ECE 100 – Linear Electronic Systems Lab

Lab 2 Report – Active Sallen-Key

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Table of Contents

Abstract

In this Lab we explore circuit design with a Butterworth filter. The report includes theoretical circuit analysis, circuit simulation as well building a circuit to model the filter. Theoretical circuit analysis started with a low pass filter, then adjust it to ripple from -1 dB to 1 dB, then back to -1 dB at our f_0. We then increase f_0 so that our transfer function clears the passband edge and the stopband edge by the same factor. We plotted our theoretical results on bode plots and compared it with our simulation as well as readings from physical circuit using an oscilloscope.

Procedure

- Analytically designed a circuit to meet certain requirements including a gain of -40 dB/decade, and break frequency of 4kHz.
- Once we analytically solved for the optimal theoretical values of the circuit components, we picked out components with values as close to our desired specs as possible while also being cost effective.
- Once we built the circuit, we put probes on the input and output of our circuit and measured the output voltage peak-to-peak value given various input voltage peak-to-peak values and different frequencies.
- We then tested our circuit for extremely high frequency values, going to the high end of the function generators frequency limit.

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Experimental Procedures and Results

Problem 1: Approximation

a)

$$-1\ dB = 10\log\left(\frac{1}{1+\left(\frac{F}{F_0}\right)^4}\right)$$

$$-.1 = \log\left(\frac{1}{1+\left(\frac{F}{F_0}\right)^4}\right) \qquad \text{when } F = 4000,\ F_0 = 5607.46\ Hz$$

$$10^{-.1} = \left(\frac{1}{1+\left(\frac{4000}{F_0}\right)^4}\right)$$

$$|H(F)|^2 = \frac{1}{1+\left(\frac{F}{5607.46}\right)^4}$$

$$10\log\left(|H(32KHz)|^2\right) = 10\log\left(\frac{1}{1+\left(\frac{32KHz}{5.607KHz}\right)^4}\right)$$

$$= -30.2609\ dB$$

$$10\log\left(|H(180KHz)|^2\right) = 10\log\left(\frac{1}{1+\left(\frac{180KHz}{5.607KHz}\right)^4}\right)$$

$$= -60.2617\ dB$$

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b)

$$H(j\omega) = \frac{\sqrt{.7943}}{1 + \frac{2\xi}{\omega_0}(j\omega) + \frac{(j\omega)^2}{\omega_0^2}}$$

$$|H(j\omega)| = \frac{\sqrt{.7943}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\xi}{\omega_0} j\omega\right)^2}} = \frac{\sqrt{.7943}}{\sqrt{1 - \frac{2\omega^2}{\omega_0^2} + \frac{\omega^4}{\omega_0^4} + \frac{4\xi^2}{\omega_0^2}\omega^2}}$$

$$= \frac{\sqrt{.7943}}{\sqrt{1 - \omega^2\left(\frac{2}{\omega_0^2} - \frac{4\xi^2}{\omega_0^2}\right) + \frac{\omega^4}{\omega_0^4}}}$$

$$\frac{d\,|H(j\omega)|}{d\omega} = -\frac{1}{2}\sqrt{.7943}\left(1 - \omega^2\left(\frac{2}{\omega_0^2} - \frac{4\xi^2}{\omega_0^2}\right) + \frac{\omega^4}{\omega_0^4}\right)^{-3/2}\left(-2\omega\left(\frac{2}{\omega_0^2} - \frac{4\xi^2}{\omega_0^2}\right) + \frac{4\omega^3}{\omega_0^4}\right)$$

$$\frac{d\,|H(j\omega)|}{d\omega} = 0 = -\frac{1}{2}\sqrt{.7943}\left(-2\omega\left(\frac{2}{\omega_0^2} - \frac{4\xi^2}{\omega_0^2}\right) + \frac{4\omega^3}{\omega_0^4}\right)$$

$$0 = -\sqrt{.7943}\left(-\omega\left(\frac{2}{\omega_0^2} - \frac{4\xi^2}{\omega_0^2}\right) + \frac{2\omega^3}{\omega_0^4}\right)$$

$$0 = \frac{2\omega\sqrt{.7943}}{\omega_0^2}\left(1 - 2\xi^2 - \frac{\omega^2}{\omega_0^2}\right)$$

$$0 = 1 - 2\xi^2 - \frac{\omega^2}{\omega_0^2}$$

$$\omega = \omega_0\sqrt{1 - 2\xi^2}$$

$$2\pi f_p = 2\pi f_0\sqrt{1 - 2\xi^2}$$

$$f_p = F_0\sqrt{1 - 2\xi^2}$$

$$|H(j\omega)| = \frac{\sqrt{.7943}}{\sqrt{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\xi}{\omega_0}\omega\right)^2}} \qquad f_p = F_0\sqrt{1-2\xi^2}$$

$$|H(j\omega)|^2 = \frac{.7943}{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\xi}{\omega_0}\omega\right)^2}$$

$$= \frac{.7943}{\left(1-\frac{f_p^2}{f_0^2}\right)^2 + \left(\frac{2\xi}{f_0}f_p\right)^2}$$

$$= \frac{.7943}{\left(1-\frac{f_0^2(1-2\xi^2)}{f_0^2}\right)^2 + \left(\frac{2\xi}{f_0}f_0\sqrt{1-2\xi^2}\right)^2}$$

$$\frac{|H(j\omega)|^2}{|H(0)|^2} = \frac{1}{4\xi^4 + 4\xi^2(1-2\xi^2)}$$

$$= \frac{1}{4\xi^4 + 4\xi^2 - 8\xi^4}$$

$$= \frac{1}{-4\xi^4 + 4\xi^2}$$

$$1\,dB = 10\log\left(\frac{1}{-4\xi^4 + 4\xi^2}\right)$$

$$10^{.1} = \frac{1}{-4\xi^4 + 4\xi^2}$$

$$\xi = .5227,\ .85245$$

$$.85245 > .7,\ \text{so } \xi = .5227$$

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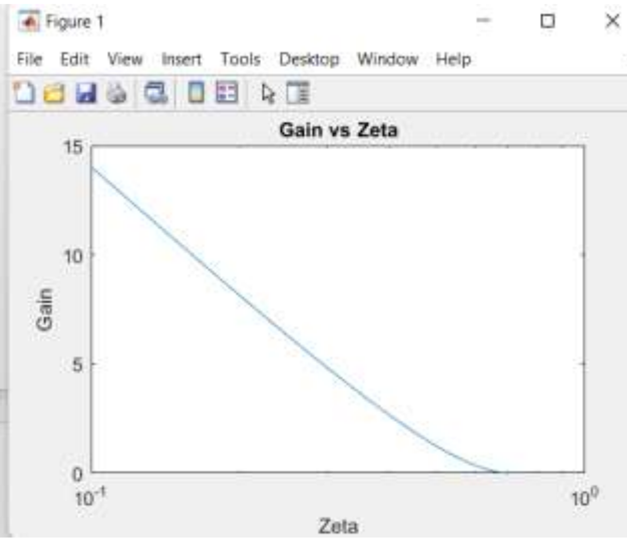
```
zeta = .1:.01:.7;
numerator = 1;
denominator = -4*zeta.^4+4*zeta.^2;

h = numerator./denominator;

h_db = 10*log10(h);
index = 1;
semilogx(zeta,h_db); %

title('Gain vs Zeta')
xlabel('Zeta')
ylabel('Gain')
```

Window
rtlb
c find()
rtlb



c)

$$-1 = 10\log\left(\frac{.7943}{(1-(\frac{f}{f_o})^2)^2+(2z\frac{f}{f_o})^2}\right)$$

$$10^{-.1} = \frac{.7943}{(1-(\frac{f}{f_o})^2)^2+4z^2(\frac{f}{f_o})^2}$$

$$10^{.1}(.7943) = \left(1-\left(\frac{f}{f_o}\right)^2\right)^2+4z^2\left(\frac{f}{f_o}\right)^2$$

$$f_o = 4200.1\ Hz$$

d)

$$|X_1 - X_2| = |-1 + .01 - (-31+30)|$$

$$= .017\ dB$$

Fo = 5650 Hz

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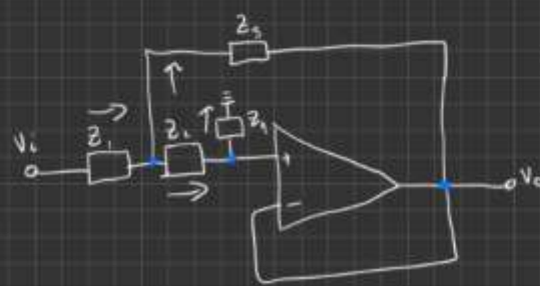
Problem 2: Realization/Simulation

a)



$R_1 = R_2 = 100k\Omega$

$f_0 = 5650Hz$

$\zeta = .522728$

$V_- = V_+ = V_o$

$$\frac{V_i - V_x}{Z_1} - \frac{V_x - V_0}{Z_3} - \frac{V_x - V_+}{Z_2} = 0$$

$$V_x = \frac{Z_2 + Z_4}{Z_4} V_+$$

$$V_x = \frac{Z_2 + Z_4}{Z_4} V_0 = \left(1 + \frac{Z_2}{Z_4}\right) V_0$$

$$\frac{V_i}{Z_1} - \left(\frac{1}{Z_1} + \frac{1}{Z_3} + \frac{1}{Z_2}\right) V_x + \left(\frac{1}{Z_3} + \frac{1}{Z_2}\right) V_0 = 0$$

$$\frac{V_i}{Z_1} = \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)\left(\frac{Z_2}{Z_4}\right) - \frac{1}{Z_3} - \frac{1}{Z_2}\right] V_0$$

$$\frac{V_i}{Z_1} = \left[\frac{1}{Z_1} + \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)\left(\frac{Z_2}{Z_4}\right)\right] V_0$$

$$V_i = \left[1 + \frac{Z_2}{Z_4} + \frac{Z_1}{Z_4} + \frac{Z_1 Z_2}{Z_3 Z_4}\right] V_0$$

$$\frac{V_0}{V_i} = \frac{1}{1 + \frac{Z_1 + Z_2}{Z_4} + \frac{Z_1 Z_2}{Z_3 Z_4}}$$

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$$\frac{V_i}{V_0} = \frac{1}{1 + sC_1(R_1+R_2) + s^2 R_1 R_2 C_1 C_2}$$

$$\frac{1}{1 + 2sC_1R + s^2 R^2 C_1 C_2} = \frac{1}{1 + \frac{2\xi}{\omega_0} s + \frac{1}{\omega_0^2} s^2}$$

$$2RC_1 = \frac{2\xi}{2\pi f_0} \qquad R^2 C_1 C_2 = \frac{1}{\omega_0^2}$$

$$C_1 = \frac{2\xi}{2R \cdot 2\pi f_0} \qquad C_2 = \frac{1}{R^2 C_1 \cdot (2\pi f_0)^2}$$

$$C_1 = \frac{2(.5227)}{2 \quad (100k)(2\pi)(5.6k)} \qquad C_2 = \frac{1}{(2\pi(5.6k))^2 (100k)^2 (.14855nF)}$$

$$C_1 = .14855 \text{ nF} \qquad C_2 = .5437 \text{nF}$$

Part B:

$$V_o = \frac{R_{out}}{R/2} \cdot V_i/2$$

$$\frac{V_o}{V_i} = \frac{R_{out}}{R}$$

$$|H| = \left|\frac{V_o}{V_i}\right| = \left|\frac{R_{out}}{R}\right| = \frac{R_{out}}{R}$$

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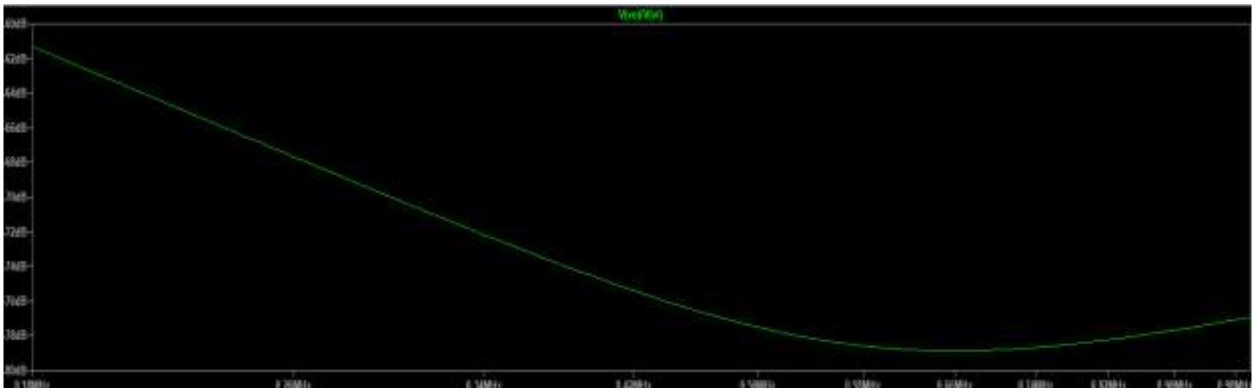
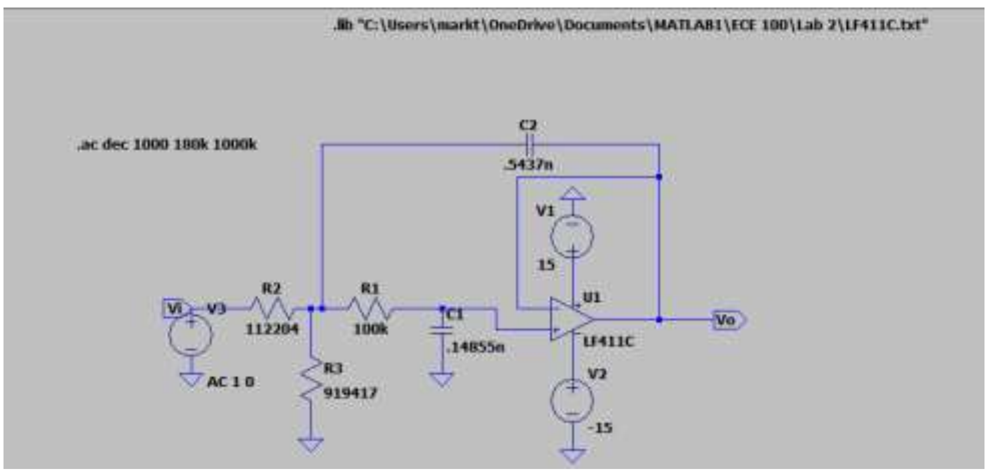
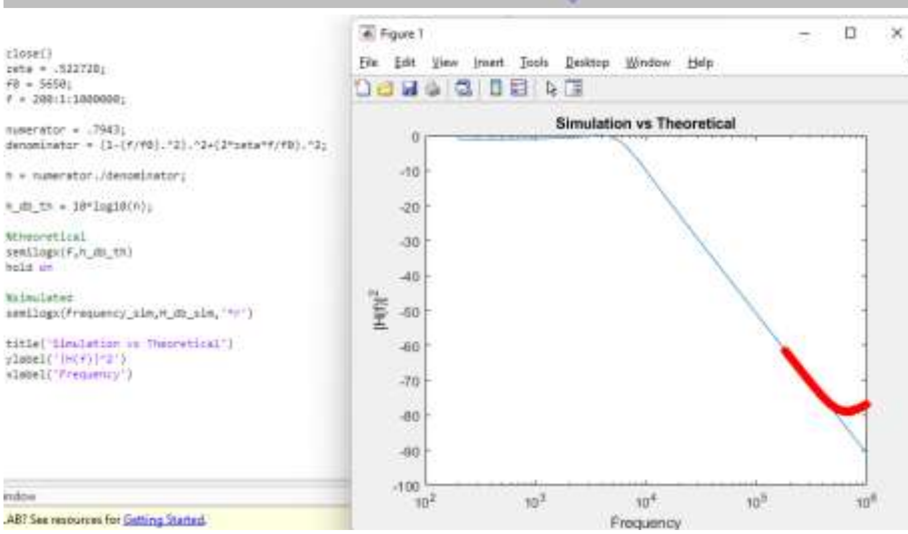
Part C:

Simulation:



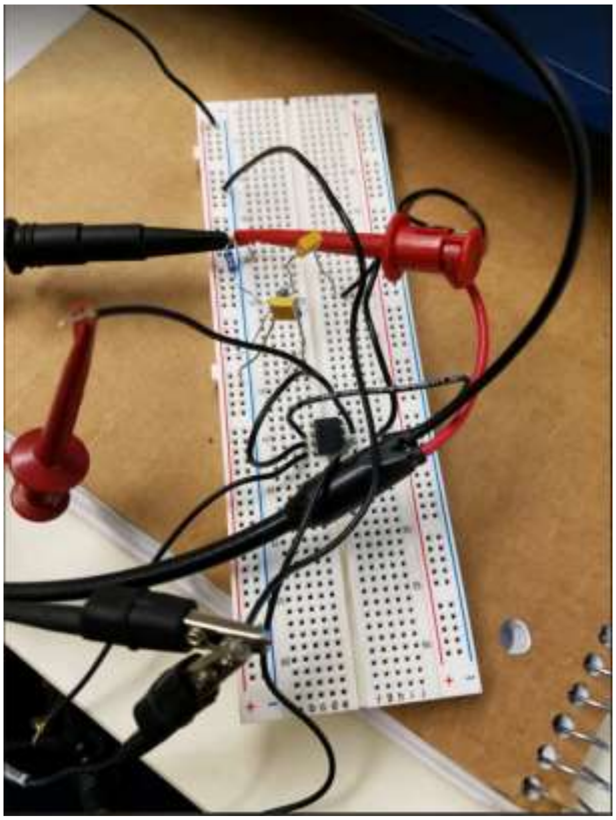
Circuit:





Problem 3: Experiment

Circuit:



Components: 1 MOhm resistor, 490 pF capacitor, 120 pF cap, 120 kOhm Resistor, LF411 Op-amp

Hz	Input (Vpp)	Output (Vpp)	Output (Vpp)	|H(f)|^2 dB
10	5	4.56		-0.800103233
10	10	9.04		-0.87663139
10	20	**15.2**	18.4	-2.383728154
5600	5	4.4		-1.110346557
5600	10	8		-1.93820026
5600	20	**15.4**	15.2	-2.270185497
4000	5	4.56		-0.800103233
4000	10	9		-0.915149811
4000	20	**15.2**	18.4	-2.383728154
32000	5	0.18		-28.87394998
32000	10	0.308		-30.22898567
32000	20	**0.56**	560mV	-31.05683937
180000	5	0.012		-52.39577517
180000	10	0.016		-55.91760035
180000	20	**0.0202**	80mV	-59.91357252
1000000	5			
1000000	10			
1000000	20			

*Oscilloscope was not reading Vout data anywhere from the range of 180kHz < f <= 1MHz. Second Output column is to double check oscilloscope readings.

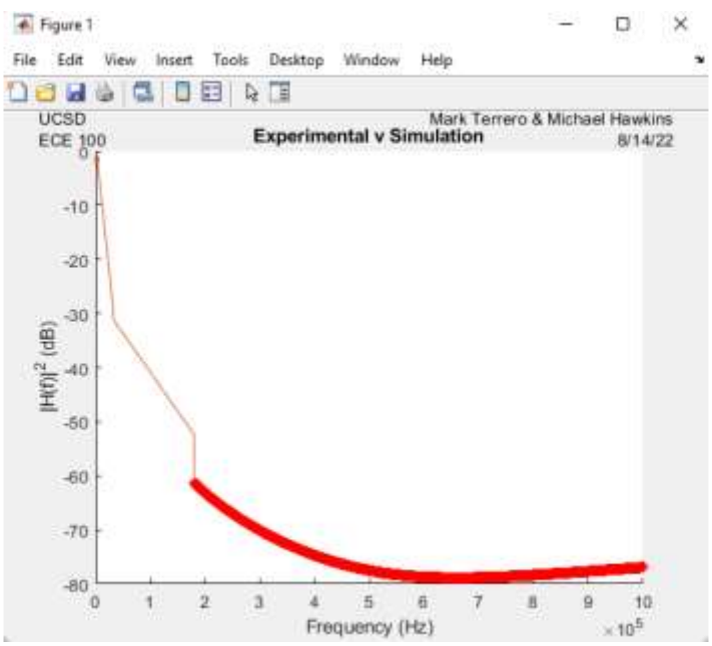
Part B

f = 5MHz
Vout is still a sine wave.

```
%simulated
semilogx(Frequency_sim,H_db_sim,'*r')

% experimental
h_real = V_out_real./V_in_real;
h_db_real = 20*log10(h_real);
semilogx(Freq_real,h_db_real)

xlabel('Frequency (Hz)')
ylabel('|H(f)|^2 (dB)')
title({'Experimental v Simulation'})
axes('position',[0 0 1 1],'visible','off')
text(.95,.97,{'Mark Terrero & Michael Hawkins';'8/14/22'},...
     'HorizontalAlignment','right')
text(.05,.97,{'UCSD';'ECE 100'})
```



Conclusion

This lab went through active circuit design of a Butterworth filter. We first do circuit analysis to find f_0 analytically and find the value of the transfer function at 32 kHz and 180 kHz and find that the filter does meet the specification but is very tight. Using the derivative of the transfer function we can find the peak frequency as a function of f_0. To add the ripple from -1 dB to 1 dB, then back to 0 dB at 4kHz, we use this peak frequency in the equation $\frac{|H(jw)|^2}{|H(0)|^2} = 1\ dB$ to find our zeta value. In our case we found our zeta ≈ 0.5227, and $f_0 \approx 4200.1\ Hz$. Adjusting our $f_0 = 5650\ Hz$ we can clear the passband edge and the stopband edge by the same factor and check the distance in gain to be < .1 dB. We now have a transfer function to model our Butterworth filter. This filter will have a gain of -1 dB in the passband region, and near f_0 will rise to 1dB then back to -1dB at f_0. It will then have a lose of -30 dB per decade.

We then create a circuit simulation based on a Sallen-Key circuit with some modification. We use a Thevenin equivalent to the resistor at our input voltage to reduce our input voltage by -1 dB, or $\sqrt{.7943}$ in magnitude. We then made the general form of the transfer function equivalent to the transfer function found with circuit analysis and found the capacitor values.

We finally build our circuit on a breadboard using $\pm 15\ V$ on the op amp rails, with varying input signals to obtain output signals and compare them with the theoretical values that should be attained by our transfer function. After 180 kHz our oscilloscope was not reading an output signal. Overall, our theoretical equation, circuit simulation, and values from our built circuit were consistent and we got good results in this lab besides the output voltage after 180 kHz.