample some rays, once these rays intersect with objects, we render them by noramlis. To check intersection with sphere, we can simply substitue the ray formula into sphere formula and compare the result  $t$  with the parameters  $t$  of ray (see Figure 1). To check intersection with triangle, I use Moller Trumbore Algorithm (see Figure 2), a faster approach based on barycentric coordinate. The basic idea is still substituding formula. If  $0 < b_1 < 1, 0 < b_2 < 1, 0 < 1 - b_1 - b_2$ , then the hit point is inside the triangle.

Solve for intersection:

$$(\mathbf{o} + t \mathbf{d} - \mathbf{c})^2 - R^2 = 0$$

$$at^2 + bt + c = 0, \text{ where}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2(\mathbf{o} - \mathbf{c}) \cdot \mathbf{d}$$

$$c = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - R^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Figure 1: Intersection with sphere

$$\vec{\mathbf{O}} + t\vec{\mathbf{D}} = (1 - b_1 - b_2)\vec{\mathbf{P}}_0 + b_1\vec{\mathbf{P}}_1 + b_2\vec{\mathbf{P}}_2$$

**Where:**

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{\mathbf{S}}_1 \cdot \vec{\mathbf{E}}_1} \begin{bmatrix} \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_1 \cdot \vec{\mathbf{S}}_1 \\ \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{D}} \end{bmatrix}$$

Cramer 法則

**Cost = (1 div, 27 mul, 17 add)**

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{P}}_1 - \vec{\mathbf{P}}_0$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_0$$

$$\vec{\mathbf{S}} = \vec{\mathbf{O}} - \vec{\mathbf{P}}_0$$

$$\vec{\mathbf{S}}_1 = \vec{\mathbf{D}} \times \vec{\mathbf{E}}_2$$

$$\vec{\mathbf{S}}_2 = \vec{\mathbf{S}} \times \vec{\mathbf{E}}_1$$

Figure 2: Moller Trumbore Algorithm

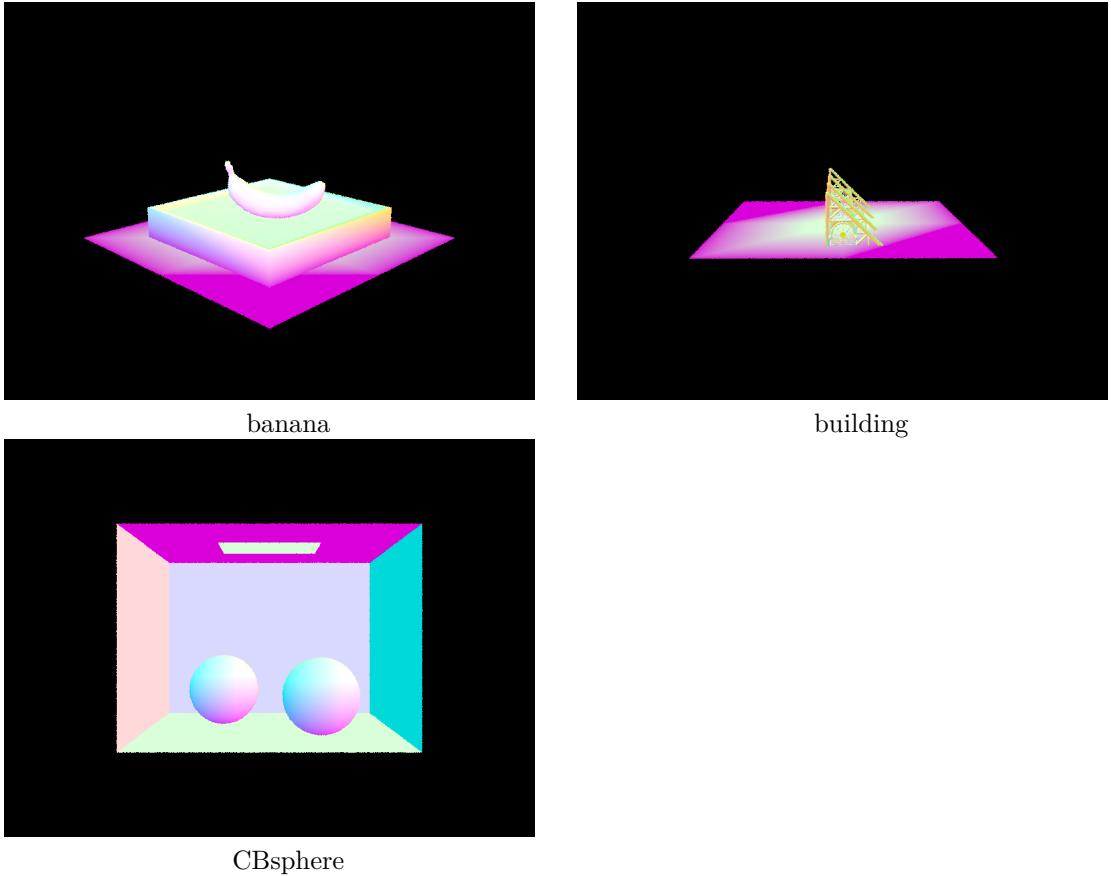


Figure 3: Normal shading for small .dae files

## 2.2 Part 2: Bounding Volume Hierarchy

This part divides the original primitives into subsection. I build a BVH tree. For every node, I find the average centroid all of its primitives. Primitives with centroids smaller than average are stored in the left child while the rest are stored in the right child. The comparison is based on the longest axis. Repeat this process until the primitives of each child are less than `max_leaf_size`. I replace the start and end iterator with a vector prim for primitives are not listed in centroid order and it is hard to assign iterator for children. However, I use dynamic memory, only leaf children have prim storing primitives. So the space complex is  $O(n)$ , the same level with given method.

**Extra Point: I implement BVH using a stack instead of recursively calling the function. This may avoid potentially stack overflow and speed up program.**

Figure 4 shows images with normal shading for a few large .dae files with BVH acceleration. For `cow.dae`, rendering times with and without BVH acceleration are 0.4s and 36s. For `maxplancky.dae`, rendering times with and without BVH acceleration are 0.58s and 390s. With BVH acceleration, once the ray does not intersect with one node, we can just cut off the whole subtree (i.e., no intersection with a set means definitely no intersection with its subset), which can save a lot of useless intersection checking.

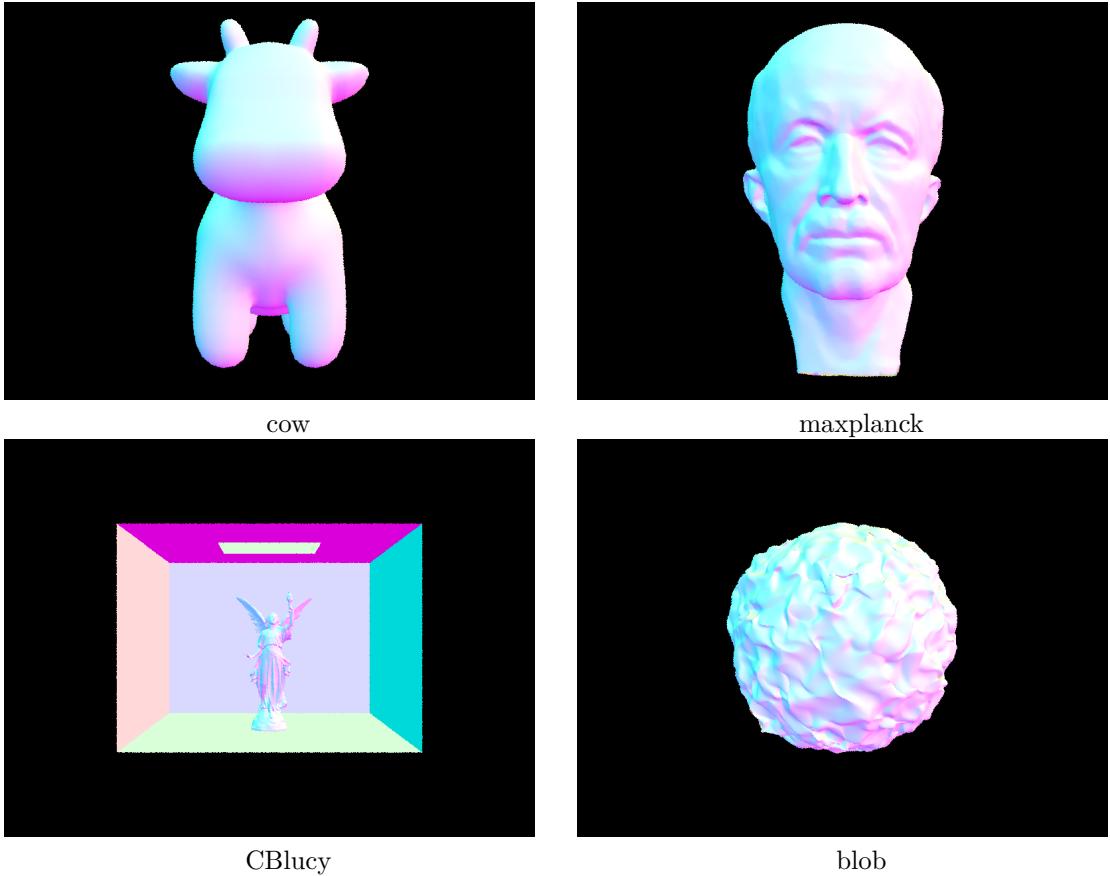


Figure 4: Normal shading for large .dae files

### 2.3 Part 3: Direct illumination

Hemisphere sampling:

1. Sample  $\omega_i$  around a hemisphere enclosing the hit point  $\text{hit\_p}$ .
2. Convert  $\omega_i$  to world space  $\omega_i_w$ . Create a ray with origin  $\text{hit\_p}$  and direction  $\omega_i_w$ . Set  $r.\min_t = \text{EPS\_F}$  and  $r.\max_t = \text{INF\_D}$  to alleviate numerical precision.
3. Find intersection by BVH.
4. Add the result of emission multiplied by BSDF, cosine between  $\omega_i$  and normal and  $(1/\text{PDF})$  to total spectrum and calculate the average value.

Light sampling:

1. Loop through the scene light of a given hit point  $\text{hit\_p}$ .
2. Convert  $\omega_i$  to object space  $\omega_{in}$ . Skip if  $\omega_{in}.z < 0$  for the sampled light point is behind surface.
3. Get automatically calculated  $\omega_i$ , PDF and  $\text{distToLight}$  when sampling light.
4. Create a ray with origin  $\text{hit\_p}$  and direction  $\omega_i$ . Set  $r.\min_t = \text{EPS\_F}$  and  $r.\max_t = \text{distToLight}$  to alleviate numerical precision.
5. Find intersection by BVH.

- If no surface is blocking, add the result of emsission multiplied by BSDF, cosine between wi and normal and (1/PDF) to total spectrum and calculate the average value.

From Figures below, we can see light sampling results in less noise. Hemisphere sampling sample around the hemisphere close to the hit point, but the incoming radiance is zero for most directions. Thus, by sampling around the light within the solid angle to the hit light can reduce this noise, especially using fewer sample points.

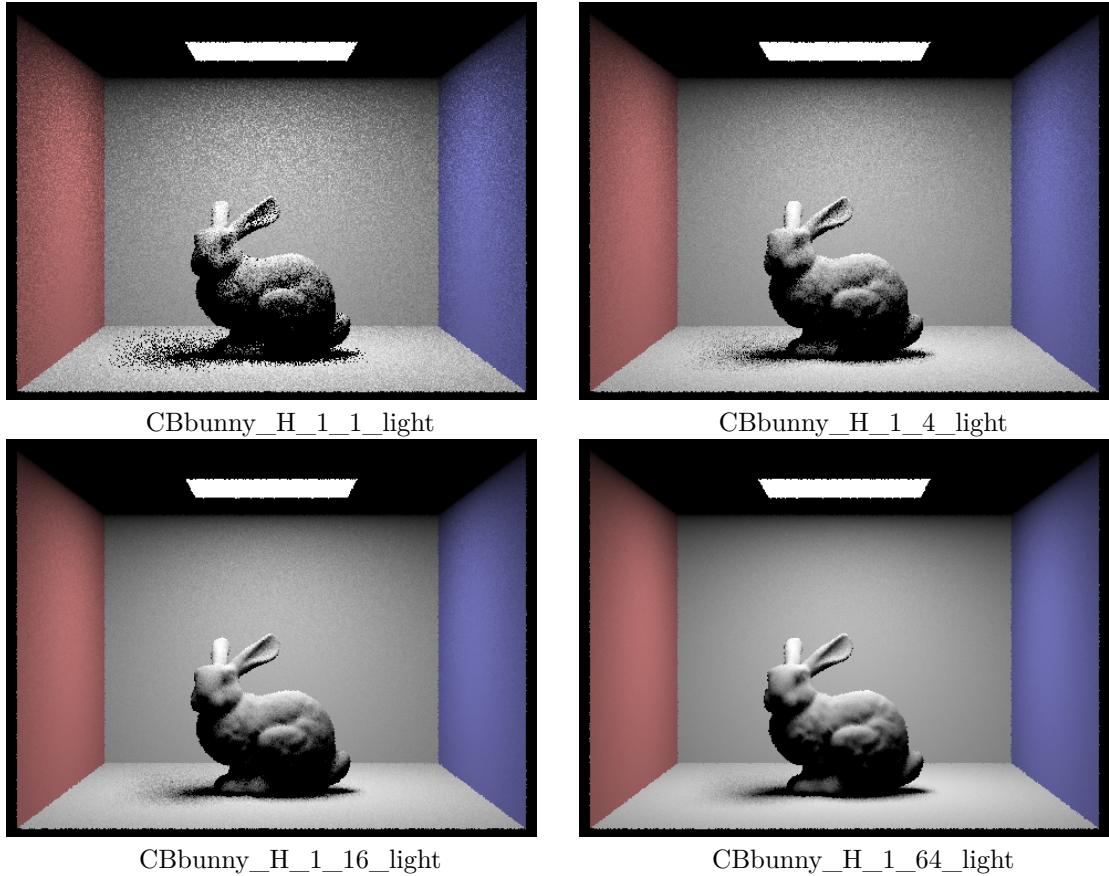
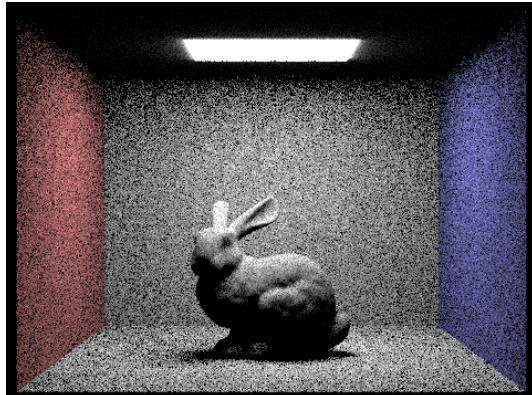
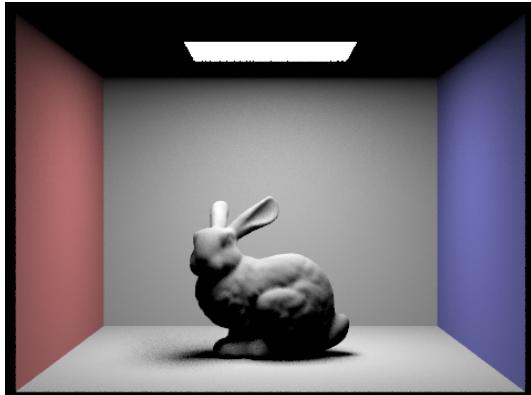


Figure 5: Rendering with 1, 4, 16, and 64 light rays  
`./pathtracer -t 8 -s 1 -l X -H -f CBbunny_H_1_X.png -r 480 360 ..../dae/sky/CBbunny.dae`



CBbunny\_H\_16\_8\_hemisphere



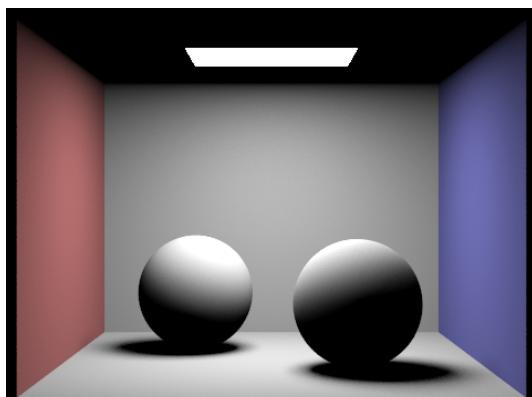
CBbunny\_H\_16\_8\_light

Figure 6: Comparison between hemisphere sampling and light sampling  
 command:./pathtracer -t 8 -s 16 -l 8 -H -f CBbunny\_H\_16\_8.png -r 480 360 ..../dae/sky/CB-bunny.dae

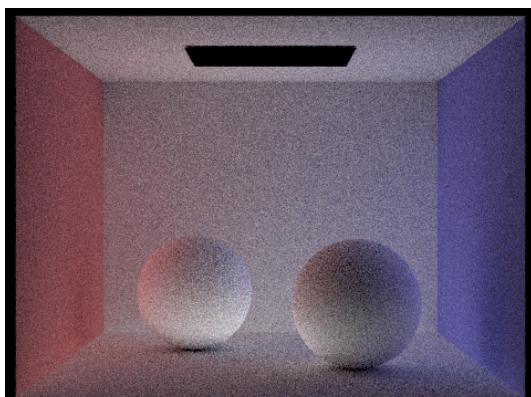
## 2.4 Part 4: Global illumination

Direct lighting includes zero\_bounce\_radiance and one\_bounce\_radiance. Indirect includes subsequent lighting bounce, i.e. at\_least\_one\_bounce\_lighting - one\_bounce\_radiance.

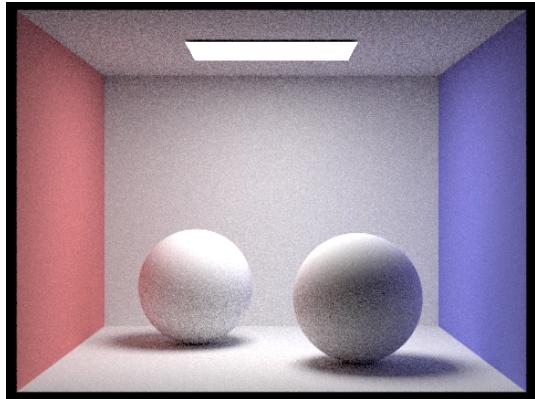
One\_bounce\_radiance can be either hemisphere sampling or light sampling. In at\_least\_one\_bounce\_lighting, I set the initial spectrum to be one\_bounce\_radiance, so it bounce at least once. Then I recursively call itself and decrease ray depth by one if ray depth is greater than one or if a Russian Roulette probability(cpdf) is satisfied. One thing to note is that when computing average spectrum, cpdf(0.6) is divided.



direct illumination



indirect illumination

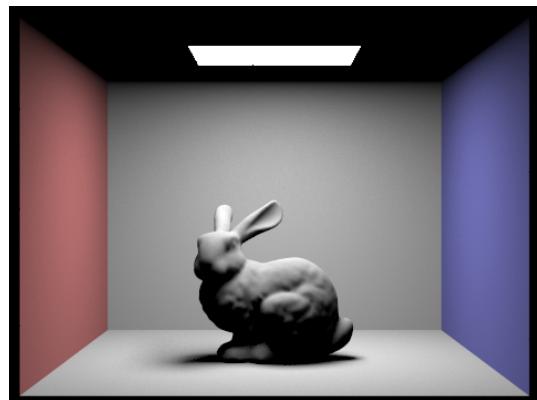


global illumination

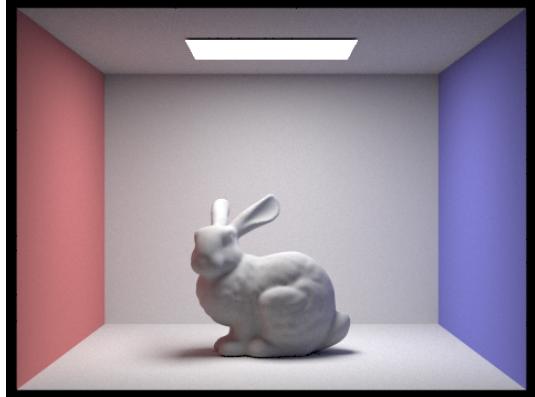
Figure 7: Comparsion between different illumination  
command:./pathtracer -t 8 -s 64 -l 16 -m 5 -r 480 360 -f X.png ..../dae/sky/CB-spheres\_lambertian.dae



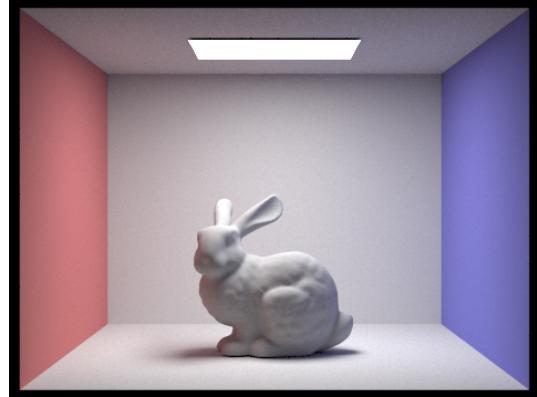
CBbunny\_m0\_



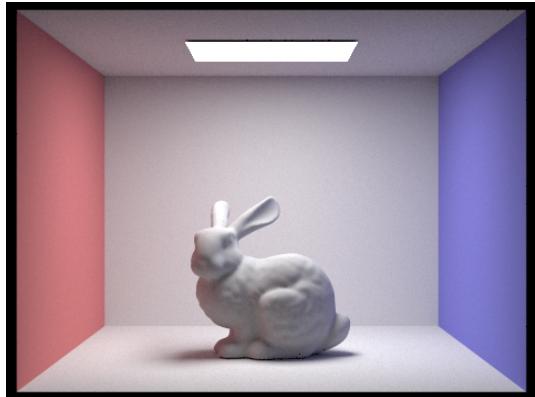
CBbunny\_m1\_



CBbunny\_m2\_

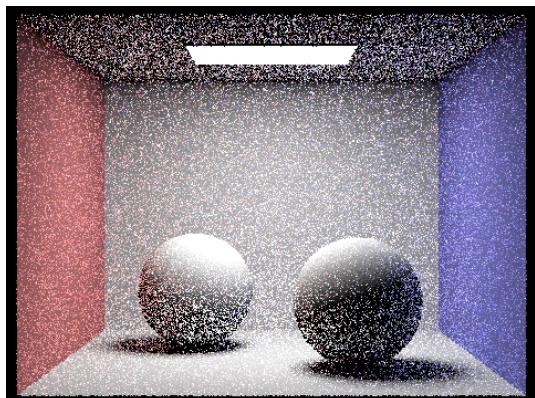


CBbunny\_m3\_

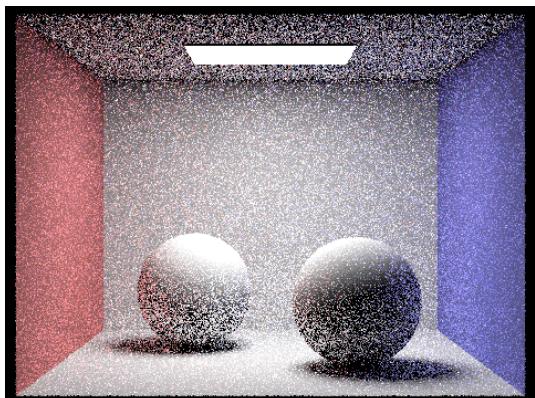


CBbunny\_m100\_

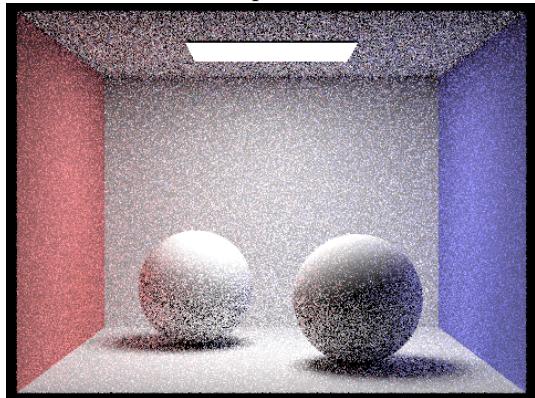
Figure 8: Comparsion between differenet max\_ray\_depth  
command:./pathtracer -t 8 -s 16 -l 8 -H -f -m X CBbunny\_mX\_.png -r 480 360 ..../dae/sky/CB-  
bunny.dae



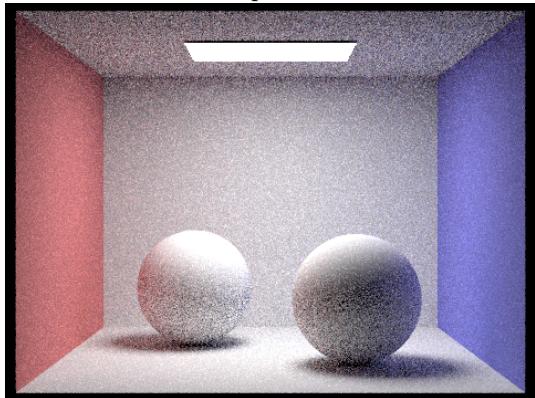
spheres1



spheres2



spheres4



spheres16

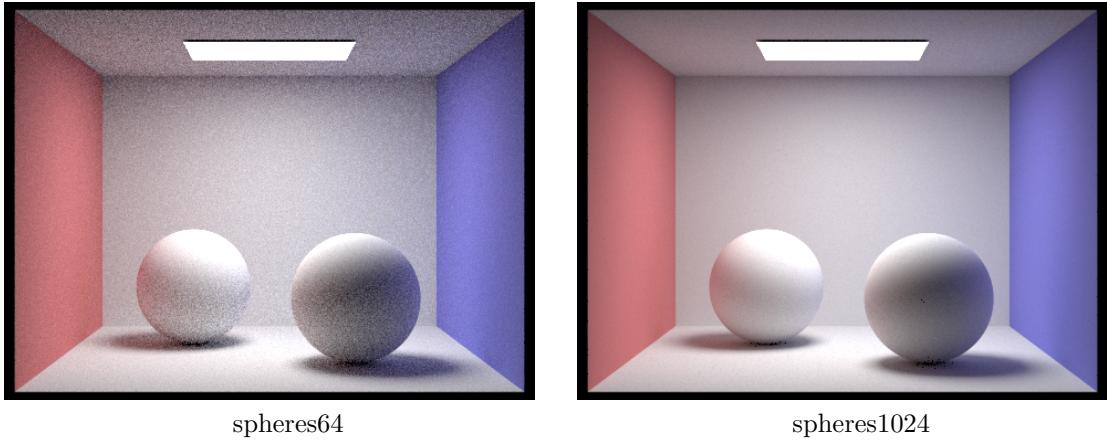


Figure 9: Comparsion between differenent sample per pixel  
 command:./pathtracer -t 8 -s X -l 4 -m 5 -r 480 360 -f spheresX.png ..../dae/sky/CB-spheres\_lambertian.dae

## 2.5 Part 5: Adaptive Sampling

To check the convergence, we simply check if  $I \leq \text{maxTolerance} \cdot \mu$  where  $\text{maxTolerance}=0.05$ ,  $I = 1.96 \cdot \frac{\sigma}{\sqrt{n}}$ ,  $\sigma$  is standard deviation and  $\mu$  is mean. To save time, we just check when  $n\%\text{samplesPerPatch} == 0$ . If condition is met, stop loop and update actual sample number.

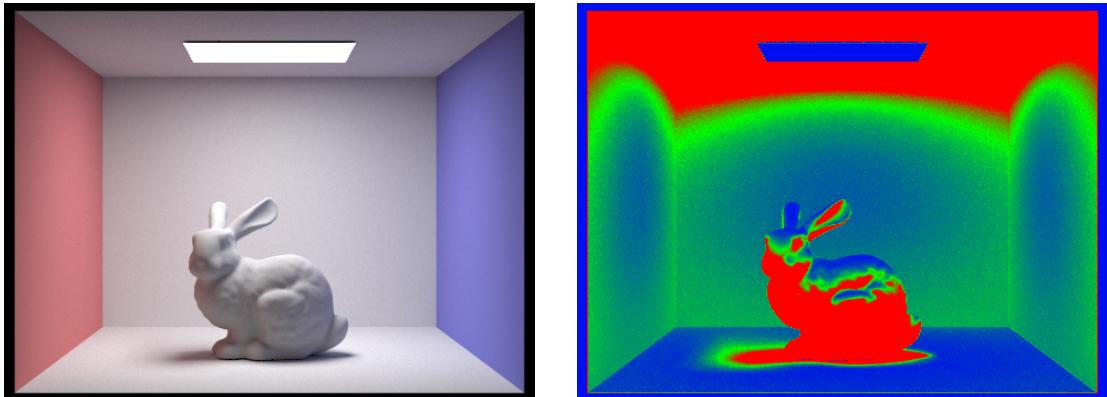


Figure 10: Adpative sampling  
 command:./pathtracer -t 8 -s 2048 -a 64 0.05 -l 1 -m 5 -r 480 360 -f bunny.png ..../dae/sky/CB-bunny.dae