

# Artificial neural network-based error compensation procedure for low-cost encoders

V K Dhar<sup>1</sup>, A K Tickoo<sup>1</sup>, S K Kaul<sup>1</sup>, R Koul<sup>1</sup> and B P Dubey<sup>2</sup>

<sup>1</sup> Astrophysical Sciences Division, Bhabha Atomic Research Centre, Mumbai-400 085, India

<sup>2</sup> Electronics and Instruments Services Division, Bhabha Atomic Research Centre, Mumbai-400 085, India

E-mail: [veer@barc.gov.in](mailto:veer@barc.gov.in)

Received 8 June 2009, in final form 10 November 2009

Published 7 December 2009

Online at [stacks.iop.org/MST/21/015112](http://stacks.iop.org/MST/21/015112)

## Abstract

An artificial neural network-based error compensation method is proposed for improving the accuracy of resolver-based 16-bit encoders by compensating for their respective systematic error profiles. The error compensation procedure, for a particular encoder, involves obtaining its error profile by calibrating it on a precision rotary table, training the neural network by using a part of these data and then determining the corrected encoder angle by subtracting the ANN-predicted error from the measured value of the encoder angle. Since it is not guaranteed that all the resolvers will have exactly similar error profiles because of the inherent differences in their construction on a micro scale, the ANN has been trained on one error profile at a time and the corresponding weight file is then used only for compensating the systematic error of this particular encoder. The systematic nature of the error profile for each of the encoders has also been validated by repeated calibration of the encoders over a period of time and it was found that the error profiles of a particular encoder recorded at different epochs show near reproducible behaviour. The ANN-based error compensation procedure has been implemented for four encoders by training the ANN with their respective error profiles and the results indicate that the accuracy of encoders can be improved by nearly an order of magnitude from quoted values of  $\sim \pm 6$  arc-min to  $\sim \pm 0.65$  arc-min when their corresponding ANN-generated weight files are used for determining the corrected encoder angle.

**Keywords:** resolver-based encoder, error compensation, artificial neural network

## 1. Introduction

A resolver [1, 2] is an electromechanical device which converts shaft angle to an absolute analogue signal. The construction of a resolver is similar to that of an ac electric motor with two sets of phase windings acting together as a variable phase transformer, whose output analogue voltage represents the input shaft angle uniquely. On excitation by a signal, one of the sense windings of the resolver develops a voltage proportional to the sine of the rotor displacement angle and the other develops a voltage proportional to the cosine of this angle. A complete encoder system is built by coupling a resolver to an excitation source and phase angle monitoring circuit known as resolver-to-digital (R/D) converter [3]. The conversion of the

resolver signal to position is achieved by using a tracking loop which monitors the phase lag between the actual and measured position. Resolvers have been available for a few decades in various forms as part of electromechanical shaft angle position measurement systems. They are robust, reliable and have a long life even while working in severe and harsh environments. High reliability and lower price compared to optical encoders make resolver-based encoders an ideal choice for applications involving moderate accuracies of  $\sim 10$  arc-min.

Keeping in view the fact that, in reality, no resolver can generate ideal sinusoidal signals because of the manufacturing tolerances; it is obvious that low-cost resolvers will be available with only modest accuracies. The non-ideal characteristics of a resolver have been discussed thoroughly in

the seminal work of Hanselman [4, 5] and it is now very well established that the most important contributors to the error profile budget are the following: (i) amplitude imbalance, (ii) quadrature error, (iii) inductive harmonics and (iv) excitation signal distortion. The non-ideal characteristics of a resolver arise because of the finite precision with which a resolver can be mechanically constructed and the non-ideal nature of the excitation signal. While low-cost resolvers are available with accuracy values of about  $\pm 10$  arc-min, improvement in their accuracy requires maintaining stringent tolerance limits in their manufacture, which can add to the cost considerably.

The easiest way to reduce or compensate position error due to mechanical misalignments and imperfections in the signal outputs of a resolver is to calibrate it against a higher accuracy sensor so that the integrated systematic error profile of the resolver along with its R/D converter can be measured and then compensated by using a suitable procedure. The results of our software-based compensation work, using the look up table approach [6], and Fourier-series-based method [7] also confirm that the accuracy of low-cost, resolver-based 16-bit encoders can indeed be improved from quoted accuracy  $\sim \pm 6$  arc-min to  $\sim \pm 0.65$  arc-min. The main aim of the present work is to study the feasibility of using artificial neural network for predicting the error profile of encoders so that appropriate correction can be applied to the measured values of the encoder angle for determining the corrected encoder angle. Since all the resolvers will not have exactly similar error profiles because of the inherent differences in their construction, the ANN needs to be trained on one error profile at a time. Hence, corresponding to each encoder, there is a unique ANN-generated weight file which one has to use for compensating the systematic error.

## 2. Summary of some compensation applications

Precision motion systems play an important and direct role in the industry. In these systems involving automated positioning machines and other machine tools, the relative position errors between the machine and the work piece directly affect the quality of the final product or the process concerned. No matter how well the machine may be designed and manufactured, these error sources are inevitable in any motion system, and hence there is an inherent limit to the achievable accuracy on these machines. While careful design and precise mechanical construction can reduce these errors, error modelling and compensation is a highly viable option in improving the system performance further at a much reduced cost. The basic motivation behind the compensation is to measure the magnitude of inaccuracy and compensate it through various compensation methods. As long as the errors are systematic, repeatable and measurable, they can be compensated by using appropriate techniques.

Conventionally, compensation methods utilize mechanical correctors, leadscrew correction etc for improving accuracy. However, these devices increase the complexity of the machine and over a period of time, due to wear and tear, degrade the error compensation. More so, even these corrective components have to be monitored,

serviced, calibrated or even replaced on a regular basis resulting in higher costs and downtime. Software-based error compensation methods include use of fuzzy error interpolation techniques, neural-based approaches, genetic algorithms, finite element analysis and multi-variant regression analysis. All of these methods work on a static geometric model of the machine errors, which are obtained (or derived) from measurements of the machine with reference standards at various predetermined points. With the present day error compensation methods, the conventional limits on the accuracy of machine tools can be overcome significantly [8]. A great deal of work in the recent past has been done in identification and elimination of sources of error in machine systems. The machine manufacturers are now able to achieve better accuracies on account of improved design followed by an appropriate error compensation methodology.

The use of ANN methodology in error compensation has been reported in many studies for various applications. A feed-forward neural network employing 11:15:5 model (11 inputs, 15 hidden nodes and 5 outputs) has been used to predict five thermally induced spindle errors with an accuracy better than  $\pm 15\%$  [9]. Tai *et al* [10] have used an ANN-based algorithm for tracking control of industrial drive systems. Although several tracking control techniques like sliding mode, variable structure, self-tuning and model reference adaptive controls have been used, ANN-based tracking controls have been found to be ideally suited for this application. Using a backpropagation-based ANN algorithm, the authors of the above work claim that the ANN-based controller can achieve real time tracking of any arbitrary prescribed trajectory with a higher degree of accuracy. The idea of using ANN-based models for physical systems has also been explored by several other researchers [11–17] to understand the characteristics of inverse dynamics of controlled systems. An ANN-based approach has been used for error compensation of machine tools also [18]. Important sources of error in CNC machine tools are the geometric motions of the individual machine elements along with the thermal errors which cause these geometric errors to vary over time. A three-layer feed-forward network with 2 inputs (corresponding to  $x$  and  $z$  coordinates in the workspace), 2 outputs (corresponding to position error components) and 12 nodes in the hidden layer has been employed by them for machine error compensation. The results of their study demonstrate that substantial improvements in positioning accuracy are obtained through the use of ANN methodology. It is also emphasized in this work that the algorithm would however fail if tested on another machine with different error characteristics. Tan *et al* [19] have applied ANN-based compensation for geometrical errors of coordinate measuring machines (CMM) to minimize the position error between end-effector and workpiece.

## 3. Measurement of encoder error profiles

Some of the main specifications of the single-turn 16-bit encoders, which are used in the present study, are the following [20]: make—CCC, USA; model—HR 90-11; size—NEMA

12; resolution— $\sim 0.33$  arc-min (16-bit); accuracy— $\pm 6.0$  arc-min. Eight such encoders are being presently used in the four-element TACTIC (TeV atmospheric Cherenkov telescope with imaging camera) array of atmospheric Cherenkov telescopes [21]. Each telescope of the array uses two encoders for monitoring its zenith and azimuth angle so that the drive control software [22] can be operated in a closed-loop configuration for tracking a celestial source. The calibration of the encoders was carried out with a rotary table of a coordinate measuring machine (make: Zeiss, Germany; model: RT 05-300). The rotary table has a resolution of  $\sim 0.5$  arc-s and an accuracy of  $\sim 2$  arc-s [23]. The calibration procedure involves rotating the resolver shaft in a  $2^\circ$  step and recording the decoded resolver angle value along with the corresponding rotary table angle value. The calibration data of a particular encoder, which are needed for training the network, thus consist of a data file of 180 values of the decoded resolver angles and the corresponding rotary table angles. The decoded resolver angles were recorded at rotary table angle values of  $0^\circ, 2^\circ, 4^\circ, 6^\circ, \dots, 358^\circ$ . Although the overall error profile of an encoder system is expected to be dominated by the non-ideal characteristics of the resolver, the error contribution from the decoder (comprising the excitation signal source, R/D converter and angle read out circuitry) cannot be completely ignored, more so, when the excitation signal source itself is contained in the decoder. Since using a single resolver with two different decoder units indicated that for the same resolver angle (as measured by the rotary table) there can be a mismatch of  $\sim 2$  arc-min between the two decoders, we have treated a particular resolver-decoder combination as a single unit and calibrated them together as a single entity. Calibration of the encoders performed in this manner and maintaining a particular resolver-decoder combination throughout its use obviates recording of the resolver and the decoder error profiles individually. The systematic nature of the error profile for each of the encoders was also validated by repeated calibration of the encoders over a period of time and it was found that the error profiles of a particular encoder recorded at different epochs show near reproducible behaviour, to within the limits of the decoder resolution. In order to check the interpolation capability of the neural network, an independent test data sample was also taken for the four encoders. These data were taken using a step size of  $2^\circ$  at rotary table angle values of  $1^\circ, 3^\circ, 5^\circ, 7^\circ, \dots, 359^\circ$ . The test data sample, for a particular encoder, thus consisted of another data file of 180 values of rotary table and the corresponding encoder angle.

A representative example of the measured error profiles for four encoders is shown in figure 1. The data presented in this figure use both the training data file and the test data file. The error at a particular encoder angle ( $\theta_{\text{ENC}}$ ) is evaluated by calculating  $\theta_{\text{ENC}} - \theta_{\text{TAB}}$ , where  $\theta_{\text{ENC}}$  represents the encoder angle and  $\theta_{\text{TAB}}$  is the corresponding angle recorded by the rotary table. It is evident from this figure that the error profiles exhibit widely different patterns. Thus, having several resolvers of the same tolerance value and from the same batch does not necessarily mean that the individual construction and behaviour (i.e. error profiles) are also exactly similar on a micro scale. Therefore, one cannot use the calibration data of

**Table 1.** Mean absolute error (MAE) and the RMS error for the four encoders.

Encoder no.	MAE (arc-min)	RMS (arc-min)
1	1.33	1.48
2	0.55	0.65
3	1.09	1.35
4	1.78	1.31

one resolver for compensating the systematic error of another resolver. The mean absolute error (MAE) and RMS error for the four encoders are given in table 1.

The minimum and maximum errors of the encoders which are observed to be between  $\sim -4.42$  arc-min (for encoder #4) and  $\sim 3.28$  arc-min (for encoder #3) are quite consistent with the tolerance range of  $\sim \pm 6$  arc-min quoted by the manufacturer.

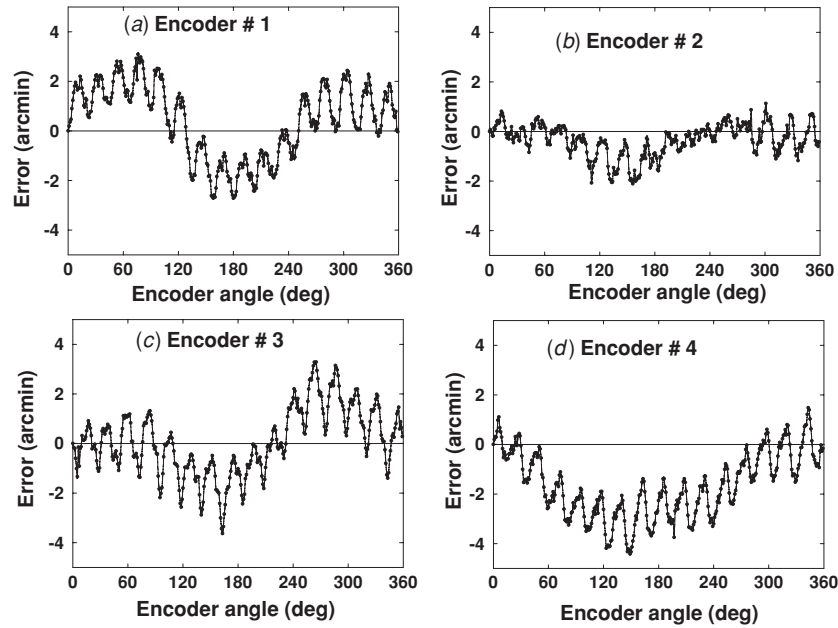
#### 4. Artificial neural network methodology and training of the network

An artificial neural network (ANN) is an interconnected group of artificial neurons that uses a mathematical model for information processing to accomplish a variety of tasks such as pattern recognition and classification. The ability of ANN to handle nonlinear data interactions, and their robustness in the presence of high noise levels, has encouraged their successful use in diverse areas of physics, biology, medicine, agriculture, computer science and astronomy [24, 25]. While the theory and the implementation of ANN have been around for more than 50 years, it is only recently that they have found widespread practical applications. This is primarily due to the advent of high-speed, low-cost computers that can support the rather computationally intensive requirement of an ANN of any real complexity.

In a feed-forward ANN the network is constructed using layers where all nodes in a given layer are connected to all nodes in a subsequent layer. The network requires at least two layers, an input layer and an output layer. In addition, the network can include any number of hidden layers with any number of hidden nodes in each layer. The signal from the input vector propagates through the network layer by layer till the output layer is reached. The output vector represents the predicted output of the ANN and has a node for each variable that is being predicted. The task of training the ANN is to find the most appropriate set of weights for each connection which minimizes the output error. All weighted inputs are summed at the neuron node and this summed value is then passed to a transfer (or scaling) function. For a feed-forward network with  $K$  input nodes described by the input vector  $(x_1, x_2, \dots)$ , one hidden layer with  $J$  nodes and  $I$  output nodes, the output  $F_i$  is given by the following equation:

$$F_i = g \left[ \sum_{j=1}^J w_{ij} g \left( \sum_{k=1}^K w_{jk} x_k + \theta_j \right) + \theta_i \right], \quad (1)$$

where  $w_{ij}$ ,  $w_{jk}$  are the weights;  $\theta_i$ ,  $\theta_j$  are the thresholds and  $(g^*)$  is the activation function. The training data sample is



**Figure 1.** (a)–(d) The error profiles (i.e.  $\theta_{\text{ENC}} - \theta_{\text{TAB}}$ ) of encoders as a function of the encoder angle ( $\theta_{\text{ENC}}$ ). A smoothed spline curve passing through the data points has also been drawn in these figures so that the periodic structure present in the error profiles can be visualized easily.

repeatedly presented to the network in a number of training cycles, and the adjustment of the free parameters ( $w_{ij}$ ,  $w_{jk}$ ,  $\theta_i$  and  $\theta_j$ ) is controlled by the learning rate  $\eta$ . The essence of the training process is to iteratively reduce the error between the predicted value and the target value. While the choice of using a particular error function is problem dependent, there is no well-defined rule for choosing the most suitable error function. We have used the mean-squared error, MSE, in this work and it is defined as

$$\text{MSE} = \frac{1}{PI} \sum_{p=1}^P \sum_{i=1}^I (D_{pi} - O_{pi})^2, \quad (2)$$

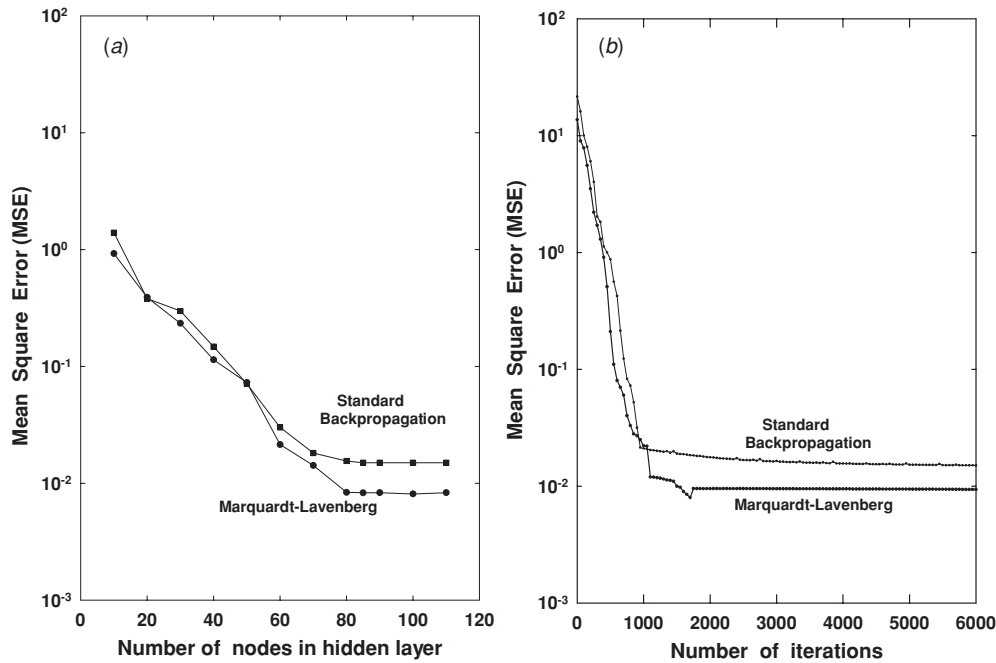
where  $D_{pi}$  and  $O_{pi}$  are the desired and the observed values and  $P$  is the number of training patterns. The MSE depicts the accuracy of the neural network mapping after a number of training cycles have been implemented. In supervised learning, the correct results (i.e. target values or desired outputs) are known and are given to the neural network during training so that it can adjust its weights to match its outputs to the target values. After training, the neural network is tested by giving it only input values, and seeing how close it comes to reproducing the correct target values.

Given the inherent power of ANN to effectively handle the multivariate data fitting, we have studied the feasibility of using it for predicting the error profiles of the encoders. The aim is to determine the correct encoder angle by applying the ANN-predicted error correction to the measured value of the encoder angle. In this work, we have used MATLAB [26] for training and testing various ANN algorithms. MATLAB is a numerical computing environment which allows easy matrix manipulation, plotting of functions and data, implementation of algorithms, creation of user interfaces and interfacing with programs in other languages. The neural network toolbox extends MATLAB with tools for designing, implementing, visualizing and simulating neural networks.

The training of the network is performed by presenting 180 values of encoder angles at one node in the input layer with one node in the output layer representing the corresponding encoder error in arc-min (i.e.  $\theta_{\text{ENC}} - \theta_{\text{TAB}}$ ). Since it is well known that one hidden layer is sufficient for approximating well-behaved functions [27], we have also used one hidden layer in the present study. The activation function chosen for the present study is the sigmoid function.

Backpropagation algorithm [28] has been, by far, the most popular and widely used learning technique for ANN training; we have also used the same algorithm to begin with. With regard to choosing the number of nodes in the hidden layer it is well known that while using too few nodes will starve the network of the resources that it needs to solve a particular problem; choosing too many nodes has the risk of potential overfitting where the network tends to remember the training cases instead of generalizing the patterns. In order to find the optimum number of nodes in the hidden layer for reproducing the error profile of a particular encoder with reasonably good accuracy, we changed the number of nodes in the hidden layer from 10 to 110 (in steps of 10 nodes) and the corresponding MSE was recorded at the end of ANN training for each configuration. The results of this study reveal that while the MSE decreased considerably when the number of nodes in the hidden layer was increased from 20 to 80; increasing the same beyond 80 resulted in only a marginal reduction in the MSE (at the cost of higher computation time). It thus seems that one hidden layer with  $\sim 80$  nodes is quite optimum for reproducing the error profile of the encoder. Using the same number of nodes in the hidden layer (i.e. 80), we then studied the performance of several other error minimization algorithms for identifying the most suitable algorithm which yields the lowest MSE. The algorithms studied were the resilient backpropagation, conjugate gradient, Fletcher–Reeves, one step secant, Levenberg–Marquardt, scaled conjugate and





**Figure 2.** (a) Mean-squared error as a function of number of nodes in the hidden layer. (b) Mean-squared error as a function of number of iterations for 80 nodes in the hidden layer.

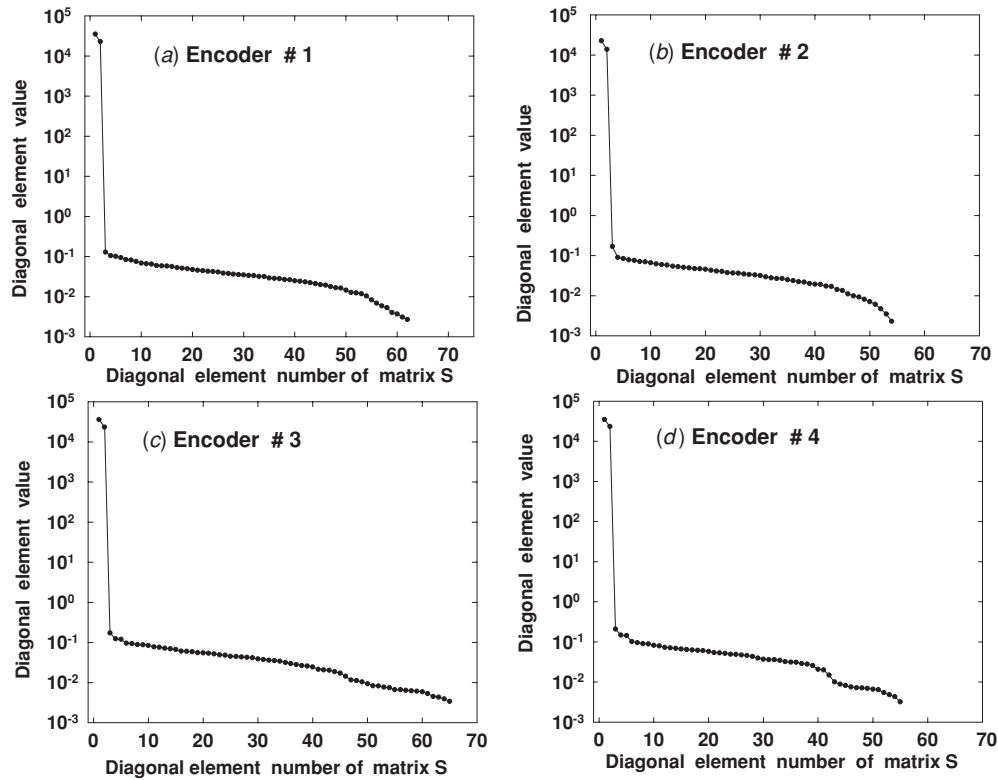
**Table 2.** Comparative MSE values for all the algorithms considered in this work.

Algorithm	MSE
Res-backpropagation	0.0150
Conj-gradient	0.0103
Fletcher-Reeves	0.0144
One step-secant	0.0147
Levenberg-Marquardt	0.0083
Scaled-conjugate	0.0096
Quasi-Newton	0.0091

quasi-Newton [29–31]. Amongst all these algorithms, it was found that Levenberg–Marquardt yields the lowest MSE value of  $\sim 0.0083$ . Comparative MSE for other algorithms considered by us are given in table 2. Figure 2(a) depicts the behaviour of the MSE as a function of number of nodes in the hidden layer for the backpropagation and Levenberg–Marquardt algorithms. The MSE as a function of number of iterations is shown in figure 2(b) for these algorithms. An examination of figure 2(b) clearly reveals that the performance of the Levenberg–Marquardt algorithm is better than that of the backpropagation algorithm. It is worth mentioning here that in order to ensure that the network has not become ‘over-trained’ [32], the ANN training is stopped when the normalized RMS error stops decreasing any further (somewhere around 6000 iterations). The MSE for the ANN configuration of 1:80:1 was also checked for the remaining three encoders and it was found that it varies from  $\sim 0.0098$  to  $\sim 0.0165$  for the remaining three encoders.

A detailed study for determining the optimum number of nodes in the hidden layer in a rigorous manner has also been conducted to make sure that the ANN configuration

chosen is not more complex than what is actually warranted. While several empirical results have been reported which provide *ad hoc*, heuristic rules for selecting the number of nodes in a hidden layer, we have followed the singular value decomposition (SVD) approach for finding the optimum number of nodes in the hidden layer by eliminating the redundant nodes [33]. Modification of the ANN structure by analysing how much each node contributes to the actual output of the neural network and dropping the nodes which do not significantly affect the output is also referred to as pruning. The basic principle of pruning relies on the fact that if two hidden nodes give the same outputs for every input vector, then the performance of the neural network will not be affected by removing one of the nodes in the hidden layer. In the SVD approach redundant hidden nodes cause singularities in the weight matrix which can be identified through inspection of its singular values. A non-zero number of small singular values indicates redundancy in the initial choice for the number of hidden layer nodes and the approach can be safely used for eliminating these nodes to attain the pruned network model. The weight matrix (denoted by  $X$  in the present work) was generated by finding the output of each of the 80 hidden nodes before subjecting them to nonlinear transformation (i.e. output of the node before sigmoid function). With a total of 180 training patterns and one hidden layer with 80 nodes, the matrix  $X$  thus has 180 rows and 80 columns. The SVD of the matrix  $X$  is given by  $X = U S V^T$ , where  $U$  and  $V$  are the orthogonal matrices and  $S$  is a diagonal matrix with 80 rows and 80 columns. The matrix  $S$  contains the singular values of  $X$  on its diagonal. The plot of non-zero diagonal elements for all four encoders is shown in figure 3 and it is evident from this figure that the number of diagonal elements with non-zero values is  $< 80$ . The number of diagonal elements with non-zero



**Figure 3.** (a)–(d) The singular values of the matrix X as a function of diagonal matrix element number.

values for the four encoders are determined to be the following: encoder #1:  $\sim 62$ ; encoder #2:  $\sim 54$ ; encoder #3:  $\sim 65$  and encoder #4:  $\sim 55$ . The above analysis clearly suggests that the number of nodes in the hidden layer can be reduced by using the SVD method. Using the optimized number of nodes in the hidden layer we have then trained the network again so that the final ANN configuration can be tested. While it took ANN about 9 min to complete 10 000 training iterations on a P-IV machine (Intel(R)core, 2 Quad CPU, 2.39 GHz, 3.25GB RAM) for 1:80:1 configuration, the training time was reduced to  $\sim 4$  min for a SVD-optimized typical configuration of 1:60:1.

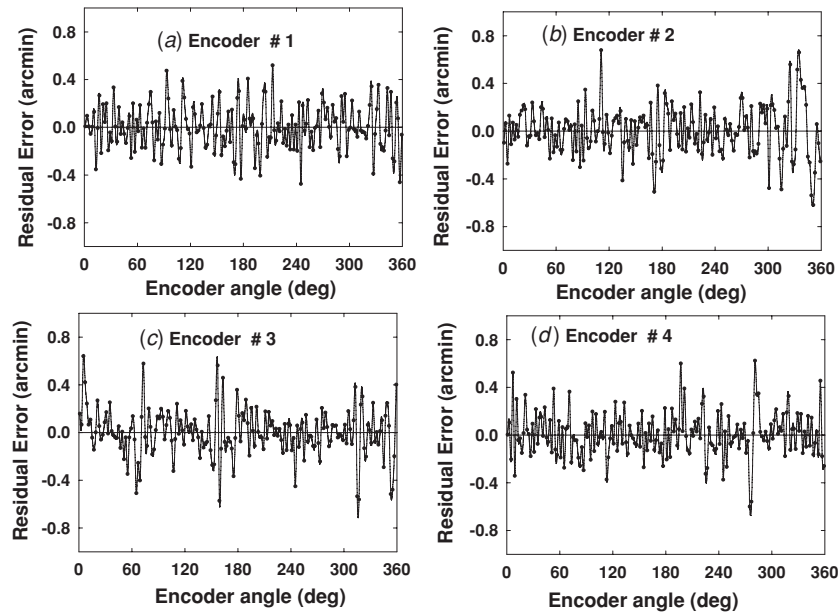
## 5. Testing of artificial neural network and results

The ANN, once properly trained with optimized number of nodes in the hidden layer for each encoder error profile separately, was tested with respective test data files comprising 180 resolver angle values each (i.e.  $1^\circ, 3^\circ, 5^\circ, 7^\circ, \dots, 359^\circ$ ) which were not used during training of the network. Denoting the ANN-predicted encoder error as  $\epsilon_{\text{ANN}}(\theta_{\text{ENC}})$ , we then calculate the residual error (i.e.  $\epsilon_{\text{ANN}}(\theta_{\text{ENC}}) - \epsilon_{\text{obs}}(\theta_{\text{ENC}})$ ) as a function of  $\theta_{\text{ENC}}$  for all 180 values of the encoder angle. The plots of the residual error as a function of  $\theta_{\text{ENC}}$  for the four encoders are shown in figure 4.

The mean absolute error and the RMS error of the resulting residual error profiles (i.e.  $\epsilon_{\text{ANN}}(\theta_{\text{ENC}}) - \epsilon_{\text{obs}}(\theta_{\text{ENC}})$ ) of the four encoders are found to be the following: encoder #1:  $\sim 0.15$  arc-min and  $\sim 0.19$  arc-min; encoder #2:  $\sim 0.16$  arc-min and  $\sim 0.21$  arc-min; encoder #3:  $\sim 0.14$  arc-min and  $\sim 0.20$  arc-min;

encoder #4:  $\sim 0.14$  arc-min and  $\sim 0.19$  arc-min. Comparison of these values with those presented in figure 1 for the pre-compensation case indicates that the pre-compensation values of mean absolute error, ranging between  $\sim 0.55$  arc-min and  $\sim 1.77$  arc-min for the four encoders, improve to  $\sim 0.15$  arc-min after compensation. The reduction in the maximum residual error is from  $\sim \pm 6$  arc-min (pre-compensation) to  $\sim \pm 0.65$  arc-min (post-compensation). The ANN-based error compensation procedure thus involves training an ANN appropriately for each of the resolver–decoder combinations by using approximately half of the calibration data and then using the optimally trained ANN configuration for predicting the error for any encoder angle in the range  $0^\circ$ – $360^\circ$ . The corresponding corrected encoder angle ( $\theta_{\text{COR}}$ ) is determined by subtracting the ANN-predicted error from the measured value of the encoder angle (i.e.  $\theta_{\text{COR}} = \theta_{\text{ENC}} - \epsilon_{\text{ANN}}(\theta_{\text{ENC}})$ ). It is important to note here that the overall mean absolute error and the RMS error values, in the full encoder angle range  $0^\circ$ – $360^\circ$ , will be marginally better than the corresponding test data because of the fact that the ANN will always yield better results around encoder angle values which were used during training (i.e.  $0^\circ, 2^\circ, 4^\circ, 6^\circ, \dots, 358^\circ$ ).

In order to compare the results of the ANN-based error compensation procedure with those obtained from the Fourier series-based approach, we will first briefly present the results of the latter method. Assuming that the error profiles of the encoders can be approximated by a Fourier series representation, as already demonstrated in our earlier work



**Figure 4.** (a)–(d) Residual error (i.e.  $\epsilon_{\text{ANN}}(\theta_{\text{ENC}}) - \epsilon_{\text{obs}}(\theta_{\text{ENC}})$ ) as a function of  $\theta_{\text{ENC}}$  for four encoders. A smoothed spline curve passing through the data points has also been drawn in these figures so that systematic structure (if any) present in the residual error profiles can be visualized easily.

[7], a general expression for the same can be written in the following manner:

$$\epsilon_{\text{fit}}(\theta_{\text{ENC}}) = a_0 + \sum_{n=1}^N [a_n \cos(n\theta_{\text{ENC}}) + b_n \sin(n\theta_{\text{ENC}})], \quad (3)$$

where  $\epsilon_{\text{fit}}(\theta_{\text{ENC}})$  is the fit to  $\epsilon_{\text{obs}}(\theta_{\text{ENC}})$  values,  $\epsilon_{\text{obs}}(\theta_{\text{ENC}})$  is the observed error in the encoder angle (i.e.  $\epsilon_{\text{obs}}(\theta_{\text{ENC}}) = \theta_{\text{ENC}} - \theta_{\text{TAB}}$ ),  $N$  is an integer indicating the maximum order of the harmonic which needs to be considered,  $a_0$  is a constant signifying the amplitude of the dc component,  $a_n$  and  $b_n$  are the even and odd Fourier coefficients, respectively. However, before approximating the error profile of an encoder with a Fourier series with manageable number of terms, it becomes important to first perform a detailed harmonic analysis of each error profile so that one can retain only those terms in the expansion which are of significant amplitude. The most prominent ten harmonics (arranged in descending order of their amplitudes) which have been used in the Fourier series expansion for the four error profiles shown in figure 1 are the following: encoder #1: (1, 2, 16, 0, 14, 48, 4, 32, 3, 84); encoder #2: (0, 1, 2, 16, 14, 32, 48, 10, 6, 4); encoder #3: (1, 16, 2, 48, 14, 12, 32, 0, 6, 10) and encoder #4: (0, 1, 16, 2, 32, 12, 4, 3, 14, 48).

To summarize the quantum of the resulting improvement when software-based error compensation is used, we have given in table 3 the mean absolute error and RMS error before and after compensation for all four encoders.

On comparing the performance of the ANN-based error compensation procedure with that of the Fourier series-based approach, it is evident from table 3 that the two methods yield very similar results. However, the main advantage of the ANN-based compensation is that it can be applied to a wide range of sensors with any arbitrary error profile. Reduction of the maximum residual error from  $\sim \pm 6$  arc-min

(pre-compensation) to  $\sim \pm 0.65$  arc-min (post-compensation), along with significant reduction in the mean absolute error and the RMS error of the residual error profiles clearly illustrates that the accuracy of low-cost, resolver-based encoders can be improved significantly by employing a suitable software-based error compensation procedure. The mean absolute error and RMS error of the residual error profiles after applying the ANN-based error compensation method are found to be in the range  $\sim 0.14$ – $\sim 0.16$  arc-min and  $\sim 0.19$ – $\sim 0.21$  arc-min, respectively. It is worth mentioning here that the use of a dedicated ANN software package is necessary only during the training of the ANN. Once satisfactory training of the ANN is achieved, the corresponding ANN-generated weight file can be easily used by an appropriate subroutine of the data acquisition program for determining the corrected encoder angle. We have also successfully implemented the ANN-based error compensation procedure in our data acquisition program, by directly using the ANN-generated weight file, so that the corrected encoder angle can be predicted directly without using the ANN software package. An added advantage of using the weight file directly is that the corrected encoder angle information can be made available ‘on-line’ if required.

## 6. Discussion

With regard to other compensation methods, while a number of methods have been proposed for improving the accuracy of resolver-based encoders, we will only present here a brief summary of some of the relevant published results. Assuming that factors such as amplitude imbalance, quadrature error, inductive harmonics and excitation signal distortion are mainly responsible for making the output signals of a resolver deviate from ideality, it has been shown in [5] that by using appropriate signal processing, quadrature error can be eliminated. It

**Table 3.** Comparison of mean absolute error (MAE) and the RMS error for pre-compensation and post-compensation cases.

Encoder no.	Before compensation MAE/RMS (arc-min)	Post compensation Fourier MAE/RMS (arc-min)	Post compensation ANN MAE/RMS (arc-min)
1	1.33/1.48	0.16/0.21	0.15/0.19
2	0.55/0.65	0.16/0.20	0.16/0.21
3	1.09/1.35	0.14/0.18	0.14/0.20
4	1.78/1.31	0.16/0.21	0.14/0.19

has also been pointed out in [5] that all even harmonics in the resolver signals can also be cancelled if the resolver is constructed with complementary phases. A compensation circuit for decreasing the quadrature error and the amplitude imbalance of a resolver is reported in [34]. The results of their study, reported for a 34-pole resolver, indicate that the accuracy can be improved from  $\sim\pm 15$  arc-min to  $\sim\pm 2$  arc-min by using the compensation circuit. As a replacement for the conventional phase-lock loop method, a resolver-to-dc converter circuit based on linearization approach has been proposed in [35], but an accuracy of only  $\sim 12$  arc-min has been achieved by using this method. A gain-phase-offset correction method using cross-correlation has been used in [36] for suppressing systematic errors of resolvers and optical encoders. However, it is also pointed out in [36] that the method is not suitable for fast point-to-point positioning with small distances (i.e. with less than one period of the resolver line signals). More recently, in an attempt to overcome the limitation of the cross-correlation method, an adaptive phase-lock loop-based R/D converter has also been proposed in [37] for estimating the angular position and speed of resolver sensor systems. Using a DSP-based system, the authors [37] claim that in addition to compensating for amplitude imbalance, quadrature error and harmonic distortion, there is also a saving in hardware. A novel ANN-based method has been used for adaptive online correction and interpolation of encoder signals [38]. The method followed by them uses a two stage RBF-based ANN model, where the first stage of the network is used for correction of incoming non-ideal encoder signals and the second stage is used to derive higher order sinusoids from corrected signals of the first stage. Although the method used by them gives similar results compared to the look up table method, it has an added advantage with respect to the memory storage requirements, since when the number of data points calibrated in a three-dimensional workspace increases by a factor of  $N$ , the number of entries in the look up table will increase by  $N^3$ . A wavelet-based algorithm [39] has been used for extracting the integral and differential nonlinearity of encoders.

On the basis of the above discussion (and other published literature on the subject) one can safely say that in addition to hardware improvements of the encoder system, there is also a strong need for developing software-based compensation methods for achieving accuracies of  $\lesssim 1$  arc-min. Significant improvement in the accuracy, from  $\sim\pm 6$  arc-min to  $\sim\pm 0.65$  arc-min, achieved in the present work, clearly demonstrates that ANN-based compensation method can indeed help in improving the accuracy of low-cost encoders. Similar error reduction for different applications, by using ANN-based

compensation method, have also been reported by others [9, 16, 19].

## 7. Conclusions

An ANN-based error compensation procedure has been developed in this work to improve the accuracy of low-cost resolver-based 16-bit encoders. The procedure has allowed us to use the existing resolvers at an accuracy which is within the limits of the encoder resolution. Reduction of the maximum residual error from  $\sim\pm 6$  arc-min (pre-compensation) to  $\sim\pm 0.65$  arc-min (post-compensation) clearly illustrates that the accuracy of low-cost, resolver-based encoders can be improved significantly by employing an ANN-based error compensation procedure. This procedure has been implemented for four encoders by training the ANN with their respective error profiles and then using their corresponding ANN-generated weight files for determining the corrected encoder angle. We believe that a little expense involved in generating the calibration data is quite justifiable keeping in mind that using optical encoders of comparable accuracy, in lieu of resolver-based encoders, is a costlier option [1]. Because of their delicate nature, optical encoders are very sensitive to hostile environment (temperature, moisture, dust etc) and are prone to reliability problems in application where the sensors need to be installed away from the control room. Furthermore, since our application needs the encoder angle data to be transferred over long cables ( $\sim 75$  m), use of a suitable driver also involves an additional cost. On the other hand, resolver-based encoders can transmit analogue angle data over much longer lengths of cable, besides having a very wide working temperature range.

## Acknowledgments

The authors would like to thank all the concerned colleagues of the Centre for Design and Manufacture of BARC for their help in calibrating the encoders using the rotary table setup of the Coordinate Measuring Machine. We would also like to thank the anonymous referees and the adjudicators for making several helpful suggestions.

## References

- [1] Gasking J (ed) 1980 *Synchro and Resolver Conversion* (<http://www.naii.com>)
- [2] Moog Components Group *Synchro and Resolver Engineering Handbook* (<http://www.moog.com>)



- [3] Gasking J (ed) *Analog Devices Application Note, AN-263* ([www.analog.com/UpdatedFiles/ApplicationNotes/394309286AN263.pdf](http://www.analog.com/UpdatedFiles/ApplicationNotes/394309286AN263.pdf))
- [4] Hanselman D 1990 Resolver signal requirements for high accuracy resolver-to-digital conversion *IEEE Trans. Ind. Electron.* **37** 556–61
- [5] Hanselman D 1991 Techniques for improving resolver-to-digital conversion accuracy *IEEE Trans. Ind. Electron.* **38** 501–4
- [6] Kaul S K et al 1997 Use of look up table improves the accuracy of low-cost resolver-based absolute shaft encoders *Meas. Sci. Technol.* **8** 329–31
- [7] Kaul S K et al 2008 Improving the accuracy of low-cost resolver-based encoders using harmonic analysis *Nucl. Instrum. Methods A* **586** 345–55
- [8] Wu S M et al 1989 Precision machining without precise machinery *CIRP Ann.* **38** 533–6
- [9] Chen J S 1996 Neural network-based modelling and error compensation of thermally-induced spindle errors *Int. J. Adv. Manuf. Technol.* **12** 303–8
- [10] Tai H M et al 1992 A neural network-based tracking control system *IEEE Trans. Ind. Electron.* **39** 504–10
- [11] Ozaki T et al 1991 Trajectory control of robotic manipulators using neural networks *IEEE Trans. Ind. Electron.* **38** 195–202
- [12] Kraft L G et al 1990 A comparison between CMAC neural networks control and two traditional adaptive control systems *IEEE Control Syst. Mag.* **10** 36–43
- [13] Psaltis D et al 1988 A multilayer neural network controller *IEEE Control Syst. Mag.* **8** 17–21
- [14] Yamada T et al 1990 An extension of neural network direct controller *Proc. Int. Workshop Intelligent Robots Systems* pp 619–26
- [15] Kawato M et al 1987 A hierarchical neural network model for control and learning of voluntary movement *Biol. Cybern.* **57** 169–85
- [16] Barto A et al 1983 Neuron like adaptive element that can solve difficult learning control problems *IEEE Trans. Syst. Man Cybern.* **SMC-13** 834–46
- [17] Guo H J 2003 Sensorless driving method of permanent-magnet synchronous motors based neural networks *IEEE Trans. Magn.* **39** 3247–9
- [18] Ziegert J C et al 1994 Error compensation in machine tools: a neural network approach *J. Intell. Manuf.* **5** 143–51
- [19] Tan K K et al 2006 Geometrical error modelling and compensation using neural networks *IEEE Trans. Syst. Man Cybern.* **36** 797–809
- [20] Computer Conversions Corporation Catalog CAT. 89
- [21] Koul R et al 2007 The TACTIC atmospheric Cherenkov imaging telescope *Nucl. Instrum. Methods A* **578** 548–64
- [22] Tickoo A K et al 1999 Drive control system for the TACTIC gamma-ray telescope *Exp. Astron.* **9** 81–101
- [23] Coordinate Measuring Machine Documentation, Zeiss <http://www.zeissmetrology.com>
- [24] Tagliaferri R et al 2003 Neural networks in astronomy *Neural Netw.* **16** 297–319
- [25] Dhar V K et al 2009 ANN based energy reconstruction procedure for TACTIC  $\gamma$ -ray telescope and its comparison with other conventional methods. *Nucl. Instrum. Methods A* **606** 795–805
- [26] <http://www.mathworks.com>
- [27] White H 1989 Learning in artificial neural networks: a statistical perspective *Neural Comput.* **1** 425–64
- [28] Rumelhart D E et al 1986 Learning representations by back-propagating errors *Nature* **323** 533–6
- [29] Hagan M T et al 2003 *Neural Network Design* (Singapore: Thomson Asia, New Delhi: Vikas Publishing)
- [30] Zurada J M 2006 *Introduction to Artificial Neural Systems* (Mumbai: Jaico Publishing House)
- [31] Press W H et al *Numerical Recipes* 2nd edn (Cambridge: Cambridge University Press)
- [32] Duda R O et al 2002 *Pattern Classification* (New York: Wiley)
- [33] Kanjilal P P et al 1993 Reduced-size neural networks through singular value decomposition and subset selection *Electron. Lett.* **29** 1516–8
- [34] Liu M et al 1999 Error analysis and compensation of multipole resolvers *Meas. Sci. Technol.* **10** 1292–5
- [35] Benammar M et al 2004 A novel resolver-to-360° linearized converter *IEEE Sensors J.* **4** 96–101
- [36] Bunte A and Beineke S 2004 High performance speed measurement by suppression of systematic resolver and encoder errors *IEEE Trans. Ind. Electron.* **51** 49–53
- [37] Sarma S and Venkateswaralu A 2009 Systematic error cancellations and fault detection of resolver angular sensors using a DSP based system *Mechatronics* to appear doi: 10.1016/j.mechatronics.2009.09.002
- [38] Tan K K et al 2005 Adaptive online correction and interpolation of quadrature encoder signals using radial basis functions *IEEE Trans. Control Syst. Technol.* **13** 370–6
- [39] Dias Pereira J M et al 2007 Wavelet techniques: a suitable tool to characterise and optimize encoders' based systems *Measurement* **40** 264–71