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ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

IE 251 Linear Programming

International Coal

GROUP 32

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TABLE OF CONTENTS

Introduction

Even though coal is a large pollutant producer, its abundancy and accessibility are required to generate electricity. Nonetheless, there have been limits set by the EU to lessen the emission of pollutants, such as generators having to pay for CO_2 emissions and having a "Sulphur Bubble", limiting the total amount of SO_2 emitted yearly, from October to October, called a Sulfur Year. This also led to blending coal with biomass, since generators receive a supplement, called the Renewables Obligation Certificate (ROC), for doing so.

The fuel-buying team for International Coal (IC) that operates a large (1,000MW) coal-fired plant, is led by Bob Manchester. Bob's goal is to maximize the profit from buying fuels whilst staying within the environmental and technological limits. Coal is bought 3 or more months ahead of planned burn time and stockpiled coal is kept in the plant as inventory. The power generated from the plant is sold to electricity markets based on future prices. Each month is divided into 4 price bands: weekend or weekday, and peak or off-peak. Peak periods consist of a 12-hour block. In addition, IC is charged a transmission rate of 0.975€ per MWh by the National Grid Company (NGC).

Bob wants to investigate power generation from the end of May till the end of October by considering three types of coal that can be ordered for burning in September and October, and wood-chip biomass which can be bought with short lead-times. Since biomass is more difficult to handle than coal, it may not provide more than 10% of the mix in any of the generating periods. The data is provided in tables 1 and 2:

	Price €/tonne	Calorific Value GJoules/tonne	SO_2 %
Coal: Stockpile	63.84	25.81	1.38%
Coal: Colombian	65.895	25.12	0.70%
Coal: Russian	65.7	24.5	0.35%
Coal: Scottish	63	26.2	1.72%
Wood Chips	110.655	18	0.01%

Period\Electricity Market Forward Prices (€/MWh)	June	July	August	September	October
Weekday Peak	54	54.525	56.475	57.525	65.55
Weekday Off-peak	40.5	40.5	42.3	42.75	47.55
Weekend Peak	50.25	51.45	53.475	53.7	58.05
Weekend Off-peak	39.3	39.45	41.25	41.475	45.15

Table 1 Prices of fuel per tonne, their calorific value, and their sulfur emission rate.

Table 2: Price of Electricity Forward Prices in €/MWh

The stockpiled inventory is 600,000 tonnes and 30% of the Sulphur bubble (which is originally 30,000 tonnes) is left this Sulphur year. In addition, each MWh of electricity produced liberates 0.8 tonnes of CO_2, and CO_2 emissions are trading at 15 Euros per tonne on the European market and the ROC is 67.5€ per MWh generated from renewables.

Assumptions and Model

To find the optimal solution to the problem presented by IC, some assumptions will be made:

1. The additivity, proportionality, divisibility, and certainty assumptions are made, which makes IC's model a linear one.
2. Since there are no explicit holding costs for holding coal and fuel from one period to another and since the prices of fuel are constant throughout the entire period, it will be assumed that the purchases were done at the beginning of June. Stockpiled coal has little lead up time so it will be assumed that the product will be available starting from June, whilst the other three types of coal will still have the usual, three-month, lead-up times. This is merely an assumption to simplify the model and the realistic buying patterns will most likely vary.
3. According to the British Government and Britain's Department for Business (2021), in 2020, the UK consumed 287 TWh (terawatt hour) of electricity. IC's power plant can produce only 8.76 GWh (gigachad hour) in a year. Thus, it will be assumed that the choice whether IC's power plant will sell power or not in a certain period will be taken purely for profit-seeking, as it cannot cause any power outages throughout the country, should it choose to not sell power.

4. Since biomass has a non-reliably varying composition, it will be assumed from the start that the calorific value is 10% of what it actually is.
5. Even though there are 600,000 tonnes of stockpiled coal available and en route, they won't be treated as free resources; a price, equivalent to their market price, will be incurred each time the stockpiled coal is used. Otherwise, the LP solving software will treat the free resources as more efficient to use than the foreign fuel types and prioritize the former's usage, defeating the purpose of this study.
6. Similar to the previous assumption, all other fuel type's fuel cost will be incurred as soon as they are used up. This also entails that there will be zero left-over fuel (except for stockpiled coal) at the end of the study.
7. It will also be assumed that the power is produced and sold uniformly over the entire period and over the month in similar periods.

Based on these assumptions, the model is as follows:

- <u>Sets and Indices</u>:

$c \in C = \{1, 2, 3, 4, 5\}$; fuel: c type: 1 ~ Stockpiled coal, 2 ~ Colombian coal, 3 ~ Russian coal, 4 ~ Scottish coal, 5 ~ Woodchips

$m \in M = \{1, 2, 3, 4, 5\}$; month m: 1 ~ June, 2 ~ July, 3 ~ August, 4 ~ September, 5 ~ October

$w \in W = \{1, 2, 3, 4\}$; period p: 1 ~ Weekday Peak, 2 ~ Weekday Off-Peak, 3 ~ Weekend Peak, 4 ~ Weekend Off-peak

- <u>Parameters</u>:

p_c^f: price of fuel c in Euros per tonne (€/tonne)

v_c: calorific value of fuel c (in GJoules/tonne)

s_c: Amount of SO_2 released after burning one tonne of fuel c

p_{wm}^e: Price of Future Electricity in period p in month m (in €/MWh)

h_{mw}: number of weekdays or weekends (respective to w) in month m

- If $w \in \{1, 2\}$, h_{mw} is the amount of weekdays in month m
- If $w \in \{3,4\}$, h_{mw} is the amount of weekends in month m

H: number of hours in a peak period and off-peak period ($H = 12$ hours)

L: Amount of energy (in MWh) produced per GJoule used up ($L = 0.278$ MWh/Gjoule)

S: Amount of the sulfur bubble limit left of the rest of the sulfur year ($S = 9{,}000$ tonnes)

T^R: transmission rate commissioned by the National Grid Company ($T^R = 0.975\ €/\text{Mwh}$)

C: Tonnes of CO_2 emitted per MWh of electricity produced ($C = 0.8\ \text{tonnes/MWh}$)

E: Power limit of International Coal's coal-fired plant ($E = 1000\text{MW}$)

ϵ: Efficiency of IC's power plant ($\epsilon = 0.35$)

I: Inventory of Stockpiled coal before the start of June ($I = 600{,}000\ tonnes$)

T^E: The price IC must pay for every tonne of CO_2 produced ($T^E = 15\ €/tonne$)

T^{ROC}: the ROC per MWh generated from renewables ($T^{ROC} = 67.5\ €/MWh$)

- <u>Decision Variables</u>:

e_{cmw}: Amount of fuel c used up in month m to sell power in period w (in tonnes)

- <u>Objective Function</u>:

$$\begin{aligned}\text{Max}\, z = &\sum_{c\in C, m\in M, w\in W} (p^e_{wm})(h_{mw})(\epsilon)(L)(v_c)(e_{cmw}) \\ &+ \sum_{m\in M, w\in W} (T^{ROC})(h_{mw})(\epsilon)(L)(v_5)(e_{5mw}) - \sum_{c\in C, m\in M, w\in W} p^f_c h_{mw} e_{cmw} \\ &- \sum_{c\in C, m\in M, w\in W} (T^E)\,(C)(h_{mw})(\epsilon)(L)(v_c)(e_{cmw}) \\ &- \sum_{c\in C, m\in M, w\in W} (T^R)(h_{mw})(\epsilon)(L)(v_c)(e_{cmw})\end{aligned}$$

$$\begin{aligned}\text{Max}\, z = &\ [\text{Revenue from selling electricity}] + [\text{ROC Revenue}] - [\text{Price of Fuel}] \\ &- [\text{CO}_2\ \text{Emissions Tax}] - [\text{Transmission Rate}]\end{aligned}$$

- <u>Subject to</u>:

$\sum_{c\in C}(\epsilon)(L)(v_c)(e_{cmw}) \le HE$ for all $m \in M$ and $w \in W$ (IC power plant limit)

- The plant's power is $P = 1000MW$, and the maximum energy it can provide is $E = P \times t = 1000MW \times 12h = 12{,}000MWh$
- Unit of $(\epsilon)(L)(v_c)(e_{cmw})$ is $1 * \frac{MWh}{GJoule} * \frac{GJoule}{tonne} * tonne = MWh$

$\sum_{m\in M, w\in W} h_{mw} e_{1mw} \le I$ (Amount of stockpiled coal used in the entire period)

- The amount of stockpile used in the entire study period cannot exceed I
- e_{1mw} represents stockpiled coal burnt in one period, and $h_{mw}e_{1mw}$ is the amount of stockpiled coal burnt in the entire month m during period w

$$\sum_{c\in C, m\in M, w\in W} s_c h_{mw} e_{cmw} \leq S \qquad \text{(Sulfur bubble limit)}$$

- The sulfur emitted cannot exceed the sulfur bubble limit, S
- Unit of $s_c h_{mw} e_{cmw}$ is $\frac{tonnes\ of\ sulfur}{tonne\ of\ fuel} * tonne\ of\ fuel = tonnes\ of\ sulfur$

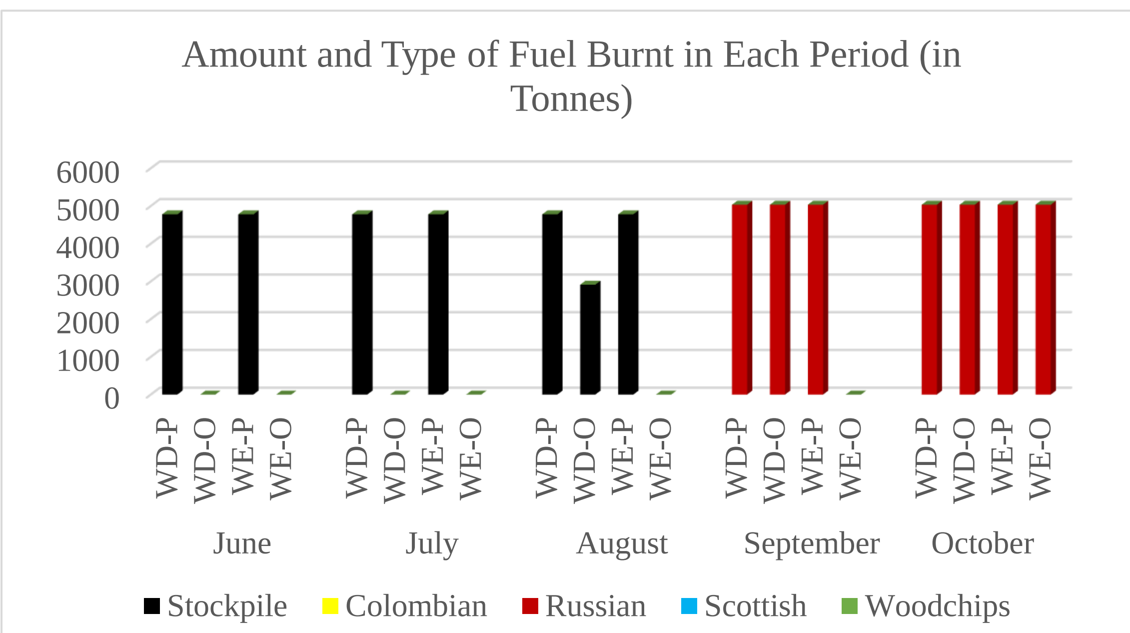
$$e_{cmw} = 0 \text{ for } c \in \{2,3,4\} \text{ and } m \in \{1,2,3\} \text{ and } w \in W \qquad \text{(Coal Lead-up time)}$$

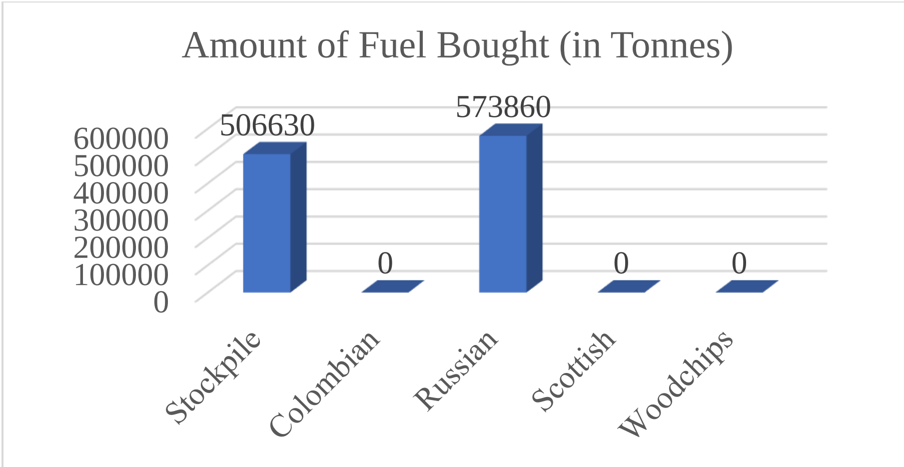
- It will take 3 months for Russian, Colombian, or Scottish coal to be delivered to the plant, so they cannot be used before month 4.
- <u>Sign Restrictions</u>:

$$e_{cmw} \geq 0$$

Results

After implementing the model into the relevant software, the optimal solution is obtained with a profit of $z = 35{,}030{,}814$ Euros, and the optimal fuel burning and buying patterns are illustrated in Figures 1 and 2:





Figures 1 and 2: The optimal solution of fuel burnt in each period in addition to the total fuel bought.

Note: WD ~ Weekday, WE ~ Weekend, P ~ Peak, O ~ Off-peak

The maximum amount of stockpiled coal burnt in each period is 4778 tonnes, and the maximum amount of Russian coal burnt in each period is 5034 tonnes. The sulfur bubble's limit is completely reached and around 84% of the stockpile inventory is used.

Even though the ROC is 67.5€, the GAMS software found it is still not profitable to buy and use woodchips. This may be due to its extremely low effective calorific value and higher price compared to the other fuel types. Moreover, even though technically a profit would still be made when selling energy in the off-peak periods, the sulfur bubble limit forces the algorithm to prioritize the peak periods.

Sensitivity Analysis

In order to determine how varying certain parameters will affect the model, sensitivity analyses will be made:

Sulfur Bubble:

As evident in the report generated by the GAMS software, the marginal improvement (increase in profit, in this case) is 710€ for every tonne the sulfur bubble limit is increased by.

Stockpiled Coal:

Since not all of the stockpiled coal was used up, then the shadow price of one more additional tonne of stockpiled coal is 0€. If any additional tonnes were available, they will remain in the inventory. This also could also be determined through the Complementary Slackness theorem, where the slackness of the stockpiled coal is non-zero, so the corresponding dual variable, which corresponds to the shadow price, is zero.

CO_2 Emission Tax:

To find how the CO_2 emission tax will affect the objective function, a new, "dummy", nonnegative variable will be created, y, with the following formula:

$$y = \sum_{c\in C, m\in M, w\in W} (C)(h_{mw})(\epsilon)(L)(v_c)(e_{cmw})$$

Then, the CO_2 tax summation term in the objective function will be altered as following:

$$\max z = \cdots - \sum_{c\in C, m\in M, w\in W} (T^E)\,(C)(h_{mw})(\epsilon)(L)(v_c)(e_{cmw}) - \cdots \rightarrow \max z = \cdots - (T^E)(y) - \cdots$$

After implementing these changes into GAMS, the value of y, which represents how many tonnes of CO_2 are being produced, is found out to be $2{,}112{,}200$ tonnes. Consequently, any additional 1€ incurred would decrease profit by $2{,}112{,}200$ euros, and any unitary decrease of the tax will increase profit by the same figure. Since all types of decision variables are affected in the same proportion by T^E, the variation of the tax would not alter the optimal solution.

ROC Revenue Unit:

To visualize the reduced cost of the woodchip consumption variable, the nonnegative variable y_{mw} will be assigned, and some changes will be done to the original GAMS model. First of all, a new constraint is added to the model:

$$y_{mw} = (h_{mw})(\epsilon)(L)(v_5)(e_{5mw})\ \forall\ m \in M, w \in W$$

Next, the following term in the objective function is changed to reflect y_{mw}:

$$\max z = \cdots + \sum_{m\in M, w\in W} (T^{ROC})(h_{mw})(\epsilon)(L)(v_5)(e_{5mw}) - \cdots \rightarrow \max z = \cdots + \sum_{m\in M, w\in W} T^{ROC} y_{mw} - \cdots$$

Upon these two changes, it appears that GAMS states that the dual value of each y_{mw} is 0. This seems to be because GAMS chooses y_{mw} as the basic variable. And if the variable is basic, then the reduced cost is 0. To combat this issue, the constraint will be altered as follows:

$$y_{mw} \leq (h_{mw})(\epsilon)(L)(v_5)(e_{5mw}) \; \forall \, m \in M, w \in W$$

This will ensure that the slack variable will be chosen as the basic variable by the GAMS solving algorithm. So, y_{mw} becomes a non-basic variable. Finally, the amount that the ROC must increase in order to make e_{5mw} a basic variable is illustrated in Figure 3:

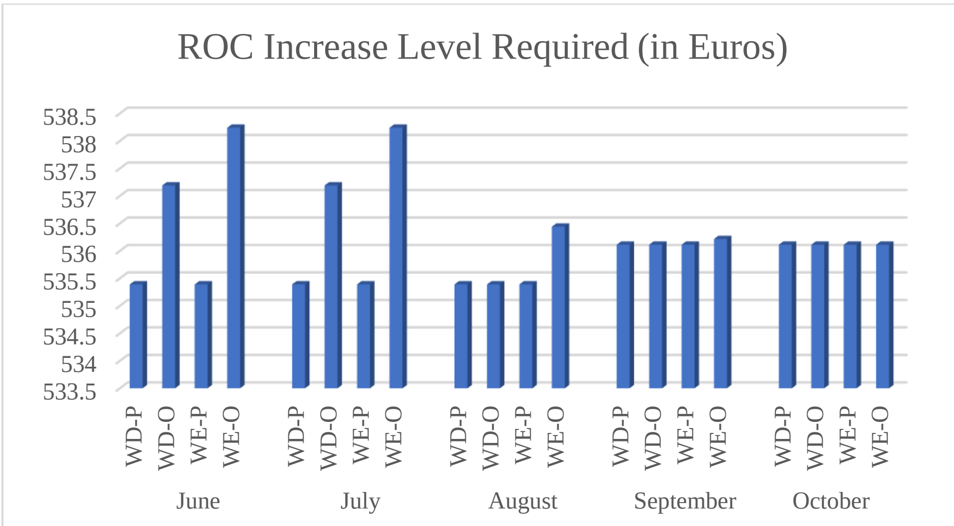


Figure 3: The increase of ROC needed to make burning woodchips optimal.

So, ROC needs to be increased about 8 folds in order to make burning woodchips profitable. This is likely due to the measly calorific value that woodchips provide, in addition to its high price per tonne.

Prices of Coal Obtained from Foreign Sources:

To find the reduced cost of the foreign coal in periods where they were not used, a new nonnegative variable, also y_c, is added to the original model. y_c represents the amount of fuel c used in the entire month of m during the periods p, and is formulated similarly to the case above, however, instead of making the LHS less than or equal to the RHS, the LHS should be greater than or equal to RHS, since the objective coefficient of y_c is negative:

$$y_{cmw} \geq (h_{mw})(e_{cmw}) \; \forall \, c \in C, m \in M, w \in W$$

$$\max z = \cdots - \sum_{c \in C, m \in M, w \in W} p_c^f h_{mw} e_{cmw} - \cdots \rightarrow \max z = \cdots - \sum_{c \in C, m \in M, w \in W} p_c^f y_{cmw} - \cdots$$

As a result, GAMS displays that the dual value of all y_{cmw} for Russian, Scottish, and Colombian coal is 0 in the periods where they are not being consumed, indicating that possibly the price is the cause of not using these fuels, but some of their other properties, like sulfur content and calorific value per unit Euro.

The Efficiency of Woodchips increasing to 68%:

Now, it is assumed that the effective calorific value of woodchips increased from 10% of the original value to 68%. Implementing this modified parameter changes the optimal solution:

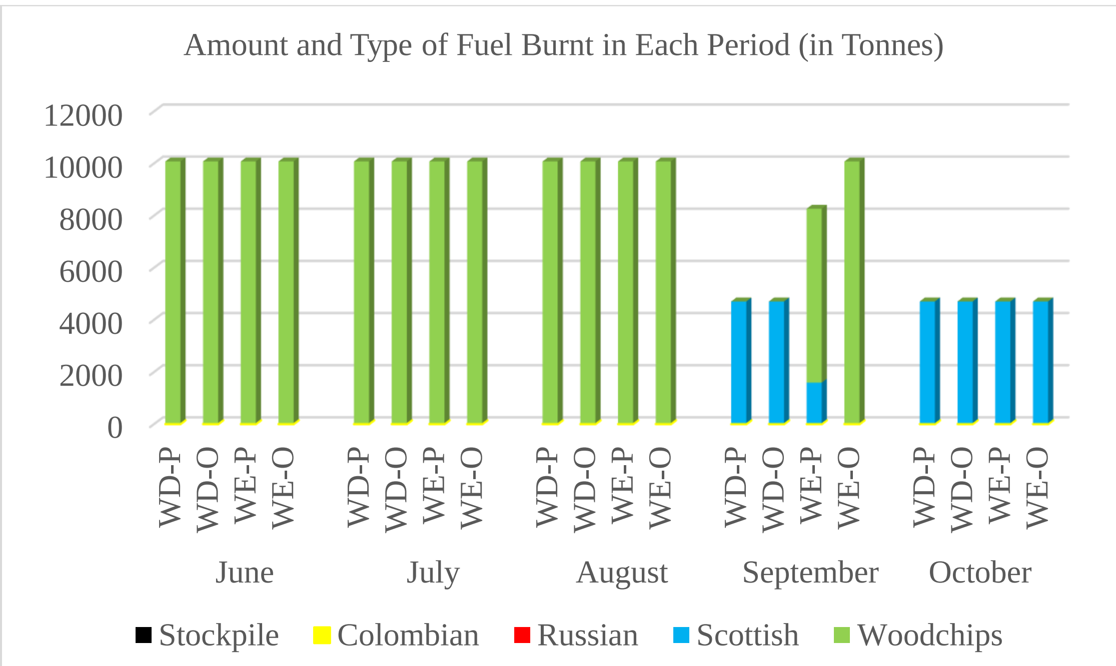


Figure 4: Optimal solution of fuel burnt per period when burning woodchips is 68% efficient

As a result, profit range increases from 35,030,814 Euros to 41,188,756.7 Euros. The sulfur bubble's limit was also reached, and none of the stockpiled coal has been used. As shown, woodchips are being prioritized over stockpiled coal.

In all of the periods where power is being generated, woodchips form the entire fuel source, with the exception of September's weekend peak period, where roughly 66% of the power is generated from renewable energy.

However, woodchips and biomass are very difficult to handle. In addition to being bad combustibles (compared to coal), wood is less dense than coal, so, more tonnes of wood are needed to provide the same amount of energy as coal does. Eventually, the cost of manipulating wood might not be negligible anymore. Thus, a constraint of how much (by mass) of woodchips is allowed with respect to other fuels will be added, to study the profit in such cases:

$$e_{5mw} \leq R \times \sum_{c \in C} e_{cmw} \ \forall \, m \in M, w \in W$$

Where R is the percentage (by mass) of woodchips allowed to be burnt per period. The optimal buying patterns for arbitrarily selected values of R are illustrated in figures A1, A2, and A3, and the profit for the selected R values are as follows:

$R = 10\%$	$R = 30\%$	$R = 70\%$
$z = 35{,}518{,}711€$	$z = 36{,}609{,}710€$	$z = 39{,}984{,}413€$

Table 3: Profit gained from arbitrarily selected values of R

Conclusion

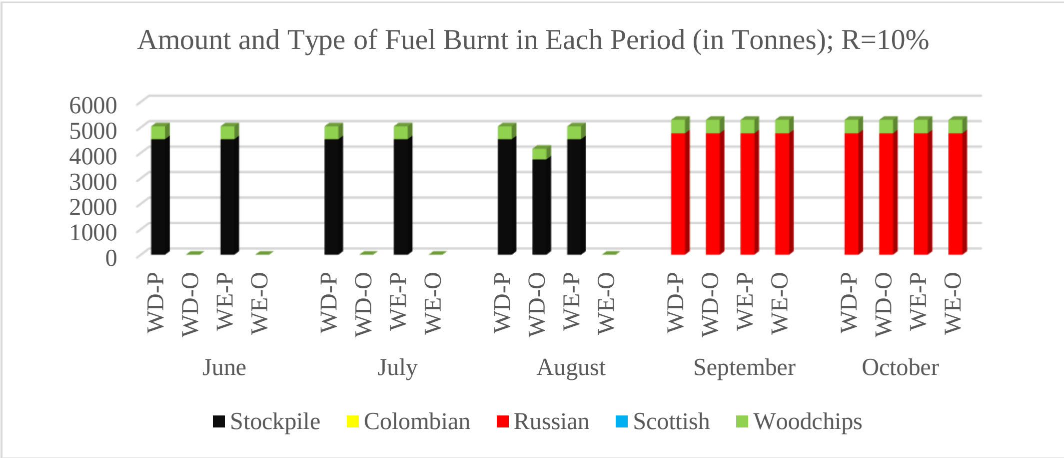
After observing the final results and optimal solution, Bob's best move is to lead IC to burn fuel according to the solution visualized in Figure 1, and order 573,360 tonnes of Russian coal for burning in September and October. Also, if the efficiency of the calorific values of woodchips increases to 68%, it is best to maximize the percentage of woodchips used (by mass) as much as possible, whilst considering the difficulties in manipulating and burning woodchips.

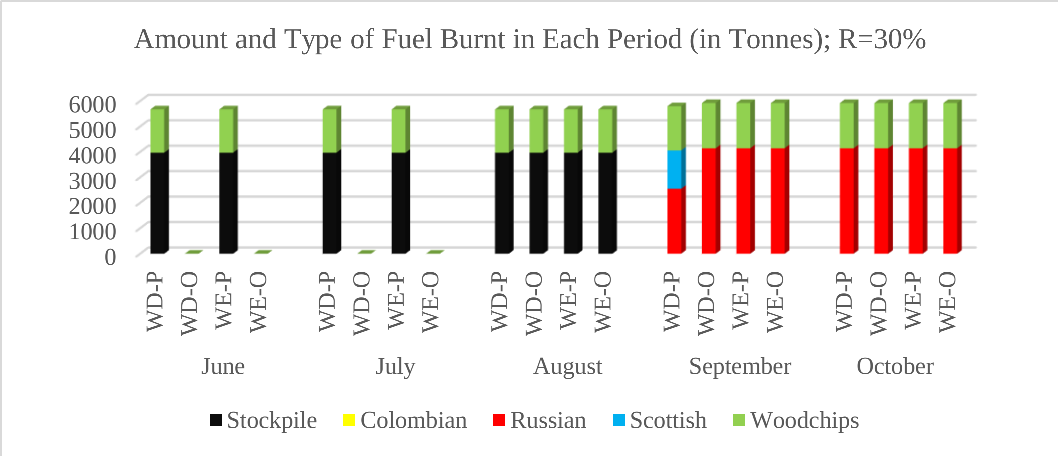
In addition, it can be beneficial to negotiate increasing the sulfur bubble with the respective authorities, as doing so will increase the profit range significantly. Although increasing ROC can increase the profit eventually, it cannot be expected realistically to have the ROC increased by 800%, so perhaps negotiating an ROC increase in out of question.

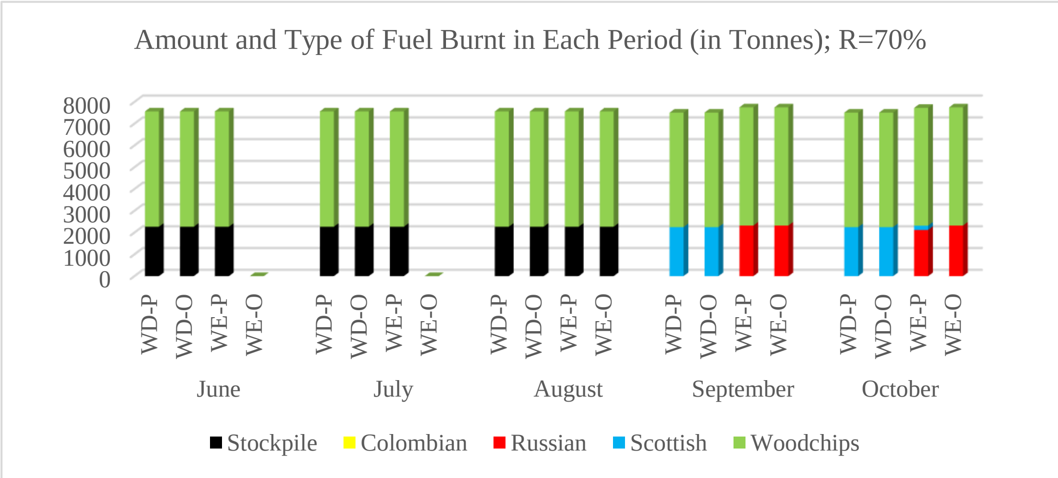
References

GOV.UK, & UK Department for Business, Energy and Industrial Strategy. (March 25, 2021). Electricity consumption from all electricity suppliers in the United Kingdom (UK) from 2002 to 2020 (in terawatt hours).

Appendix







Figures A1,A2, and A3: Optimal solution for arbitrarily selected values of *R*