University of Moratuwa

Faculty of Engineering

Department of Computer Science and Engineering

B.Sc. Engineering

09 Semester 2 Examination 2010

CS 2022 – Data Structures and Algorithms

Time allowed: **2 Hours** August 2010

INSTRUCTIONS TO CANDIDATES:

This paper contains **FOUR (4) questions** on 5 pages, including this page.

Answer **ALL** questions.

This is a **closed book** examination.

Mobile phones or any other communication devices are not permitted.

The marks for each question and section thereof are shown in square brackets.

This paper contributes to 70% of the total marks of the subject.

Clearly state the assumptions you made. If you have any doubt regarding the interpretation of the wording of a question, make your own decision, but clearly state it on the script.

Q1 **[25 marks]**

a) Which of the following problems can be solved using an algorithm? Explain why or why not for each problem.

i. Given your horoscope, find the number of children you will have.

ii. Given a date in the future, find the position of Saturn in the sky at that time.

iii. Build a vehicle.

iv. Given a picture of a vehicle number plate, identify the number on the plate. [4]

b) Give an example of an Internet site which depends on algorithms, and explain how it uses them. [2]

c) Algorithm A takes 500 ms to solve a problem for a given input, while algorithm B takes 300 ms with the same input. Which algorithm is better? Explain. [4]

d) What operations does the *queue* data type support? [3]

e) Show the implementation of a queue in pseudo-code using (i) an array and (ii) a dynamic data structure. For each implementation, show the code for creating an empty queue, and for each operation supported by the data type. [12]

Q2 **[25 marks]**

a) A *heap* is a special type of binary tree.

What are the properties of a max-heap? [3]

b) Show, using a suitable diagram, a max-heap formed by the following numbers:

9 6 22 4 3 12 16 10 20 21 12.

Also show how the heap may be stored in an array. [5]

c) What is the *height* of the heap in b) above? [hint: a heap comprising a single node has height zero.] What is the maximum number of nodes which can be in a heap of that height? [2]

d) Explain how an unsorted array of length *n* stored in array *A* may be formed into a heap using the procedure MAX_HEAPIFY(*A*, *i*), and show the pseudo-code for forming the heap. [8]

[Assume that the procedure MAX_HEAPIFY(*A*, *i*), which converts the tree rooted at i into a heap in time O(lg n), is already implemented. You do not need to show its code]

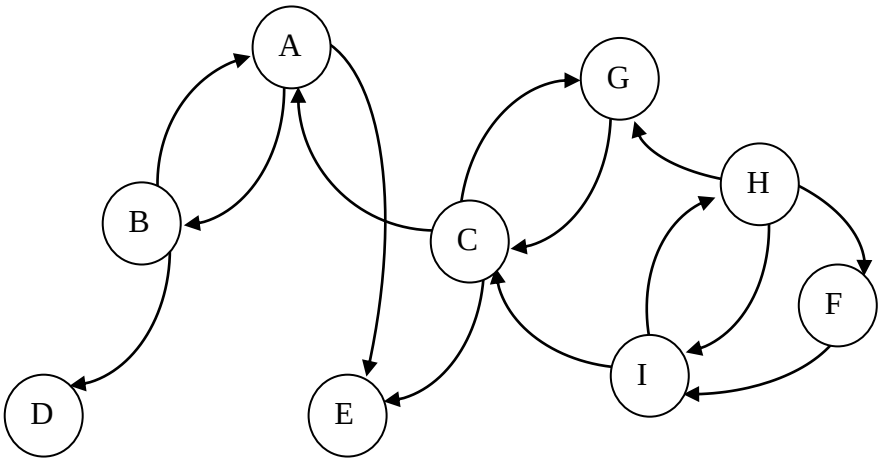
e) Explain, using pseudo-code, how you can sort 10 million 32-bit integers in linear time. [Hint: a 32-bit integer can be considered as four 8-bit base-256 digits.] [5]

f) Assuming you have a computer which performs 2 million calculations per second, approximately how long would it take to sort the 10 million numbers?

[You only need to give an approximate time, and can ignore operations which take a non-significant time.] [2]

Q3 **[25 marks]**

a) Generate the Breadth First Search Tree for the following graph starting from vertex C. [5]



Sections b) to f) are based on the following scenario:

The diagram below (Figure Q3) shows the homes of six friends (Andun, Asitha, Chathuranga, Hasini, Rashmika and Vatsala) and the roads which connect the homes. Some of the roads in the area are one-way roads (arrow heads show the direction/directions of traffic allowed). The distances are shown next to the arrows. Hasini is hosting a party after the exam and all the friends are attending the party. Each friend is planning to take the shortest path to Hasini's home and pickup the friends who are on his path (assume all of them have cars which can accommodate 5 people).

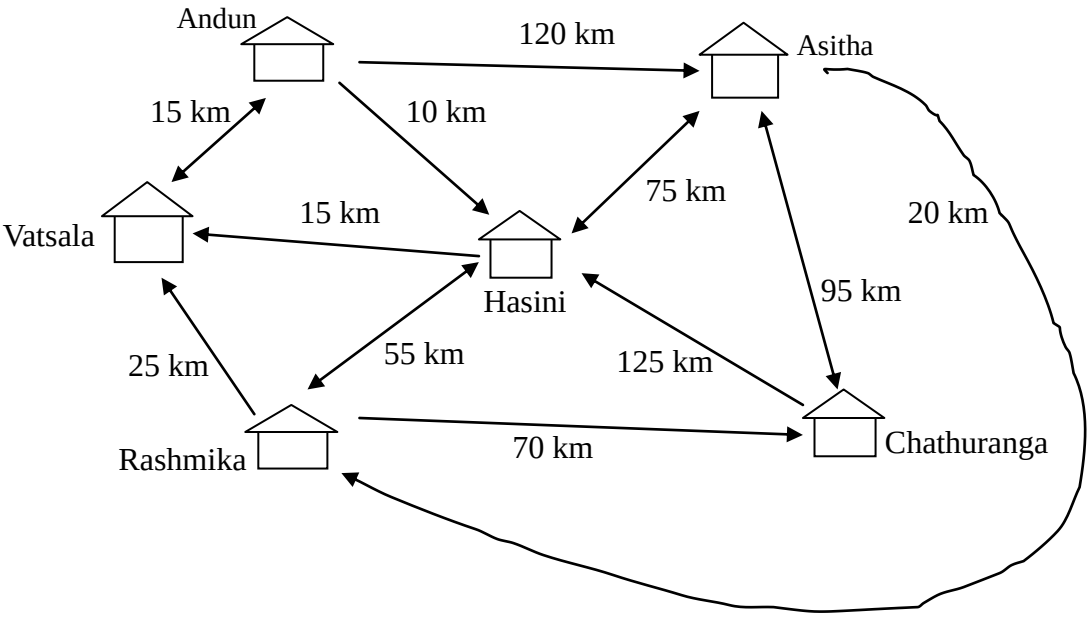


Figure Q3: Homes and the Connecting Roads

b) Hasini wants to find out the number of cars the friends will use to reach her home. Briefly explain how she can use any Single Source Shortest Path (SSSP) algorithm (we discussed in the class) to solve the above problem. You do not have to specify the SSSP algorithm. [4]

c) Using the method described as answer of part b) and a SSSP algorithm, find the number of cars that will be there at Hasini's home on the day of the party. State the SSSP algorithms you are using and show step by step workings you followed in calculating the SSSP and obtaining the answer. [8]

d) You are given that you can solve the SSSP problem in worst case time complexity of O (|V| |E|), where |V| is the number of vertices in the graph and |E| is the number of edges in the graph. What will be the time complexity of the solution you proposed in part b)? Please explain. [2]

e) Can Rashmika use the above calculations to obtain the shortest path from his home to Andun's home? Please explain. [3]

f) Illustrate the Adjacency List representation of the graph that you used in part b). [3]

Question 4 is on the next page.

Q4 **[25 marks]**

a) Give the asymptotic growth in "big oh" notation for the following functions. Please show how you obtained your answer.

i. $T(n) = 10^{20}\, n + (3\times10^{-10})\, n^2 + 0.9n \log n$ [2]

ii. $T(n) = 3T(n/2) + n \log n$ [2]

iii. $T(n) = T(\lfloor \sqrt{n} \rfloor) + n$ [4]

b) Dynamic Programming is a technique which lets you solve certain set of problems efficiently compared to recursion. In order to take advantages by applying the dynamic programming technique, the problem should have some properties. Briefly explain each of those properties. [3]

Sections c) to f) are based on the following scenario:

You are hired as the project manager of ABC organization with a probation period of 12 weeks. You are required to select a set of projects from the available projects and complete them within the 12 weeks. Table Q4 shown below lists the available projects. You have to fully complete the project to obtain the specified profit and partial completion of project will not earn any profit. One of your friends says that you can solve this problem using the dynamic programming approach.

c) Specify the problem in a mathematical form assuming that x_i denotes the selecting or not selecting of the i^{th} project. [2]

d) Considering the selection or non selection of the first project, identify how you can decide to select or not select the first project. [2]

e) Let $P(i,k)$ denote the maximum profit possible by selecting a subset of projects from projects $i, i+1,\ldots,n$ and time limit k. Derive the recursive solution to the problem of selecting the optimal set of projects that can be completed within a given amount of time and would maximize the profit. [2]

f) Compute the solution to the problem using the dynamic programming approach in either top-down or bottom-up approach. Specify the approach you used. [8]

Table Q4: Project Details

Project Identifier	**Completion Time (weeks)**	**Expected Revenue (Rs. Millions)**
1	4	150
2	3	80
3	3	60
4	4	70
5	2	60
6	1	10