MTH20017 Mathematical Methods and Statistics for Engineering

Week 2 Lab Curves in 2d and 3d, and visualisation of 2d fields

In this lab you will familiarise yourself with some commands for plotting curves, surfaces, level curves, vector fields, and basic operations for manipulating functions.

One of the main goals is to help build intuition regarding the relationship between a scalar field $f(x, y)$ and its gradient $\nabla f(x, y)$.

When you see

Complete the assessment task now!

you should turn to the last page of this worksheet and complete and submit the lab assessment task described there.

We recommend that you create a livescript (`.mlx`) file for this lab.

`MATLAB` summary

Plotting commands in `MATLAB` are listed on the reference page: https://au.mathworks.com/help/matlab/2-and-3d-plots.html

To define functions the anonymous function construction is convenient: https://au.mathworks.com/help/matlab/matlab_prog/anonymous-functions.html

One thing to be careful of is the definitions of various operations. `MATLAB` typically assumes that operations involve *matrices*, so $X * X$ is treated as matrix multiplication. To multiply *elementwise* you need to place a "." in front of the relevant operation, e.g. $X. * X$, or $Y./X$, or $X.\hat{}3$. (Anonymous functions eliminate the need to specify elementwise operations, because they are always defined elementwise and we have to use the function `arrayfun` to apply them to matrices.)

Repeat below for different functions, and then do things like scale x by 0.5 or 2, multiply the whole thing by -1.

`MATLAB` has many commands to plot numerical data, and a slightly smaller set of commands to plot functions that have been implicitly defined. For example, to plot vector fields it is only possible to do so using numerical data via `quiver` and `quiver3`.

We will use the numerical plotting commands, but you are encouraged to experiment with the implicit commands such as `fcontour`, `fplot`, and `fsurf`.

Useful `MATLAB` commands; use doc for more information.

```
anonymous functions (@ command)
arrayfun
axis equal
clabel
contour3
contour (and fcontour)
contourf
figure
gradient
hold off
hold on
meshgrid
plot3 (and fplot3)
plot (and fplot)
quiver
quiver3
surf (and fsurf)
surfc
title
```

Note: `MATLAB` produces interactive graphics. You can zoom in or out, or rotate the view.

Curves in 2d and 3d

Practice plotting curves in 2d and 3d via the plot, fplot, plot3, and fplot3 commands.

Lissajou curves in 2d

See https://en.wikipedia.org/wiki/Lissajous_curve

Plot Lissajou curves of the form

$$x(t) = \sin(at + \delta)$$
$$y(t) = \sin(bt)$$

for various choices of the parameters.

- What happens as you change a, b, and δ?
- If a and b are integers can you figure out how what is the period of the curve?
- What happens if you choose an irrational value for one of a or b? (Hint: curves are qualitatively different depending on whether a/b is *rational* or *irrational*. Why?)

Record the parameters for your favourite discoveries and share the resulting images to the MTH20017 Discord server!

Sample code.

```
% t: equally spaced points between 0 and 20*pi
% This interval will need to be larger for some choices of a and b.
% Also, you may wish to decrease the spacing from the default of pi/50.
t = 0:pi/50:20*pi;
a = 3;
b = 4;
d = pi/2.;
x = sin(a*t+d);
y = sin(b*t);
plot(x,y);
```

Helices in 3d

Plot helices for various choices of parameters. How does the curve change when $z = t$ is changed to $z = 2t$? What about $z = -t$? Can you construct a helix which revolves around the y axis with radius 3?

Sample code.

```
% t: equally spaced points between 0 and 6*pi
t = 0:pi/50:6*pi;
% plot first helix
x = cos(t);
y = sin(t);
z = t;
plot3(x,y,z)
% multiple plots on the one figure
hold on
% plot another helix
x2 = cos(t);
y2 = sin(t);
z2 = 2*t;
plot3(x2,y2,z2)
hold off
```

Visualising 2d scalar fields

Overview

In the following sections you will work through `MATLAB` tools to visualise 2d scalar fields and their associated gradient fields.

Experiment with different choices of scalar field.

Each subsection below should have a different `MATLAB` section in your livescript, so that you can edit the code and run it independently.

Functions to try include: $f_1(x, y) = x^2 + y^2$, $f_2(x, y) = -0.2y^3 + 0.1x^2y + 0.4x^2$, $f_3(x, y) = \sin x + \sin y$, $f_4(x, y) = \sin x \sin y$, $f_5(x, y) = x^2 + 2y^2$, $f_6(x, y) = (0.3 * x^2 + 0.3 * y^2 - 1)^2$. Or, better yet, construct your own weird functions and do your best to visualise them. (If you construct any neat functions please share with your lab teacher and Nathan.)

What happens to the scalar fields and associated gradient field if you:

- Change sgrid to make the scalar grid finer or coarser.
- Change vgrid to make the vector grid finer or coarser.
- Replace x by $2x$ or $x/2$?
- Scale x and y by the same amount?
- Replace f by $-f$?
- Replace x by $x + y$, or $x - y$?

Some sample setup code, creating a set of function values on a domain of $[-2, 2] \times [-2, 2]$.

```
% Define grid spacing for vector fields
vgrid = 1.0/3.0;
% Define grid spacing for scalar fields
sgrid = 0.05;

% Define function of interest as anonymous functions
% leave only 1 function uncommented
f = @(x,y) x^2 + y^2 ;
%f = @(x,y) -0.2*y^3 + 0.1*x^2*y + 0.4*x^2 ;

% Define meshes for 2d scalar and vector fields
[Xs,Ys] = meshgrid(-2:sgrid:2,-2:sgrid:2);
[Xv,Yv] = meshgrid(-2:vgrid:2,-2:vgrid:2);

% Choose one function to be the main function to be plotted
% arrayfun applies the anonymous function to matrices
Fs = arrayfun(f,Xs,Ys);
Fv = arrayfun(f,Xv,Yv);
```

Note: we have created two separate meshes, one for the contour and surface plots, the other for the gradient plots, as typically the former plots look better with a fine mesh while the gradient plots are more intelligible with a coarser mesh.

Surfaces

Plot the surface $z = f(x, y)$ using the `surf command`.

Sample code

```
% Plotting surface
% doc surf
surf(Xs,Ys,Fs);
surf(Xs,Ys,Fs,EdgeColor="none");
```

Level curves / contour maps

Plot the level curves, aka the contour map, experimenting with different options to

- Fill in the contours with colour.
- Change the number of level curves / contours.

Complete the assessment task now!

- Display the surface and contour map simultaneously via `surfc`.
- Display the contours in 3d via `contour3`.

```
% Plotting contours
% doc contour
% doc contourf
% doc contour3
% doc surfc
contour(Xs,Ys,Fs);
contourf(Xs,Ys,Fs);
[C, h] = contour(Xs,Ys,Fs,15);
title("Plot of f(x,y) = x^2 + y^2","This is a subtitle")
clabel(C,h)
contour3(Xs,Ys,Fs);
surfc(Xs,Ys,Fs);
```

Calculating and plotting the gradient of a scalar field

Remember: the gradient vector field ∇f points in the direction of steepest increase, and has magnitude equal to the rate of increase.

In this subsection you will be able to see this in action by combining what you have seen of $f(x, y)$ via surface and contour plots, together with a `quiver` plot of $\nabla f(x, y)$.

To calculate the gradient use the `gradient` function, and plot the result using quiver. Note: here we are using the vector grid.

```
% Plotting gradient
% doc gradient
% doc quiver
[DX,DY]=gradient(Fv,vgrid);
quiver(Xv,Yv,DX,DY)
```

To get a real sense for how the level curves and gradient are related you should plot them on the same graph. For example

```
% Plotting gradient and contour on the same graph
contour(Xs,Ys,Fs,15);
hold on
[DX,DY]=gradient(Fv,vgrid);
quiver(Xv,Yv,DX,DY)
hold off
```

Week 2 Lab assessment task

Use the `contour` command to produce a plot of the level curves of a function of your choice.

Please try to find a "nice" function that produces interesting level curves, and choose the number of contours so that the plot has an appropriate level of detail.

Use the `title` command to add a title in the same format as the image shown below.

Note: the title *must* include your name and ID as a "subtitle". If you do not do this you will receive zero marks for the assessment. (The purpose of this requirement is to ensure that you are indeed producing the plot yourself.)

Save the plot as a pdf file, by clicking on the "Save" menu of the plot, and ensuring that you save the plot in pdf format.

Finally, submit your plot file to the Week 2 Lab assessment in Canvas. (Click on "Assignments" in the left-hand menu.)

Ideally you should submit the plot during the lab, but you have until 5pm on Friday to submit in case you need more time.