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Signal Restoration and Pattern Recognition in Ultra High Resolution Volcanic and Isotopic Signals

Thea Quistgaard

Supervisors: Vasileios Gkinis & James Avery

University of Copenhagen

September 9, 2021

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Figure: J. M. W. Turner: The Eruption of the Soufriere Mountains in the Island of St Vincent, 30 April 1812, 1815

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Water Isotopes in Greenlandic Ice Cores

$$\delta^{18}{\rm O}=\frac{^{18}R_{sample}}{^{18}R_{reference}}-1$$
 , where $^{18}R=\frac{n_{18}{\rm O}}{n_{16}{\rm O}}$

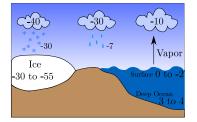


Figure: Fractionation through evaporation, transportation and precipitation along with typical water isotopic ratios in [‰]

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Water Isotopes in Greenlandic Ice Cores: Crête

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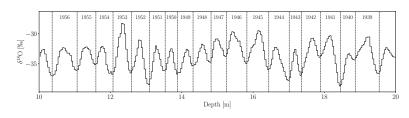


Figure: Ten meters of the top of Cretê ice core, with identification and dating of 19 annual layers. Dating as from https:

//www.iceandclimate.nbi.ku.dk/research/strat_dating/annual_layer_count/ice_core_dating/

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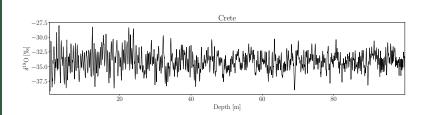


Figure: 100 meters of the Crête ice core, with diffusion visible.

- Diffusion: Displacement of water isotopes in the firn column
- Parameter of interest: Diffusion length, σ
- σ dependent on temperature T

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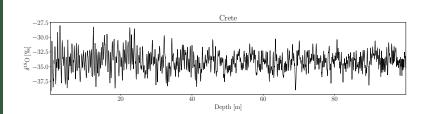


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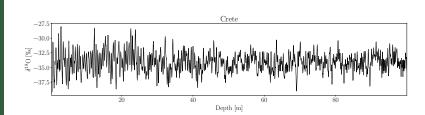


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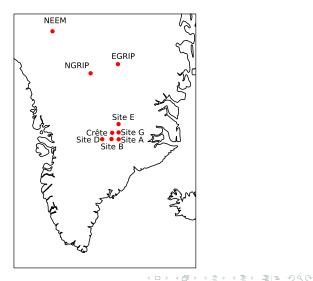
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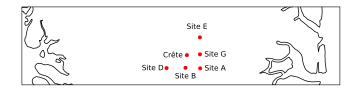


Figure: Map of spatial locations of the shallow Alphabet cores and deep core Crête analyzed in this thesis.

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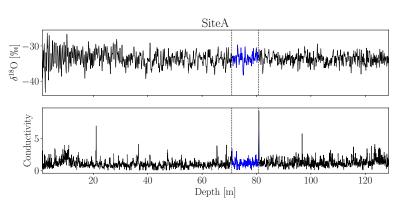


Figure: Site A, full $\delta^{18}\mathrm{O}$ and ECM data series.

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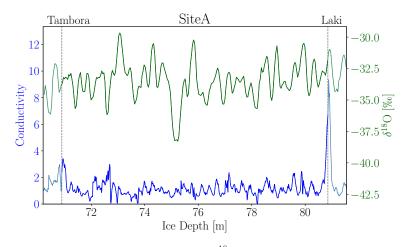


Figure: Site A, Laki to Tambora δ^{18} O and ECM data series.

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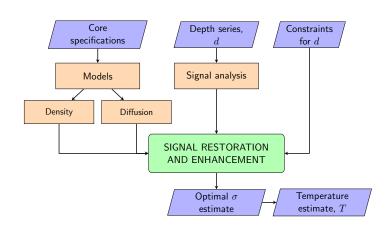
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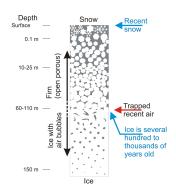


Figure: Illustration of densification process in firn column, from snow to glacial ice. Image source Blunier, T., and J. Schwander (2000), and https://www.iceandclimate.nbi.ku.dk/research/drill_analysing/cutting_and_analysing_ice_cores/analysing_reses/firn_zone/.

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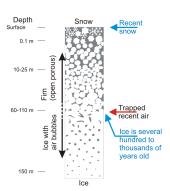


Figure: Illustration of densification process in firm column, from snow to glacial ice. Image source Blunier, T., and J. Schwander (2000), and https: //www.iceandclimate.nbi.ku.dk/research/ drill_analysing/cutting_and_analysing_ice_ cores/analysing_gasses/firn_zone/.

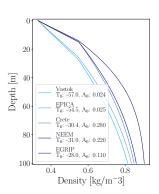


Figure: Density profile examples given five different initial conditions representing present day conditions at the five different ice core locations. Temperature. T_0 , is in ${}^{\circ}C$ and accumulation, A_0 , is in meter of water equivalent per year.

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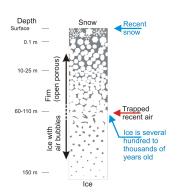


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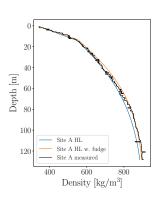


Figure: Density profiles from ice core Site A near Crête. Both purely modelled profile and profile fitted to the inputted depth-density measurements are presented. Computed through method developed in [M. M. Herron and C. C. Langway, 1980].

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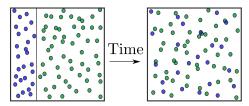


Figure: Example of diffusion over time, where two materials are mixed.

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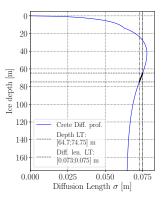


Figure: Diffusion length profile for ice core Crête.

- $\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)]$
- $\delta(z)$: Measured isotopic signal
- $\delta'(z)$: Initial isotopic signal
- $\mathcal{G}(z)$: Gaussian filter dependent on depth and diffusion length
- S(z): Thinning function

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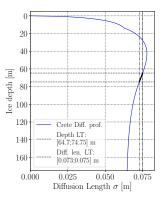


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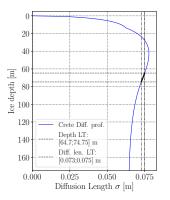


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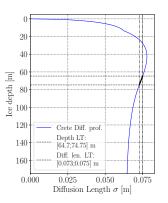


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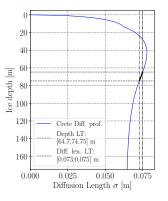


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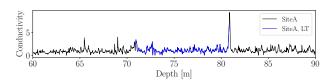


Figure: Site A, full ECM profile.

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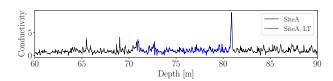
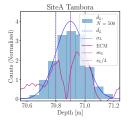
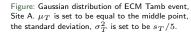


Figure: Site A, full ECM profile.





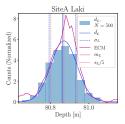


Figure: Gaussian distribution of ECM Laki event, Site A. μ_L is set to be equal to the middle point, the standard deviation, σ_L^2 is set to be $s_L/5$.

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Data: Isotopic Depth Series

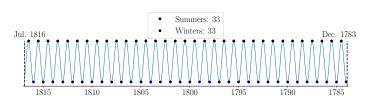


Figure: Illustration of number of summers and winters between Laki and Tambora eruption depositions.

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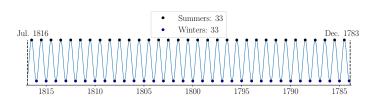


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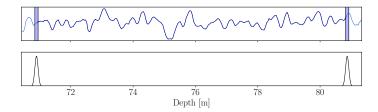


Figure: Isotopic depth series for Site A with a Gaussian approximating a 2 month time span illustrating the distribution from where the Laki and Tambora positions are drawn.

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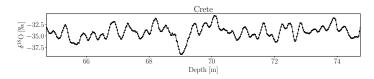


Figure: Raw data from drill site Crête between estimated Laki and Tambora event locations.

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Figure: Raw data from drill site Crête between estimated Laki and Tambora event locations.

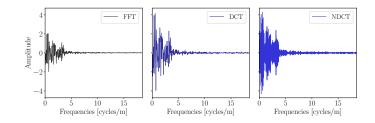


Figure: Examples of three different spectral transforms, FFT, DCT, NDCT, performed on the depth series between Tambora and Laki eruptions from Crête.

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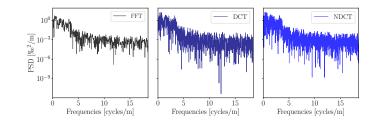


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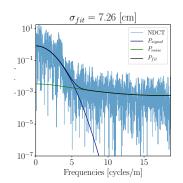


Figure: Noise, signal and total fit to PSD, illustrating the construction of the Wiener Filter.

•
$$\delta_M(z) = \delta_m(z) + \eta(z)$$

•
$$\tilde{F}(f) = \frac{|\tilde{\delta_m}(f)|^2}{|\tilde{\delta_m}(f)|^2 + |\tilde{\eta}(f)|^2}$$

•
$$|\tilde{\delta}_m(f)|^2 = P_0 e^{-k^2 \sigma_{\text{tot}}^2}$$

• $|\tilde{\eta}(f)|^2$ is assumed red noise cf. [C. Holme, V. Gkinis and B. M. Vinthe 2018].

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SNR and Wiener Filtering

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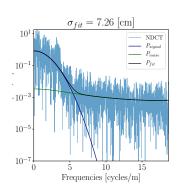


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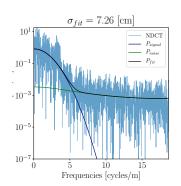


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SNR and Wiener Filtering

Spectral Analysis

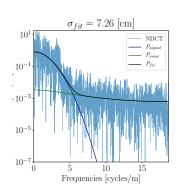


Figure: Noise, signal and total fit to PSD, illustrating the construction of the Wiener Filter.

•
$$\delta_M(z) = \delta_m(z) + \eta(z)$$

•
$$\tilde{F}(f) = \frac{|\tilde{\delta_m}(f)|^2}{|\tilde{\delta_m}(f)|^2 + |\tilde{\eta}(f)|^2}$$

•
$$|\tilde{\delta}_m(f)|^2 = P_0 e^{-k^2 \sigma_{\text{tot}}^2}$$

• $|\tilde{\eta}(f)|^2$ is assumed red noise cf. [C. Holme, V. Gkinis and B. M. Vinther. 2018].

Spectral Analysis

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Wiener Filter

Spectral Analysis

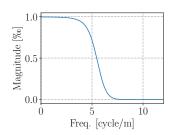


Figure: Wiener filter on linear scale.

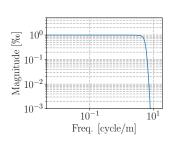


Figure: Wiener filter on double logarithmic scale.

Final Frequency Restoration Filter

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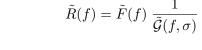
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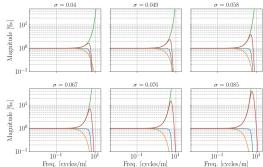


Figure: Frequency filter examples ranging from diffusion length 0.04 m to 0.085 m.

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General Idea: Back Diffusion

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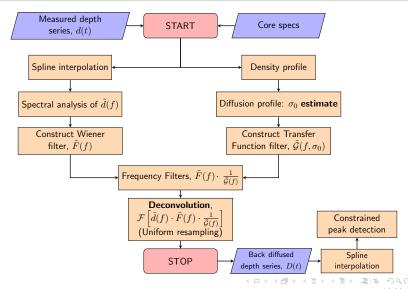
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Imposing Constraints

•
$$N_P = 33$$

- Peak(trough) distance

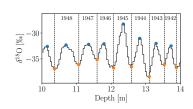


Figure: 7 years of signal from the top 10 to 15 m of the Crête ice core, with clearly visible seasonal cycles. Blue marks peaks and orange marks troughs.

Imposing Constraints

• $N_P = 33$

• $N_T = 33 \pm 1$

Peak(trough) distance

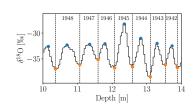


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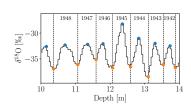


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Back Diffusion

• $N_P = 33$

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• P/T prominence min. 50 % of SD_{signal}

 Peak(trough) distance minimum 50 % of λ_A

Pattern: ...PTPTP...

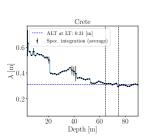


Figure: Annual layer thickness for 90 m of the Crête ice core. Estimated through spectral analysis

Imposing Constraints

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- $[\sigma_{min}, \sigma_{max}]$ in [0, 15] cm
- Pattern: ...PTPTP...

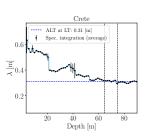


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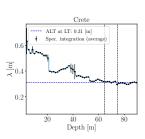


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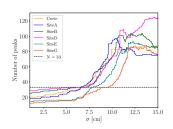


Figure: Number of peaks estimated versus diffusion length, based on diffusion lengths in the interval [0.01; 0.15] m.

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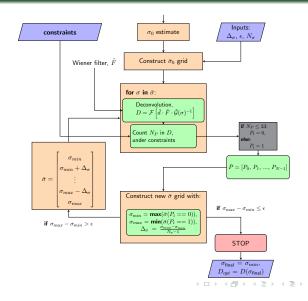
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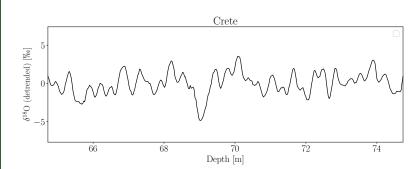


Figure: Crête ice core, section between Laki and Tambora events, raw data. Not presented in thesis.

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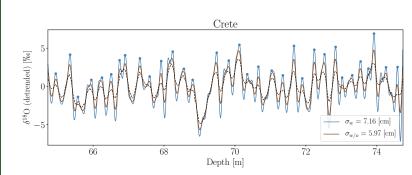


Figure: Crete ice core, with back diffused data (constrained and unconstrained).

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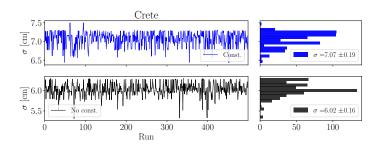


Figure: Optimal diffusion length estimates, constrained and unconstrained, for Crête ice core, 500 runs with Laki and Tambora positions drawn from 2 month Gaussian location distributions.

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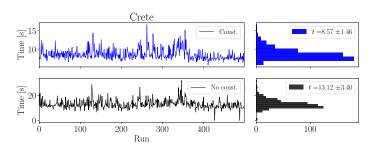


Figure: Computational time for algorithm, constrained and unconstrained, 500 runs with Laki and Tambora positions drawn from 2 month Gaussian location distributions.

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• Restore lost information from signal

- Develop a method to find optimal diffusion length at a depth section
- Use optimal diffusion length to estimate temperature

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Unintended Result: Number of Peaks

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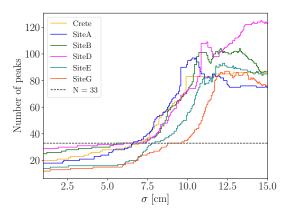


Figure: Number of peaks versus optimal diffusion length estimate, all investigated cores.

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Number of Peaks

SiteB Crete SiteA 60 60 50 50 40 40 30 30 20 20 20 8 6 6 SiteD SiteE SiteG 60 60 60 50 50 40 40 30 30 20 20 20

Figure: Number of peaks versus optimal diffusion length estimate, all cores separately with plateaus visible especially for Site B and G. Plateaus may show a way to date undated ice cores.

Diffusion Length [cm]

Diffusion Length [cm]

4 D > 4 B > 4 E > 4 E | 4 D | 9 C

Diffusion Length [cm]

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Estimated Optimal Diffusion Lengths

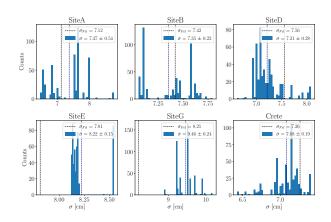


Figure: Optimal diffusion length estimate along with diffusion length estimate from spectral fit.

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Estimated Firn Diffusion Lengths

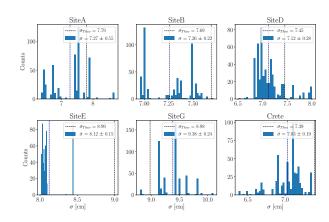


Figure: Corrected firn diffusion length estimates along with theoretical estimate from diffusion length profiles.

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Estimated Firn Temperatures

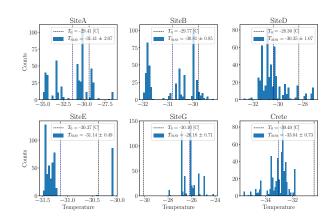


Figure: Firn temperature estimates, based on firn diffusion lengths from previous slide. Along with measured surface temperature from [H. B. Clausen, N. S. Gundestrup and S. J. Johnsen 1988].

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Algorithm Updates

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- More sensitive and sophisticated peak detection/pattern recognition

- Examine σ distribution instead of single σ

Algorithm Updates

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- More sensitive and sophisticated peak detection/pattern recognition
- Further improvement of the direct grid search optimization routine
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- More sensitive and sophisticated peak detection/pattern recognition
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More sensitive and sophisticated peak detection/pattern recognition

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- Examine σ distribution instead of single σ
- Implement a better and faster $\sigma(z)$ estimation method

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Further Iso-CFM possibilities

Investigate model further through results, utilizing the possibilities of the iso-CFM, like...

- Varying the input steady state accumulation rate
- Introduce seasonal cycles
- Investigate non-steady state models
- Improve results through statistical analysis with random variations

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Further Iso-CFM possibilities

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Improvements and Future work

• Examine σ plateaus as a variable to investigate non-dated depth series

- More intricate and complex analysis and decisions

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- Examine σ plateaus as a variable to investigate non-dated depth series
- More extensive analysis of final temperature results
- More intricate and complex analysis and decisions

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Future Work

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• Examine σ plateaus as a variable to investigate non-dated depth series

- More extensive analysis of final temperature results
- Investigate method stability by introducing new ice cores and different volcanic events
- More intricate and complex analysis and decisions

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- Examine σ plateaus as a variable to investigate non-dated depth series
- More extensive analysis of final temperature results
- Investigate method stability by introducing new ice cores and different volcanic events
- More intricate and complex analysis and decisions incorporated in the constraints and optimization

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Take-Away Messages

This work has successfully developed and presented the following key points:

- A **general method** for temperature estimation through diffusion length estimates
- A test of the method on a range of different ice cores

Take-Away Messages

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- An **in-depth stability analysis** of the methods used by isotopic data analysts
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Take-Away Messages

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- A **general method** for temperature estimation through diffusion length estimates
- An **in-depth stability analysis** of the methods used by isotopic data analysts
- A test of the method on a range of different ice cores
- Developed a **stepping stone for the further analysis** and understanding of isotopic signals, diffusion lengths and paleotemperatures

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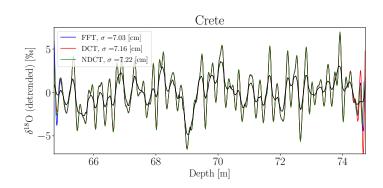
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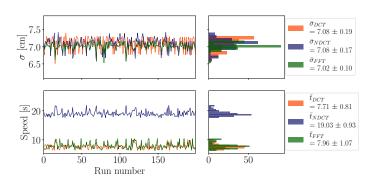


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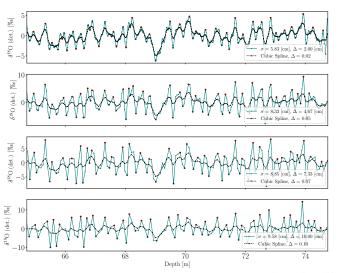
Spectral Transforms



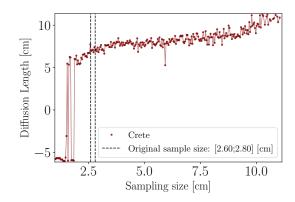
Spectral Transforms



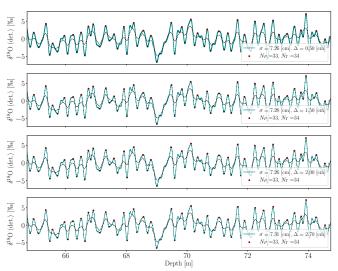
Interpolations: Before Deconvolution



Interpolations: Before Deconvolution



Interpolations: After Deconvolution



Interpolations: After Deconvolution

