Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

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[1]University of Copenhagen

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Outline of talk

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Water Isotopes in Ice Cores

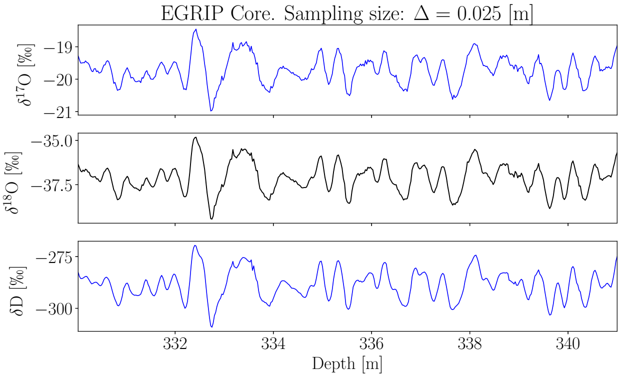

Figure: Examples of three water isotopes measured from the EGRIP core in Greenland.

Diffusion in Firn

- Fick's 2^{nd} law:

$$\frac{\partial \delta}{\partial t} = D(t)\frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}}, \quad \text{a Gaussian filter,} \tag{3}$$

and

$$\mathcal{S}(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'}, \quad \text{the thinning function} \tag{4}$$

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Actual Total Diffusion

Total diffusion in ice and firn

$$\sigma_{\text{tot}}^2 = [S(z)\sigma_{\text{firn}}]^2 + \sigma_{\text{ice}}^2(z) \quad (5)$$

Giving an actual measured diffusion length of

$$\hat{\sigma}_i^2 = \sigma_{\text{firn}}^2 S(z) + \sigma_{\text{ice}}^2 + \sigma_{\text{dis}}^2 \quad (6)$$

with

$$\sigma_{\text{dis}}^2 = \frac{2\Delta^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \quad (7)$$

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Laki to Tambora
T. Quistgaard
So Far...
Water Isotopes
Data
Volcanic Horizons
Back Diffusion
And now?
Detrend and Standardize
Layer Counting Algorithm
Outlook
Layer Counting Algorithm, Cont.

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Example Data: Site A

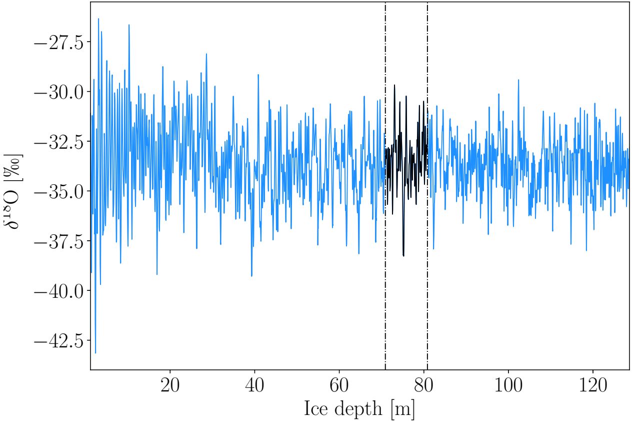


Figure: Example data from Alphabet Core drilled at site A near Crête.

Unevenly Sampled Data: Spline Interpolation

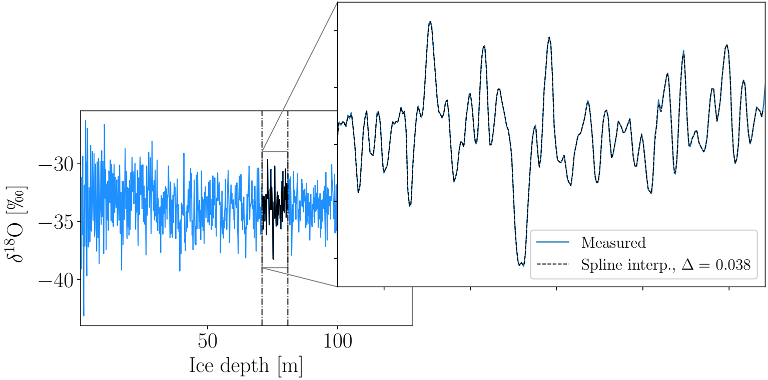
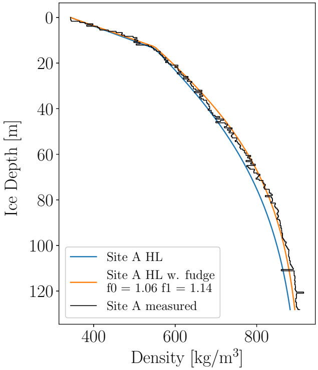
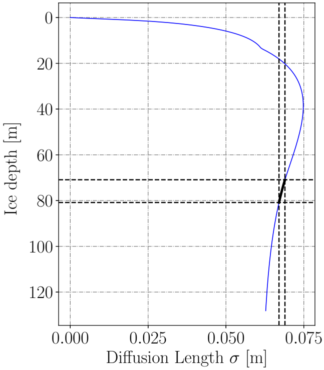


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.

Community Firn Model



(a) Density-depth profiles based on analytical Herron-Langway model. Black is empirical data, blue is purely analytical fit and orange is fudged analytical fit



(b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.

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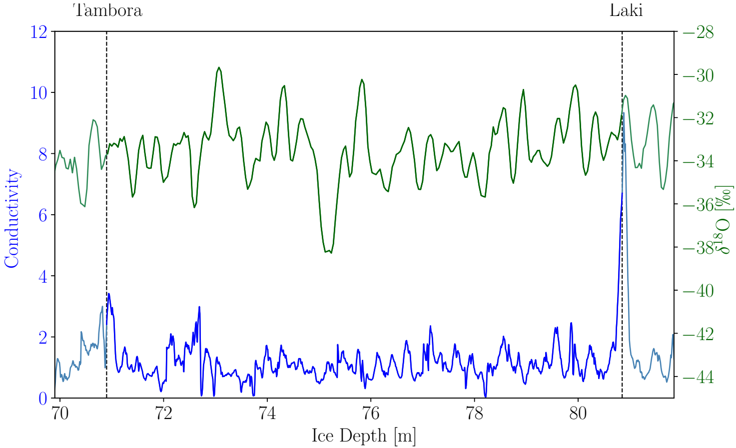
Laki and Tambora

- **Electrical Conductivity Measurements (ECM)**
- **Dielectric Profiling (DEP)**



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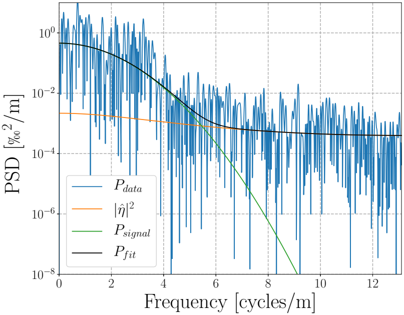
Spectral Analysis with DCT



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_\eta^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\text{signal}} = P_0 e^{-k^2 \sigma^2}$$

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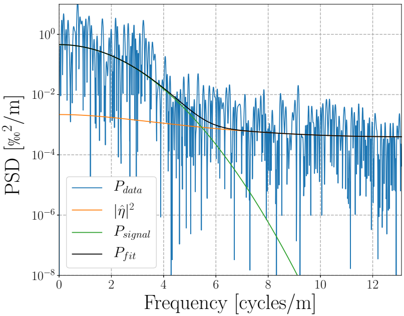

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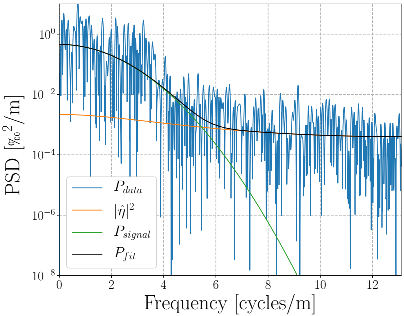

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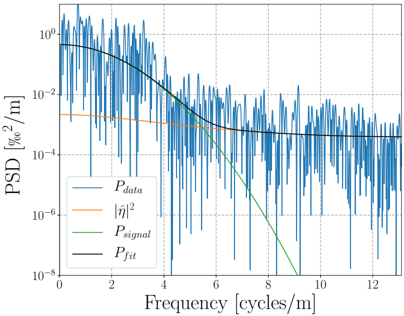

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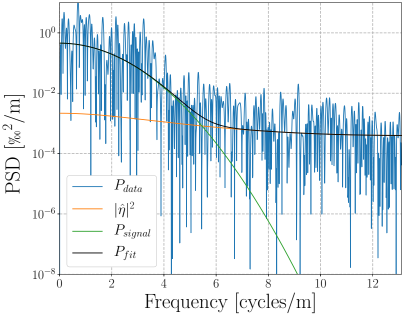

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Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1} \tag{8}$$

Add an optimal Wiener filter to enhance signal and minimize noise:

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

yielding a restoration filter as

$$\tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\text{meas}} \cdot \tilde{R} \tag{10}$$

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Filtering

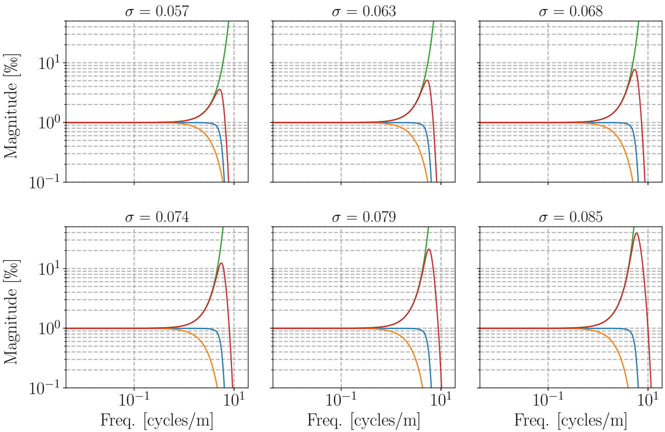

Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

Deconvolution

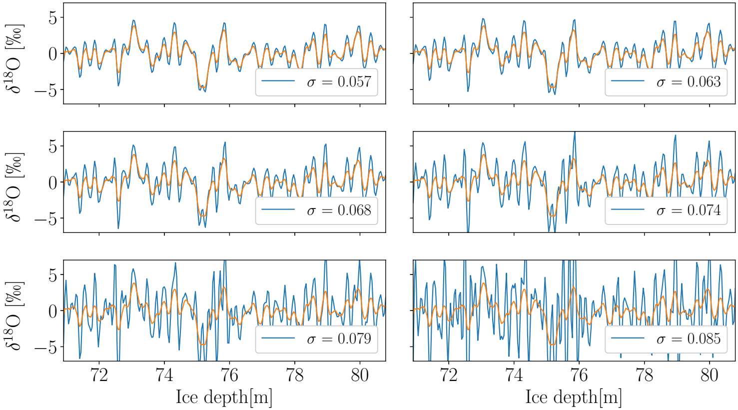

Figure: The estimated restored signal (blue) given diffusion length. Plotted along with original measured data (orange).

Enhanced Signal, Minimized Noise

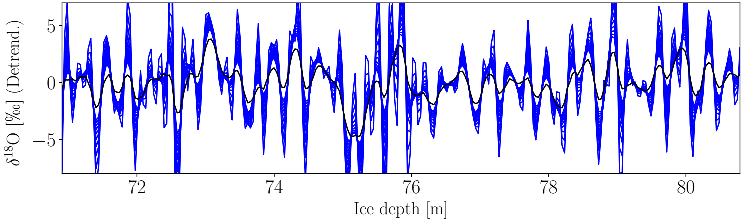

Figure: The original data plotted along with each estimate of the restored data with diffusion lengths ranging from 0.057 to 0.085.

Enhanced Signal, Minimized Noise

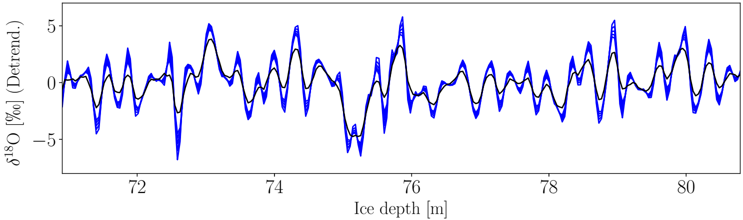

Figure: The original data plotted along with each estimate of the restored data with diffusion length $\sigma^2 < 0.07$.

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Estimating Cycle Length - ACF

$$R_k = \frac{1}{(n-k)\sigma^2} \sum_{i=1}^{n-k} (x_i - \mu)(x_{i+k} - \mu) \qquad (11)$$

The estimated cycle length, l, will be the first k such that $R_{k-1} > R_{k-2}$ and $R_k < R(k-1)$

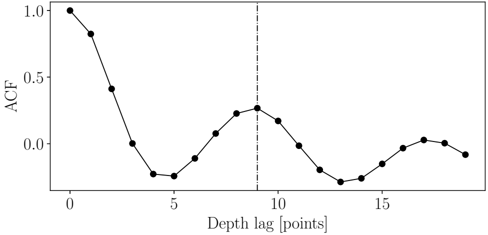

Figure: Autocorrelation as a function of (pointwise-) depth lag.

Estimating Cycle Length - Adjustment

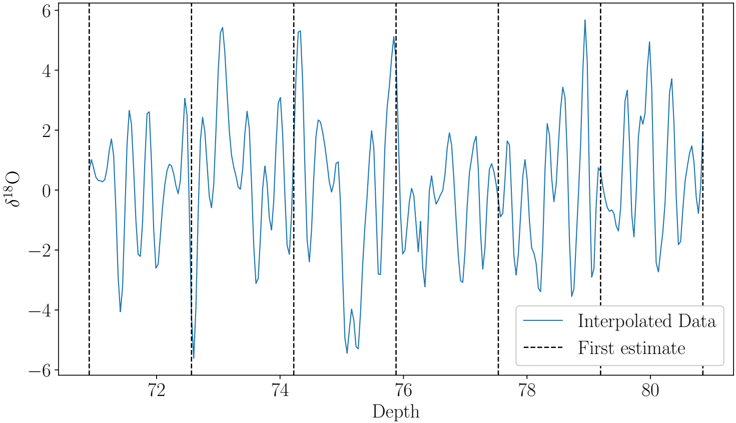

Figure: First estimate of sections.

Estimating Cycle Length - Adjustment

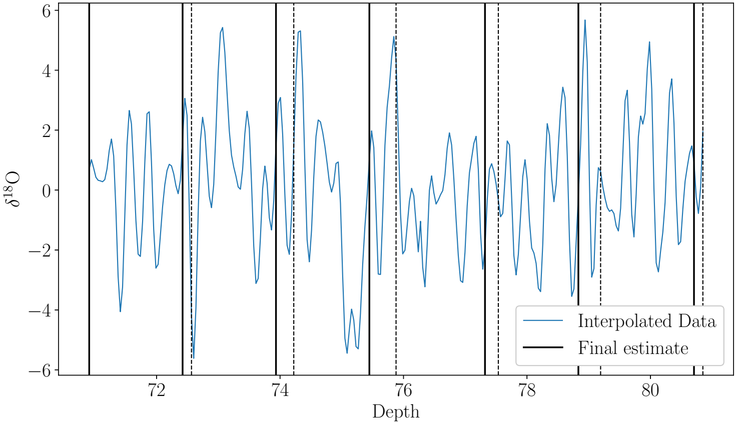

Figure: Final estimate of sections.

Detrending with Moving Average

Moving average:

$$\mu_i = \sum_{j=i-l_i/2}^{i+l_i/2} \frac{x_i}{l_i + 1} \tag{12}$$

Moving standard deviation

$$\sigma_i^2 = \sum_{j=i-l_i/2}^{i+l_i/2} \frac{(x_i - \mu_i)^2}{l_i + 1} \tag{13}$$

New standardized signal $\bar{s}$:

$$s_i = \frac{x_i - \mu_i}{\sqrt{2}\sigma_i} \tag{14}$$

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Detrending and standardize

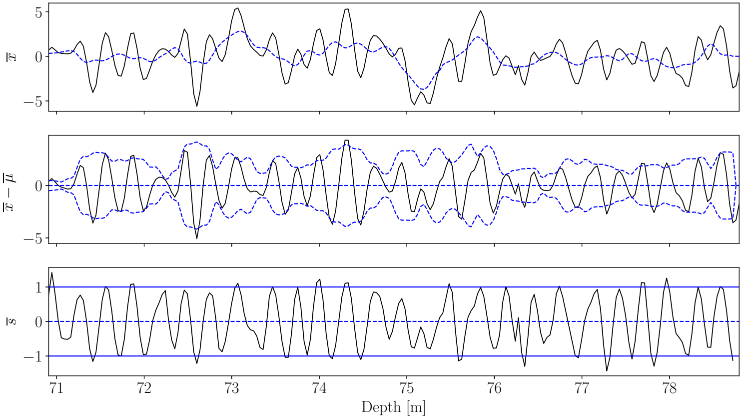

Figure: Standardization through moving average, $\bar{\mu}$, and standard deviation, $\bar{\sigma}^2$

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Peak Detection by Pattern Recognition

- Prior information: Typical annual cycle (noisy sine)
- Convolutional Neural Networks
- Kalman Filtering, MCMC or something else entirely

Layer Counting Algorithm

Prior to estimation:

- Diffusion and densification models
- Noisy sine signal

Outcome:

- Peak counting
- Dating by years
- Layer thickness approximation

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Layer Counting Algorithm

- In Different Cores, Same (Known) Age
- Down entire (Dated) Core
- Combination

Thank you!

Any questions?