Forme Trigonométrique
Corrigé

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Exercices.

Exercice 7.1 [♦◇◇]

Calculer $(1+i)^2 023$.
On a :

$$(1+i)^{2023} = (\sqrt{2}e^{i\frac{\pi}{4}})^{2023} = \sqrt{2}^{2023} e^{i\frac{2023\pi}{4}} = \sqrt{2}^{2023} e^{-i\frac{\pi}{4}}$$

□

Exercice 7.2 [♦♦◇]

Soient trois réels x, y, z tels que $e^{ix} + e^{iy} + e^{iz} = 0$. Montrer que $e^{2ix} + e^{2iy} + e^{2iz} = 0$.
On a :

$$\begin{aligned} & e^{ix} + e^{iy} + e^{iz} = 0 \\ \iff & e^{-ix} + e^{-iy} + e^{-iz} = 0 \end{aligned}$$

Et :

$$\begin{aligned} & (e^{ix} + e^{iy} + e^{iz})^2 = e^{2ix} + e^{2iy} + e^{2iz} + 2(e^{ixy} + e^{ixz} + e^{iyz}) \\ \iff & e^{2ix} + e^{2iy} + e^{2iz} = -2(e^{ixy} + e^{ixz} + e^{iyz}) \end{aligned}$$

Or :

$$2(e^{ixy} + e^{ixz} + e^{iyz}) = 2e^{i(x+y+z)}(e^{-ix} + e^{-iy} + e^{-iz}) = 0$$

Ainsi,

$$e^{2ix} + e^{2iy} + e^{2iz} = 0$$

□