Combinatorics 2018 HW 2.1

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1. How many different permutations for "Combinatorics"? (Case sensitive)

Using the letters of the given word, we can construct the following multiset:

$$S = \{1 \cdot a, 1 \cdot b, 1 \cdot c, 1 \cdot C, 2 \cdot i, 1 \cdot m, 1 \cdot n, 2 \cdot o, 1 \cdot r, 1 \cdot s, 1 \cdot t\}$$

$$= \{n_i \cdot l_i \mid n_i \text{ is the number of } l_i \text{ characters in "Combinatorics"}\}$$

If $|S| = n$, the number of permutations of these characters is

$$\frac{n!}{\prod n_i!} = \frac{13!}{1! \cdot 1! \cdot 1! \cdot 1! \cdot 2! \cdot 1! \cdot 1! \cdot 2! \cdot 1! \cdot 1! \cdot 1!} = \frac{13!}{4} = 1{,}556{,}755{,}200$$

(Explanation: First, we calculate the number of different permutations assuming that all 13 characters are different. As the two characters *i* and *o* appear twice each, we counted each unique permutation 4 times. Using the *division principle*, we divide by 4 to get the correct result.)

2. The coefficient number of $a^2b^2c^2$ in the expanded equation of（$2a+b+c$）^{6}?

Let us first expand the expression:

$$(2a + b + c)^6 = (2a + b + c) \cdot \ldots \cdot (2a + b + c) \quad \text{(6 times)}$$

To get $a^2b^2c^2$, we must choose each variable twice from the 6 terms above.

Let us first choose the 2 a's from the 6 terms, then the 2 b's from the remaining 4 terms. Note that the choice of a's and b's determine the c's. The number of possibilities:

$$N = C(6,2) \cdot C(4,2) = 90$$

However, every time we choose a we also choose the corresponding 2. As a result, we have to adjust N:

$$N' = 2^2 \cdot N = 360$$

An alternative way to count these would be this: Imagine that we use red balls to represent a's, green balls to represent b's, and blue balls to represent c's. Then we could assign one ball to each term, e.g.:

$$(2a + b + c) \cdot (2a + b + c) \cdot (2a + b + c) \cdot (2a + b + c) \cdot (2a + b + c) \cdot (2a + b + c)$$

The number of ways we could make this arrangement is:

$$N = \frac{P(6,6)}{P(2,2) \cdot P(2,2) \cdot P(2,2)} = \frac{6!}{2!^3} = 90$$

The coefficient number of $a^2b^2c^2$ in the above expressions is 360.

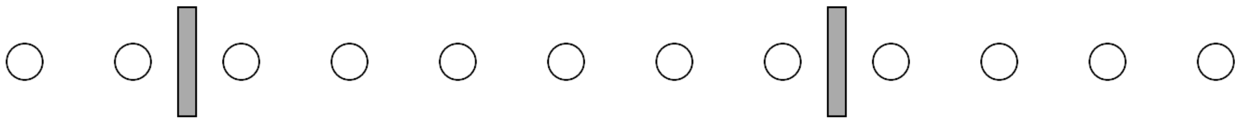
3. For the case of giving fruits to 3 kids, in total there are 12 identical apples, each child may at least have one apple, how many different ways to give apples to 3 kids?

We can view the 12 apples as *balls* and the 3 kids as 3 groups of these balls defined by 2 *bars*.

$$r = 12$$

$$k = 3$$

One possible configuration would be this:



Meaning that

$$n_1 = 2 \quad n_1 = 6 \quad n_1 = 4$$

… where n_i is the number of apples we give to kid number i. In this arrangement, there are $n - 1$ possible *slots* for the two bars. (Given that we want to give each kid at least one apple, bars cannot be placed at the ends and two bars cannot be in the same slot.) Then, the number of possibilities is given by:

$$C(n-1, k-1) = C(11, 2) = 55$$

4. What is the number of integral solutions of the equation x1+x2+x3=30, in which x1≥5, x2≥-8, x3≥5.

First, let us *normalize* the equation, i.e. introduce new variables so that we have simpler constrains:

$$y_1 + y_2 + y_3 = 28 \qquad \forall\, i \in \{1,2,3\}: y_i \geq 0$$

… where

$$\begin{aligned} y_1 &= x_1 - 5 \\ y_2 &= x_2 + 8 \\ y_3 &= x_3 - 5 \end{aligned}$$

We could represent this problems with 28 1's and sections representing the three variables, defined by 2 separating *bars*.

$$1 \quad 1 \mid 1 \quad 1 \quad 1 \quad 1 \ldots 1 \quad 1 \quad 1 \mid 1 \quad 1 \quad 1 \quad 1$$

In the above example, y_1 is 2, y_2 is 22, and y_3 is 4, defined by the number of 1's in each section. Note that numbers can be zero, represented by bars on the side or sharing the same slot, e.g.

$$\mid \ \mid 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \ldots 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

Here, y_1 is 0, y_2 is 0, and y_3 is 28. For counting the possibilities, we first take the 30 elements (28 1's and 2 bars) and choose which two should be the bars. The order does not matter. The number of possibilities:

$$C(30, 2) = 435$$

There is a bijection (one-to-one correspondence) between solutions to the new (y_i) and old (x_i) equations. Thus, **the number of integral solutions to the given equation with the given constraints is 435.**