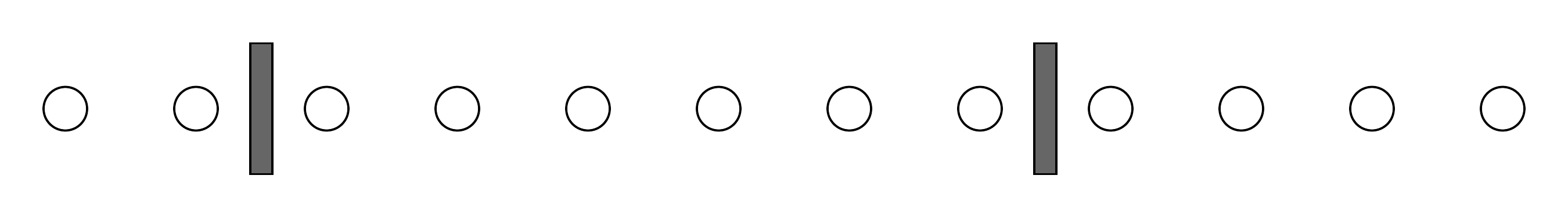
Combinatorics HW 5-2

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1. **Integer composition: Integer 5 is partitioned into orderly partitions which are made up by numbers 1,2,3,4. Such as (1+1+3, or 1+3+1 or 2+3, 4+1,....) How many different ways are there?**

Let us first discuss the problem of partitioningintopartitions. An equivalent problem is: Givenidentical balls, how do we partition them intogroups? We can get the answer by placingbars into thegaps between the balls:



We can easily calculate the number of ways to do this:

Now let us go back to the original question. When partitioning the number 5, we can partition it into 2, 3, 4, or 5 groups (1 is not considered as we cannot use the element 5). These are independent cases, so we have to use the *addition principle* to get the final result:

**Thus, there are 15 ways to partition 5 into orderly partitions of 1, 2, 3, 4.**

1. **Integer partition: How many ways to partition n into several numbers that the order between numbers is ignored. Please write the corresponding generating function.**

Ways of choosing number 1:

Ways of choosing number 2:

… etc. These can be viewed as steps, so we can use the *multiplication principle* to get the corresponding generating function:

Alternatively, we can represent these terms as fractions:

To get the partition number of, we need to find the coefficient of.

(Note: In practice, it’s enough to go until, as for partitioningwe certainly cannot use numbers larger than it.)

1. **Provide proof that the partition number of the summation of the partitioning of integer *n* into odd numbers, is equaled to the partition number of *n* being partitioned into the self-conjugated Ferrers Diagram.**

Letdenote the set of partitions ofconsisting of distinct odd numbers only.  
Letdenote the set of self-conjugated Ferrers diagrams corresponding to.

To prove that, I will show that there is a one-to-one correspondence between elements of these two sets.

For any element or(i.e. a partitioning of using distinct odd numbers only) we can construct a self-conjugated Ferrers diagram of. Let us consider the partitioning

From this, we can directly construct a Ferrers diagramsuch that row of the diagram containssquares. Then, we can transform this into a self-conjugated Ferrers Diagramby *folding* the rows and rearranging them like this:



As the number of squares in each row ofis odd, this kinds of symmetric folding is always possible.

As the number of squares in the rows ofis guaranteed to be decreasing (is a valid Ferrer diagram and the numbersare all distinct),is also guaranteed to be a valid Ferrers diagram.

On the other hand, given a self-conjugated Ferrers diagram, we can always construct the corresponding partitioningby simply reversing this process:



Asis self-conjugated (i.e. symmetric w.r.t. the diagonal), each “**Γ**”section of(corresponding to one row of ) is guaranteed to have an odd number of squares.

It is also easy to see that the number of squares inwill be decreasing.

**Conclusion**

I have shown that for each partitioning ofconsisting of distinct odd numbers only, there is a corresponding self-conjugated Ferrers diagram of and vice-versa. This means that there is a one-to-one correspondence between the elements of these two sets, which in turn **proves that the number of elements in these two sets are equal.**