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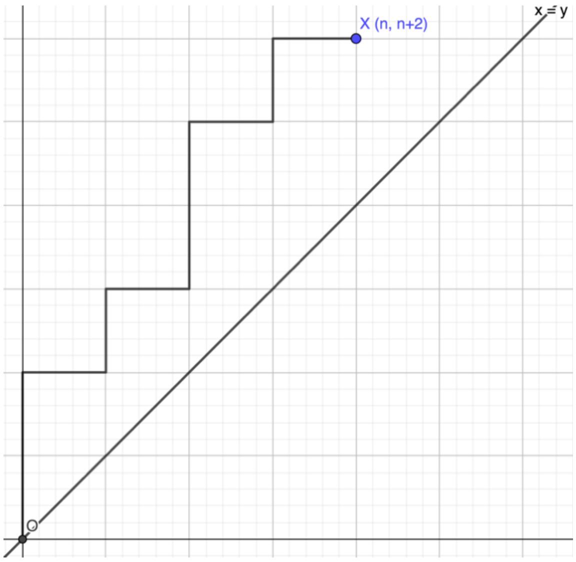
Note: For ease of explanation, I reordered the problems (2-1-3).

2. Find out the number of lattice paths from (0,0) to (n,n+2), which are above but do not touch y=x line? List the formula with n.

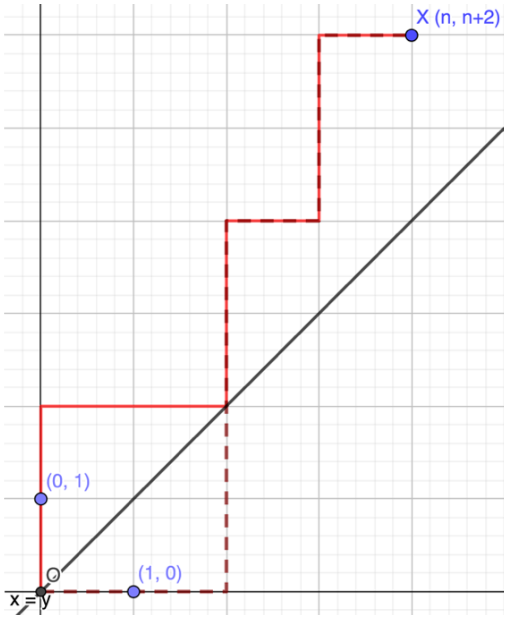
We would like to count the number of lattice paths from $(0,0)$ to $(n, n+2)$ that are above and do not touch the line. An example for this is depicted on the right.



As the first move is guaranteed to be ↑, **it's sufficient to only regard paths starting at $(\mathbf{0}, \mathbf{1})$**. To count valid paths, we can use the *subtraction principle*: calculate the number of all possible paths and subtract the number of invalid paths.

The number of all lattice paths between $(0,1)$ and $(n, n+2)$ is $C(2n+1, n)$.

But how do we enumerate the invalid ones? The crucial realization is this: **for every invalid path from $(\mathbf{0}, \mathbf{1})$, we have a corresponding path from $(\mathbf{1}, \mathbf{0})$**. Moreover, there is a one-to-one correspondence between these paths. To get the corresponding path, just reflect the segment between O and the touching point, as depicted on the figure.



Thus, instead of directly counting the invalid paths from $(0,1)$ to X, we can count the corresponding paths from $(1,0)$ to X. The number of these paths is $C(2n+1, n-1)$.

Combining these formulas, we get the final result:

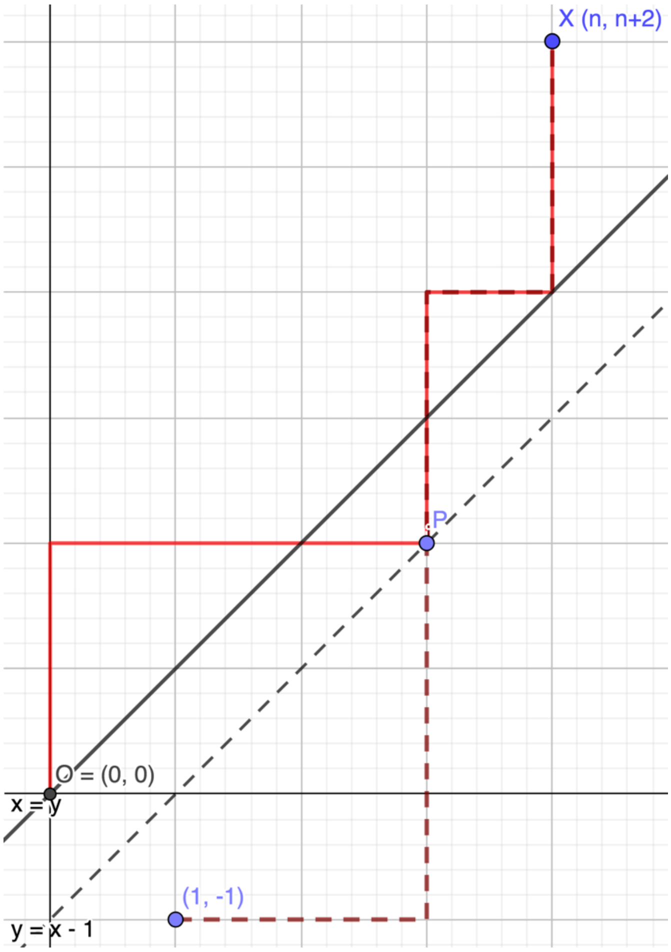
$$\boldsymbol{C(2n+1, n) - C(2n+1, n-1)}$$

1. Find out the number of lattice paths from (0,0) to (n,n+2), which are above but do not cross y=x line? List the formula with n.

We can solve this problem using a similar method as in the previous problem. In this case, the number of all possibilities is $C(2n+2, n)$.

Again, we notice: **For each invalid path starting at $(\mathbf{0}, \mathbf{0})$, there is a corresponding path from $(\mathbf{1}, -\mathbf{1})$.** Moreover, there is a one-to-one correspondence between these paths. To get the corresponding path, just reflect the segment from O to P on the shifted line $y = x - 1$.

Thus, instead of directly counting the invalid paths from $(0,0)$ to X, we can count the number of corresponding paths from $(1, -1)$ to X. The number of these paths is $C(2n+2, n-1)$.



Combining these formulas, we get the final result:

$$\boldsymbol{C(2n+2, n) - C(2n+2, n-1)}$$

3. **If we want to use positive integers from 1 until 7 to form a ring in order. Since 1 and 7 are adjacent to each other in the ring. Due to their neighboring position, 1 and 7 are also considered as neighbor numbers. Then if we want to pick 3 non-neighboring numbers from this ring of 7 numbers, how many different solutions are there?**

How many different ways are there to choose 3 non-neighboring numbers from

1

7 2

6 3

5 4

The easiest way is to enumerate all of them:

1-3-5	2-4-6	3-5-7
1-3-6	2-4-7	
1-4-6	2-5-7	

There are 7 possibilities.