T. Quistgaa

### 50 Far...

Data
Volcanic Horizor

#### And now?

Layer Count Algorithm

### Outlook

Layer Counting Algorithm, Cont

# Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

Thea Quistgaard<sup>1</sup>

<sup>1</sup>University of Copenhagen

December 11, 2020

# Outline of talk

T. Quistgaa

Water Isoto

Data
Volcanic Horizon
Back Diffusion

And now

Layer Coun Algorithm

Outlook

Layer Counting Algorithm, Cont 1 So Far...

Water Isotopes

Data

Volcanic Horizons

**Back Diffusion** 

2 And now?

Peak Detection
Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.



## Table of Contents

T. Quistgaa

Water Isotopes

1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Peak Detection
Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.

Lavo

# Water Isotopes in Ice Cores

T. Quistgaa

So Far...

Water Isotopes

----Volcanic Horiz

And nou.

- - -

Layer Coun

Outlook

Layer Counting

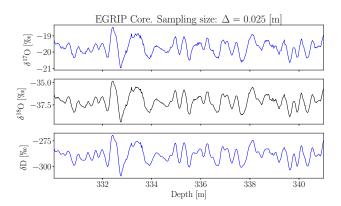


Figure: Examples of three water isotopes measured from the EGRIP core in Greenland.



### vvater isoto

Laki to Tambora

### T. Quistgaa

So Far...

#### Water Isotopes

Volcanic Horiz

Back Diffusion

#### And now

Peak Dete Layer Cour Algorithm

#### Outloo

Layer Counting Algorithm, Con

## Diffusion in Firn

• Fick's 2<sup>nd</sup> law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with steady state solution

$$\delta_{\text{meas}}(z) = S(z)[\delta_{\text{init}}(z) * \mathcal{G}(z)]$$
 (2)

where  $\delta_{\rm meas}(z)$  is the measured signal,  $\delta_{\rm init}(z)$  is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \quad (3)$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)

### T. Quistgaa

### 50 Far...

Water Isotopes

Volcanic Hori:

Back Diffusion

#### And now

Layer Coun

### Outlool

Layer Counting

### Diffusion in Firn

• Fick's 2<sup>nd</sup> law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with steady state solution

$$\delta_{\mathsf{meas}}(z) = S(z)[\delta_{\mathsf{init}}(z) * \mathcal{G}(z)] \tag{2}$$

where  $\delta_{\rm meas}(z)$  is the measured signal,  $\delta_{\rm init}(z)$  is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \quad (3)$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)

### Water Isotopes

### Laki to Tambora

### T. Quistgaa

### 30 Fai...

Water Isotopes

Volcanic Hori:

Back Diffusion

#### And now

Layer Coun

### Outlook

Layer Counting Algorithm, Cont

## Diffusion in Firn

• Fick's 2<sup>nd</sup> law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with steady state solution

$$\delta_{\text{meas}}(z) = S(z)[\delta_{\text{init}}(z) * \mathcal{G}(z)] \tag{2}$$

where  $\delta_{\rm meas}(z)$  is the measured signal,  $\delta_{\rm init}(z)$  is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \quad (3)$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (

### Water Isotopes

### Diffusion in Firn

Fick's 2<sup>nd</sup> law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

$$\delta_{\mathsf{meas}}(z) = S(z)[\delta_{\mathsf{init}}(z) * \mathcal{G}(z)] \tag{2}$$

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \quad (3)$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)

### Water Isotopes

Laki to Tambora

### T. Quistgaard

### Water Isotopes

Data

Back Diffusion

#### And now

Layer Coun Algorithm

### Outlool

Layer Counting Algorithm, Cont

## Diffusion in Firn

• Fick's 2<sup>nd</sup> law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with steady state solution

$$\delta_{\text{meas}}(z) = S(z)[\delta_{\text{init}}(z) * \mathcal{G}(z)]$$
 (2)

where  $\delta_{\rm meas}(z)$  is the measured signal,  $\delta_{\rm init}(z)$  is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \quad (3)$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'}, \quad \text{the thinning function} \tag{4}$$

### T. Quistgaard

### Water Isotopes

## Diffusion in Firn

Fick's 2<sup>nd</sup> law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

$$\delta_{\mathsf{meas}}(z) = S(z)[\delta_{\mathsf{init}}(z) * \mathcal{G}(z)] \tag{2}$$

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \quad (3)$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)



## Actual Total Diffusion

T. Quistgaa

Water Isotopes

vvater isoto

Volcanic Horiz

And now

Peak Detec Layer Count

Dutlook

Layer Counting

Total diffusion in ice and firn

$$\sigma_{\text{tot}}(z)^2 = [S(z)\sigma_{\text{firm}}(z)]^2 + \sigma_{\text{ice}}(z)^2$$
 (5)

Giving an actual measured diffusion length at  $z_i$  of

$$\sigma(z_i)^2 = \sigma_{\text{firn}}(z_i)^2 S(z_i) + \sigma_{\text{ice}}(z_i)^2 + \sigma_{\text{dis}}(z_i)^2$$
 (6)

with

$$\sigma_{\mathsf{dis}}(z_i)^2 = \frac{2\Delta(z_i)^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

## **Actual Total Diffusion**

T. Quistgaard

Water Isotopes

Volcanic Horizo

And now?

Peak Detect Layer Counti Algorithm

Outlook

Layer Counting Algorithm, Con Total diffusion in ice and firn

$$\sigma_{\text{tot}}(z)^2 = [S(z)\sigma_{\text{firm}}(z)]^2 + \sigma_{\text{ice}}(z)^2$$
 (5)

Giving an actual measured diffusion length at  $z_i$  of

$$\sigma(z_i)^2 = \sigma_{\mathsf{firn}}(z_i)^2 S(z_i) + \sigma_{\mathsf{ice}}(z_i)^2 + \sigma_{\mathsf{dis}}(z_i)^2 \tag{6}$$

with

$$\sigma_{\mathsf{dis}}(z_i)^2 = \frac{2\Delta(z_i)^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

# Table of Contents

T. Quistgaa

So Far...
Water Isotopes

Data Volcanic Horizo

Back Diffusion

Posk Detecti

Layer Cour Algorithm

Outlool

Layer Counting Algorithm, Cont So Far...

Water Isotopes

Data

Volcanic Horizons
Back Diffusion

2 And now?

Peak Detection
Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.

# Example Data: Site A

I. Quistgaar

So Far... Water Isotope

Data Volcanic Horizo

And now

Layer Count

Outlook

Layer Counting Algorithm, Con

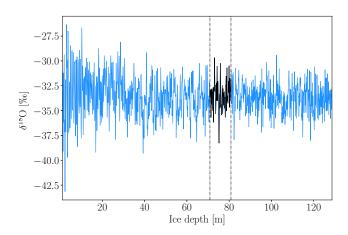


Figure: Example data from Alphabet Core drilled at site A near Crête.

So Far...

# Unevenly Sampled Data: Spline Interpolation

Data

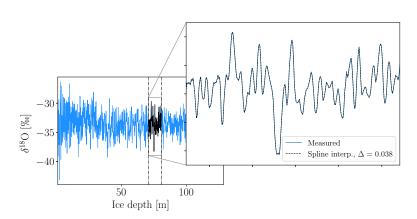


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.



# Community Firn Model

Laki to Tambora T. Quistgaard

So Far... Water Isotope

Data

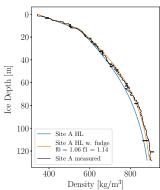
Volcanic Horizo Back Diffusion

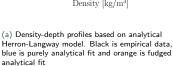
And now

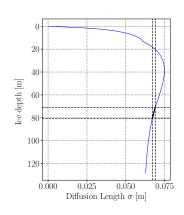
Peak Detect

Outlook

Layer Counting







(b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.



## Table of Contents

T. Quistgaa

Water Isoto

Volcanic Horizons

And now?

Peak Detec Layer Count Algorithm

Outlool

Layer Counting Algorithm, Cont 1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Peak Detection
Layer Counting Algorithm

3 Outlook

Layer Counting Algorithm, Cont.

So Far...

# Laki and Tambora

### T Quietes

So Far...

Volcanic Horizons

### Volcanic Horizo

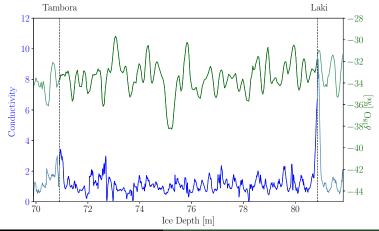
### And now

Layer Counti

### Outlook

Layer Counting

- Electrical Conductivity Measurements (ECM)
- Dielectric Profiling (DEP)



So Far...

## Laki and Tambora

T. Quistgaar

So Far...

Volcanic Horizons

Back Diffusio

#### And now

Peak Detecti

#### Outloo

Layer Counting Algorithm, Cont



(a) (1815) J. M. W. Turner: "The Eruption of the Soufriére Mountains in the Island of St. Vincent, 30 April 1812"



(b) (1828) J. M. W. Turner: "Landcaster Sands"

So Far...

# Table of Contents

Back Diffusion

So Far...

Back Diffusion

And now?

Outlook



# Spectral Analysis with DCT

T. Quistgaar

So Far...
Water Isoto

Water Isotop Data

Volcanic Hori

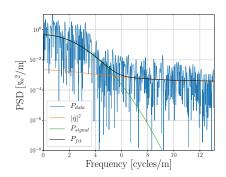
Back Diffusion

#### And now

Peak Detect

### Outlook

Layer Counting



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

# Spectral Analysis with DCT

T. Quistgaar

So Far...

Data Data

Back Diffusion

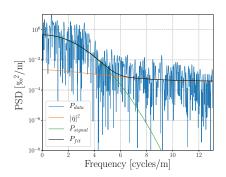
. .

Peak Detecti

Layer Countii

Outlook

Layer Counting



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

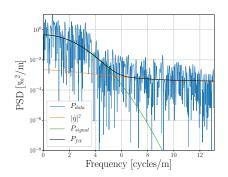
$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

So Far...

# Spectral Analysis with DCT

Back Diffusion



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

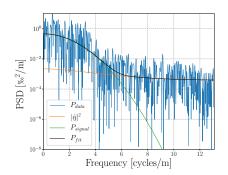
$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

So Far...

# Spectral Analysis with DCT

Back Diffusion



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

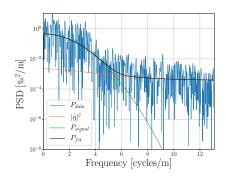
$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

So Far...

# Spectral Analysis with DCT

Back Diffusion



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 {\sigma^2 \over \sigma^2}}$$

### T. Quistgaa

Vater Isotope

Volcanic Hori

Back Diffusion

#### And now

Peak Detec Layer Coun Algorithm

#### utlook

Layer Counting Algorithm, Cont

# Diffusion Lengths and Transfer Functions

$$\hat{\delta}_{\text{meas}} = \hat{\delta}_{\text{init}} \cdot \hat{M} \Leftrightarrow \hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\hat{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{F} \cdot \hat{M}^{-1} = \hat{\delta}_{\text{meas}} \cdot \hat{R}$$
 (10)

### T. Quistgaard

Water Isotop

Volcanic Hori

Back Diffusion

#### And now

Peak Detect Layer Counti Algorithm

· · · ·

Outlook

Layer Counting

# Diffusion Lengths and Transfer Functions

$$\hat{\delta}_{\text{meas}} = \hat{\delta}_{\text{init}} \cdot \hat{M} \Leftrightarrow \hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\hat{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{F} \cdot \hat{M}^{-1} = \hat{\delta}_{\text{meas}} \cdot \hat{R}$$
 (10)

### T. Quistgaard

Nater Isotop Data

Back Diffusion

Dack Dillusio

### Alla llow

Layer Count Algorithm

#### Outlook

Layer Counting Algorithm, Cont

# Diffusion Lengths and Transfer Functions

$$\hat{\delta}_{\text{meas}} = \hat{\delta}_{\text{init}} \cdot \hat{M} \Leftrightarrow \hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\hat{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\hat{\delta}_{\mathsf{init}} = \hat{\delta}_{\mathsf{meas}} \cdot \hat{F} \cdot \hat{M}^{-1} = \hat{\delta}_{\mathsf{meas}} \cdot \hat{R} \tag{10}$$

### T. Quistgaa

Water Isotop Data

Back Diffusion

Back Dillusion

### Dark Datas

Layer Counting

#### utlook

Layer Counting Algorithm, Cont

# Diffusion Lengths and Transfer Functions

$$\hat{\delta}_{\text{meas}} = \hat{\delta}_{\text{init}} \cdot \hat{M} \Leftrightarrow \hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\hat{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\hat{\delta}_{\mathsf{init}} = \hat{\delta}_{\mathsf{meas}} \cdot \hat{F} \cdot \hat{M}^{-1} = \hat{\delta}_{\mathsf{meas}} \cdot \hat{R} \tag{10}$$

# **Filtering**

T. Quistgaa

So Far... Water Isotope

Volcanic Hori

Back Diffusion

And now

Layer Count

Outlook

Layer Counting Algorithm, Con

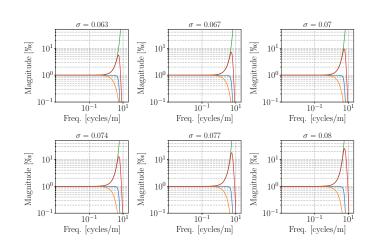


Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

### **Back Diffusion**

Laki to Tambora

# Deconvolution

T. Quistgaa

So Far... Water Isotope

Volcanic Hor

Back Diffusion

### And now

Layer Counti Algorithm

### Outlook

Layer Counting

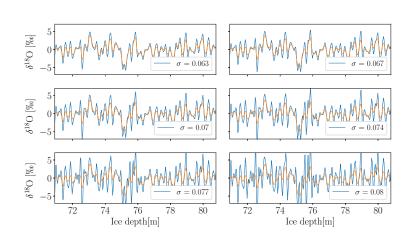


Figure: The estimated restored signal (blue) given diffusion length. Plotted along with original measured data (orange).



# Enhanced Signal, Minimized Noise

Back Diffusion

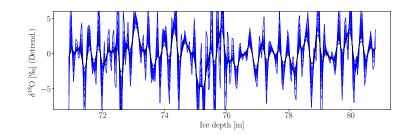


Figure: The original data plotted along with each estimate of the restored data with diffusion lengths ranging from 0.057 to 0.085.

### T. Quistgaa

So Far...

Vater Isotop

Volcanic Ho

Back Diffusion

#### And now

Peak Detect Layer Count

### Outlook

Layer Counting

# Enhanced Signal, Minimized Noise

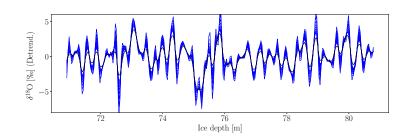


Figure: The original data plotted along with each estimate of the restored data with diffusion length  $\sigma^2 < 0.075$ .

So Far...

## Table of Contents

T. Quistga

Water Isoto

Volcanic Horizon
Back Diffusion

And now?

Peak Detection

Layer Counting

Outlook

Layer Counting Algorithm, Cont 1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Peak Detection

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.

So Far...

# Peak Detection SciPy.signal.find\_peaks

T. Quistgaa

So Far... Water Isoto

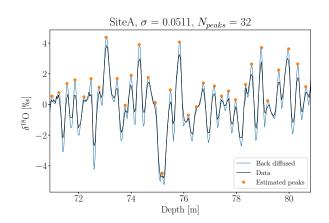
Volcanic Horizo

And nour

Peak Detection

. . .

Layer Counting



So Far...

## More Robust Peak Detection

۰ -

SO Far.

Data

/olcanic Hori

And now

Peak Detection

Layer Countil

Outlook

Layer Counting

- Signal representation by piecewise polynomial interpolation
- Least squares analytic fitting to cubic splines

So Far...

# Table of Contents

T. Quistgaa

30 Far..

Data Volcanic Horizo

And now?

Peak Detecti

Layer Counting Algorithm

Outlook

Layer Counting

1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Peak Detection

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.



# Methods

. .

Water Isot

Volcanic Horiz

. .

#### And now

Layer Counting Algorithm

### Outlook

Layer Counting

- Prior information: Typical annual cycle (noisy sine)
- Convolutional Neural Networks
- Kalman Filtering, MCMC or something else entirely

# Hopeful Outcome

T. Quistgaa

### Water Isote

Data Volcanic Horizo

### And now

Layer Counting
Algorithm

Algorithm

Layer Counting

### Prior to estimation:

- Diffusion and densification models
- Noisy sine signal

### Outcome:

- Peak counting
- Dating by years
- Layer thickness approximation

# Hopeful Outcome

T. Quistga

### So Far

Data
Volcanic Horizo

Back Diffusion

#### And now

Peak Detection Layer Counting Algorithm

### Outlook

Layer Counting

### Prior to estimation:

- Diffusion and densification models
- Noisy sine signal

### Outcome:

- Peak counting
- Dating by years
- Layer thickness approximation

## Table of Contents

T. Quistgaa

So Far...

Data

Back Diffusion

And now

Layer Coun

Outloo

Layer Counting Algorithm, Cont. So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Peak Detection
Layer Counting Algorithm

OutlookLayer Counting Algorithm, Cont.

## Further Work

i. Quistga

So Far..

Data

/olcanic Horiz

Back Diffusio

#### And now

Layer Coun

Outlook

Layer Counting Algorithm, Cont.

- In Different Cores, Same (Known) Age
- Down entire (Dated) Core
- Combination

And now?

Outlook

Layer Counting Algorithm, Cont.

Laki to Tambora

# Thank you!

T. Quistga

,, , , , ,

Water Isoto

Volcanic Hor

Back Diffusio

And now

Peak Detectio

Jutlook

Layer Counting Algorithm. Cont. Any questions?