Part A

Exercise 10

a) Let x be a variable. Then, either x is a local variable, or x is a global variable
d) The expression e evaluates to value v. When e is evaluated, there are no variables, global or local, that are changed when e is evaluated.

Exercise 11

a) Even if x $\notin \xi$, there exists an expression e so that, $< e, \xi, \phi, \rho > \Downarrow < v, \xi', \phi, \rho' >$ holds true.
c) There exists an expression e so that $< e, \xi, \phi, \rho > \Downarrow < v, \xi, \phi, \rho' >$ is true.
d) There exists an expression e so that $< e, \xi, \phi, \rho > \Downarrow < v, \xi', \phi, \rho' >$ (with no new variables $\in \xi$) is true.

Exercise R

The alternative WhileEnd rule of Impcore is effectively the same, since the value "0" is found where "v1" is originally found. In this new rule, $e1$ still evaluates to "0", meaning that "v1" is still 0 as well.

The alternative FormalAssign rule of Impcore are not the same. The difference is that the original rule specifies that ρ will change because a new mapping is being added. In this new rule, ρ is not specified that it WILL change, it only states that it MAY change. Thus, it is possible that the mapping of x is not changed in this new rule, while it is required in the original one. The new binding hides previous bindings of x. An example code which will behave different with both rules is (set x 5), where x is already bound to another value.

Part B

Exercise 16

a)

$$\frac{x \notin dom\ \xi \quad x \notin dom\ \rho}{< VAR(x), \xi, \phi, \rho > \Downarrow < 0, \xi'(x \Rightarrow 0), \phi, \rho >}$$

$$\frac{x \notin dom\ \xi \quad x \notin dom\ \rho \quad < e, \xi, \rho, \phi > \Downarrow < v, \xi', \phi, \rho' >}{< SET(x, e), \xi, \phi, \rho > \Downarrow < v, \xi'(x \Rightarrow v), \phi, \rho' >}$$

b)

$$\frac{x \notin dom\ \xi \quad x \notin dom\ \rho}{< VAR(x), \xi, \phi, \rho > \Downarrow < 0, \xi, \phi, \rho'(x \Rightarrow 0) >}$$

$$\frac{x \notin dom\ \xi \quad x \notin dom\ \rho \quad < e, \xi, \rho, \phi > \Downarrow < v, \xi', \phi, \rho' >}{< SET(x, e), \xi, \phi, \rho > \Downarrow < v, \xi', \phi, \rho'(x \Rightarrow v) >}$$

c)
I think I prefer the changes in part b), since it doesn't create global variables but rather local ones. It is common practice to never use global variables, which can be used in very small programs, but not at all when programs are expanded, which is the reason some languages like Java don't even have global variables. Thus, making sure that the amount of global variables is close to 0 is important, and the rules in part b) serve this purpose better, although I can see the rules in part a) being used if its used for a small program.

PART C

Exercise 12

12)

$$\dfrac{\dfrac{x \in \text{DOM } P}{\langle \text{VAR}(x), \xi, \phi, P\rangle \Downarrow \langle 99, \xi, \phi, P\rangle}\ \text{FORMAL VARIABLE} \qquad \dfrac{}{\langle \text{LIT}(3), \xi, \phi, P\rangle \Downarrow \langle 3, \xi, \phi, P\rangle}\ \text{LITERAL} \qquad 99 \in \text{DOM } P}{\langle \text{SET}(\text{VAR}(x), \text{LIT}(3)), \xi, \phi, P\rangle \Downarrow \langle 3, \xi, \phi, P\{x \to 3\}\rangle}\ \text{FORMAL ASSIGN}$$

$$\dfrac{\dfrac{x \in \text{DOM } P}{\langle \text{VAR}(x), \xi, \phi, P\{x \to 3\}\rangle \Downarrow \langle 3, \xi, \phi, P\rangle}\ \text{FORMALVAR}}{\langle \text{BEGIN}(\text{SET}(\text{VAR}(x), \text{LIT}(3)), \text{VAR}(x)), \xi, \phi, P\rangle \Downarrow \langle 3, \xi, \phi, P\{x \to 3\}\rangle}\ \text{BEGIN}$$

Exercise 13

a)

13) A)

IF $x \neq 0$, THEN

$$\dfrac{\dfrac{x \in \mathrm{dom}\, P}{\langle \mathrm{VAR}(x), \xi, \phi, P \rangle \Downarrow \langle x, \xi, \phi, P \rangle}\ \text{FORMAL VAR} \qquad x \neq 0 \qquad \dfrac{x \in \mathrm{dom}\, P}{\langle \mathrm{VAR}(x), \xi, \phi, P \rangle \Downarrow \langle x, \xi, \phi, P \rangle}\ \text{FORMAL VAR}}{\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LIT}(0)), \xi, \phi, P \rangle \Downarrow \langle x, \xi, \phi, P \rangle}\ \text{IF TRUE}$$

IF $x = 0$, THEN

$$\dfrac{\dfrac{x \in \mathrm{DOM}\, P}{\langle \mathrm{VAR}(x), \xi, \phi, P \rangle \Downarrow \langle x, \xi, \phi, P \rangle}\ \text{FORMAL VAR} \qquad x = 0 \qquad \dfrac{x \in \mathrm{DOM}\, P}{\langle \mathrm{VAR}(x), \xi, \phi, P \rangle \Downarrow \langle x, \xi, \phi, P \rangle}\ \text{FORMAL VAR}}{\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LIT}(0)), \xi, \phi, P \rangle \Downarrow \langle x, \xi, \phi, P \rangle}\ \text{IF FALSE}$$

Exercise F

F) A) ACCORDING TO THE CONJECTURE:

WHENEVER $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$ AND $y \in dom\ \xi$ AND $y \notin fv(e)$, THEN $\xi'(y) = \xi(y)$.

HOWEVER, IF $\xi = \{y \to 5\}$, $\rho = \{\}$,

$e =$ (if 1 (set y 3) 0), $v = 3$,

$\xi' = \{y \to 3\}$, $\rho' = \{\}$, ϕ CONTAINS ALL THE FUNCTIONS IN THE INITIAL BASIS

AS SEEN, WHEN EVALUATING e, THE VALUE OF y CHANGES IN ξ, SO, $\xi'(y) \neq \xi(y)$. FURTHER, $y \in dom\ \xi$ AND $y \notin fv(e)$, SINCE IF e CALLS FUNCTION f, THEN f'S VARIABLES ARE NOT CONSIDERED FREE IN e. e CALLS SET, WHICH HAS y AS A PARAMETER, BUT y IS NOT FREE IN e. THUS, I HAVE PROVIDED A COUNTEREXAMPLE TO THE CLAIM.

B) CASE 1: FORMAL ASSIGN

$$D = \dfrac{x \in \text{DOM } P \qquad \dfrac{D_1}{\langle e, \xi, \phi, P\rangle \Downarrow \langle v, \xi', \phi, P\rangle}}{\langle \text{SET}(x, e), \xi, \phi, P\rangle \Downarrow \langle v, \xi', \phi, P'\{x \to v\}\rangle} \text{ FORMAL ASSIGN}$$

BECAUSE D_1 IS SMALLER THAN D, WE ARE ALLOWED TO USE THE INDUCTION HYPOTHESIS, WHICH TELL US DOM ξ = DOM ξ'. SINCE NO GLOBAL VARIABLES ARE CHANGED, THIS CASE HOLDS TRUE

CASE 2: GLOBAL ASSIGN

$$D = \dfrac{x \notin \text{DOM } P \qquad x \in \text{DOM } \xi \qquad \dfrac{D_1}{\langle e, \xi, \phi, P\rangle \Downarrow \langle v, \xi', \phi, P\rangle}}{\langle \text{SET}(x, e), \xi, \phi, P\rangle \Downarrow \langle v, \xi'\{x \to v\}, \phi, P'\rangle} \text{ GLOBAL ASSIGN}$$

AGAIN, USING THE INDUCTIVE HYPOTHESIS ON D_1, WE PROVE THAT $\xi \neq \xi'$. HOWEVER, THE ONLY DIFFERENCE IS $\{x \to v\}$. SINCE THE VALUE OF OUR y DOESN'T CHANGE, THEN $\xi(y) = \xi'(y)$ HOLDS TRUE.

C)

$$\text{LITERAL}\ \frac{}{\langle \text{LIT}(3), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\rangle}$$

$$\text{GLOBAL VAR}\ \frac{Y \notin \text{dom}\ \rho \quad Y \in \text{dom}\ \xi}{\langle \text{VAR}(Y), \xi, \phi, \rho\rangle \Downarrow \langle \xi(Y), \xi, \phi, \rho\rangle}$$

$$D = \text{IF TRUE}\ \frac{\xi(Y)=5 \quad \text{LITERAL}\ \frac{}{\langle \text{LIT}(1), \xi, \phi, \rho\rangle \Downarrow \langle 1, \xi', \phi, \rho'\rangle} \quad \frac{Y \notin \text{dom}\ \rho \quad Y \in \text{dom}\ \xi \quad \langle \text{LIT}(3), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\rangle}{\langle \text{SET}(Y,3), \xi', \phi, \rho'\rangle \Downarrow \langle 3, \xi'', \phi, \rho''\rangle}}{\langle \text{IF}(\text{LIT}(1), \text{SET}(Y,3), \text{LIT}(0)), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi''\{Y \mapsto 3\}, \phi, \rho''\rangle}$$

IN THIS CASE, WE SEE $Y \in \text{dom}\ \xi$. WE ALSO SEE THAT $Y \notin FV(e)$. BEFORE EVALUATING e, $\xi(Y)=5$, BUT ONCE e IS EVALUATED, THE RESULTING ENVIRONMENT IS $\xi''(Y)=3$. SINCE $\xi(Y) \neq \xi''(Y)$, THEN THE GLOBAL VARIABLE Y HAS CHANGED WHEN EVALUATING e, EVEN THOUGH Y DOES NOT APPEAR IN EXPRESSION e.

D) THE CONJECTURE STATES A GLOBAL VARIABLE Y WILL NOT CHANGE WHEN A PROGRAM RUNS A SNIPPET OF CODE THAT DOESN'T CONTAIN Y DIRECTLY. HOWEVER, AS SHOWN IN THE EXAMPLE ABOVE, THERE EXISTS EXAMPLES WHERE THIS CONJECTURE DOES NOT HOLD TRUE.