Combinatorics HW w6-1

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1. **Please prove the following equation of fibonacci sequence Fi:**

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$$

Let us remember the fact that

$$F_{n-1} = F_n - F_{n-2}$$

Let us apply this formula to all F_i where i is an odd number:

$$\begin{aligned} F_1 &= F_2 - F_0 \\ F_3 &= F_4 - F_2 \\ F_5 &= F_6 - F_4 \\ &\vdots \\ F_{2n-1} &= F_{2n} - F_{2n-2} \end{aligned}$$

Adding these together, almost all terms on the right are canceled. We get:

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = (F_2 - F_0) + (F_4 - F_2) + (F_6 - F_4) + \cdots$$
$$= F_{2n} - F_0 = F_{2n}$$

2. **Please provide the corresponding characteristic equations for the following recurrence relation:**

$$a_n = 2a_{n-1} + 4a_{n-2} - 5a_{n-3}$$

Let us rearrange the equation:

$$a_n - 2a_{n-1} - 4a_{n-2} + 5a_{n-3} = 0$$

The corresponding characteristic equation is

$$q^3 - 2q^2 - 4q + 5 = 0$$

3. Solve the recurrence relation $h_n=2h_{n-1}+8h_{n-2}$, $n\geq 2$, $h_1=1$, $h_2=10$

Let us first rearrange the relation:

$$h_n - 2h_{n-1} - 8h_{n-2} = 0$$

This is a linear, homogeneous recurrence relation. As a consequence, we can directly derive its characteristic equation:

$$q^2 - 2q - 8 = 0$$

Proceeding from here, we can find the roots by rewriting the left-hand side as a product:

$$(q+2)(q-4) = 0$$

Thus, the roots of this equation are -2 and $+4$. Building on this, let us formulate our candidate for h_n's closed-form solution:

$$h_n = c_1 \cdot (-2)^n + c_2 \cdot 4^n$$

We can find the value of the two constants using the initial conditions.

$$h_1 = -2c_1 + 4c_2 = 1$$

$$h_2 = 4c_1 + 16c_2 = 10$$

Solving this simple, fully defined system of linear equations, we get

$$c_1 = \frac{1}{2} \quad c_2 = \frac{1}{2}$$

… i.e. the closed-form solution for the given recurrence relation is

$$h_n = \frac{1}{2} \cdot (-2)^n + \frac{1}{2} \cdot 4^n \quad n \geq 1$$