IE361 Stochastic Models in Operations Research

Case Study on Dynamic Programming:

Pricing in Airline Revenue Management

Middle East Technical University

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- **For the computations, you may use any tool that you feel comfortable with.**
- **Along with the hard copy of your report, you are supposed to submit e-copies of your files to ODTUClass. Please give all of your documents in one file and upload it as a compressed file. Name the file with the names of your group members. Please name all your Python, Excel and Matlab (or other) files properly, and refer to these files in the appropriate parts of your report.**

1 Setting

This case study is on developing and experimenting with a Dynamic Programming (DP) formulation for price-based Revenue Management (RM) in airline industry. The practice of price discrimination is almost as old as commerce itself and its basis could be spatial differences (location of market), temporal differences (time of sales) or income differences (customer wealth). Temporal differences lead to dynamically setting the product price through a complex decision process or just seller's intuition. RM principally exploits temporal differences in customer valuations to increase revenues and its focus is on industries where customers' willingness to pay tends to increase as the sales proceeds. In this case study, the seller's tradeoff is between setting lower prices to benefit from earlier arrivals and sparing inventory for possible future customers who are willing to pay higher prices.

In the dynamic pricing problems, the control tool of the seller for managing the demand is the price and the objective is to determine the optimal price as a function of time and other relevant factors. Alternatively, it is possible to work with the probability of selling one item to the current customer. Under the assumption that the reservation price distribution of the customer is known, the probability of sale can be represented as a function of the posted price. (For real life problems, the seller's incentive for selling a ticket gets smaller with less time to departure or with more seat inventory.) With this alternative approach, you are supposed to give a DP formulation for the single-leg airline RM problem in this case study. The DP formulation is to find the optimal ticket price at a given state. The monopolist and risk-neutral seller's objective is to maximize the total revenue over the sales horizon and the marginal service costs are ignored. Batch bookings and booking cancellations are not allowed; that is, booking requests are processed individually until all seats are sold.

At most one individual sales transaction is considered at each stage of the recursive model you are supposed to give. In this respect, the sales horizon is divided into small time intervals of unit length during which at most one customer may arrive and the customer books ticket for the flight or not depending on the price posted by the seller. Hence, the aggregate demand is disintegrated into the following two components.

- Customer Arrival Rate: For the arbitrarily small time intervals of length ϵ, the time-dependent probability of a single customer arrival in period t corresponding to time interval $[t, t-\epsilon]$ is estimated as ρ_t. Note that the arrival probabilities are assumed to be independent of the prices posted by the seller. t denotes the time to flight departure.

- Reservation Price Distribution: Each customer arrival does not necessarily result in a booking. Whenever the posted price is above the maximum amount the customer is willing to pay, sales does not take place. Thus, the customer reservation price, P_t, is considered as a time-dependent (continuous) random variable to represent the randomness in customers' willingness to pay and the change in customer valuation along the sales horizon.

Overbooking is a commonly applied practice by almost every airline today and seat allocation models in airline RM literature are used for finding overbooking limits and ticket prices. The prospective loss for selling extra seats is due to denied boarding penalties that will be incurred if the number of no-shows (the booked customers who do not show up at boarding) are less than the number of overbooked seats. The denied boarding penalties include monetary fines determined by civil aviation regulations (independently of the ticket's price), compensation of connected flight legs (if any) and accommodation and other expenses of traveller due to waiting for the next flight. Let $\pi(\cdot)$ denote this penalty amount incurred as a function of the number of no-shows that would allow us to determine the number of denied customers at the boarding. The probability of no-show at the time of the flight for a booked customer is β.

The sales horizons in the airline industry commonly vary between 90 days and 360 days prior to flight; yet, majority of the seats are sold in the last 30 days. That is, pricing in the final month is critical.

2 Questions

Question 1. (60 points) Consider maximization of the expected revenue over the entire sales horizon of length T when the number of total available seats is S. Assume T is the total number of sufficiently (arbitrarily) small time intervals of size ϵ.

a) (10 points) Give a DP formulation assuming that overbooking is not allowed.

b) (20 points) Revise the DP formulation in part (a) for the following extension: the overbooking limit (the maximum number of overbooked customers) is η. Let d be the amount of penalty incurred for each booked customer denied at the boarding.

c) (30 points) Solve the DP model you give for question 1(b) assuming that the overbooking limit (the maximum number of overbooked customers) is $\eta = 10$. Let $\beta = 0.05$, $d = 200$, $S = 90$ and $T = 144$. Find the optimal prices for all time intervals and every possible value of seat inventory. Consider (time-stationary) $\rho_t = 0.9$ and $P_t \sim U(41, 710)$ for $t = 1,\ 2,\ ...,\ 144$.

(c-i) Plot the optimal price as a function of time to flight for given seat inventory $s = -5$ and $s = 20$. Comment on the behaviour of the optimal price in time to flight.

(c-ii) Plot the optimal price as a function of seat inventory for given time to flight $t = 1$ and $t = 20$ periods (time intervals). Comment on the behaviour of the optimal price in seat inventory.

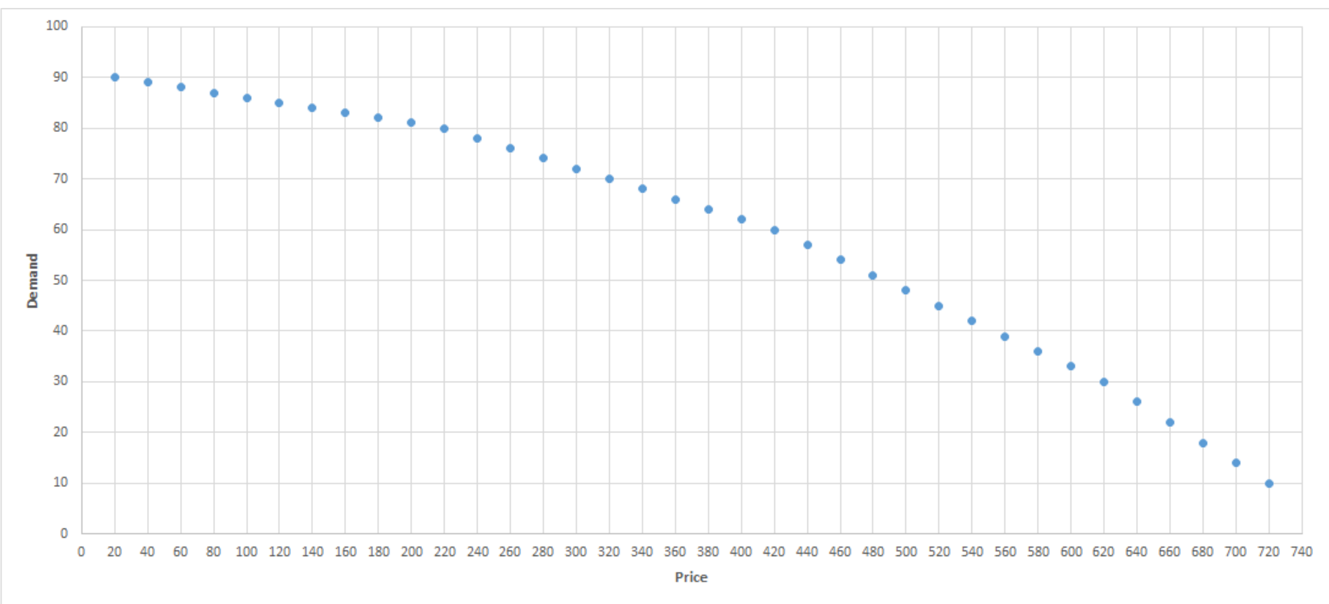
Question 2. (40 points) Consider the set of alternative (predetermined) price levels, p_j, that can be posted: $\{p_1, p_2, ..., p_m\}$.

a) (15 points) Give a Mathematical Programming (MP) model in order to determine the number of seats to be sold at the posted price level j. Note that price is posted for the whole sales horizon (booking period).

MP model adopts demand that is aggregated over the sales horizon. Let the parameter μ_j define demand-price relationship in the MP model, it denotes the (expected) number of seats that can be sold during the sales horizon when the price posted is p_j, $j = 1, \ldots, m$.

b) (15 points) Solve the MP model for the following numerical setting: $S = 90$, $m = 36$, $p_j = 20j$ for $j = 1, 2, \ldots, 36$. The relation between μ_j and p_j is given below.

$$\mu_j = \begin{cases} 90 - \frac{p_j - 20}{20} & \text{for } 1 \leq j \leq 10, \\ 80 - \frac{(p_j - 220)}{10} & \text{for } 11 \leq j \leq 20, \\ 60 - \frac{3 \cdot (p_j - 420)}{20} & \text{for } 21 \leq j \leq 30, \\ 30 - \frac{p_j - 620}{5} & \text{for } 31 \leq j \leq 36. \end{cases}$$



c) (10 points) Compare the use of DP and MP models in terms of the price behaviour over the sales horizon. In particular, compare the pricing policies you give in questions 1(c) and 2(b). Note that the numerical settings in questions 1(c) and 2(b) are comparable to a certain extent.

How would you work with MP in order for its performance to be comparable with DP? Note that estimates for expected demand (number of seats) to be sold at alternative price levels can be updated at different points in time for the remaining part of the sales horizon.