

# MACHINE LEARNING

## 02 REGRESSION WITH MULTIPLE INPUT VARIABLES



# UNE SÉRIE DE FORMATIONS

## PARTIE 1 : SUPERVISED MACHINE LEARNING : REGRESSION AND CLASSIFICATION

Introduction to Machine Learning  
**Regression with multiple input**  
Classification

## PARTIE 2 : ADVANCED LEARNING ALGORITHMS

Neural Networks  
Neural Networks training  
Advice for applying Machine Learning  
Decision Trees

## PARTIE 3 : UNSUPERVISED LEARNING, RECOMMENDERS, REINFORCEMENT LEARNING

Unsupervised Learning  
Recommender Systems  
Reinforcement Learning



# 01 SUPERVISED MACHINE LEARNING : REGRESSION AND CLASSIFICATION

## SUPERVISED VS. UNSUPERVISED ML

What is Machine Learning ?  
Supervised Learning  
Unsupervised Learning  
Jupyter Notebooks  
**Lab** : Python and Jupyter Notebooks

## REGRESSION MODEL

Linear regression model  
**Lab** : Model representation  
Cost function formula  
Cost function intuition  
Visualizing the cost function  
Visualization examples  
**Lab** : Cost function

## TRAIN THE MODEL WITH GRADIENT DESCENT

Gradient descent  
Implementing gradient descent  
Gradient descent intuition  
Learning rate  
Gradient descent for linear regression  
Running gradient descent  
**Lab** : Gradient descent



## RAPPEL

### Supervised learning

$x$   
input

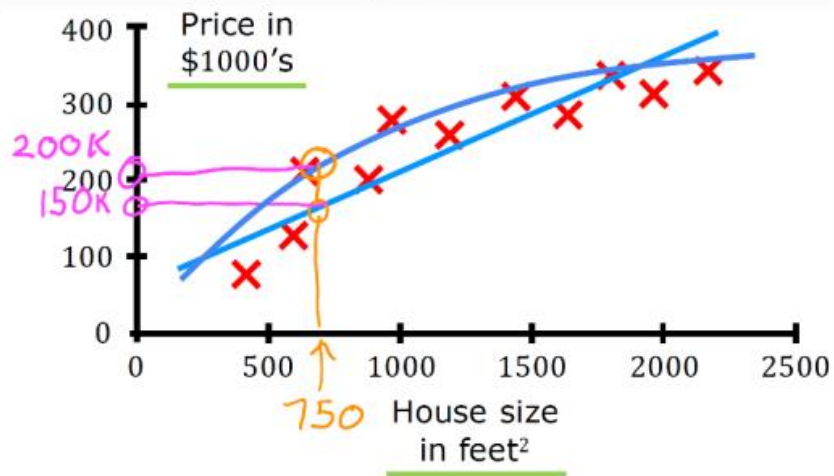
$y$   
output label

learns from being given data labeled with the "right answers"

#### Regression

Predict a number

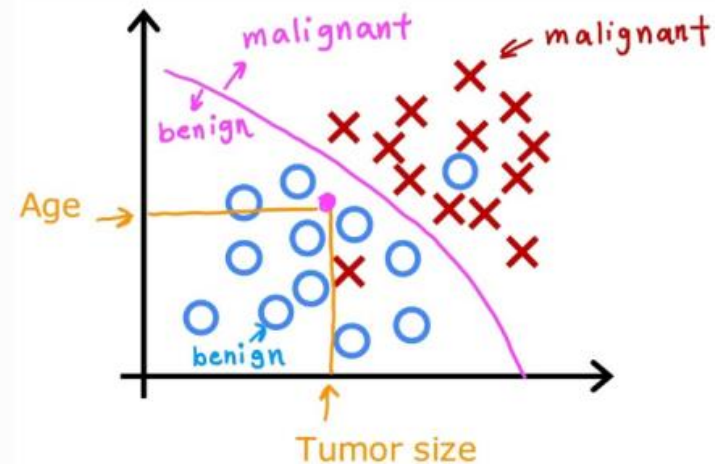
Infinitely many possible outcomes



#### Classification

Predict categories

Small number of possible outputs



## RAPPEL

### Unsupervised Learning

Data only comes with inputs  $x$ , but not output labels  $y$   
Algorithm has to find *structure* (= something interesting in *unlabeled* data)

#### Clustering

Group similar data points together.

#### Anomaly detection

Find unusual data points.

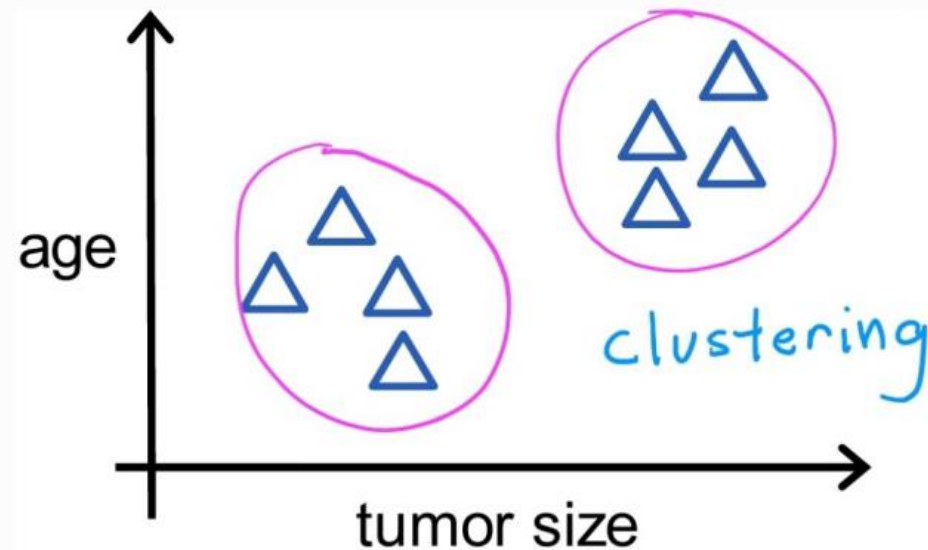
#### Dimensionality reduction

Compress data using fewer numbers.

### Clustering



Example : Google News



## RAPPEL

## Linear Regression Model

## Terminology

$x$	$y$
$x^{(1)}$	$y^{(1)}$
$\vdots$	$\vdots$
$x^{(m)}$	$y^{(m)}$
(training set)	

$x$  = "input" variable / feature

$y$  = "output" variable / "target" variable

$m$  = number of variables in the training set

$w$  = weight /  $b$  = bias

$(x, y)$  = single training example

$(x^{(i)}, y^{(i)})$  =  $i^{\text{th}}$  training example

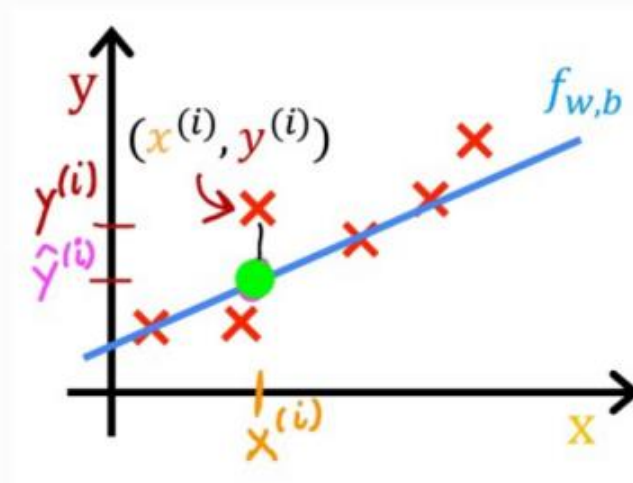


## RAPPEL

## Cost Function

$$\hat{y}^{(i)} = f_{w,b}(x^{(i)}) = wx^{(i)} + b$$

Find  $(w, b)$  such as  $\hat{y}^{(i)} \approx y^{(i)} \forall (x, y)$



## Squared error cost function

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m \left( \underbrace{f_{w,b}(x^{(i)})}_{\hat{y}^{(i)}} - y^{(i)} \right)^2$$

goal: to minimize  $J(w, b)$

## RAPPEL

## Gradient Descent Algorithm

Repeat until convergence :

$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$b = b - \underbrace{\alpha}_{\text{learning rate}} \frac{\partial}{\partial b} J(w, b)$$

Simultaneous update :



$$\text{tmp-}w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$\text{tmp-}b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

$$w = \text{tmp-}w$$

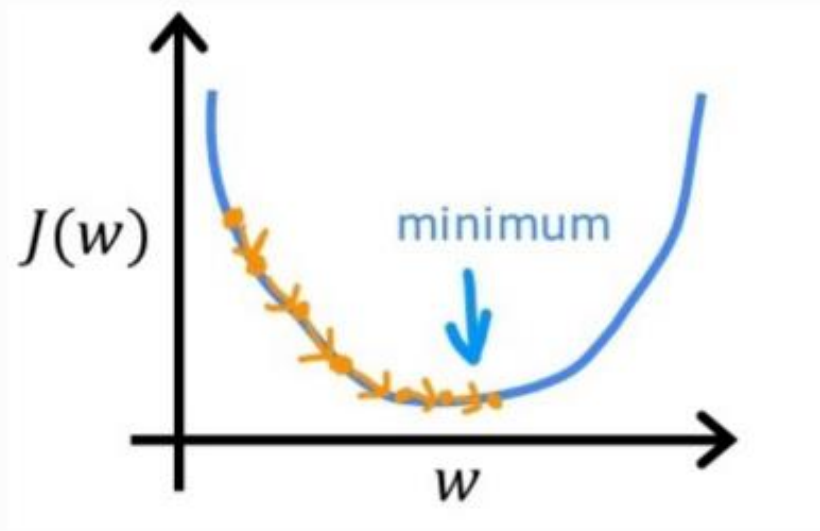
$$b = \text{tmp-}b$$

Batch = each step of gradient descent uses all the training examples

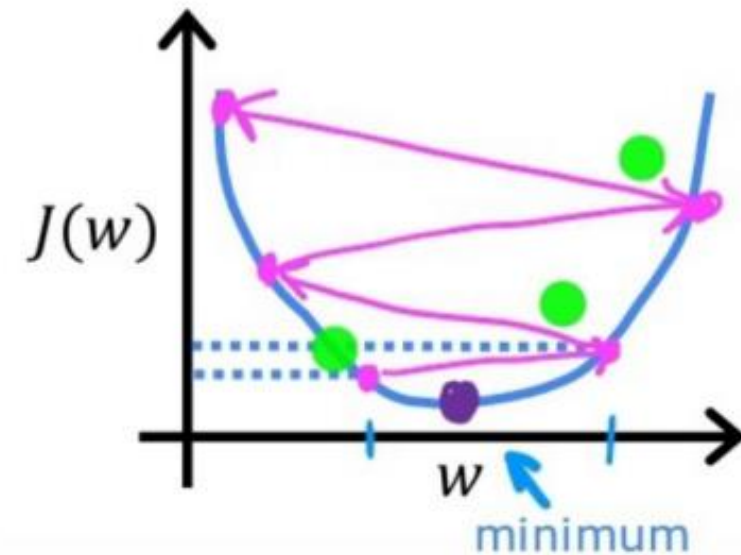


## RAPPEL

## Learning Rate



If  $\alpha$  is too small...  
Gradient descent may be slow.



If  $\alpha$  is too large...

Gradient descent may:

- Overshoot, never reach minimum
- Fail to converge, diverge

# 02

## REGRESSION WITH MULTIPLE INPUT VARIABLES

### INTRODUCTION TO MULTIPLE LINEAR REGRESSION

Multiple features

Vectorization

**Lab** : Python, NumPy and vectorization

Gradient descent for multiple linear regression

**Lab** : Multiple linear regression

### GRADIENT DESCENT

Feature scaling

Checking gradient descent for convergence

Choosing the learning rate

Feature engineering

Polynomial regression

**Lab** : Feature scaling and learning rate

**Lab** : Feature engineering and polynomial regression

**Lab** : Linear regression with scikit-learn

**Lab** : Linear regression

# 101

## INTRO TO MULTIPLE LINEAR REGRESSION



## Multiple Features

$x_j$  =  $j^{\text{th}}$  feature

$n$  = number of features

$\vec{x}^{(i)}$  = features of  $i^{\text{th}}$  training example

$x_j^{(i)}$  = value of feature  $j$  in  $i^{\text{th}}$  training example

## Model

$$f_{\vec{w}, b}(\vec{x}) = \sum_{i=1}^n w_i x_i + b = w_1 x_1 + \dots + w_n x_n$$

$$\left. \begin{array}{l} \vec{x} = [x_1 \ x_2 \ \dots \ x_n] \\ \vec{w} = [w_1 \ w_2 \ \dots \ w_n] \end{array} \right\} \quad f_{\vec{w}, b}(\vec{x}) = \vec{w} \overset{\text{dot product}}{\cdot} \vec{x} + b$$

## Vectorization

## Without vectorization

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + \dots + w_n x_n$$

$f = 0$   
 for  $i$  in range( $n$ ):  
 $f += w[i] * x[i]$   
 $f += b$



## Vectorization

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$

$f = \text{np.dot}(w, x) + b$   
 multiplies in parallel



## Gradient Descent

$$\vec{w} = \vec{w} - \alpha \vec{d} \quad \text{where} \quad \vec{d} = \left[ \frac{\partial J_{w_1}}{\partial w_1} \quad \frac{\partial J_{w_2}}{\partial w_2} \quad \dots \quad \frac{\partial J_{w_n}}{\partial w_n} \right]$$

# LAB-01

PYTHON, NUMPY AND VECTORIZATION



# Gradient Descent for Multiple Linear Regression

Parameters

$$\vec{w} = [w_1 \dots w_n]$$

$b$  still a number

Model

$$f_{\vec{w}, b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$

Cost

$$J(\vec{w}, b)$$

Gradient  
Descent

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

Gradient  
Descent

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

$$j=1 \quad w_1 = w_1 - \alpha \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$j=n \quad w_n = w_n - \alpha \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)}) x_n^{(i)}$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^m (f_{\vec{w}, b}(\vec{x}^{(i)}) - y^{(i)})$$

## Alternative to gradient descent : Normal Equation Method

### Normal equation

- Only for linear regression
- Solve for  $w$ ,  $b$  without iterations

### Disadvantages

- Doesn't generalize to other learning algorithms.
- Slow when number of features is large ( $> 10,000$ )

### What you need to know

- Normal equation method may be used in machine learning libraries that implement linear regression.
- Gradient descent is the recommended method for finding parameters  $w, b$



# LAB-02

## MULTIPLE LINEAR REGRESSION

# QUESTIONS ?

SUR UN CONCEPT ? UNE IDÉE ?  
SUR UN DÉTAIL DU CODE ?  
(ENVIE D'UNE PAUSE ?)

N'HÉSITEZ PAS !

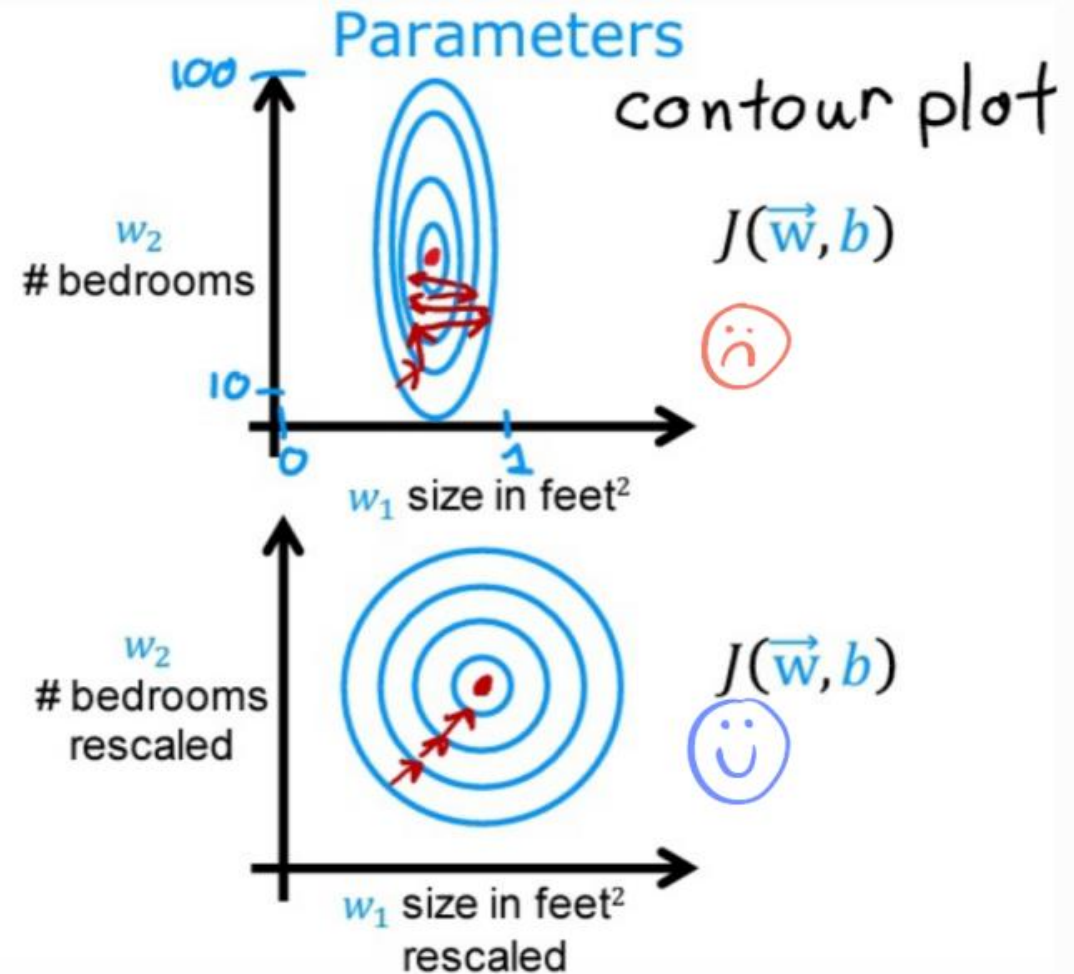
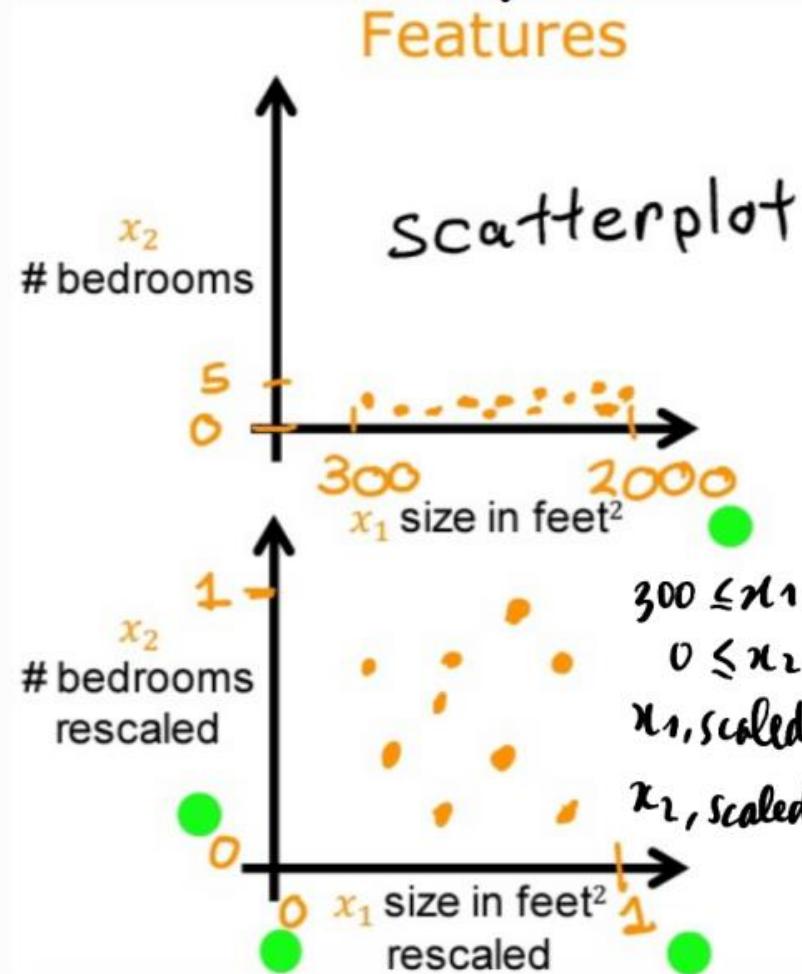
IL N'Y A PAS DE QUESTION BÊTE, SI VOUS AVEZ UN DOUTE, D'AUTRES ONT SÛREMENT LE MÊME

# 102

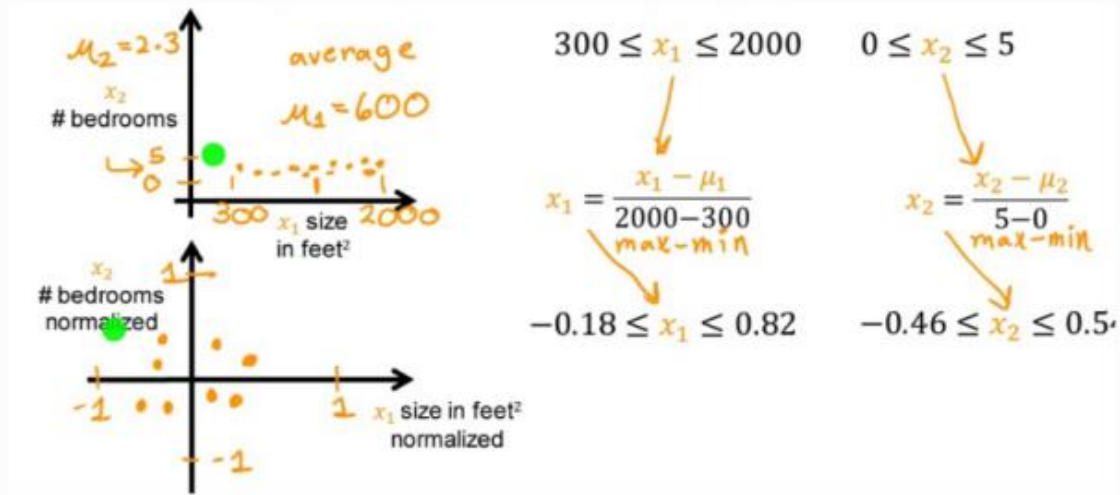
## GRADIENT DESCENT IN PRACTICE



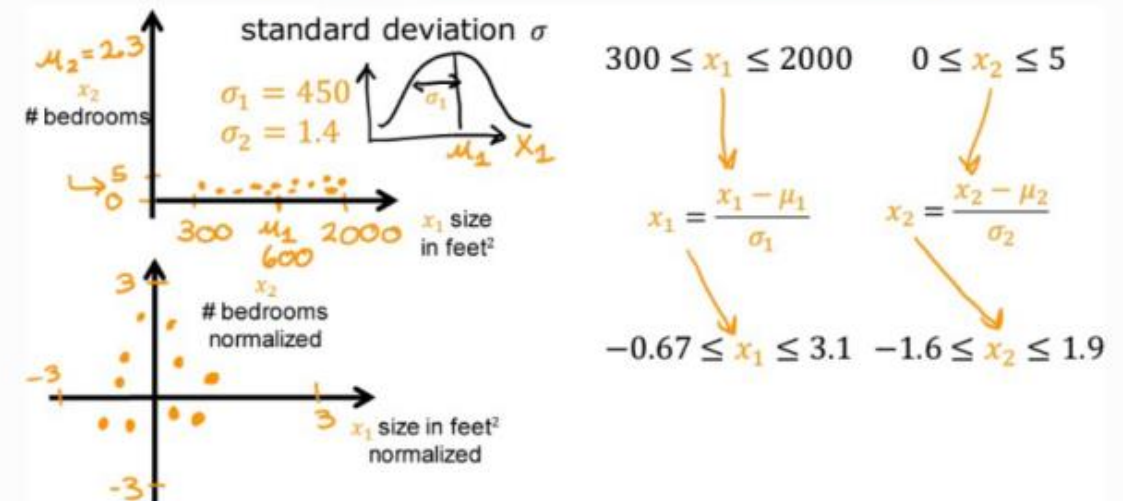
# Feature Scaling



## Mean Normalization



## Z-score Normalization



**Feature Engineering** Using intuition to design **new features** by using **original features**.

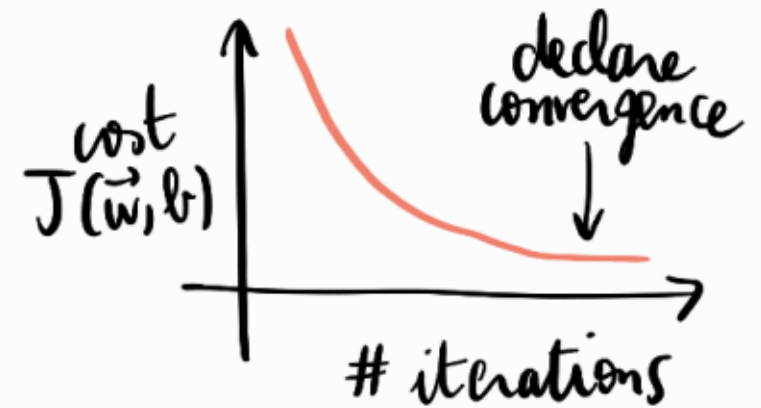
Example: if  $x_1$  = frontage,  $x_2$  = depth, then we can **create**  $x_3 = x_1 \times x_2$  = area of the house, which is more relevant for price prediction  $\Rightarrow f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + w_3 x_3$

## Gradient Descent Convergence

$J(\vec{w}, b)$  should **decrease** after each iteration

## Automatic Convergence Test

Let  $\epsilon = 10^{-3}$ . If  $J(\vec{w}, b)$  decreases by  $\leq \epsilon$  in 1 iteration, declare **convergence** (found parameters  $\vec{w}, b$  to get close to global minimum)





# LAB-03

## FEATURE SCALING AND LEARNING RATE



# QUESTIONS ?

SUR UN CONCEPT ? UNE IDÉE ?  
SUR UN DÉTAIL DU CODE ?  
(ENVIE D'UNE PAUSE ?)

N'HÉSITEZ PAS !

IL N'Y A PAS DE QUESTION BÊTE, SI VOUS AVEZ UN DOUTE, D'AUTRES ONT SÛREMENT LE MÊME

# LAB-04

FEATURE ENGINEERING AND  
POLYNOMIAL REGRESSION

# LAB-05

LINEAR REGRESSION WITH SCIKIT-LEARN

# LAB-06

## LINEAR REGRESSION



# 02 REGRESSION WITH MULTIPLE INPUT VARIABLES

## INTRODUCTION TO MULTIPLE LINEAR REGRESSION

Multiple features

Vectorization

**Lab** : Python, NumPy and vectorization

Gradient descent for multiple linear regression

**Lab** : Multiple linear regression

## GRADIENT DESCENT

Feature scaling

Checking gradient descent for convergence

Choosing the learning rate

Feature engineering

Polynomial regression

**Lab** : Feature scaling and learning rate

**Lab** : Feature engineering and polynomial regression

**Lab** : Linear regression with scikit-learn

**Lab** : Linear regression



# UNE SÉRIE DE FORMATIONS

## PARTIE 1 : SUPERVISED MACHINE LEARNING : REGRESSION AND CLASSIFICATION

Introduction to Machine Learning  
**Regression with multiple input**  
Classification

## PARTIE 2 : ADVANCED LEARNING ALGORITHMS

Neural Networks  
Neural Networks training  
Advice for applying Machine Learning  
Decision Trees

## PARTIE 3 : UNSUPERVISED LEARNING, RECOMMENDERS, REINFORCEMENT LEARNING

Unsupervised Learning  
Recommender Systems  
Reinforcement Learning

# MACHINE LEARNING

## 02 REGRESSION WITH MULTIPLE INPUT VARIABLES