BST Examples

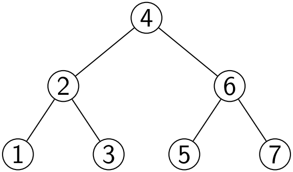
Example: Height of a BST

Input

- First line: `7`
- Second line: `4 2 6 1 3 5 7`   (insert into an empty BST in this order)

Task   Output the *height in edges* of the resulting BST. (Convention: height(empty) $= -1$, height(leaf) $= 0$.)

Expected Output   `2`



Height of a BST

Convention:

$$\mathrm{height}(\mathsf{empty}) = -1, \qquad \mathrm{height}(\mathsf{leaf}) = 0.$$

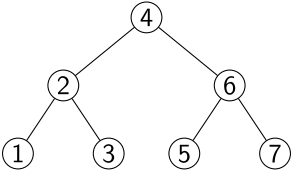
Postorder (bottom-up) rule for any node v:

$$h(v) = 1 + \max\left(h(\mathrm{left}(v)),\ h(\mathrm{right}(v))\right).$$

Algorithm idea (no code):

1. Visit children first (postorder): compute each child's height.
2. Apply the rule $h(v) = 1 + \max(h_L, h_R)$.
3. The root's height is the tree height.

Walkthrough on the example $\{4, 2, 6, 1, 3, 5, 7\}$



Bottom-up computation:

$$h(1) = h(3) = h(5) = h(7) = 0 \quad \text{(leaves)}$$

$$h(2) = 1 + \max(h(1), h(3)) = 1 + \max(0, 0) = 1$$

$$h(6) = 1 + \max(h(5), h(7)) = 1 + \max(0, 0) = 1$$

$$\boxed{h(4) = 1 + \max(h(2), h(6)) = 1 + \max(1, 1) = 2}$$

Conclusion: Tree height (in edges) is $\boxed{2}$.