T. Quistgaar

So Far...

Diffusion and Densification

Volcanic Horizo

Voicanic Honzon

Standardize

Peak Detecti

ام ماد

Layer Counting Algorithm, Cont

Laki to Tambora

Pattern Recognition in High Resolution Volcanic and Isotopic Signals

Thea Quistgaard¹

¹University of Copenhagen

November 19, 2020

Outline of talk

T. Quistgaard

So Far..

Diffusion and Densification Back Diffusion

And now

Standardize
Peak Detection
Layer Counting

Outlool

Layer Counting Algorithm, Cont So Far...

Water Isotopes
Diffusion and Densification
Back Diffusion
Volcanic Horizons

2 And now?

Detrend and Standardize Peak Detection Layer Counting Algorithm

Outlook Layer Counting Algorithm, Cont.



Table of Contents

T. Quistgaa

Water Isotopes

Diffusion and Densification Back Diffusion

_ _ _

Detrend and Standardize

Peak Detectio

Outlook

Layer Counting Algorithm, Cont. So Far...

Water Isotopes

Diffusion and Densification Back Diffusion Volcanic Horizons

2 And now?

Detrend and Standardize Peak Detection Layer Counting Algorithn

3 Outlook

Layer Counting Algorithm, Cont.

Water Isotopes in Ice Cores

T. Quistgaa

30 Far...

Water Isotopes

Diffusion and Densification

Volcanic Horizo

And now?

Standardize
Peak Detectio

A ..

Layer Counting Algorithm, Cont.

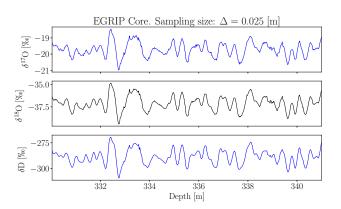


Figure: Examples of three water isotopes measured from the EGRIP core in Greenland.

T. Quistgaaı

So Far..

Water Isotopes

Diffusion ar

Back Diffus

Volcanic

And now?

Detrend and

Standardize

Layer Cou

Outlool

Layer Counting Algorithm, Cont

Diffusion in Firn

• Fick's 2nd law:

$$\frac{\partial \delta}{\partial t} = D(t) \frac{\partial^2 \delta}{\partial z^2} - \dot{\epsilon}_z(t) z \frac{\partial \delta}{\partial z} \tag{1}$$

with solution

$$\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)] \tag{2}$$

where $\delta(z)$ is the measured signal, $\delta'(z)$ is the initial isotopic signal

$$\mathcal{G}(z) = rac{1}{\sigma\sqrt{2\pi}}e^{-rac{z^2}{2\sigma^2}},$$
 a Gaussian filter, (3)

$$S(z) = e^{\int_0^z \dot{\epsilon_z}(z')dz'},$$
 the thinning function (4)



Laki to

Diffusion in Firn Tambora

Water Isotopes

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Laki to Diffusion in Firn

T. Quistgaard

50 Far..

Water Isotopes

Diffusion an Densification

Back Diffus

And now!

Detrend and Standardize

Peak Detect

Layer Coun

)utlool

Layer Counting Algorithm, Con • Fick's 2nd law:

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Diffusion in Firn

T. Quistgaa

So Far..

Water Isotopes

Diffusion an Densification Back Diffusi

And now?

Detrend and

Peak Detect

Layer Coun

)utlool

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Laki to

Diffusion in Firn Tambora

T. Quistgaard

Water Isotopes

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Diffusion in Firn

T. Quistgaard

So Far..

Water Isotopes

Diffusion an Densification

Back Diffus

Dotrond and

Standardize

Peak Detect

Algorithm

Jutlook

Layer Counting Algorithm, Cont • Fick's 2nd law:

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Diffusion in Firn and Ice

i. Quistgaa

30 Far...

Water Isotopes

Diffusion and Densification

Back Diffusion

Volcanic Horizor

Detrend and Standardize

D I D .

Layer Count

Outlook

Layer Counting Algorithm, Cont With a total diffusion given as

$$\sigma_{\mathsf{tot}}^2 = [S(z)\sigma_{\mathsf{firn}}]^2 + \sigma_{\mathsf{ice}}^2(z) \tag{5}$$

T. Quistgaard

So Far...

Water Isotopes

Diffusion and Densification

And now

Standardize
Peak Detection

Layer Countin Algorithm

)utlool

Layer Counting Algorithm, Cont

Diffusion During Measurements

Measured diffusion length is actually

$$\hat{\sigma}_i^2 = \sigma_{\mathsf{firn}}^2 S(z) + \sigma_{\mathsf{ice}}^2 + \sigma_{\mathsf{dis}}^2 \tag{6}$$

with

$$\sigma_{\mathsf{dis}}^2 = \frac{2\Delta^2}{\pi^2} \ln\left(\frac{\pi}{2}\right) \tag{7}$$

So Far...

Table of Contents

T. Quistgaa

Water Isotop

Water Isotopes
Diffusion and

Densification
Back Diffusion

Volcanic Horizon

And now?

Detrend and Standardize

Peak Detection

Outloo

Layer Counting Algorithm. Cont. So Far...

Water Isotopes

Diffusion and Densification

Back Diffusion
Volcanic Horizons

2 And now?

Detrend and Standardize Peak Detection Layer Counting Algorithn

3 Outlook

Layer Counting Algorithm, Cont.

Community Firn Model

T. Quistgaard

So Far...

So Far..

Diffusion and Densification

Back Diffusion

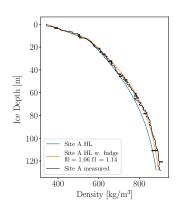
Volcanic Horizon

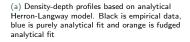
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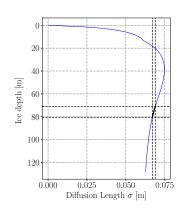
Peak Detecti Layer Counti

Outloo

Layer Counting







(b) Modeled diffusion length profile based on empirically computed density profile. Black dashed lines indicate ice depth corresponding to date Laki and Tambora eruptions.



Laki to

Tambora

Back Diffusion

Table of Contents

So Far...

Back Diffusion

And now?

Outlook

Example Data: Site A

Back Diffusion

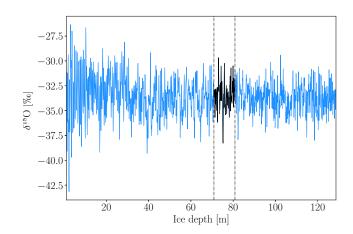


Figure: Example data from Alphabet Core drilled at site A near Crête.

So Far...

Unevenly Sampled Data: Spline Interpolation

So Far..

Diffusion a

Back Diffusion

Dack Dillusi

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Detrend and Standardize Peak Detecti

Algorithn

Layer Counting Algorithm, Cont

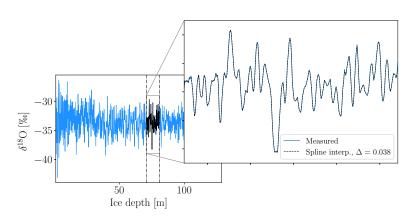


Figure: Example data from Alphabet Core drilled at site A near Crête. Shows zoom in of data from Laki to Tambora along with spline interpolated data.



Spectral Analysis with DCT

T. Quistgaar

So Far.

Water Isotope

Densificatio

Back Diffusion

And now

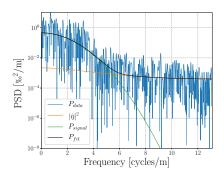
Detrend and Standardize

Peak Detect

Algorithn

Jutloo

Layer Counting Algorithm, Cont



$$P_{\text{tot}} = P_{\text{signal}} + |\hat{\eta}|^2$$

$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|^2}$$

$$P_{\mathsf{signal}} = P_0 e^{-k^2 \sigma^2}$$

Spectral Analysis with DCT

T. Quistgaar

So Far.

Diffusion and

Back Diffusion

Dack Dillus

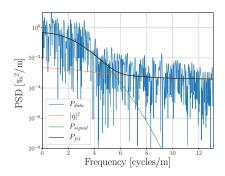
And now

Detrend and

Peak Detect

. . . .

Layer Counting



$$P_{\rm tot} = P_{\rm signal} \! + \! |\hat{\eta}|^2$$

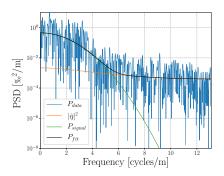
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So Far...

Spectral Analysis with DCT

Back Diffusion



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So Far...

Spectral Analysis with DCT

T. Quistgaar

So Far..

Diffusion and

Back Diffusion

Dack Dillus

And now

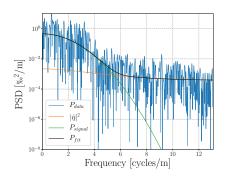
Detrend and

Peak Detect

Layer Count

Outloo

Layer Counting



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Spectral Analysis with DCT

T. Quistgaar

So Far.

Diffusion and

Back Diffusion

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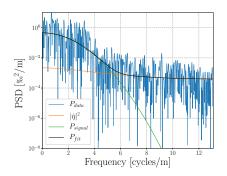
And now

Detrend and Standardize

Peak Detecti

Jutlaal

Layer Counting



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$$|\hat{\eta}|^2 = \frac{\sigma_{\eta}^2 \Delta}{|1 - a_1 e^{-ik\Delta}|}$$

$$P_{\rm signal} = P_0 e^{-k^2 {\sigma^2 \over \sigma^2}}$$

So Far...

T. Quistgaard

So Far.

Water Iso Diffusion

Densification

Back Diffusion

Back Diffusi

And now?

Detrend and Standardize

Peak Detecti

Outlool

Layer Counting Algorithm, Cont

Diffusion Lengths and Transfer Functions

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1}$$
 (8)

Add an optimal Wiener filter to enhance signal and minimize

$$\tilde{F} = \frac{P_{\text{signal}}}{P_{\text{signal}} + |\hat{\eta}|^2} \tag{9}$$

yielding a restoration filter as

$$\tilde{\delta}_{\mathsf{init}} = \tilde{\delta}_{\mathsf{meas}} \cdot \tilde{F} \cdot \tilde{M}^{-1} = \tilde{\delta}_{\mathsf{meas}} \cdot \tilde{R} \tag{10}$$

So Far...

T. Quistgaard

Back Diffusion

And now?

Diffusion Lengths and Transfer Functions

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So Far...

Diffusion Lengths and Transfer Functions

T. Quistgaard

So Far.

Diffusion a

Back Diffusion

Volcanic Hori

And now?

Allu llow

Standardize

Peak Detecti

Layer Countir Algorithm

Outlool

Layer Counting Algorithm, Cont.

$$\tilde{\delta}_{\text{meas}} = \tilde{\delta}_{\text{init}} \cdot \tilde{M} \Leftrightarrow \tilde{\delta}_{\text{init}} = \tilde{\delta}_{\text{meas}} \cdot \tilde{M}^{-1}$$
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So Far...

Diffusion Lengths and Transfer Functions

T. Quistgaard

Back Diffusion

And now?

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Back Diffusion

Laki to Tambora

So Far...

Filtering

Back Diffusion

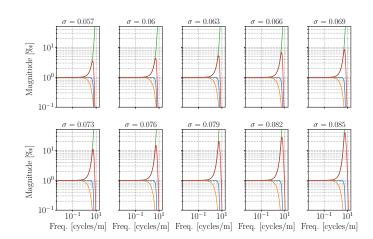


Figure: Frequency filters: The optimal filter found from the PSD (blue), the transfer function (orange), the inverse of the transfer function (green) and the combined signal restoration filter (red).

So Far...

Deconvolution

T. Quistgaa

So Far

Diffusion a

Back Diffusion

Detrend and Standardize Peak Detecti

Jutloo

Layer Counting Algorithm, Cont

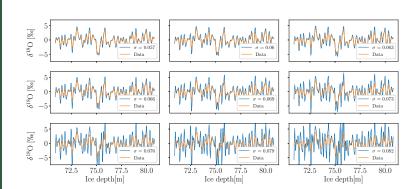


Figure: The estimated restored signal (blue) given diffusion length. Plotted along with original measured data (orange).

Enhanced Signal, Minimized Noise

i. Wuistgaa

So Far..

Diffusion a

Densification

Back Diffusion

And now?

Standardize

Peak Detecti Layer Countii

Dutloo

Layer Counting Algorithm, Cont

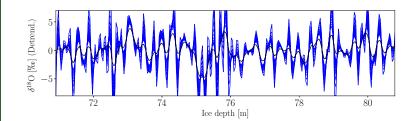


Figure: The original data plotted along with each estimate of the restored data, given diffusion length.

Enhanced Signal, Minimized Noise

So Far..

Diffusion a

Back Diffusion

Back Diffusi

Detrend and Standardize

Peak Detecti Layer Countii

utloo

Layer Counting

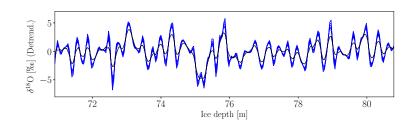


Figure: The original data plotted along with each estimate of the restored data, given diffusion length.

So Far...

Table of Contents

T. Quistgaa

So Far..

Diffusion and

Densification

Volcanic Horizons

Voicume Horizon

Detrend and Standardize

Peak Detection

Outlook

Layer Counting Algorithm, Cont. So Far...

Water Isotopes

Diffusion and Densification

Back Diffusion

Volcanic Horizons

2 And now?

Detrend and Standardize Peak Detection Layer Counting Algorithn

3 Outlook

Layer Counting Algorithm, Cont.

Laki to

Tambora

So Far...

DEP and ECM

Volcanic Horizons

- Electrical Conductivity Measurements (ECM)
- Dielectric Profiling (DEP)

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Laki to Tambora

So Far...

DEP and ECM

So Far.

Diffusion ar

Densificatio

Volcanic Horizons

Detrend and Standardize

Peak Detecti

Layer Coun Algorithm

utloo

Layer Counting Algorithm, Cont

- Electrical Conductivity Measurements (ECM)
- Dielectric Profiling (DEP)

So Far...

Laki and Tambora

T. Quistgaar

So Far..

Water Isotop

Densification

Volcanic Horizons

Voicanic Horizo

Detrend and Standardize

Peak Detect Layer Count Algorithm

Dutloo

Layer Counting Algorithm, Cont

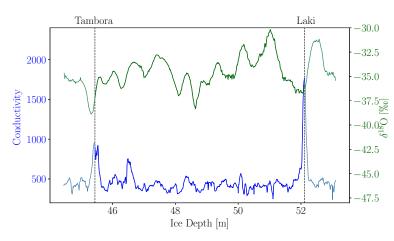


Figure: Example of volcanic horizons used for dating of cores, core B22.



Table of Contents

T. Quistgaa

So Far..

Diffusion and Densification

And now?

Detrend and Standardize

Peak Detect Layer Count

Outlool

Layer Counting Algorithm, Cont. So Far...

Water Isotopes
Diffusion and Densification
Back Diffusion
Volcanic Horizons

2 And now?

Detrend and Standardize

Peak Detection
Layer Counting Algorithm

Outlook
 Laver Counting Algorithm Cont

Estimating Cycle Length

T. Quistgaa

So Far.

Diffusion and

Densification

Back Diffusion

Volcanic Horizo

And now

Detrend and Standardize

Peak Detection

.

Layer Counting

Detrend and Standardize

Laki to Tambora

Detrending with Moving Average

So Far..

Diffusion and

Densification

...

Volcanic Horizor

Detrend and

Standardize

Peak Detectio

Outloo

Layer Counting Algorithm, Con



Outlook

Detrend and Standardize

Tambora

Standardize Amplitude Laki to

Detrend and

Standardize

So Far...

Table of Contents

T. Quistgaa

So Far..

Diffusion and Densification

Volcanic Ho

And now?

Standardize

Peak Detection

Laver Coun

Algorithm

Lavor Countin

So Far...

Water Isotopes

Diffusion and Densification

Back Diffusion

Volcanic Horizons

2 And now?

Detrend and Standardize

Peak Detection

Layer Counting Algorithm

3 Outlook

Layer Counting Algorithm, Cont.



Peak Detection

Laki to Tambora

MLP Neural Networks

So Far..

Water Isoto

Densificatio

Back Diffusion

Volcanic Horizon

And now?

Detrend ar

Peak Detection

Laver Counti

· · ·

Layer Counting Algorithm, Con

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So Far...

Kalman Filtering

_

So Far..

Water Isot

Diffusion ar

D 1 D:W :

Back Diffusi

Voicanic monze

Detrend and

Standardize

Peak Detection

Layer Count

0.....

Layer Counting Algorithm, Con

Table of Contents

T. Quistgaa

So Far..

Diffusion and Densification

Voicanic Hori

Detrend and

Peak Detecti

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont So Far...

Water Isotopes
Diffusion and Densification
Back Diffusion

And now?

Detrend and Standardize Peak Detection

Layer Counting Algorithm

Outlook

Layer Counting Algorithm, Cont.

Ouistaa

So Far.

Water Isoto

Diffusion an

Densification

Back Diffusi

Volcanic H

And now

Detrend and

Standardize

Peak Detection

Laver Counting

Algorithm

Outlook

Layer Counting Algorithm, Cor

- Counting by Peaks (Given diff. len. and est. cycle length)
- Counting by Layer Thickness

Table of Contents

T. Quistgaa

So Far...

Diffusion and Densification

Voicume 11011

Detrend and Standardize

Peak Detection

Outloo

Layer Counting Algorithm, Cont. So Far...

Water Isotopes
Diffusion and Densification
Back Diffusion
Volcanic Horizons

2 And now?

Detrend and Standardize Peak Detection Layer Counting Algorithn

Outlook Layer Counting Algorithm, Cont.

Layer Counting Algorithm

..........

So Far.

Diffusion and

Densification

Back Diffusion

Volcanic Horizo

Detrend and

Standardize Peak Detecti

Layer Counti

Dutloo

Layer Counting Algorithm, Cont

- In Different Cores, Same (Known) Age
- Down entire (Dated) Core
- Combination

References I

T. Quistgaard