Binary Search:
Shipping Capacity Example

The Problem

You have packages with weights $w_1, \ldots, w_n$ in order. Each day a ship with capacity C loads packages from left to right (keeping order) until the next package does not fit; then it sails and the next day continues.

Goal: Given D (days), find the *minimum* capacity C so that all packages are shipped within D days.

Constraints: $w_i \geq 0$, order cannot change.

Key Insight: Monotone Feasibility

Define predicate $\texttt{can}(C)$: "Can we ship all packages in at most D days if daily capacity is C?"

We can **binary search** the smallest feasible $C^\star$.

Search Bounds

- Lower bound $L = \max_i w_i$ (must fit the heaviest package in one day).
- Upper bound $R = \sum_i w_i$ (do everything in one day).

We binary search on integers in $[L, R]$.

Example Instance

Weights: $[3, 2, 2, 4, 1, 4]$, $D = 3$.

Bounds: $L = \max = 4$, $R = \sum = 16$.

Try $C = 10$: loads 1st day 3+2+2, 2nd day 4+1+4. Total 2 days $\Rightarrow$ feasible.

Try smaller C and continue until you find $C^\star = 6$ (for this instance, answer is 6).