Python iterated functions analysing Switzerland's Covid-19 pandemic

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ABSTRACT

The novel coronavirus (COVID-19) was first reported at the end of 2019 and has impacted almost every aspect of life as we know it. This study models data for positive covid-19 cases and deaths in Switzerland—one of the high performing countries in the world. Using two iterative map models—the exponential map model and the logistic map model, we model the daily and cumulative incidence of COVID-19 in Switzerland during the early stage of the outbreak up to July 2021. None of the peaks in the data used fits well with the 3-dimensional iterative map model and the predictive ability of the logistic map model was found to give a better fit.

1 Introduction

The novel coronavirus (COVID-19) was widely reported to have first been detected in Wuhan (Hebei province, China) in December 2019. After the initial outbreak, COVID-19 continued to spread to all provinces in China and very quickly spread to other countries within and outside of Asia. At present, 226 844 344 cases of infected individuals have been confirmed globally with 4 666 334 deaths and 5 634 533 040 vaccine doses administered[1]. Although the foundations of this disease are very similar to the severe acute respiratory syndrome (SARS)[2] virus that took hold of Asia in 2003, it is shown to spread much more easily because majority of infected people do not experience severe symptoms, thus making it more likely for them to remain mobile, and hence to infect others[3]. New cases and deaths have seemed to slow down recently with the implication of lockdown measures and availability of vaccines in almost all countries globally.

At the time of this writing Switzerland had 226 844 344 confirmed cases with 4 666 334 deaths and 5 634 533 040 vaccines administered[1]. They have been one of the countries to successfully manage the pandemic especially the first phase and have been voted as the safest country in the world for Covid-19 around June 2020 in a 200-country study by Hong-Kong based Deep Knowledge Group.They owe that to the careful ways in which they are attempting to relax lockdown and economic freezing mandates in a fact and science-based manner, without sacrificing public health and safety[4]. The following were some of the activities authorised under their lockdown measures (i) classroom courses in compulsory schools (primary and secondary I levels) (ii) classroom courses for groups of up to 5 persons (including the teacher) in upper secondary schools, tertiary schools and other educational institutions (driving school, language courses). Non-contact amateur sport activities practised individually or in groups of not more than five persons (including the coach, trainer or supervisor); including the use of facilities and establishments necessary for this purpose, provided that protection plans exist and are respected, Training of competitive athletes who are members of a national framework / national sports federation or who train individually, in a group of up to five people (including the coach, trainer or supervisor) or in a fixed group (elite athletes); protection plans exist and are respected;Training, with physical contact, of members of teams belonging to a league whose competitions are mainly professional (elite athletes); they must comply with strict hygiene rules set out in a protection plan[5].

This study is based on the simple discrete 3-dimensional iterative map model which shares some similarities with the classic SIR model[6] and focuses solely on Switzerland. We show that due to the measures explained above used by the country to manage Covid-19, only a small portion of the data collected from the World Health Organisation (WHO) agrees (to some extent) with the 3-dimensional exponential iterative map. Further, we use the same model to show that the logistic map model provides a good fit for Switzerland's Covid-19 data. The latter agrees with the results shown for Switzerland in Ref.[7].

2 Some background on iterative maps

The properties of one-dimensional iterative maps have been extensively studied and are by now very well understood. An one-dimensional iterative map is of the form

$$x_{n+1} = f(x_n) \tag{1}$$

where f is some function that acts on the nth iterand to produce the $(n=1)$th iterand. A classic example of such a map is the logistic map, for which

$$x_{n+1} = rx_n(1-x_n), \tag{2}$$

where x_n is a number between zero and one that represents the ratio of existing population to the maximum possible population. Logistic model is mainly used in epidemiology. It is commonly used to explore the risk factors of a certain disease, and predict the probability of occurrence of a certain disease according to the risk factors. We can roughly predict the development and transmission law of epidemiology through logistic regression analysis

$$Q_t = \frac{a}{1+\exp^{b-c(t-t_0)}}, \tag{3}$$

where Q_t is the cumulative confirmed cases/deaths; a is the predicted maximum of confirmed cases/deaths; b and c are fitting parameters; t is the number of days since the first case and t_0 is the time when the first case occurred[8].

Another very simple one-dimensional iterative map is given by

$$x_{n+1} = \alpha x_n, \tag{4}$$

For values of $\alpha > 1$ this leads to exponential growth in x_n as the map is iterated. The exponential map is also known as the Gompertz model (Gompertz,1825) and is widely used as an animal population growth model to describe the extinction law of the population. The development of infectious diseases is similar to the growth of individuals and populations, hence, can be used to model Covid-19. For regression analysis it is given by the following equation

$$Q_t = ae^{-be^{-c(t-t_0)}}, \tag{5}$$

where Q_t is the cumulative confirmed cases/deaths; a is the predicted maximum of confirmed cases/deaths; b and c are fitting parameters; t is the number of days since the first case and t_0 is the time when the first case occurred[8].

3 Method

All data related to COVID-19 was downloaded from the publicly available JHU-CSSE (2020) data source provided continuously by the Johns Hopkins University Center for Systems Science and Engineering (JHU CSSE)[9] and stored in a csv file where it was then extracted using a Python script. For the purpose of this report, the data covered the dates from the 22^{nd} January 2020 (as Day 1 of the pandemic) up to the 29^{th} of July 2021 (Day 553).

The Levenberg–Marquardt (least squares) optimisation was then used to optimise the fitting parameters $alpha$ and r for the exponential and logistic models, respectively. The data used for the exponential map has been given in Figure1 and the data used for the logistic map has been given in Figure2.

4 The Levenberg–Marquardt (least squares) Method

The Levenberg-Marquardt least squares method is a mathematical optimization technique. It finds the best function match of data by minimizing the sum of squareed errors. Using the least squares method, unknown data can be easily obtained, and the sum of squares of errors between these obtained data and actual data is minimized.

It uses the regression coefficient R_2 to evaluate the fitting ability of various modelling methods and can be obtained by the following equation

$$R^2 = 1 - \frac{\sum(y_i-\hat{y}_i)^2}{\sum(y_i-\bar{y})^2}, \tag{6}$$

where y_i is the actual cumulative confirmed Covid-19 cases; $\hat{y}_i$ is the predicted cumulative confirmed COVID-19 cases, and $\bar{y}$ the average of the actual cumulative confirmed Covid-19 cases[10]. Here a fitting parameter that is much closer to 1 gives a better prediction.

The model equations used in this study have been explained in detail in Ref.[6] where in equation 1 the first equation updates the total number of cases by setting it equal to the previous total number of cases, plus the number of new cases; in the second equation the number of new cases is assumed to be proportional to the previous number of new cases multiplied by the previous number of susceptible people (people not yet exposed to Covid-19); and the third equation is used to keep track of the global population, by subtracting the estimated number of people who have died each day, based on the fertility rate c. The fertility rate in Switzerland is estimated to be about 1.3% with an estimated population of 58 million people in 2021.

180	33634	108	1
181	33742	141	0
182	33883	117	3
183	34000	154	2
184	34154	148	0
185	34302	110	0
186	34412	65	1
187	34477	132	0
188	34609	193	1
189	34802	220	1
190	35022	210	1
191	35232	180	0
192	35412	138	0
193	35550	66	0
194	35616	130	0
195	35746	181	3
196	35927	181	1
197	36108	161	1
198	36269	182	0
199	36451	152	0
200	36603	105	1

201	36708	187	3
202	36895	274	1
203	37169	234	0
204	37403	268	0
205	37671	253	0
206	37924	200	0
207	38124	128	0
208	38252	197	1
209	38449	311	4
210	38760	266	2
211	39026	306	2
212	39332	295	0
213	39627	276	1
214	39903	157	0
215	40060	202	1
216	40262	383	1
217	40645	361	0
218	41006	340	1
219	41346	376	1
220	41722	292	0

221	42014	163	1
222	42177	216	5
223	42393	370	0
224	42763	364	2
225	43127	405	0
226	43532	425	0
227	43957	444	0
228	44401	191	1
229	44592	245	4
230	44837	469	1
231	45306	405	1
232	45711	528	0
233	46239	465	0
234	46704	475	1
235	47179	257	4
236	47436	315	3
237	47751	514	11
238	48265	530	3
239	48795	488	3
240	49283	0	0

Figure 1. Data used for the Levenberg–Marquardt (least squares) optimisation of the parameter $alpha$ of the 3-D model given in Ref.[6] to show exponential fit. The first columns represent the Day number (column 1), the total number of cases (column 2), the number of new cases (column 3), and the total number of deaths (column 4).

60	7474	1321	22
61	8795	1082	2
62	9877	1020	31
63	10897	914	38
64	11811	1117	40
65	12928	1148	33
66	14076	753	36
67	14829	1093	59
68	15922	683	74
69	16605	1163	55
70	17768	1059	48
71	18827	779	55
72	19606	899	75
73	20505	595	49
74	21100	557	50
75	21657	596	56
76	22253	1027	74
77	23280	771	53
78	24051	500	54
79	24551	556	34
80	25107	308	70
81	25415	273	32
82	25688	248	36
83	25936	400	65
84	26336	396	42
85	26732	346	46
86	27078	326	41
87	27404	336	25
88	27740	204	36
89	27944	119	49
90	28063	205	31

91	28268	228	40
92	28496	181	40
93	28677	217	10
94	28894	167	11
95	29061	103	55
96	29164	100	34
97	29264	143	17
98	29407	179	21
99	29586	119	17
100	29705	112	8
101	29817	88	0
102	29905	76	22
103	29981	28	11
104	30009	51	10
105	30060	66	5
106	30126	81	13
107	30207	44	7
108	30251	54	3
109	30305	39	12
110	30344	36	22

111	30380	33	3
112	30413	50	2
113	30463	51	6
114	30514	58	1
115	30572	15	2
116	30587	10	5
117	30597	21	5
118	30618	40	1
119	30658	36	6
120	30694	13	5
121	30707	18	2
122	30725	11	1
123	30736	10	7
124	30746	15	2
125	30761	15	2
126	30776	20	2
127	30796	32	0
128	30828	17	0
129	30845	17	1
130	30862	9	0

Figure 2. Data used for the Levenberg–Marquardt (least squares) optimisation of the parameter r of the 3-D model given in Ref.[6] to show logistic fit. The first columns represent the Day number (column 1), the total number of cases (column 2), the number of new cases (column 3), and the total number of deaths (column 4).

5 Results

We have analysed how the data given in Table1 and Table2 fits with the 3-D iterative map in Ref. citebot20. The results have been shown in Figure3 and Figure4, respectively. There were many instances where there were no new Covid-19 daily cases/deaths reported, and thus the cope of the analysis were over short periods of time. Since Switzerland's data is chaotic in nature, it becomes vividly clear that the model better predicts how the decease is going to continue is the logistic model. Several studies made before modelling Covid-19 in Switzerland have all seemed to agree with the fact that the logistic model gives a better prediction.

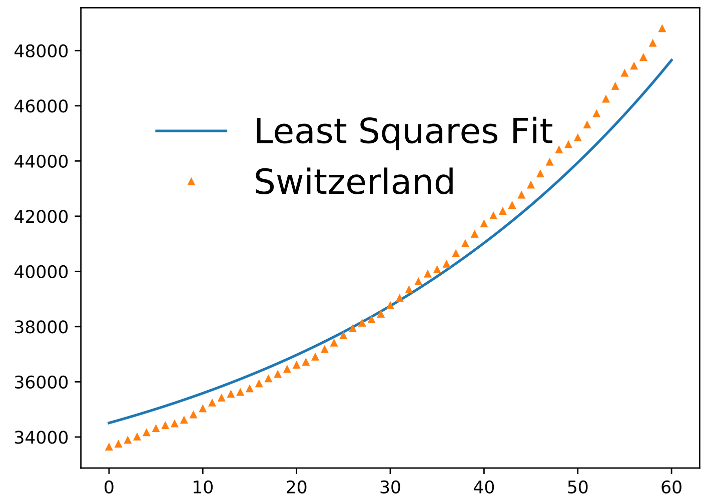

Figure 3. A comparison of the data with the 3-D iterative map model showing how the model fits the Switzerland's data for the total number of confirmed cases x_i and new cases y_i.

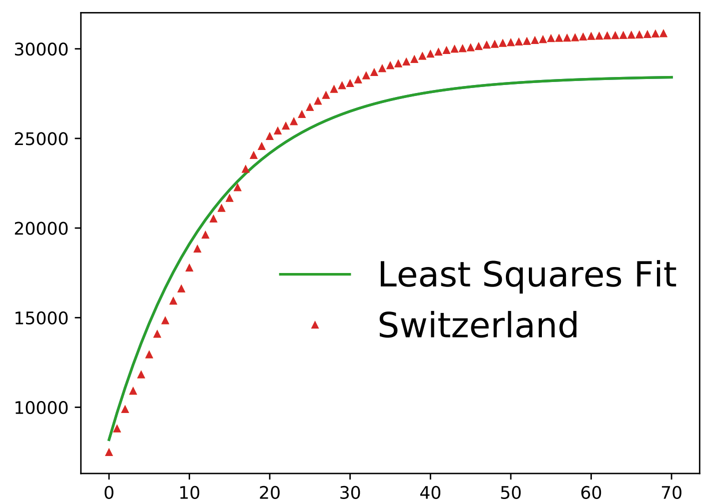

Figure 4. A comparison of the data with the 3-D iterative map model showing how the model fits the Switzerland's data for the total number of confirmed cases x_i and new cases y_i.

6 Conclusion

The data for covid-19 cases is not always very accurate. In particular, if one looks at the number of excess deaths, for example, one notices that the pandemic has caused many more deaths than are being reported. However, based on our prediction which suggest logistic growth in the decease and the public awareness as well availability of resources in Switzerland. These include government funding measures and the availability of vaccines, we predict that the spread of the decease will come to a stop very soon in Switzerland. Their highest peak which was in the second phase of Covid-19 has only had about 10 000 new reported daily cases.

Appendix A: Extra material

Most mathematical expressions and symbols used in this report were done with the help of this website[11].

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