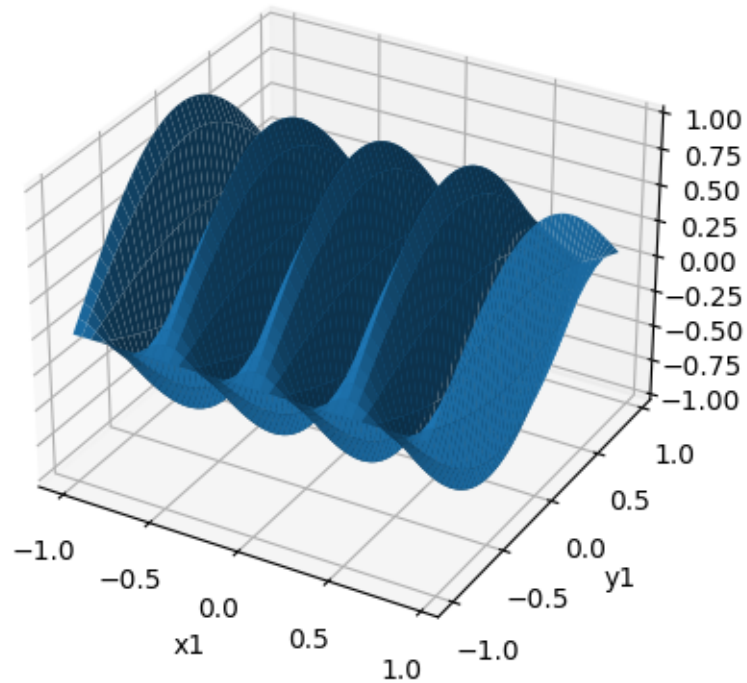


# laplace aggregation many

May 5, 2023

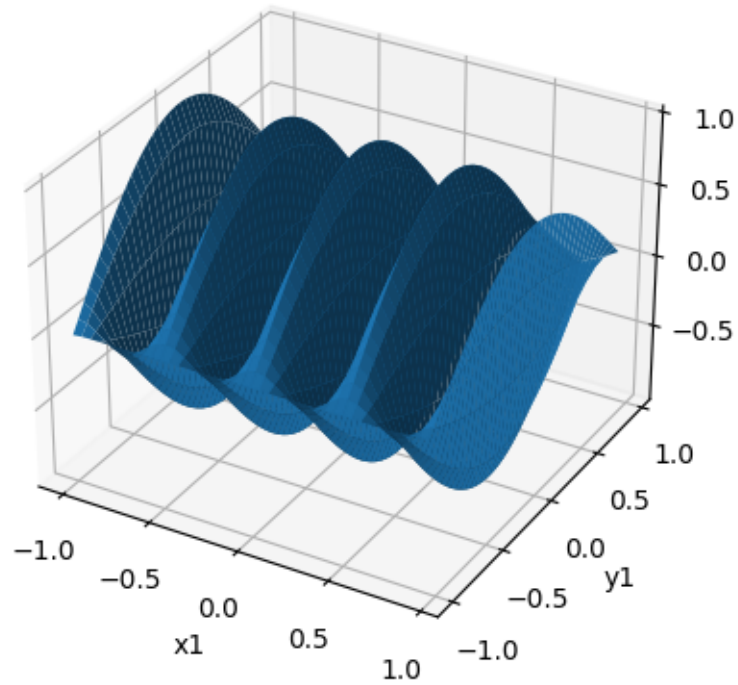
```
[ ]: fig=plot_function(real_u)  
plt.title('Real solution of the PDE')  
plt.show()
```

Real solution of the PDE



```
[ ]: fig=plot_vectorized_function(u)  
plt.title('Numerical solution of the PDE')  
plt.show()
```

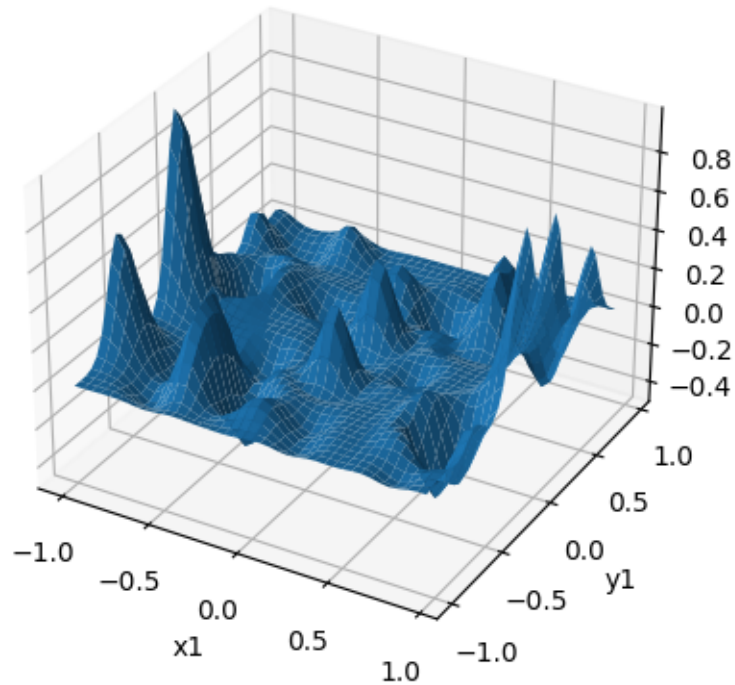
## Numerical solution of the PDE



```
[ ]: models=[  
    ↳  
    ↳PDESolverLaplace(X_int=make_points_inside(50),X_boundary=make_points_boundary(10),sigma=0.  
    ↳1,name=f'model {k}') for k in range(100)  
]  
for model in models:  
    model.fit(f,real_u,nugget=1e-5)
```

```
[ ]: fig=plot_vectorized_function(models[46])  
plt.title('Numerical solution of the PDE with only a few points')  
plt.show()
```

## Numerical solution of the PDE with only a few points



```
[ ]: aggregate=AggregateLaplace(models,nugget=1e-5)
mean_sol=lambda x: np.mean(np.array(list(map(lambda model:model(x),models)))).
    ↳T,axis=1)
```

```
[ ]:
```

```
M [[ 9.08423728e-03  1.75461673e-03  4.83645062e-06 ...  3.25619875e-03
      7.62002450e-03  7.47020411e-02]
 [ 1.20567571e-02  1.69898948e-03  1.68019268e-05 ...  4.40530443e-03
      9.40376173e-03  7.22746091e-02]
 [ 1.79136950e-03  1.55815710e-03  4.99210918e-05 ...  4.92808352e-03
      5.10355982e-03  5.31622702e-02]
 ...
 [-1.13199155e-02  3.90027159e-03  9.48465644e-04 ... -2.26158928e-03
      2.28082406e-02 -4.76468294e-03]
 [ 2.36361027e-03  2.36846851e-03  5.03270041e-04 ... -6.62870993e-04
      4.08380513e-02  1.59135498e-02]
 [ 1.13894051e-02  8.57158463e-04  1.90641943e-04 ... -1.36317887e-04
      5.88050041e-02  3.84325603e-02]]
COV mat [[[ 1.42468761e-02 -5.82573138e-05 -1.43245404e-07 ...  1.90846375e-04
      2.72045888e-04  3.83081859e-02]
 [-5.82573138e-05  4.71857185e-05 -7.03186218e-08 ...  1.02684866e-05
```

```

-3.86113815e-05  7.87846741e-05]
[-1.43245404e-07 -7.03186218e-08  1.00194056e-05 ... -1.92989385e-07
 1.67983981e-07  5.83148868e-07]
...
[ 1.90846375e-04  1.02684866e-05 -1.92989385e-07 ...  3.09392031e-04
-1.30331757e-04  3.50695222e-04]
[ 2.72045888e-04 -3.86113815e-05  1.67983981e-07 ... -1.30331757e-04
 5.89425303e-03  4.33554804e-03]
[ 3.83081859e-02  7.87846741e-05  5.83148868e-07 ...  3.50695222e-04
 4.33554804e-03  5.40486775e-01]]

[[ 1.99652312e-02 -9.69498857e-05 -8.24394213e-07 ...  4.27117126e-04
 3.73451153e-03  5.32736980e-02]
[-9.69498857e-05  3.82850645e-04 -1.22839553e-06 ...  1.40182781e-05
-5.42855970e-05  1.03445932e-04]
[-8.24394213e-07 -1.22839553e-06  1.04049753e-05 ... -1.47708075e-06
-1.59261896e-06  3.45329858e-07]
...
[ 4.27117126e-04  1.40182781e-05 -1.47708075e-06 ...  8.07639686e-04
 4.14640549e-04  1.06447319e-03]
[ 3.73451153e-03 -5.42855970e-05 -1.59261896e-06 ...  4.14640549e-04
 2.56772137e-02  1.24787446e-02]
[ 5.32736980e-02  1.03445932e-04  3.45329858e-07 ...  1.06447319e-03
 1.24787446e-02  3.97704286e-01]]

[[ 2.77040845e-03 -8.27528210e-05 -1.45624913e-06 ...  1.70413620e-04
 2.16549256e-03  1.56898691e-02]
[-8.27528210e-05  2.61138569e-03 -1.15706550e-05 ... -3.61114041e-05
 4.78685791e-04  2.73196020e-04]
[-1.45624913e-06 -1.15706550e-05  1.50538543e-05 ... -7.06577245e-06
-1.10393470e-05 -3.91619721e-06]
...
[ 1.70413620e-04 -3.61114041e-05 -7.06577245e-06 ...  1.53957592e-03
 1.46010416e-03  1.44707644e-03]
[ 2.16549256e-03  4.78685791e-04 -1.10393470e-05 ...  1.46010416e-03
 8.48843381e-02  2.90098711e-02]
[ 1.56898691e-02  2.73196020e-04 -3.91619721e-06 ...  1.44707644e-03
 2.90098711e-02  1.96953119e-01]]

...

[[ 3.59082451e-03  2.14171461e-04  3.30005976e-04 ...  1.11574593e-05
-7.65403020e-04 -1.26682501e-03]
[ 2.14171461e-04  1.92097212e-02  3.98161937e-04 ... -4.46265520e-04
 4.06061285e-04  9.97983846e-04]
[ 3.30005976e-04  3.98161937e-04  4.69333107e-02 ... -3.18957100e-04
 2.80646961e-04 -5.46286567e-03]
...

```

```

[ 1.11574593e-05 -4.46265520e-04 -3.18957100e-04 ... 3.98976417e-04
-8.93068531e-06 -1.31279957e-04]
[-7.65403020e-04 4.06061285e-04 2.80646961e-04 ... -8.93068531e-06
9.95011478e-02 -4.40895982e-03]
[-1.26682501e-03 9.97983846e-04 -5.46286567e-03 ... -1.31279957e-04
-4.40895982e-03 7.90241995e-02]]

[[ 1.17374954e-03 -6.23697319e-05 -4.90914979e-05 ... -8.42918084e-07
7.72552518e-04 4.13759524e-03]
[-6.23697319e-05 7.09963673e-03 3.85134517e-05 ... -8.53528626e-05
4.51365860e-04 3.01373332e-04]
[-4.90914979e-05 3.85134517e-05 6.35258204e-03 ... -4.04106972e-05
1.89936769e-04 -2.02313890e-03]
...
[-8.42918084e-07 -8.53528626e-05 -4.04106972e-05 ... 4.40479332e-05
-7.54544307e-07 -3.86367903e-05]
[ 7.72552518e-04 4.51365860e-04 1.89936769e-04 ... -7.54544307e-07
3.47154681e-01 4.31859287e-02]
[ 4.13759524e-03 3.01373332e-04 -2.02313890e-03 ... -3.86367903e-05
4.31859287e-02 8.66111257e-02]]

[[ 7.99216942e-03 -7.53997633e-05 -4.38305639e-05 ... -7.63628633e-07
3.20094768e-03 2.04250062e-02]
[-7.53997633e-05 1.15562775e-03 7.79938775e-06 ... -7.65180964e-06
2.61302885e-04 -1.62242307e-04]
[-4.38305639e-05 7.79938775e-06 4.99486217e-04 ... -2.64528783e-06
7.66054822e-05 -3.38505604e-04]
...
[-7.63628633e-07 -7.65180964e-06 -2.64528783e-06 ... 1.16423827e-05
8.00215155e-08 -4.61283740e-06]
[ 3.20094768e-03 2.61302885e-04 7.66054822e-05 ... 8.00215155e-08
7.34900302e-01 1.41844123e-01]
[ 2.04250062e-02 -1.62242307e-04 -3.38505604e-04 ... -4.61283740e-06
1.41844123e-01 1.28422954e-01]]]
COV Y [[1.42368761e-02 3.71857185e-05 1.94056316e-08 ... 2.99392031e-04
5.88425303e-03 5.40476775e-01]
[1.99552312e-02 3.72850645e-04 4.04975314e-07 ... 7.97639686e-04
2.56672137e-02 3.97694286e-01]
[2.76040845e-03 2.60138569e-03 5.05385429e-06 ... 1.52957592e-03
8.48743381e-02 1.96943119e-01]
...
[3.58082451e-03 1.91997212e-02 4.69233107e-02 ... 3.88976417e-04
9.94911478e-02 7.90141995e-02]
[1.16374954e-03 7.08963673e-03 6.34258204e-03 ... 3.40479332e-05
3.47144681e-01 8.66011257e-02]
[7.98216942e-03 1.14562775e-03 4.89486217e-04 ... 1.64238266e-06
7.34890302e-01 1.28412954e-01]]]
alpha [[-1.09373046e-01 -4.71697620e-02 -2.67634098e-03 ... 1.94654926e-02

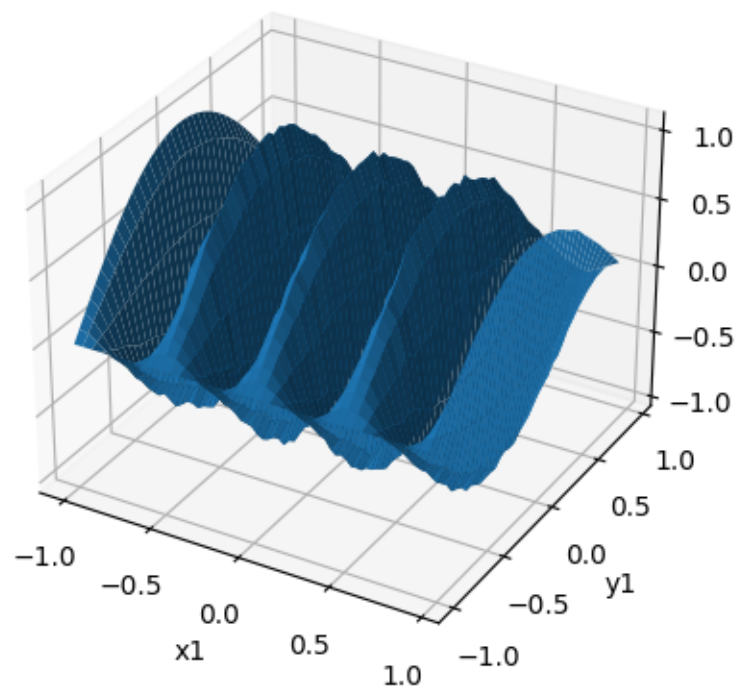
```

```

-3.77201893e-02  7.05086407e-02]
[ 1.94216935e-02  3.05505777e-02  1.54534069e-03 ...  2.90641486e-03
 1.45906881e-02  5.88170417e-04]
[ 1.88778650e-02 -9.89742515e-02  4.41131920e-03 ...  4.42184849e-03
 1.17618008e-02 -5.78924683e-02]
...
[ 4.93513303e-02  8.07794824e-02  9.32976237e-02 ... -5.05723363e-03
-6.34610859e-02  2.11293556e-01]
[ 5.64732558e-01  1.17603789e-01  1.31742416e-01 ... -4.20979819e-01
 9.56146251e-05  2.92160961e-01]
[ 3.79622543e-01  4.99359220e-02 -9.62809724e-02 ... -1.82745769e-01
 4.16008561e-04  2.23503692e-02]]

```

Numerical solution of the PDE

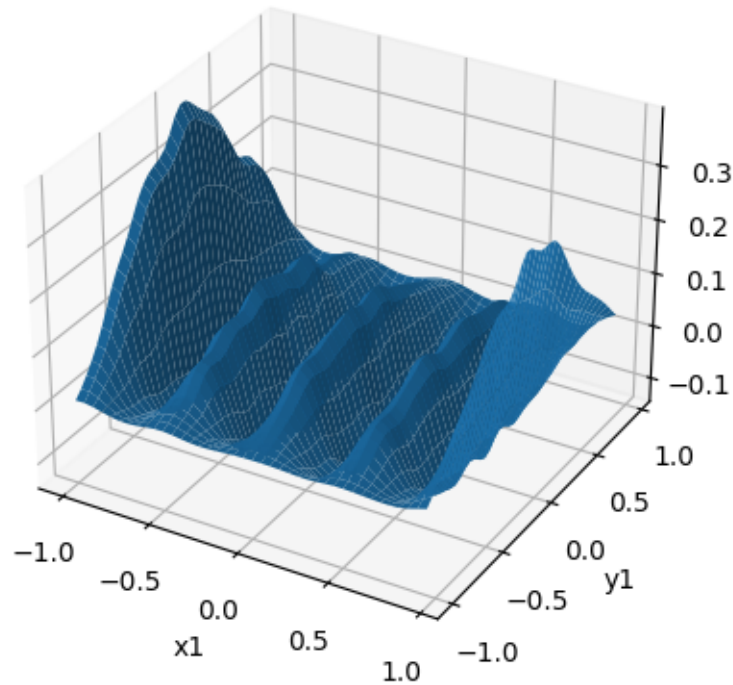


```

[ ]: fig=plot_vectorized_function(mean_sol,0.05)
plt.title('Mean solution of the PDE')
plt.show()

```

## Numerical solution of the PDE



```
[ ]: print(np.linalg.norm(U-U_real,ord=np.inf))
      np.linalg.norm(U_agg-U_real,ord=np.inf)
```

0.014706481303046434

```
[ ]: 0.1546384132101291
```

```
[ ]: np.linalg.norm(np.mean(U_indivs,axis=0)-U_real,ord=np.inf)
```

```
[ ]: 0.9287964838396056
```

```
[ ]: x_sample=np.array([[-0.7,-0.6]])
      print(real_u(x_sample[0]))
      aggregate(x_sample)
```

-0.47552825814757677

```
M [[-0.0492976  -0.01808238 -0.0475608  -0.03160416 -0.00652555 -0.01995485
      -0.05836062  0.01487124  0.00700029 -0.03186571 -0.02011299 -0.02936412
      -0.01593538  0.00414316 -0.07581913 -0.03206507 -0.0249035  0.02392429
      -0.03385113 -0.01870284 -0.1891493  -0.03282358  0.00543214 -0.04555898
      -0.04967589 -0.00417274  0.00281238 -0.17545846  0.00156952 -0.17768556
      0.02117816 -0.0169925  -0.04930442 -0.18037675 -0.04168831 -0.00882301
```

```

0.00834389 -0.19481758 0.00805644 0.01169081 0.00438257 -0.08741213
-0.05066567 -0.01389079 -0.14827093 -0.00093711 -0.09243108 -0.16920922
0.04592385 -0.05465784 -0.05181101 0.01051667 -0.12203037 -0.01061689
-0.04704995 -0.00201993 -0.01310456 -0.01576684 -0.02055987 0.01319246
-0.10966741 0.00753863 -0.24385947 0.00377382 -0.12157246 -0.1178463
-0.02469506 -0.14397355 0.01957776 -0.0926597 -0.00540361 -0.04762236
0.029722 -0.0014133 -0.03518548 -0.02729242 -0.02198009 -0.15485132
0.00638634 -0.03691947 -0.197742 -0.04319756 0.01389219 -0.09503486
-0.16590584 -0.23997209 -0.08113865 -0.2124152 -0.04442653 -0.15312914
-0.06355354 0.00612935 -0.11071308 -0.01988077 -0.01368733 -0.01398563
0.00144078 -0.0986291 0.00368526 -0.11276985]]
COV mat [[[ 8.29973981e-03 2.75106238e-03 2.58019734e-03 ... 2.92814856e-04
1.57907329e-03 -5.64388142e-04]
[ 2.75106238e-03 3.07703932e-03 -3.90391832e-05 ... -4.29544096e-04
1.63229799e-04 2.16760589e-05]
[ 2.58019734e-03 -3.90391832e-05 1.07091952e-01 ... -1.36737885e-02
3.04846251e-03 -1.30418847e-03]
...
[ 2.92814856e-04 -4.29544096e-04 -1.36737885e-02 ... 1.65799183e-01
8.44108320e-03 7.04026527e-02]
[ 1.57907329e-03 1.63229799e-04 3.04846251e-03 ... 8.44108320e-03
1.38514114e-02 3.69146291e-03]
[-5.64388142e-04 2.16760589e-05 -1.30418847e-03 ... 7.04026527e-02
3.69146291e-03 6.36484736e-02]]]
COV Y [[8.28973981e-03 3.06703932e-03 1.07081952e-01 3.06635341e-02
8.99892614e-03 1.52034874e-01 2.97917642e-02 7.77073728e-03
2.03096789e-02 1.80989349e-02 7.53229926e-03 1.65258254e-02
1.49847705e-03 1.71520128e-03 2.86060140e-02 2.87906487e-02
1.87907300e-01 6.58692438e-03 1.72993947e-02 1.53973912e-03
4.18027162e-01 2.20381078e-02 2.57695340e-03 3.13524783e-02
2.58592531e-01 3.45121425e-02 2.18095624e-04 2.06948974e-01
1.40279680e-02 2.95317706e-01 1.29349070e-02 1.43638491e-02
4.53940069e-02 1.87645423e-01 2.35514140e-02 9.55534277e-03
5.56061492e-03 3.35856518e-01 1.04869001e-02 3.65135159e-03
4.63931612e-03 2.42736046e-01 2.69820671e-02 3.96461615e-03
1.83527615e-01 1.01420231e-02 3.85861345e-02 2.68165171e-01
1.92428160e-02 1.57783229e-02 7.18992280e-02 1.14419838e-02
1.51113645e-01 1.12123331e-02 2.01770154e-02 7.81575461e-03
5.33490618e-02 5.66844058e-03 6.39553504e-03 3.34465512e-02
1.33013842e-01 2.53283464e-03 3.78183551e-01 1.92602206e-02
2.69776632e-01 1.30958218e-01 1.54633995e-02 2.14569789e-01
9.39995336e-03 2.44680853e-01 7.70030932e-04 8.77777985e-02
8.98724632e-03 5.04688203e-03 1.39417418e-01 9.17028997e-03
2.36536694e-02 2.82205319e-01 4.25877138e-02 1.21989385e-01
3.03795672e-01 1.49827022e-02 1.26051770e-02 1.33971635e-01
4.09075934e-01 3.28656167e-01 3.27825647e-01 4.61502506e-01
1.42374946e-02 3.03268060e-01 1.33819857e-01 2.04145151e-03
1.26611315e-01 5.23426164e-03 1.18844260e-01 8.65302922e-02

```



```

1.97780800e-02 1.65789183e-01 1.38414114e-02 6.36384736e-02]]
alpha [[-0.23890194  0.01681445 -0.26679003 -0.45205939  0.10721399  1.42822934
-0.58219274 -2.79878015 -1.20970417 -2.18225717 -3.13511902 -1.46805988
-2.0422377  2.17004072 -1.20041781 -0.74340338  1.59266376  0.45112275
-0.50726997 -1.82555191  0.8727413  1.17631177 -2.7077272 -0.30475232
-1.11654026  1.12113287 -6.32437953 -0.14974331 -1.72958755  1.08098044
 0.63554359 -3.6599343 -0.0700554  0.18684425 -0.58777087 -3.74403856
 3.26326124 -2.32804354  0.07955543 -3.00942824 -1.76718409 -0.53757937
-2.26915413  0.19146782 -0.96822933 -1.17573367  0.56937992 -0.95074992
-1.59780021  0.50897896  0.60328084  2.1407687 -0.83609368 -0.77812631
 0.72897516  5.202751 -0.96145395  2.85268783 -2.31424833 -0.42309577
-0.42772949 -1.82574249  0.3166862 -2.60887634 -0.07818005 -0.85152834
-3.53570629  3.5623061 -0.60043789 -1.15461443  0.92532247  1.2458825
 0.93627413 -0.51902527  1.29144251  1.61465895 -1.53159023  0.42816142
-3.01291508 -0.99216844  0.19248322  2.61353231  0.60013211  0.73326983
 1.0774124  0.26422344 -0.11586153  0.51037958 -0.05901469  0.42040728
-0.45576305 -3.03934478  1.79876837 -4.29905963  0.94395802 -1.6253199
-1.37491581  1.36216599 -0.02130969  1.00459232]]

```

```
[ ]: array([-0.44695107])
```

```

[ ]: ax=plt.subplot()
clip_error=np.linalg.norm(U_agg-U_real,ord=np.inf)
im = ax.imshow(np.clip(np.abs(U_agg-U_real),0,clip_error).reshape(X.
    ↪shape),extent=[-1,1,1,-1])
#ax.scatter(u_right.X_int[:,0],u_right.X_int[:,1],c='r',s=4)
#ax.scatter(u_left.X_int[:,0],u_left.X_int[:,1],c='g',s=2)
plt.colorbar(im)

```

```
[ ]: <matplotlib.colorbar.Colorbar at 0x1388a6d10>
```

