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T1 ________________	**49118**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**E**	F4 ________________

2016

MCM/ICM

Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Water Problem Must Be Solved

Water scarcity is becoming a crucial problem of every country and region all over the world. In order to look for a solution to the problem of water supply scarcity, we build a series of models to solve the problem. Our studies mainly include three processes: 1) evaluate and analyze2) forecast 3) intervention plan.

Based on the traditional model for Water Resources Carrying Capacity, we build up a evaluation system and name it as ***Water Supply Ability model (WSA)***. Our model focuses on two sides of water scarcity: physical and economic, picking up the regional water resources amount, the regional water use amount and other indicators as statistic data. We use improved ***TOPSIS Method***, which is based on ***Entropy Method***, to evaluate the relative water supply ability of several regions in a same period. Taking China for example, we obtain the comprehensive scores of different regions, which shows that Southwest China ranks first with the score of 0.104276 and Northwest China ranks last with the score of 0.003572. We pick up Northwest China as our analysis sample, and according to the indicators we set and the reality situation, we explain the physical scarcity and economic scarcity of local water resources.

Then, we adopt ***Grey Forecasting Method*** and randomness analysis of indicators to build up a model to predict the future water supply ability. Taking Northwest China for example, we use our forecast system to predict every indicators (i.e. population, GDP, etc.) during the period from 2014 to 2025 and randomly analyze the probably fluctuate indicators like precipitation. Combining the WSA model we obtain the comprehensive scores of Northwest China in the future. From the evaluation scores and the concrete changes in every indicators, we can see the water supply capacity of this region will improve in the future 13 years, but there will still exist some restrains by the local natural situation.

To figure out the solution to water scarcity problem in Northwest China, we build a ***Water Cycle Model*** to study the famous ***Demise of Lake Lop Lor*** case and find out the fundamental reason for demise of lake is the system itself and human activity. Combining current South-to-North Water Diversion Project in China, we conceive a plan to establish artificial water cycle system, and gradually restore the ecological system under the circumstance of limitation on human activity. We later examine the feasibility of the plan and build an ***Economic Growth Model*** within environment constraints to demonstrate positive effect that improved environment have on economic growth, combining the WSA model, we forecast the improvement that our intervention plan brings to the water supply ability of Northwest China.

Lastly, we evaluate the strengths and weakness of our model and suggest improvement plan.

Key words: WSA Model, Water Cycle Model, Demise of Lop Lor, Economic Growth Model

Content

1. Introduction

Water is the source of life, every community and ecosystem on Earth depends on water for sanitation, hygiene, and daily survival. Critical as water is, with the rapid increase of population and development of economy, it is under great threat around the world. Water scarcity can be divided into physical scarcity and economic scarcity. The former means water demand in one country or one region exceeds water supply, namely, resource-oriented water shortage, while the latter means quality-oriented water shortage. However, currently, water use is increasing at twice the rate of population suggests that there is another cause of scarcity. Our main aim is to create a water supply ability measuring model which give consideration to not only physical factors (i.e. natural water source, technological advances) and environment constraints, but also social factors such as sanitation condition and human population, etc.

Previous research has studied the measurement of Water Resource Carrying Capacity (WRCC). Many experts and scholars use various methods such as Principle Component Analysis (Wang Xuequan, 2005) and Fuzzy Comprehensive Evaluation (Xu Lang) to measure and evaluate Water Resource Carrying Capacity in one region. Their researches provides foundation for us to study the water supply ability and evaluation of water resources. However, previous study didn’t give much consideration to social factors in measurement of Water Resource Carrying Capacity, and we focus on modeling a measure of the ability of a region to provide clean water to meet the needs of its population, thus, based on existent materials, we use Entropy and TOPSIS method to develop a model for evaluating the ability of a region to provide clean water to meet the needs of its population and named it as water supply ability model (WSA).

According to tasks given, our detail work is as follows:

- Develop a water supply ability measuring model (WSA) based on Entropy and TOPSIS method.
- Evaluate the water situation in northwest China using our WSA model.
- Forecast the water situation in northeast China using Grey Method and its impact on citizens.
- Put forward an intervention plan using a micro water cycle model.
- Build an economic growth model within environment constrains.
- Evaluate intervention plan’s impact on northwest China.

2. Hypothesis

In order to guarantee the accuracy and reliability of our models, we put forward the following assumptions:

- We assume that the sanitation infrastructure, water resources management level and other issues are manifested by economic power, in other words, economic power has positive effect on water supply ability.

- ◆ We ignore natural disasters, war and other irresistible factors' impact on water supply ability.

4. Water Supply Ability Model (WSA)

In this section, we develop a model for measuring water supply ability. We first carefully choose 8 Indicators. Then we use the Entropy and improved TOPSIS method to create a model for evaluating supply ability of clean water.

4.1 Selection of Indicators

There are many factors that directly or indirectly influence a region or a country's supply ability of fresh water. It is of much significance to select accurate and related indicators to suggest and evaluate water supply ability. Many studies hold different preference for different indicators when measuring Water Resource Carrying Capacity or measuring water supply. Considering there is no agreement on indicator selection, we give our priority to indicators that meet the following requirements.

- ◆ The indicators should incorporate both environmental and social factors that can exert influence on clean water supply ability. Namely, the indicators should affect water supply ability either physically or socially.
- ◆ The indicators should indeed influence water supply ability whether directly or indirectly.
- ◆ The indicators had better be quantifiable for the sake of directly measuring clean water supply ability and easy comparison.
- ◆ The indicators had better be tested through previous research and shows impact on water supply ability.

Based on above requirements, we choose the following 8 indicators which can be classified into two main types and eight hypo-types as shown in the following table. We then explained their meaning and effect on water supply ability.

Table 1. Indicator notation.

Type	Indicator	Unit	Effect
Environmental Factors	Per capital water resources	100 million cu.m	Positive
	Annual Precipitation	mm	Positive
	Water distribution space imbalance	mm	Negative

	degree		
Social Factors	Per capital GDP	yuan	Positive
	Water use	(cu.m/person)	Negative
	Population	10000 persons	Negative
	Waste water treatment rate	%	Positive
	Local exploitation rate	%	Negative

1. Environmental factors definition

- **Per capital water resources**: this indicator represents water resources that one person possesses. Per capital water resource is one of the most direct indicator that can evaluate water supply of one region or a country.
- **Annual precipitation**: the total precipitation of one region, precipitation can influence water resource thus exert positive effect on water supply ability.
- **Water distribution space imbalance degree:** degree of water space distribution imbalance, it is related to variance of different region's precipitation.

2. Social factors definition

- **Per capital GDP**: average GDP for every person in one region or a country. According to our previous assumption, we adopt Per Capital GDP as the measurement of sanitation and management issues since there is no quantified indicator for these issues.
- **Water use:** total amount of water that one region or a country uses. we choose water use indicator which in a year to measure the water capacity that a region or a country has to provide in order for survival and development,
- **Population**: number of people in a given year in one region or a country
- **Waste water treatment ratio:** the ratio of treated waste water to total water. In view of the prevalence of water pollution and water waste, we select waste water treatment rate to measure the technology advancement's influence on clean water supply ability. Last but not least, effective irrigated area rate
- **Local exploitation rate:** the ratio of the water resources exploited from local area to total amount of water resources. This indicator measures the degree of using local water resources. Usually, the alarm value is 50% internationally, the smaller the number is, the better.

3. Indicator's effect on water supply ability

Different indicator exerts different effect on water supply ability, some are positive, some are negative, while other factors have two-way impact. For those factors that have two-way effect, weight of negative effect and positive effect is usually different, therefore, we can determine the weight and specify the effect direction. For instance, high per capital GDP can enhance sanitation level, water exploitation ability, while it also means more water use, however, through historical experience we can know that the positive effect weighs more than negative effect. Therefore, based on previous studies, report and research, it is obvious that in most cases, per capital water resources, annual precipitation, per capital GDP and waste water treatment rate have positive effect on water supply ability, while the other indicators tend to have negative effect on water supply ability.

4.2 Model Building

After selection of social and environmental factors, we begin to develop our model on the basis of Entropy method and TOPSIS method. Our detail modelling work is as follows:

1. Original Matrix Establishing

Assume there is *m* evaluation objects (*j* =1, 2…,m) namely *m* regions or countries, *n* evaluation indicators (*i*=1,2,3,⋯,n) for each evaluation indicator, each evaluation object has a corresponding value. All values form a original matrix as follows:

$$X = (x_{ij})_{n\times m}$$

2. Data Normalization

As different indicator uses different units or dimensions and effect that indicators have on water supply ability vary from individual to individual, it's not appropriate to directly use it in model. Therefore, we need to normalize and adjust the data before modelling. We can use the following equation to standardize the original matrix.

$$y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}{}^{2}}}$$

After normalization of data, we can obtain the following standard matrix. We put the positive factor in previous columns for convenience of analysis.

$$Y = (y_{ij})_{n\times m}$$

3. Indicator Weight Determination

We adopt Entropy method to specify weight for all evaluation indicators, we can obtain the entropy of each evaluation indicator which is:

$$H_i = -\frac{1}{\ln m}\left(\sum_{j=1}^{m} f_{ij} \ln f_{ij}\right) \qquad (i = 1,2,\cdots,n;\ j = 1,2,\cdots,m)$$

$$f_{ij} = \frac{1+y_{ij}}{\sum_{j=1}^{m}(1+y_{ij})}$$

We can obtain entropy weight for each indicator by the following equation:

$$w_i = \frac{1 - H_i}{n - \sum_{i=1}^{n} H_i}$$

Now we've got weight of each indicator and obtain vector $W = (w_1, w_2, \cdots, w_n)$

4. Calculate Comprehensive Performance

According to entropy weight of each indicator that we obtained above, we can calculate comprehensive performance of water supply ability for each region. Do math operation to entropy vector ***W*** and matrix *y* using the following algorithm and we can obtain weighting matrix $\boldsymbol{Z_{ij}}$..

$$Z = (z_{ij})_{n \times m,}$$

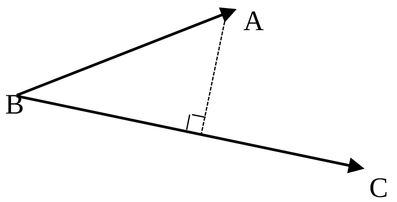
$$z_{ij} = y_{ij} \cdot w_{ij} \qquad i = 1, 2, \cdots, n; j = 1, 2, \cdots m$$

Each row vector of matrix ***Z*** can be seen as coordinate of weighted indicators in *n*-dimension space and we adopt TOPSIS method to calculate the optimal and worst solution under ideal situation which is presented as shown below:
The first *k* indicators are positive indicators while the rest are negative indicators.

$$Z_{optimal} = \left\{ \max_i z_{i1}, \max_i z_{i2}, \cdots, \max_i z_{ik}, \min_i z_{ik+1}, \cdots, \min_i z_{in} \right\}$$

$$Z_{worst} = \left\{ \min_i z_{i1}, \min_i z_{i2}, \cdots, \min_i z_{ik}, \max_i z_{ik+1}, \cdots, \max_i z_{in} \right\}$$

Unlike traditional TOPSIS method, we optimize and transfer the previous measurement which uses Euclidean distance into measuring at the ratio of one point's vertical distance from worst solution to distance between worst and optimal solution. Let's first examine this method in two-dimension space, the method can be illustrate in figure below.



As the figure shows, point B and point C respectively represents worst and optimal solution under ideal situation. Point A is the coordinate of row vector that composed of every region's evaluation value for *n* indicators. The distance from A to B can be calculated by the following equation:

$$d_1 = \frac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{\left| \overrightarrow{BC} \right|}$$

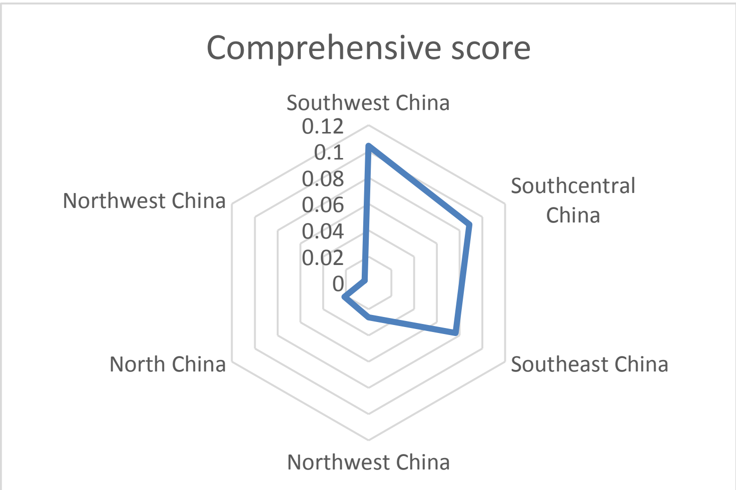
In a similar way, we can obtain the distance from worst solution point B to optimal solution point C named as d_2.In the perspective of two-dimension space, this method is reasonable and efficient, it's not hard to extend this method to *n*-dimension space. Therefore, the ratio of d_1 to

d_2 is what we use to evaluate the comprehensive performance of each research object, in other words, different regions or countries' comprehensive performance in regard to water supply ability.

5. Model Solution

We use our WSA model to analyze the data of different regions in China as an example and obtain the comprehensive score, namely, water supply ability, the data (from *Statistical Book of China, 2013*) and results are as follows:

	Annual Precipitation (mm)	Per Capital Water Resources(100 million cu.m)	Local Exploitation Rate	Per Capital GDP （yuan）	Water use (100milion, cu.m)	Population （10000 persons）	Waste Water Treatment Ratio(%)
North China	2440.00	0.0764	0.0601	91606.75	508.41	17047.03	90.01
Northeast China	1850.00	0.2269	0.2236	54442.04	635.91	10976.3	76.30
Southeast China	8634.00	0.1177	0.3987	228153.70	1881.83	39911.98	91.62
South-central China	8439.00	0.1941	0.2537	161014.15	1659.34	38161.23	84.37
Southwest China	5239.00	0.5019	0.3745	59452.83	598.39	19577.86	73.46
Northwest China	1634.00	0.2271	0.4005	35339.587	899.56	9842.46	74.76



As is shown in the graph, Southwest China ranks first in water supply ability while Northwest ranks the last in water supply ability, which is in accordance with China's actual condition.

6. Dynamic Analysis

Take a further look at our indicators, we can easily find that many factors are dynamic. For instance, annual precipitation changes randomly, high amount precipitation can improve supply of water while low precipitation will exacerbate water scarcity. Our further research will study the randomness of annual precipitation and total water resources and further illustrate the dynamic influence of these factors.

4.3 Region Evaluation---Northwest China

We choose northwest area of China as our research object, as is vividly shown in the Water Stress Map, this region stands out as one of the over-exploited regions. Northwest China includes the autonomous regions of Xinjiang and Ningxia and the provinces of Shanxi, Gansu, and Qinghai. Based on the indicator system we built which incorporates both environmental and social factors, we can analyze the reason and type of water scarcity in this area. We collect data from *Statistical Book of China, 2013* as shown in the above figure to analyze the basic information of northwest area of China and make comparison with other areas. Following is the analysis of the environmental drivers and social factors that cause water scarcity in Northwest China.

◆ Environmental Drivers

We first analyze the environmental factors to explore the reason for water scarcity in northwest China. From the table above we can easily find that the value for precipitation is the lower than any other regions in China. Value of per capital water resources in Northwest China is not the lowest because of small population in this area, however the total amount water resources is still lay behind other regions. Obviously, we can conclude that northwest China displays physical water scarcity on first thought. To take further thinking, we analyze the root environmental drivers of water scarcity, namely, the reason for low precipitation and water resources.
Environmental drivers can be summarized as climate, topography and river which are the determining factor of physical water supply. The latitude scope of northwest China is from 34 °N to 45 °N and the longitude range from 105°E to 115 °E which is far from sea. Location determines this region's typical monsoonal climate with small rainfall and low amount of water resources. Additional, the unique topography further decrease the physical availability of water. Three east-west mountains------ Tianshan mountain, Kunlun mountain and the Altai mountains stands in this area and block the moisture from the Indian Ocean and Arctic Ocean which further decrease the precipitation. What's more, northwest China locates in typical interior drainage area which means little river water supply.

◆ Social factors

Compared to other regions in China, waste water treatment rate in northwest China is pretty low and per capital GDP is also lower than other regions. The population is smaller than other regions while the water use is a bit high considering the population scale.

From the value of social factors, we can conclude that there also exists economic water scarcity to some degree. The waste water treatment rate don't catch up with the national level which results from the backward in technology and education. Low per capital GDP displays economic weakness which in turn results in poor-equipped sanitation infrastructure and management of water resources. As population grows, amount of water use increases at the same time which causes water pressure in this region. Water usage increase is the result of large irrigation area. In all, lack of management and sanitation infrastructure which manifested by per capital GDP and increasing population growth and water usage cause economic water scarcity.

5. Water Supply Ability Forecast Model

5.1 Problem Restatement

Our task is to forecast the water situation of northwest China in 2025.Based on previous built water supply ability model (WSA), the data can be forecasted according to the following process: In a nutshell, we first 1) use the current data to generate predictive value through forecast model, then 2) input the predictive value into the Water Supply Ability model (WSA) and we can 3) obtain the WSA predictive value. In order to know the water supply ability 15 years later in northwest area of China, we use the Grey Forecasting Model to forecast water situation. We first made the following hypothesis.

(1) There exists no big change for climate in this region in 20 years.

(2) Weight for each indicator doesn't change much as the data changes, in other words, the data used for testing doesn't change the entropy of previous indictor much.

5.2 Model Solution

5.2.1 Calculate Future Predicted Value for Each Indicator

Some indicators have relatively clear mathematical model (i.e. population growth, economic growth model), however, considering the small sample and the complexity of factors that affect each indicator, it's difficult for many factors to draw a specific mathematical expressions, such as *Water Exploitation Rate* (the ratio of water quantity exploited from local area to total amount of water resources). Additionally, some indicators such as precipitation are random variables which behave according to Gaussian distribution.

◆ Forecast for random variables:

When forecasting random variables such as Precipitation (p) and Total Amount of Water Resources (q), use the computer to randomly select some data as predicting data (relying on the distribution of these random variables). We can obtain sample variance and mean by calculating on the basis of historical data. Since we assumed that the climate won't change much from 2005 to 2015, the variance and mean can be seen as constant, so we can assume X_i (Sample of precipitation) and Y_i(Sample of total amount of water resources) is independent identically distributed. Obviously, the two random variables obey the normal distribution. From the knowledge of mathematical statistics, sample mean and variance is Uniformly Minimum Variance Unbiased estimate (UMVUE) of normal distribution, therefore, we use the two figures to estimate their mean and variance of their distribution as shown as follows:

$$p \sim N(\mu_1, \theta_1^2) \qquad\qquad q \sim N(\mu_2, \theta_2^2)$$

$$\mu_1 = \sum_{i=1}^{n} \frac{X_i}{n}, \theta_1^{\,2} = \sum_{i=1}^{n} \frac{(X_i - \mu_1)^2}{n-1} \quad \text{and,} \quad \mu_2 = \sum_{i=1}^{n} \frac{Y_i}{n}, \theta_2^{\,2} = \sum_{i=1}^{n} \frac{(Y_i - \mu_2)^2}{n-1}$$

Xi, ***Yi*** represents sample. Randomly select *m* figures according to distribution $N(\mu_1, \theta^2)$ using the computer. After observation and analysis of data, we find that there exists close relationship between the value of annual precipitation and total amount of water resources. However, we didn't look into the correlation between annual precipitation and total amount of water resources, instead, we adopt a simplified method which can help to obtain the predictive value of total amount of water resources.

Set the predictive value of annual precipitation that obtained through above method as *a*;
Assume CDF of **p** is $F_1(x)$, and for **q** is $F_2(x)$, calculate according to the following equation:

$$F_2(x) = F_1(a)$$

Assume b is the solution to the equation, obviously, b is the lower quantile of F_1(a), and, the value a and b respectively represent predictive precipitation and total amount of water resources.

- Forecast for other indicators

We use Grey Model to forecast value of other indicators. Grey Model Forecast can help to use data to identify the development tendency difference for system factors and do correlation analysis, display the data tendency characteristic through some algorithm. After that, we can forecast according to the characteristic, finally, build a differential equation to work out the corresponding predictive original value.

Specific steps are as follows:

(1) Data testing and modification

The original data is:

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))$$

Define: $\lambda(k) = \dfrac{x^{(0)}(k-1)}{x^{(0)}(k)}, k = 2,3,\cdots,n$ and $X = (e^{\frac{-2}{n+1}}, e^{\frac{2}{n+2}})$

If $\forall k = 2,3,\cdots,n$, $\lambda(k) \in X$, then we can use the grey model for forecast.

Or we should select sequence: $y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \cdots, y^{(0)}(n))$

$$y^{(0)}(k) = x^{(0)}(k) + c \qquad k = 1,2,\cdots,n$$

Notice that *c* is a moderate constant which can make the class ratio of $y^{(0)}$ fall into the range of X set. Therefore, y_0 can be used for the grey model.

(2) Use 1-AGO process to generate sequence:

$$x^{(1)}(k) = x^{(0)}(1) + \cdots + x^{(0)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

Define: $d(k) = x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1)$

$$z^{(1)}(k)=\alpha x^{(1)}(k)+(1-\alpha)x^{(1)}(k-1)$$

Therefore, GM (1,1) grey differential equation is as follows:

$$d(k)+az^{(1)}(k)=b$$

Then we can obtain the following equation set:

Set $U=\begin{bmatrix} a \\ b \end{bmatrix}$, then $Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{bmatrix}$ and B = $\begin{pmatrix} -Z^{(1)}(2) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(n) & 1 \end{pmatrix}$

The above equation set can also be described as:

Y=BU

Through simple linear regression, using the least square method, we can obtain the following estimate value:

$$\hat{u}=[\hat{a},\hat{b}]^T=(B^TB)^{-1}B^TY$$

Then we can obtain the corresponding albinism differential equation:

$$\frac{dx^{(1)}}{d(t)}+ax^{(1)}(t)=b$$

Solve the equation and we can obtain the solution:

$$x^{(1)}(t)=(x^{(0)}(1)-\frac{b}{a})e^{-\alpha(t-1)}+\frac{b}{a}$$

Forecast original value:

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \qquad k=1,2,3,\cdots,n-1$$

(3) Analyze the estimate value.

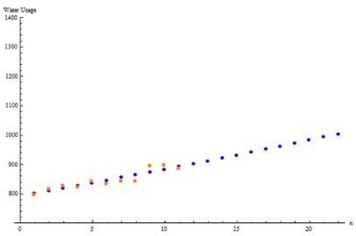
In this step, we examine the residual. The relative residual equation is:

$$\varepsilon(k)=\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}, k=1,2,\cdots,n$$

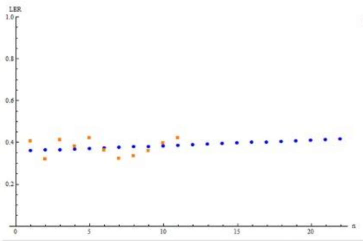
If $|\varepsilon(k)|<0.1$, then our model can meet high requirement and can be used for forecast.

(4) Input actual data and forecast.

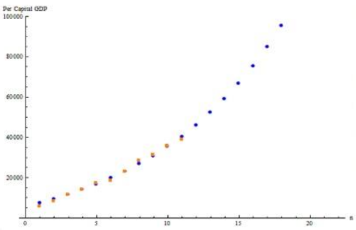
Following is the comparison between predictive value and original value for each indicators. The orange point stands for actual point while blue point stands for predictive value. From the figure we can know that the fitting degree between actual value and predictive value is pretty high. After residual analysis, our prediction proves to be effective.



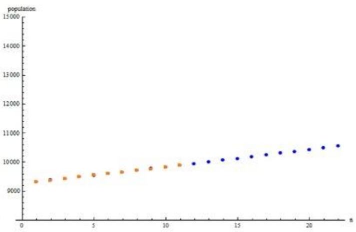
Water Usage



Local Exploitation Rate



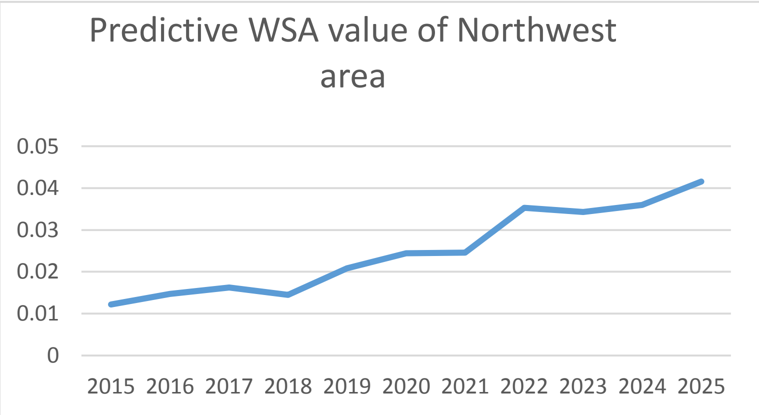
Per Capital GDP



Population

5.2.2 Calculate Predictive Water Supply Ability.

Considering that our first model is based on comparison, in a similar way, we forecast future water situation by comparison with that of six main regions' performance in 2013. In other words, based on the data matrix of the six main regions in 2013, we obtain a predictive score which is a relative value to score of 2013. Take predictive value of next few years into the WSA model and we can obtain the predictive value namely comprehensive performance of water supply ability, we can view the changes of predictive value. Following is the predictive comprehensive value for northeast China and the predictive value for each indicator is mentioned above.



The predictive WSA value for northwest China generally shows an upward trend though there exists small fluctuations which resulted from the influence of random variable such as precipitation and amount of water resources. The predictive WSA value in 2025 is higher than that of northeast and north area of China in 2013, however, it still lay behind the value of southeast area in 2013.

5.2.3 Impact on Lives of Citizens

Let's take a further look at the predictive value for different indicators and analyze the impact on the lives of citizens of this region. In the future 10 years, the precipitation and total amount of resources will stay at nearly the same level, while per capital GDP and total amount of water use will increase a lot with fast population growth. The local exploitation rate remains high in the future years.

Increase of WSA value generally shows a promising future water supply ability in northwest area which will raise people's standard of living, people can get more water through more accesses. The enhancement in per capital GDP indirectly shows improvement in sanitation and management of water resources, namely, water resources are better organized. Therefore, people can enjoy higher level of social service in regard to water. However, as the figure shows, the quantity of water resources and precipitation won't change much in the future years, which, demonstrate that physical water scarcity will remain to be a serious problem in this region. In all, as time goes by, the economic water scarcity can be alleviated to some degree, people can have access to high-quality and well-organized water resources. However on the other hand, the physical water scarcity still needs to be solved or improved, the physical water supply ability is limited to a certain level.

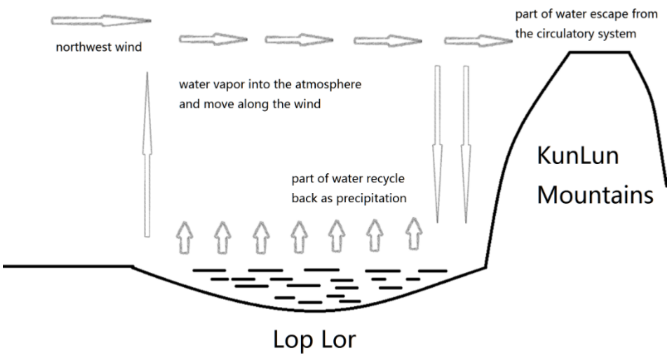
6. Our Intervention Plan

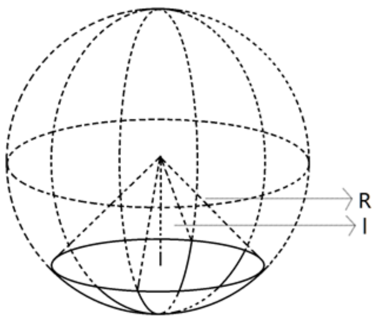
Now that we've forecasted the future water situation in northwest area and find out that physical water scarcity is the long-time problem for northwest area, so we are committed to put forward a plan which can help to alleviate physical water scarcity in northwest area. The water scarcity has long been a serious problem in this area, in order to put forward a intervention plan that can inherently improve the water scarcity in northwest region of China, we studied the famous demise of Lop Lor case.

6.1 Case Study of Demise of Lop Lor lake

In the 30s of last century, the Lop Lor lake covers an area of 3000 square kilometers, however, it came to a complete end in the 70s. Our analysis will demonstrate that one reason for demise of Lop Lor Lake is due to natural causes, but human activity exacerbated the process.

Lop Lor is located on the north of the Kunlun mountains and controlled by the northwest cold air all the year round. The water vapor generally doesn't float out of the basin itself, but rather generate rain fall in north of the Kunlun mountain along with the northwest wind in the direction toward southeast area. On the one hand, the precipitation added glaciers to the Kunlun mountain, on the one hand, it provides the Tarim river with rich water resources, thus forming an inland water cycle.（As shown in the following left figure）. As time goes by, part of the water cycle escapes from the Lop Nor area reducing the total amount of water cycle. While there exists no other water supplement, so water resources gradually reduced. Last century, due to a surge of population in Tarim area, the river was blindly exploited and come to the end of water break, thus, making the Lop Nor lake a dead lake.





6.1.1 Inland Water Cycle Model

In order to vividly show the relationship between human activity and demise of Lop Lor, we develop the following model.

(1) Nomenclatures `

Symbol	Notation
Q	Lake water quantity
S	Area of Lake
l	R-maximum depth
R	Lake sphere radius
α	Net evaporation rate

Variable with a Subscript 0 means the initial value of the function.

(2) Additional hypothesis.

① Assume the entire lake bed is a camber surface with radius equal to R as shown in the above right figure.

②Assume precipitation is direct in proportion to evaporation e, proportionality coefficient is a.

(3) Model building

Model 1: water cycle model without human activity.

Differential for Q meets: $-dQ = S \cdot dl$

Through Pythagorean theorem and the area of formula:

$$S = \pi(R^2 - l^2)$$

The speed of lake depth decline meets: $\frac{dl}{dt} = \alpha$

Initially $l(0) = l_0$

Through the equation above, we can obtain the function of water quantity change without human activities as follows:

$$Q = -\alpha\pi(-\frac{\alpha^2 t^3}{3} - \alpha l_0 t^2 + (R^2 - l_0{}^2)t) + C, \qquad C = Q_0$$

Model 2: water cycle model with human participation

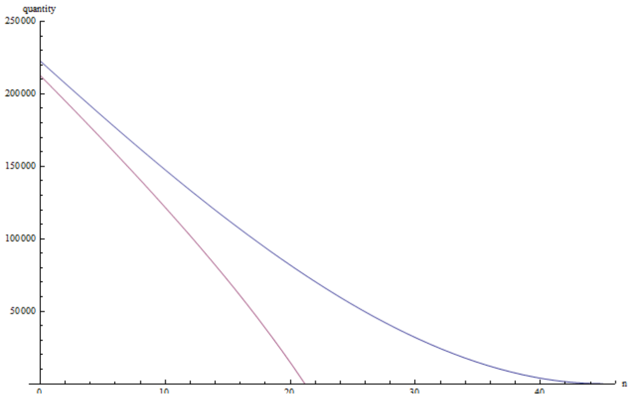
Now we consider the demise situation of Lop Nor under the influences of human activities. X_0 represents the initial number of population, and X (k) means the number of population in Kth years. Since the demise period of Lop Lor is from 1942 to 1970 which is a golden population growth period, therefore, we adopt the basic population growth model without resistance, namely:

$$X(k) = X_0(1+r)^k \quad (r \text{ means growth rate })$$

This is a discrete population growth model without resistance.

$$Q_1 = \beta \bullet x(k) \quad (\beta \text{ stands for per capital water resources})$$

Remained water quantity; $Q_2 = Q - Q_1$



As shown in the left figure, the blue line represents water quantity change without human activity, while the purple line is water quantity change with human participation.

From the figure above, we can easily conclude that human activity exacerbate the demise of Lop Lor lake.

6.1.2 Conclusion

From the case of Lop Nor, we can learn from that if we want to improve the water scarcity water in northwest area fundamentally. On the one hand, we should improve legislation about the exploitation of water resources, while on the other hand, we should increase supplementary water and increase the total amount of water cycle, make the addition to water cycle bigger than disappearing amount thus generating a virtuous cycle. In all, the main points of our intervention plan includes:

1. strictly limit the local exploitation degree of water resources
2. add water vapor namely increase precipitation.

6.2 Intervention Plan

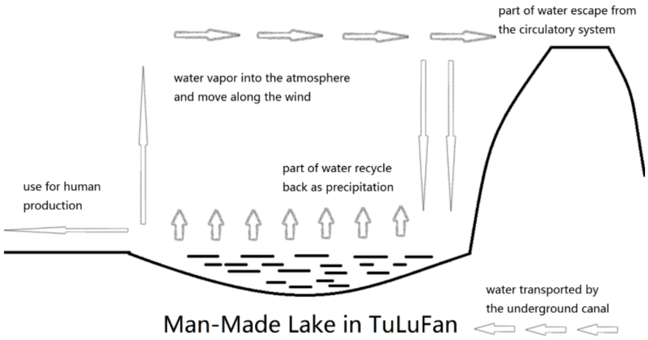
Existing south-western water transfer can only improve the irrigation water scarcity in middle and downstream area of the three river in northeast China which will not change the drought and desertification situation in northwest China. So we put forward a east-to west water transfer plan, in a nutshell, dig underground canal, get water in middle or downstream of Yangtze River and directly transfer it to Turpan Depression, thus forming a permanent artificial connected lake (it can be viewed as a connector).

We first consider the topography of Turpan Depression area, it's the lowest inland depression of China, height for the Bogurda mountain and Kara Ukraine mountain in the back is about 3500 meters to 4000 meters. Take the ridgeline of the mountains as boarder and we can learn that it covers an area of 50140 km^2 and the area below the sea level is about 4050 km^2.

Hongze Lake is the collection of Huaihe and Yangtze River and is a flood discharge for the Yangtze River. Usually, the water depth is about 5.4 meters, the altitude of lakebed is about 10 meters. Annually, there are approximately 40 billion cubic meters of water flowing from Yangtze river or discharge channels to the sea. Therefore, we select Hongze Lake as the channel of water transfer to Turpan Depression.

6.3 Concrete Operation Plan.

Transfer water from Hongze Lake to Turpan Depression through underground canal and automatic water flow. Dig the canal from the depth of minus 10 meters from the sea level and establish a closed underground canal in the whole river basin which is about 3300 km^2. According to the altitude and water-storage depth, we can calculate and find that when the water cycle is in balance, the area of the lake in Turpan Depression can cover an area of 15000km^2 with a height of 15 meters above the sea level.



Figure

6.4 Feasibility Analysis

We build a micro water cycle water to guarantee the feasibility of our intervention plan.
(1) Hypothesis: after control legislation of water management, the water exploited amount can be in an appropriate range.

(2) Nomenclatures:

Name Q as the transferred water quantity from Hongze lake.
S as the artificial river area in balance condition.
e as the annual evaporation amount of water.
p as the escaped water vapor that didn't generate rainfall or glacial water.
m as the amount of exploited water from local water resources.

(3) Foundation of Model

If we want to the artificial lake circulation system reaches a virtuous cycle, the water flow into the system should be higher than the water that escape from the system.

Namely:

$$s \times e \times p + m \leq Q$$

As we assumed before: $m \leq M$

Then the system can be into virtuous cycle when:

$$Q \geq M + S \times e \times p$$

Assume the total transfer amount is 50 billion m^3, among which 40 billion m^3 water is evaporated and go into the cycle system, 10 billion m^3 water is used for production and living. The amount of evaporated water is about 3000 mm the south of Turpan Depression. Then the amount of evaporation can be calculated:

$$3000 \text{ mm} \times 15000 \text{ km}^2 = 45 \text{ billion m}^3.$$

Considering the water supplement by rainfall is about 50 billion m^3, then the amount of water inflow and outflow can reach a balance.

It's easily calculated that the transferred water of 50 billion m^3 goes into the inland water cycle system and will enhance the precipitation by 100 mm. Part of it permeate and supply the underground water, part of it flows into river and increase river volume of run off, part of it is supplied to glacier, on the other hand increase of precipitation will decrease water usage to some degree. To make it short and simple, we inject 50 billion m^3 water into this inland cycle system. Part of the water is stored as surface water in lakes and dams, increasing water filed in turn increase precipitation through participation of water cycle system, finally, a brand-new inland water cycle balance is reached.

6.5 Economic Benefit Analyzing

6.5.1 Nomenclatures

Symbol	**Notation**
Q(t)	Output of time t
c	Coefficient concerning technology,>0
K(t)	Capital at time t
L(t)	Labor force at time t
α	Ratio of the capital to output
β	Ratio of labor force to population
r(x)	Population growth rate
r	Population fixed growth rate
s	Resistance from environment
x(t)	Population at time t
X(m)	Maximum environment intake capacity
λ	Ratio of capital to output

Variable with a Subscript 0 means the initial value of the function.

6.5.2 Economic Growth Model with Population Resistance

According to Cobb-Douglas economic growth model:

$$Q = cK^{\alpha}L^{1-\alpha}\text{ , } 0<\alpha<1$$

(1) β is the ratio of labor quantity to total population quantity, assume β is set.

$$L(t) = \beta \bullet x(t)\text{ ,}$$

Considering environment resistance, assume

$$r(x) = r - sx \quad , \quad s = \frac{r}{x_m}$$

Through population growth differential equation:

$$\frac{dx}{dt} = rx(1 - \frac{x}{x_m}), \quad x(0) = x_0$$

We can obtain $x(t) = \dfrac{x_m}{1 + (\frac{x_m}{x_0} - 1)e^{-rt}}$

$$\therefore \; L^{1-\alpha}(t) = \beta^{1-\alpha}\left(\frac{x_m}{1 + (\frac{x_m}{x_0} - 1)e^{-rt}}\right)^{1-\alpha} \qquad \text{(Eq.1)}$$

（2）Assume that K is in direct proportion to output, the ratio is λ .

.

$$\begin{cases} \dfrac{dk}{dt} = \lambda Q \\ Q = cK^{\alpha}L^{1-\alpha} \end{cases}$$

Use Mathmatical 9.0 to solve and we obtain;

$$K^{\alpha}(t) = \left(\frac{1}{-\alpha\lambda c \cdot m(t) + C}\right)^{\frac{\alpha}{\alpha-1}} \qquad \text{(Eq.2)}$$

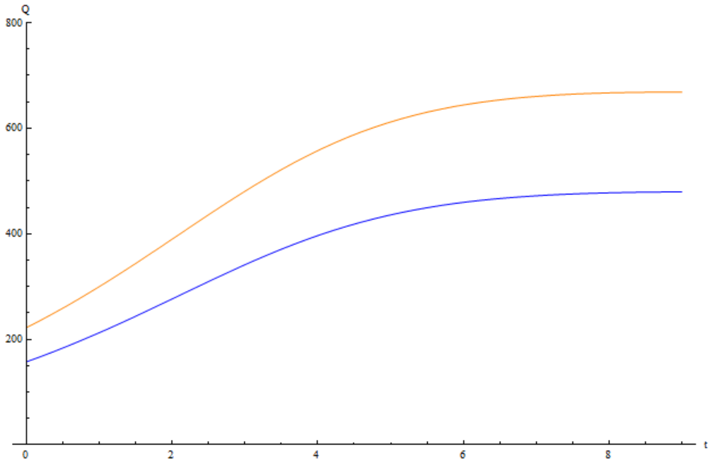
And $m(t) = \dfrac{\left(\frac{x_0 x_m}{x_0 + (x_m - x_0)e^{-rt}}\right)^{1-\alpha}\left(1 + \frac{e^{rt}}{1-\alpha}\right)^{1-\alpha} H\left(1-\alpha, 1-\alpha, 2-\alpha, -\frac{x_0 e^{rt}}{x_m - x_0}\right)}{r(1-\alpha)}$

H is a Hypergeomatric Function

$$C = \frac{1}{K_0^{\alpha-1}} + \alpha\lambda c \cdot m(0)$$

$$\therefore Q(t) = c \cdot K^{\alpha}(t) \cdot L^{1-\alpha}(t) \qquad \text{(Eq.3)}\backslash$$

6.5.3 Model Analysis



This figure shows the economic output change with time passing by, when population intake capacity doubles, the function curve for Q transfer from the below blue curve to the above orange curve, which demonstrate that when ecosystem gets better, population intake capacity increases, economic output can be conspicuously improved.

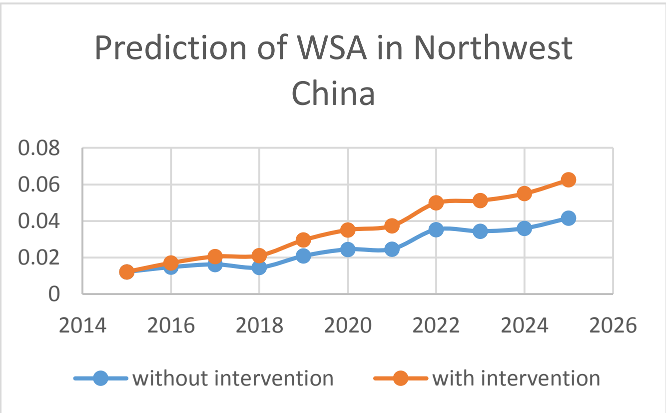
6.5.4 Conclusion

Our intervention plan will improve the environment which will also improve the economic output. Therefore, efficient intervention is indispensable both for ecologic and economic use.

6.6 Future WSA under Intervention

Now we predict the influence of our prevention plan to the water supply ability in Northwest China. Because we can't evaluate the time spend on building up the water project, we assume it would be finished and be in stable operation before 2013.

In order to evaluate more accurately, we adopt the Control Variable Method to merely consider these indicators that change directly because of our intervention plan, that is, the precipitation and the total amount of water resources. We compare the future's comprehensive score change in areas that are not adopted our solution in Northwest China with areas that have adopted our prevention plan.



From above we can see that the overall data of Northwest China in 2025 is nearly equal to North China, and is obviously get better comprehensive scores than areas that not adopted any measurements.

The result merely shows the increase when we change the natural factors, while in the real operation process, the enormous improvements on natural factors will greatly improve the maximum amount of population capacity. According to the economic growth model, the maximum amount of population capacity is in proportion to the economic development.

From what have analyzed above, our solution will not only converse the local physical scarcity, but also will largely promote local economic development situation and enormously improve the economic scarcity of water resources. We estimate that, with the implementation of our solution, the water supply capacity of Northwest China in 2025 will reach or even exceed China's South-East coastal areas.

7. Strengths and Weakness

Strengths

- Easy to calculate and predict

There are many factors to influence water supply capacity, and we, get start based on the two sides of physical scarcity and economic scarcity, pick up several indicators which accurately reflect regional water supply situation to large extent. These indicators are representative and easy to be statistics, which will enormously decrease the difficulty for analysis staff to collect data.

- Accuracy

In the WSA model, we use TOPSIS method, and we believe we have great reliability on proportional distribution to every indicator. In the model of predicting water supply situation in the future, we adopt grey model based on the randomness analysis. Thus, we can get the relatively accurate result with just a small amount of statistics, and the random analyze can take full consideration to the active natural factors like precipitation.

- Relate to reality

By making reference to the case of disappearance of Lop Nor, and considering the real natural and economic situation in Northwest China, we put forward a solution that refers to natural and social factors. We also make feasibility and benefits analyze to the solution by using the model for the relationship between population and economic development. Our solution can be promote and apply to the global areas with water scarcity problem, which have similar natural condition to Northwest China.

Weakness

In the predicting model, we just used statistics to predict the changing of several regional indicators in the future without considering the regularities of these statistics and inner relationship between statistics. If we were to make more progress on this model, we can make more accurate analyze of the main contains on every indicator to find out the promoting or restrained relationship among these indicators.

8.References

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