

Exam of the Course: SOD314

16 May 2023

Notes: this exam lasts 2 hours, you can use the material that was distributed during the classes. There are 4 exercises. You can refer to Theorems that are present in the slides by indicating their number and slide page. In the interest of time, keep your answers concise and to the point.

Exercise 1. Consider $N = 7$ personal computers connected as indicated in Figure 1, with bi-directional and synchronous links. Each computer i has m sub-images of a large image. Each sub-image j has $a \times b$ pixels, and each pixel is a RGB vector of 3 dimensions.

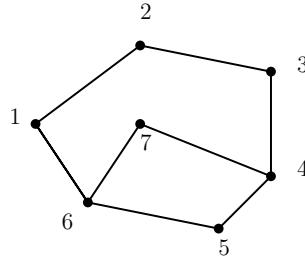


Figure 1: 7 computers in a network.

Vectorize the sub-images into a vector of dimensions $3ab$ and let $y_{i,j}$ the resulting vectorized j -th sub-image of the i -th computer. Imagine that for each sub-image, you have a selection matrix $S_{i,j} \in \mathbf{R}^{3ab \times 3a'b'}$, such that you can select the pixels of the larger image (of $a' \times b'$ pixels) that belong to the sub-image.

Your task is to reconstruct the large image starting from the sub-images solving the problem,

$$\min_{x \in \mathbf{R}^{3a'b'}} \frac{1}{2} \|x\|_2^2 + \sum_{i=1}^N \sum_{j=1}^m \|S_{i,j}x - y_{i,j}\|_2^2.$$

1. Write each step of the gradient tracking algorithm, paying special attention that x is a vector. If needed, choose a mixing matrix accordingly.
2. What are the convergence and convergence rate guarantees that you can give. Argue your answer based on the problem characteristics (is it strongly convex? Smooth? If so compute the constants).
3. Have you chosen a step size? If so how?
4. Consider a peer-to-peer ADMM approach to solve the problem. Write the resulting algorithm with great detail, and discuss its convergence and convergence rate guarantees.
5. How the above discussions would change if we substitute $\|S_{i,j}x - y_{i,j}\|_2^2$ with $\|S_{i,j}x - y_{i,j}\|_1$?
6. Consider adding now a new *directed* link from computer 1 to computer 4. Write a decentralized gradient descent with a Push-Sum protocol to solve the problem.

Exercise 2. Reconsider the problem in Exercise 1. We look now at cloud settings.

1. Write a cloud-based ADMM algorithm to solve the problem. Write the resulting algorithm with great detail, and discuss its convergence and convergence rate guarantees.
2. Write a FedAvg algorithm to solve the problem. Consider B batches in the SGD iterations, each batch randomly constructed with $p < m$ sub-images.
3. When $p = m$, compare the FedAvg algorithm with the cloud-based ADMM algorithm step by step.
4. Describe briefly differential privacy and write a mechanism to render FedAvg differentially private.