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ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

IE 251 Linear Programming

CHARY

GROUP 32

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INTRODUCTION

A silicon shortage has caused chip scarcity in 2022 and CHARY, an electronics company, has requested a group of industrial engineers to alleviate the shortage at minimum cost. CHARY has developed a plan to supply chips of three different quality grades: excellent, good, and mediocre. Also, CHARY divided the demand into two six-month periods. The demand of each quality grade chip is illustrated in table 1. In addition, CHARY has contracted four different suppliers who are able to sell CHARY chips in a bulk of 100 called lots, each containing a set amount of each quality grade chip. The exact distributions and price of each lot, as well as the limit of lots that are to be supplied are also illustrated in table 2.

	Requirements of Chips of quality grade		
	Excellent	Good	Mediocre
Period 1	2000	1500	400
Period 2	5000	1500	600

Table 1: Requirements of chips of each quality grade to be supplied in each period.

	Number of chips in a lot of 100 chips			Price per lot, in $		Supply Limit, Lots per year
Supplier	Excellent	Good	Mediocre	Period 1	Period 2	
1	60	30	10	400	440	30
2	50	35	15	300	330	20
3	40	20	40	250	250	30
4	50	25	25	275	300	50

Table 2: The amount of chips of each quality grade in each lot, and the price of each lot in each period, along with the supply limit over the year.

According to the contracts, each supplier is to supply the lots uniformly over the year. In other words, only half the total supply limit can be provided per period. For example: supplier 1 has a limit of 30 lots per year. Supplier 1 is to supply only half that amount at max in each period, so 15 lots at max in period 1 and 15 lots at max in period 2.

In order to accommodate for excess chips bought in the first period, CHARY has estimated that the holding costs of chips is $0.1, in order to transfer said chips to be supplied to the second period.

CHARY has also contracted local retailers in order to make specific purchases of each chip from any quality grade. The chips are sold in single units and there is no limit to the amount of chips sold through special orders. The cost of each special order chip is: $10 for an excellent chip, $6 for a good chip, and $4 for a mediocre chip.

CHARY has evaluated a penalty of $1 for each dollar spent on lots and special orders in which the total amount that is paid will exceed the total budget, $25,000, provided by the company.

MODELING

In order to alleviate the chip shortage effectively, several assumptions need to be taken, which include the following:

1- As CHARY buys chips from the suppliers and retailers, it costs the company $0.1 to hold inventory for each chip no matter the quality within every subsequent period. As there is a fitting number of chips purchased during each period, the limit of the inventory to hold the chips isn't a drawback. Thus, it is to be assumed that the capacity of this inventory has no effective limit that restricts the amount of chips held in inventory. This allows the company to prioritize reaching the required amount of chips without worrying about inventory complications.

2-The chips supplied are either free of defects or the defective units will be replaced within the same period, free of cost.

3-To meet the requirements of chips from each quality while having the cost at a minimum, it will be assumed that there can be an excess of chips. Having no excess of chips will most likely result in more special-order purchases which will drastically increase the total cost of reaching the chip requirement.

4-The previous assumption creates another assumption of what will happen to the excess chips that have been purchased. The first possibility is that the excess chips are simply disposed of to avoid handling inventory costs. The other possibility is holding these chips in inventory and using them in following periods, if any.

Based on these assumptions, the model is as follows:

-Sets and Indices:

$i \in I = \{1, 2, 3\}$: Chip of i quality where; Excellent ~ 1; Good ~ 2; Mediocre ~ 3

$s \in S = \{1, 2, 3, 4\}$: Supplier s

$p \in P = \{1, 2\}$: Period p

-Parameters:

D_{ip}: Demand of chip of quality i in period p

l_{is}: Number of chips of quality i per lot from supplier s

L_s: Supply limit of lots from supplier s per year

c_{sp}: Cost of a lot of chips from supplier s in period p

k_i: Cost of special order of a single chip of quality i

h: Cost of holding one chip from period 1 to period 2

B: Total budget

g: Penalty per dollar caused when the total budget is exceeded

G: Total penalty accumulated

H: Total holding costs accumulated

-Decision Variables:

x_{sp}: Number of lots from supplier s *purchased* in period p

o_{ip}: Amount of specially-ordered chips of quality i *purchased* in period p

-Objective Function and Constraints:

$$\textbf{Min}\, z = \sum_{s \in S, p \in P} (c_{sp} x_{sp}) + \sum_{i \in I, p \in P} (k_i o_{ip}) + H + G$$

Subject to:

$$\Sigma_{s\in S}(l_{is}x_{s1}) + o_{i1} \geq D_{i1} \text{ for } i \in I \quad \text{(Period 1 Chip Demand)}$$

$$\Sigma_{s\in S}(l_{is}x_{s2} + l_{is}x_{s1}) + \Sigma_{p\in P}\left(o_{ip}\right) \geq \Sigma_{p\in P}\left(D_{ip}\right) \text{ for } i \in I \quad \text{(Total Chip Demand)}$$

$$x_{sp} = \frac{L_s}{2} \text{ for } s \in S \text{ and } p \in P \quad \text{(Limit of Lots per period per Supplier)}$$

$$g\left(\Sigma_{s\in S, p\in P}\left(c_{sp}x_{sp}\right) + \Sigma_{i\in I, p\in P}\left(k_i o_{ip}\right) - B\right) = G \quad \text{(Accumulated Penalty)}$$

$$h\left(\Sigma_{i\in I}\left(\sum_{s\in S}(l_{is}x_{s1}) + o_{i1} - D_{i1}\right)\right) = H \quad \text{(Total Holding Costs)}$$

-Sign Restrictions:

$$x_{sp} \geq 0 \ \forall \ s \in S, p \in P$$

$$o_{ip} \geq 0 \ \forall \ i \in I, p \in P$$

RESULTS

After adding the parameters and data into Microsoft's Excel Solver module, the objective optimal objective function's value is at $66,810, with all the supplier's lots completely sold out in all periods, and no special orders except 500 excellent quality grade chips ordered in period 2, as illustrated:

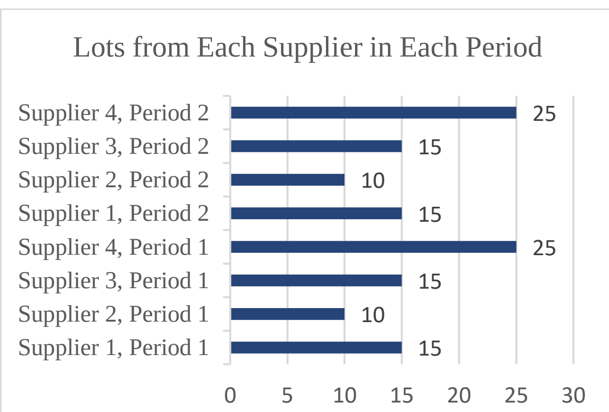

Figure 3 (Left): Optimal solution for lots purchased from each supplier in each period.

Figure 4 (Right): Optimal solution for special orders on each quality grade chip in each period.

This result shows that there is a lot of excess mediocre and good quality grade chips (2050 and 450, respectively) but no excess in the excellent quality grade chips. In fact, it seems that the software found it more optimal and less costly to maximize the amount of excellent chips in lots obtained from suppliers rather than from retailers in special orders, even after accounting for holding costs for the excess chips. After further analysis, this can be explained by special order chips being more expensive on a per-unit-basis than chips bought in bulk from suppliers when excluding the other two quality grade chips (when they are in excess) and accounting for holding costs for chips bought in the 1st period:

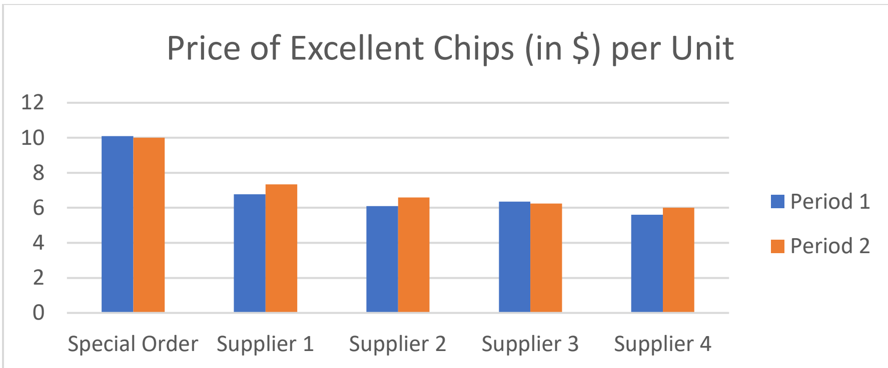

Figure 5: The per-unit-price of each excellent chip from different sources whilst excluding the two other quality grade chips that are in excess.

An interesting note is that even if holding costs were included for excess chips at the end of period 2, the final result in that case, $67,060, will still be less than the case were no excess chips are allowed, $101,650, as shown in Figures A1 and A2. So, whether the chips would be thrown away or stored for any subsequent periods does not change the fact that buying in excess is less costly then the zero excess case.

DISCUSSION

The certainty of the events in our lives is changing day by day and cannot be kept up to date. Unfortunately, the validity of the studies carried out in this century, where conditions and needs change instantaneously, is short-lived. However, there is also a way to adapt to the ever-changing conditions. In addition to solving the problem, the Excel solver module also provides the necessary tools to perform the sensitivity analysis easily and effectively. Thanks to this analysis, any changes to the data and what these changes entail can be mostly interpreted without solving the model again. Thus, adapting to problems and sudden shifts is easier with the SIMPLEX Algorithm and a relevant software, like the Excel Solver module.

As an example of these possible disruptions, a situation such the fluctuation of the prices of the lots purchased from supplier 1 in period 1 can be tackled. In such cases, the table titles *Allowable increase* and *Allowable Decrease* illustrate the range in which the optimal variables do not change. For example, if the objective coefficient is 810, and if this value increases by 390 or less, or decreases indefinitely, the optimal solution will remain valid.

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
J3	Amount of Lots bought from Supplier 1 in Period 1 Values	15	0	810	390	1E+30
J4	Amount of Lots bought from Supplier 2 in Period 1 Values	10	0	610	390	1E+30
J5	Amount of Lots bought from Supplier 3 in Period 1 Values	15	0	510	290	1E+30
J6	Amount of Lots bought from Supplier 4 in Period 1 Values	25	0	558	442	1E+30

Figure 6: Part of the sensitivity report produced by the Excel Solver module.

The reduced cost is an important aspect when it comes to sensitivity analysis. The reduced cost shows how much the objective function can change in order to make the current solution sub-optimal. It also indicates that our optimal solution is still valid if the objective coefficient is altered by less than the reduced cost.

An interesting observation is that if the cost of the special order excellent chips in period 2 where increased by 0.1, then the optimal solution will change. In addition, the special order chips of the other two quality grades in both period will only remain optimal when their objective coefficient is 0, i.e.: when the special orders are free, which is not profitable for the retailers.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Solver V2.xlsx]Sheet1
Report Created: 12/25/2021 3:58:15 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
J3	Amount of Lots bought from Supplier 1 in Period 1 Values	15	0	810	390	1E+30
J4	Amount of Lots bought from Supplier 2 in Period 1 Values	10	0	610	390	1E+30
J5	Amount of Lots bought from Supplier 3 in Period 1 Values	15	0	510	290	1E+30
J6	Amount of Lots bought from Supplier 4 in Period 1 Values	25	0	558	442	1E+30
J7	Amount of Lots bought from Supplier 1 in Period 2 Values	15	0	880	320	1E+30
J8	Amount of Lots bought from Supplier 2 in Period 2 Values	10	0	660	340	1E+30
J9	Amount of Lots bought from Supplier 3 in Period 2 Values	15	0	500	300	1E+30
J10	Amount of Lots bought from Supplier 4 in Period 2 Values	25	0	600	400	1E+30
J11	Amount of Special Order Chips of Excellent quality bought in Period 1 Value	0	0.1	20.1	1E+30	0.1
J12	Amount of Special Order Chips of Good quality bought in Period 1 Values	0	12.1	12.1	1E+30	12.1
J13	Amount of Special Order Chips of Mediocre quality bought in Period 1 Value	0	8.1	8.1	1E+30	8.1
J14	Amount of Special Order Chips of Excellent quality bought in Period 2 Value	500	0	20	0.1	5.33333333
J15	Amount of Special Order Chips of Good quality bought in Period 2 Values	0	12	12	1E+30	12
J16	Amount of Special Order Chips of Mediocre quality bought in Period 2 Value	0	8	8	1E+30	8

Figure 7: Part of the sensitivity analysis report generated by the Excel Solver module, showing the reduced cost, objective coefficient, and final values of the decision variables.

Shadow price is the amount that the objective function value improves for every unitary increase in a right hand side value. The most notable shadow price is the supply limit of chips from supplier 4 in period 1 with the most negative shadow price of -440. It indicates that every increase in the supply limit by 1 lot decreases the objective function value by $440. It is also worth noting that the shadow price of the chip demand are 0 except for the total excellent chip demand. This can be justified due to the fact that an excess of chips are found in every scenario except period 2 excellent chips.

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
V3	Demand of Chips of Excellent quality in Period 1 LHS	3250	0	2000	1250	1E+30
V4	Demand of Chips of Good quality in Period 1 LHS	1725	0	1500	225	1E+30
V5	Demand of Chips of Mediocre quality in Period 1 LHS	1525	0	400	1125	1E+30
V6	Total Chip Demand of Chips of Excellent quality over the Entire Year LHS	7000	20	7000	1E+30	500
V7	Total Chip Demand of Chips of Good quality over the Entire Year LHS	3450	0	3000	450	1E+30
V8	Total Chip Demand of Chips of Mediocre quality over the Entire Year LHS	3050	0	1000	2050	1E+30
V9	Supply Limit of Lots of Chips from Supplier 1 in Period 1 LHS	15	-390	15	8.333333333	7.5
V10	Supply Limit of Lots of Chips from Supplier 2 in Period 1 LHS	10	-390	10	10	6.428571429
V11	Supply Limit of Lots of Chips from Supplier 3 in Period 1 LHS	15	-290	15	12.5	11.25
V12	Supply Limit of Lots of Chips from Supplier 4 in Period 1 LHS	25	-440	25	10	9
V13	Supply Limit of Lots of Chips from Supplier 1 in Period 2 LHS	15	-320	15	8.333333333	15
V14	Supply Limit of Lots of Chips from Supplier 2 in Period 2 LHS	10	-340	10	10	10
V15	Supply Limit of Lots of Chips from Supplier 3 in Period 2 LHS	15	-300	15	12.5	15
V16	Supply Limit of Lots of Chips from Supplier 4 in Period 2 LHS	25	-400	25	10	18

Figure 8: Part of the sensitivity analysis report generated by the Excel Solver module, displaying the shadow price and the right hand side of the constraints.

After analyzing the parameters, some suggestions come to mind for minimizing the cost even further. CHARY should negotiate with the suppliers to have a higher limit of lots per year to lessen the amount of special-order chips purchased, since the special order prices are relatively higher than ones bought in bulks. In addition, it is possible to calculate which supplier will yield the highest cost decrease. This is done by multiplying the shadow cost of each supply limit by the allowable increase and taking the absolute value. As a result and as shown in the following figure, it will be best to negotiate with supplier 4 to increase their supply limit in period 1 to decrease the cost as much as possible:

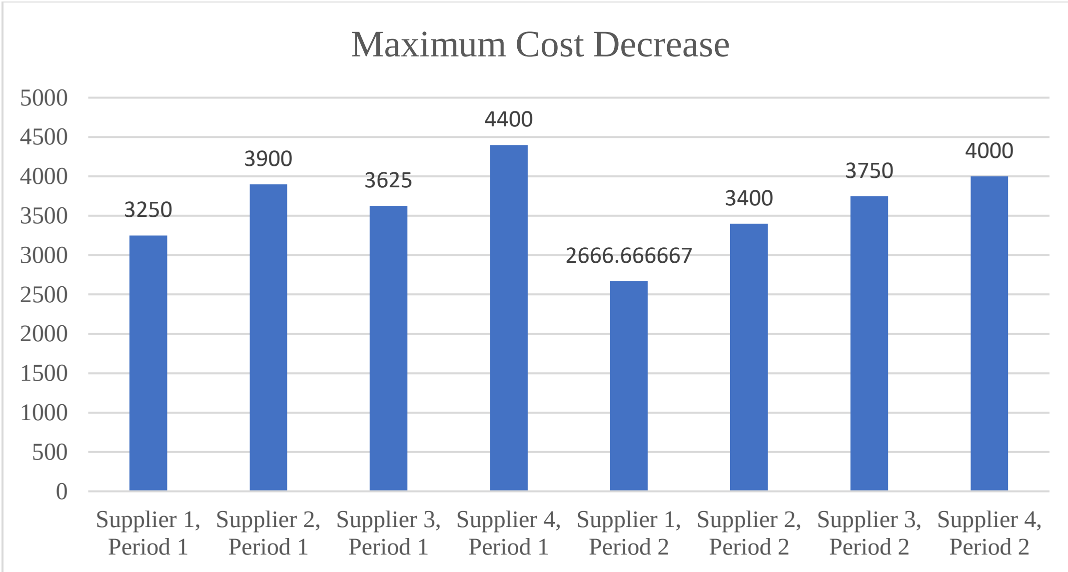

Figure 9: Maximum amount that the objective function after successful negotiations.

Another suggestion that should be mentioned is CHARY negotiating the special-order prices especially when bought in bulks. As a customer buys a huge amount of the product, the customer should receive a wholesale price in order to improve CHARY's objective function value, as well as to build positive business relationships between both the seller and the buyer. So, CHARY should ask for a wholesale and discounted price since they are buying great amounts of chips.

As CHARY obtains excess chips from purchasing the maximum number of lots possible, it is suggested to sell these chips instead of them being held in inventory and creating an extra cost. The chips can be sold to local retailers for a cheaper price than to ensure them being sold. Briefly, holding excess chips will cause another responsibility of maintenance and costs to take care of when the company could sell them for a wholesale price that would benefit both the company and the buyer.

APPENDIX A

Objective Function	
$$\min z = \sum_{s\in S, p\in P} (c_{sp} x_{sp}) + \sum_{i\in I, p\in P} (k_i o_{ip}) + H + G$$	
$z =$	101650

Figure A1: Optimal Solution in case of no excess chips

	Decision Variables	Values
x_{sp}	Amount of Lots bought from Supplier 1 in Period 1	15
	Amount of Lots bought from Supplier 2 in Period 1	10
	Amount of Lots bought from Supplier 3 in Period 1	0
	Amount of Lots bought from Supplier 4 in Period 1	16
	Amount of Lots bought from Supplier 1 in Period 2	15
	Amount of Lots bought from Supplier 2 in Period 2	10
	Amount of Lots bought from Supplier 3 in Period 2	0
	Amount of Lots bought from Supplier 4 in Period 2	0
o_{ip}	Amount of Special Order Chips of Excellent quality bought in Period 1	0
	Amount of Special Order Chips of Good quality bought in Period 1	300
	Amount of Special Order Chips of Mediocre quality bought in Period 1	0
	Amount of Special Order Chips of Excellent quality bought in Period 2	3400
	Amount of Special Order Chips of Good quality bought in Period 2	700
	Amount of Special Order Chips of Mediocre quality bought in Period 2	0

Figure A2: Optimal Variables in case of no excess chips

	Decision Variables	Values
x_{sp}	Amount of Lots bought from Supplier 1 in Period 1	15
	Amount of Lots bought from Supplier 2 in Period 1	10
	Amount of Lots bought from Supplier 3 in Period 1	0
	Amount of Lots bought from Supplier 4 in Period 1	16
	Amount of Lots bought from Supplier 1 in Period 2	15
	Amount of Lots bought from Supplier 2 in Period 2	10
	Amount of Lots bought from Supplier 3 in Period 2	0
	Amount of Lots bought from Supplier 4 in Period 2	0
o_{ip}	Amount of Special Order Chips of Excellent quality bought in Period 1	0
	Amount of Special Order Chips of Good quality bought in Period 1	300
	Amount of Special Order Chips of Mediocre quality bought in Period 1	0
	Amount of Special Order Chips of Excellent quality bought in Period 2	3400
	Amount of Special Order Chips of Good quality bought in Period 2	700
	Amount of Special Order Chips of Mediocre quality bought in Period 2	0

	Constraints	LHS	Sign	RHS
D_{ip} $p = 1$	Demand of Chips of Excellent quality in Period 1	2200	≥	2000
	Demand of Chips of Good quality in Period 1	1500	≥	1500
	Demand of Chips of Mediocre quality in Period 1	700	≥	400
$\sum_{p \in P} D_{ip}$	Total Chip Demand of Chips of Excellent quality over the Entire Year	7000	=	7000
	Total Chip Demand of Chips of Good quality over the Entire Year	3000	=	3000
	Total Chip Demand of Chips of Mediocre quality over the Entire Year	1000	=	1000
$\frac{L_{is}}{2}$	Supply Limit of Lots of Chips from Supplier 1 in Period 1	15	≤	15
	Supply Limit of Lots of Chips from Supplier 2 in Period 1	10	≤	10
	Supply Limit of Lots of Chips from Supplier 3 in Period 1	0	≤	15
	Supply Limit of Lots of Chips from Supplier 4 in Period 1	16	≤	25
	Supply Limit of Lots of Chips from Supplier 1 in Period 2	15	≤	15
	Supply Limit of Lots of Chips from Supplier 2 in Period 2	10	≤	10
	Supply Limit of Lots of Chips from Supplier 3 in Period 2	0	≤	15
	Supply Limit of Lots of Chips from Supplier 4 in Period 2	0	≤	25

Objective Function
$\min z = \sum_{s \in S, p \in P} (c_{sp} x_{sp}) + \sum_{i \in I, p \in P} (k_i o_{ip}) + H + G$
$z =$ 101650

Total Holding Costs of Remaining Chips from Period 1 to Period 2	$H = h\left(\sum_{i \in I, s \in S} (l_{is} x_{s1} + o_{i1} - D_{i1})\right)$	$H =$	50
Total Penalty Accumulated for Exceeding the $25,000 budget on Suppliers and Retailers	$G = g\left(\sum_{s \in S, p \in P} (l_{sp} x_{sp}) + \sum_{i \in I, p \in P} (k_i o_{ip}) - B\right)$	$G =$	38300

Figure A3: The LP model in the case of no excess chips, visualized in Microsoft's Excel application.