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Team Control Number

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**Summary Sheet**

U.S. airports have the world's most stringent security procedures, but the long and unpredictable security checking time has also caused a strong dissatisfaction with TSA. The existing security system did not significantly reduce the waiting time of passengers. Besides, the waiting time has large fluctuations, which is difficult to predict.

By analyzing the given data and modeling, this paper will figure out the following tasks:

1. Bottlenecks of the whole system
2. Causes of huge variance of the waiting time
3. Advantages and disadvantages of Pre-check lanes
4. Impacts of cultural differences on specific security processes
5. Corresponding improvement measures and policies to all the problems above

In question A, we set up a single queue model (M/E<sub>k</sub>/1 Model) to reflect the situation of getting through the whole security checking system. We first use the chi square test to verify the distribution that arrival time interval and the leaving time interval obey. Then, according to the given data, we calculate the expected value of the average length of the queue and the expected value of a single passenger's waiting time. Based on the model, we find it is that the input rate of queueing system is greater than the output rate, which leads to the uncertainty of queue length and waiting time. This is also the bottleneck.

Then, we established model 2. We give the definition of Buffer Area, and found that the size of this area can significantly affect the security rate, actually, the bigger it is, and the higher the rate will be.

In question B, we put forward two schemes, which raises the system output efficiency without changing the existing manpower. According to the improved schemes, we quantitatively analyzed the most reasonable ratio of lanes.

In question C, from the view of cultural differences, this paper theoretically analyzed the causes of high frequent airport congestion in the United States, and puts forward the practical solutions. However, the similar situation appears less in China.

In question D, we put forward the corresponding policies, such as reducing the baggage charges. What's more, according to our models, we theoretically analyzed that these kinds of policies are effective.

In the end, based on the models and analysis, we summary all the recommended policies and suggestions to ameliorate the congestion for airline companies and airports

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

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# 1 Introduction

## 1.1 Problem Statment

There are two things calling for great patience and perseverance before take-off: one is flight delay; the other is the long waiting time at a security check-point. Actually, hourslong lines at some airports have provoked public outrage. To tackle the problem, the TSA produced a solution providing improved equipments and increasing security staffing to promote the efficiency of security check. Although the solution could help to reduce waiting time, it seemed that it didn't work very well. High cost reduced the utility of the approach, and the unexpected great variance is another weakness.

Since there is often a trade off between guaranteeing passengers' safety and saving their time, airlines can hardly make an optimum choice. We are asked to build a model to explore the flow of passengers through a security check point, and use the model to identify the check points. Since we have understood the weakness of the initial process, we can make some modifications to the current process to improve passenger throughput and shorten variance in the waiting time. In addition, we should take cultural norms into consideration to make our model adopted to different culture environment. Based on our model, our last task is proposing policy and procedural recommendations for the security managers.

## 1.2 Our Work

1. We read the title carefully and make preliminary analysis of the data in the excel form. We find that there is a remarkable difference between the service rate of zone A and zone B.
2. Based on the preliminary analysis, we get the conclusion that although Security check is a complex process with several periods, we can split it into several parts. So we analyze the data of different periods to explore the data distribution patterns and the bottlenecks respectively.
3. As the whole system is mainly effected by the service rate of Zone B, we could mainly focus on Zone B. According to the process of Zone B and based on the data analysis, we could set up an  $M/E_k/1$  model of queuing theory.
4. Our propose is to offer some measures which could increase passenger throughput and reduce waiting time variance, so we simulate the process of Zone B with computer and apply our modifications to the original process.
5. We analyzed the output rate of Pre-Check lane, and set up equations to calculate the best ratio of regular lanes and Pre-Check lanes.

6. We set up Model 2 to quantitatively analyze the impacts on output rate, which is determined by the size of Buffer Area. And then, we obtained the optimal size.
7. We analyzed the impact of cultural differences on our model, and then designed a better shape of the Buffer Area to improve the efficiency of the system.
8. In order to solve the bottleneck of the existing queuing system, we put forward two policy recommendations which can be used in most countries and regions. Indirectly improve the throughput and the output rate of the queuing system.

## 2 Data Analysis

### 2.1 Basic Analysis

Calculate the average value of each index, after observing, we could tell the following information:

- For every single passenger, the average Milimeter Wave Scan time is far more than the average time a baggage gets through the X-ray scanner, the former time is 11.6 seconds while the latter time is 6.71 seconds. So the security efficiency is mainly effected by the Milimeter Wave Scan time.
- Calculate the time of ID check, the average time is 11.23 seconds, which is approximately equal to the interval time of Milimeter Wave Scanning. According to our daily life, the actual time of ID check is really short, so the speed of getting through the ID Check point is decided by the speed of getting through the Milimeter Wave Scanning.
- The expectation of the time interval of Pre-Check arrival times and regular arrival times is 9.2 seconds and 12.95seconds. This result means that although the passengers choosing to go through Pre-Check lanes account for 45% of all passengers according to the information offered, in this sample, at a given time, there are more Pre-Check passengers entering the queues than regular passengers.
- About 90% of passengers finished milimeter wave scan within 13 seconds, while 5% of them spend more than 20 on the process. It's obvious that a small amount of passengers encumbers the whole system.
- We subtract the expectation of milimeter wave scan times(11 seconds) from the expectation of the time to get scanned property (29 seconds) , then we get the result (18 seconds) which refer to the time passengers packing their luggage.

## 2.2 the Distribution of the Input Rate

This case is typically a complex queuing problem. To apply the queuing model, we have to obtain the regularities of data distribution.

By observing the data in the excel form, we find it is likely that the number of passengers arriving in a certain period obeys the Poisson distribution. And the number obey the Poisson distribution iff the time interval obeys an exponential distribution. So we use Chi square test to prove that.

First we set up the original hypothesis  $H_0$ : the time interval obey an exponential distribution. Thus we can get alternative hypothesis  $H_1$ : the time interval doesn't obey an exponential distribution.

Next we divide the region from 0 to 50 seconds into  $m$  intervals. After that we count the number of the samples falling in every interval, and use the following equation to get the statistical quantity:

$$V_i = \sum_{i=1}^m \frac{(n_i - p_i N)^2}{p_i N} \quad (1)$$

Where:

$N$  is the total number of the samples;

$n_i$  is the number of the samples which fall in the  $i$ th interval;

$p_i$  is the theoretical probability of the samples falling in the  $i$ th interval under the original hypothesis;

Given significant level 0.05, we can get the critical value  $\chi_{0.05}^2$ . By comparing it and the statistical quantity, we can obtain the final conclusion.

Using the data for column A and B, we apply the above method to figure out the statistical quantities 16.62943 and 21.16591 which fall in the region between the two critical values 2.7 and 19.023, so we accept the original hypothesis, that is to say, We may reasonably believe that the number of passengers arriving in a certain period obeys a Poisson distribution.

## 2.3 the Distribution of the Output Rate

Similar to last subsection, we apply chi-square test to the data for column E, then we can prove that the output rate of passengers obey the Erlang Distribution.

### 3 Terminologies

#### Terms

input rate: the rate of the passengers entering the system

output rate: the rate of the passengers leaving the system

buffer area: the area where passengers get prepared to pass through milimeter wave scan

#### Variables

Variables	Descriptions
$X_i$	the time interval of regular arrival(s)
$X'_i$	the time interval of pre-check arrival(s)
$\bar{X}$	the average value of $X_i$ (s)
$\bar{X}'$	the average value of $X'_i$ (s)
$\lambda$	system input rate
$\mu$	system output rate
$L_q$	the equation of the length of the queue(exclude service time)
$W_q$	the equation of the length of waiting time(exclude service time)
$L_s$	the equation of the length of queues(include service time)
$W_s$	the equation of the length of waiting time(include service time)
$\rho$	the ratio of $\lambda$ to $\mu$

### 4 The Passager Flow Model

#### 4.1 Assumptions

- We assume that every passenger bring only one baggage with he or she to simplify the process.
- Assuming that at different times, the rate of the passengers arriving at the airport obeys the same distribution type.
- The output rate of pre-check lanes and regular lanes are assumed to obey the same kind of distribution. In practice, although their average value varies, they have similar distribution features.
- Assuming that the ID-Check time is very short, after that passengers need to wait for some time before entering the Zone B.
- Assuming that all of the pre-check passengers are using the pre-check lanes while all the regular passengers are using regular lanes.

We also make some assumptions about how we interpreted the data set.

- Assuming that the X - ray scanning time interval, millimeter scanning time interval and property scanning time interval all come from regular passenger lanes.

## 4.2 Embedded Markov Chain

In order to derive the  $M/E_k/1$  queuing model, we firstly explain the method of embedded Markov chain. The concept of embedded Markov chain is to use the total number of customers at a certain time when one customer come or go to define. Try to find the system transfer matrix from state  $n$  to state  $n+1$  probability, so that a non Markov problem can be simplified to a discrete Markovchain, thus we get solutions.

Several assumptions are made in the method as follows:

1. The input parameters of the system obey Poisson distribution
2. The service time  $t$  for every customer is a random variable with the same distribution independent of each other, The probability distribution function is  $F(t)$ . The expectation value and variance are:

$$E(t) = \int_0^{\infty} t dF(t) = \frac{1}{\mu}$$

$$Var(t) = \sigma^2$$

3.  $\lambda < 1/E(t)$  or  $\rho = \frac{\lambda}{\mu}$

4. Only one service site in queue

Based on the assumptions described above, we can get the Pollazack-Khintchine equation:

$$L_s = E(n) = \frac{2\rho - \rho^2 + \lambda^2\sigma^2}{2(1 - \rho)} \quad (2)$$

Other index can be deduced as follows:

$$L_q = L_s - \rho = \frac{\rho^2 + \lambda^2\sigma^2}{2(1 - \rho)} \quad (3)$$

$$W_q = \frac{L_q}{\lambda} = \frac{\rho^2 + \lambda^2\sigma^2}{2\lambda(1 - \rho)} \quad (4)$$

$$W_s = W_q + \frac{1}{\mu} = \frac{\rho^2 + \lambda^2\sigma^2}{2\lambda(1 - \rho)} + \frac{1}{\mu} \quad (5)$$

From the equations above, in the average service time  $1/\mu$ ,  $L_s$ ,  $l_q$ ,  $W_q$ ,  $W_s$  all get bigger as gets bigger, which means that when the servicetime of each customer is relatively close, the queuing system is better. In the case of the deviation of the service time distribution is very large, the workindex is poor.

### 4.3 $M/E_k/1$ Model

Assuming  $T_1 + T_2 \dots + T_k$  obey the same negative exponent distribution and they are independent, we can get their probability density as follows:

$$f(t_i) = k\mu e^{-k\mu t_i}, t_i \geq 0, i = 1, 2, \dots, k \quad (6)$$

Then  $T = T_1 + T_2 \dots + T_k$  obey the Erlang distribution with parameters  $k$  and  $\mu$ :

$$f(t) = \frac{(\mu k)^k}{(k-1)!} t^{k-1} e^{-k\mu t}, t \geq 0 \quad (7)$$

where  $k$  and  $\mu$  are positive and  $k$  is a round number.

Above all, if the service that the customers receive is a process with several periods in turn rather than a single one, and the time of every period obey the same negative exponent distribution, then the total service time obey the Erlang distribution. We can get its expectation and variance as follows:

$$E(t) = \frac{1}{\mu} \quad (8)$$

$$\sigma = \frac{1}{\sqrt{k}\mu} \quad (9)$$

We define a queueing model in a state of health: the input rate of the system is smaller than the output rate, that is, the average rate of arriving at airport is smaller than the rate of leaving, which also means that  $\lambda < \mu$ . In this case, the average queue length and the average waiting time of a single passenger can be estimated. For example, we use the time series of the passengers getting through the regular lane to estimate the input rate of a single queue

$$\lambda = \frac{1}{\bar{X}}$$

$$\bar{X} = \frac{\sum_{i=1}^N X_i}{N}$$

Using millimeter wave scan timing sequence to estimate the queue output rate

$$\lambda = \frac{1}{\bar{B}}$$

$$\bar{B} = \frac{\sum_{i=1}^N b_i}{N}$$

Get into the formula, get the average queue length and the average waiting time of a single passenger:



$$L_q = \frac{\lambda^2/k\mu^2 + \rho^2}{2(1-\rho)} = \frac{1+k}{2k} \cdot \frac{\lambda^2}{\mu(\mu-\lambda)} \quad (10)$$

$$W_q = \frac{1+k}{2k} \cdot \frac{\lambda}{\mu(\mu-\lambda)} \quad (11)$$

$L_q$  : the equation of the length of queues.

$W_q$  : the equation of the length of waiting time.

$\lambda$  : System input rate. We could use the reciprocal of the average value of the arrival time interval to estimate it.

$\mu$  : System output rate. We could use the reciprocal of the average time of getting through the Millimeter Wave Scan to estimate it. After calculating, we found that this model is consistent with the 7th order Erlang distribution.

#### 4.4 Take Pre-check into Consideration

For the program Pre-Check, as there are no output efficiency and time getting through the Millimeter Wave Scan, we have to make some assumptions. Firstly, the arrival time interval also obey the 7th order Erlang distribution. Secondly, the output efficiency is bigger than  $\mu$ .

Then We can obtain the expected queue length in the steady state. After calculating, we can get the expectation of queue length.

$$L'_q = \frac{\lambda'^2/k\mu'^2 + \rho^2}{2(1-\rho)} = \frac{1+k}{2k} \cdot \frac{\lambda'^2}{\mu'(\mu'-\lambda')} \quad (12)$$

$$W'_q = \frac{1+k}{2k} \cdot \frac{\lambda'}{\mu'(\mu'-\lambda')} \quad (13)$$

Where:

$L'_q$  is the average queue length of precheck lanes

$W'_q$  is the average wasting time of precheck lanes

$k = 7$

#### 4.5 Bottlenecks Analysis

Through the analysis above, we can find that there are two bottlenecks in the system:

- Since the efficiency of the whole system is determined by the system output efficiency, however, the system output rate is not enough, that is, the efficiency of passengers getting through the millimeter scan is limited Only

if we improve the output efficiency of the system can we check more passengers per unit time.

- Unreasonable distribution ratio of lanes. Since approximately 45% of passengers have enrolled in this program, almost account for half of the total number, although there are no output efficiency given, however, obviously, the efficiency of Pre-Check Lane is smaller than three times of the efficiency of Regular Lanes, so the ratio of one Pre-Check lane open for every three regular lanes is absolutely unreasonable. In order to solve this problem, increasing the total number of Pre-Check Lanes will work, and the exact number and modifications will be stated below.

## 5 Simulation Results and Analysis

According to Markov chain, when input rate is bigger than output rate, the state is of non-recurrence, the average queue length and the average wasting time don't exist. Thus to study the properties of the queue under such a state, we need to use computer programs to simulate the situations in reality. In this way, we can figure out the average length and average wasting time given a fixed time or fixed passenger. In addition, by comparing the simulation results and the former model results, we can get to know whether our simulation is consistent with the reality.

### Simulation in the state of health

we use computer programs to simulate the whole process and apply our modifications to the virtual process and obtain the following results.

For the state of health, assuming that the system capacity of 0 from the beginning of the simulation, we obtain the average queue length and average waiting time under different simulation time. We simulate the process for 20 times, the results are shown in the following table:

$W_{q0}$	$L_{q0}$	$W_{q0}$	$L_{q0}$
0.677085	5.02418	1.1124	8.83078
0.723112	5.44587	0.702563	5.12632
0.731401	5.35464	0.693245	5.05266
0.722835	5.40682	0.668199	4.87569
0.654154	4.85742	0.724631	5.44146
0.728794	5.43805	0.85262	6.4958
0.905095	6.85681	0.691886	5.1383
0.667775	4.87845	0.796	5.96653
0.630124	4.61113	0.635673	4.5555
0.608649	4.34158	0.926677	7.11041

According to the table, we can calculate that the average queue length and waiting time is 5.54042 persons and 0.7426459s respectively. Since the actual value of the average queue length and waiting time are 4.265762 persons and 0.9205753 respectively, We can get the conclusion that our simulation is consistent with the reality.

The results are in agreement with the theoretical data. In this case, the length of the queue and the waiting time can be estimated, that is, the airport queue can be in the best condition.

### Simulation in the state of unhealth

The state of unhealth refers to the state that the output rate is faster than the input rate. Under unhealthy state,  $W_q$  and  $L_q$  are negative. In other words, We can't estimate the length of lines and wasting time under the circumstance. Huge difference may exist in the wasting time among passengers depending on different arrival times.

During the rush hour, the input rate is faster than the output rate remarkably. Statistics show that approximately Atlanta airport has an average throughput of 80000 passenger on Friday, and the throughput during the rush hours of the busy day account for 1/10 of the daily throughput.

According to the data in reality, we assume that 8000 passengers arrive at the airport at the same time period and use computer programs to simulate the reality. We use different  $\lambda$  to reflect different input rate. Then the simulation can tell us the average length of the line, the average value of waiting time and the variance of waiting time at different input rate level. The simulation results was shown in the following form:

$\lambda$	Variance
0.13	9826343
0.12	7857062
0.11	5337220
0.105	3951764
0.1	1767126
0.095	562593.5
0.09	70044.22
0.085	2469.613
0.08	677.1242
0.075	273.4898
0.07	227.735
0.06	128.3813

We found that when  $\lambda < \mu = 0.08647$ , the variance of the average waiting time of a single passenger can be controlled in a very small range. However, when  $\lambda > \mu$ , that is, the input rate is greater than the output rate, the variance of the average waiting time will increase dramatically. Therefore, we can deter-

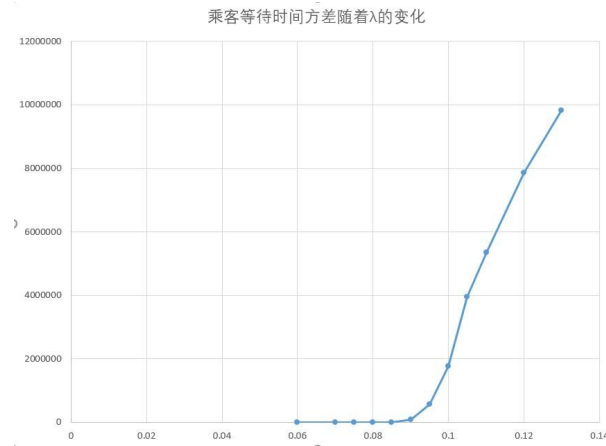


Figure 1: the variance of waiting time

mine that it is the input rate is greater than the output rate that mainly cause the existence of large variance of waiting time.

## 6 Complementary Model

In reality, many passengers need to remove there computers or take off their shoes and belts before they receive millimeter wave scan. This is easily to result in congest at the entrance of zone B, which is the main cause of low efficiency. So we select out the problem separately and set up a buffer area model. We use this model to quantify the effect of offering the passengers a zone to remove their own things.

### 6.1 Assumptions

- According to data analysis 6, we know that it takes a passenger 18 seconds to remove their personal things averagely. We assume the time of this behavior obey the exponential distribution with parameter  $\lambda = 1/18$ , that is,  $\xi \sim E(\lambda = 1/18)$
- To simplify the calculation of the model, we ignore the time of passing through the millimeter wave scan.
- We assume that if a passenger remove his metal items and leave the area, another passenger come into the passenger immediately.

### 6.2 the Effect on Different Sizes of Zones to Output Rate

Capacity of one person

As is shown in the following picture, if the area can only hold one person, then the efficiency of the single one can reflect the efficiency of the system. Then the expectation of the number of people passing through the site within unit time is  $\lambda$

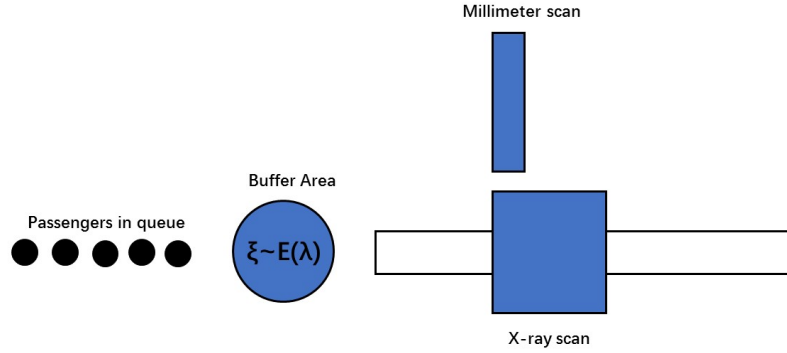


Figure 2: capacity of one person

### Capacity of two people

Given any time stamp  $t$ , the time they spend on the process obey the same exponential distribution with the parameter  $\lambda$  independent with each other:

$$\xi_1 \sim E(\lambda)$$

$$\xi_2 \sim E(\lambda)$$

Thus the time length  $Z$  of any of the two passengers leaving the area equals to the minor value between  $\xi_1$  and  $\xi_2$ . Since  $Z = \min(\xi_1, \xi_2)$ , We can prove that  $\xi_3 \sim E(2\lambda)$

Proof:

$$Z = \min(X, Y) \text{ , where } X, Y \sim E(\lambda)$$

$$\begin{aligned} \therefore F(Z) &= P(Z < z) \\ &= P(\min(X, Y) < z) \\ &= P(X < z, Y > x) + P(X < Y, Y < z) \\ &= \int_0^z \int_X^{+\infty} \lambda^2 e^{-\lambda(x+y)} dy dx + \int_0^z \int_Y^{+\infty} \lambda^2 e^{-\lambda(x+y)} dx dy \\ &= 1 - e^{-2\lambda z} \end{aligned}$$

$$\therefore Z \sim E(2\lambda)$$

Similar to this, if

$$\xi = \min\{\xi_1, \xi_2, \dots, \xi_k\}$$

where  $\xi_i \sim E(\lambda)$ ,  $i = 1, 2, \dots, k$

We have:

$$\xi \sim E(k\lambda)$$

Then the expectation of the number of people passing through the site within unit time is  $2\lambda$

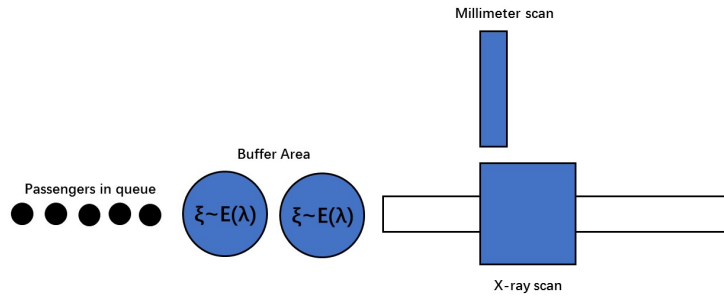


Figure 3: capacity of two people

### Capacity of n people

Similar to the last circumstance, we can derive that under the circumstance that the capacity of the zone is n people, Zobe the exponential distribution with the parameter  $n\lambda$ . Then the expectation of the number of people passing through the site within unit time is  $n\lambda$ .

## 7 Modifications to the Current Process

Based on the model we have built, we discover that low efficiency of the system is mainly caused by Zone B, So we make some modifications in Zone B from different aspects.

- **More Precheck lanes**

Based on our own collection, we can regard a Pre-check lane as an " $M/E_k/1$  model", whose parameters obey the following conditions:  $k = 7, \mu' = 1.25\mu$  Get Pre-check channel.

Based on equation (9) and (10), we are going to calculate the best ratio of lanes:  $m/n = L_q/L_q$  where n is the number of pre-chen lanes and m is the

number of regular lanes. We get that  $m/n = 2.09$ , that is, when one Pre-Check lane open for every two regular lanes, the whole system gets more efficient.

- **Pick out the passengers with potential danger**

Passengers were diverted after millimeter scan to increase the number of passengers per unit of time. According to data analysis 4, a small amount of passengers encumbers the whole system. So we can pick out the passengers with potential danger and send them to another zone to be checked separately.

Passengers are divided into two categories after the millimeter scan: about 85% passengers can directly get through the metal detector while the rest need additional checks. For passengers who need to undergo additional inspections, extra work will be done in Zone D, which makes the output rate of the whole queue system ( $\mu$ ) increased. For the data of those 85% people, after chi square test, we found that they accord with the 7th order Erlangian distribution whose parameter ( $\mu$ ) equals 0.09425. After calculating, we can get the expectation of queue length and the waiting time of a single passenger. ( $L_q = 2.1246$ ,  $W_q = 27.5110$ ) It can be seen that the output efficiency of the queuing system can be significantly improved by this method.

- **Only one queue before the entrance**

Assigning passengers to several queues is less efficient than only one queue. When there are multi-service windows with single queue waiting, the passenger reaching the front of this queue can be served as long as any window is available for them. In contrast, multi-queueing model is more ineffective. The number of people in different queues may be imbalanced, which may easily cause crowding in front of some counters while other counters are idle.[1]

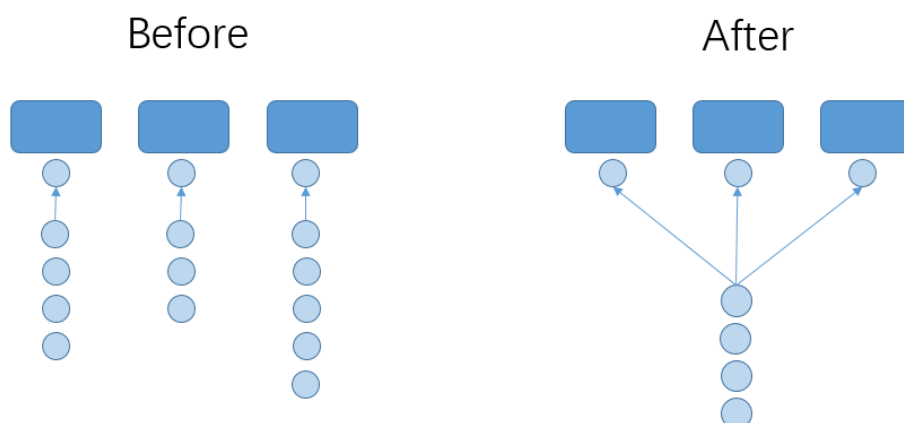


Figure 4: multi-queueing model and single-queueing

Also we can prove that in a mathematical way: It is assumed that several queues are subject to the same queue model and are independent of each

other. In the steady state, the idle probability of each service point is  $P_0$ . So the probability of that there is at least one service point is idle while the total number of queues is  $n$ :

$$P = \binom{n}{1} \cdot P_0 \cdot (1 - P_0)^{n-1}$$

In the new way, only if there is no one waiting in the queue, there can be at least one idle service point. In this case,

$$P' = \binom{n}{1} \cdot P_0 \cdot P_1^{(n-1)} < P$$

$P_1$  is the probability of only one person receiving service. Therefore, this improvement can reduce the idle probability of the system, so as to reduce the waste of resources.

- **Larger Buffer region** Increase the area of Buffer region before Zone B. We assume that the time getting through the millimeter scan equals constant  $P$ , then set the maximum capacity  $n$  must meet the following conditions:

$$1/(n\lambda) < p$$

$$n > 1/(p\lambda)$$

We take  $n$  as a larger value in order to ensure that the system in the face of some larger deviation of the value of a certain buffer capacity, to ensure that the whole system can have some adjustments for itself when facing the extreme situation.

## 8 Modeling the Cultural Difference

- Modelling the cultural difference

### (1) USA

Americans respect private space and public order, and they also have a more stringent security process when compared with other countries. But this may cause the inefficiency in security check. Especially when they are taking off their belts or shoes, they may need more space and time. So that we need a larger Buffer area to accommodate more people. Since Americans are not willing to cut in line, even if people in the front of the line have not yet completed taking off their clothes while people behind who have done all of the job will not beat them to get through the millimeter scan, which may cause low efficiency.

### (2) Nations of personal efficiency

Contrary to America, people in these countries are more likely to cut in line. This kind of culture makes the whole system more efficient. According to



the actual situation, Chinese people rarely miss their flights because of the long waiting time for security check.

- Improvements for USAs

We set a Buffer area as shown below. According to the Buffer area, all the passengers in this area can get checked at any time, and eliminates the pressure of cutting in line. At the same time, the passengers behind can also enter the area at any time.

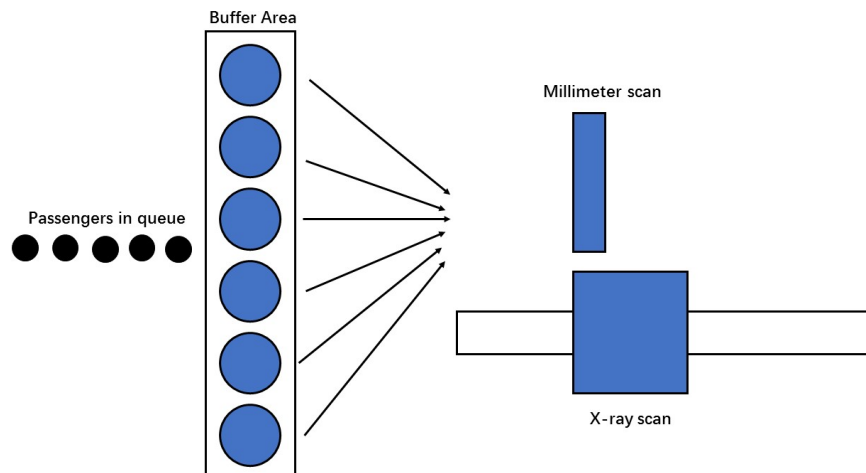


Figure 5: the Buffer area for America

## 9 Strengths and Weakness

### 9.1 Strengths

- The model we established:  $M/E_k/1$  model can accurately reflect the bottleneck of the queuing system, and help us to understand that when the input rate is greater than the output rate, the variance of waiting time may be extremely big.
- At the same time, our model helped to analyze how to set the most optimal buffer area, and how to set the best ratio of lanes. Finally, our model proved that the single queue model is more effective than the multi queue model.
- We effectively solved the doubts raised before, and gave the reasons for the bottleneck and the huge variance. Corresponding improvements are also stated.

## 9.2 Weaknesses

- The system input and output rate are treated ideally as exponential distribution and the 7 order Erlang distribution. However, in fact, the actual situation may not consist with our hypothesis.
- We regarded Zone B as the output of the whole system, while ignoring that there may also be some congestions in Zone C and Zone D, which may affect the accuracy of the model.
- We didn't take the changes in the parameters of input rate distribution into consideration.

## 10 Policy and Procedural Recommendations for the Security Managers

- Bottleneck analysis

Based on the analysis above, we know that the final purpose is to improve the output efficiency of the whole queuing system. We can reduce the total number of carrying baggage for each passenger, and make passengers with no baggage quickly get through the queuing system to achieve the final purpose.

- Policies proposed

On the one hand, increase the maximum weight of passengers' free consigned baggage, in the other hand, reduce excessing charges. In this way, more passengers will be encouraged to consign their baggage, thereby reducing the amount of work required to get through the security check.

Open the exclusive lanes for baggage free passengers This policy will enable these passengers get through the security check even more quickly, thereby improving the throughput of the whole system. At the same time, passengers will also be encouraged to carry less baggage or consign their luggage more often, thereby improving the efficiency of the whole system.

In summary, we offer the following advice to security managers:

1. According to the number of expected passengers, control and make the security check rate equal to passengers' arrival rate, by the ways of increasing or reducing staffing and security check lanes.
2. Set up a security check lane for passengers with no bags and reduce the charge for consigning their luggage. Airline companies should also increase the weight of free-charge consigned luggage. All these policies are to encourage passengers to consign their luggage as much as possible, and

then lead to a decrease of the time wasted when passenger going through baggage check.

3. Adjust the ratio of number of pre-check lanes to the number of regular security check lanes according to actual condition and models mentioned above, try to match their average queue length.
4. Set up a buffering area in X-ray check site, to reduce the time wasted when passengers taking off and putting on their clothes and shoes.

## References

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