Combinatorics HW 1.2

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1. **How many odd numbers between 1000 and 9999 whose digits are distinct with each other?**

Let us represent the four digits with four variables:

$$\underline{a} \quad \underline{b} \quad \underline{c} \quad \underline{d}$$

1. First, let us choose the last digit d. We have 5 options: 1, 3, 5, 7, 9.
2. After this, we have 8 options for a, as we cannot choose 0 or d.
3. After this, we have 8 options for b, as we cannot choose a or d.
4. After this, we have 7 options for c, as we cannot choose a, b or d.

$$\overline{8} \quad \overline{8} \quad \overline{7} \quad \overline{5}$$

Using the *multiplication principle*, the result is

$$|A| = 8 \cdot 8 \cdot 7 \cdot 5 = 2240$$

2. How many 7-digit numbers are there such that the digits are distinct integers taken from {1, 2, …, 9} and such that the digits 5 and 6 do not appear consecutively in either order?

$$D = \{1, 2, \dots, 9\}$$

The number of all 7-digit numbers with distinct digits drawn from D:

$$|U| = P(9, 7) = \frac{9!}{2!} = 181{,}440$$

Let us regard the complement set: 7-digit numbers with distinct digits where 5 and 6 appear consecutively. To enumerate this set, let us regard 5 and 6 as one unit X:

$$X \in \{(5, 6), (6, 5)\} \qquad |X| = 2!$$

The number of ways to choose the remaining 5 digits from $D \setminus \{5, 6\}$:

$$C(7, 5) = \frac{7!}{2! \cdot 5!} = 21$$

Finally, the number of permutations of these 6 units is $6!$. Applying the *multiplication principle*, the number of all such numbers is

$$|\bar{A}| = 2! \cdot C(7, 5) \cdot 6! = 30{,}240$$

Another way to calculate this would be to first get the permutations for the 5 remaining digits, then put X in one of the 6 slots between these numbers:

$$|\bar{A}| = 2! \cdot P(7, 5) \cdot 6 = 30{,}240$$

To get the target set, we use the *subtraction principle*:

$$|A| = |U| - |\bar{A}| = 181{,}440 - 30{,}240 = 151{,}200$$

Thus, **there are 151,200 7-digit numbers with distinct digits drawn from *D* where the digits 5 and 6 do not appear consecutively.**

3. How many different lattice paths from (-1,1) to (5,4)?

Using the formula from the lecture:

$$|(a,b)(c,d)| = \binom{(c-a)+(d-b)}{(c-a)}$$

Here we have

$$(a,b) = (-1,1) \qquad (c,d) = (5,4)$$

Thus, the number of different lattice paths between these the points is

$$\binom{\big(5-(-1)\big)+(4-1)}{\big(5-(-1)\big)} = \binom{9}{6} = \frac{9!}{6!\cdot 3!} = 84$$

Explanation: Basically, we need to arrange $n \rightarrow$ symbols and $m \uparrow$ symbols in some order, where

$$n = c - a = 6$$
$$m = d - b = 3$$

We have $n+m$ slots and we need to choose n for the $\rightarrow$ symbols, the number of ways we can do this is

$$C(n+m, n) = \binom{9}{6}$$