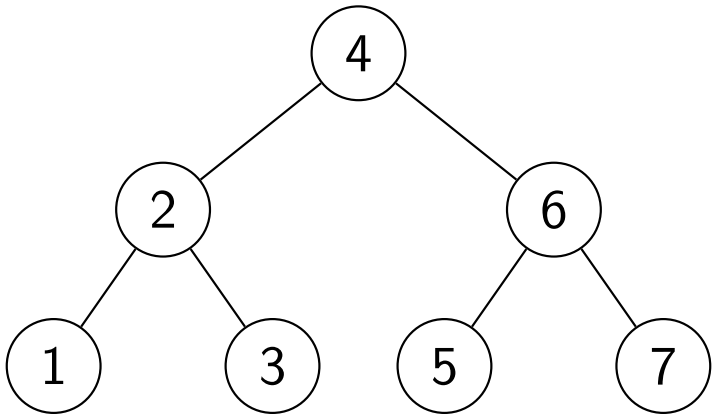
BFS (Breadth First Search) on a Simple BST: Step-by-Step

The example BST

We will run BFS (level-order traversal) on this BST:



Levels (root has level 1):

Level 1: $\{4\}$ Level 2: $\{2, 6\}$ Level 3: $\{1, 3, 5, 7\}$

BFS idea

Goal. Visit nodes level by level from the root, left to right within each level.

1. Put the root (4) with its level (1) into a queue.
2. While the queue is not empty:
   - Take the front node out and *visit* it (append to output).
   - Put its left child (if any) into the queue with level+1.
   - Put its right child (if any) into the queue with level+1.

Notation: we write a node value x at level ℓ as x_ℓ (e.g., 4_1, 2_2).

BFS on the example: queue evolution (step-by-step)

Step	Queue before pop	Action	Queue after pop / Output
0	$[4_1]$	(init)	$[4_1]$ / Output $= []$
1	$[4_1]$	pop 4_1; enqueue $2_2, 6_2$	$[2_2, 6_2]$ / Output $= [4]$
2	$[2_2, 6_2]$	pop 2_2; enqueue $1_3, 3_3$	$[6_2, 1_3, 3_3]$ / Output $= [4, 2]$
3	$[6_2, 1_3, 3_3]$	pop 6_2; enqueue $5_3, 7_3$	$[1_3, 3_3, 5_3, 7_3]$ / Output $= [4, 2, 6]$
4	$[1_3, 3_3, 5_3, 7_3]$	pop 1_3; no children	$[3_3, 5_3, 7_3]$ / Output $= [4, 2, 6, 1]$
5	$[3_3, 5_3, 7_3]$	pop 3_3; no children	$[5_3, 7_3]$ / Output $= [4, 2, 6, 1, 3]$
6	$[5_3, 7_3]$	pop 5_3; no children	$[7_3]$ / Output $= [4, 2, 6, 1, 3, 5]$
7	$[7_3]$	pop 7_3; no children	$[]$ / Output $= [4, 2, 6, 1, 3, 5, 7]$

Final BFS order: 4, 2, 6, 1, 3, 5, 7 (level by level, left to right).

Level counts and sums (step outcome)

From the BFS view we can aggregate per level:

- **Level 1:** nodes $\{4\}$ $\Rightarrow$ count $= 1$, sum $= 4$
- **Level 2:** nodes $\{2, 6\}$ $\Rightarrow$ count $= 2$, sum $= 2 + 6 = 8$
- **Level 3:** nodes $\{1, 3, 5, 7\}$ $\Rightarrow$ count $= 4$, sum $= 1 + 3 + 5 + 7 = 16$

Number of levels: 3 **Level sums:** [4, 8, 16] **Maximum level width:** 4 (at level 3)