DEVELOPMENT OF THE FIRST HYDRAULIC ANALOG COMPUTER

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Mihailo Petrovic Alas (1868-1943), inventor of the first hydraulic analog computer for resolving differential equations, was professor of mathematics at Belgrade University, a member of the Serbian Royal Academy of Sciences and a member of the Czech, Polish and Romanian academies of sciences. He started his studies at the Department of Natural and Mathematical Sciences of the Great School in Belgrade, where he obtained a comprehensive knowledge in mathematics and natural sciences – physics, chemistry, geology, biology, psychology and philosophy[1]. Petrovic continued his studies in Paris, at the École Normale Supérieure where scientific universality was strongly encouraged. His professors of mathematics, mechanics and physics had a strong influence on his turning toward phenomenology and general problems of the mathematical modeling of diverse processes[2]. At the time of Petrovic's studying, the French school of mathematics was at its peak. He graduated first in mathematics (1892), then in physics (1893) and soon after, in 1894 he was awarded his Ph.D. degree. His thesis, *Sur les zéros et les infinis des intégrales des équations différentielles algébriques*[3], committed to the intensive exploration of differential equations characteristics from equations themselves, was examined by Charles Hermite, Emile Picard and Paul Prudent Painlevé.

From 1894 to 1938 Mihailo Petrovic was professor of theoretical mathematics at Belgrade Great School (in 1905 it became an University), he taught 45 generations of students, published 257 papers and books on mathematics, obtained 5 patents in France[4] and founded a very active mathematical school which succes-

1. The most important influence on him had his professor of mechanics, Ljubomir Kleric (1844-1910), who designed and constructed several cinematic computing instruments (1875-1899). Petrovic did not construct nor invent new types of cinemators, but dealt with changes in their structure, trying to get more flexible conditions for solving wider classes of differential equations. He worked on a cinematic rationalizator for calculating ephemerides, enlarged the possibilities of professor Kleric's tractoriograph, established the relation between the retification (curvimetre) and the quadrature (integraph), and showed how an integraph could be replaced by a curvimetre.

2. Here should be mentioned his professor of Mechanics, M. Koenigs, who taught Calculating Technics, and the professor of Physics, Gabriel Lippmann, later a Nobel Prize winner (1908), who liked Petrovic and considered him his best student.

3. Published by Gauthier – Villars, Paris, 1894.

4. Telemetre a sextant (Distance Measuring Device, 1910), Engrenages en vrille (Automatic Car Gearshift, 1913), Cadran calendrier pour objects d'horlogerie (Éternal Calendar, 1916), Submersibilité du navire (Depth Measuring Device, presented to the British Admirality in 1919), Aiguille aimantée dans un champ magnétique mobile (Minesweeper, 1920).

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sful development was radically slowed down after the breakdown of the Kingdom of Yugoslavia. He was appointed the educator to the crown-prince George Karadjordjevic, who wrote long, enthusiastic letters about Einstein's lectures in Paris to his teacher in 1936 ; also he was the scientific mentor to his assistant-professor, Sima Markovic, later the Secretary General of the Communist Party of Yugoslavia, who obtained his doctor's degree with the thesis *Common Riccati's Equation of the First Order* in 1914. The crown-prince was forced to abdicate and the secretary disappeared in Moscow as a victim of Stalin's purges and was replaced by Josip Broz. Petrovic allocated 13 doctorates, but only one of his students, Konstantin Orlov, continued the work on his theory. Among mathematicians, his best known student was Jovan Karamata (1902-1967), later a professor at the University of Geneva[5].

Professor Mihailo Petrovic

Seemingly simple, the biography of Mihailo Petrovic has a " renaissance " dimension as well. The same year Petrovic received his PhD degree, he also passed the exam for a master fisherman, and became a member of the Société mathématique de France. His doctoral and fisherman's diplomas were hung side by side on the wall of his study where the only furniture consisted of an iron martial bed, a writing table and a chair[6]. He wrote many articles on fish and fishing, constructed some new types of fishing tools, and prepared exhibitions on fishing in Serbia in some European capitals ; he set up some piscatorial societies, wrote laws and international treaties on fishery as well as its history. Until his death he

5. See : Aleksandar Nikolic, " Dealing with Two Famous Results of Jovan Karamata ", *Archives internationales d'histoire des sciences*, 48, 141 (1998), 353

6. Petrovic's nickname " Alas ", which could not be separated from his real name, means " a river fisherman ".

alternatively taught mathematics, did his scientific work, and went into fishing with his fellow fishermen on the Danube and the Sava rivers[7].

Petrovic took part in five French scientific expeditions, and traveled across the Atlantic ocean, from the far Arctic north (1931, 1933) to the far Antarctic south (1934) ; he also visited distant islands in the south of the Indian ocean (1935). He depicted the memoirs from these journeys into interesting travelogues[8]. For a very long time he passionately tried to resolve the secret of the procreation of European and African eels, explored as far back by Aristotle. In 1932 he joined an expedition to the Sargasso Sea and afterwards, persuaded by friends, wrote *The Novel on the Eel*[9], never wanting to publish its last chapter.

Professor Petrovic with his " Musical Band-Suz "

He lived very modestly, but from time to time he spent nights in Belgrade's restaurants playing on the violin traditional songs and melodies with his " Musical Band-Suz " and recorded about 1000 of those for Radio Belgrade (all this records disappeared on April 6th 1941 in the German bombing). He did not like public promotions[10], but occasionally prepared feasts for numerous friends – members of the Academy and professors as well as fishermen and restaurant musicians. In 1941, at the age of 73, he was captured and deported to a prison camp as lieutenant colonel of the Yugoslav Royal Army. He grimly survived the breakdown of the Kingdom of Yugoslavia and therefore by pure decision, not undergoing any disease, died in 1943, one year after his return from Germany. What could not be destroyed are – his ideas. " His life is proof of his honest devotion to mathematics, fishing, science expeditions, music, people... He sank into

7. He lived on the rivers, but never learned to swim : he nearly drowned 11 times, even though Milutin Milankovic (author of *The Canon of Insolation*) tried to teach him how to swim.

8. *Through the Polar Field* Belgrade, 1932, *In the Kingdom of Pirates* Belgrade, 1933, *With Ocean Fishermen* Belgrade, 1935, *On Remote Islands*, Belgrade, 1936.

9. *The Novel on the Eel*, Belgrade, 1940

10. His only public statement was : " God knows that I am not responsible for this ", given in the moment of receiving Doctor Honoris Causa diploma of the Belgrade University.

that multiple life and whatever set him apart from that, stressing form over essence – he rejected pitilessly, as something that endangers his right to live and act in freedom "[11].

I

Although educated in the best traditions of the French school, Petrovic created a science which flew upstream, for it has an inexorably holistic approach in the time when specialization was spreading irresistibly. Moreover, his dream about building mathematical Arcadia was interlaced by whirlpools of wars. In spite of that, from 1912 to 1918, although he was not taking off his captain's uniform, he published tens of scientific papers and patents.

As a mathematician, Petrovic was devoted to the Classical Analysis which is, generally speaking, the exploration of the properties of various real or complex functions defined by power series, definite integrals or solutions of differential equations. He was the founder of two new mathematical disciplines : the first was the theory of Mathematical Spectra for resolving problems in arithmetic, algebra and infinitesimal calculus, similar to the spectral method in chemical analyses[12]. This theory was founded as a result of successful work on cryptography during the First World War. " It consists of dispersing prior or auxiliary unknown quantities in a certain problem into a numeral spectrum, much like a prism in the spectral analysis dispersing a bundle of light rays into a spectrum of light. Unknown quantities are dispersed, separated and determined in the same way as it is done with unknown chemical elements in spectral analysis. The number that gives a spectrum through its own dispersing plays the same role as analyzed bundle of rays : one special analytical function, a spectrum generatrix, plays the same role as a prism that disperses rays into a spectrum "[13]. This theory could be considered an anticipation of two important mathematical principles : the data linearization of a certain problem throughout its mathematical processing ; and " goedelization ", i.e. making use of natural numbers to articulate all symbols, formulas, theorems and proofs of a mathematical theory. In 1928 Petrovic gave a course " Mathematical Spectra " at the Faculté des sciences – Université de Paris. He proposed the use of spectra to astronomers because of the economy of calculations – multiplying the great mass of ordered pairs of real numbers could be reduced to multiplying one only. However, this theory attracted only a few followers, because spectra are natural numbers of vast length. The theory could be applied very efficiently today in the digital environment.

11. Prince George Karadjordjevic, *The Truth About My Life*, Belgrade, 1969, 423

12. Mihailo Petrovic, *Les spectres numériques*, Gauthier Villars, Paris, 1919, VII+110 p ; M. Petrovic, *Leçons sur les Spectres mathématiques* (professées à la Sorbonne en 1928), Gauthier Villars, Paris, 1928, II+90 p.

13. Mihailo Petrovic, *Phenomenology of Correspondence*, Serbian Royal Academy of Science, Belgrade, 1933, 58.

Second discipline established by Petrovic was Mathematical Phenomenology which basically attempts to comprise and to aggregate phenomena previously inaccessible to mathematical research[14]. He wanted to construct a genuine mathematical picture of the world, which would not be only theoretical, but which would make mechanics of phenomena conductible, no matter of what kind the phenomena are. In doing so, he comprehended mathematically mechanics, physics, chemistry, biochemistry, electrodynamics as well as descriptive sciences, such as economics, sociology and medicine. This extensive mathematical discipline has never been accepted under the name of " mathematical phenomenology ", but as " dynamic systems " or " theory of chaos " it is still close to the ideas of Petrovic.

The fundamental idea of his mathematical phenomenology was that one can affirm, by an adequate mathematical apparatus, the correspondence of facts and ensures the linking of apparently unconnected phenomena belonging to different levels of natural events. Petrovic believed that, " apart from numbers, dimensions and orders, modern mathematics should capture other general details in the world of facts that are independent from the concrete nature of their bearers "[15].

Petrovic's basic concern was the insight that different names were given to the same phenomena in different domains of our perception. Because of that, the quantity of presumptions is large and tends to be even larger. Thus, he attempted to make it theoretically as small as possible, trying to find as many analogies between phenomena as possible and to observe " the nucleus of analogy ", the identity of mathematical relations which describe different phenomena. That identity is comprehensive and includes a certain number of equations (differential or explicit) by which these relations are expressed, analytical form of elements and parameters of the phenomena, their differentials or some other combinations, as well as constants in these equations. This identity makes quantitative descriptions of phenomena (no matter how much they differ) to be the same analytical problem – integration, discussion and interpretation of the same equations. The nucleus of analogy is a direct consequence of the " phenomenological reduction ", noticing the analytical equivalence of different phenomena. The effect of the nucleus of analogy is so complete and thorough that every result obtained for one group of phenomena can be allocated to another group, by attaching one definite meaning.

The main idea of Mihailo Petrovic was that " mathematical analogies can perform an additional kind of service : they are an appropriate auxiliary means for the materialization of analytical problems. Materialization here means finding, for a given analytical form, a real phenomenon on which the same mathematical relations could be applied, so that by following materialized process the analytical

14. Mihailo Petrovic, *Mécanismes communs aux phénomènes disparates*, Libraire Félix Alcan, Paris, 1921, 279, (Nouvelle collection scientifique). Serbian edition : *Elements of Mathematical Phenomenology*, Serbian Royal Academy of Science, Belgrade, 1911, XIII+774 p.

15. Mihailo Petrovic, *Phenomenology of Correspondence*, 9.

solution could be obtained. In this materialization the kind of relation or the kind of particular detail that is hidden in the equations of the analytical problem and which is difficult to highlight by purely analytical means, becomes obvious in the definite phenomenon which the problem materializes "[16].

Mathematical analogies enable the materialization of analytical problems by finding concrete individual phenomena which are developing under the laws described by the same mathematical relations. Starting from this point, Petrovic imagined diverse calculating machines which work according to the relations described by differential equations. First, he conceptualized a chemical computing device and proposed some types of chemical reactions as the arithmetical device of an analog computer for solving analytically unintegrable differential equations, namely Riccati's equations with constant and variable parameters[17].

Riccati's equation could be " materialized " by one bimolecular chemical reaction at such a speed which varies through time following the changes in mixture concentrations, active components and physical conditions (for instance, temperature). By direct measurements of the formed products reaction in time, and by interpolation of table (or graph) of the data so acquired, a chemical integration of Riccati's equation would be realized, and by that also a further integration of homogenous differential equations which generally could not be integrated analytically. Here, for the first time Petrovic expressed a precept on simultaneous differential equations, i.e. the theorem on differential inequalities which S.A. Chapligin articulated in a significantly reduced form in 1917[18].

He was also thinking about an electric analog computer, because in his phenomenology he had developed all the necessary theoretical instruments required for its realization. Petrovic noticed the existence of " the simple analogy between electric and hydraulic phenomena ", because " electric inertia, manifested in self-induction, plays the same role in electric phenomena as the inertia of a fluid in a hydraulic phenomena ; in such a view, the electric resistance plays the same role as the mechanical resistance in a flow of fluid. And all that always implies the same mathematical relations and facts in both types of these so disparate phenomena "[19]. Therefore " the oscillatory motion of a fluid in a bended pipe, at first opened at one side and closed at the other, after a sudden opening of the closed side could be materialized, by analogy, through discharge of an electric condenser with a constant or a variable resistance and a certain coefficient of self-induction... That makes the mathematical description of these facts, no matter how different they are, represent one and the same analytical problem, which consists of solving, a discussion and an interpretation of the same equations, which

16. Mihailo Petrovic, *Elements of Mathematical Phenomenology*, 755.

17. Mihailo Petrovic, " Sur l'equation différentielle de Riccati et ses applications chimiques ", *Sitzungberichte der Königl. - Böhmischen gesellschaft der Wissenschaften*, Prague, 1896, 1 -25.

18. Other priorities could also be appointed to Petrovic because his book *Calculating with numerical intervals* (1932) represents a monograph on inequalities, published before the well known book *Inequalities* (1934) by Hardy and Littlewood.

19. Mihailo Petrovic, *Electric Analogies*, Science and Technique, Belgrade 1941, 141-151.

makes the adequate facts analytically equal among themselves "[20]. According to such prepositions, it is not hard to imagine and to construct electric analog computer, but the difficulties of practical accomplishment moved him away from that idea. But the same idea was concretized in the hydrodynamical group of phenomena, already familiar to Petrovic because of, among other things, his frequent and passionate contact with water.

At the end of XIX century hydrodynamic models had been already used for the construction of diverse arithmetical devices. However previous attempts of applying hydraulics in computing technics were conceptualized very reductively and did not exceed bare implementation of Archimedes's hydrostatic principle[21]. Petrovic was the first to propose an efficient computing machine called Hydro Integrator for solving differential equations based on analogy with hydrodynamics. He was dissatisfied with contemporary mechanical calculating apparatuses and their unreliable system of wheels, cogs, wires, levers and cams ; also he was not contented because existing computers were reduced to solving one particular task or task type. They did not tolerate systematic change of their modules, component parts, but only allowed a shift of the input data. He edited the shape and the disposition of his computer's parts and by such a programming made it possible to solve a significantly wider range of differential equations as well as to construct various algebraic and transcendental curves.

The principles applied in the construction of this Hydro Integrator had never been applied in any calculating machine before. Petrovic himself said " that all the intergraphs and apparatus for graphic integration of individual types of differential equations proposed before, had been based on entirely different principles of a purely cinematic nature, which are far less suitable for realization and produce types of equations that are less general, than the hydraulic principle "[22]. He constructed his Hydro Integrator in order to perform " the materialization of the Riccati's differential equation which is not eligible for analytical integration in a general case "[23]. It was a foremost challenge for him and since he observed that " its integration is related to the numerous questions concerning advanced geometry, mechanics and physics "[24] he wanted to attempt its graphical integration. In accordance to this, he wanted his computer to be deduced from the logic of natural phenomena, and not to be its mechanical imitation. As such, it had a very wide-ranging mathematical relevance and served for " the graphical integration of extensive classes of differential equations, including types that nowadays could

20. *Phenomenology of Correspondence*, 71-72.

21. Petrovic owned a Walther Dyck's *Catalog mathematischer und mathematisch – physikalischer Modelle – Apparate und Instrumente*. (München, 1892-3) in which various mathematically designed instruments were described.

22. Mihailo Petrovic, " Sur l'intégration hydraulique des équations différentielles ", *American Journal of Mathematics*, XX, 4 (1898), Baltimore, 293.

23. Mihailo Petrovic, *Phenomenology of Correspondence*, 78.

24. Mihailo Petrovic, " On Hydraulic Integration ", *Serbian Technical Journal*, IX (1898), Belgrade, 1.

not be integrated by any known method ; for determination of definite integral value ; for construction of curves of the equations which contain indefinite integrals ; for construction of various algebraic and transcendental curves, and generally, for graphical resolving of various problems "[25].

Before Petrovic, nobody had come to the idea of such a computer because analogies were perceived only practically, *ad hoc*. For instance, his professor, Gabriel Lipmann, pointed out to the analogies between thermodynamic and electric phenomena. James Clerk Maxwell, John V.S. Rayleigh, Ludwig Boltzmann, Hendrik A. Lorentz also illustrated different phenomena by analogies. However, Petrovic was the first who actually made a general mathematical theory of analogies[26]. Lord Kelvin observed analogies between different natural phenomena stating that the real understanding of one natural phenomenon means the ability for making its mechanical model. He successfully implemented perceived analogies and on that basis in 1876 he realized his Harmonic Synthesizer, analog computer conceptualized by the need to predict the heights of the tide in ports. It was a specialized device where summation is performed by a wire and pulley system, and resultant water height is recorded, like later in Petrovic's Hydro Integrator, as a continuous curve on a paper roll. Moreover, Kelvin said that he was " led to a quite unexpected conclusion : it seems very remarkable to me that the general differential equations of the second order with variable coefficients may be rigorously, continuously, and in a single process solved by a machine "[27].

Lord Kelvin was the first to grasp the significance of the mechanization of integration. Most subsequent analog machines can be seen as realizations of his ideas. He really discovered the basic feedback principle by which an integrator mechanism can be applied to solving differential equations. But he could not practically implement this idea, because the torque output from the wheel of his integrator was very slight and inadequate to drive further integrating mechanism. Theoretically speaking, problem was that Kelvin never developed a general model for the coupling of analogies - his analogue device was created for one specific purpose and was not suitable for general mathematical aims. As a contrast, Petrovic's apparatus was designed according to mathematical needs and it was not appropriate for industrial objectives.

II

Mihailo Petrovic presented the idea of his Hydro Integrator in the *Comptes rendus* of the Paris Academy of Science in 1897[28]. Because of the importance of

25. *Ibid*, 2.

26. He published the theory in French in the work *Mécanismes communs aux phénomènes disparates*, but unfortunately the complete mathematical apparatus on approximately 800 pages was given only in the Serbian edition of *Elements of Mathematical Phenomenology* in 1911.

27. W. Thompson, *Treatise on Natural Philosophy*, Vol. I, Cambridge Un. Press, 1890, 498.

28. Mihailo Petrovic, " *Sur un procédé d'intégration graphique des équations différentielles* ", *Comptes rendus des séances de l'Académie des Sciences*, CXXIV, 20 (1897), Paris, 1081 - 1084.

the paper, the Society of French Physicists made an exception and in the same year reprinted that already published paper in its journal[29]. The success of the discovery was proved by the bronze medal given to Petrovic at the Universal Exhibition in Paris in 1900. For the invention of the Hydro Integrator, he was also awarded a diploma from the mathematicians of London in 1907, and a prize in Rome, in 1911. The response to Petrovic's paper in contemporary professional literature was also significant : his discovery was frequently mentioned and described in detail. The originality of the invention was first confirmed by A. Price in 1900[30], who explained it in detail in a monograph on instrumental mathematics. In 1913, N. De Morih[31] gave a separate presentation of Petrovic's discovery as a special method of the mechanical integration of differential equations. In 1943, A. Willers[32] mentioned Petrovic's result as a completely new solution in analog technique, as did Kamke[33] in 1942.

Petrovic founded the construction of his hydro-analog computing machine on the laws of hydrodynamics stating that the level of liquid[34] in a vessel depends on the size and the shape of the vessel and the submerged body. It consisted of two main parts : an arithmetical device and an input/output unit.

The arithmetical device consisted of a liquid, a vessel of a certain shape, a body of a certain shape, and a float. Before Petrovic, only the weight of a body was implemented in the computing process and his original contribution was the use of the shape of a body as the carrier of information.

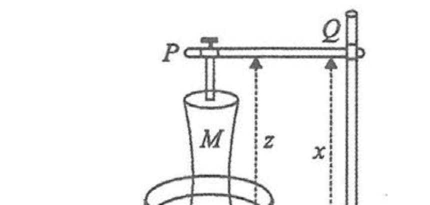

Fig. 1 The simplest visual presentation
of the arithmetical device of Petrovic's computer (1898).

29. *Ibid, Journal de Physique*, Paris, 1897, 476 - 479.

30. A.W. Price, " Petrovic Apparates for Integrating Differential Equations of the First Order ", *Philosophical Magazine*, 49 (1900), 487-490.

31. H. De Morih, *Les appareils d'intégration*, Paris, 1913, 6, 194-197.

32. A.F. Willers, *Mathematische Instrumente*, Berlin, 1943, s III, 305.

33. E. Kamke, *Differentialgleichungen Lösungsmethoden und Lösingen*, Bd. 1, Leipzig, 1942, 642.

34. In his first paper concerning hydraulic computer (1897) Petrovic indicates that the vessel is filled with mercury ; in all following papers he writes " any liquid ".

A liquid is pored into the vessel B. When submerging a solid body M in a liquid, its level would change according to a certain law, depending on the shapes of the vessel and the body, and, when they are determined, on changing x, i.e. the distance between the holder of the body M and the bottom of the vessel. Variable y is further defined as the distance between the bottom and the level of the liquid, and z as $z = x - y$. If $F(y)$ is the surface of the horizontal section of the vessel B at level y, and $F(z)$ the horizontal section of body M on the same level, then, by infinite submerging of body M, the dimension x changes to $x - dz$, and y into $y + dy$, so that the volume raised above the level y will be $[F(y) - F(z)]\, dy$.

This volume must be equal to the volume of the liquid displaced up by the body M while being submerged by dz, i.e. $F(z)\, dz$, i.e. :

$$[F(y)-F(z)]\, dy = F(z)\, dz$$

that is, because $z = x - y$,

$$F(y)\, dy \,/\, dx = F(x-y)$$

This is the differential equation of the problem, and by its integration the relation between the variables x and y can be obtained.

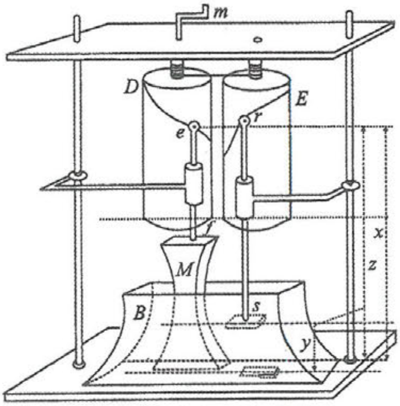

Fig. 2 Petrovic's blueprint of a possible construction of the computer (arithmetical device, input and output unit), which can be used in various ways – as integraph, integrator, pantograph, and so applied to all the problems demanding exploration of integral curves. Submerging the body M (*dx*) follows the curve drawn on cylinder D, while the law governing the changes of the liquid level (*dy*) can be registered by a peripheral device, here a pen which draws on the paper attached to cylinder E.

Petrovic based the construction of his Hydro Integrator on this fundamental principle, introducing afterwards some sophisticated details : The input unit was a dynamic body of a certain shape connected with a rotating cylinder on which an additional input function $\eta = f(\xi)$ was graphically described. Petrovic further abandoned use of the arm m at the input unit and introduced a clock mechanism with permanent angular speed so that the data at the input cylinder entered the arithmetical device at constant speed. The input and the output cylinders were connected, so they both moved at the same speed. In this way he ensured a fully automatic computing process, a unique innovation at the highest level of automation so far reached by contemporary computing machines. He added yet another possibility : the liquid from the vessel could flow out continually through a hole at the bottom of the vessel, and by that, the number of the input equations became even larger.

The output unit consisted of one or more rotating cylinders (directly connected with the float in the arithmetical device) on which the output data – an integral curve of the set differential equation was " printed ". The first idea of a printing cylinder Petrovic got from the lectures of professor Koenigs in Paris in 1892 and he drew sketches of a cylinder as the output computing unit in his notebook.

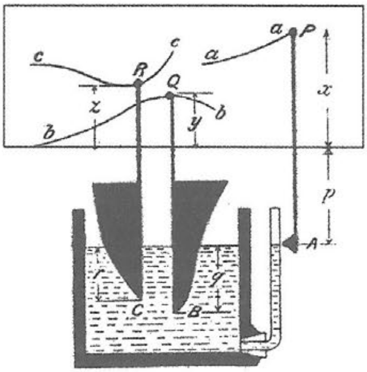

Fig. 3 Petrovic's scheme of the advanced Hydro Integrator displaying possibilities of a more complex input system (1899).

This Hydro Integrator was suitable for solving two classes of differential equations. By changing the constructive elements on the apparatus, i.e. the shapes of the vessel, and/or of the body which is submerged into the vessel, by uniting one, two or more vessels, submerging bodies and integral cylinders, Petrovic considerably extended the possibility of the hydraulic integration to a broader class of differential equations. He obtained conditions for the integration of Riccati's equations, as well as some other types of differential equations. " The principle of integrations, emphasized in this paper ", wrote Petrovic, " can be achieved in other very diverse ways as well, and the field for combinations of this kind is very broad. Whole classes of differential equations of the first order, which can be integrated, correspond to every manner of its realization and a whole family of curves, which can be continually constructed "[35].

After the presentation of the Petrovic's Hydro Integrator in 1900, various types of Hydro Integrators were developed for solving differential equations.

35. Mihailo Petrovic, *Elements of Mathematical Phenomenology*, 760.

Among them, maybe the most important was the computer of V.S. Lukyanov, developed in 1936, in the Institute for Mechanical Engineering in Moscow[36]. This computer was used for solving Fourier's partial differential equations, and was founded on two phenomena : the flow of a liquid through capillary tubes, and the moving of heat through a certain material. The level of water in its valves stands for stored numbers, and the dynamic of flow between them symbolizes mathematical operations. Even that type of analog computer was anticipated by M. Petrovic, who mentioned, before 1900, the construction of an arithmetical device based on a system of many interconnected vessels[37], and discussed capillar hydrodynamic phenomena as a possibility for materialization of analytical problems related to the curves' integrals[38].

III

Petrovic started to work theoretically on the development of his Hydro Integrator after his return from Paris in 1894. He published his last paper on hydraulic computers in 1899[39]. The fact that he did not attempt to protect his invention, although it had a higher inventive level than his inventions which were granted French patents, implies that commercialization was not on his mind. The main aim was the advancing of mathematics and University education. Only one sample of the computer was made, since for specified purposes it was not essential to make more[40]. There are no reliable data of what happened to the computer, but it seems feasible that it was destroyed in 1944, when the Nazi forces burnt the building where the Mathematical Institute had its premises.

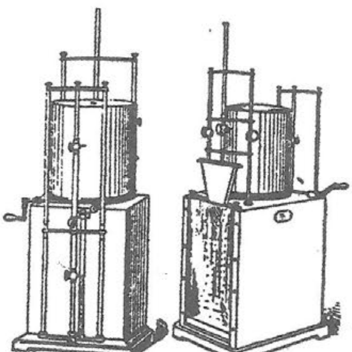

Fig. 4. Drawing of Petrovic's Hydro Integrator

36. V.S. Lukyanov, " Hydraulic Instruments for Technical Calculations ", *News, Academy of Sciences of USSR*, 2 (1939), Moscow, 53-67.

37. Mihailo Petrovic, *On Hydraulic Integration.*

38. Elements of Mathematical Phenomenology.

39. "Appareil à liquide pour l'intégration graphique de certains types d'équations différentielles", *American Journal of Mathematics*, 22 (1899), 1, 1-12.

40. In 1980, a replica of the Hydro Integrator was made at Belgrade University.

displayed on the Universal Exhibition in Paris, 1900. The name of the French engineer who actually made it still remains unknown. This is the only known drawing from the A.W. Price's article in the Philosophical Magazine.

Petrovic made a prototype which was not only a realization of his mathematical system, but also a step forward for mathematics itself. It was not the application of the computer that was put forward in the front line, but the materialization itself, proof of the validity of his mathematical theory of analogies. For him, the computer was " an auxiliary means for materialization of analytical problems. Materialization here means the finding, for an analytical problem, a concrete phenomenon subjected to the same relations and same laws obtained by analytical solving of that problem ". That device serves for " determining purely analytical, calculative facts, hidden, for instance, in differential equations of some analytical problem, which in some cases facilitates the exact as well as approximative integration of these equations "[41]. Such a computer did not fit into industrial, but into mathematical needs, and therefore it had no influence on industry, especially because it was too advanced for the practical needs of the time.

Probably one of the reasons for abandoning the work on the Hydro Integrator, after only a few years work, was that Petrovic came to the idea of *virtual computers*. " Instead of concrete phenomena, as they act in reality, it is possible to represent in mind materialization of fictional phenomena, with appropriately adjusted assumptions concerning their mechanism. Phenomena may not correspond to reality, but they should be of such character that, if they happen according to such assumptions, laws of phenomenon would be, on one hand, included in the given analytical problem, and, on the other hand, certain special details of these laws would be self-evident in that factual phenomenon "[42].

Because of that it would not be an exaggeration if we affirm that Mihailo Petrovic in 1897 opened new horizons in the development of scientific thinking about analog computing. The mathematical and overall conceptual flexibility of his Hydro Integrator was not achieved for a long time, until the appearance of advanced electronic analog devices[43]. He deserves the credit for advancing the principle of analog computer systems based on a specific physical model, as well as for the specific use of the law of hydrodynamics in its practical realization.

41. Mihailo Petrovic, *Phenomenology of Correspondence*, 78.

42. *Ibid*, 77-78.

43. The Hydro Integrator could be further conceived. Petrovic in *Elements* (p. 550) pointed that " there are also some special analogies between thermodynamical phenomena and the flowing out of water from a vessel. Homologous elements of such analogies are presented in the table :

Phenomenon	I element	II element	III element
hydrodynamical	height of the water	mechanical energy of falling water	quantity of the flowed out water
thermodynamical	temperature	quantity of heat	entropy"

This means that (1) it is possible to imagine a computer based on the principles of thermodynamics and (2) Petrovic's existing computer can be used for simple measuring of the entropy of a system.

Petrovic's was the first hydraulic model that could be applied for solving various classes of differential equations. This refers also to the technique of using hydraulics and pneumatics in the construction of analog computers, which encompasses a series of elements applied later in similar apparatuses (integral cylinder, elementary vessel, capillarity).

Above all, generally speaking Petrovic was the first who turned to the simulation of differential functions. He made 3-D models for a variety of such functions and implemented them as the shapes of the vessel and the submerging body which were the parts of the arithmetical device of his computer. The interactive usage of the shapes (vessel/submerging body) as the carrier of information was his original contribution. In the construction of peripheral devices, he also made some original contributions. First was the introduction of a cylinder with the graphic of a certain function imprinted on it as a device for entering information (besides the shapes of the bodies). By upgrading the input system with a clock mechanism with a permanent angular speed he reached fully automatic calculating process (the input cylinder was connected with an output cylinder on which the integral curve was drawn, and so they both moved at the same speed).

Apart from this, Petrovic's computer with its elegance, simplicity and clarity of applied principles still represents an outstanding utility for the visualizing and better understanding of differential equations and hydrodynamics in the educational process. It shared the destiny of Petrovic's theory in general which, in spite of its originality, consistency and applicability, did not receive a wider response, because it originated from genuine philosophical synthesis demanding more than a plain mathematical knowledge. Although his conception of mathematical analogical supervision was never comprehended in the context of cybernetics, probably it is not mistaken to say that Petrovic's work belongs more to the history of great ideas than to the history of computer technics.