Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

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Outline of talk

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Water Isotopes in Ice Cores

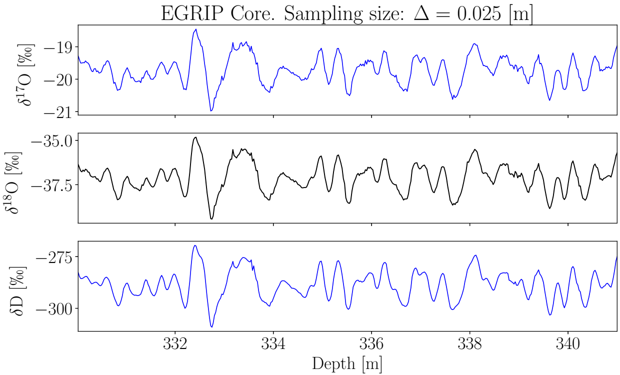

Figure: Examples of three water isotopes measured from the EGRIP core in Greenland.

Diffusion in Firn

- Fick's 2^{nd} law:

$$\frac{\partial \delta}{\partial t} = D(t)\frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

and

$$\mathcal{S}(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'}, \quad \text{the thinning function} \tag{4}$$

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Diffusion in Firn and Ice

With a total diffusion given as

$$\sigma^2_{\text{tot}} = [S(z)\sigma_{\text{firn}}]^2 + \sigma^2_{\text{ice}}(z) \tag{5}$$

Diffusion During Measurements

Measured diffusion length is actually

$$\hat{\sigma}_i^2 = \sigma_{\mathsf{firn}}^2 S(z) + \sigma_{\mathsf{ice}}^2 + \sigma_{\mathsf{dis}}^2 \tag{6}$$

with

$$\sigma_{\mathsf{dis}}^2 = \frac{2\Delta^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

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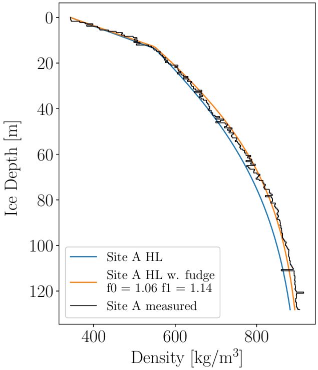
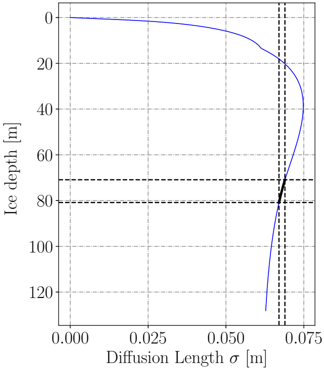
Community Firn Model



(a) Density-depth profiles based on analytical Herron-Langway model. Black is empirical data, blue is purely analytical fit and orange is fudged analytical fit



(b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.

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Example Data: Site A

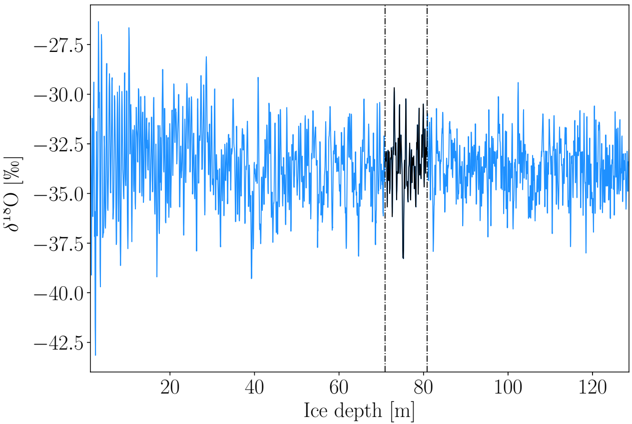


Figure: Example data from Alphabet Core drilled at site A near Crête.

Unevenly Sampled Data: Spline Interpolation

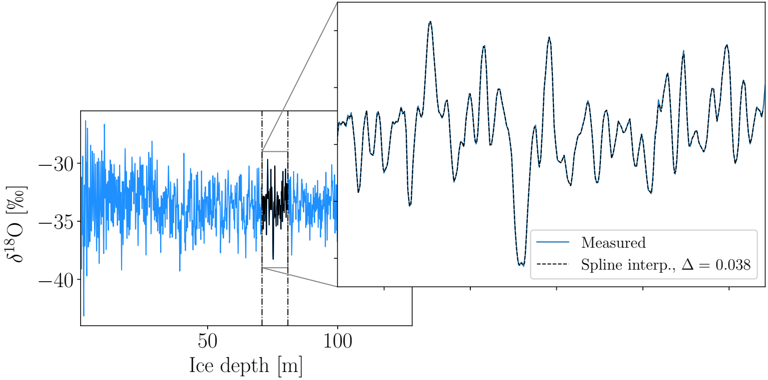
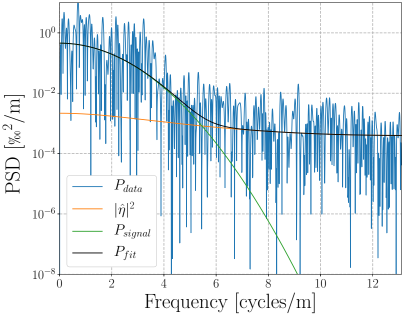


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.

Spectral Analysis with DCT



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_\eta^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\text{signal}} = P_0 e^{-k^2\sigma^2}$$

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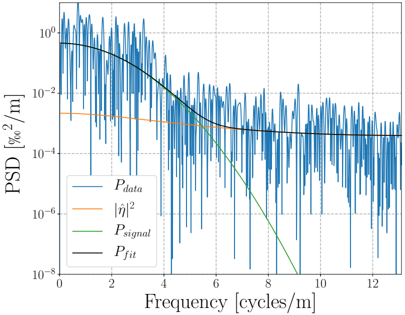

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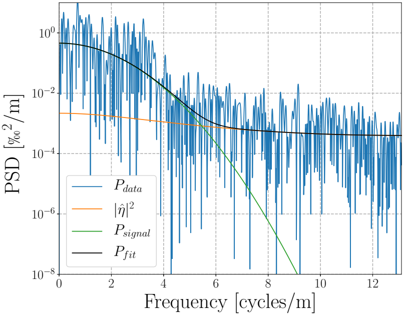

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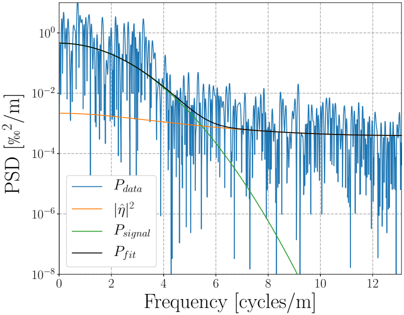

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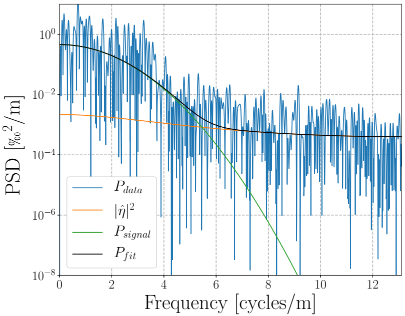

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Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1} \tag{8}$$

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

yielding a restoration filter as

$$\tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\text{meas}} \cdot \tilde{R} \tag{10}$$

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Filtering

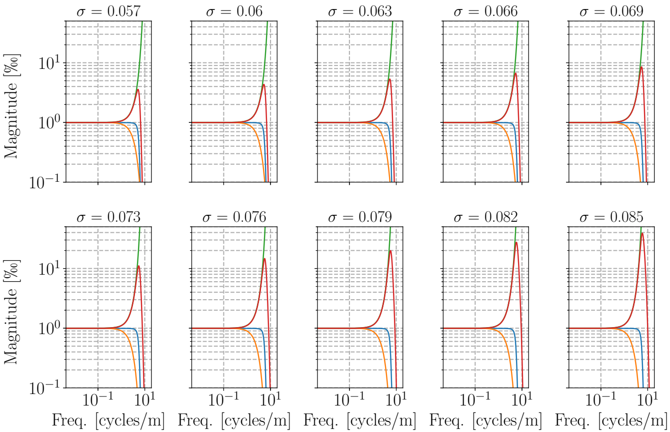

Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

Deconvolution

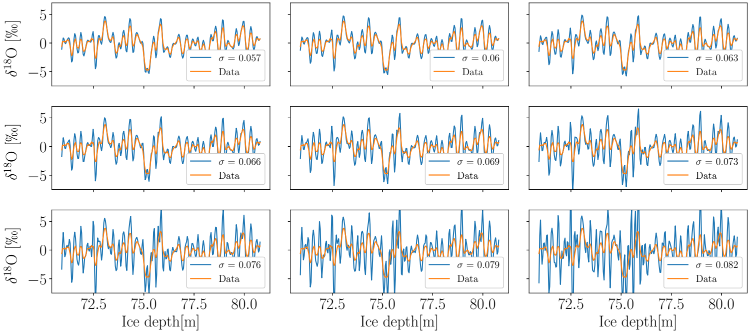

Figure: The estimated restored signal (blue) given diffusion length. Plotted along with original measured data (orange).

Enhanced Signal, Minimized Noise

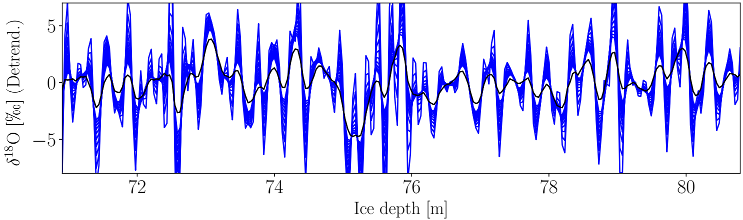

Figure: The original data plotted along with each estimate of the restored data, given diffusion length.

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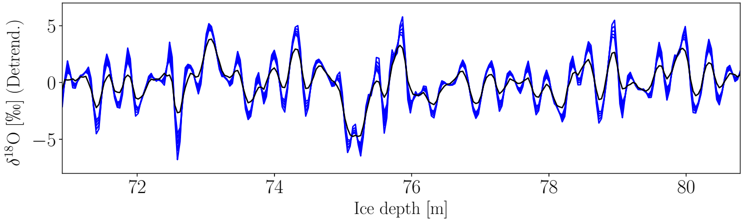

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DEP and ECM

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- **Dielectric Profiling (DEP)**

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Laki and Tambora

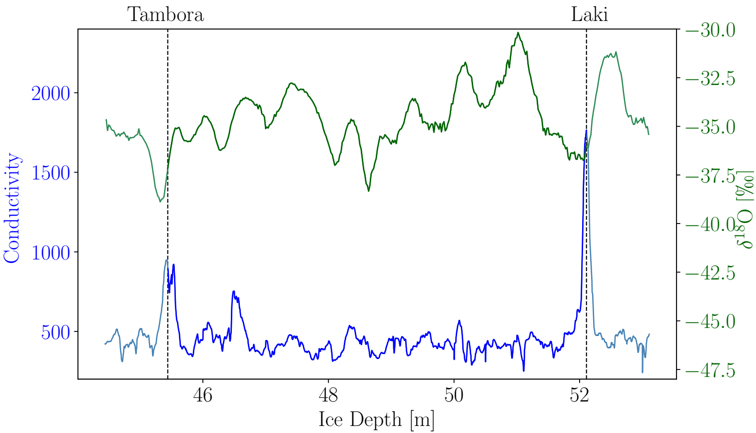

Figure: Example of volcanic horizons used for dating of cores, core B22.

Laki to Tambora
T. Quistgaard
So Far...
Water Isotopes
Diffusion and Densification
Back Diffusion
Volcanic Horizons
And now?
Detrend and Standardize
Peak Detection
Layer Counting Algorithm
Outlook
Layer Counting Algorithm, Cont.

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Estimating Cycle Length

Detrending with Moving Average

Standardize Amplitude

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MLP Neural Networks

Kalman Filtering

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- Counting by Peaks (Given diff. len. and est. cycle length)
- Counting by Layer Thickness

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Layer Counting Algorithm

- In Different Cores, Same (Known) Age
- Down entire (Dated) Core
- Combination

References I