Node-screening pour le problème des moindres carrés avec pénalité ℓ_0

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ℓ_0-penalized problems

Sparse problem

Ingredients of the problem

- A target y

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Rough formulation

Problem

Find x sparse such that $y \simeq \text{Model}(Ax)$

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Remark : Entries of x weight each atom in the linear combination.

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where $\lambda > 0$ is a tuning parameter and M is a big-enough constant.

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Properties :

- Continuous and integer variables
- Combinatorial problem
- Can be addressed with Branch-and-Bound (BnB) algorithms

Branch-and-bound algorithms

Branch-and-bound principle

Idea :

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Node $\nu = (\mathcal{S}_0, \mathcal{S}_1, \bar{\mathcal{S}})$ **where :**

- $\mathcal{S}_0$: indices of x fixed to zero
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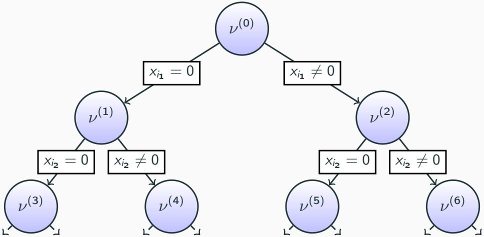

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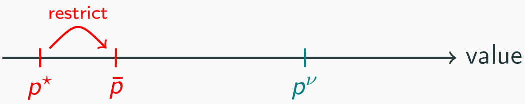

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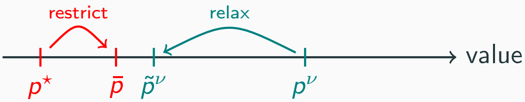
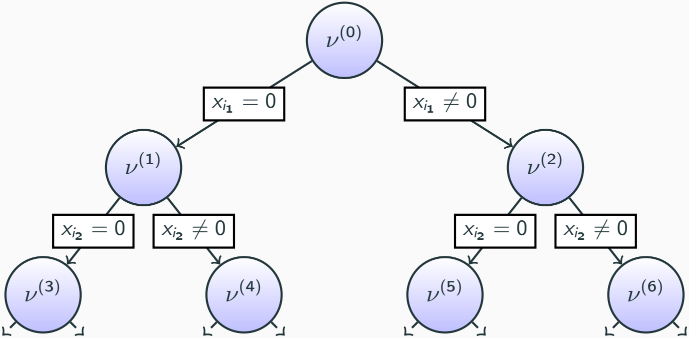


Exploration and pruning process



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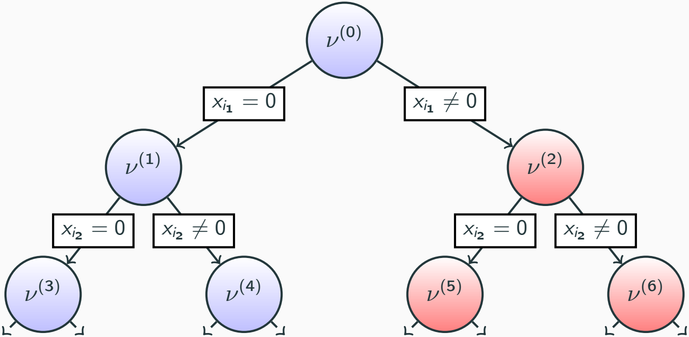

BnB efficiency

The efficiency of the BnB algorithm depends on :

- The number of nodes processed
- The ability to process nodes quickly

Node-screening improves both of these things !

Node-screening

Main idea

Testing if a node can be pruned $\equiv$ Solving convex problems

Sometimes it is obvious !

This is costly

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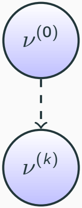
Question : How to detect prunable nodes in a more economic way ?

Dual problem



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- Sub-problem
- Relaxed problem

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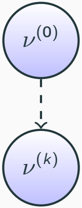

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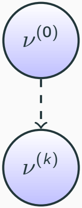

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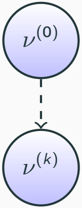

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- Terms depending on the current node constraints
- The pivot function is defined as $\gamma(t) = M|t| - \lambda$

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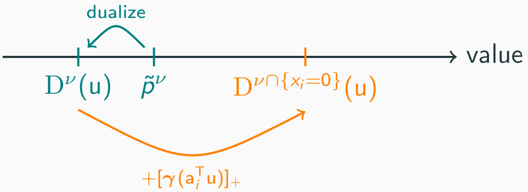

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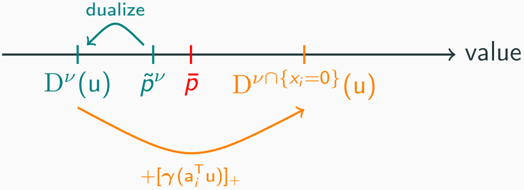

Node-screening test

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Given some point u,

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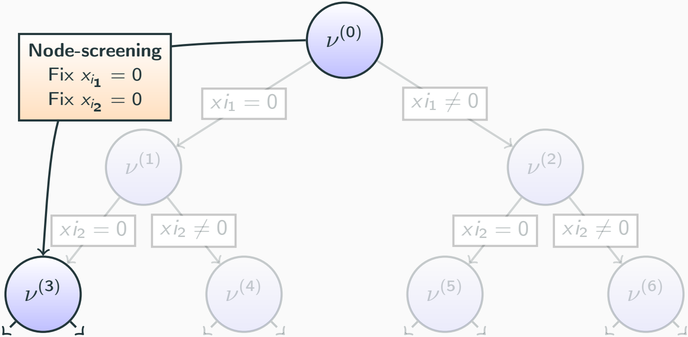
Nesting property : If multiple node-screening tests are passed, the corresponding variables can be fixed simultaneously.

Consequence of passing a node-screening test



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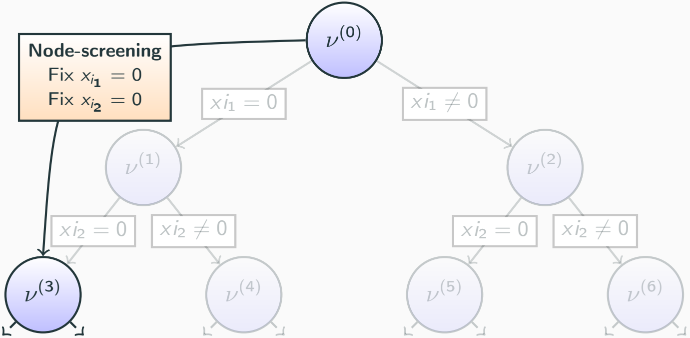

Consequence : Less nodes are explored by the BnB algorithm.

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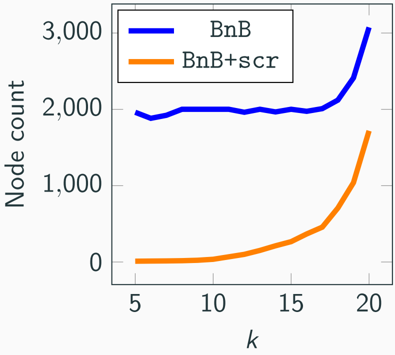

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