Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

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Outline of talk

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Water Isotopes in Ice Cores

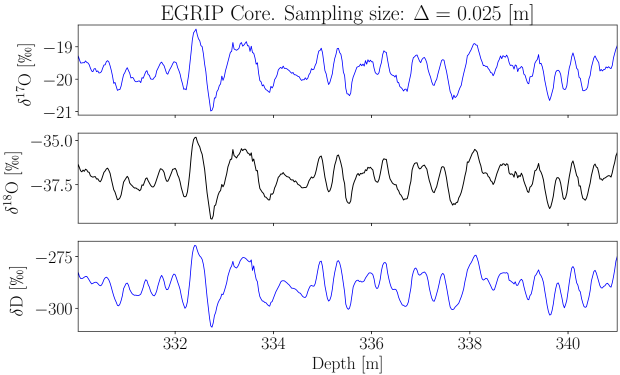

Figure: Examples of three water isotopes measured from the EGRIP core in Greenland.

Diffusion in Firn

- Fick's 2^{nd} law:

$$\frac{\partial \delta}{\partial t} = D(t)\frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with steady state solution

$$\delta_{\text{meas}}(z) = S(z)[\delta_{\text{init}}(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta_{\text{meas}}(z)$ is the measured signal, $\delta_{\text{init}}(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

and

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Layer Counting Algorithm, Cont.

Actual Total Diffusion

Total diffusion in ice and firn

$$\sigma_{\text{tot}}(z)^2 = [S(z)\sigma_{\text{firn}}(z)]^2 + \sigma_{\text{ice}}(z)^2 \tag{5}$$

Giving an actual measured diffusion length at z_i of

$$\sigma(z_i)^2 = \sigma_{\text{firn}}(z_i)^2\, S(z_i) + \sigma_{\text{ice}}(z_i)^2 + \sigma_{\text{dis}}(z_i)^2 \tag{6}$$

with

$$\sigma_{\text{dis}}(z_i)^2 = \frac{2\Delta(z_i)^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

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Example Data: Site A

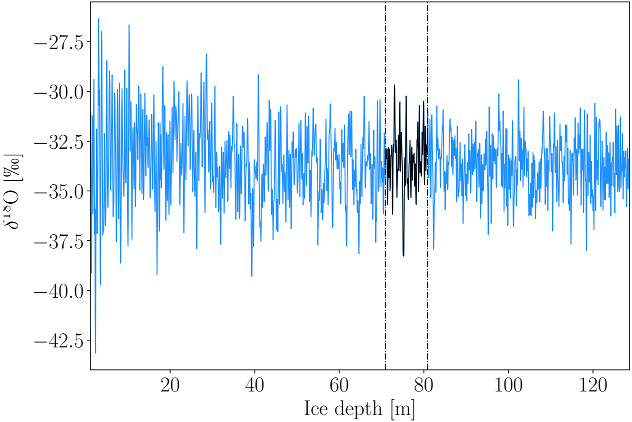


Figure: Example data from Alphabet Core drilled at site A near Crête.

Unevenly Sampled Data: Spline Interpolation

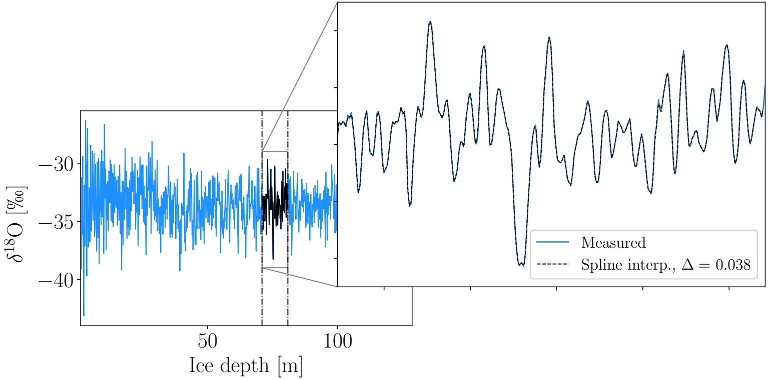
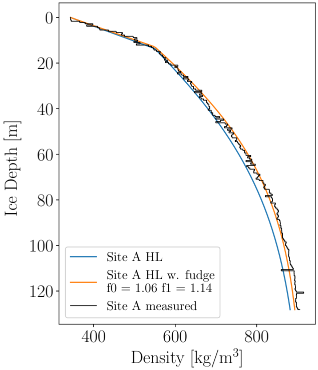
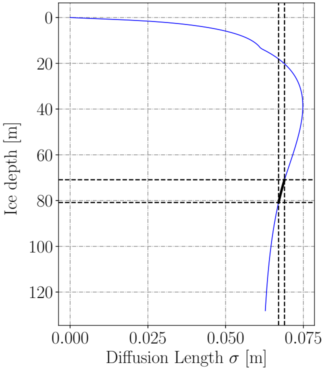


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.

Community Firn Model



(a) Density-depth profiles based on analytical Herron-Langway model. Black is empirical data, blue is purely analytical fit and orange is fudged analytical fit



(b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.

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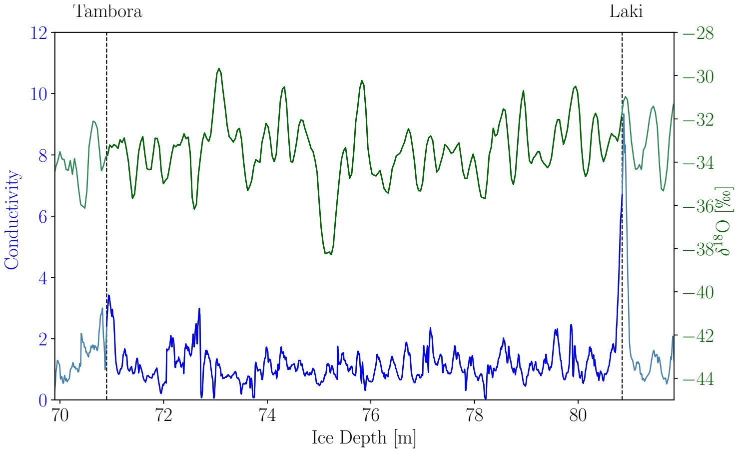
Laki and Tambora

- **Electrical Conductivity Measurements (ECM)**
- **Dielectric Profiling (DEP)**



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Laki and Tambora

(a) (1815) J. M. W. Turner: "*The Eruption of the Soufriére Mountains in the Island of St. Vincent, 30 April 1812*"

(b) (1828) J. M. W. Turner: "*Landcaster Sands*"

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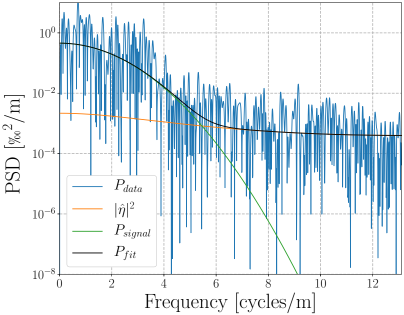
Spectral Analysis with DCT



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_\eta^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\text{signal}} = P_0 e^{-k^2\sigma^2}$$

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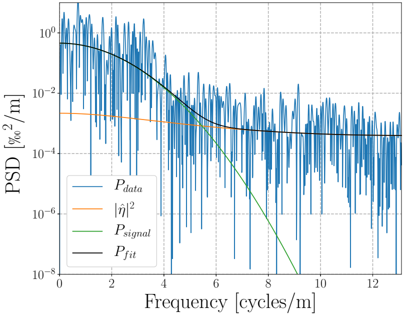

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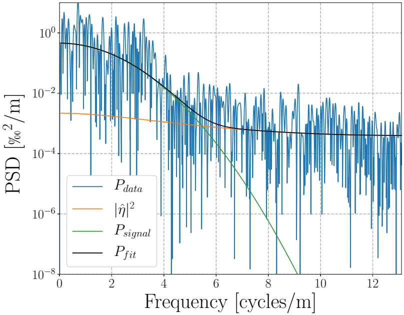

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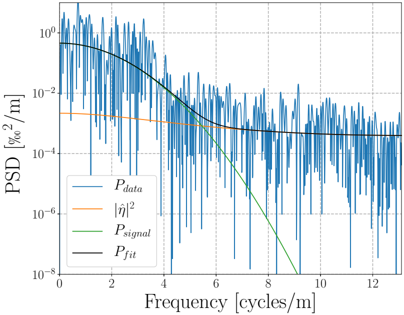

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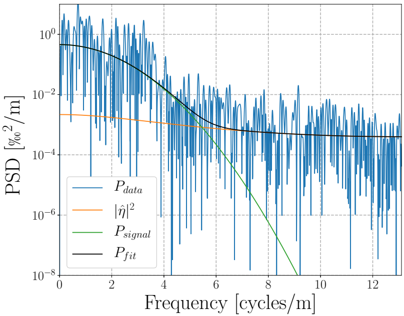

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Diffusion Lengths and Transfer Functions

$$\hat{\delta}_{\text{meas}} = \hat{\delta}_{\text{init}} \cdot \hat{M} \Leftrightarrow \hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{M}^{-1} \quad (8)$$

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\hat{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \quad (9)$$

yielding a restoration filter as

$$\hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{F} \cdot \hat{M}^{-1} = \hat{\delta}_{\text{meas}} \cdot \hat{R} \quad (10)$$

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Filtering

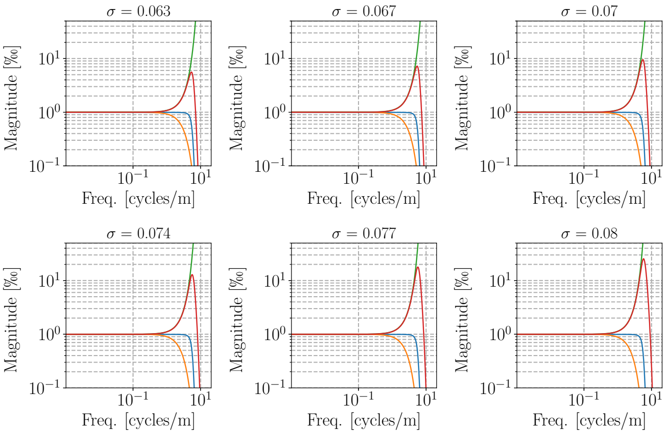

Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

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Deconvolution

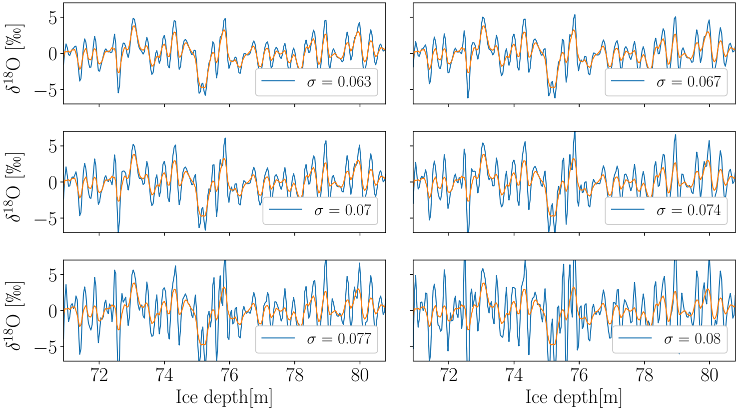

Figure: The estimated restored signal (blue) given diffusion length. Plotted along with original measured data (orange).

Enhanced Signal, Minimized Noise

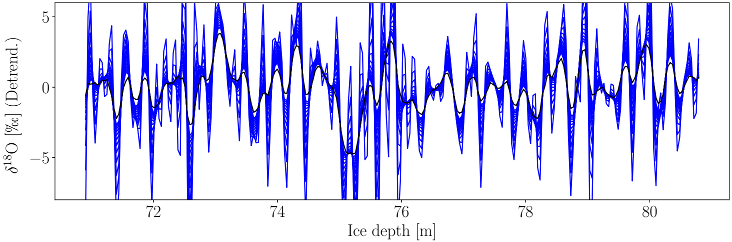

Figure: The original data plotted along with each estimate of the restored data with diffusion lengths ranging from 0.057 to 0.085.

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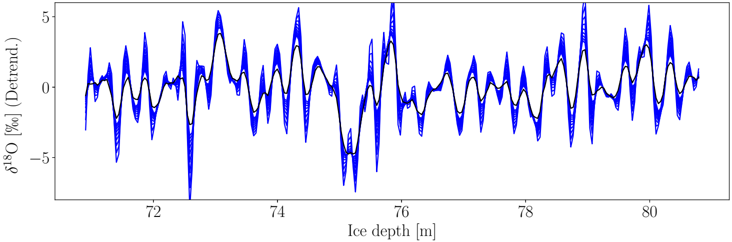

Figure: The original data plotted along with each estimate of the restored data with diffusion length $\sigma^2 < 0.075$.

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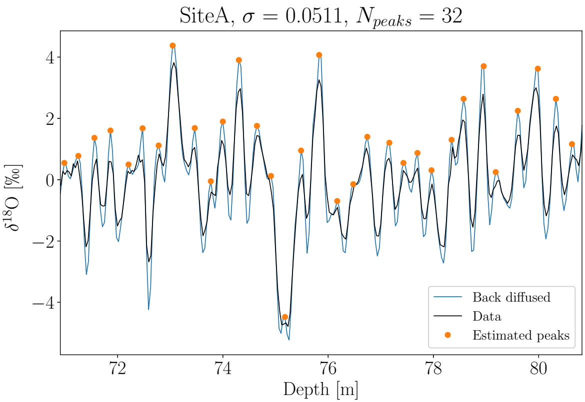
Peak Detection SciPy.signal.find_peaks



More Robust Peak Detection

- Signal representation by piecewise polynomial interpolation
- Least squares analytic fitting to cubic splines

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Methods

- Prior information: Typical annual cycle (noisy sine)
- Convolutional Neural Networks
- Kalman Filtering, MCMC or something else entirely

Hopeful Outcome

Prior to estimation:

- Diffusion and densification models
- Noisy sine signal

Outcome:

- Peak counting
- Dating by years
- Layer thickness approximation

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Further Work

- In Different Cores, Same (Known) Age
- Down entire (Dated) Core
- Combination

Thank you!

Any questions?