Exercise 22

Inductive hypothesis: (append (append xs ys) zs) = (append xs (append ys zs))

Case 1, when xs = '()
(append (append xs ys) zs)
= {by assumption, that xs = '()}
(append (append '() ys) zs)
= {append-nil}
(append ys zs)
= { append-nil, by assumption that xs = '()}
(append xs (append ys zs))

Case 2, when xs = (cons x qs)
(append (append xs ys) zs)
= {by assumption that xs = (cons x qs)
(append (append (cons x qs) ys) zs)
= { append-cons }
(append (cons x (append qs ys)) zs)
= {append-cons}
(cons x (append (append qs ys) zs))
= {by inductive hypothesis}
(cons x (append qs (append ys zs)))
= {append-cons}
(append (cons x qs) (append ys zs))
{by assumption that xs = (cons x qs)}
(append xs (append ys zs))

Exercise A

a)

$$\text{CONS}\ \frac{\langle \text{CONS}, \rho, \sigma_3\rangle \Downarrow \langle \text{PRIMITIVE}(\text{CONS}), \sigma_4\rangle \quad \langle X, \rho, \sigma_4\rangle \Downarrow \langle v_1, \sigma_5\rangle \quad \langle XS, \rho, \sigma_5\rangle \Downarrow \langle v_2, \rho, \sigma_6\rangle \qquad \ell_1 \notin \text{DOM}\ \sigma_6 \quad \ell_2 \notin \text{DOM}\ \sigma_5 \quad \ell_1 \neq \ell_2}{\langle \text{APPLY}(\text{CONS}, X, XS), \rho, \sigma_0\rangle \Downarrow \langle \text{PAIR}\langle \ell_1, \ell_2\rangle, \sigma_6\{\ell_1 \mapsto x, \ell_2 \mapsto xs\}\rangle}$$

$$\text{CDR}\ \frac{\langle \text{CDR}, \rho, \sigma_0\rangle \Downarrow \langle \text{PRIMITIVE}(\text{CAR}), \sigma_1\rangle \quad \langle (\text{CONS}\ X\ XS), \rho, \sigma_0\rangle \Downarrow \langle \text{PAIR}\langle \ell_1, \ell_2\rangle, \sigma_2\rangle}{\langle \text{APPLY}(\text{CDR}, (\text{CONS}\ X\ XS)), \rho, \sigma_0\rangle \Downarrow \langle \sigma_2(\ell_2), \sigma_2\rangle}$$

b)

$e_1 = $ (if 1 (set ys xs))

$e_2 = $ ys

$\rho = \{(xs \mapsto \ell_1), (ys \mapsto \ell_2)\}$

$\sigma = \{(\ell_1 \mapsto \text{'}(1\ 2)), (\ell_2 \mapsto \text{'}())\}$

AT FIRST, $e_2 = ys = \text{'}()$.

HOWEVER, AFTER EVALUATING BOTH EXPRESSIONS,

$e_2 = ys = \text{'}(1\ 2)$