KERNEL REGRESSION APPROACH FOR NON-PARAMETRIC TWO-WAY ANOVA (KRSS)

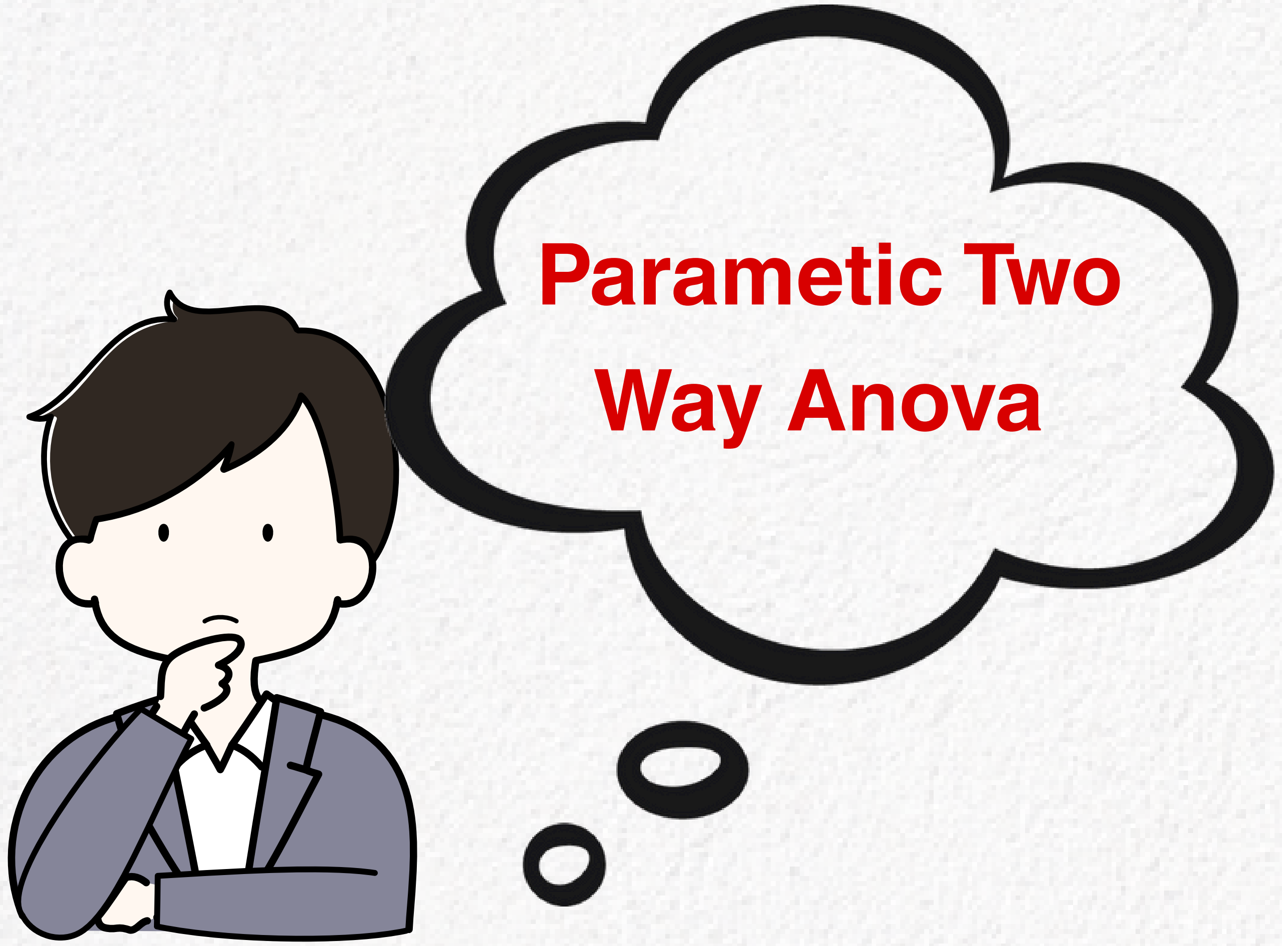
Group 01

Thenuka Yatawara - s16383
Maheesha Sewmini - s16349
Lakmini Thamodya - s16340
Amidu Yohan - s16385

EXAMPLE

You're a young Force researcher at the Jedi Academy, trying to uncover what influences Midichlorian levels - the key to Force sensitivity. You suspect two factors might be important.

1. **Location (Olympia,Venture)**
2. **Tribe (Jedi,Sith)**



Assumption

- **Independence**
- **Normality**
- **Linearity**
- **Homogeneity of variance**

Existing non-parametric alternatives to two-way ANOVA

1. Scheirer-Ray-Hare test

- This is the two factor version of the Kruskal-Wallis test

2. Puri and Sen tests

- This is overly conservative in the presence of other non null effects, which can reduce statistical power.

3. Rank Transform Approach (RT)

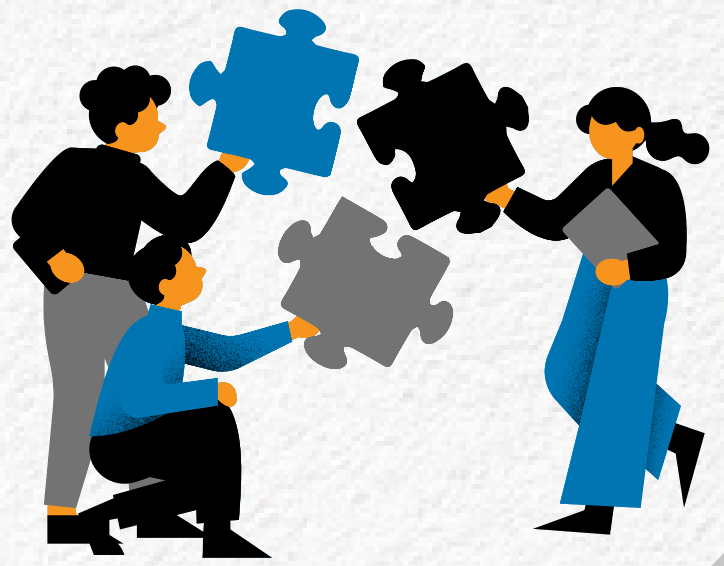
- This is carried out by replacing original observations with their respective ranks
- This is poorly control over Type 1 error rates
- The rank transform provides misleading results in testing significance of interactions when there are large main effects involving both factors but no true interctions.

4. Aligned Rank Transform (ART) ANOVA

- Aligned Rank Tests (ART) handle the issue of annoying effects (like other non-null factors) by removing their influence before ranking, ensuring cleaner, more accurate nonparametric analysis.
- This makes ART suitable for testing interaction effects in factorial designs, even when traditional nonparametric tests (like PS or RT) struggle.
- ART can change the shape of the data when there are lots of similar values (ties) or when the data is very uneven (skewed), which might hide important patterns.

Kernel Ridge Regression based on Statistical Significance Testing (KRSS)

Methodology of KRSS



1. Kernel-Based Modeling
2. Fit Kernel Ridge Regression and Compute RSS
3. Apply Permutation Test
4. Computing P- value and Check Significance

1 Kernel-Based Modelling

1.1 Choice of the Kernel → Gaussian Kernel

Gaussian Kernel Function

$$K(x, y) = \exp\left(-\frac{(x-y)^2}{2\sigma^2}\right)$$

sigma is the bandwidth parameter that controls the ‘width’ of the kernel.

1.2 Compute Kernel Matrices for Categorical Factors

Let $\mathbf{A}$ and $\mathbf{B}$ be two categorical factors with levels $a1, a2, \ldots, ak$ and $b1, b2, \ldots, bm$;
For each categorical factor, we construct a kernel matrix that encodes the pairwise similarities between its levels.

Kernel Matrix for Factor A (K1)
Kernel Matrix for Factor B (K2)
Kernel Matrix for Interaction (K12)

$$K1_{ij} = K(a_i, a_j)$$

$$K2_{ij} = K(b_i, b_j)$$

Calculate kernel matrices using the best sigma.

1.3 Best Bandwidth Parameter Selection (Sigma)

- Reasons :
  - Controls the "smoothness" of the kernel function
  - Too small: Overfitting (high variance)
  - Too large: Underfitting (high bias)

- Selection Method :

  K-Fold Cross Validation

- **Procedure** :

  Step 1 :Tests σ values from 0.1 to 10 (in 0.1 increments)

  Step 2 : for each σ:
  - Computes kernel matrices (K1, K2, K12)
  - Fits kernel ridge regression model
  - Evaluates prediction error on test folds

  Step 3: Selects σ with lowest average MSE

2 Fit Kernel Ridge Regression and Compute RSS

2.1 Fit Kernel Ridge Regression to the Data

Model Comparison By

1. **Fitting Reduced Model**
2. **Fitting Full Model**

2.2 Test Statistic

- The residual sum of squares (RSS) is computed for both the full and reduced models

$$\mathrm{RSS} = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

- RSS measures the difference between the observed response (Y) and the predicted response ($\hat{Y}$)

Test Statistic = RSS_reduced - RSS_full

3 Apply Permutation Test

To test Observed effect is real or due to random noice

Step 3.1 Shuffle the Response (Break Associations)

- Randomly reorders the response values while keeping the factor levels unchanged.

📌 Null Hypothesis : effect is 0 (Not significant)

Step 3.2 Re-compute RSS for Shuffled Data

- Measures how much "better" the full model fits even when no true interaction exists.
- Uses the same kernel matrices for fitting KRR

Step 3.3 Store Statistic

perm_statistics = RSS_reduced_perm - RSS_full_perm

NOTE:

- If H_0 is true, this ΔRSS should be small
- Large values occur only by random chance.

4 Computing P- value and Check Significance

Repeat the permutation process 1000 times (1000 Permutations)

To generate a distribution of the test statistic under the null hypothesis.

$$\text{4.1 p-value} = \frac{\text{Number of permuted test statistics} \geq \text{Observed test statistic}}{\text{Total Number of permutations}}$$

Calculate P values for main effects and interaction effects to check the significance

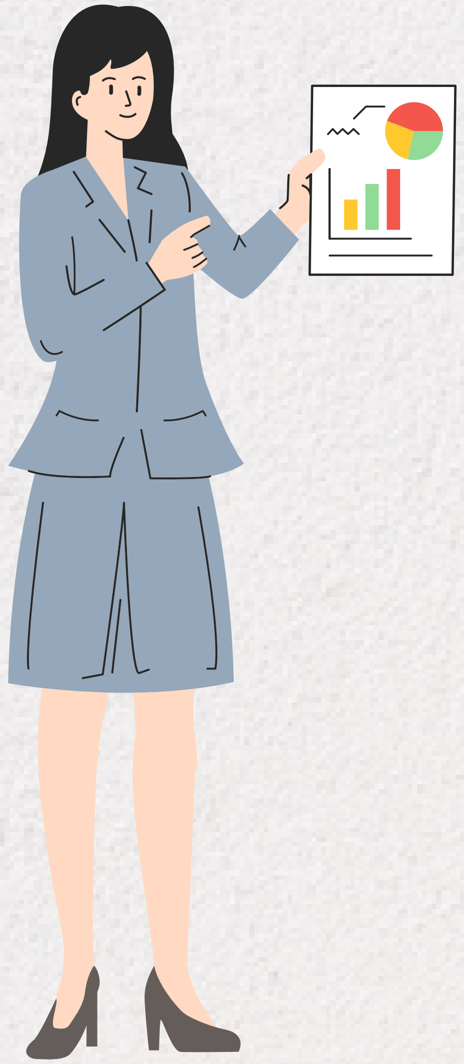
i.e

1. Main effect of A (compares K1+K2 vs K2)
2. Main effect of B (compares K1+K2 vs K1)
3. Interaction effect (compares K1+K2+K12 vs K1+K2)

4.2 Check The Significance of Each Factor

If p-value <0.05 → Reject H_0 (effect is significant)

COMPARE WITH EXISTING PARAMETRIC TWO-WAY ANOVA



Parametric Two-way ANOVA.

VS

krss

TWO WAY ANOVA TABLE

Parametric Two-way ANOVA		
Location	1.53e-09	***
Tribe	0.102	
Location:Tribe	3.22e-06	***

krss Two-way ANOVA		
Location	0.000	***
Tribe	0.751	
Location:Tribe	0.001	***

SAME !!!!

POWER ANALYSIS

Evaluate the sensitivity of parametric vs. Krss methods across sample sizes

POWER ANALYSIS RESULTS

Sample Size (n)	Krss Non-Parametric Method Power	Parametric Two-Way ANOVA Power
20	0.38	0.9679
40	0.42	0.9999
60	0.48	0.9999
80	0.56	1.0000
100	0.62	1.0000

1. The parametric two-way ANOVA exhibits **high power** for small sample sizes, indicating a strong ability to detect true effects when parametric assumptions are met.
2. In contrast, the Krss method demonstrates **lower power** (0.38 to 0.42) for small sample sizes, suggesting it is less sensitive under these conditions.

TEST ROBUSTNESS UNDER VARIOUS VIOLATIONS

Validate Krss under violations of Normality and Homoscedasticity.

TEST ROBUSTNESS UNDER VARIOUS VIOLATIONS RESULTS

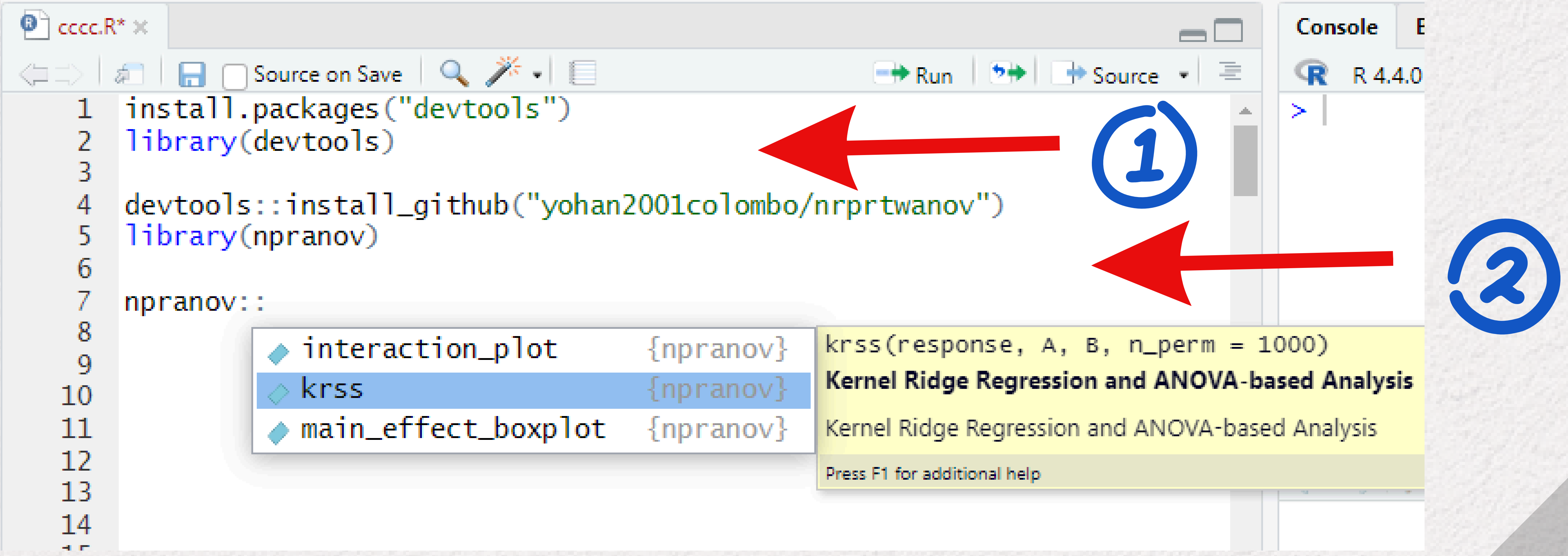
Data set	Factor A	Factor B	Interaction
Original	0.957	0.827	0.694
Non-Normal	0.939	0.581	0.810
Heterosce-dastic	0.835	0.578	0.748

The p-values for Non-normal data and Heteroscedastic data were very close to those for the original data.

Krss function demonstrated strong robustness to violations of Normality and Homoscedasticity.

Implementation and Interpretation

Loading npranov Package



Diet vs. Exercise: Impact on Weight Loss

To study the effect of diet type and exercise intensity on weight loss after 6 weeks

Experimental Design

- Factors:
    - Diet Type (3 levels): Low Carb, Low Fat, Mediterranean
    - Exercise Intensity (2 levels): Low, High
- Response Variable: Weight Loss (kg)
- Participants: 60 individuals, 10 per group (3 diets × 2 exercise levels)

Both diet type and exercise intensity affect weight loss, with a potential interaction between them.

Diet vs. Exercise: Impact on Weight Loss

Weight Loss is not normally distributed

Shapiro-Wilk Test: p-value = 2.753e-05

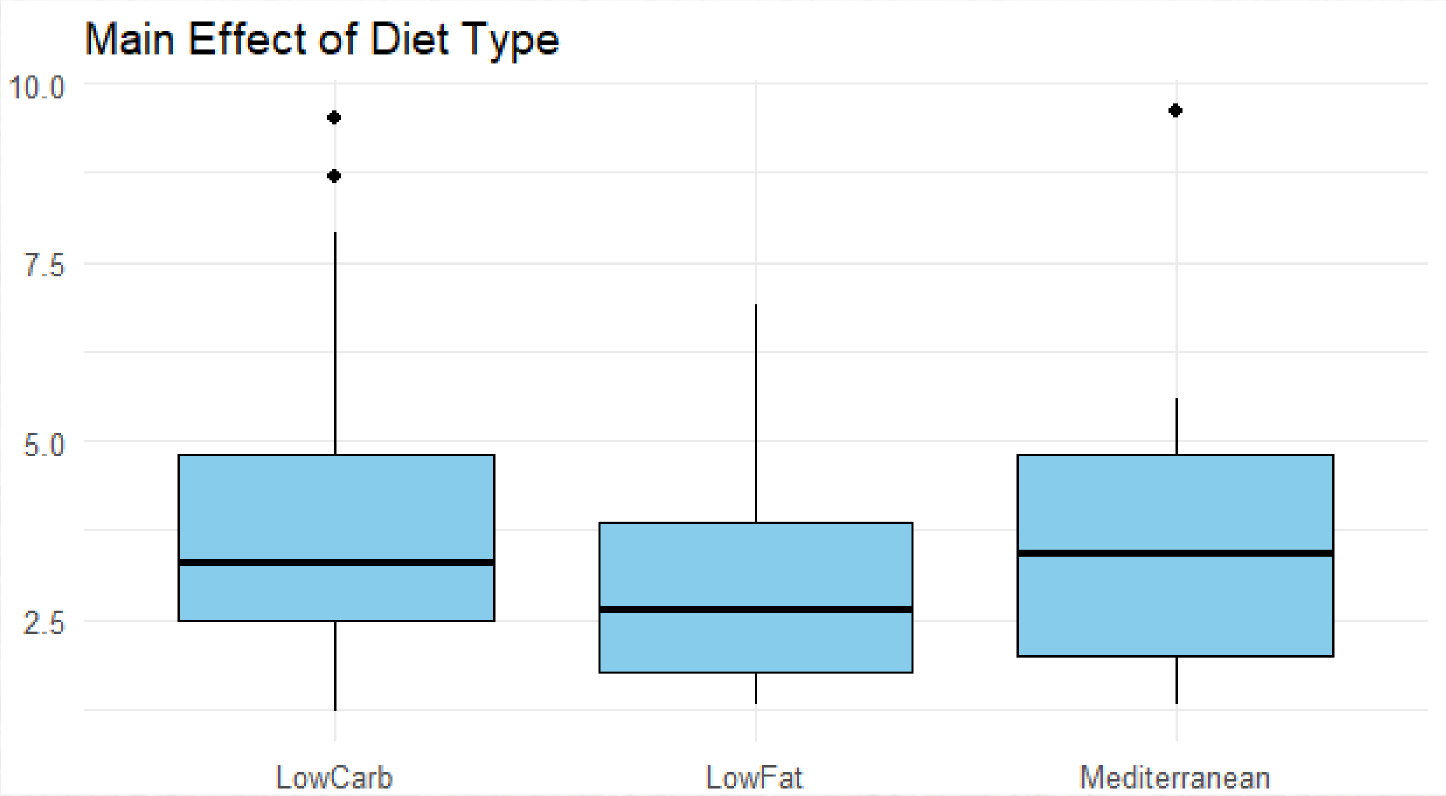
apply KRSS method

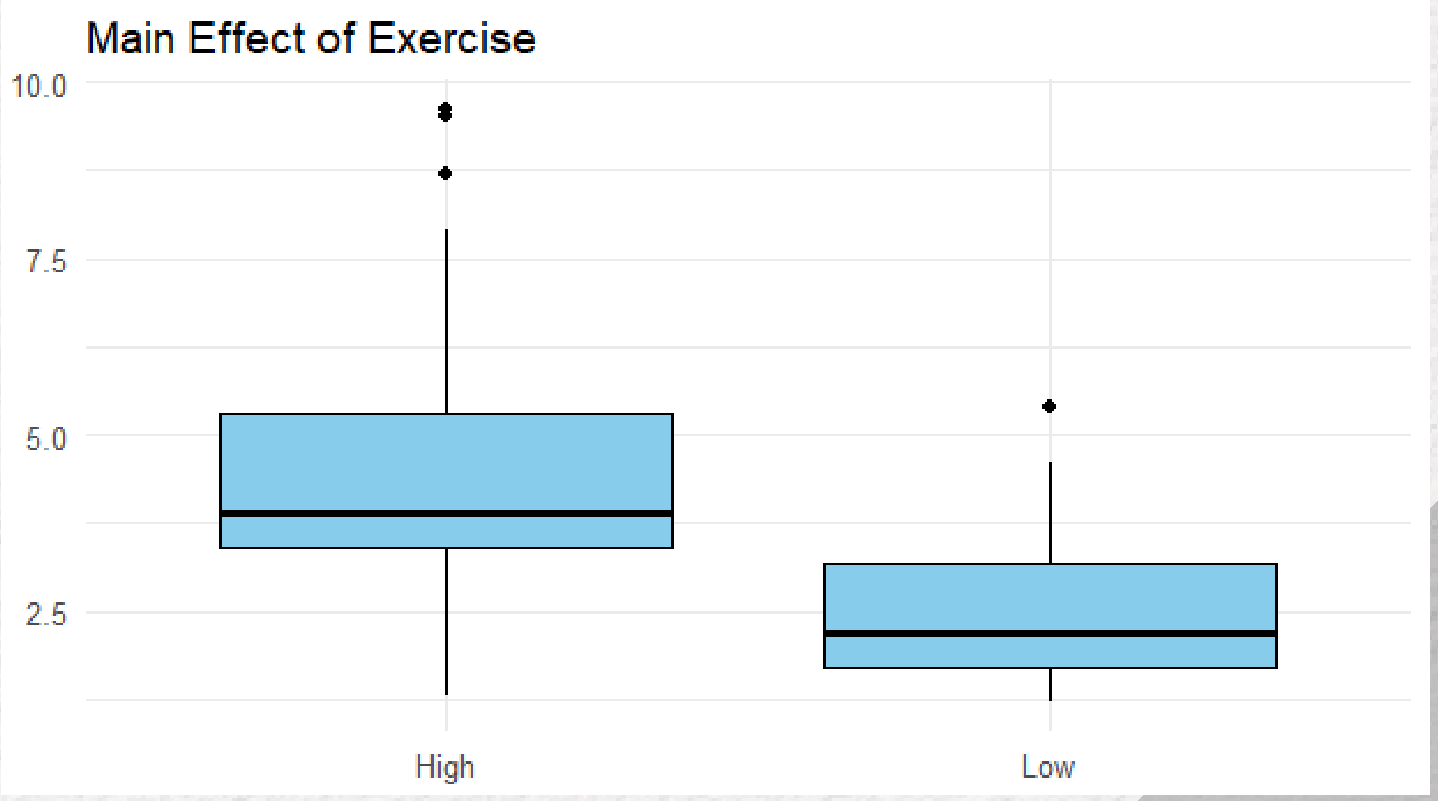
```
Response: Weight_Loss
> krss_result$anova_table
            P_value Significance
Diet          0.224
Exercise      0.001          ***
Interaction   0.124
```

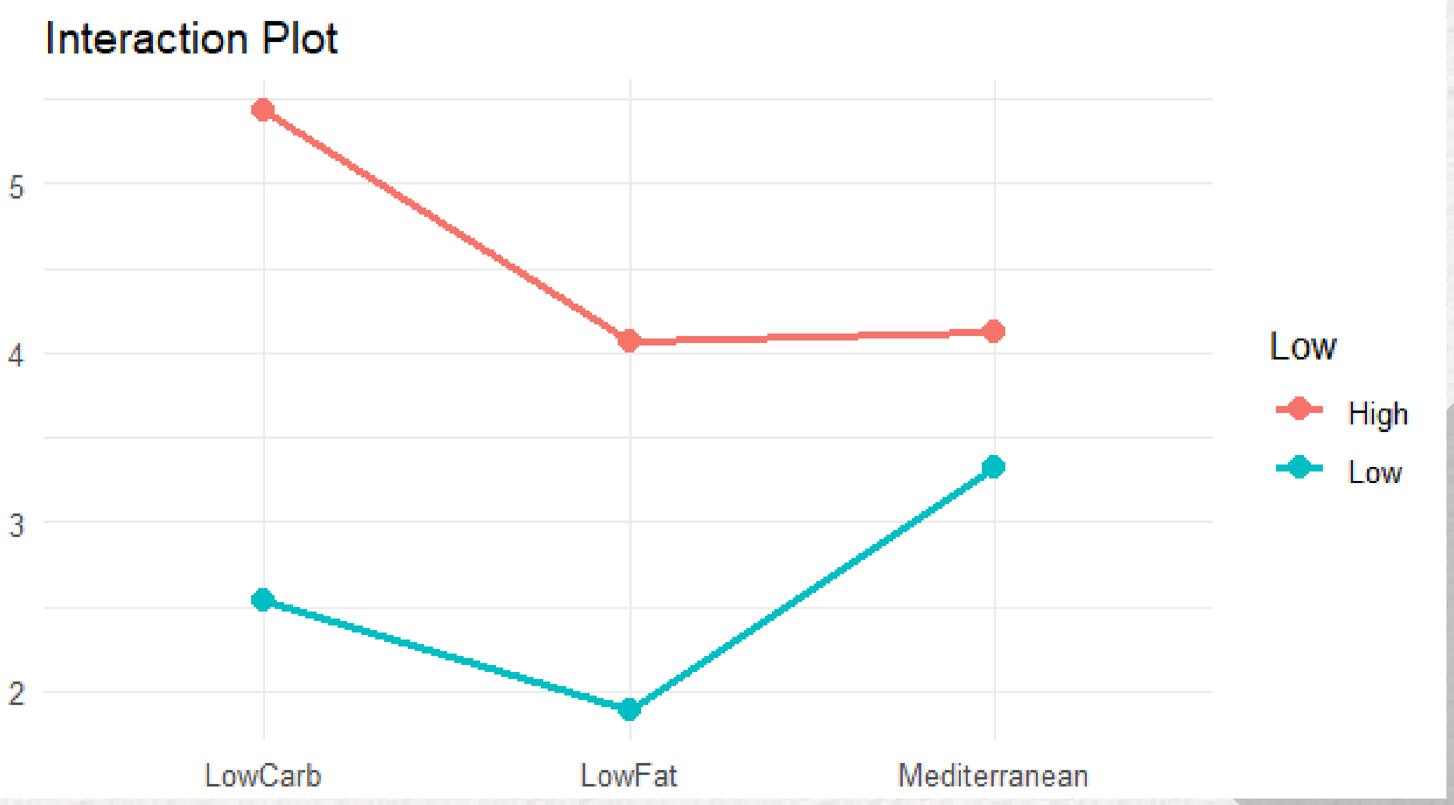
Significant impact!
($p < 0.05$)

higher intensity exercise leads to more significant weight loss.

Diet vs. Exercise: Impact on Weight Loss







Implementation Challenges

- Computational Complexity → 1000+ permutations increase runtime
- Parameter Sensitivity → Bandwidth (σ) selection via cross-validation

Conclusion

- Robust non-parametric alternative for non-normal/unbalanced data
- Validates real-world findings (e.g., dose effect $p < 0.001$ in Tooth Growth data)

- Lower power for small samples vs. parametric ANOVA
- Computational cost (permutation tests).

KRSS complements traditional ANOVA, bridging gaps in real-world data analysis.

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THANK YOU