GDZ Vehicle Assignment and Rerouting Systems Design Project 2023 – 2024 January 10th, 2024

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As part of the IE497 course, a project sought by students is to be tackled and reported on. This report tackles such a problem, related to an electrical distribution company having issues allocating its field teams throughout the day.

2) Introduction

2-1) Presify

Specializing in crafting sophisticated big data analytics software catered explicitly to energy markets and systems, Presify harnesses the prowess of high-level statistics, advanced data analysis, and profound engineering expertise. The bedrock of Presify's unparalleled ability to engineer advanced big data analytics software lies within the collective expertise and deep-seated knowledge of its astute staff of Industrial Engineers.

2-2) GDZ and Service Distribution

GDZ, one of the 21 electricity distribution companies in Türkiye and Presify's customer, ensures reliable distribution of electricity in Izmir and Manisa covering 13,123 km^2 of area across 47 districts and 2,383 neighborhoods and serving 3.6 million customers. The firm's headquarters lie in Bornova, Izmir. Their services, aside electricity distribution, include:

- Preventive and General Maintenance: Periodic inspection and maintenance of electrical infrastructure, such as transmission lines, transformers, substations, and distribution equipment, to prevent breakdowns and ensure smooth operation.
- **Fault Repair and Troubleshooting**: Responding to power outages and quickly addressing power outages and electrical faults; emergency responses done by mobilizing teams to immediately resolve critical errors and security risks.
- **Meter Reading and Billing**: Collecting data from your customers' electricity meters to accurately track electricity consumption; calculating customer bills.
- **Customer Service**: Handling customer inquiries, complaints, and service requests; enabling new power connections for residential and business customers.
- **Regulatory Compliance**: Compliance with regulations and standards established by regulatory authorities to ensure safety, environmental responsibility, and fair pricing.
- **Environmental and Safety Commitment**: Promotion of employee and public safety practices; implementation of environmentally friendly initiatives; services

given by field service technicians work; testing and calibration of electrical equipment.

These jobs are maintained by different sets of teams, and the concerned service of this study is **fault repair and troubleshooting.** This is done by dispatching some on-site teams to these problems throughout their shifts. The firm has distributed the fault repair area into multiple regions (Table 1):

| Regions | Number of Service Centers |
|----------------|---------------------------|
| Izmir Metropol | 5 |
| North Izmir | 6 |
| South Izmir | 12 |
| Manisa | 16 |

Table 1: Number of service centers in each region

2-3) Field Teams and Vehicles

Each service center houses multiple on-site teams that are tasked with going around the respective region and completing the tasks allotted to them. There are a total of 106 teams working one, two, or three shifts and composed of two or more workers. At least one of these workers should have what is called an EKAT (Elektrik Kuvvetli Akım Tesisleri) Certificate.

If a team works more than one shift, then for that team, the individuals working in that team change from one shift to the next. GDZ also has the capacity to dispatch extra teams when the workload becomes too high or too many urgent jobs are at hand. These teams are made up of workers who are called from their homes during out-of-work hours and are usually paid at overtime rate.

Moreover, GDZ has two main types of vehicles, which are boom trucks (trucks equipped with cranes) and 4x2 cars, with two models of each: Atlas and Fuso (boom trucks) and Duster and L200 (4x2 models). There are a total of 155 vehicles. The vehicles are assigned to each service center respectively, meaning it is difficult to trade vehicles between service centers.

2-4) Job Types

Some of the jobs done by the field teams include but not limited to: cable repairing, tree cutting, and most importantly, breakdown fault repairing.

Some jobs are penalizable by law, such that, if a certain deadline is passed, then GDZ is fined a penalty. These jobs are:

- **Breakdowns**: If the resolution of a breakdown event passes the 10-hour mark, then GDZ is fined a penalty.
- **Street lighting**: The resolution limit for these jobs is 24 hours.
- **Branch Repairs**: Within the city, the limit for these jobs is 24 hours. In the countryside, this number is 48 hours.

These penalties and limits are also imposed for all other electrical distribution firms within Türkiye.

2-5) Customer Types

When it comes to the company's customer base, there are five types of subscriptions: domestic, industrial, agricultural, commercial, and general street illumination.

The urgency of jobs is affected by the nature of the subscription. For example, faults in areas with many commercial subscriptions means that there is a high chance that a hospital or fire station may have suffered a blackout, which would need immediate attention.

3) Current System

3-1) Teams

Teams are considered to be uniform, in the sense that any two teams working on the same job using the same vehicle are considered to have the same parameters (service time, travelling time, etc.).

3-2) Vehicles

Some jobs are preferentially done by certain types of vehicles. Different types of vehicles complete jobs in different amounts of time, and each vehicle has a certain average speed and different fuel consumption rate. The average distance crossed by each vehicle type and the average fuel consumption in terms of price are in Figure 1 and Figure 2:

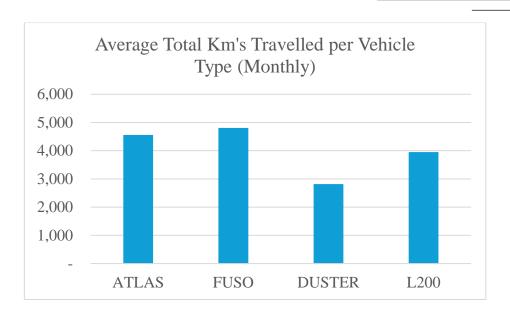


Figure 1: Average total kilometers crossed by each vehicle of a certain type over a month.

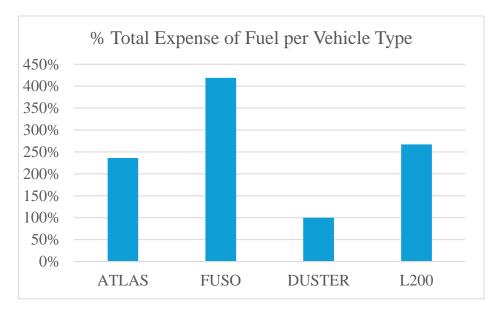


Figure 2: Average price of fuel consumed by each vehicle of a certain type in a month (in terms of percentage of the least costly vehicle). Real data concealed for privacy concerns.

3-3) Job Cycle

The lifetime of a job can be expressed in Figure 3:

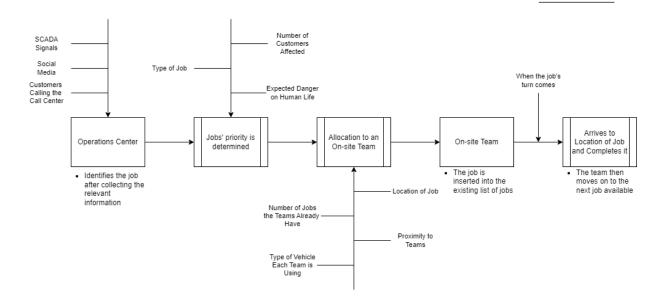


Figure 3: Flow chart of a task.

3-4) Job Priority Types

There are two main types of job priorities defined by GDZ: Routing priority and operator priority.

Routing priority: This is a weight given to each of the job types and ranges from 100 to 1000 in multiples of 100, where a weight of 100 represents the most urgent and important jobs. These weights do not change and are static. Some examples of these weights are shown in Table 2:

| Job Type | Routing Priority |
|---------------------|-------------------------|
| Breakdown | 100 |
| Street Lighting | 200 |
| Branch Repairs | 200 |
| Planned Maintenance | 200 |
| Measuring | 300 |
| ••• | ••• |

Table 2: Some Routing Priorities of Certain Jobs.

Operator Priority: This is a ranking system used by the operators in the Operations Coordination Center and ranks are mostly subjective. They range from 1 to 10, with 1 being the most urgent job. This ranking takes into account the number of people affected, the sensitivity of affected facilities (hospitals, fire stations, etc.), and any possible mortal and/or monetary losses. In practice, this type of priority is more significant and important.

3-5) Dynamicity

The crux of the system is its dynamicity. Some jobs do not even exist at the start of the shifts, and randomly appear over time, forcing GDZ and its Operations Center to make multiple reassignments and reroutings in one single shift. Initial assignment and routing decisions are made at the beginning of the shifts, then reassignment and rerouting decisions are made throughout the shifts. Taking Bornova as an example (Figure 4):

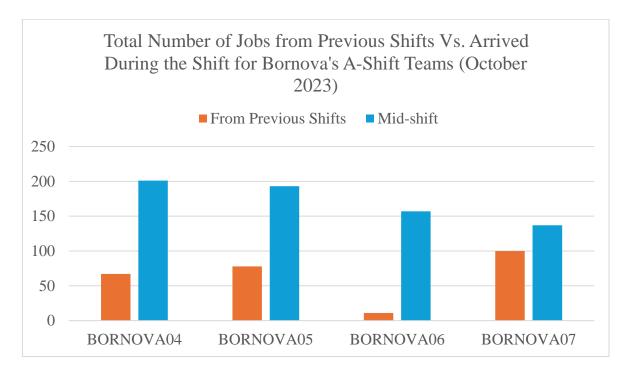


Figure 4: Total jobs from previous shifts vs. mid-shift jobs for Bornova's a-shift teams over the month of October 2023.

4) Company Expectations

GDZ expects a vehicle routing/rerouting system that:

- 1. Reduces costs
- 2. Increases customer satisfaction
- 3. Is able to assign newly incoming jobs to the field teams effectively

5) Problem Symptoms

The problem symptoms of the current system are:

- 1. High overtime costs
- 2. Failure of performing reassignments and reroutings
- 3. Failure of meeting deadlines of penalizable jobs

5-1) Overtime

Upon examining historical data, it was possible to observe that the teams have been working a significant number of overtime hours. Figure 5 shows the additional working hours of the four Bornova teams over a period of 21 days:

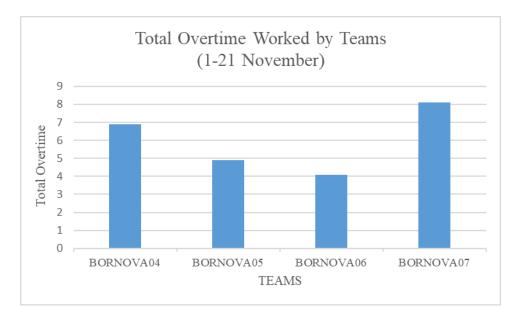


Figure 5: Overtime hours for selected teams between 1-21 November 2023.

Excessive overtime hours entail two disadvantages for the company:

- 1. Compensating the workers' overtime hours with overtime hour costs, which is higher than that of regular times.
- 2. Delaying the start of the teams in the next shift. The team working overtime may deliver the vehicle to the next shift late, and the next team waits for the vehicle at the center during this period. One such example is shown in Table 3:

| Team | Shift | Normal Shift Hours | Start Time of the Shift | Ending Time of the Shift | Shift Time |
|-----------|-------|--------------------|-------------------------|--------------------------|------------|
| BORNOVA05 | А | 07:00-15:00 | 07:32:13 | 17:29:32 | 09:57:19 |
| BORNOVA05 | В | 15:00-23:00 | 17:41:47 | 23:03:39 | 05:21:52 |

Table 3: An extreme case of a shift's commencement being delayed due to the preceding team's tardiness.

Workers working in shift A were paid extra because they worked overtime for about 2 hours. In addition, the normal hours were paid for the shift B team, which was idle for about 2.6 hours while waiting for the first team to arrive to base, despite having worked nothing.

5-2) Reassignment and Rerouting

GDZ has a systematic way of making initial assignments and routings but no method for whenever a new job is created. Because of this, GDZ has no system that assigns the teams to the new job or how the new job will be added to existing routing plan and change it accordingly.

To exemplify, in Figure 6, the initial assignments to the teams BORNOVA-04, BORNOVA-05, BORNOVA-07 can be seen. During the shift, a new job is created (designated in yellow), and it is entirely left up to the operator to choose which team the new job shall be assigned to.

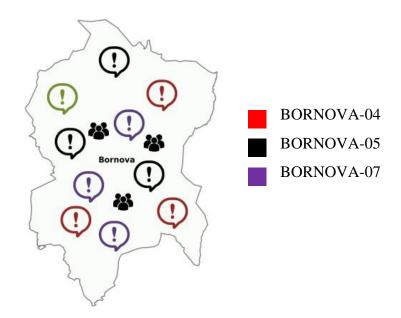


Figure 6: Visualization by map of the BORNOVA-04, BORNOVA-05 and BORNOVA-07 teams and their assigned jobs.

Generally, this decision is made by assigning the team closest to that work order, taking into account only the distance. However, in this case and in the current system, no systematic thought is made into how much the team will change their route for the new job or in which position they will go to the new job, nor how other teams' schedules, routes, and assignments will be affected as a result.

5-3) Penalizable Jobs

The state imposes some regulations on electricity distribution firms, which the firm has to abide by. Violating these rules imposes monetary penalties, and these rules come in the form of deadlines for completing certain jobs, as mentioned before. The deadlines are reiterated in the table below:

| Job Type | Maximum Allowable Duration | |
|-----------------|--|--|
| Breakdown | 10 hours | |
| Street Lighting | 24 hours | |
| Branch Repairs | 24 hours in the city centers, 48 hours in the rural areas | |

Table 4: Penalizable jobs and their respective resolution limit.

The exact monetary penalty is constant for each type of job and does not increase with the amount of tardiness. However, because these jobs are sensitive, delaying their resolution causes customer satisfaction to decrease. If a customer has suffered injuries or monetary losses due to these delayed resolutions, they may choose to sue the firm.

To exemplify from the data at hand, there were breakdown events that were resolved 15 hours after the fact. By the time it is resolved, food and other organic material may start to go bad and rot. If that specific customer had sued the firm and the court filed for the customer, GDZ could have suffered even more losses.

When looking at the data belonging to November, the number of jobs for those types, penalized jobs and their percentages are given in the Table 5:

| Job Type | Total Jobs | Total Violations | % Violations |
|-----------------------|-------------------|-------------------------|--------------|
| Breakdown | 21,371 | 316 | %2 |
| Street Lighting | 21,061 | 8,054 | %38 |
| Branch Repairs | 2,548 | 221 | %8 |

Table 5: Total and percent violations of the aforementioned penalizable jobs in November 2023.

6) Problem Elements

6-1) Six Elements of the Problem

A problem is a structured situation involving a decision maker who is dissatisfied with the current situation and aims to address unmet goals or needs. The overall problem

structure consists of several key elements, each of which plays a distinct role in effectively defining and solving the problem, these are: decision maker, decision maker's objective, decision criteria, performance measures, control inputs (decision variables or courses of action), and problem context.

1. Decision Maker: GDZ EDAŞ Fault Coordination Center

2. Decision Maker's Objective:

- Increasing customer satisfaction
- Reducing operational costs as a result of improving the existing vehicle assignment and routing system
- Developing a vehicle reassignment and rerouting system

3. Decision Criteria:

- Maximizing customer satisfaction
- Minimizing operational costs
- Effective assignment of incoming jobs to vehicles/teams

4. Performance Measures:

- Weighted completion times of jobs
- Total distance covered
- Costs due to overtime and delay
- Number of jobs assigned to a different vehicle in reassignments

5. Courses of Actions:

- Assignment and reassignment of jobs to vehicles/teams
- Routing and rerouting of jobs
- Jobs to be left to the next shifts

6. Problem Context:

- Fault repairment job orders (demand)
- Location of job orders, centers and vehicles/teams
- Types and number of vehicles in centers
- Number of the teams and workers in centers
- Responsibility areas of centers and teams
- Uniformity of teams
- Regulations

6-2) Influence Diagram

Influence diagrams contribute to a structured and systematic approach to problem definition. It helps in decision-making and provides a clear understanding of the relationships within a complex system. In the diagram provided in Figure 7, Rectangles represent control inputs, ellipses represent outputs, circles represent system components and clouds represent uncontrollable inputs:

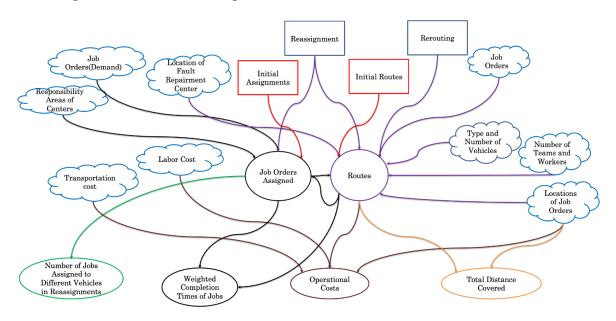


Figure 7: Influence diagram of the system.

6-3) Hierarchy of Systems

The hierarchy of systems is provided in Figure 8:

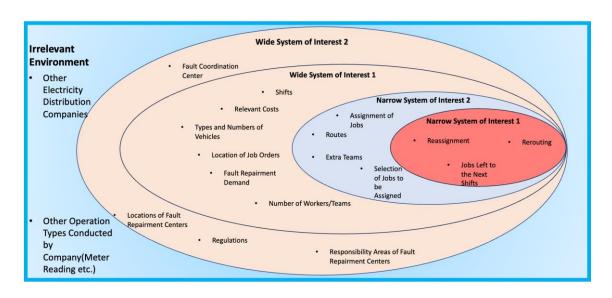


Figure 8: Hierarchy of systems figure.

It is worth noting that since the initial assignment and routing schedules are to be used as inputs to the reassignment and rerouting system, the narrow system for reassignment and rerouting is encapsulated by the narrow system of interest 2, which contains the initial assignment and routing model.

7) Problem Definition

In light of these analyses, the problem definition is then to develop a vehicle assignment, reassignment, routing, and rerouting system that:

- 1. Reduces overall costs, which include transportation and overtime costs, and incurred penalties.
- 2. Reduces weighted life time of jobs.
- 3. Respects the previous schedule

8) Mathematical Model

The following mathematical model is a reassignment and rerouting model, that takes the previous schedule as an input. The initial model and schedule is a standard VRP mathematical model, which is a generalization of the following model. The flow of the schedules and mathematical models is described in Figure 9:

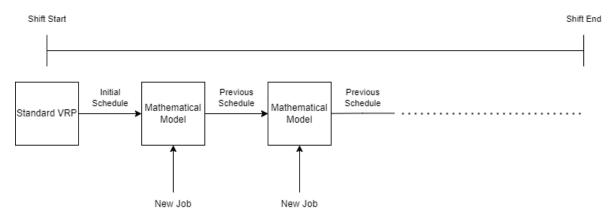


Figure 9: Sequence of computing mathematical models with respect to shift timeline.

8-1) Sets and Indices

- $c \in C$: Set of vehicles (teams)
- $j \in J$: Set of jobs (nodes)
 - Note: Let node H represent the center, which is the same for all vehicles.

- Note: Let node 0_c represent the location of vehicle c. If the model is initialized at time 0, then node $H = 0_c \ \forall \ c \in C$. If the vehicle is on the way to another job, then 0_c is represented as a phantom job with 0 service time. If the vehicle is at a certain job, then 0_c is that job with the remaining service time.
- $j^N \in J^N \subset J$: Set of jobs (nodes) that are new to the shift, i.e.: did not exist in the previous iterations.
- $j^* \in J^* \subset J$: Set of jobs (nodes) that have deadlines and are penalizable.
 - O Note that a job may belong to both j^* and j^N , and this is likely the case because most incoming jobs are urgent and are penalizable.
- $(i,j) \in A$: Set of arcs between all jobs, for $i,j \in J + H + \{0_c \mid c \in C\}$.

8-2) Parameters

- \mathbb{C}_{ij}^T : Transportation cost from job (node) i to job j. (TL)
- \mathbb{C}_{c}^{0} : Cost of overtime for a team c. (TL/team/hour)
 - O Note: takes into account the number of team members.
- $\mathbb{C}_{j^*}^V$: **Constant** cost of (penalty incurred) when violating the deadline for a job j^* . (TL/job)
- r_i : Creation time of job j, in hours since shift start.
 - o Note: If a job was created before the shift starts, this value is negative.
 - Note: This parameter's system-equivalent is a timestamp, such as 9 a.m.,
 October 18th, 2023, and this timestamp is always before the time the model runs, because if it weren't, then the job doesn't exist yet.
- s_i : Service time required by job j.
- d_{j*} : Deadline of job j^* , in hours since shift start.
- t_{ij}^c : Time to approach job j from job i by vehicle c.
- w_i : Weight of job j.
- w_j^D : Weight/importance of completing job j in the current shift.
 - This provides an implicit (non-monetary) cost in deferring jobs to the next shift(s).
- \hat{C}_i : Completion time of job j in the previous schedule
- $\hat{Z}_{jc} = \begin{cases} 1 & \text{if vehicle c is assigned to job } j \in J J^N \text{ in the previous schedule} \\ 0 & \text{otherwise} \end{cases}$

- TC: Current Time, in hours since shift start.

8-3) Decision Variables

- $y_{ij}^c = \begin{cases} 1 & \text{if vehicle } k \text{ visits job } j \text{ directly after job } i \text{ in the current model} \\ 0 & \text{otherwise} \end{cases}$
- $z_{jc} = \begin{cases} 1 & \text{if job j is assigned to vehicle c in the current schedule} \\ 0 & \text{otherwise} \end{cases}$
- $n_j = \begin{cases} 1 & \text{if job j will be deferred to the next shift(s)} \\ 0 & \text{otherwise} \end{cases}$
- $v_{j^*} = \begin{cases} 1 & \textit{job } {j^*}' \textit{s deadline has been violated} \\ 0 & \textit{otherwise} \end{cases}$
- C_j : Completion time of job j, in hours since shift start.
 - Note: This variable's system equivalent is a timestamp, for example: 8 a.m.,
 October 18th, 2023.
- C_j^+ , C_j^- : Positive and negative deviation from the completion time of job j between the previous and the current schedule, defined for jobs $j \in J J^N$.
- L_j : Life time of a job j; i.e.: The time spanned from the creation of a job until it is resolved, in hours.
- o_c : Overtime accumulated by team (using vehicle) c, in hours.

8-4) Objective Functions

$$\min F_1 = \sum_{j \in J} w_j L_j$$

$$\min F_2 = \sum_{c \in C} \sum_{(i,j) \in A} \mathbb{C}_{ij}^T y_{ij}^c + \sum_{c \in C} \mathbb{C}_c^O o_c + \sum_{j^* \in J^*} \mathbb{C}_{j^*}^V v_{j^*}$$

$$\min F_3 = \sum_{c \in C} \sum_{j \in J - J^N} (1 - \hat{Z}_{jc}) z_{jc}$$

$$\min F_4 = \sum_{j \in J - J^N} (C_j^+ + C_j^-)$$

$$\min F_5 = \sum_{i \in J} w_j^D n_j$$

- F_1 : Represents the minimization of weighted life time of all jobs.
- F_2 : Represents the minimization of total costs, which are: transportation, overtime, and penalties.

- F_3 : Represents the minimization of reassigning jobs to different teams.

 Note: This objective function is in order to reduce team confusion about the tasks they have at each stage of the shift; if their set of tasks keep changing, then human error may rise.

- F_4 : Represents the minimization of the deviation of completion times for jobs in order to maximize stability.

o Note: This objective function *could* be given a low weight, to serve as a tiebreaker, according to the firm's preferences.

- F_5 : Represents the minimization of weighted deferring of jobs to the next shifts.

8-5) Constraints

$$\sum_{c \in C} \sum_{j \in J} y_{ij}^c + n_i = 1 \,\forall \, i \in J$$

A job is either:

- Assigned to one and only one vehicle

- Or deferred to the next shift(s)

$$-\sum_{j\in J}y_{0_cj}^c=1\ \forall\ c\in C$$

$$\sum_{i \in J} y_{ij}^{c} - \sum_{i \in J} y_{ji}^{c} = 0$$

$$\forall j \in J, \forall c \in C$$

Intermediate nodes for each vehicle

$$-\sum_{i\in I}y_{iH}^c=1\ \forall\ c\in C$$

$$-\sum_{i\in J}y_{ij}^c\leq z_{jc}\;\forall\;j\in J,c\in\mathcal{C}$$

$$C_{j} \ge C_{i} + s_{j} + t_{ij}^{c} - M(1 - y_{ij}^{c})$$

$$\forall c \in C, (i, j) \in A$$

Note for $C_{initial} = C_{0c}$, $\forall c \in C$:

$$C_{0_c} = TC$$

or

$$C_{0_c} = TC + s_{0_c}$$

depending on the status of the team.

Note: This also eliminates subtours (Appendix A)

$$L_j \ge C_j - r_j \ \forall \ j \in J$$

$$C_{j} - \hat{C}_{j} = C_{j}^{+} - C_{j}^{-} \ \forall \ j \in J - J^{N}$$

$$- o_c \ge C_j + t_{jH}^c - 8 - M(1 - y_{jH}^c)$$
$$\forall (j, H) \in A, c \in C$$

$$- C_{i^*} - d_{i^*} \leq M v_{i^*} \ \forall \ j^* \in J^*$$

8-6) Domain

-
$$y_{ij}^c \in \{0,1\} \, \forall \, c \in C, (i,j) \in A$$

-
$$z_{jc} \in \{0,1\} \forall j \in J, c \in C$$

-
$$n_i$$
 ∈ {0,1} $\forall j$ ∈ J

-
$$v_{j^*} \in \{0,1\} \, \forall \, j^* \in J^*$$

-
$$C_j^c \ge 0 \ \forall \ j \in J, c \in C$$

-
$$o_c \ge \forall c \in C$$

9) Moving Forward

The operational problems that GDZ faces have been examined in this research, with a particular emphasis on the effective distribution of field teams for fault repair. The goal is to work together with Presify to optimize GDZ's dynamic system. Important findings highlight problems including exorbitant overtime expenses and reassignment difficulties. The suggested mathematical model aims to improve operational efficiency, decrease costs, and shorten task completion times. GDZ's operating efficiency and customer happiness might be enhanced by finding a practical solution, as this study concludes. Subsequent efforts must encompass the pragmatic application, ongoing oversight, and cooperation with operational groups to enhance the suggested model in real-world scenarios. This succinct synopsis captures the main ideas, methodology, recommendations, and findings of the project.

The lifetime of a job is the creation time less release.

The deviation of the completion times between the current and old schedules is calculated in terms of positive and negative deviations for each job that is not new.

The overtime value for a team c is the completion time of the last job (not including H), plus the required time to go from that job to the center using a vehicle c. This constraint is only binding if $y_{jH}^c = 1$. Note that $s_H = 0$.

Determines whether job j^* 's deadline has been violated or not. If violated, incur a **constant** penalty.

Moving Forward | Systems Design Report

Tasks October November December January February March April

Analyzing the Current Situation

Examining the Problem Symptoms

Problem Definition

Literature Review

Modelling

Developing the Solution Approachs

Table 6: Progress plan.

10) References

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 https://doi.org/10.1016/j.trc.2015.03.002

11) Appendix A

11-1) Miller-Tucker-Zemlin Variable

Based on the Miller-Tucker-Zemlin formulation (AIMMS, 2020) to avoid subtours, the use of an additional continuous variable $u_i \ge 0$, is used in the following constraint:

$$u_j - u_i \ge q_j - Q(1 - y_{ij}^c)$$

Where q_j is the demand of node j, and Q is the total demand. If demand is determined to be in terms of time, then u_j could be thought of as completion time, q_j could be thought of as service time of job j (in addition to time needed to reach j from i), and Q could be substituted with big M:

$$C_j - C_i \ge s_j + t_{ij} - M(1 - y_{ij}^c)$$

$$\downarrow$$

$$C_j \ge C_i + s_j + t_{ij} - M(1 - y_{ij}^c)$$

Which could be generalized as the following:

$$C_j \geq C_i + s_j + t_{ij}^c - M \big(1 - y_{ij}^c\big)$$

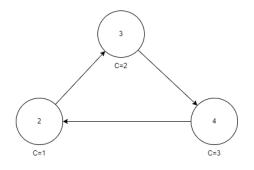
Along with a constraint such as:

$$\sum_{c \in C} \sum_{i \in I} y_{ij}^c + n_i = 1 \ \forall \ i \in J$$

Which are the constraints used in the model.

11-2) Further proof

Suppose a solution that contains subtours and completion times for a certain vehicle, c (Figure A1):



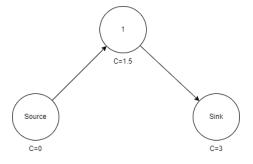


Figure A1: Solution containing subtours.

The constraints for the jobs in the detached subtour are then:

$$C_3 \ge C_2 + s_3 + t_{2,3}^c$$

$$C_4 \ge C_3 + s_4 + t_{3,4}^c$$

$$C_2 \ge C_4 + s_1 + t_{4,2}^c$$

And if these constraints are added together:

$$\begin{aligned} C_2 + C_3 + C_4 &\geq C_2 + C_3 + C_4 + s_3 + s_4 + s_1 + t_{2,3}^c + t_{3,4}^c + t_{4,2}^c \\ \downarrow \\ 0 &\geq s_3 + s_4 + s_1 + t_{2,3}^c + t_{3,4}^c + t_{4,2}^c \end{aligned}$$

Which cannot hold since each of $t_{ij}^c > 0 \ \forall (i,j) \in A, c \in C$. Thus, subtours are not possible under this formulation.