T. Quistgaa

50 Far...

Data

ack Diffusion

And now

Standardize

Layer Countin

Outlook

Layer Counting Algorithm, Cont.

Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

Thea Quistgaard¹

¹University of Copenhagen

November 25, 2020

Outline of talk

T. Quistgaard

So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

And now?

Detrend and Standardize Layer Counting Algorithm

Outlook

Table of Contents

T. Quistgaa

Water Isotopes

Data

olcanic Horizo

And now

Standardize

Layer Countin Algorithm

Laver Counti

ayer Counting Igorithm, Cont. So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Detrend and Standardize Layer Counting Algorithm

Outlook

Water Isotopes in Ice Cores

T. Quistgaa

So Far...

Water Isotopes

Data Volcanie Horiz

Back Diffusion

And now

Standardize

Layer Countin Algorithm

Outlook

Layer Counting Algorithm, Cont.

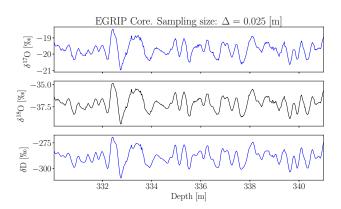


Figure: Examples of three water isotopes measured from the EGRIP core in Greenland.

. ..

Laki to Tambora

T. Quistgaa

So Far...

Water Isotopes

Data

VOICAIIIC FIORIZ

And now

Standardize

Algorithm

Outlook

Layer Counting Algorithm, Cont

Diffusion in Firn

• Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}},$$
 a Gaussian filter, (3)

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)



Water Isotopes

Laki to Tambora

T. Quistgaa

30 Far...

Water Isotopes

Data

olcanic Horiz

And now

, ...a ..o..

Layer Countin

Outlook

Layer Counting Algorithm, Cont.

Diffusion in Firn

• Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = rac{1}{\sigma\sqrt{2\pi}}e^{-rac{z^2}{2\sigma^2}},$$
 a Gaussian filter, (3)

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)



Diffusion in Firn

T. Quistgaard

So Far...

Water Isotopes

Volcanic Horiz

And now

Standardize

Layer Countin Algorithm

Uutlook

ayer Counting

• Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)

vvater isotope

Laki to Tambora

T. Quistgaa

30 Far...

Water Isotopes

Volcanic Horiz

Back Diffusion

And now

Standardize Laver Countin

Algorithm

Outlook

Layer Counting Algorithm, Cont

Diffusion in Firn

• Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)

T. Quistgaard

Water Isotopes

Diffusion in Firn

Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

$$\mathcal{G}(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)



vvater isotope

Laki to Tambora

T. Quistgaard

30 Fai...

Water Isotopes

Volcanic Horiz

Back Diffusion

And now

Standardize

Layer Countin Algorithm

Outlook

Layer Counting Algorithm, Cont

Diffusion in Firn

• Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2\sigma^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)



Actual Total Diffusion

T. Quistga

Water Isotopes

vvater isoto

/olcanic Horiz

Back Diffusion

And now

Standardize

Outlook

Laver Coun

Total diffusion in ice and firn

$$\sigma_{\text{tot}}^2 = [S(z)\sigma_{\text{firn}}]^2 + \sigma_{\text{ice}}^2(z)$$
 (5)

Giving an actual measured diffusion length of

$$\hat{\sigma}_i^2 = \sigma_{\text{firn}}^2 S(z) + \sigma_{\text{ice}}^2 + \sigma_{\text{dis}}^2 \tag{6}$$

with

$$\sigma_{\mathsf{dis}}^2 = \frac{2\Delta^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

Actual Total Diffusion

T. Quistgaard

Water Isotopes

Volcanic Horiz

, ...a ..o..

Layer Counting

Outlook

Layer Counting Algorithm, Cont. Total diffusion in ice and firn

$$\sigma_{\text{tot}}^2 = [S(z)\sigma_{\text{firn}}]^2 + \sigma_{\text{ice}}^2(z)$$
 (5)

Giving an actual measured diffusion length of

$$\hat{\sigma}_i^2 = \sigma_{\text{firn}}^2 S(z) + \sigma_{\text{ice}}^2 + \sigma_{\text{dis}}^2 \tag{6}$$

with

$$\sigma_{\mathsf{dis}}^2 = \frac{2\Delta^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

Table of Contents

T. Quistgaa

Data

o Far...

1 So Far...

Water Isotopes

Data

Volcanic Horizons
Back Diffusion

2 And now?

Detrend and Standardize Layer Counting Algorithm

Outlook

Example Data: Site A

Data

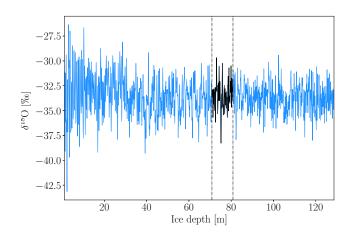


Figure: Example data from Alphabet Core drilled at site A near Crête.

So Far...

Unevenly Sampled Data: Spline Interpolation

Data

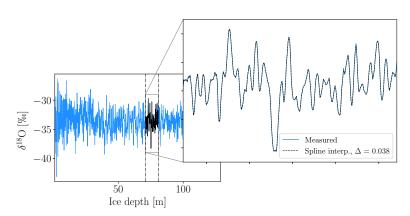


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.



uistgaard

Water Isotope

olcanic Horizo

And now

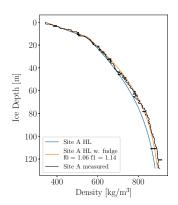
Allu llow

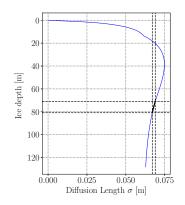
Layer Counting

Outlook

Layer Counting Algorithm, Cont.

Community Firn Model





- (a) Density-depth profiles based on analytical Herron-Langway model. Black is empirical data, blue is purely analytical fit and orange is fudged analytical fit
- (b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.



So Far...

Table of Contents

T. Quistga

Water Isoto

Volcanic Horizons

And now

Standardize Layer Countin

Outlook

ayer Counting Ilgorithm, Cont. So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Detrend and Standardize Layer Counting Algorithm

Outlook

So Far...

Laki and Tambora

T Ouistaa

So Far...
Water Isotop

Volcanic Horizons

And now

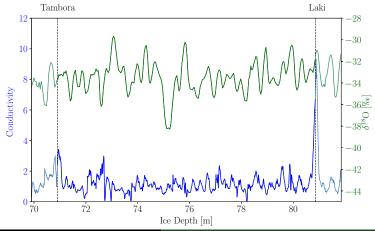
Standardize Layer Counting

Outlook

Layer Counting

• Electrical Conductivity Measurements (ECM)

• Dielectric Profiling (DEP)



So Far...

Table of Contents

T. Quistgaa

Water Isotop

)ata

Back Diffusion

And now

Standardize Layer Countin

Algorithm

Layer Countin

1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Detrend and Standardize Layer Counting Algorithm

Outlook

Spectral Analysis with DCT

T. Quistgaar

So Far...

Water Isotop

Volcanic Hori

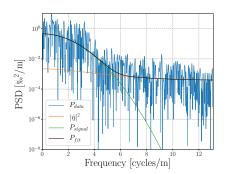
Back Diffusion

And now

Standardize

Layer Counting

Outlook



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

Spectral Analysis with DCT

T. Quistgaai

So Far...

Water Isotop

Volcanic Hori

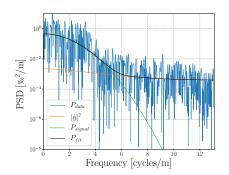
Back Diffusion

And now

Standardize

Layer Counting

Outlook



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

Spectral Analysis with DCT

T. Quistgaa

So Far...

Water Isotop

Volcanic Hor

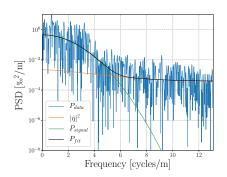
Back Diffusion

And now

Standardize

Layer Counting

Outlook



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

Back Diffusion

Laki to Tambora

Spectral Analysis with DCT

T. Quistgaai

So Far...

Volcanic Hori

Back Diffusion

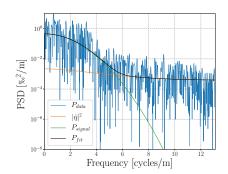
And now

Detrend and

Layer Counting

Outlook

ayer Counting



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 \sigma^2}$$

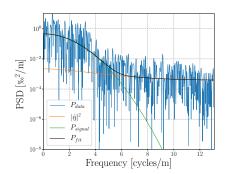
Back Diffusion

Laki to Tambora

So Far...

Spectral Analysis with DCT

Back Diffusion



$$P_{\mathsf{tot}} = P_{\mathsf{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\rm signal} = P_0 e^{-k^2 {\sigma^2 \over \sigma^2}}$$

So Far...

T. Quistgaard

Water Isotop

Volcanic Hor

Back Diffusion

And now

Standardize

Algorithm

Outlook

Layer Counting Algorithm, Cont.

Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise.

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\text{meas}} \cdot \tilde{R}$$
 (10)

So Far...

T. Quistgaard

Nater Isotop

Volcanic Hori

Back Diffusion

And now

Standardize

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont

Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\text{meas}} \cdot \tilde{R}$$
 (10)

So Far...

T. Quistgaard

Water Isotope

Volcanic Hori

Back Diffusion

Allu llow

Layer Counting

Algorithm

Outlook

Layer Counting Algorithm, Cont.

Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\tilde{\delta}_{\mathsf{init}} = \tilde{\delta}_{\mathsf{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\mathsf{meas}} \cdot \tilde{R} \tag{10}$$

T. Quistgaard

Water Isotop

Volcanic Hor

Back Diffusion

And now

Standardize

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.

Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

$$\tilde{\delta}_{\mathsf{init}} = \tilde{\delta}_{\mathsf{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\mathsf{meas}} \cdot \tilde{R} \tag{10}$$

Filtering

T. Quistgaa

So Far...
Water Isotop

Volcanic Hor

Back Diffusion

And now

Standardize

Layer Countin Algorithm

Outlool

Layer Counting Algorithm, Cont.

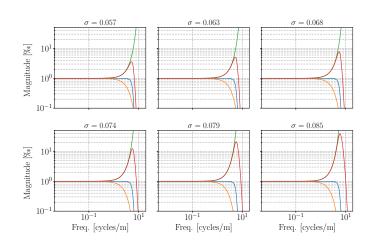


Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

So Far...

Laki to Tambora

Deconvolution

T. Quistgaar

So Far...
Water Isotope

Volcanic Hor

Back Diffusion

And now

Allu llow

Standardize Layer Countin

A .. .

Layer Counting Algorithm, Cont

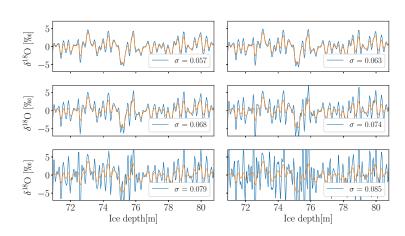


Figure: The estimated restored signal (blue) given diffusion length. Plotted along with original measured data (orange).



Enhanced Signal, Minimized Noise

T. Quistgaa

So Far...

Data

Voicanic mor

Back Diffusion

And now

.....

Layer Counti

o ..

Layer Countii

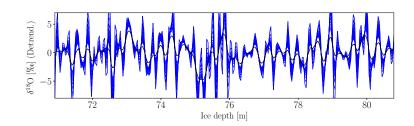


Figure: The original data plotted along with each estimate of the restored data with diffusion lengths ranging from 0.057 to 0.085.

T. Quistgaa

_

Water Isotop

Data

voicanic mon

Back Diffusion

And now

Dotrond an

Layer Countii

A .. .

Layer Count

Enhanced Signal, Minimized Noise

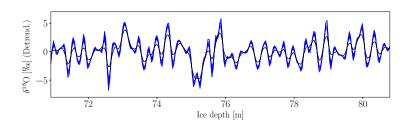


Figure: The original data plotted along with each estimate of the restored data with diffusion length $\sigma^2 < 0.07$.

Table of Contents

T. Quistgaa

So Far...

1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

Detrend and Standardize

Layer Countin

Outlook

ayer Counting Igorithm, Cont. 2 And now?

Detrend and Standardize

Layer Counting Algorithm

Outlook

So Far...

Estimating Cycle Length - ACF

T. Quistgaard

Water Isotop

Volcanic Horiz

Back Diffusion

And now

Detrend and Standardize

Layer Countin

Outlool

Layer Counting Algorithm, Cont.

$$R_k = \frac{1}{(n-k)\sigma^2} \sum_{i=1}^{n-k} (x_i - \mu)(x_{i+k} - \mu)$$
 (11)

The estimated cycle length, l, will be the first k such that $R_{k-1}>R_{k-2}$ and $R_k< R(k-1)$

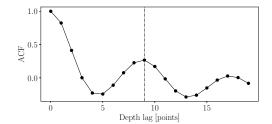


Figure: Autocorrelation as a function of (pointwise-) depth lag.

Estimating Cycle Length - Adjustment

T. Quistgaard

Water Isoto

Volcanic Horizo

And now

Detrend and Standardize

Layer Countin

Outlook

Layer Counting Algorithm, Cont.

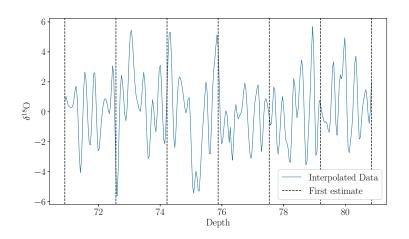


Figure: First estimate of sections.



Estimating Cycle Length - Adjustment

T. Quistgaard

So Far... Water Isoto

Volcanic Horizo

And now

Standardize Layer Countin

Outlook

Layer Counting Algorithm, Cont.

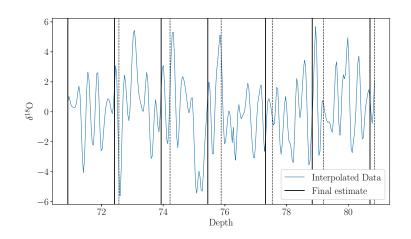


Figure: Final estimate of sections.



Detrending with Moving Average

T. Quistgaard

So Far..

Data Volcanic Hori:

Back Diffusion

And now

Detrend and Standardize

Layer Countin Algorithm

Outlook

Layer Counting Algorithm, Cont Moving average:

$$\mu_i = \sum_{i=i-l_i/2}^{i+l_i/2} \frac{x_i}{l_i+1} \tag{12}$$

Moving standard deviation

$$\sigma_i^2 = \sum_{j=i-l_i/2}^{i+l_i/2} \frac{(x_i - \mu_i)^2}{l_i + 1}$$
 (13)

New standardized signal \bar{s} :

$$s_i = \frac{x_i - \mu_i}{\sqrt{2}\sigma_i} \tag{14}$$

So Far...

Detrending and standardize

T. Quistgaa

So Far...

Water Isotope Data

Volcanic Horiz

And now

And now Detrend and

Standardize Layer Countii

Outlook

Layer Coun

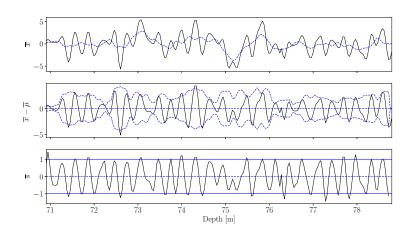


Figure: Standardization through moving average, $\bar{\mu}$, and standard deviation, $\bar{\sigma}^2$

So Far...

Table of Contents

T. Quistgaa

. .

Water Isoto

Volcanic Horiz

And now?

....

Layer Counting

Algorithm

Layer Counti

1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Detrend and Standardize

Layer Counting Algorithm

Outlook

So Far...

Peak Detection by Pattern Recognition

T. Quistga

Water lente

.

/olcanic Hori

Back Diffusio

And nov

Alla llow

Layer Counting

Algorithm

Layer Counti

- Prior information: Typical annual cycle (noisy sine)
- Convolutional Neural Networks
- Kalman Filtering, MCMC or something else entirely

Layer Counting Algorithm

T. Quistgaa

Water Isotop

Data

ocanic Horizi ack Diffusion

And nov

Standardize

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Con

Prior to estimation:

- Diffusion and densification models
- Noisy sine signal

Outcome:

- Peak counting
- Dating by years
- Layer thickness approximation

And now?

Laki to Tambora

Layer Counting Algorithm

T. Quistgaard

So Far...

Water Isotop

Volcanic Horiz

Back Diffusion

And non

Layer Counting Algorithm

Outlook

Layer Count

Prior to estimation:

- Diffusion and densification models
- Noisy sine signal

Outcome:

- Peak counting
- Dating by years
- Layer thickness approximation

So Far...

Table of Contents

T. Quistgaa

30 Far..

Data

Back Diffusion

And now

Standardize Layer Countin

Algorithm

Layer Counting
Algorithm, Cont.

1 So Far...

Water Isotopes

Data

Volcanic Horizons

Back Diffusion

2 And now?

Detrend and Standardize Layer Counting Algorithm

Outlook Layer Counting Algorithm, Cont.

Layer Counting Algorithm

T. Quistgaa

So Far..

Water Iso

Volcanic Hori

olcanic monz

And now

Alla llow

Lanca Canada

Algorithm

Outlook

- In Different Cores, Same (Known) Age
- Down entire (Dated) Core
- Combination

So Far...

Thank you!

i. Quistgaai

Water Isoto

Data

Volcanic Hori

Back Diffusio

And now

Detrend and

Layer Countin

Outlook

Layer Counting Algorithm, Cont. Any questions?