2.4 Ensemble aggregation with no hypothesis

We give ourselves a time series

$$u : (t,x) \in \mathbb{R} \times D \mapsto u(t,x) \in \mathbb{R} \tag{11}$$

and models M_1,...M_n of u. We can aggregate them using

$$\alpha(t,x) = \mathbb{E}_{(}\tau,\chi)\left[M_i(\tau,\chi)M_j(\tau,\chi)\right]^{-1}\mathbb{E}_{(}\tau,\chi)\left[M_i(\tau,\chi)u(\tau,\chi)\right] \tag{12}$$

Suppose we have access to a validation set $V = \{(t_i, U(t_i)) \in \mathbb{R}\times\mathbb{R}^{N_i} | U(t_i) = (u(t_i, x_i^1), ...u(t_i, x_i^{N_i}))\}$, and let $T = (t_1, ...t_{|V|})$

2.4.1 Time and space independant

If the data is relatively sparse (or we wish to spend little on this method), we compute

$$\alpha = \left[\frac{1}{|V|}\sum_{(\tau,\chi)\in V} M_i(\tau,\chi)M_j(\tau,\chi)\right]^{-1}\left[\frac{1}{|V|}\sum_{(\tau,\chi)\in V} M_i(\tau,\chi)u(\tau,\chi)\right] \tag{13}$$

2.4.2 Time dependant

Since covariances should be stable in time, we can expect fitting with a simple GP in time to be possible. We define

$$\alpha_i = \left[\frac{1}{N^i}\sum_{\chi\in(x_i^1,...,x_i^{N_i})} M_i(t_i,\chi)M_j(t_i,\chi)\right]^{-1}\left[\frac{1}{N^i}\sum_{\chi\in(x_i^1,...,x_i^{N_i})} M_i(t_i,\chi)u(t_i,\chi)\right] \tag{14}$$

We then define $\xi \sim \mathcal{N}(\mu, K)$ for $K : (t,t) \mapsto \mathbb{R}$. Then

$$\alpha(t) = \mathbb{E}[\xi|\alpha_i] = \mu + K(t,T)K(T,T)^{-1}\begin{pmatrix}(\alpha_1-\mu)^T\\ ...\\ (\alpha_{|V|}-\mu)^T\end{pmatrix} \tag{15}$$

We can choose $\mu = \frac{1}{n}\mathbb{1}$ or $\mu = \alpha$ defined in (13)

2.4.3 Space dependant

If there are enough points in the validation set, one can average space points locally, ie defining for $k \in \mathbb{N}$, $B_i(x,k) = \{k\ nearest\ neighbors\ of\ x\ in\ (x_i^1, ..., x_i^{N_i}\}$,

$$\alpha(x) = \left[\frac{1}{k|V|}\sum_i\sum_{\chi\in B_i(x,k)} M_i(t_i,\chi)M_j(t_i,\chi)\right]^{-1}\left[\frac{1}{k|V|}\sum_i\sum_{\chi\in B_i(x,k)} M_i(t_i,\chi)u(t_i,\chi)\right] \tag{16}$$

2.4.4 Time and space dependent

We can thus define, without averaging in time in (16)

$$\alpha(t,x) = \mathbb{E}[\xi|\alpha_i] = \mu + K(t,T)K(T,T)^{-1}\begin{pmatrix}(\alpha_1(x)-\mu)^T\\ ...\\ (\alpha_{|V|}(x)-\mu)^T\end{pmatrix} \tag{17}$$

References