Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

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February 9, 2021

Outline of talk

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Diffusion in Firn

- Fick's 2^{nd} law:

$$\frac{\partial \delta}{\partial t} = D(t)\frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with steady state solution

$$\delta_{\text{meas}}(z) = S(z)[\delta_{\text{init}}(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta_{\text{meas}}(z)$ is the measured signal, $\delta_{\text{init}}(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma(z)\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma(z)^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

and

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Example Data: Site A

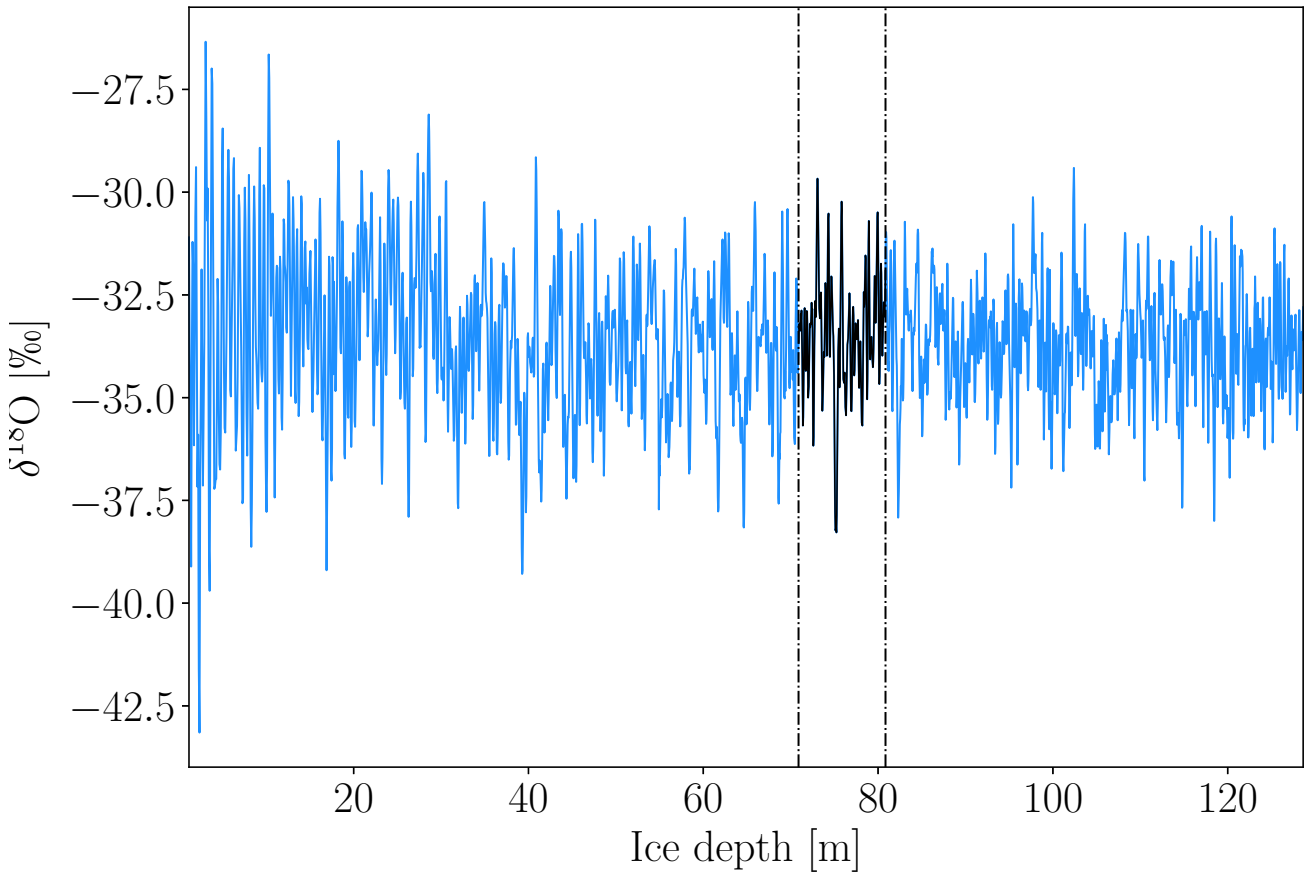


Figure: Example data from Alphabet Core drilled at site A near Crête.

Unevenly Sampled Data: Spline Interpolation

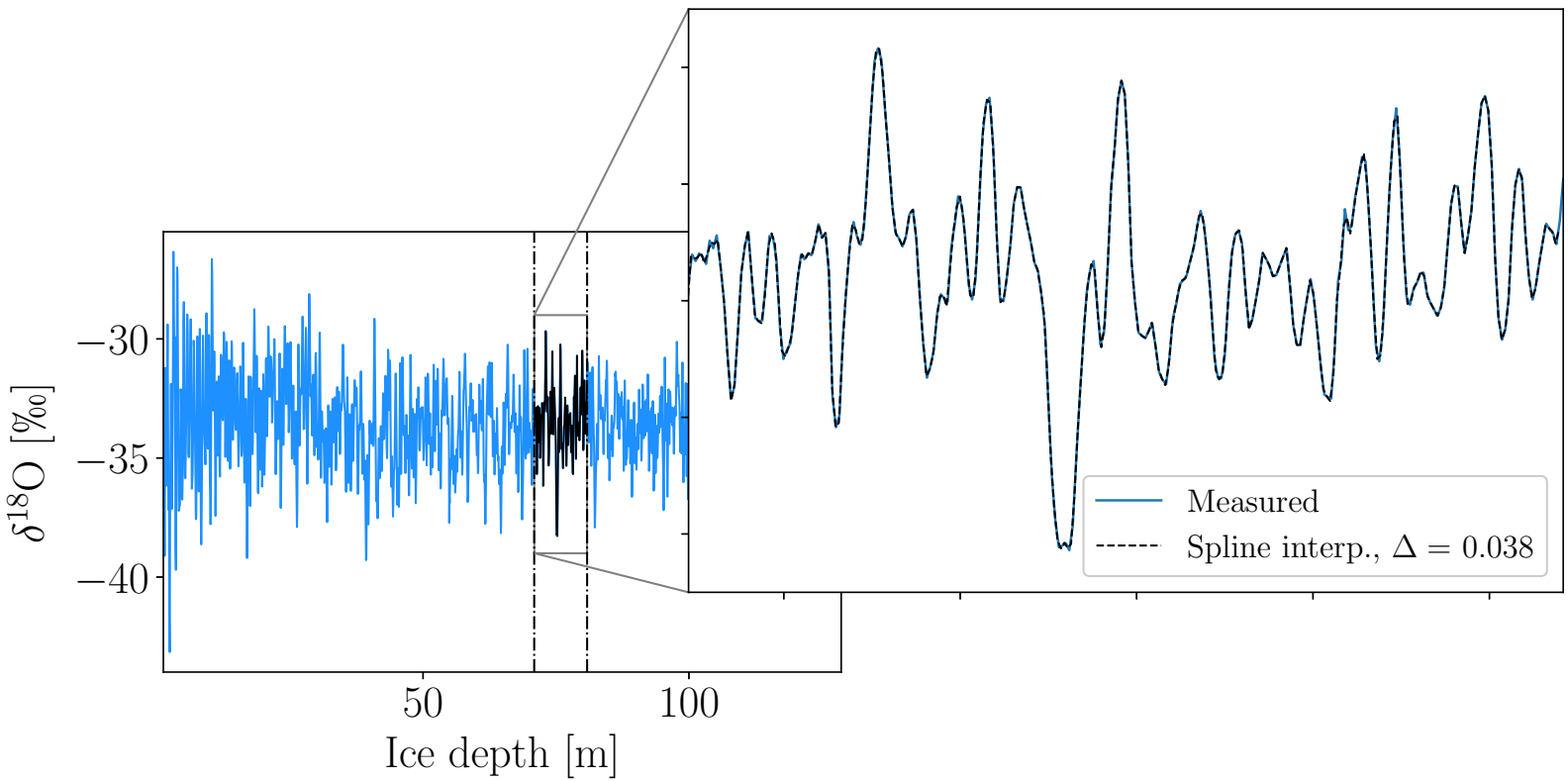


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.

Site A

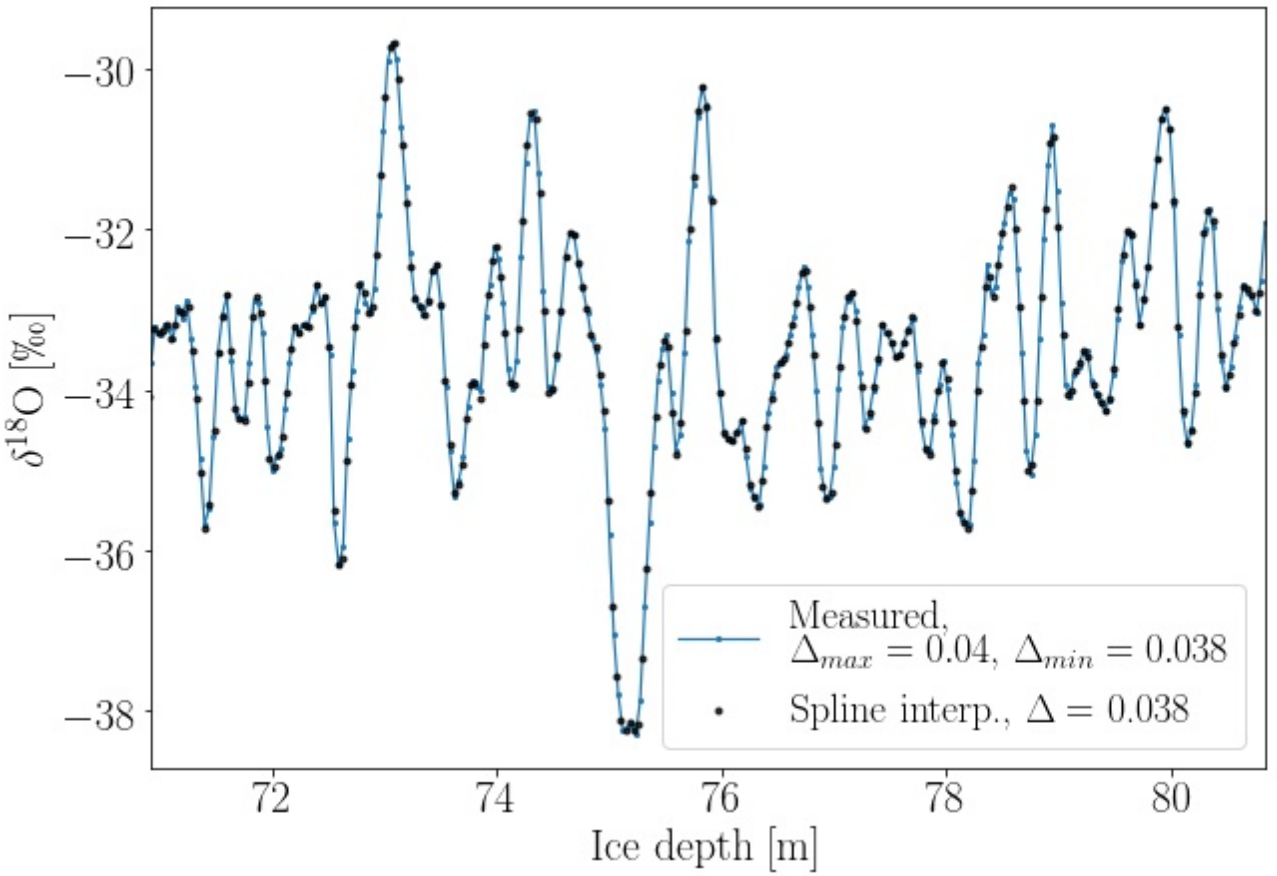


Figure: Site A, raw and cubic spline interpolated data.

Site G

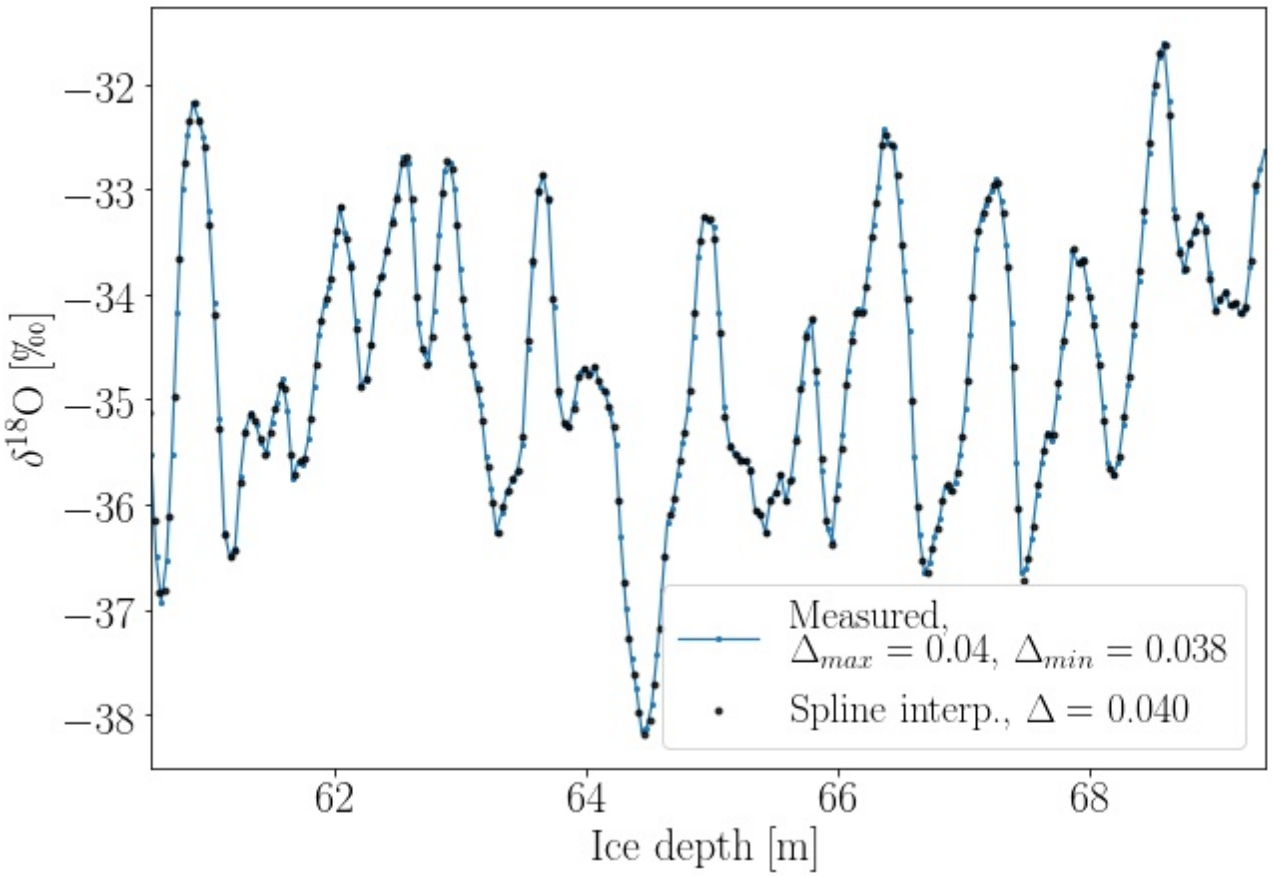
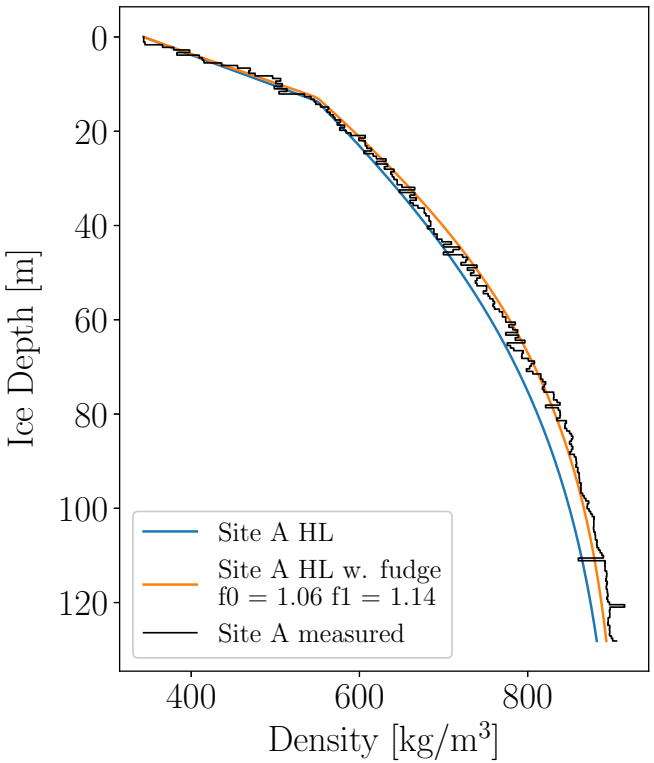
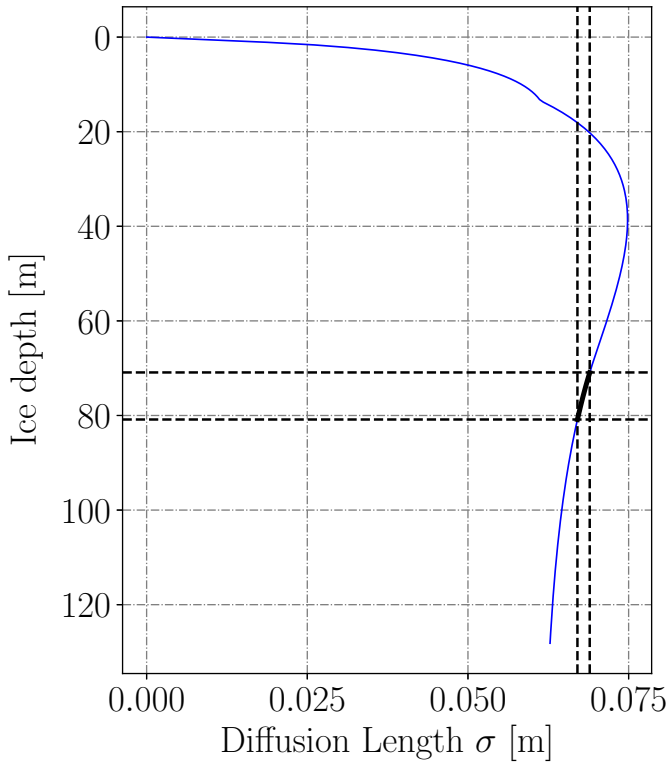


Figure: Site G, raw and cubic spline interpolated data.

Site A: Density and Diffusion Profiles

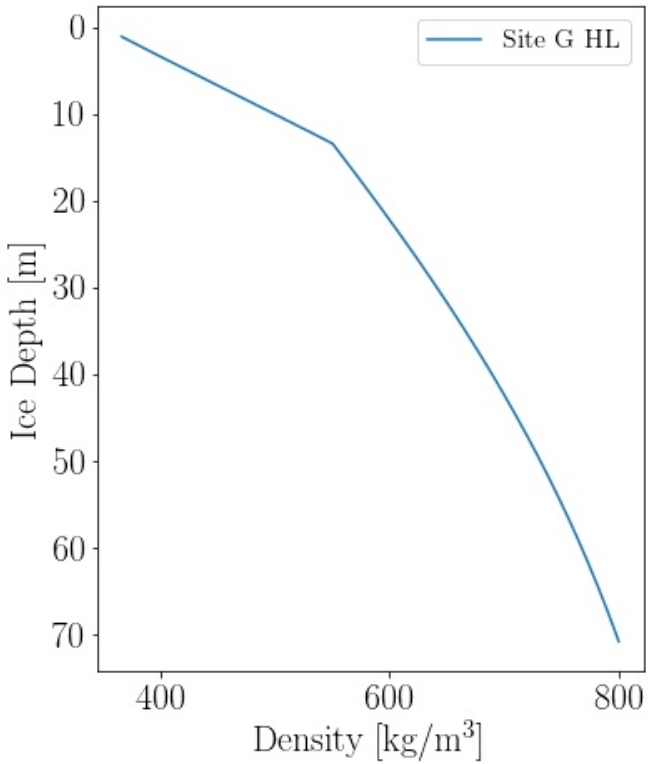


(a) Density-depth profiles based on analytical Herron-Langway model. Black is empirical data, blue is purely analytical fit and orange is fudged analytical fit



(b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.

Site G: Density and Diffusion Profiles



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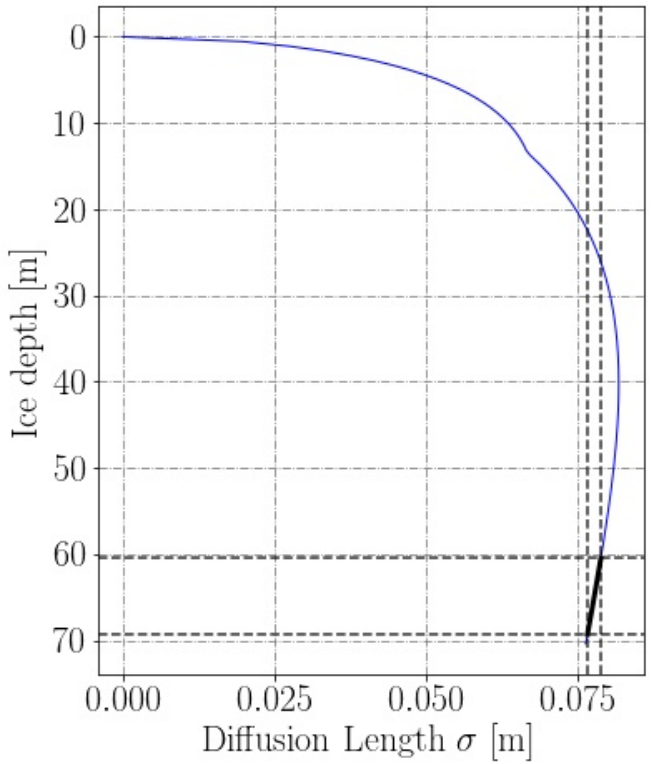

(b) Modelled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.

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Laki and Tambora

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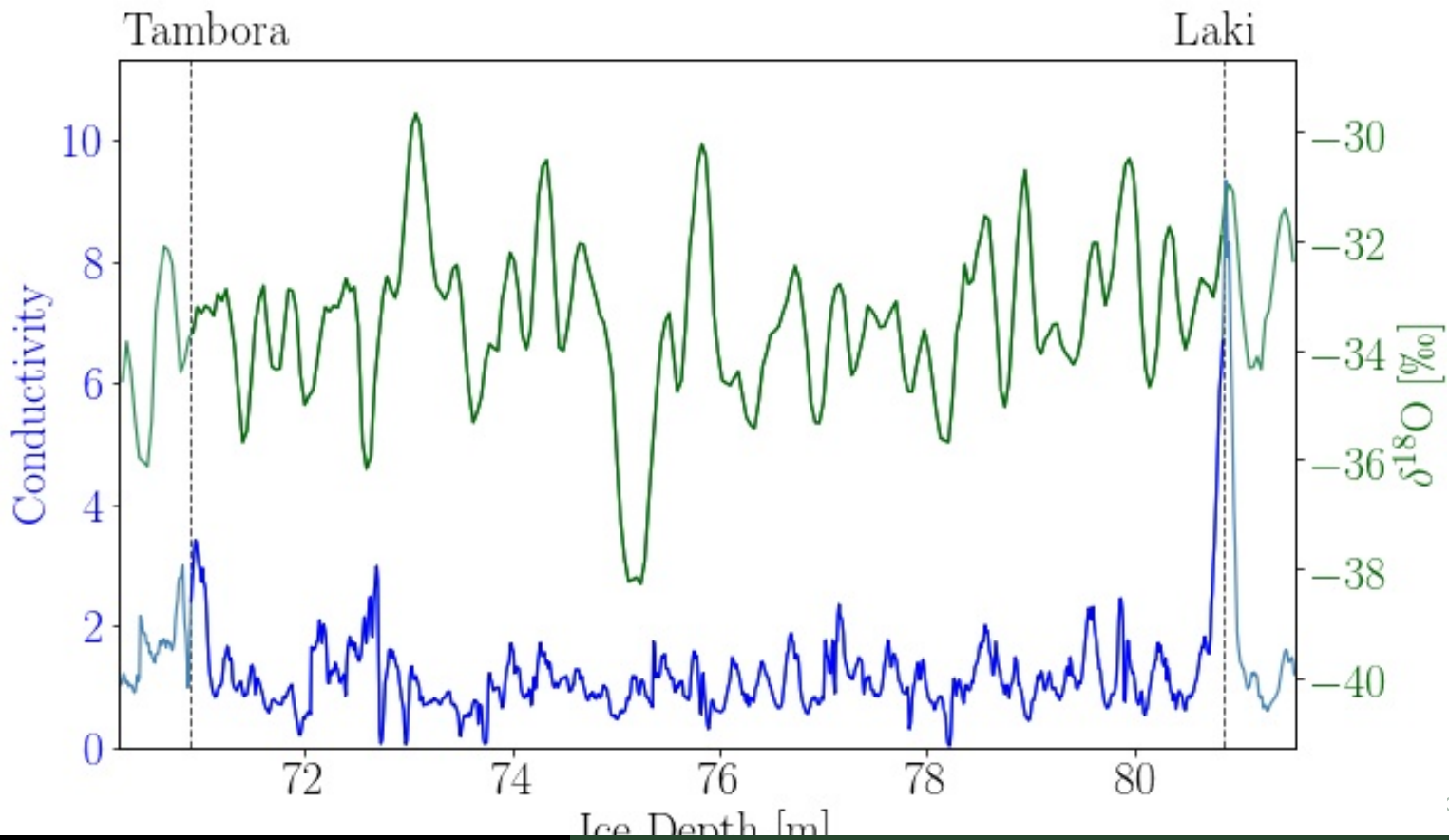
- **(Dielectric Profiling (DEP))**



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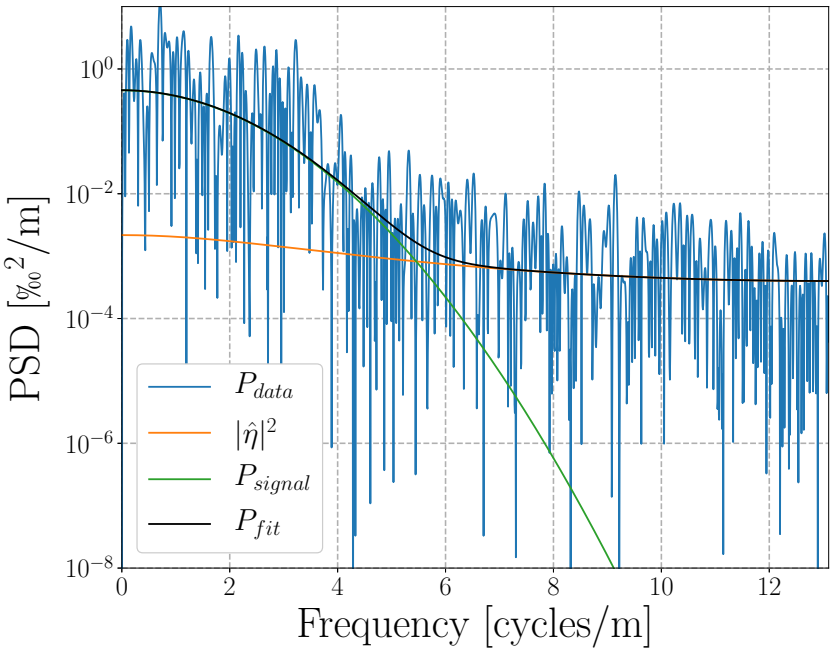
Spectral Analysis with DCT



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_\eta^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

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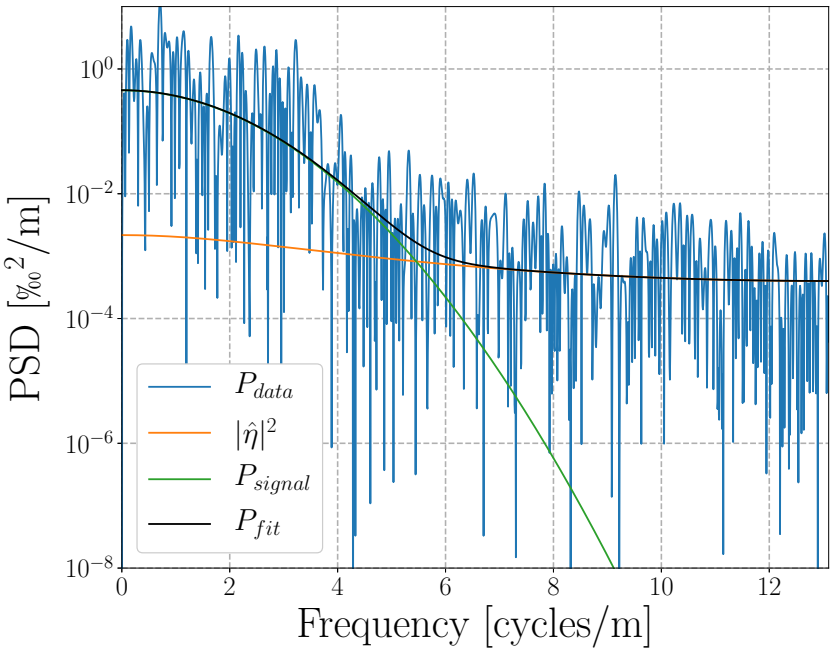

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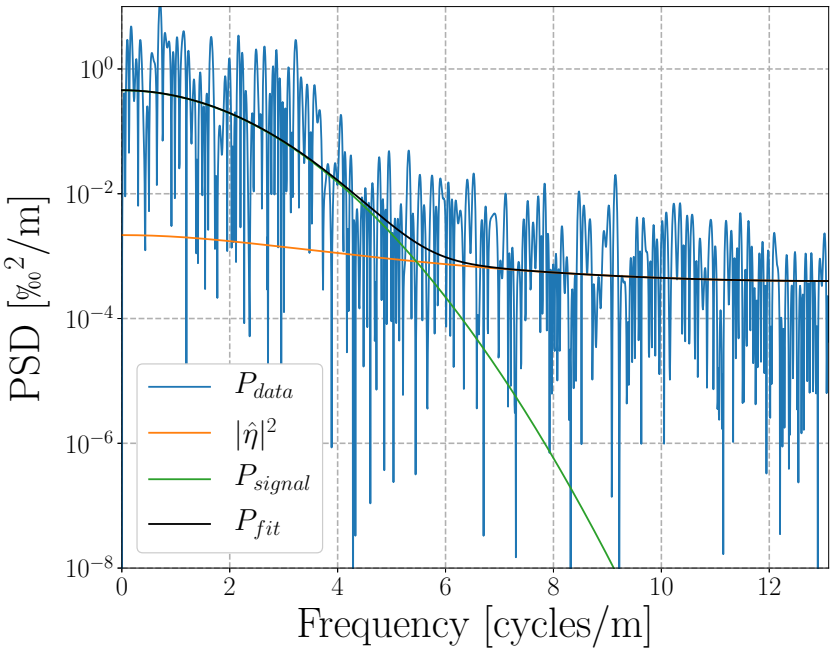

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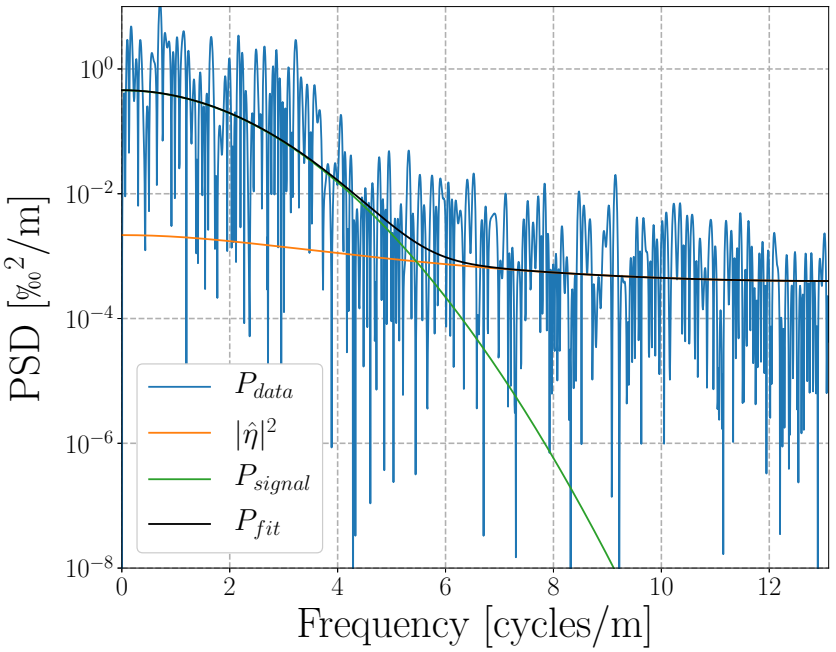

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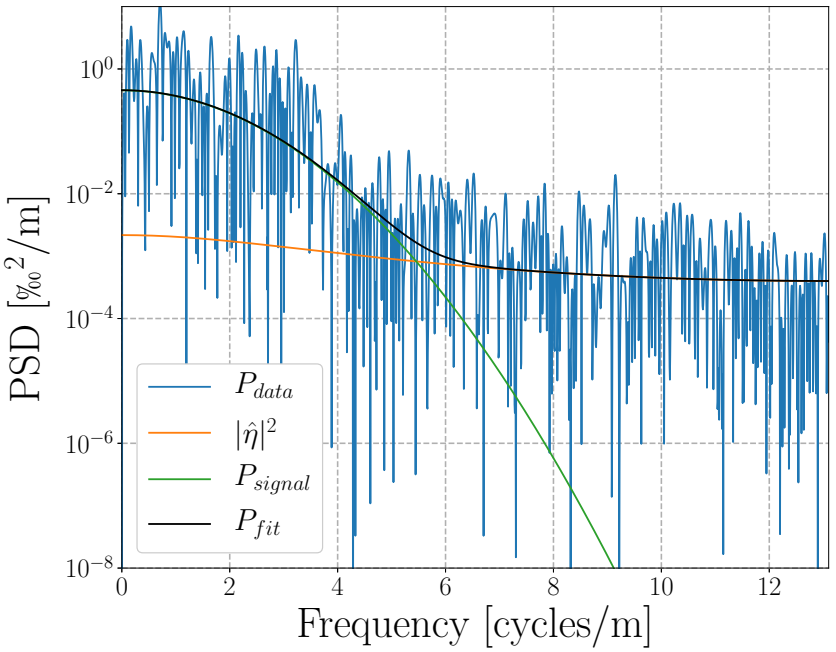

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Diffusion Lengths and Transfer Functions

$$\hat{\delta}_{\text{meas}} = \hat{\delta}_{\text{init}} \cdot \hat{M} \Leftrightarrow \hat{\delta}_{\text{init}} = \hat{\delta}_{\text{meas}} \cdot \hat{M}^{-1} \tag{5}$$

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\hat{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{6}$$

yielding a restoration filter as

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Filtering

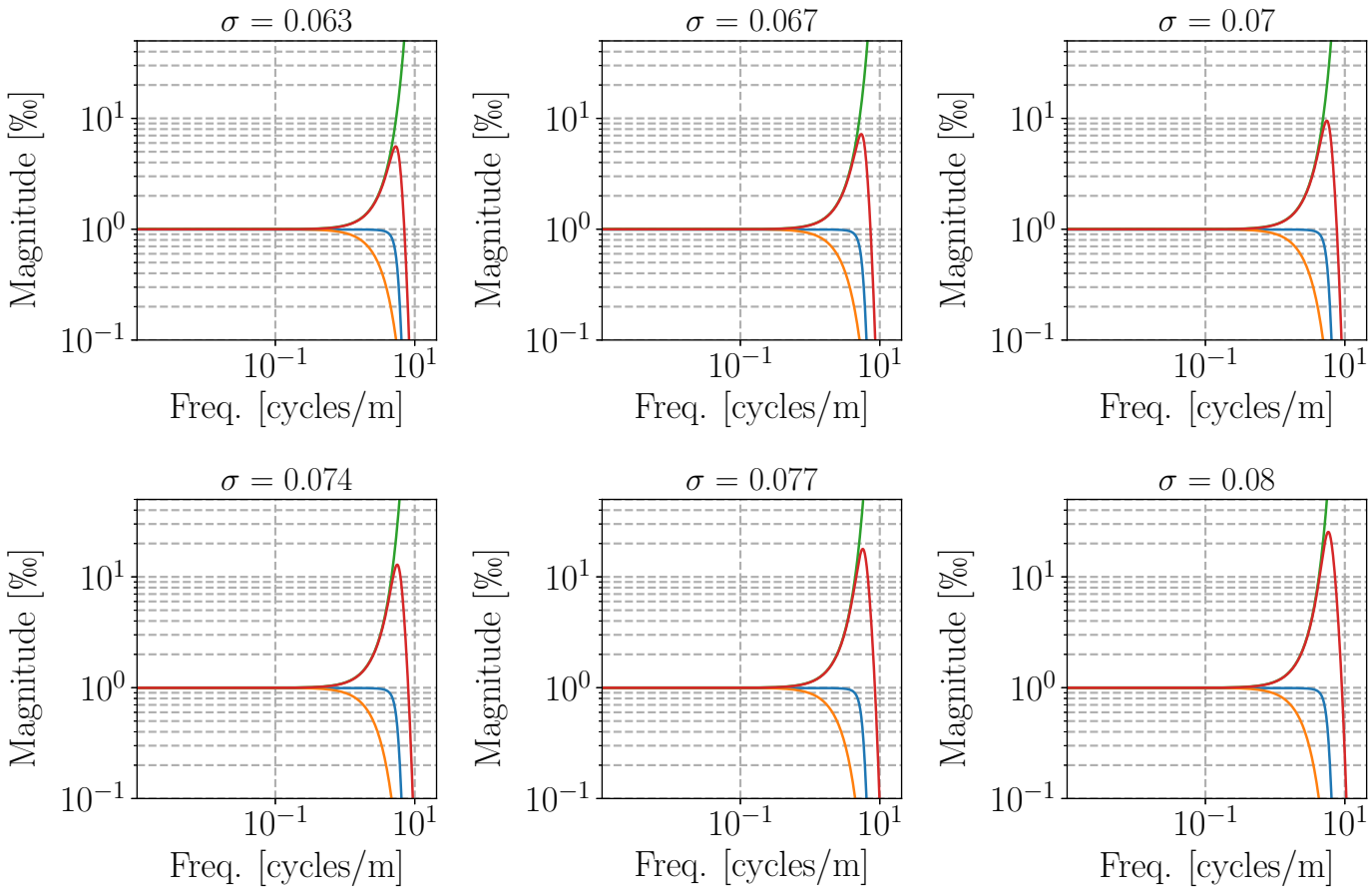

Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

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Peak Detection

- SciPy.signal.find_peaks
- N = 32 years btw Tambora and Laki Eruptions
- Best diffusion length estimate algorithm
- Interpolations and resampling

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Diffusion Length Estimate Algorithm

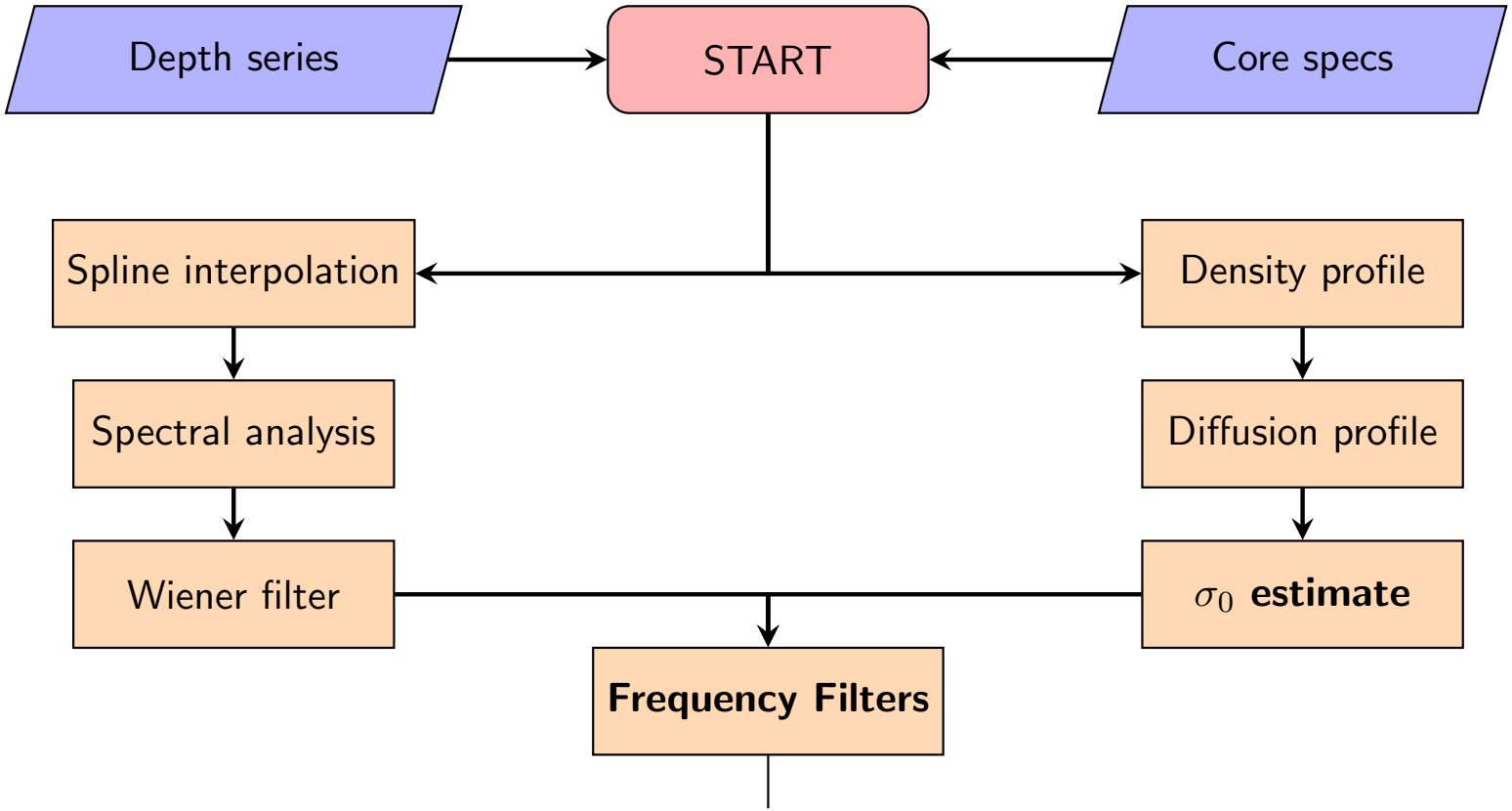

Figure: Flowchart of method for diffusion length computation, preliminary analysis steps.

Diffusion Length Estimate Algorithm

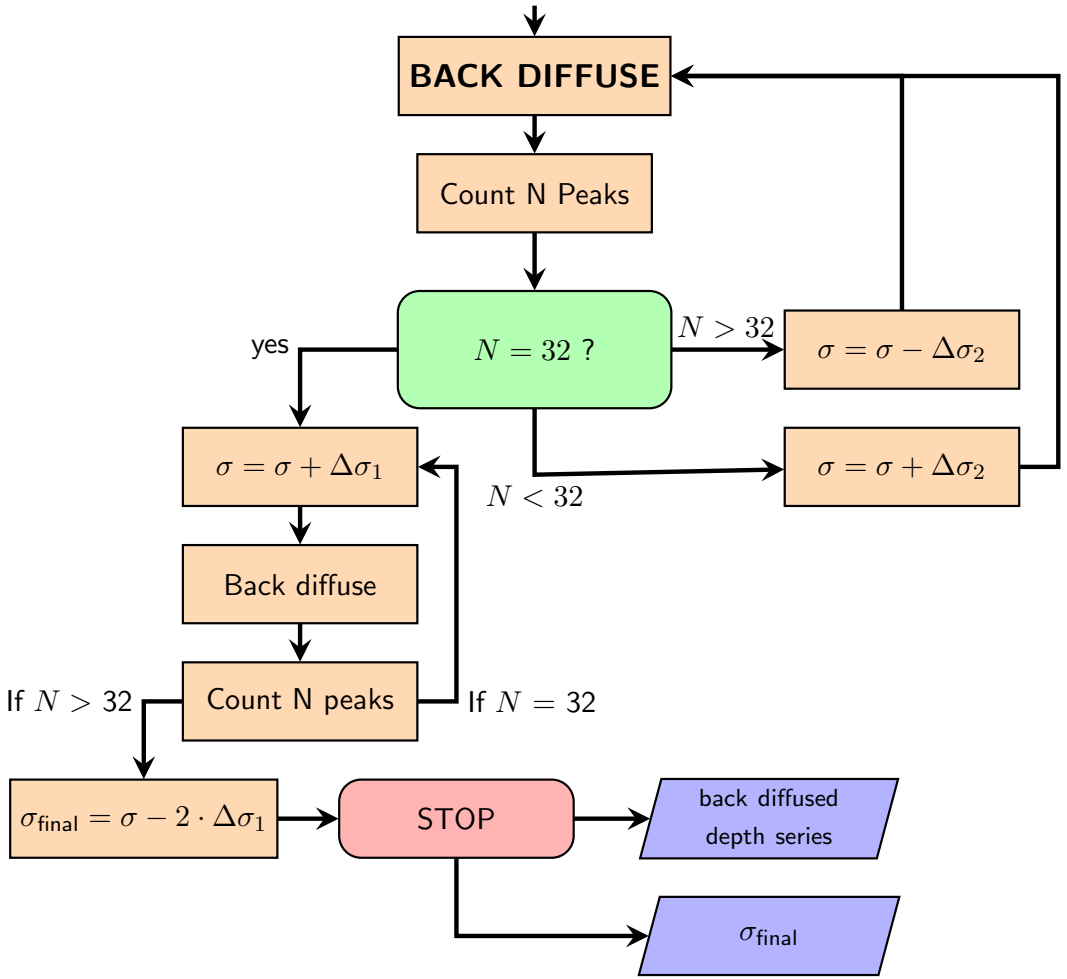

Figure: Flowchart of method for diffusion length computation, decision chart.

Diffusion Length V. Peaks - No Limit

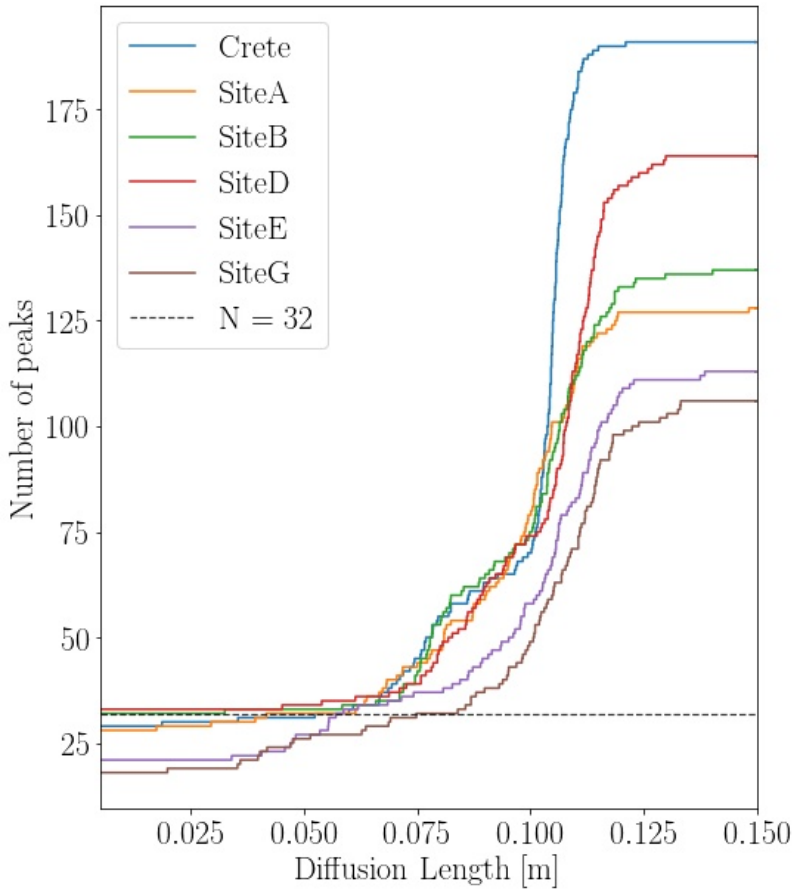

Figure: Diffusion length used in back diffusion versus counted number of peaks in data series for all cores.

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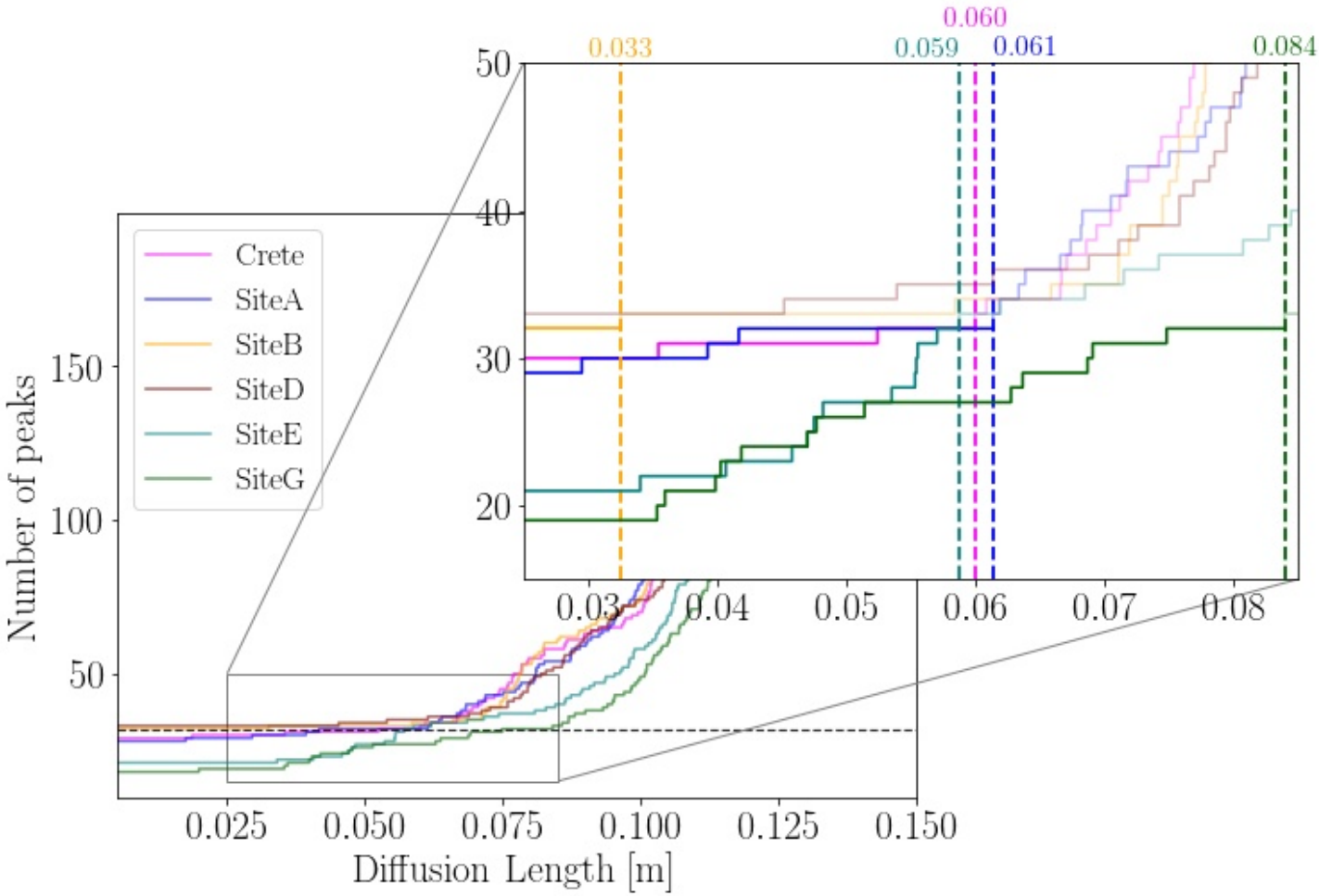

Figure: Zoom-in around N = 32 peaks and corresponding diffusion length used in back diffusion.

Cubic Spline Interpolation: Before Deconvolution

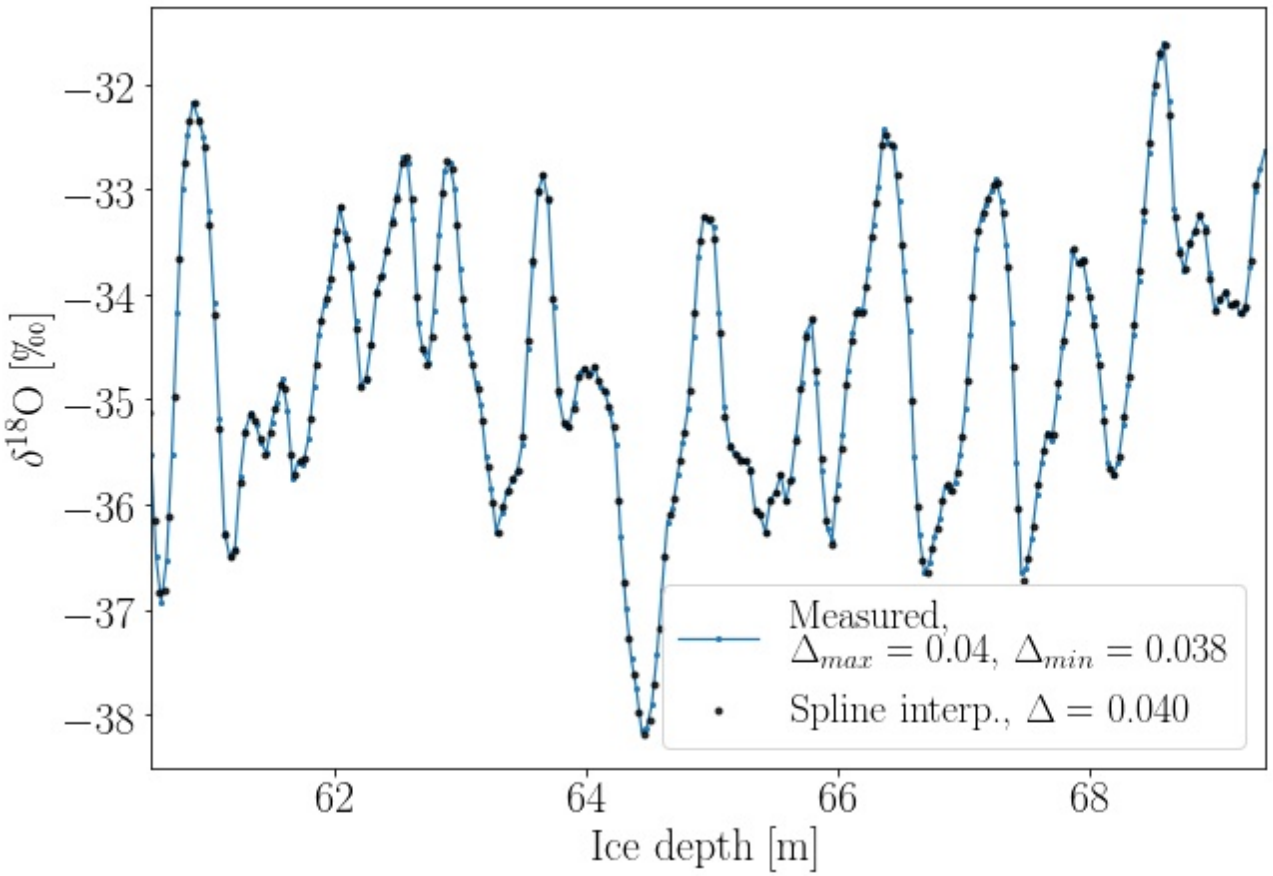

Figure: Cubic spline resampling of raw data.

Resampling size V. Diffusion Length Estimate

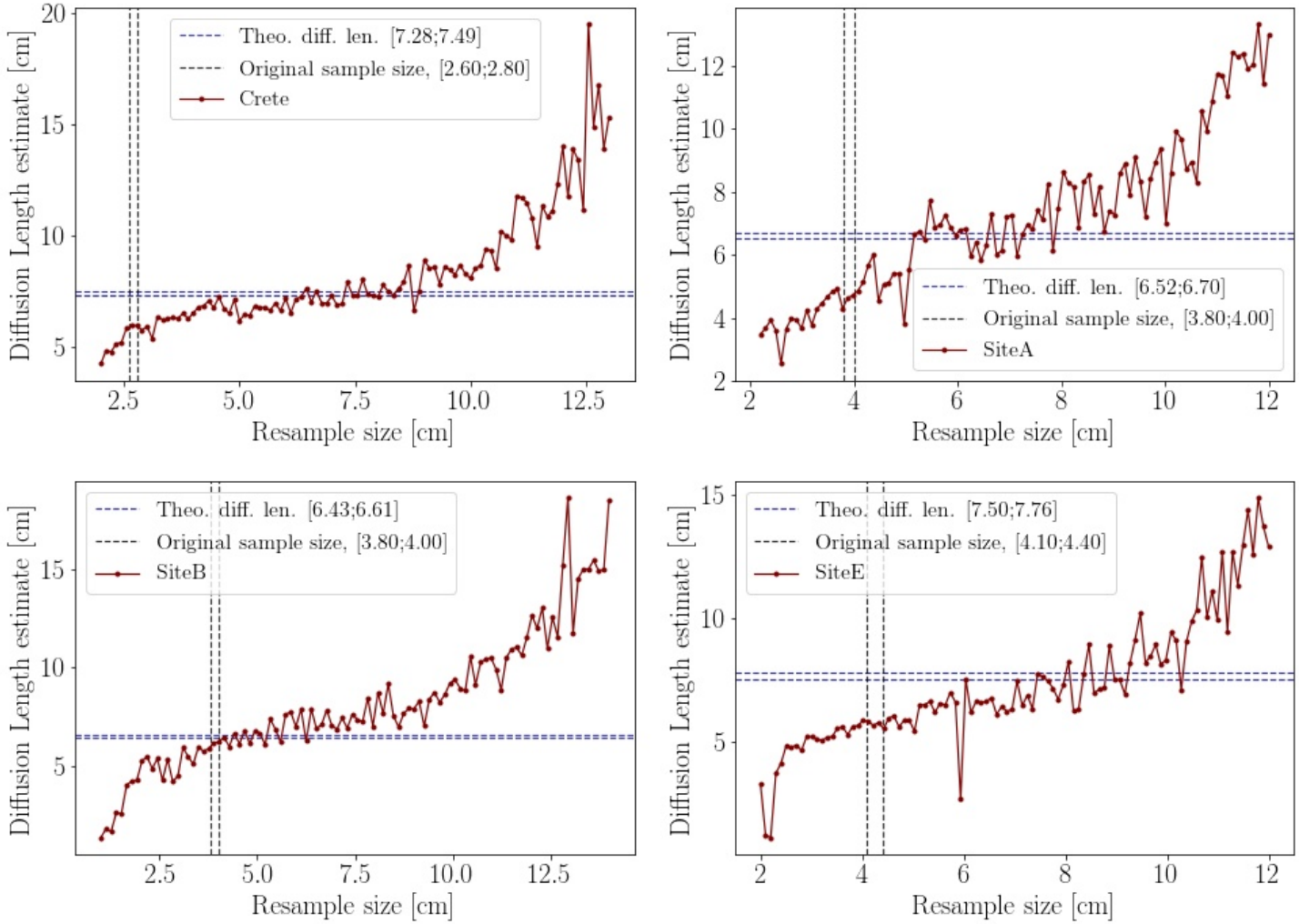

Figure: Resampling size versus diffusion length estimate to result in N = 32 peaks.

Cubic Spline Interpolation: After Deconvolution

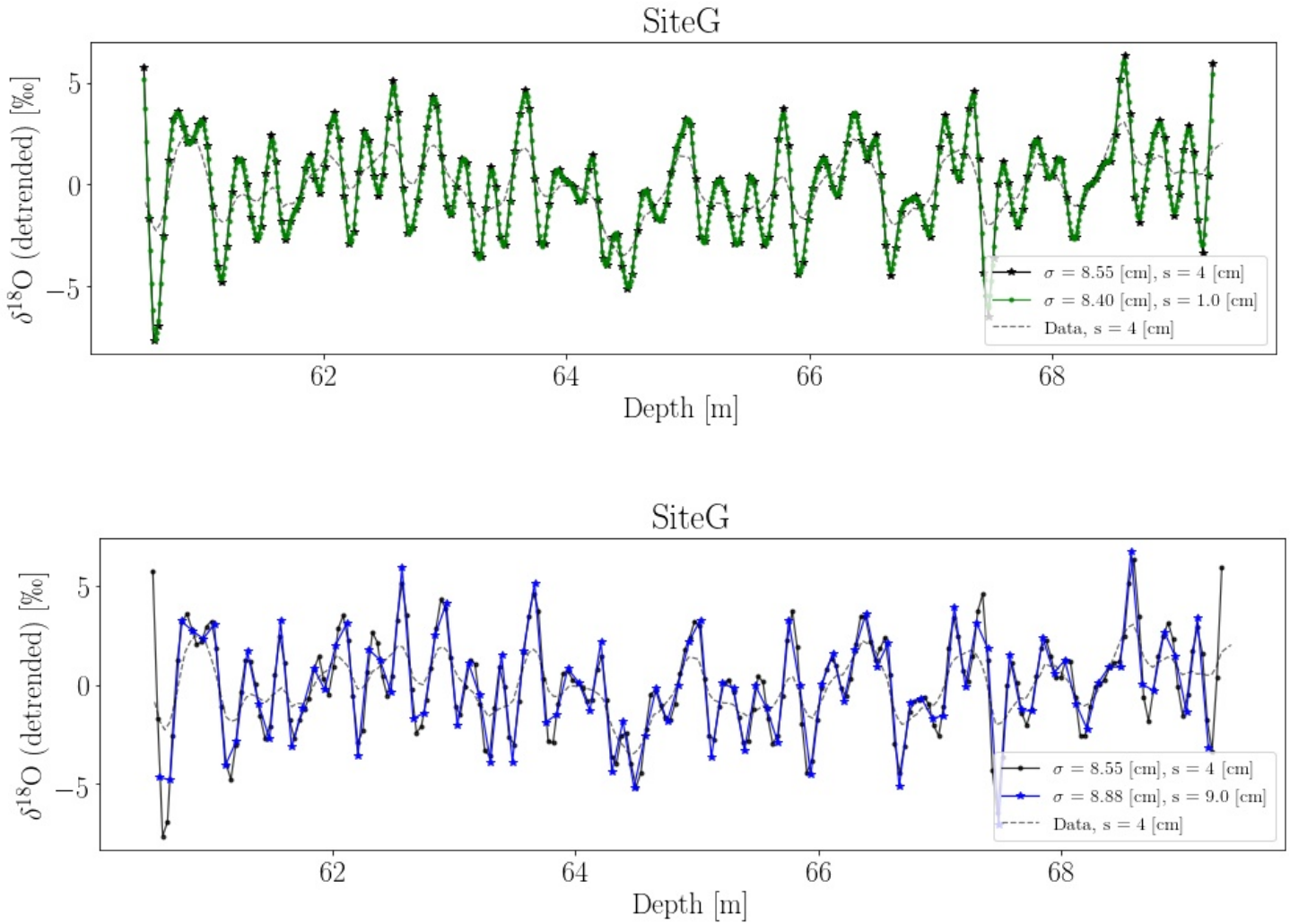

Figure: Deconvoluted data with resampling of 1 and 9 cm intervals after deconvolution, but before peak finding

Resampling size V. Diffusion Length Estimate

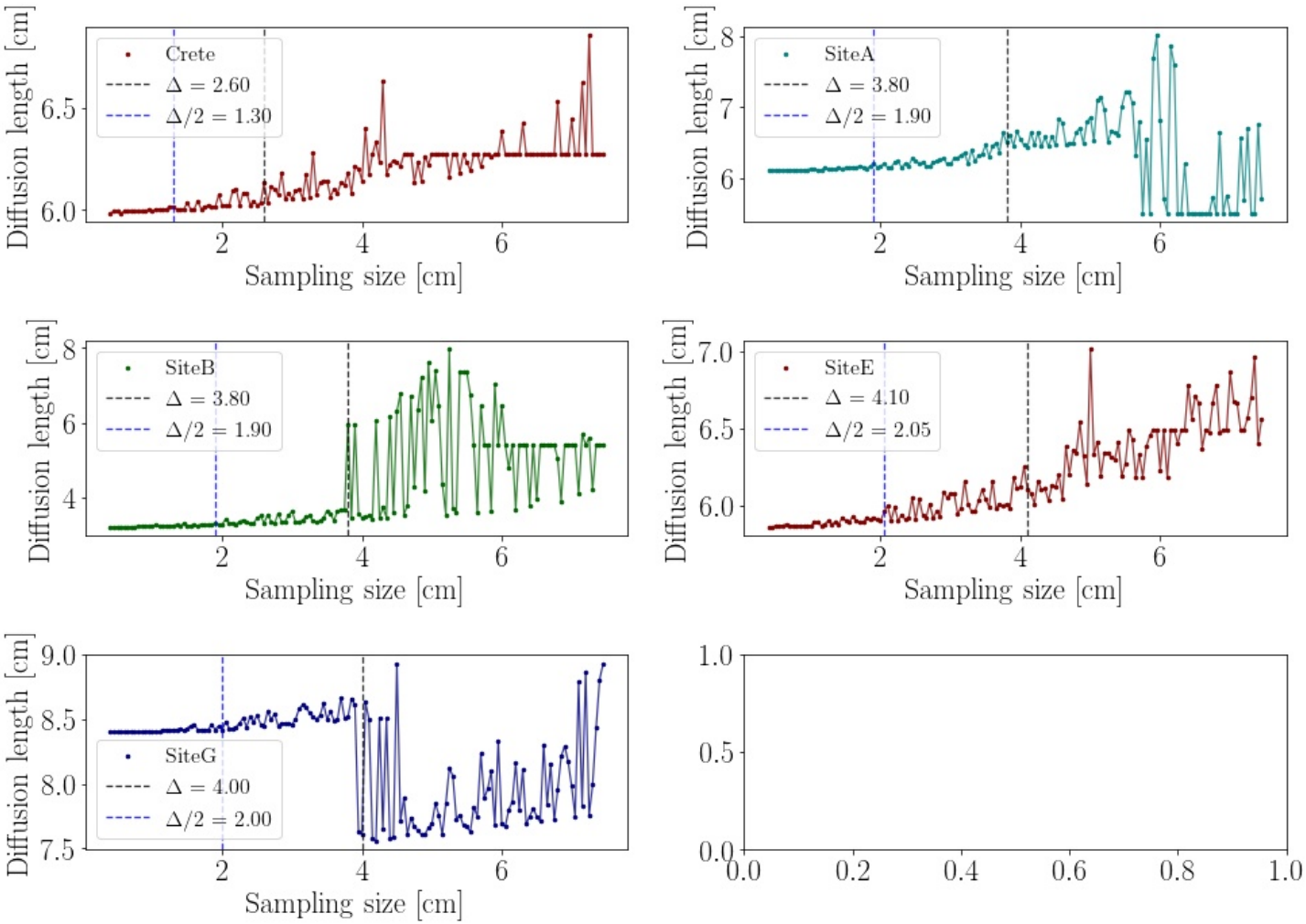

Figure: Resampling size after deconvolution versus diffusion length estimate to result in N = 32 peaks.

Site A: Theoretical V. Estimated Diffusion Length

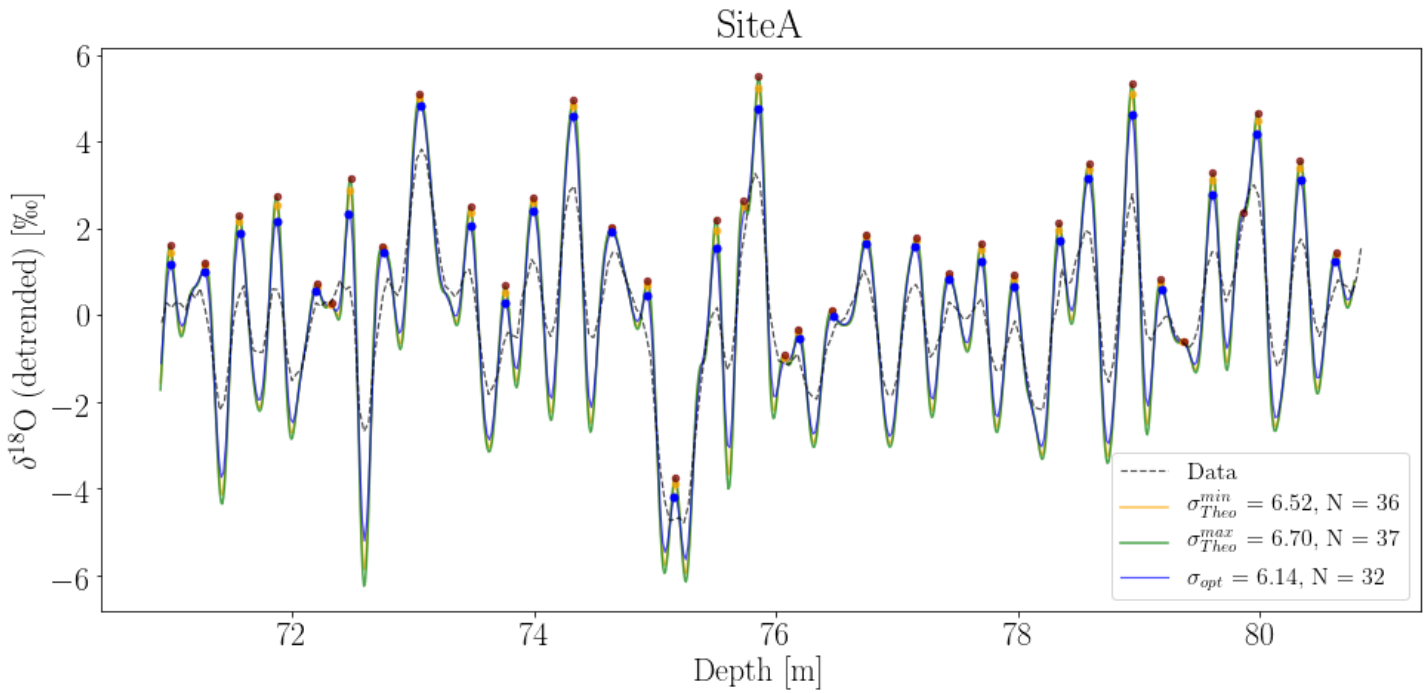

Figure: Data and back diffused signal, using theoretically predicted diffusion lengths and diffusion length estimated through analysis, Site A.

Site G: Theoretical V. Estimated Diffusion Length

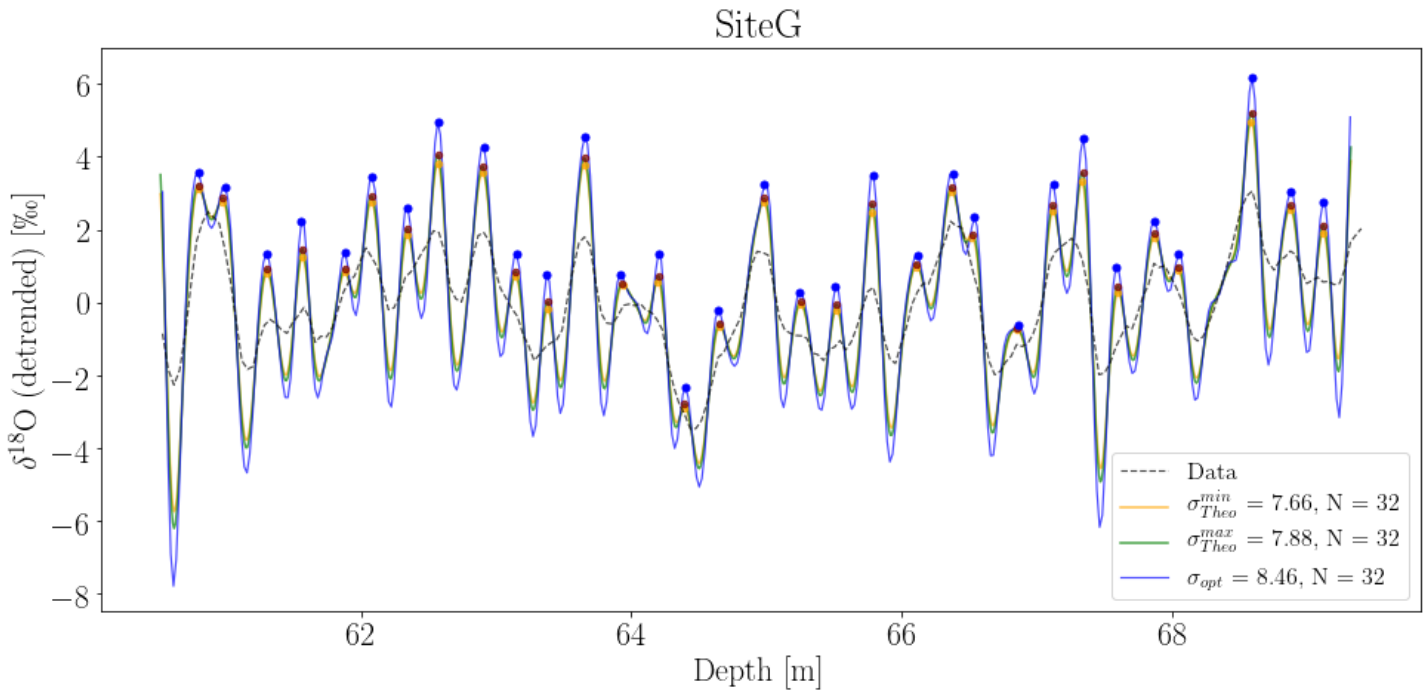

Figure: Data and back diffused signal, using theoretically predicted diffusion lengths and diffusion length estimated through analysis, Site G.

Diffusion Lengths, All Cores

	Crete	Site A	Site B	Site E	Site G
σ_{Theo}^{min} [cm]	7.28	6.52	6.43	7.50	7.66
σ_{Theo}^{max} [cm]	7.49	6.70	6.61	7.76	7.88
σ_{est} [cm]	6.02	6.14	3.27 (N = 32) 5.85 (N = 33)	5.95	8.46

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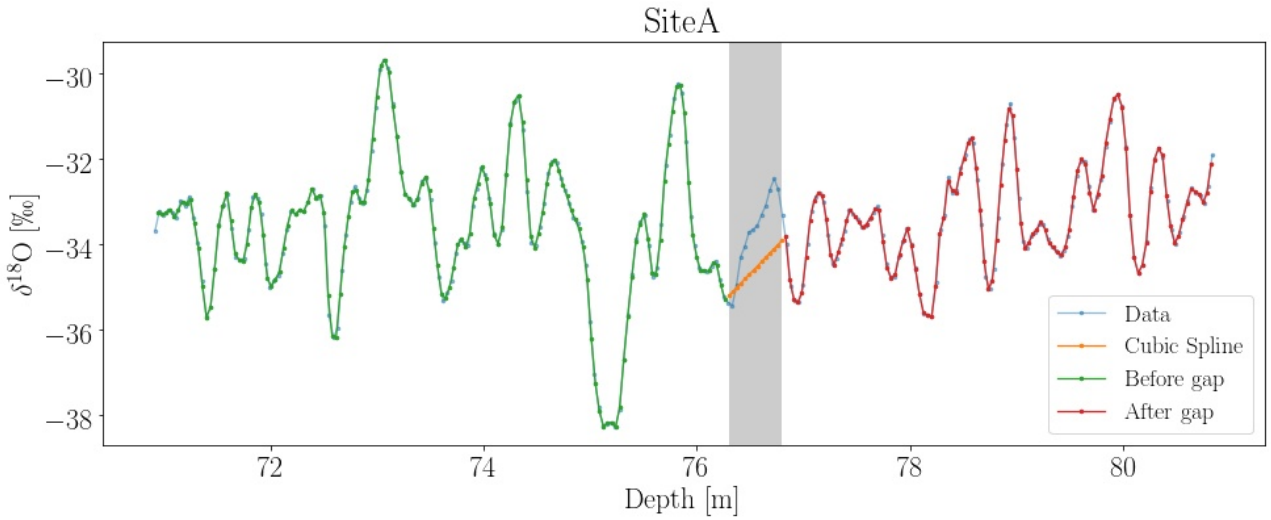
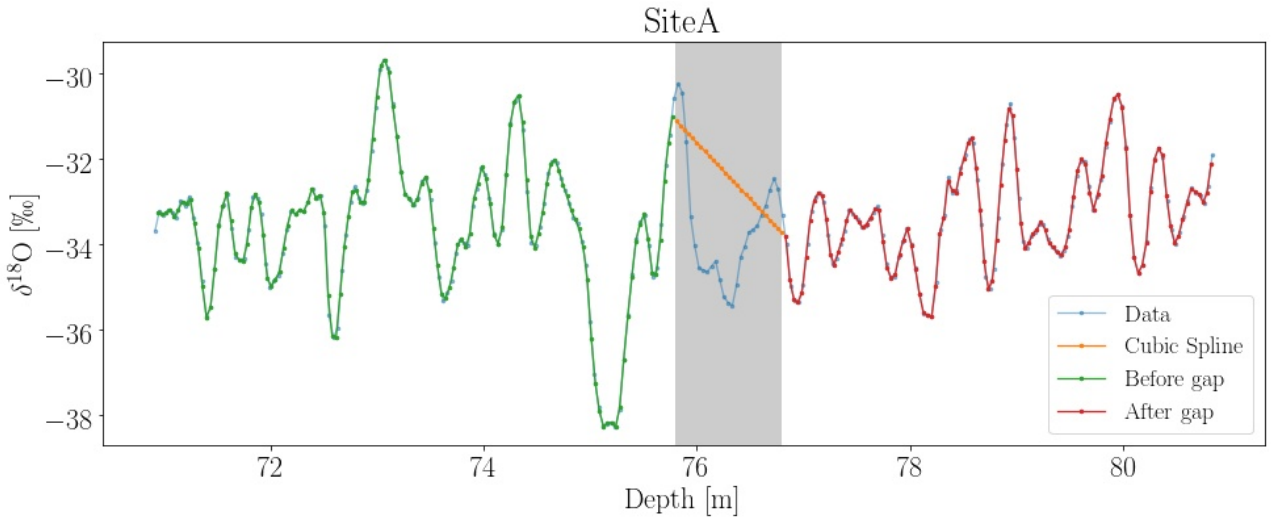
Linear Interpolation, 50 and 100 cm gaps





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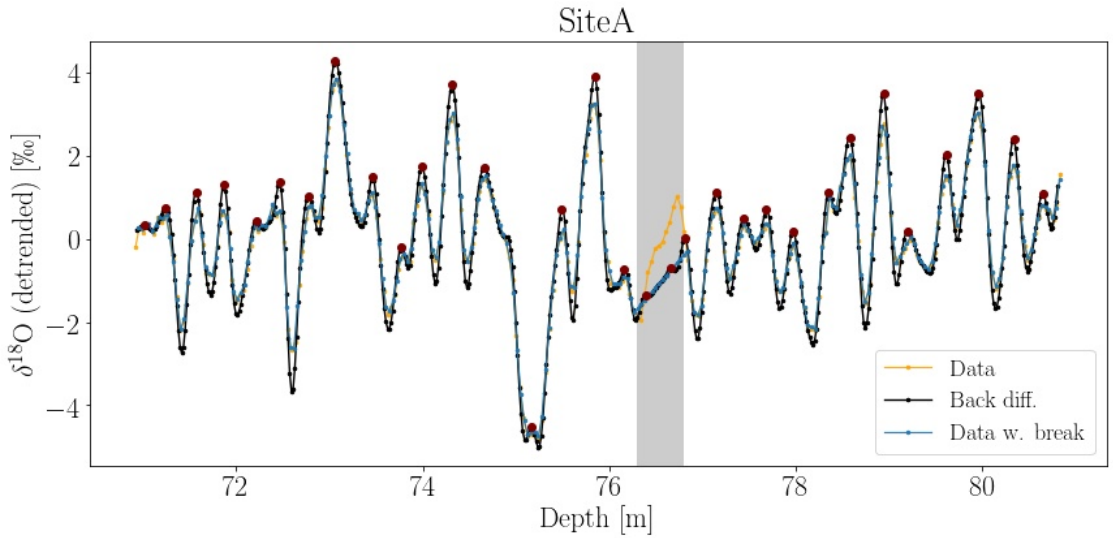

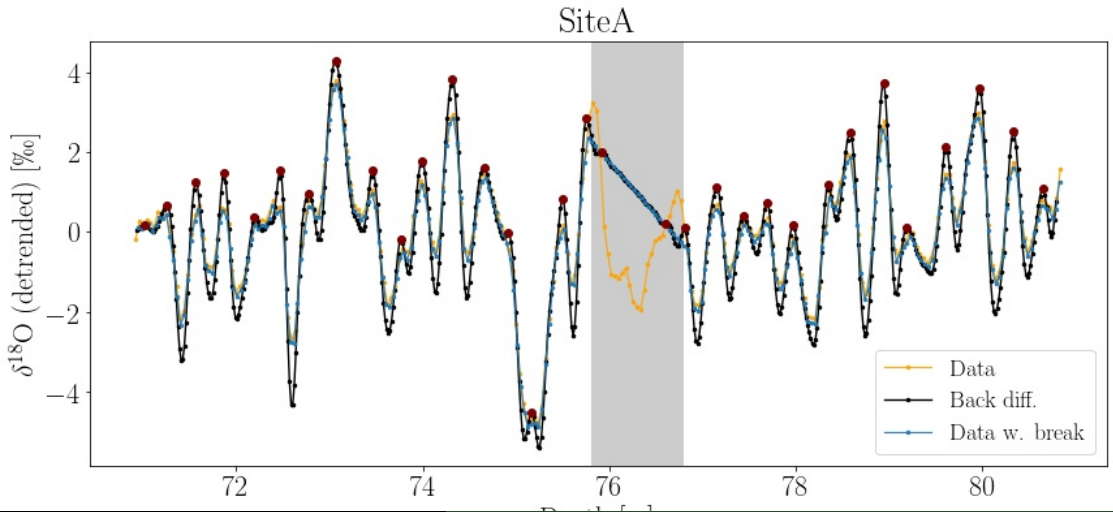

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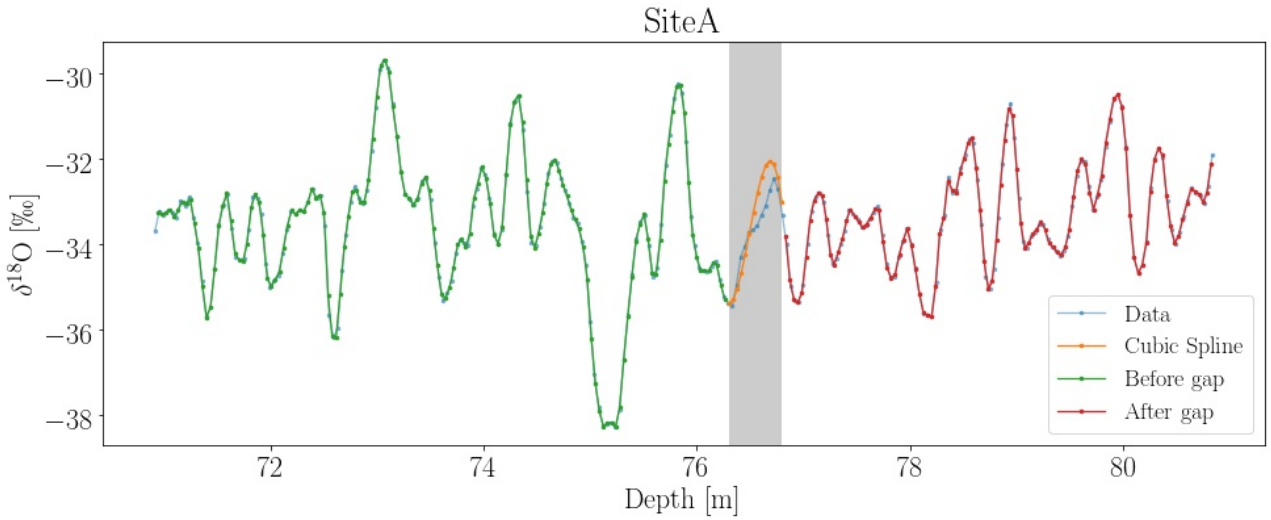

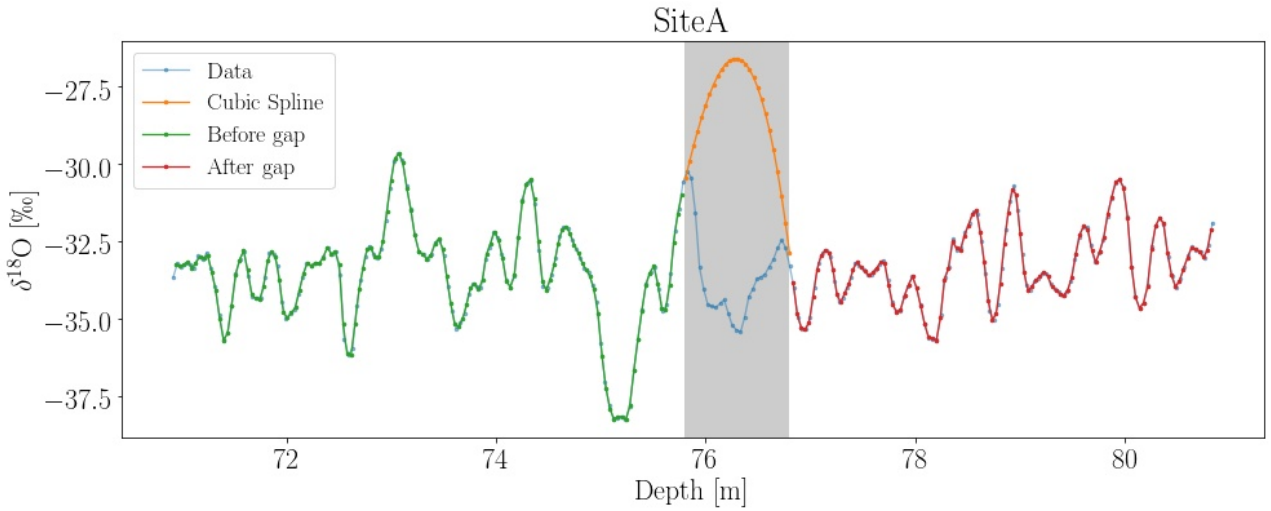

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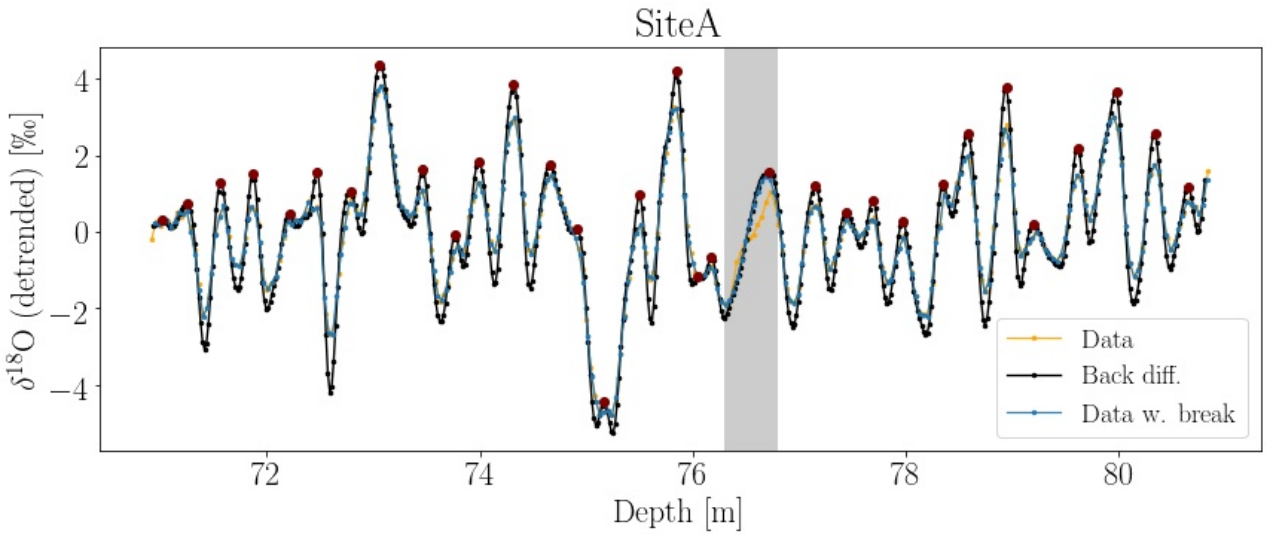

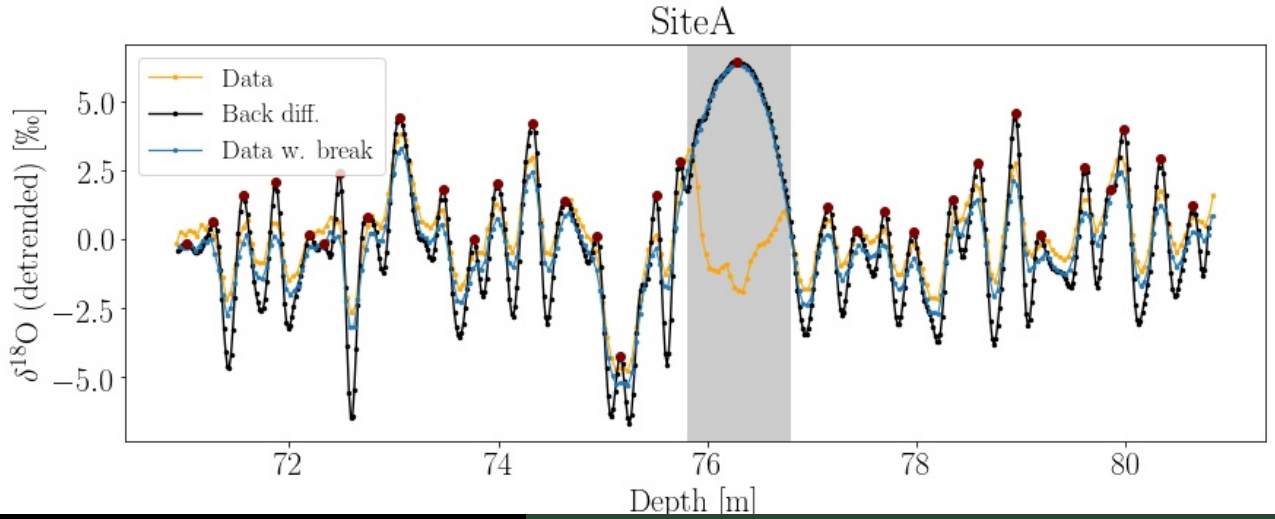

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Crete

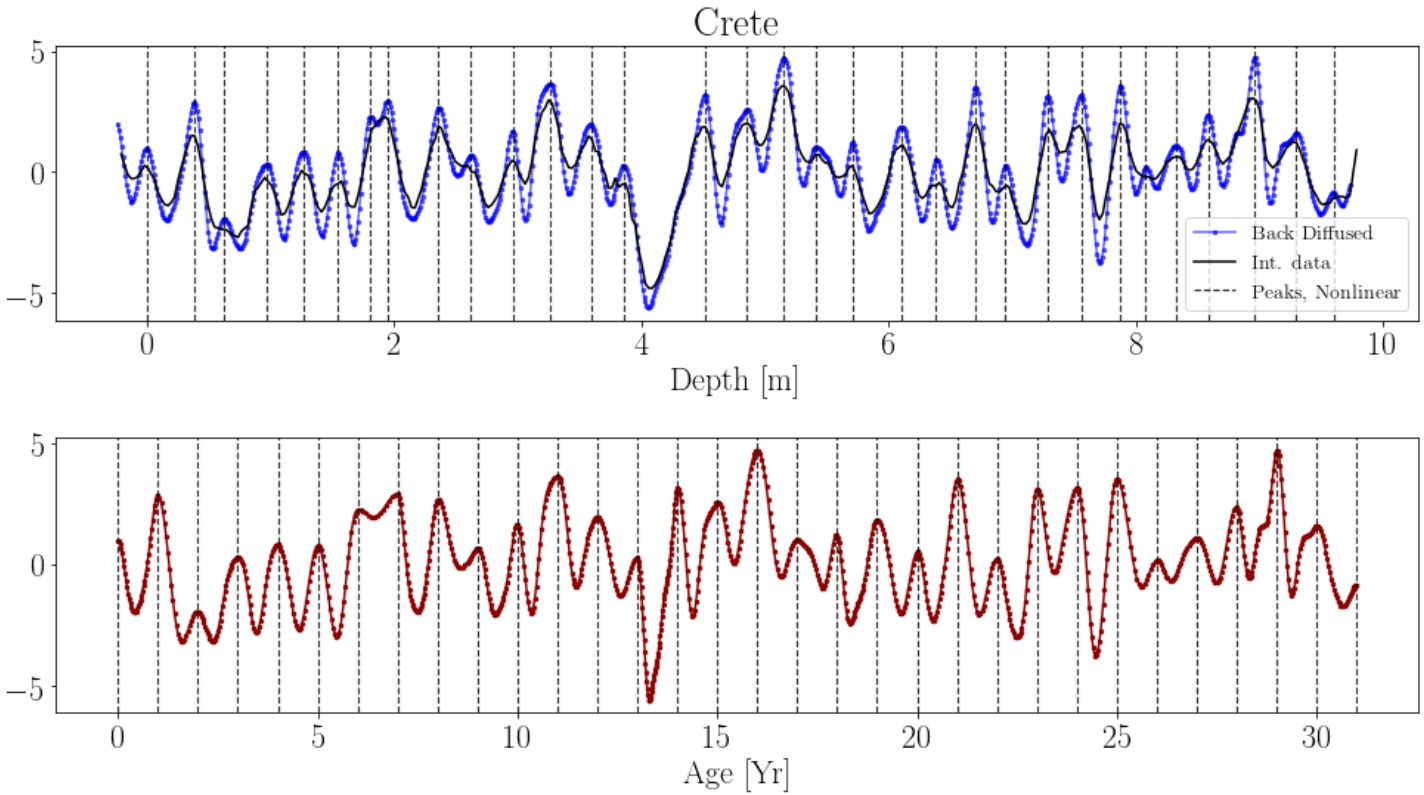

Figure: Data series on nonlinear and linearized timescales, Crete.

Site A

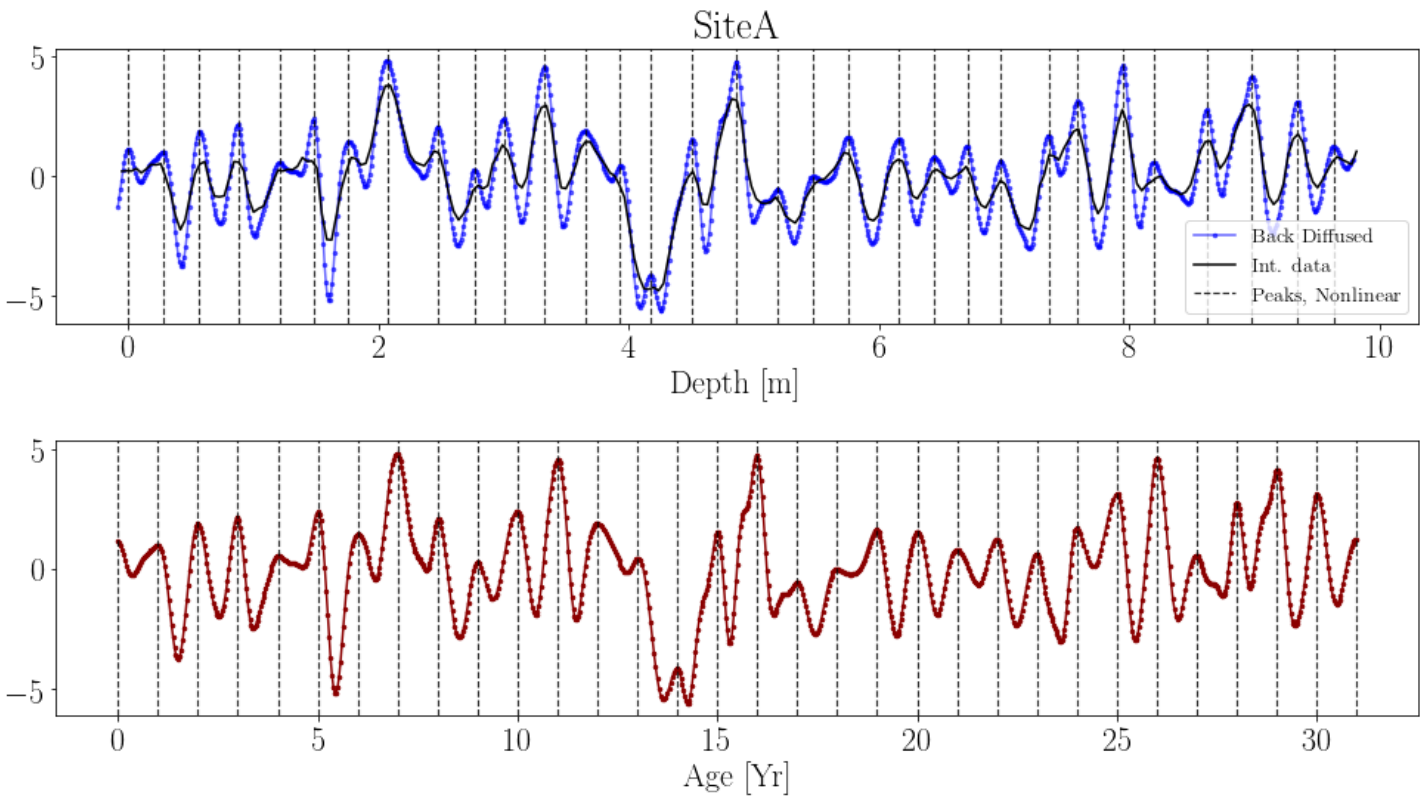

Figure: Data series on nonlinear and linearized timescales, Site A.

Site G

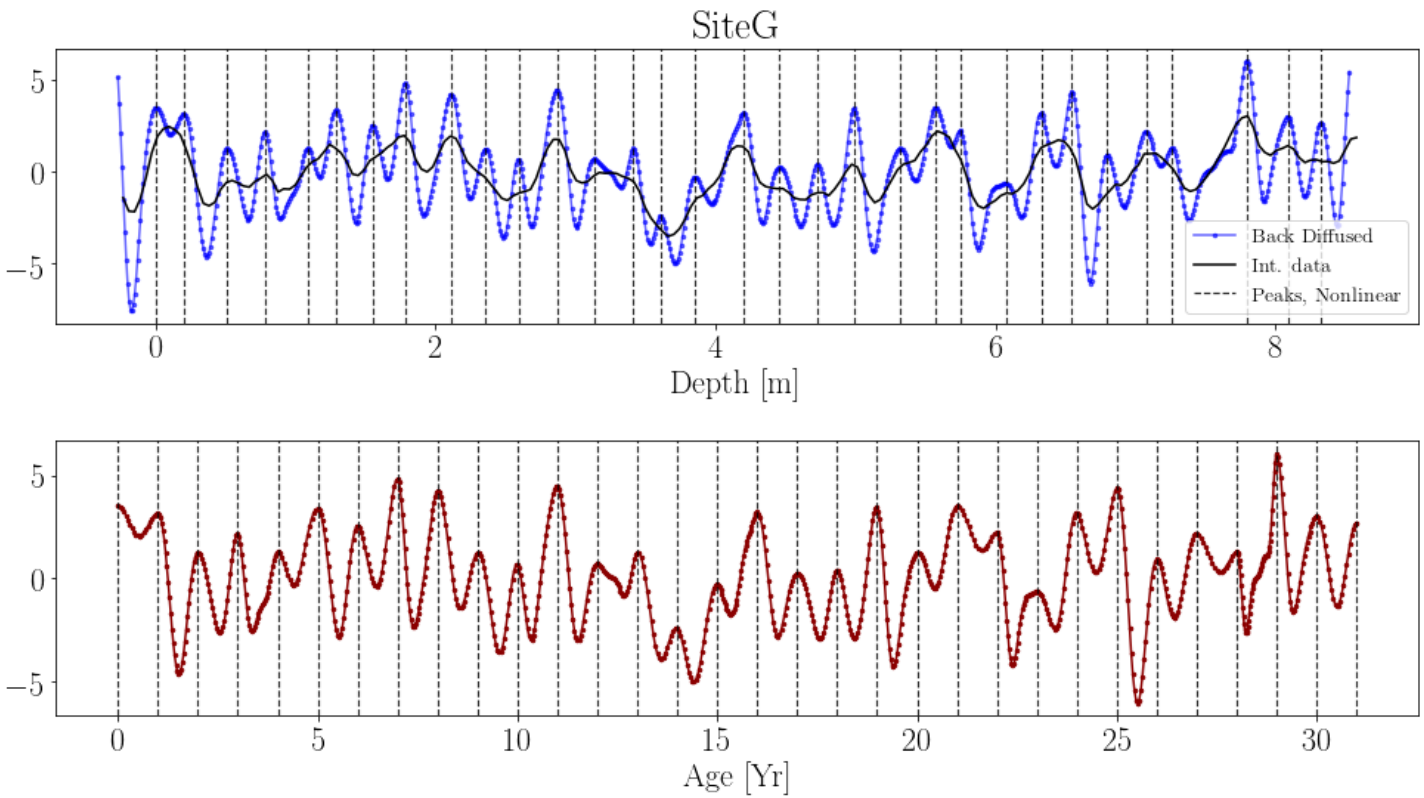

Figure: Data series on nonlinear and linearized timescales, Site G.

Further Work

- Peaks and troughs
- Accumulation seasonality
- ECM data back diffusion
- Missing data reconstruction
- Peak/cycle detection through standardization and classification

Thank you!

Any questions?

Actual Total Diffusion

Total diffusion in ice and firn

$$\sigma_{\text{tot}}(z)^2 = [S(z)\sigma_{\text{firn}}(z)]^2 + \sigma_{\text{ice}}(z)^2 \tag{8}$$

Giving an actual measured diffusion length at z_i of

$$\sigma(z_i)^2 = \sigma_{\text{firn}}(z_i)^2\, S(z_i) + \sigma_{\text{ice}}(z_i)^2 + \sigma_{\text{dis}}(z_i)^2 \tag{9}$$

with

$$\sigma_{\text{dis}}(z_i)^2 = \frac{2\Delta(z_i)^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{10}$$

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Laki and Tambora

- **Electrical Conductivity Measurements (ECM)**
- **Dielectric Profiling (DEP)**