Exam of 5OD14

25 March 2025

Notes: this exam lasts 2 hours, you can use the lecture notes that were distributed during the classes. There are 2 exercises and a 2 bonus questions. You can refer to Theorems that are present in the notes by indicating their number and page. In the interest of time, keep your answers concise and to the point.

Exercise 1. Control of flexible structures

Consider a modern aircraft, whereby we cover the wing with smart material actuators. The actuators can change the shape of the wing by contracting or extending. Consider the position of actuator i at time k as $x_{i,k} \in \mathbf{R}$. Stack the positions of all actuators $i = 1, \dots, n$ into a vector $x_k \in \mathbf{R}^n$. We can model the position of actuators at time k, as the following linear dynamical system,

$$x_k = x_{k-1} + \begin{bmatrix} \alpha & \beta & & \mathbf{0} \\ \beta & \alpha & \beta & \\ & \ddots & \ddots & \ddots \\ \mathbf{0} & & \beta & \alpha \end{bmatrix} u_k = x_{k-1} + Bu_k, \tag{1}$$

where $u_k = [u_{1,k}, \dots, u_{n,k}]^\top \in \mathbf{R}^n$ is the control action (contraction, extension) of the actuators, and the tridiagonal matrix is composed of coefficients α, β.

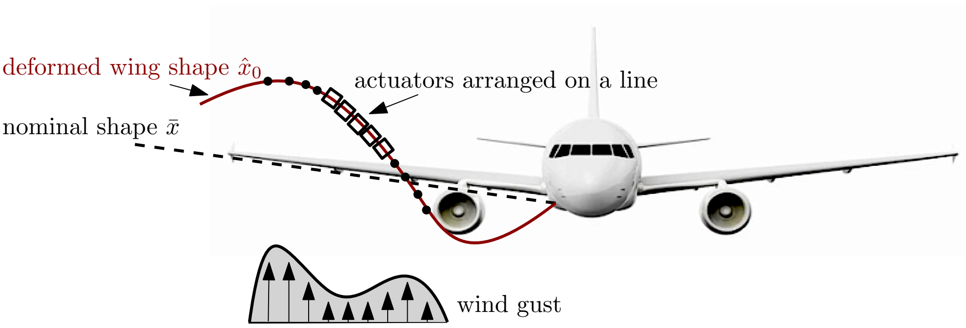

Figure 1: A deformed wing with smart actuators arranged on a line.

A wind gust perturbs the wing and we use the smart actuators to control it back to its original place. We can write this as the following optimization problem,

$$\begin{aligned} \min_{(x_k)_{k=0,\dots,T},(u_k)_{k=1,\dots,T}} \quad & \frac{1}{2}\left(\sum_{k=1}^{T} \|x_k\|_2^2 + \|u_k\|_2^2\right) + \frac{\gamma}{2}\|x_T - \bar{x}\|^2, & (2) \\ \text{subject to} \quad & x_k = x_{k-1} + Bu_k, \forall k \in [1,T] & (3) \\ & x_0 = \hat{x}_0, & (4) \end{aligned}$$

for $\gamma > 0$.

1. Is the problem convex? Strongly convex? Smooth?
2. Write the Lagrangian dual problem of the optimization problem and prove that it can be written as,
$$\max_{\lambda} \sum_{i=1}^{n} q_i(\lambda),$$
where q_i are dual functions only containing the primal variables associated to actuator i, i.e., $(x_{i,k}, u_{i,k})$ for all k's. What is the dimension of λ?
3. (**1st bonus question**) Write the analytical expression for q_i.

4. Suppose that the actuators can communicate with each other on a line graph. Write a decentralized gradient descent algorithm that can solve **the dual problem** in a cooperative fashion. How does the error depends on the number of actuators? How can you retrieve the primal variables?

5. Write a cloud-based ADMM that can solve the dual problem in a cloud-based fashion. Write as much details that you can on what the single actuators have to compute, what they need to communicate, etc. Write also the convergence guarantees that we can expect.

Exercise 2. Self-driving vehicles

You are the CEO of a new startup based in Paris, offering taxi rides via a fleet of self-driving vehicles. Imagine that the vehicles are all the same (say a FIAT 500), and they are equipped with cameras and onboard processor units. Your original idea is to adapt the driving profile of the vehicles to the average customer. In this context, each vehicle can learn a model on how they should drive (stressed, sporty, aggressive, calmly, ..) by collecting labels from the customers.

Imagine that the training optimization problem is convex. You don't want all the vehicles to share all the data with a centralized server, since they don't have the bandwidth to share the camera feeds. However, you would still like that each vehicle improved their model with other vehicles information.

1. Look at this problem from a point of view of federated learning. How would you formulate it and which algorithm would you use to solve it.

2. Your company is doing very good, and you expand your business in other smaller cities like Troyes, Lille, and Tours. You know that the customer type there will be quite different from a typical Parisian. You would still like to share all the models: How would the federated learning approach change?

3. You have now a significant fleet of taxis. You have a proposition to expand your business in Italy, say in Milano. However there they require you to share your models (as they are acquired online) to a local company, for transparency and fair competition. You clearly don't want to share your models, but you think you can reach a compromise, since you would also like to aggregate their models to yours. Which algorithm would you use to share an obfuscated model?

4. Back in Paris, with the lesson you have learned in Italy, you want to start a luxury spin-off, offering taxi drives with Ferraris instead of FIAT 500s. The cars are very different and the customers too. But you still think you can make use of the FIAT 500 and average customer profile to your advantage. How would you personalize the general model to the new spin-off?

2nd bonus question

Describe the literature project you have done. Which algorithm have you studied? What differences does it bring to the algorithms we have studied in the course?