Index Number:								

UNIVERSITY OF MORATUWA

FACULTY OF ENGINEERING

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

BSc Engineering Honours Degree
2013 Intake Semester 4 Examination

CS2022: DATA STRUCTURES & ALGORITHMS

Time allowed: 2 Hours   February 2016

ADDITIONAL MATERIAL: *None*

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of **two (2) Sections** in **13** pages.

2. Answer **all** questions in the question paper itself **in the space provided**. If you make a mistake or need additional space you may attach additional sheets.

3. Section **A** contains **twenty-five (25) MCQ and/or short answer questions**. **Answer these questions in the space provided in the paper**.

4. **For each MCQ question** in section A, there is **only one correct answer** and you are expected to **clearly mark only one choice**.

5. Each question in section A carries 2 marks.

6. Section B contains **two (2)** essay type questions.

7. The maximum attainable mark for each question in section B is given in brackets.

8. The maximum attainable mark for this paper is 100. This examination accounts for 60% of the module assessment.

9. This is a closed book examination.

   NB: It is an offence to be in possession of unauthorized material during the examination.

10. Only calculators approved and labelled by the Faculty of Engineering are permitted.

11. Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions made on the script.

12. In case of any doubt as to the interpretation of the wording of a question, make suitable assumptions and clearly state them on the script.

13. This paper should be answered only in English.

SECTION A

1) What is a computational task?

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2) If bubble sort is performed on an array with *n* elements, what will be the maximum possible number of **swaps** that will be performed by the bubble sort?

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3) Draw a flowchart to output whether a given array of numbers is in the ascending order.

4) If $f(x) = 2x^2 + 4x$ and $g(x) = 3x^3 + 9x + 27$ and $h(x) = 5x^5 + 5$ which of the following statements are correct?

    I. $f(x) \in O(g(x))$ and $g(x) \in O(h(x))$ and $h(x) \in \Omega(f(x))$

    II. $f(x) \in o(g(x))$ and $g(x) \in o(h(x))$ and $h(x) \in \omega(f(x))$

    III. $f(x) \in \Theta(g(x))$ and $g(x) \in o(h(x))$ and $h(x) \in \omega(f(x))$

a) II Only
b) I and II Only
c) II and III Only
d) All three

5) Which of the following statements are correct?

    I.   If $f(x) \in o(g(x))$ then $f(x) \in \Omega(g(x))$ can never be true.

    II.   If $f(x) \in o(g(x))$ then $f(x) \in O(g(x))$ is always true.

    III.   If $f(x) \in O(g(x))$ then $f(x) \in \Omega(g(x))$ can never be true.

a) II Only
b) I and II Only
c) I and III Only
d) All three

6) Which of the following statements are correct regarding the sorting algorithms?

    I.   For a mobile phone application, using Heapsort is better than using Mergesort.

    II.   The worst case time complexity of Mergesort is better than that of Heapsort.

    III.   The space complexity of Heapsort and Insertion sort are the same.

a) II Only
b) I and II Only
c) I and III Only
d) All three

7) Which of the following statements are correct?

    I.   If $f(x) \in o(g(x))$ and $g(x) \in \Theta(h(x))$, then $f(x) \in o(h(x))$ is always true.

    II.   If $f(x) \in o(g(x))$ and $g(x) \in \Theta(h(x))$, then $f(x) \in O(h(x))$ is always true.

    III.   If $f(x) \in \omega(g(x))$ and $g(x) \in \omega(h(x))$, then $h(x) \in O(f(x))$ is always true.

a) I Only
b) I and II Only
c) II and III Only
d) All three

8) Suppose a divide and conquer algorithm solves a problem by dividing the problem into **5** sub-problems and then solving **3** sub-problems out of it. The dividing of the problem and combining solutions to sub problems to obtain the final solution takes constant time. The recursive function that represents the time complexity of the algorithm, $T(n)$ could be;

a) $T(n) = T\left(\frac{n}{3}\right) + 5c$

b) $T(n) = 5T\left(\frac{n}{3}\right) + c$

c) $T(n) = T\left(\frac{n}{5}\right) + 3c$

d) $T(n) = 3T\left(\frac{n}{5}\right) + c$

Continued …

9) Which of the following statements are correct regarding Mergesort?

    I.   In the recursive scenario, Mergesort always divides the array into two subarrays of exactly the same size.

    II.   The terminating conditions for merge sort is array with only one element.

    III.   Worst case time complexity of Mergesort is $O(n\log n)$.

    a) I Only

    b) I and II Only

    c) II and III Only

    d) All three

10) List two approaches that can be used to express **recursive** algorithms.

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11) Relaxation is a crucial step for any Single Source Shortest Path algorithm. Fill in the blanks of the following Pseudo-code to complete relaxation of edge (u,v) with a weight of w.

RELAX(u, v, w)
if ………. **(a)** ……….
    ………. **(b)** ……….
    $v.\pi = u$

(a) ………

(b) ………

12) Spanning tree is:

    a) A connected, acyclic subgraph with subset of vertices

    b) A connected graph with all vertices which may include cycles

    c) A connected, acyclic subgraph with all vertices

    d) An acyclic subset of edges that connect a subset of vertices

13) It is possible to generate a forest of trees from

    I.   Depth First Search

    II.   Breadth First Search

    a) I Only

    b) II Only

    c) Both I and II

    d) None of the above

14) An example sorting algorithm for divide and conquer approach is:

    a) Heap Sort

    b) Merge Sort

    c) Both Heap Sort and Merge Sort

    d) Neither of Heap Sort and Merge Sort

15) Select the correct statement of heap sort for an array of n elements

    a) Heap sort is conducted in bottom up manner
    b) At the beginning of the Heap Sort we cannot assume any sub tree is a heap
    c) Running time of heap sort is $\Theta(n)$
    d) In a heap, any sub tree can have only n/2 nodes

16) Write an example software component, which uses priority queue data structure.

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17) What is the meaning of 'in place sorting'?

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18) Name an example sorting algorithm which does 'in place sorting'.

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19) Which of the following mechanism does not include a collision resolution technique?

    a) Direct addressing
    b) Open addressing
    c) Chaining
    d) All of the above are hashing techniques

20) Write two potential strategies to increase efficiency of a hash table.

    (a) ……………………………………………………………………………………………

    (b) ……………………………………………………………………………………………

21) If the index of the first probe location is 2, write the next four probe locations for the following hash function.

$$H(k, i) = (h'(k) + i + i^2) \bmod m$$

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22) Given below is Dijkstra algorithm for SSSP.

```
DIJKSTRA( G, w, s)
1. INITIALIZE-SINGLE-SOURCE(G, s)
2. S = Ø
3. Q = G.V
4. while Q ≠ Ø
5.     u = EXTRACT-MIN(Q)
6.     S = S ∪ {u}
7.     for each v ∈ G.Adj[u]
8.         RELAX (u, v, w)
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Explain the role of **Q** and the functionality of call EXTRACT-MIN(Q).

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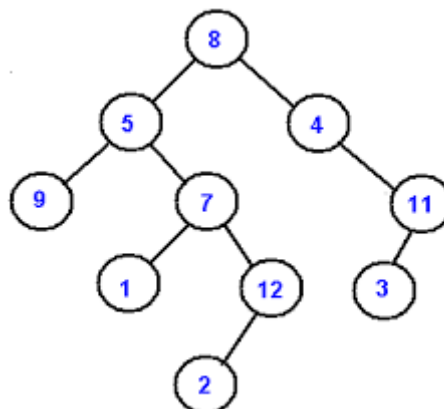Answer the questions 23, 24 and 25 using the following binary-tree structure:

23) What is the result of in-order traversal?

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24) What is the result of pre-order traversal?

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25) If the tree is converted to a Binary Search Tree what is the result of in-order traversal?

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SECTION B

Q1.   **[25 *marks*]**

(a) Give the asymptotic growth in "big oh" notation for the following functions. **Please show how you obtained your answer**.

    i) $T(n) = 100\,n + 10^{-22}\,n^2 + 0.9n^2 \log n$ [2 *marks*]

    ii) $T(n) = T\left(\frac{n}{2}\right) + n \log n$ [3 *marks*]

    iii) $T(n) = 2T\left(\frac{n}{2}\right) + 3T\left(\frac{n}{3}\right) + c$ [3 *marks*]

(b) You are hired as the project manager of ABC organization with a probation period of 20 weeks. You are required to select a set of projects from the available projects and complete them within the 20 weeks. Table (Table QB1) shown below lists the available projects. You have to fully complete the project to obtain the specified profit and partial completion of project will not earn any profit. One of your friends says that you can solve this problem using the dynamic programming approach.

Table QB1: Project Details

Project Identifier	Completion Time (weeks)	Expected Revenue (Rs. Millions)
1	4	14
2	6	15
3	8	24
4	10	25
5	12	12

i)   Specify the problem in a mathematical form assuming that x_i denotes the selecting or not selecting of the i^{th} project. [2 *marks*]

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ii) Let $P(i,k)$ denote the maximum profit possible by selecting a subset of projects from projects $\underline{i}$, $i+1$,…,n with a time limit of k weeks. Write the recursive solution to the problem of selecting the optimal set of projects that can be completed within a given amount of time and would maximize the profit. [4 *marks*]

iii) Compute the solution to the problem using the dynamic programming approach in either top-down or bottom-up approach. [8 *marks*]

iv) If it is possible to obtain partial profit for partially completed projects (profit will be proportional to the percentage of work completed), **briefly explain** a faster method of selecting the best possible set of projects. [3 *marks*]

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Q2. [25 *marks*]

The pseudocode of BFS algorithm is shown below. Answer questions (a), (b) and (c) based on BFS.

```
BFS(G,s)
1. for each vertex u in V[G] -  {s}
2      do color[u] ← white
3           d[u] ← ∝
4           π[u] ← nil
5   color[s] ← gray
6   d[s] ← 0
7    π[s] ← nil
8   Q ← Φ
9   enqueue(Q,s)
10  while Q ≠ Φ
11      do u ← dequeue(Q)
12         for each v in Adj[u]
13           do if color[v] = white
14             then color[v] ← gray
15                 d[v] ← d[u] + 1
16                 π[v] ← u
17                 enqueue(Q,v)
18         color[u] ← black
```

(a) Explain the following terminology, in relation to BFS. [2 *marks*]

    i)   d[v]

………

………

    ii)   Π[v]

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(b) What is the worst case time complexity of BFS? [2 *marks*]

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(c) Prove the time complexity you stated in part (b) using pseudocode provided above.[4 *marks*]

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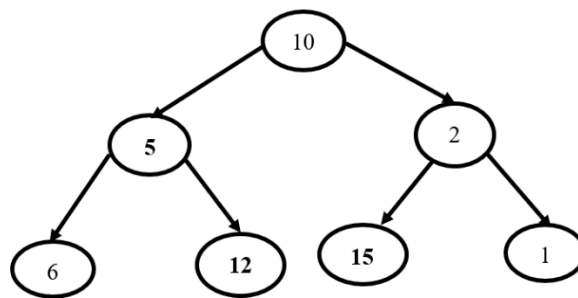
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(d) Apply BUILD-MAX-HEAP to the binary tree below. Show/explain the steps followed.

[5 *marks*]

(e) Write two differences of adjacency-list representation and matrix representation of a graph.

[4 *marks*]

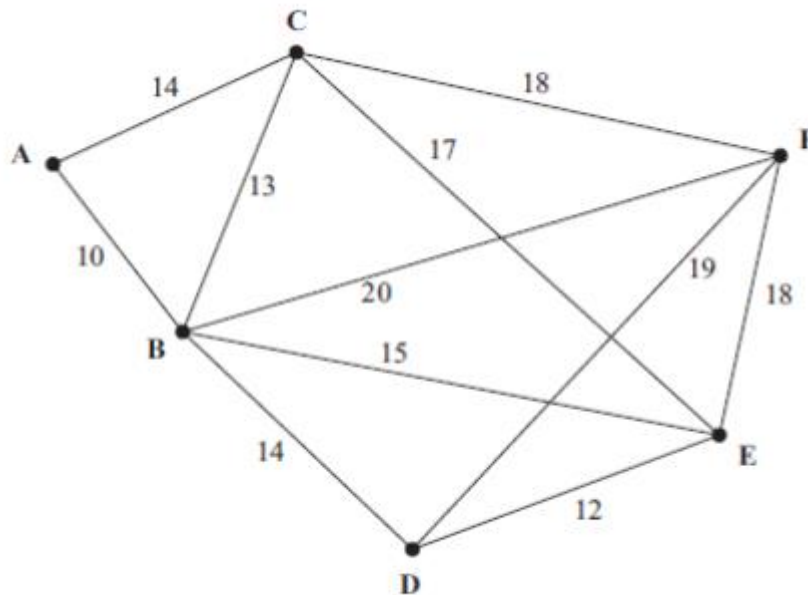………

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(f) Use Kruskal's algorithm to find a minimum spanning tree for the graph below. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree. Draw the resulting minimum spanning tree.[5 *marks*]

(g) State whether your minimum spanning tree is unique. Justify your answer. [3 *marks*]

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--- End of the Paper ---