

Fitch Group

CQF Module 1 Lecture 4

Introduction to Stochastic Calculus

Stochastic Calculus and Itô's Lemma

- * Construction of a Brownian motion
 - * Properties
 - * Calculus for a fn. of a B.M.
 - * Develop rules
 - * Look at how to simulate a B.M. in Excel
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By the end of this lecture you will be able to

- understand where Brownian motion and diffusion processes come from
 - manipulate functions of random variables (simple cases)
 - simulate a B.M. sample path
-

Introduction



Stochastic calculus is very important in the mathematical modeling of financial processes. This is because of the underlying (assumed) random nature of financial markets.

Stoch. process

$$\left\{ Y_t : t \in \mathbb{R}^+ \right\}$$

Stochastic calculus: A motivating example

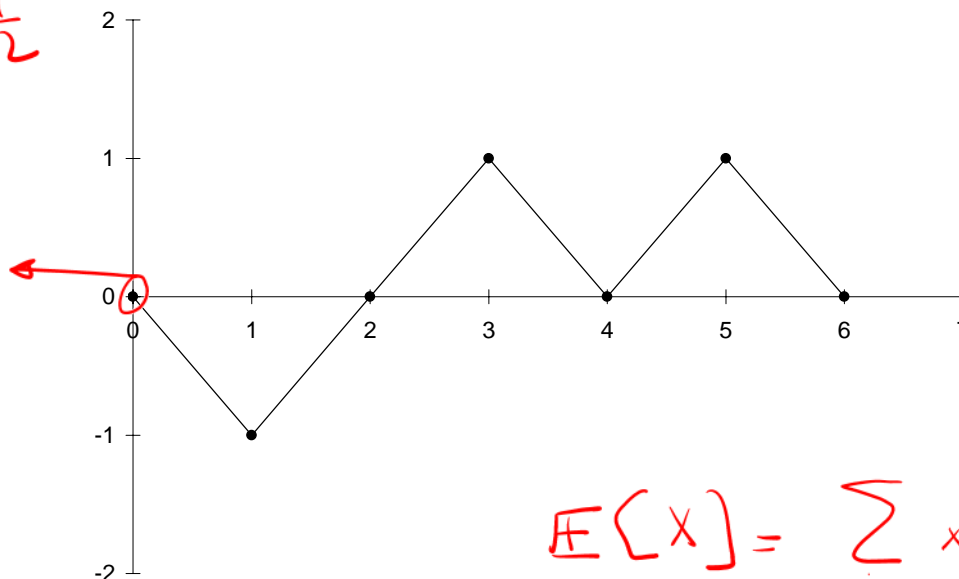
Toss a ^{fair} coin. Every time you throw a head you receive \$1, every time you throw a tail you pay out \$1. In the experiment below the sequence was THHTHT, and you finished even.

$$P(H) = P(+1) = \frac{1}{2}$$

$$P(T) = P(-1) = \frac{1}{2}$$

$$R_i = \begin{cases} +1 & H \\ -1 & T \end{cases}$$

important
condition



$$E[X] = \sum_{i=1} x_i f_i = \mu$$

$$V[X] = \sum_{i=1} x_i^2 f_i - \mu^2$$

- R_i denotes the random amount, either \$1 or -\$1, you make on the i th toss:

$$E[R_i] = 0, \quad E[R_i^2] = 1 \quad \text{and} \quad E[R_i R_j] = 0. \quad \text{indep.}$$

$E[R_i] = (+1)\frac{1}{2} + (-1)\frac{1}{2} = 0$
 $E[R_i^2] = (+1)^2\frac{1}{2} + (-1)^2\frac{1}{2} = 1$

In this example it doesn't matter whether or not these expectations are conditional on the past. In other words, if you threw five heads in a row it does not affect the outcome of the sixth toss.

new R.V

- Introduce S_i to mean the total amount of money you have won up to and including the i th toss so that

$$S_i = \sum_{j=1}^i R_j \quad \text{subject to } S_0 = 0$$

Later on it will be useful if we have $S_0 = 0$, i.e., you start with no money.

$$E\left[\sum_{j=1}^i R_j\right] = \sum_{j=1}^i E[R_j] = i \times 0 = 0$$

If we now calculate expectations of S_i it *does* matter what information we have. If we calculate expectations of future events before the experiment has even begun then

$$E[S_i] = 0 \quad \text{and} \quad E[S_i^2] = E[R_1^2 + 2R_1R_2 + \dots] = \underline{i}.$$

On the other hand, suppose there have been five tosses already, can we use this information and what can we say about expectations for the sixth toss?

$$E[S_i] = E\left[\sum_{j=1}^i R_j\right] = i \times 1 = i$$

• This is the **conditional expectation**.

$$E\left[\left(\sum_{j=1}^i R_j\right)^2\right] = E\left[R_1^2 + \dots + R_i^2 + 2R_1R_2 + \dots\right]$$

$i \times 1 = i$ $\rightarrow 0$

Variation $V^N = \sum_{j=1}^N (S_j - S_{j-1})^2$

Quadratic variation

$N=2$ Quadratic variation
Sum of squared changes Q or V^2

The **quadratic variation** of the random walk is defined by

$$Q = \sum_{j=1}^i (S_j - S_{j-1})^2.$$

Because you either win or lose an amount \$1 after each toss, $\underbrace{|S_j - S_{j-1}|}_{=1} = 1$. Thus the quadratic variation is always i :

Issue with this

game is the
bet size

$\lim_{i \rightarrow \infty}$

means both infinite variance and variation

$$\sum_{j=1}^i \underbrace{(S_j - S_{j-1})^2}_{(\pm 1)^2} = i.$$

We are going to use the coin-tossing experiment for one more demonstration. And that will lead us to a continuous-time random walk.

Introduce time dep: we play the game in time t .

Brownian motion

Change the rules of the coin-tossing experiment slightly.

We still start with zero winnings (certain)

First of all restrict the time allowed for the six tosses to a period t , so each toss will take a time $t/6$. Second, the size of the bet will not be \$1 but $\sqrt{t/6}$.

$$P(\sqrt{t/6}) = \frac{1}{2}$$
$$P(-\sqrt{t/6}) = \frac{1}{2}$$

The quadratic variation measured over the whole experiment is

$$Q = \sum_{j=1}^6 \underbrace{(S_j - S_{j-1})^2}_{6 \times \left(\pm \sqrt{\frac{t}{6}}\right)^2} = 6 \times \left(\sqrt{\frac{t}{6}}\right)^2 = t.$$

finite

Change the rules again, to speed up the game.

$$n \gg 6$$

- We will have n tosses in the allowed time t , with an amount $\pm \sqrt{t/n}$ riding on each throw.

The quadratic variation is still

$$\text{Time} = \frac{t}{n} = t \times \frac{1}{n} \rightarrow o\left(\frac{1}{n}\right) \leftarrow$$

$$\text{bet size} = \sqrt{t} \times \frac{1}{\sqrt{n}} \rightarrow o\left(\frac{1}{\sqrt{n}}\right) \sum_{j=1}^n \underbrace{(S_j - S_{j-1})^2}_{\left(\pm \sqrt{t/n}\right)^2} = n \times \left(\sqrt{\frac{t}{n}}\right)^2 = t. \leftarrow \text{finite time}$$

Now make n larger and larger. This speeds up the game, decreasing the time between tosses, with a smaller amount for each bet. But the new scalings have been chosen very carefully, the time step is decreasing like n^{-1} but the bet size only decreases by $n^{-1/2}$.

$$\pm \sqrt{\frac{t}{n}}$$

$$\pm \sqrt{dt}$$

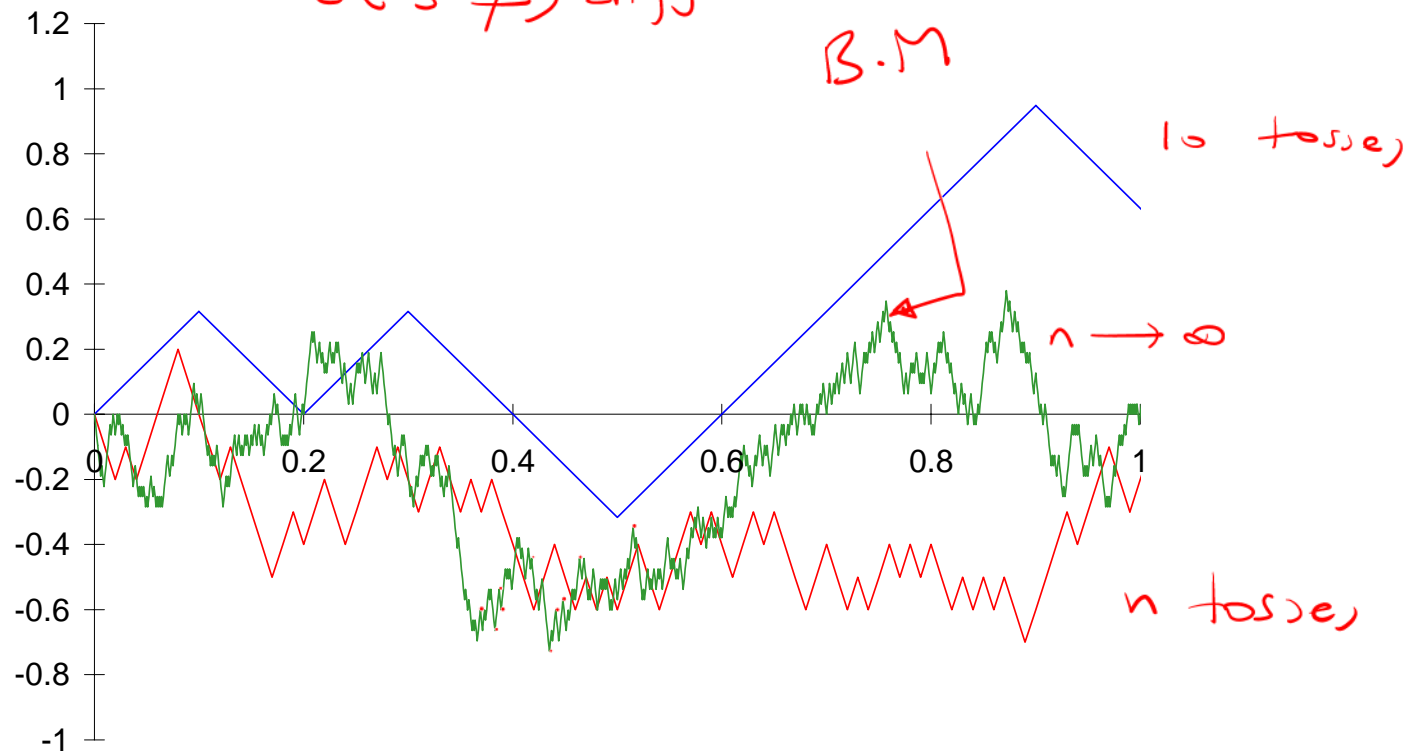
$$t$$

$$\sqrt{\frac{t}{n}}$$

$$-\sqrt{\frac{t}{n}}$$

diff \Rightarrow cts

cts \nRightarrow diff



$$f'(x) = \lim_{\Delta x \rightarrow 0}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

provided limit exists (and is unique)

A series of coin tossing experiments.

$$\phi \sim N(0,1) \text{ scale with } \sqrt{dt}$$

As we go to the limit $n = \infty$, the resulting random walk stays finite. It has an expectation, conditional on a starting value of zero, of $\mathbb{E}[dX] = 0$

$$\mathbb{V}[dX] = dt \quad E[S(t)] = 0 \quad dX \sim N(0, dt)$$

and a variance

$$\mathbb{E}[\phi \sqrt{dt}] = \sqrt{dt} \mathbb{E}[\phi] = 0$$

$$E[S(t)^2] = t. \quad \mathbb{V}[\phi \sqrt{dt}] = dt \mathbb{V}[\phi] = dt \times 1$$

We use $S(t)$ to denote the amount you have won or the value of the random variable after a time t .

W_t Wiener process

B_t

- The limiting process for this random walk as the time steps go to zero is called **Brownian motion**, and we will denote it by $X(t)$.

$$\{X_t : t \in \mathbb{R}^+\} \quad \mathbb{E}[X_t] = 0 \quad \mathbb{V}[X_t] = t$$

Standard B.M

$$1. X_0 = 0 \quad \text{a.s.}$$

$$2. t \mapsto X_t \text{ is continuous}$$

$$3. X_t - X_{t_i} \text{ is a Gaussian}$$

$$\text{a.s. } \phi \sqrt{dt} \quad N(0,1)$$

4. Increments of a B.M are indep.

$$dX_t = X_{t_1} - X_{t_0} \text{ indep. } \rightarrow dX_{t_2} = X_{t_2} - X_{t_1}$$

Having built up the idea and properties of Brownian motion from a series of experiments, we can discard the experiments, to leave the Brownian motion that is defined by its properties. These properties will be very important for our financial models.

$X_{t_i} - X_{t_{i-1}}$ is indep of $X_{t_{i-1}} - X_{t_{i-2}}$

$$dX = X_{t+dt} - X_t \sim N(0, dt)$$

mean
variance

Let $t > s$

$$X_t - X_s \sim N(0, |t-s|)$$

trans pdf for $X_t - X_s$

$$\frac{1}{\sqrt{2\pi|t-s|}} \exp \left[-\frac{(x_t - x_s)^2}{2|t-s|} \right]$$

Very Important Notation

We have seen X as the 'end result' of a random walk, up to some time t .

We will often work with the amount by which X changes from moment to moment.

- Think of dX as being an increment in X , i.e. a Normal random variable with mean zero and standard deviation $dt^{1/2}$.

Consider a naive approach here. Suppose we try and apply everyday diff to X_t

$$\frac{dX_t}{dt} = \lim_{dt \rightarrow 0} \frac{X(t+dt) - X(t)}{dt} = \frac{dX_t}{dt} \quad \frac{\sqrt{dt}}{dt} = \frac{1}{\sqrt{dt}}$$

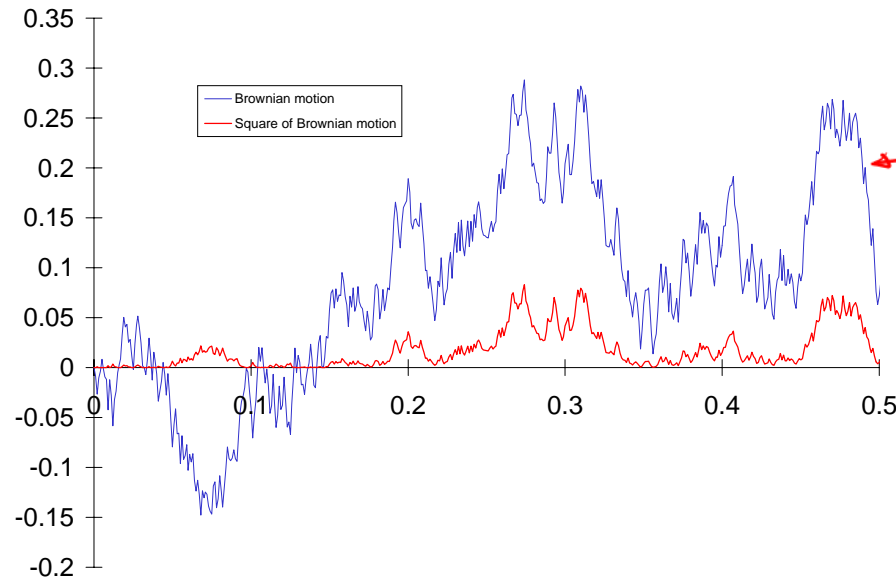
$\lim_{dt \rightarrow 0} \frac{1}{\sqrt{dt}}$ does not exist

Functions of stochastic variables and Itô's lemma

Now we'll see the idea of a function of a stochastic variable. Below is shown a realization of a Brownian motion $X(t)$ and the function $F(X) = X^2$.

$$F(x) = e^x$$

$$F(x) = \sin x$$



→ we just simulated
this in
Excel

Whenever we have functions of a variable it is natural to want to know how to differentiate and manipulate these functions.

What are the rules of calculus when variables are stochastic?

X_t is now our stochastic var. and we
say $F = F(X_t)$ is a fn. of a stoch. variable
↑

The first point to note is that in the stochastic world we really have two 'variables.'

These are time t and the Brownian motion X . $F(X_t)$

t is implicitly defined as part of X

$\therefore F = F(X)$ or $F = F(X_t)$; F is a fn. of one variable

We are used to writing ordinary and partial differential equations in the form

$$\frac{dF}{d\cdot}$$

or

$$\frac{\partial F}{\partial \cdot}$$

where the quantities on the bottom are the independent variables.

So might expect something similar in the stochastic world.

Note:

We immediately hit a problem, however.

Because dX is of size \sqrt{dt} it is much bigger than dt .

$$\sqrt{dt} > dt$$

This means that we have to be careful whenever we think about gradients/slopes/derivatives/sensitivities, since these are limits as dt goes to zero.

$$\text{If } dX \sim O(\sqrt{dt})$$

$$\lim_{dt \rightarrow 0} dX^2 \rightarrow dt \quad \text{or} \quad \mathbb{E}(dX^2) = dt$$

Ho Mult
tasc

x	dt	dX
dt	dt^2 ~ 0	$dt^{3/2} < dt$ ~ 0
dX	$dt^{3/2}$ ~ 0	dX^2 $= dt$

	dt	dX
dt	0	0
dX	0	dt

For this reason, in the stochastic world we instead work with stochastic differential equations.

These take the form

differential $\underbrace{dF = \dots dt + \dots dX.}_{\text{O.D.E.}}$

$$dF = A dt \quad \text{O.D.E.}$$

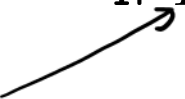
A. S.D.E. is an O.D.E. + a random cpt.

$$\rightarrow dF = A dt + \boxed{\beta dx} \quad \text{random cpt.}$$

So, what are the rules of calculus?

Since X is stochastic, so is F , and we can ask 'what is the stochastic differential equation for F ?'

If $F(X) = X^2$ what is the equation for dF ?


$$\frac{dF}{dX} = 2X \Rightarrow dF = 2X dX \text{ in everyday calculus}$$

But what about when X represents a B.M.?

If $F = X^2$ is it true that $dF = 2X dX$?

No.

- The ordinary rules of calculus do not generally hold in a stochastic environment.

Then what are the rules of calculus?



We are going to throw caution to the wind, pretend that there are no problems or subtleties, use Taylor series... and see what happens!



Taylor Series ... and Itô

If we were to do a naive Taylor series expansion of F , completely disregarding the nature of X , and treating dX as a small increment in X , we would get

T.S.E. $F(X + dX) = F(X) + \frac{dF}{dX}dX + \frac{1}{2}\frac{d^2F}{dX^2}dX^2,$

ignoring higher-order terms.

dF differential

We could argue that $F(X + dX) - F(X)$ was just the 'change in' F and so

$\boxed{Itô I}$

$$dF = \frac{dF}{dX}dX + \frac{1}{2}\frac{d^2F}{dX^2}dX^2.$$

$\nearrow dt$

This is *almost* correct.

most basic form

use for fns. like X_t^{\wedge} , e^{X_t} , $\sin X_t$, $\log X_t$

Because of the way that we have defined Brownian motion, and have seen how the quadratic variation behaves, it turns out that the dX^2 term isn't really random at all.

The dX^2 term becomes (as all time steps become smaller and smaller) the same as its average value, dt .

Taylor series and the ‘proper’ Itô are very similar. The only difference being that the correct Itô’s lemma has a dt instead of a dX^2 .

In 'everyday' calculus if $\Delta x \ll 1$ then Δx^2 is negligible

- You can, with little risk of error, use Taylor series with the 'rule of thumb'

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\boxed{dX^2 = dt.}$$

Now dX^2 cannot be ignored \therefore
 $dX^2 \sim O(dt)$

and in practice you will get the right result.

$$\mathbb{E}[dX^2] = dt$$

$$1 \times dt$$

Let's get some intuition now, and then shortly we will do Itô's lemma properly!

$$dF = \frac{dF}{dX} dX + \frac{1}{2} \frac{d^2 F}{dX^2} dt$$

$$dF = \underbrace{2X dX}_{\text{drift}=1} + \underbrace{\sqrt{dt}}_{\text{time-step}}$$

$$\boxed{F = X^2}$$

$$\frac{dF}{dX} = 2X$$

$$\frac{d^2 F}{dX^2} = 2$$

$$dF = A dt + B dX$$

time-step \leftarrow

$\rightarrow \sqrt{\text{time-step}}$

We can now answer the question, "If $F = X^2$ what is dF ?" In this example

$$\frac{dF}{dX} = 2X \quad \text{and} \quad \frac{d^2F}{dX^2} = 2.$$

Therefore Itô's lemma tells us that

$$dF = dt + 2X dX.$$

This is an example of a **stochastic differential equation**.

Now suppose $F = F(t, X_t)$ $t \rightarrow t + dt$ $X_t \rightarrow X_t + dX_t$ 2D TSE

$$F(t+dt, X_t+dX_t) = F(t, X_t) + \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X_t} dX_t + \frac{1}{2} \frac{\partial^2 F}{\partial X_t^2} dX_t^2 \rightarrow dt$$


$$dF = \left(\frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial X_t^2} \right) dt + \frac{\partial F}{\partial X_t} dX_t$$

$$\log t + \sin X_t$$

Stochastic differential equations

Stochastic differential equations are used to model random quantities, a stock price for example.

They have two parts to them, a **deterministic** and a **random**.



The diagram illustrates the decomposition of a stochastic process into a deterministic drift component and a random diffusion component. A red jagged line on the left is equated to a red smooth curve in the middle plus a green jagged line on the right.

$$dF = \underbrace{A(F, t) dt}_{\text{drift/growth, det.}} + \underbrace{B(F, t) dX_t}_{\text{diffusion/volatility, random}}$$

SDE for F

Suppose we want to model a stock price as a random quantity. Let's use S to denote that stock price.

A stochastic differential equation for S would look something like this:

$$dS = \overset{\mu S}{\text{Deterministic}} dt + \overset{\sigma S}{\text{Random}} dX$$

In words: "The change in the stock price has a predictable component and a random component."

More precisely

$$dS = \text{Something } dt + \text{Something else } dX.$$

The randomness is captured by the dX term.

But what are these 'somethings' ?

In the standard models they would be functions of S and time, t .

$$dS = \overset{\mu S}{f(S, t) dt} + \overset{\sigma S}{g(S, t) dX}.$$

The function $f(S, t)$ captures how the predictable bit of the stock model varies with S and t and the $g(S, t)$ function captures the randomness.

↪ vol.

↗ trend

$$dS = f(S, t) dt + g(S, t) dX.$$

We sometimes call the $f(S, t)$ function the **growth rate** or the **drift**.

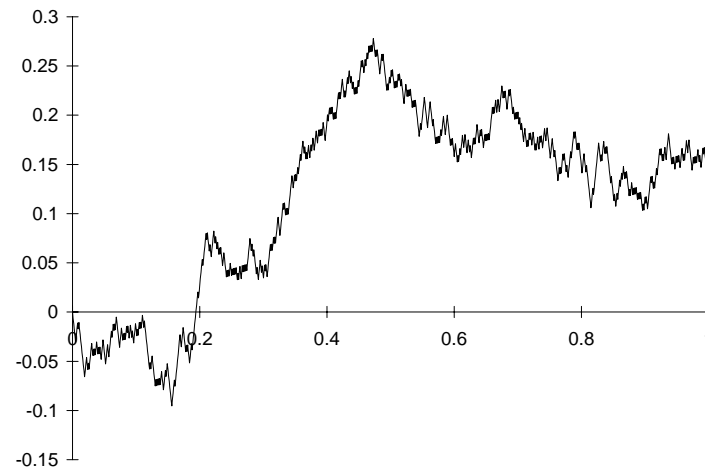
The $g(S, t)$ is related to the **volatility** of S .

Some pertinent examples

The first example simple Brownian motion but with a drift:

A.B.M

$$dS = \mu dt + \sigma dX.$$



In this realization S has gone negative.

Our second example is similar to the above but the drift and randomness scale with S :

G.B.M

→

$$dS = \mu S dt + \sigma S dX.$$

To appreciate the dynamics of a SDE we can do so in the absence of randomness. That is set $\sigma=0$ to "switch-off" randomness,

$$dS = \mu S dt$$

$$\int \frac{dS}{S} = \mu \int dt$$

$$\log S = \mu t + C$$

$$S(t) = \underline{A} e^{\mu t}$$

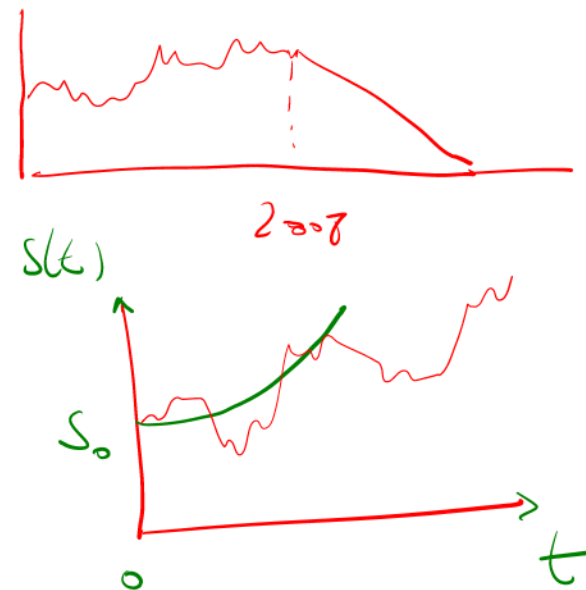
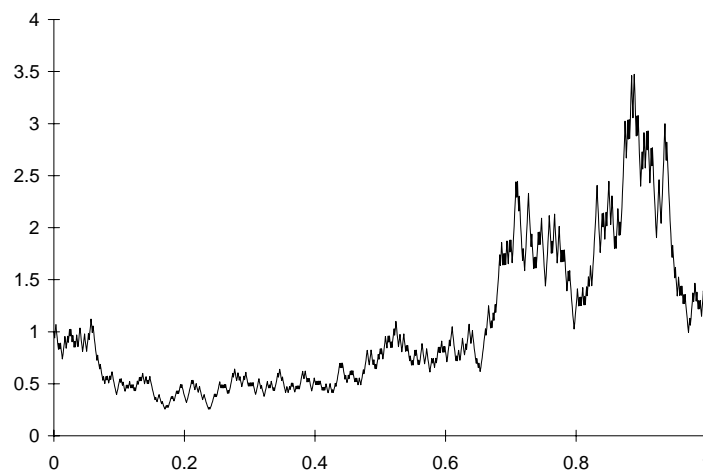
$$S(t) = A e^{rt}$$

$$S(0) = S_0$$

$$\mu = r$$

$$A = S_0$$

$$S(t) = \underline{S_0 e^{rt}}$$



If S starts out positive it can never go negative; the closer that S gets to zero the smaller the increments dS .

The mean square limit

This is useful in the precise definition of stochastic integration.

Examine the quantity *defⁿ*

$$E \left[\left(\sum_{j=1}^n (X(t_j) - X(t_{j-1}))^2 - t \right)^2 \right] \rightarrow 0$$

where

$$t_j = \frac{jt}{n}.$$

$$E \left[\sum (\quad)^2 \right] = t$$

This can be expanded as

$$\begin{aligned} & E \left[\sum_{j=1}^n (X(t_j) - X(t_{j-1}))^4 \right. \\ & + 2 \sum_{i=1}^n \sum_{j < i} (X(t_i) - X(t_{i-1}))^2 (X(t_j) - X(t_{j-1}))^2 \\ & \left. - 2t \sum_{j=1}^n (X(t_j) - X(t_{j-1}))^2 + t^2 \right]. \end{aligned}$$

Since $X(t_j) - X(t_{j-1})$ is Normally distributed with mean zero and variance t/n we have

$$E \left[(X(t_j) - X(t_{j-1}))^2 \right] = \frac{t}{n}$$

and

$$E \left[(X(t_j) - X(t_{j-1}))^4 \right] = \frac{3t^2}{n^2}.$$

Thus the required expectation becomes

$$n \frac{3t^2}{n^2} + n(n-1) \frac{t^2}{n^2} - 2tn \frac{t}{n} + t^2 = O\left(\frac{1}{n}\right).$$

As $n \rightarrow \infty$ this tends to zero. We therefore say that

$$\sum_{j=1}^n (X(t_j) - X(t_{j-1}))^2 = t$$

in the ‘mean square limit.’ This is often written, for obvious reasons, as

$$\int_0^t (dX)^2 = t.$$

Whenever we talk about ‘equality’ in the following ‘proof’ we mean equality in the mean square sense.

Stochastic Integration Formulas

Starting with H_0^1 I

$$dF = \frac{dF}{dX} dX_t + \frac{1}{2} \frac{d^2 F}{dX^2} dt$$

$$\underbrace{F(X_t)}$$

$$\frac{dF}{dX_t} dX_t = dF - \frac{1}{2} \frac{d^2 F}{dX^2} dt \quad \text{Now integrate } \int_0^t$$

$$\int_0^t \frac{dF}{dX_s} dX_s = \int_0^t dF - \frac{1}{2} \int_0^t \frac{d^2 F}{dX_s^2} ds$$

$$\underbrace{\int_0^t \frac{dF}{dX_s} dX_s}_{\text{It\^o Integral}} = F(X_t) - F(X_0) - \frac{1}{2} \int_0^t \frac{d^2 F}{dX_s^2} ds$$

It\^o Integral $\int_0^t X_s^2 dX_s$ we see $\left(\frac{dF}{dX} = X^2 \right) \rightarrow F = \frac{X^3}{3}$
 $\rightarrow \frac{d^2 F}{dX^2} = 2X$
 $\rightarrow \frac{1}{3}(X_t^3 - X_0^3) - \underbrace{\int_0^t X_s ds}_{\text{we now stop}}$

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$F(t, X_t)$

$$dF = \left(\frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial X_t^2} \right) dt + \frac{\partial F}{\partial X_t} dX_t$$

$$\int_0^t \frac{\partial F}{\partial X_s} dX_s = \int_0^t dF - \int_0^t \left(\frac{\partial F}{\partial s} + \frac{1}{2} \frac{\partial^2 F}{\partial X_s^2} \right) ds$$

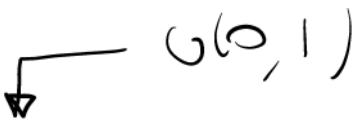
$$\int_0^t \frac{\partial F}{\partial X_s} dX_s = F(X_t, t) - F(X_0, 0) - \int_0^t \left(\frac{\partial F}{\partial s} + \frac{1}{2} \frac{\partial^2 F}{\partial X_s^2} \right) ds$$

Simulating a B.M using a scaled standard Normal

Slow but accurate

The Excel spreadsheet function `RAND()` gives a uniformly-distributed random variable. $RAND() \sim U(0,1)$

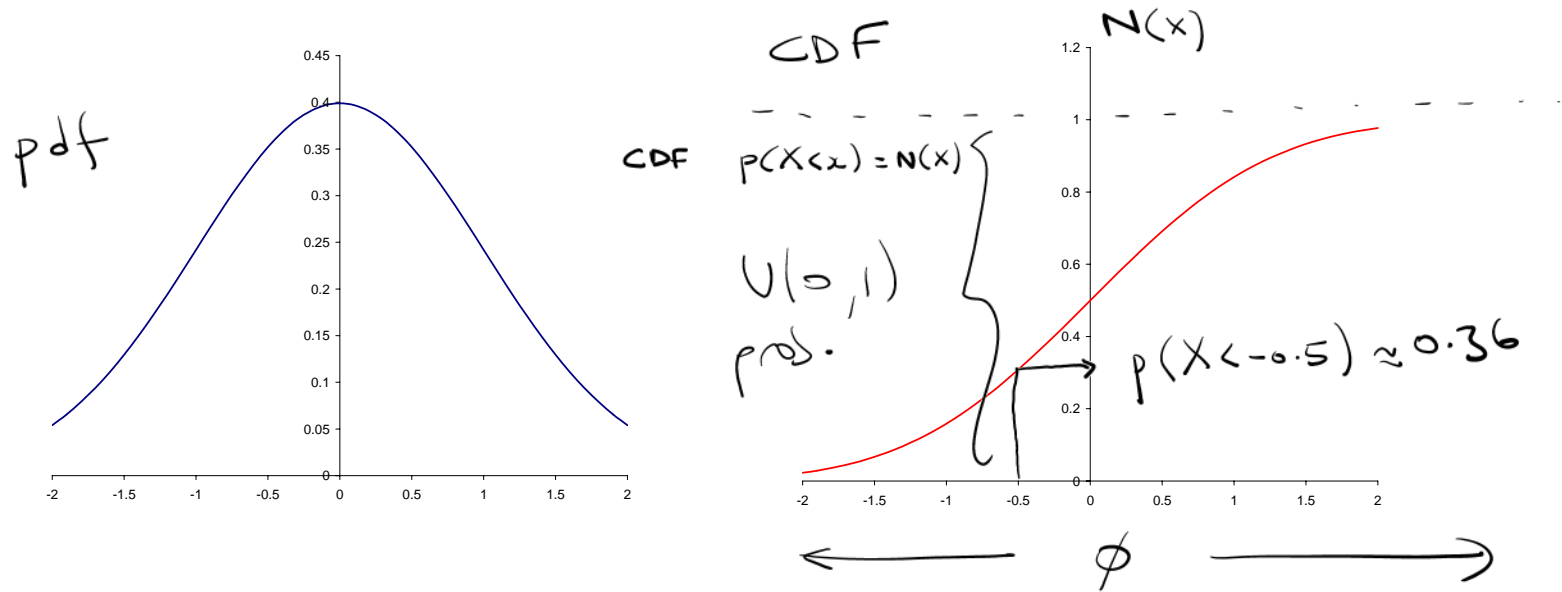
This can be used, together with the inverse cumulative distribution function `NORMSINV` to give a genuinely Normally distributed number:

- $$NORMSINV(RAND()) \sim N(0,1)$$


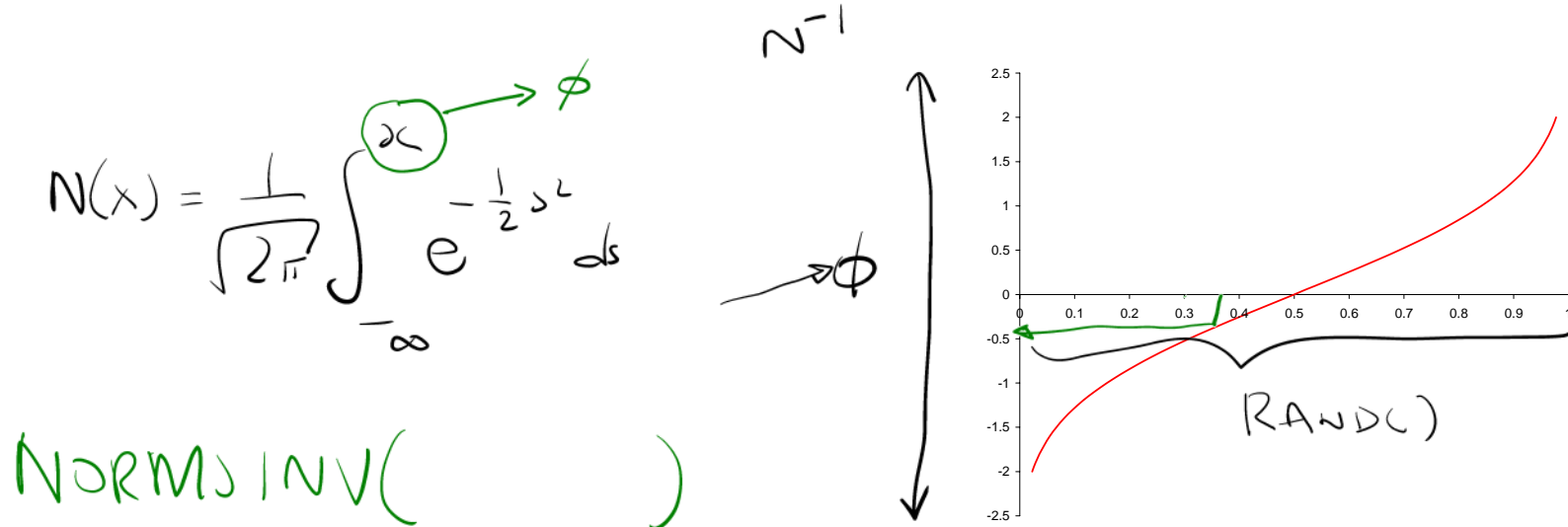
Why does this work?

$$\phi \sim N(0, 1)$$

The pdf and cdf for the Normal distribution



The inverse cumulative distribution function



Fast but inaccurate

An approximation to a Normal variable that is fast in a spreadsheet, and quite accurate, is simply to add up twelve random variables drawn from a uniform distribution over zero to one, and subtract six:

Sum up 12 lots of `RAND()`
subtract 6

get a crude approx[^] for $\phi \sim \mathcal{N}(0,1)$

$$\left(\sum_{i=1}^{12} \text{RAND}() \right) - 6.$$

Why 12?

Any 'large' number will do. The larger the number, the closer the end result will be to being normal, but the slower it is.

Why subtract off 6?

The random number must have a mean of zero.

And the standard deviation?

Must be 1.

① Sum n lots of RAND() $\sum_{i=1}^n \underbrace{\text{RAND()}}_{\psi_i}$ $\psi_i \sim U(0,1)$

② Calculate the mean and variance. Reminder $E[\psi_i] = \frac{1}{2}$; $V[\psi_i] = \frac{1}{12}$
 $E\left[\sum_{i=1}^n \psi_i\right] = \sum_{i=1}^n E[\psi_i] = \frac{n}{2} \neq 0 \quad \therefore \text{subtract off } \frac{n}{2}$

③ Our updated recipe becomes $\sum_{i=1}^n \psi_i - \frac{n}{2}$

④ Variance? $V\left[\sum_{i=1}^n \psi_i - \frac{n}{2}\right] = \sum_{i=1}^n V[\psi_i] = \frac{n}{12} \neq 1$

\therefore consider a normalising constant

⑤ $V\left[\alpha \left(\sum \psi_i - \frac{n}{2}\right)\right] = 1$ $\alpha \in \mathbb{R}$

$$\alpha^2 \underbrace{V\left[\sum_{i=1}^n \psi_i - \frac{n}{2}\right]}_{\frac{n}{12}} = 1 \quad \therefore \quad \alpha^2 \frac{n}{12} = 1 \Rightarrow \alpha = \sqrt{\frac{12}{n}}$$

⑥ $\sqrt{\frac{12}{n}} \left[\sum_{i=1}^n \text{RAND()} - \frac{n}{2} \right] \sim N(0,1)$

Let's take $\sqrt{\frac{12}{n}} \left[\sum_{i=1}^n \text{RAND}() - \frac{n}{2} \right]$ and rewrite as

$$\frac{\sum_{i=1}^n \text{RAND}() - n \times \left(\frac{1}{2}\right)^{\mu}}{\underbrace{\left(\sqrt{\frac{12}{n}}\right)^{\sigma}}_{\sigma} \sqrt{n}} \equiv \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n Y_i - n\mu}{\sigma \sqrt{n}}$$

hence $\sqrt{\frac{12}{n}} \left(\sum_{i=1}^n \text{RAND}() - \frac{n}{2} \right)$ is
Gaussian.

Summary

Please take away the following important ideas

- A B.M. is a standard normal with a special scaling
 - Functions of random variables can't be differentiated in quite the same way as functions of deterministic variables.
 - Instead of using Taylor series you must use Itô's lemma. However, they are very similar and a simple rule of thumb can usually be used to get from Taylor to Itô.
 - Random number generation in Excel.
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