Laki to Tambora

Signal Restoration and Pattern Recognition in Ultra High Resolution Volcanic and Isotopic Signals

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Laki to Tambora

Figure: J. M. W. Turner: *The Eruption of the Soufriere Mountains in the Island of St Vincent, 30 April 1812*, 1815

Outline

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Water Isotopes in Greenlandic Ice Cores

$$\delta^{18}\mathrm{O} = \frac{^{18}R_{sample}}{^{18}R_{reference}} - 1, \text{ where } {^{18}R} = \frac{n_{^{18}\mathrm{O}}}{n_{^{16}\mathrm{O}}}$$

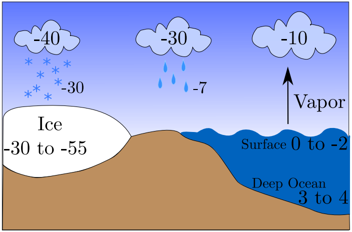

Figure: Fractionation through evaporation, transportation and precipitation along with typical water isotopic ratios in [‰]

Water Isotopes in Greenlandic Ice Cores: Crête

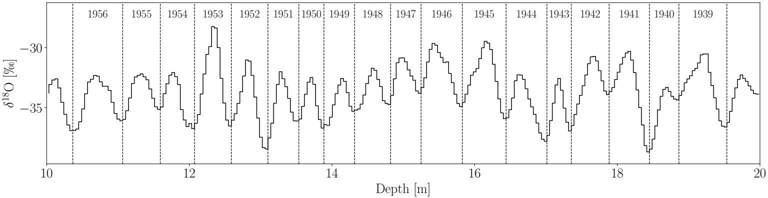

Figure: Ten meters of the top of Cretê ice core, with identification and dating of 19 annual layers. Dating as from `https://www.iceandclimate.nbi.ku.dk/research/strat_dating/annual_layer_count/ice_core_dating/`

Water Isotopes in Greenlandic Ice Cores: Diffusion´

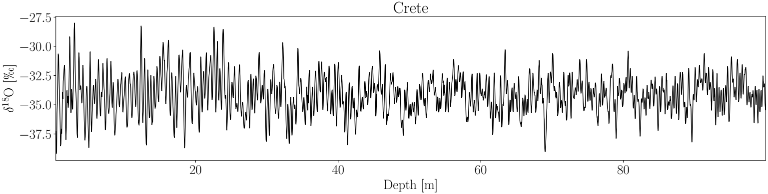

Figure: 100 meters of the Crête ice core, with diffusion visible.

- Diffusion: Displacement of water isotopes in the firn column
- **Parameter of interest**: Diffusion length, σ
- σ dependent on temperature T

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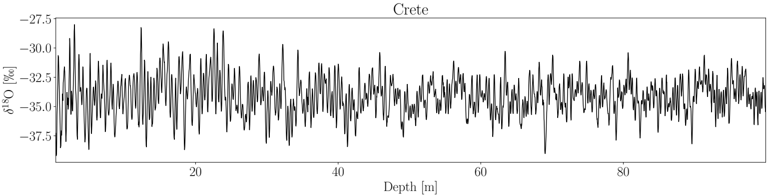

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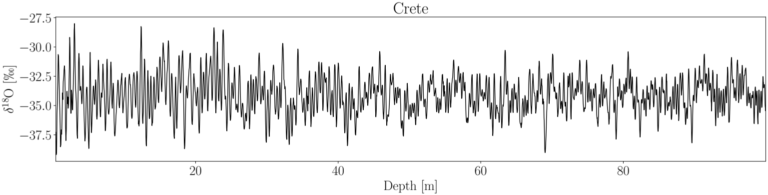

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Ice Cores in this Project

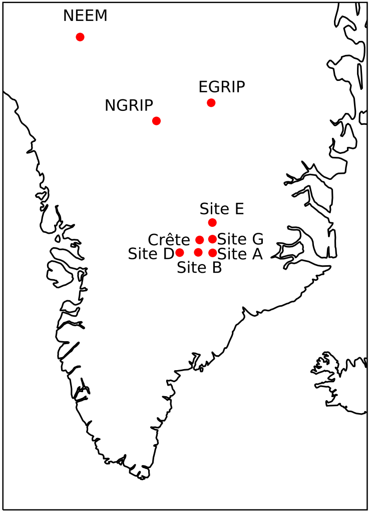

Figure: Location of Alphabet cores along with some major ice cores, NEEM, EGRIP and NGRIP.

Ice Cores in this Project

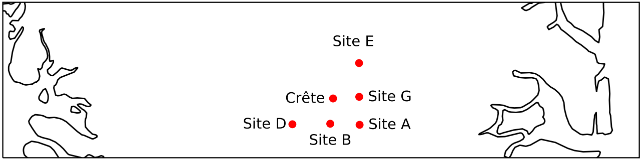

Figure: Map of spatial locations of the shallow Alphabet cores and deep core Crête analyzed in this thesis.

A Rare Gem of Knowledge

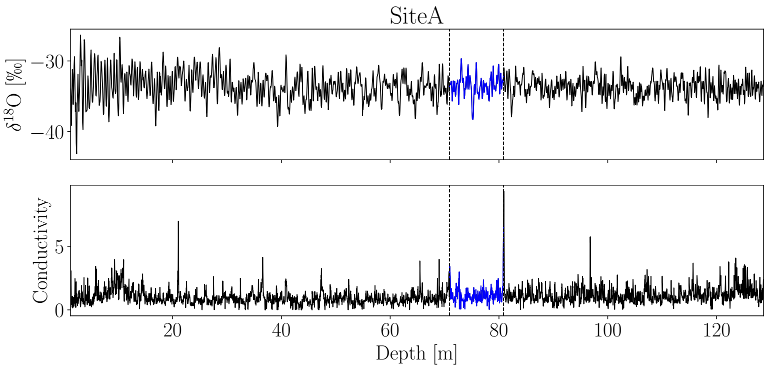

Figure: Site A, full δ^{18}O and ECM data series.

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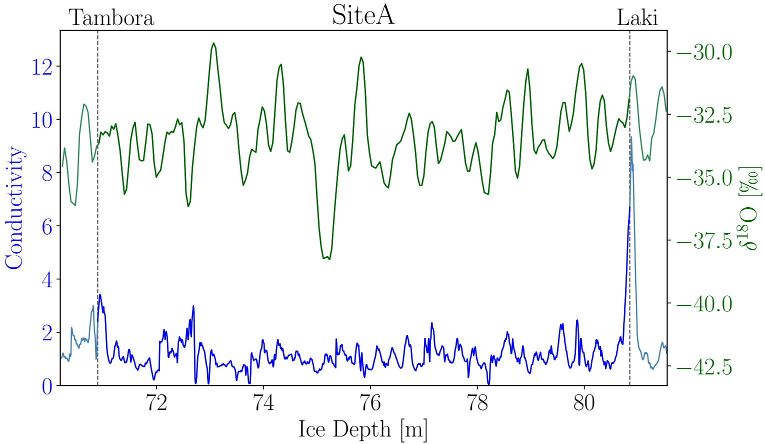

Figure: Site A, Laki to Tambora δ^{18}O and ECM data series.

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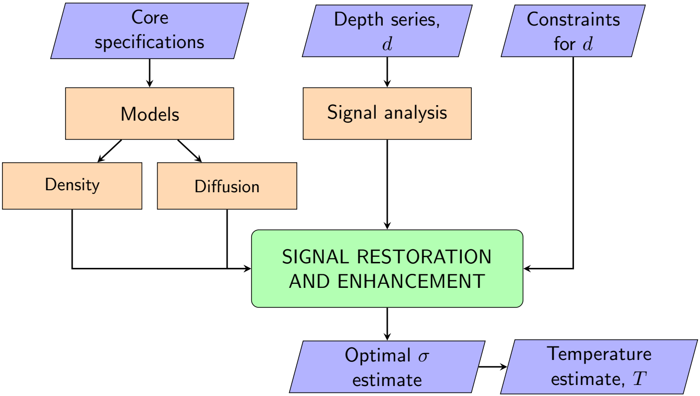
Goal and Objective



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Densification

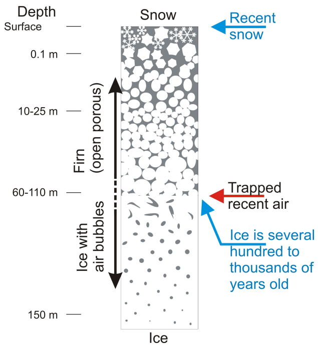

Figure: Illustration of densification process in firn column, from snow to glacial ice. Image source Blunier, T., and J. Schwander (2000), and `https://www.iceandclimate.nbi.ku.dk/research/drill_analysing/cutting_and_analysing_ice_cores/analysing_gasses/firn_zone/`.

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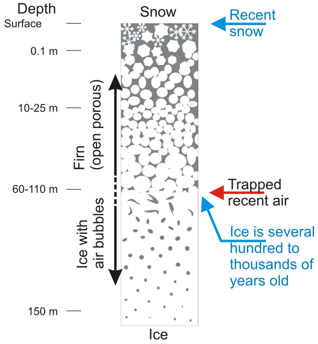

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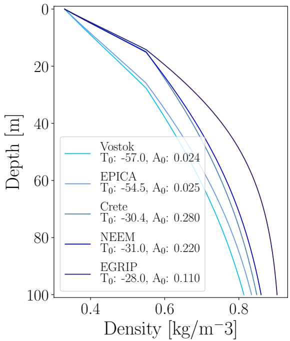

Figure: Density profile examples given five different initial conditions representing present day conditions at the five different ice core locations. Temperature, T_0, is in °C and accumulation, A_0, is in meter of water equivalent per year.

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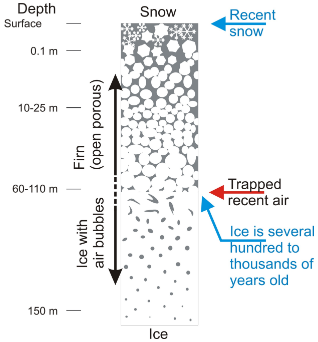

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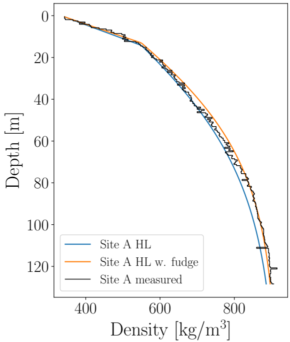

Figure: Density profiles from ice core Site A near Crête. Both purely modelled profile and profile fitted to the inputted depth-density measurements are presented. Computed through method developed in [M. M. Herron and C. C. Langway, 1980].

Diffusion

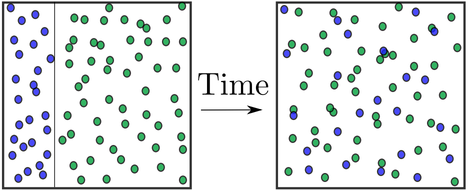

Figure: Example of diffusion over time, where two materials are mixed.

Laki to Tambora
T. Quistgaard
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Basic Problem
Goal
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Densification and Diffusion
Data
Depth Series
Signal Restoration
Spectral Analysis
Back Diffusion
Optimal Restoration
Testing
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Results
Outlook
Improvements and Future work
Conclusion
Take-Away Messages

Diffusion Length Profiles from the Iso-CFM

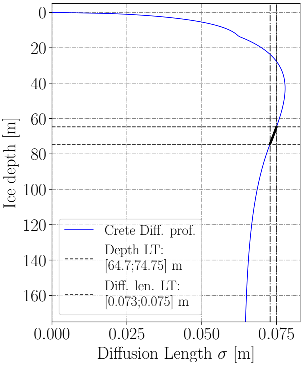

Figure: Diffusion length profile for ice core Crête.

- $\delta(z) = S(z)[\delta'(z) * \mathcal{G}(z)]$
- $\delta(z)$: Measured isotopic signal
- $\delta'(z)$: Initial isotopic signal
- $\mathcal{G}(z)$: Gaussian filter dependent on depth and diffusion length
- $S(z)$: Thinning function

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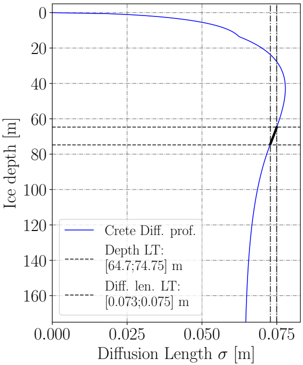

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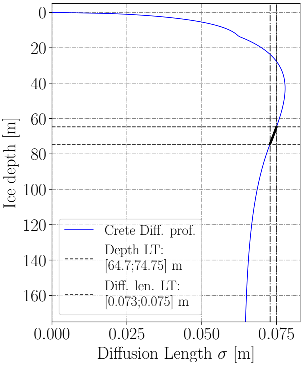

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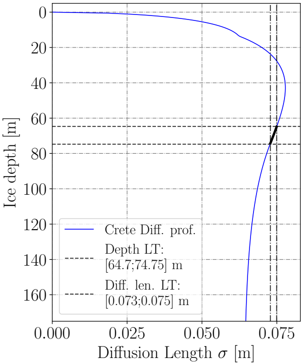

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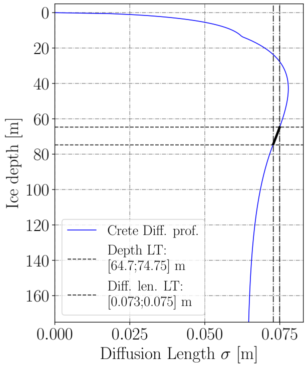

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Data: ECM Depth Series

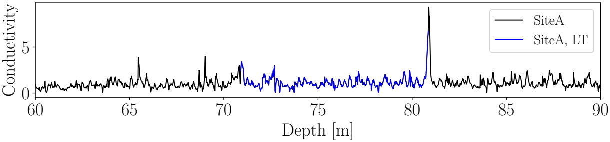

Figure: Site A, full ECM profile.

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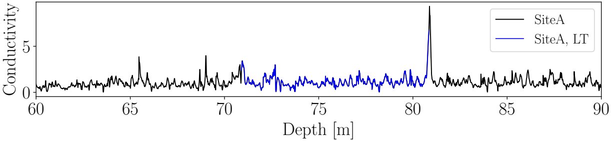

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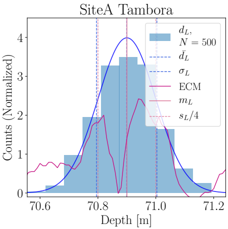

Figure: Gaussian distribution of ECM Tamb event, Site A. μ_T is set to be equal to the middle point, the standard deviation, σ_T^2 is set to be $s_T/5$.

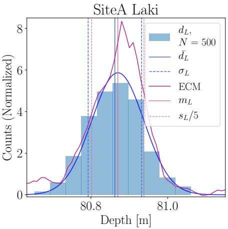

Figure: Gaussian distribution of ECM Laki event, Site A. μ_L is set to be equal to the middle point, the standard deviation, σ_L^2 is set to be $s_L/5$.

Data: Isotopic Depth Series

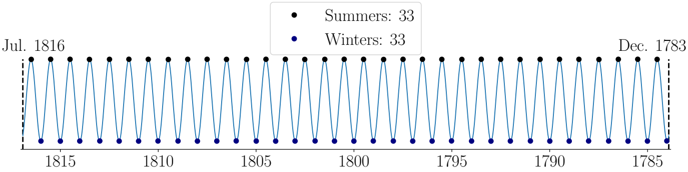

Figure: Illustration of number of summers and winters between Laki and Tambora eruption depositions.

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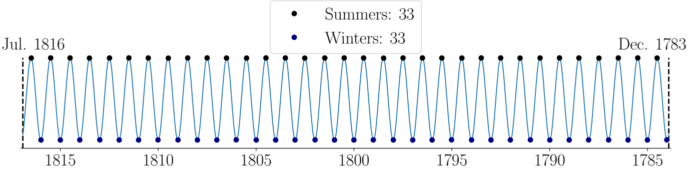

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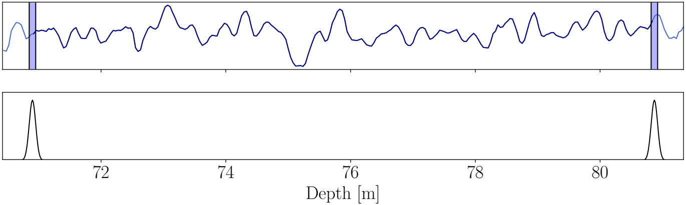

Figure: Isotopic depth series for Site A with a Gaussian approximating a 2 month time span illustrating the distribution from where the Laki and Tambora positions are drawn.

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Time Series and Spectral Analysis

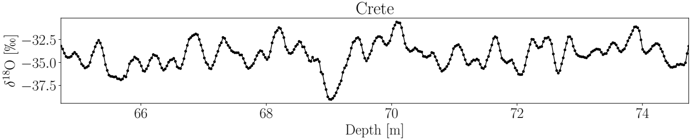

Figure: Raw data from drill site Crête between estimated Laki and Tambora event locations.

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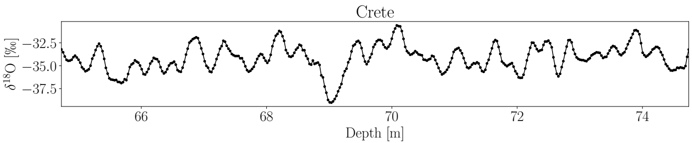

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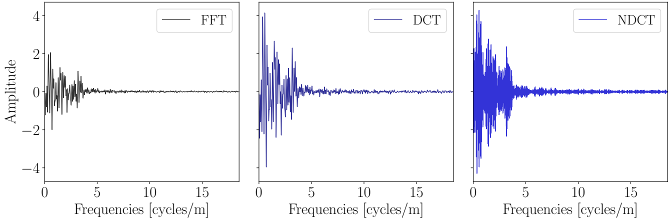

Figure: Examples of three different spectral transforms, FFT, DCT, NDCT, performed on the depth series between Tambora and Laki eruptions from Crête.

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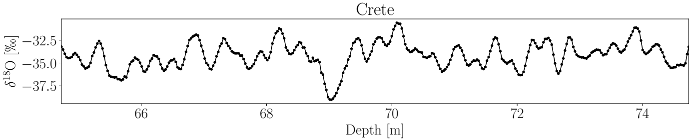

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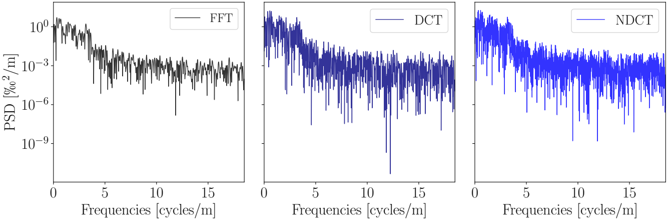

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SNR and Wiener Filtering

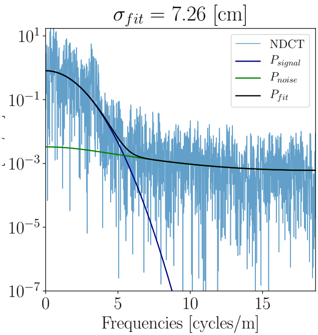

Figure: Noise, signal and total fit to PSD, illustrating the construction of the Wiener Filter.

- $\delta_M(z) = \delta_m(z) + \eta(z)$
- $\tilde{F}(f) = \frac{|\tilde{\delta_m}(f)|^2}{|\tilde{\delta_m}(f)|^2 + |\tilde{\eta}(f)|^2}$
- $|\tilde{\delta}_m(f)|^2 = P_0 \, e^{-k^2 \sigma_{\text{tot}}^2}$
- $|\tilde{\eta}(f)|^2$ is assumed red noise cf. [C. Holme, V. Gkinis and B. M. Vinther, 2018].

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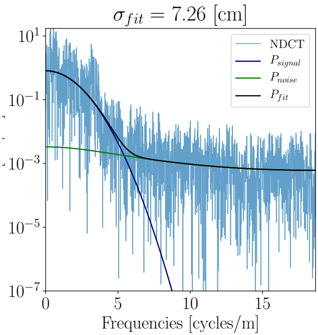

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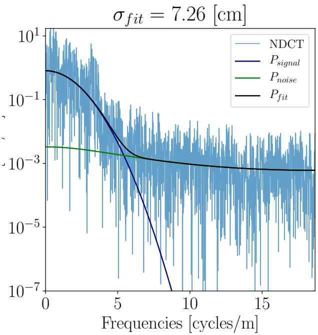

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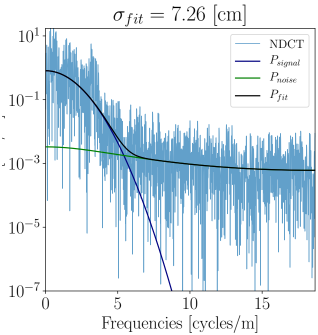

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Wiener Filter

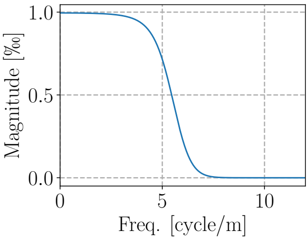

Figure: Wiener filter on linear scale.

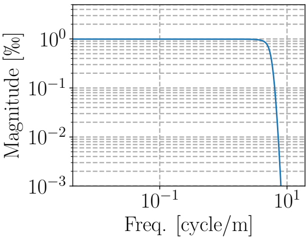

Figure: Wiener filter on double logarithmic scale.

Final Frequency Restoration Filter

$$\tilde{R}(f) = \tilde{F}(f)\,\frac{1}{\tilde{\mathcal{G}}(f,\sigma)}$$

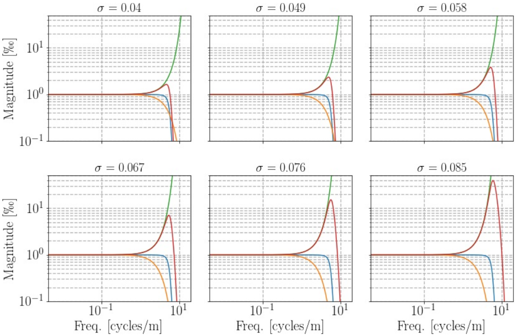

Figure: Frequency filter examples ranging from diffusion length 0.04 m to 0.085 m.

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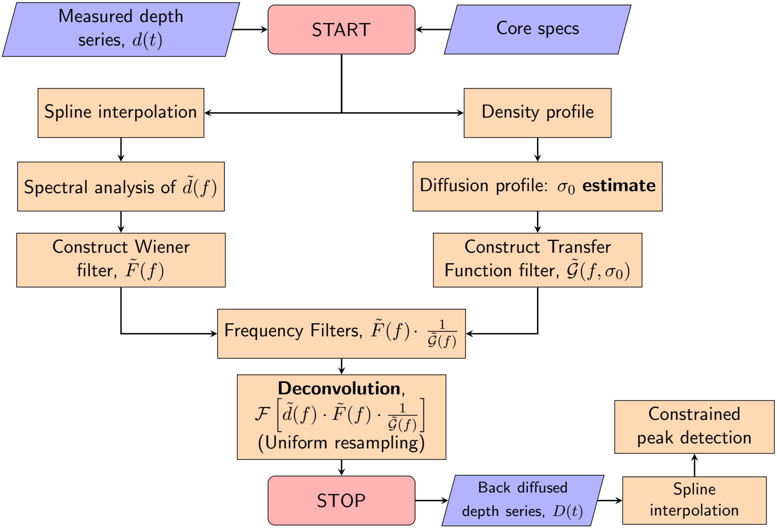
General Idea: Back Diffusion



Imposing Constraints

- $N_P = 33$
- $N_T = 33 \pm 1$
- P/T prominence min. 50 % of SD_{signal}
- Peak(trough) distance minimum 50 % of λ_A
- $[\sigma_{min}, \sigma_{max}]$ in [0, 15] cm
- Pattern: ...$PTPTP$...

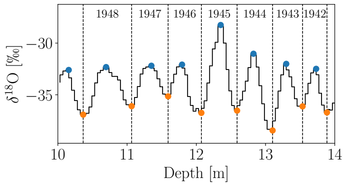


Figure: 7 years of signal from the top 10 to 15 m of the Crête ice core, with clearly visible seasonal cycles. Blue marks peaks and orange marks troughs.

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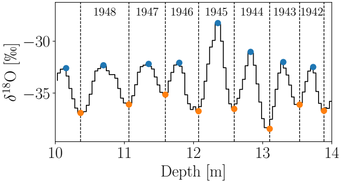

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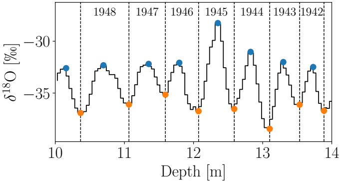

Figure: 7 years of signal from the top 10 to 15 m of the Crête ice core, with clearly visible seasonal cycles. Blue marks peaks and orange marks troughs.

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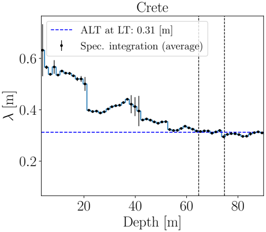

Figure: Annual layer thickness for 90 m of the Crête ice core. Estimated through spectral analysis

Imposing Constraints

- $N_P = 33$
- $N_T = 33 \pm 1$
- P/T prominence min. 50 % of SD_{signal}
- Peak(trough) distance minimum 50 % of λ_A
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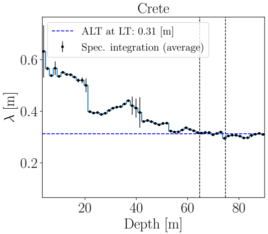

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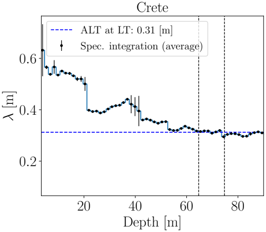

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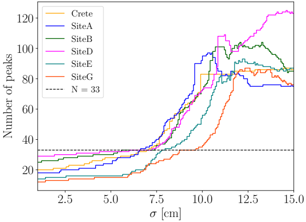

Figure: Number of peaks estimated versus diffusion length, based on diffusion lengths in the interval [0.01; 0.15] m.

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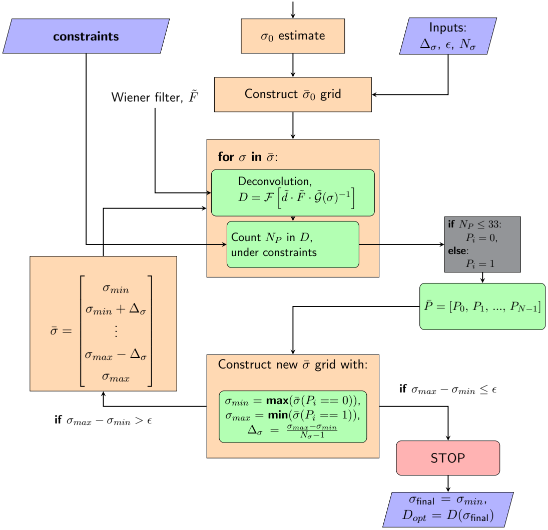
Optimal Signal Restoration Algorithm



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Constraints, Qualitatively

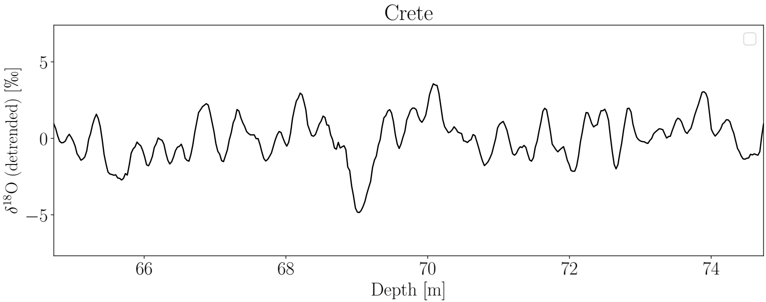

Figure: Crête ice core, section between Laki and Tambora events, raw data. Not presented in thesis.

Constraints, Qualitatively

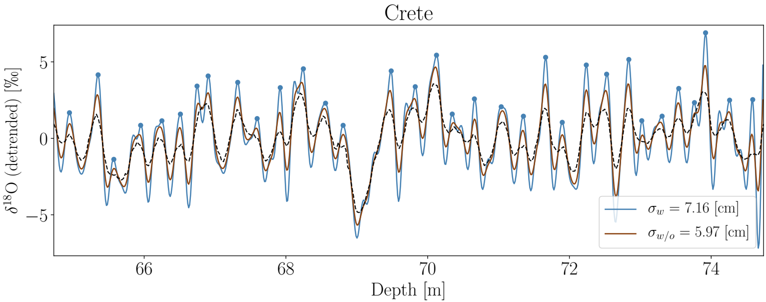

Figure: Crete ice core, with back diffused data (constrained and unconstrained).

Constraints, Quantitatively

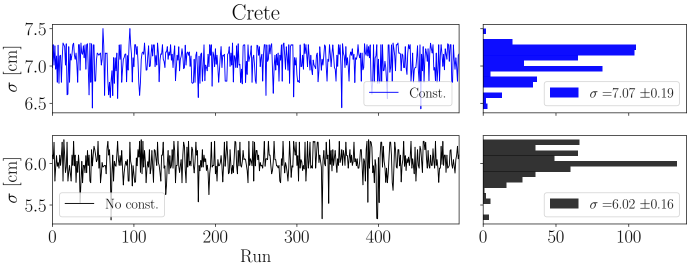

Figure: Optimal diffusion length estimates, constrained and unconstrained, for Crête ice core, 500 runs with Laki and Tambora positions drawn from 2 month Gaussian location distributions.

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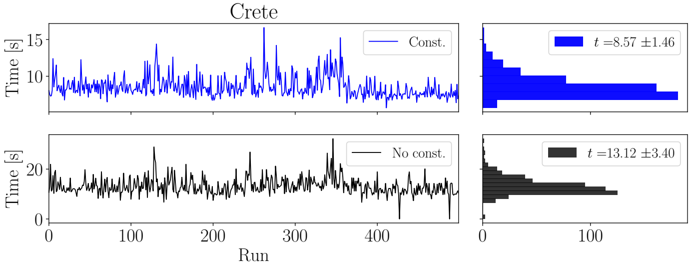

Figure: Computational time for algorithm, constrained and unconstrained, 500 runs with Laki and Tambora positions drawn from 2 month Gaussian location distributions.

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Goals to Results

- Restore lost information from signal
- Develop a method to find optimal diffusion length at a depth section
- Use optimal diffusion length to estimate temperature

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Unintended Result: Number of Peaks

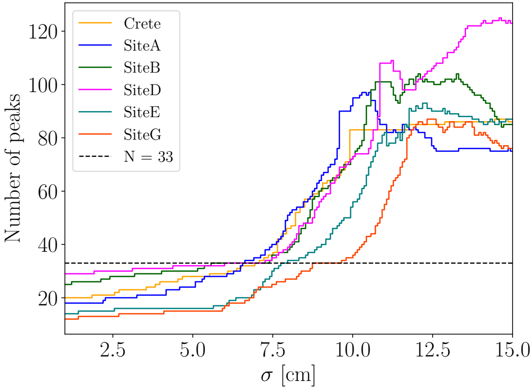

Figure: Number of peaks versus optimal diffusion length estimate, all investigated cores.

Number of Peaks

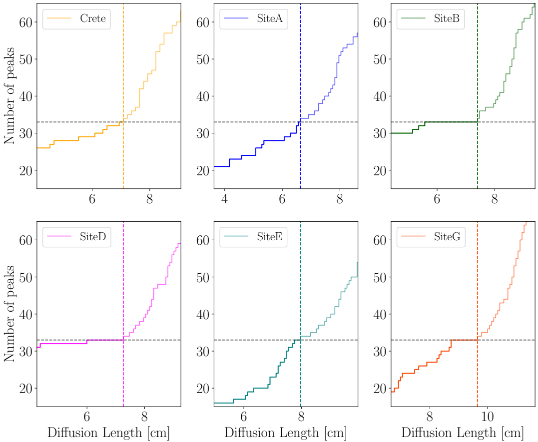

Figure: Number of peaks versus optimal diffusion length estimate, all cores separately with plateaus visible especially for Site B and G. Plateaus may show a way to date undated ice cores.

Estimated Optimal Diffusion Lengths

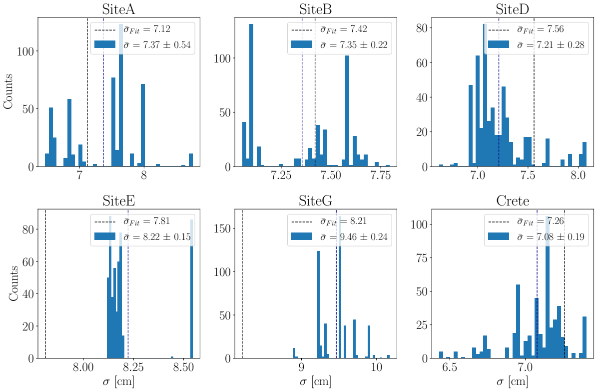

Figure: Optimal diffusion length estimate along with diffusion length estimate from spectral fit.

Estimated Firn Diffusion Lengths

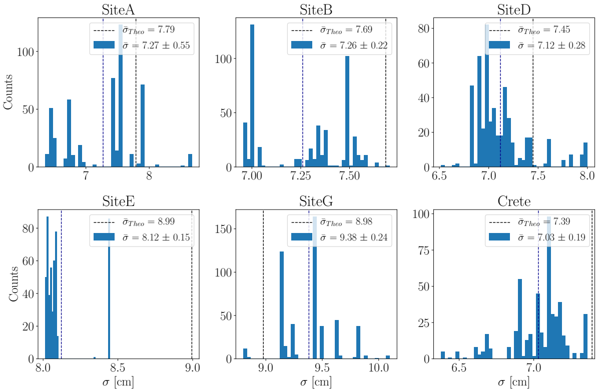

Figure: Corrected firn diffusion length estimates along with theoretical estimate from diffusion length profiles.

Estimated Firn Temperatures

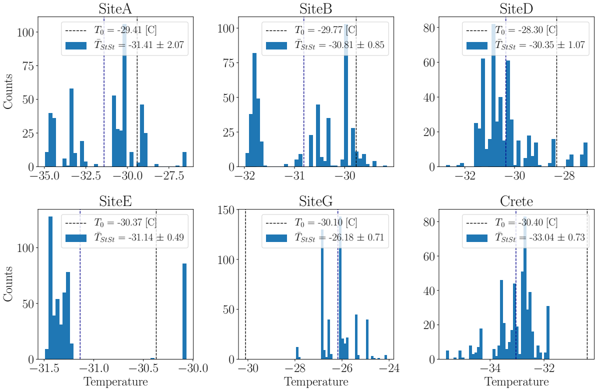

Figure: Firn temperature estimates, based on firn diffusion lengths from previous slide. Along with measured surface temperature from [H. B. Clausen, N. S. Gundestrup and S. J. Johnsen 1988].

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Algorithm Updates

- More sensitive and sophisticated peak detection/pattern recognition
- Further improvement of the direct grid search optimization routine
- Shifting volcanic event location should shift the imposed constraints
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Investigate model further through results, utilizingn the possibilities of the iso-CFM, like...

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- Investigate non-steady state models
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- More extensive analysis of final temperature results
- Investigate method stability by introducing new ice cores and different volcanic events
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Take-Away Messages

This work has successfully developed and presented the following key points:

- A **general method** for temperature estimation through diffusion length estimates
- An **in-depth stability analysis** of the methods used by isotopic data analysts
- A test of the method on a range of **different ice cores**
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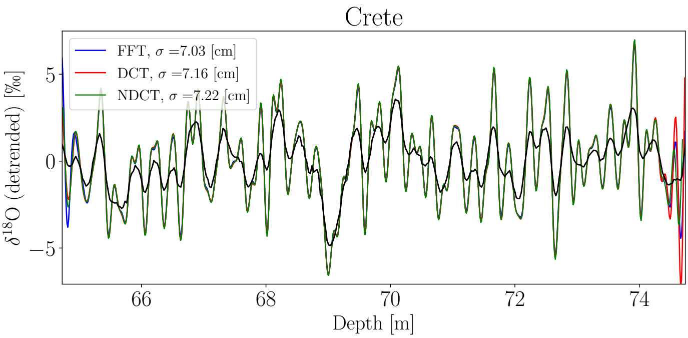
Thank you!

Questions?

Appendix

Spectral Transforms



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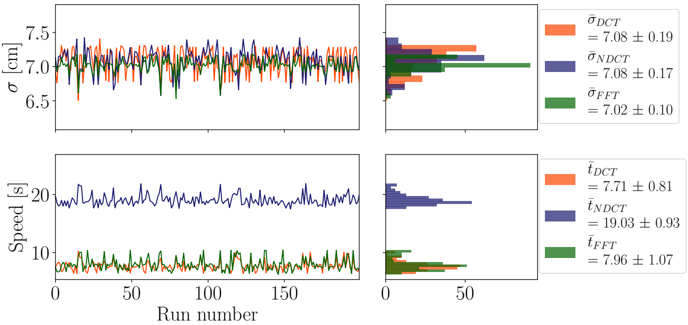
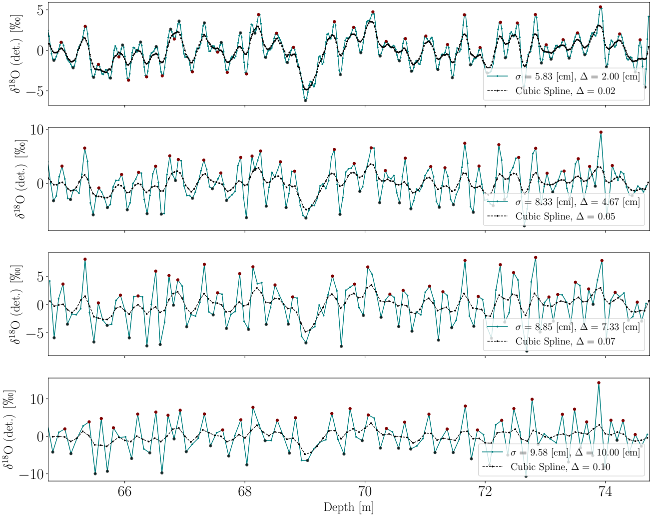


Interpolations: Before Deconvolution



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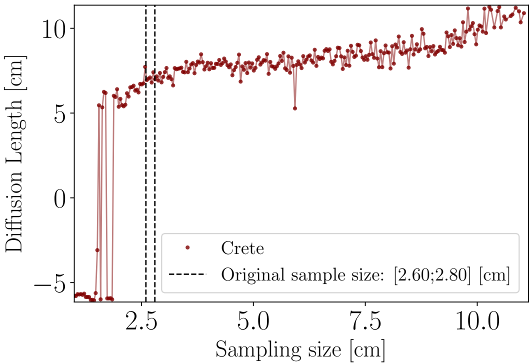

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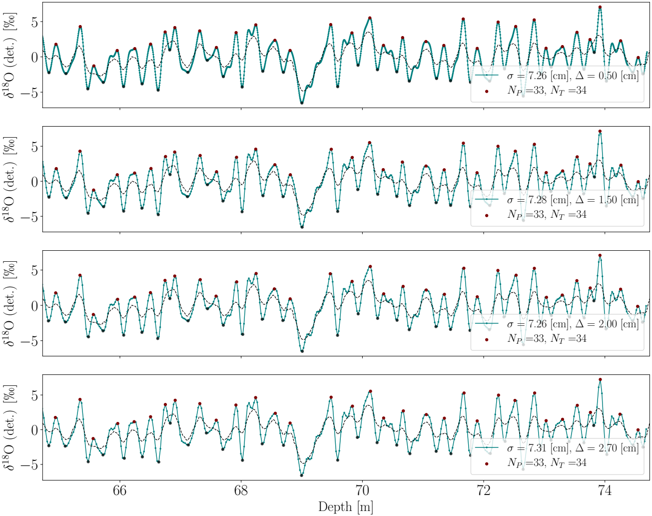

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