An efficient solver for L0-penalized problems

Theo Guyard with C. Herzet, A. N. Arslan and C. Elvira

INRIA and INSA Rennes

ROADEF | 21 Feb 2023

L0-penalized problems

Objectives

L0-penalized problems

Minimize a loss ⟷ **Objectives**

L0-penalized problems

Minimize a loss $\xleftrightarrow{\text{Objectives}}$ Sparse solution

L0-penalized problems

Minimize a loss $\longleftrightarrow$ **Sparse solution**

Objectives

Machine Learning
Signal processing
Network design
...

L0-penalized problems

Minimize a loss ⟷ **Objectives** ⟷ **Sparse solution**

Machine Learning
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ℓ_0-penalized problem

$$\min_x \; F(Ax) + \lambda\|x\|_0 + G(x)$$

L0-penalized problems

Minimize a loss ⟷ **Objectives** ⟷ **Sparse solution**

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$F(Ax)$: loss

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$F(Ax)$: loss
$\|x\|_0$: sparsity

L0-penalized problems

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$$\min_x \; F(Ax) + \lambda \|x\|_0 + G(x)$$

$F(Ax)$: loss
$\|x\|_0$: sparsity
λ : trade-off

L0-penalized problems

Minimize a loss ⟷ **Objectives** ⟷ **Sparse solution**

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$$\begin{aligned} F(Ax) &: \text{loss} \\ \|x\|_0 &: \text{sparsity} \\ \lambda &: \text{trade-off} \\ G(x) &: \text{modelling} \end{aligned}$$

Contributions

ℓ_0-penalized problem

$$\min_{x} \; F(Ax) + \lambda\|x\|_0 + G(x)$$

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ℓ_0-penalized problem

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NP-hard

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NP-hard

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- D. Bertsimas (2016)
- S. Bourguignon (2017)
- A. Atamtürk (2020)
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- This talk (2023)
- Extended paper (202?)

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✓ BnB algorithm
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✓ Generalize w.r.t. F and G

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✓ BnB algorithm
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✓ Generalize w.r.t. F and G
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Branch-and-Bound


N^0

Branch-and-Bound

$\min_{\mathbf{x}} \;\; F(\mathbf{A}\mathbf{x}) + \lambda\|\mathbf{x}\|_{\mathbf{0}} + G(\mathbf{x})$

N^0

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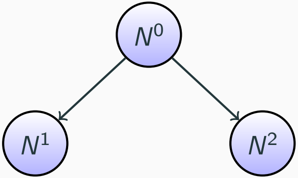

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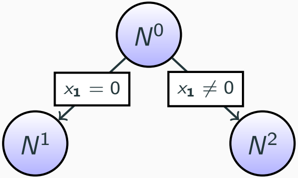

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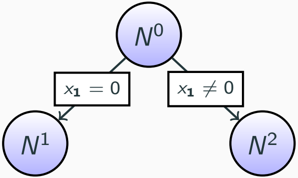

$$\begin{array}{ll} \min_{\mathbf{x}} & \text{Same obj.} \\ \text{s.t.} & x_{\mathbf{1}} = 0 \end{array}$$

$$\begin{array}{ll} \min_{\mathbf{x}} & \text{Same obj.} \\ \text{s.t.} & x_{\mathbf{1}} \neq 0 \end{array}$$

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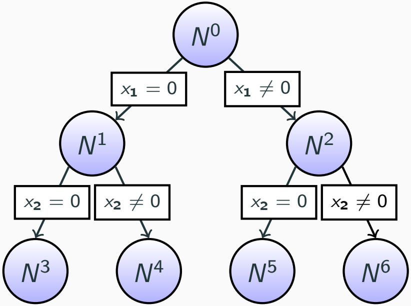

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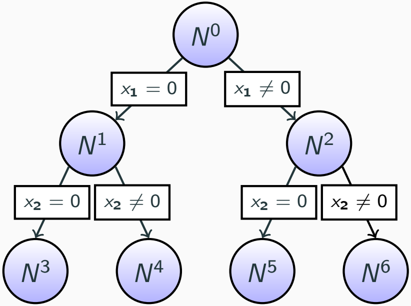

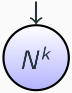

Subproblem
Still NP-hard

Branch-and-Bound

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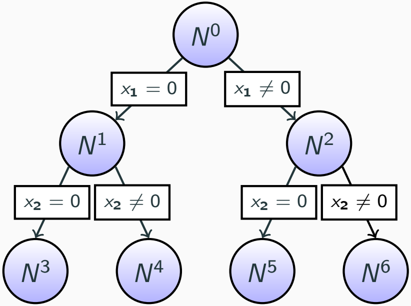

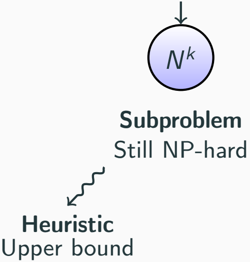

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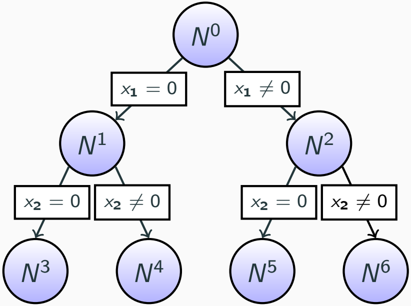

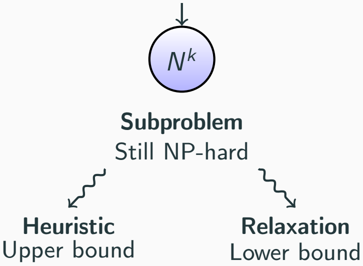

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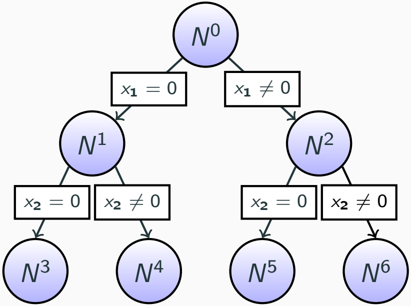

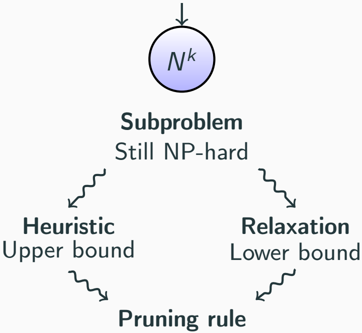

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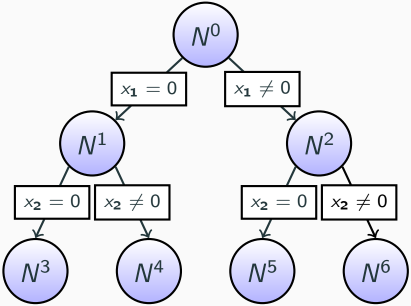

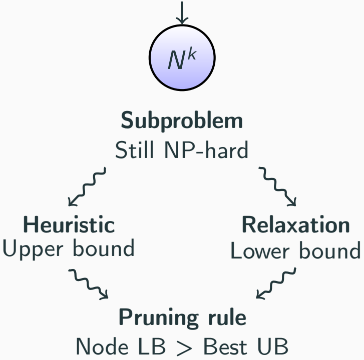

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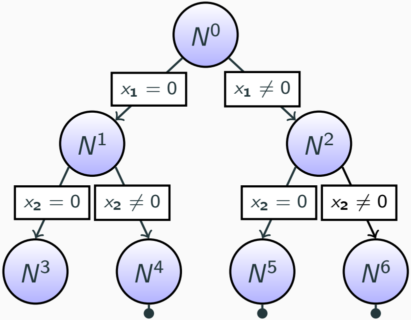


N^k

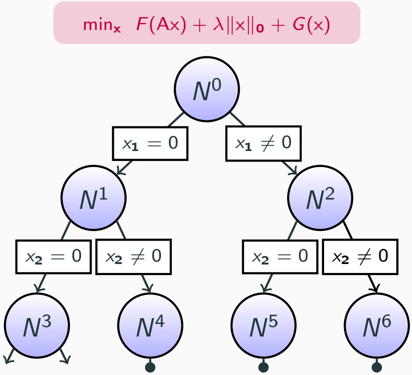
Subproblem
Still NP-hard

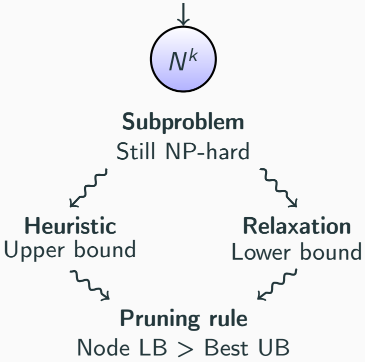
Heuristic
Upper bound

Relaxation
Lower bound

Pruning rule
Node LB > Best UB

Branch-and-Bound





Structural bottleneck

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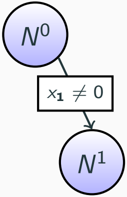

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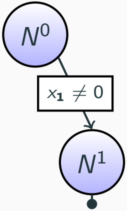

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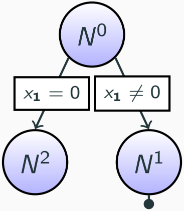

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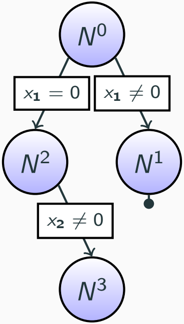

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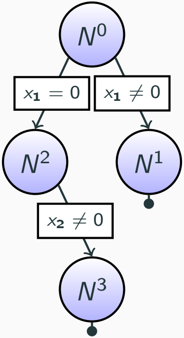

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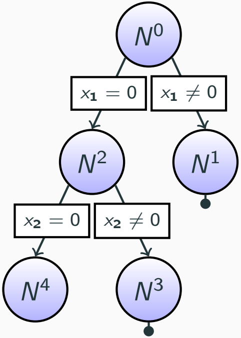

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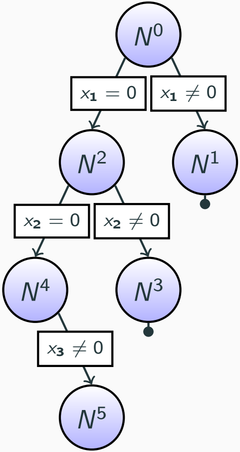

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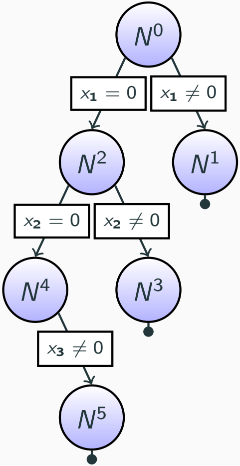

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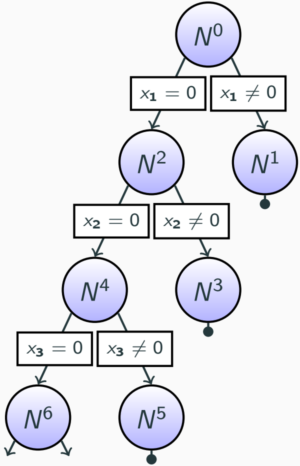

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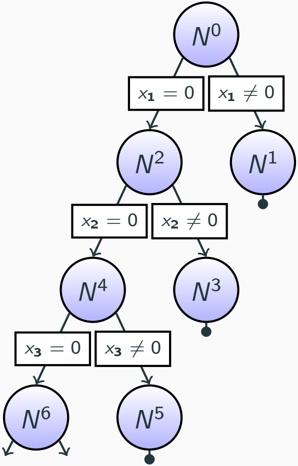
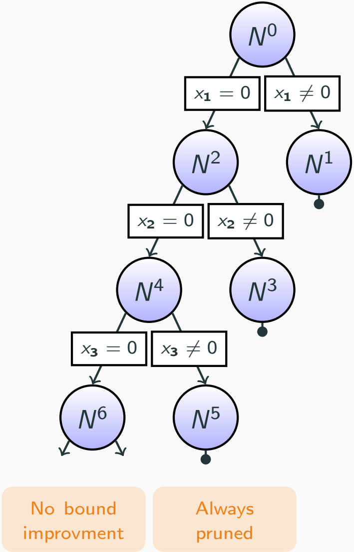


Always pruned

Structural bottleneck



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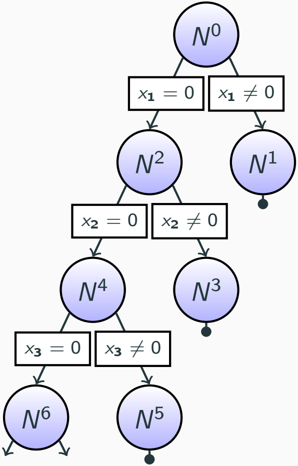

No bound improvment

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Waste of computational power !

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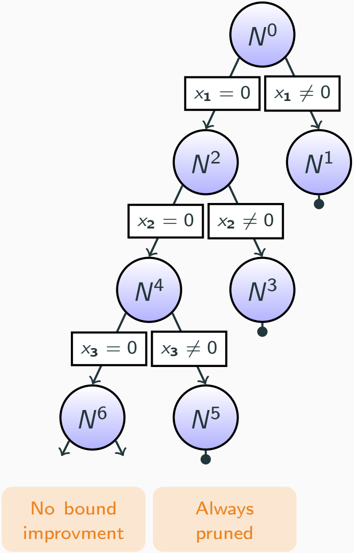
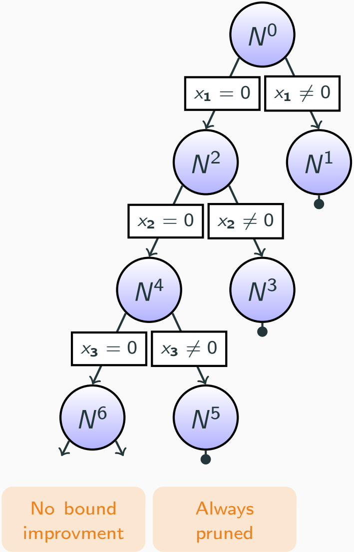


Waste of computational power !

$\downarrow$

How to avoid such situations ?

Structural bottleneck



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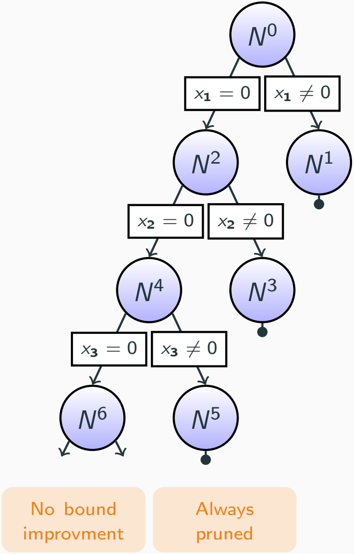
Leverage duality link between nodes

Structural bottleneck



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Leverage duality link between nodes

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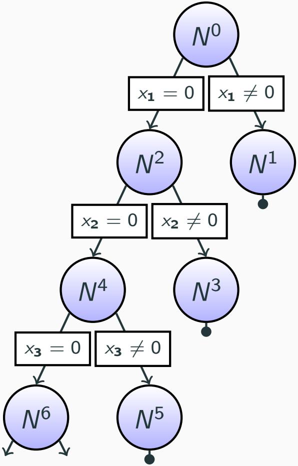

No bound improvment

Always pruned

Waste of computational power !

↓

How to avoid such situations ?

↓

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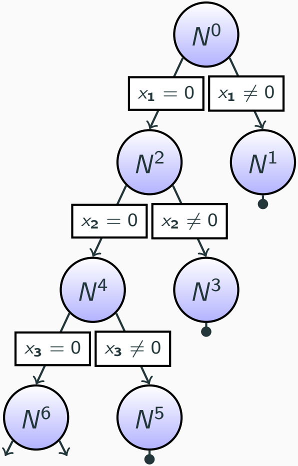

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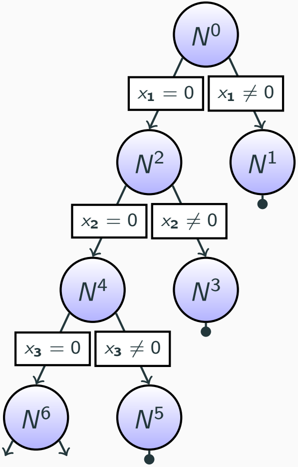

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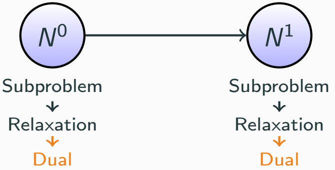

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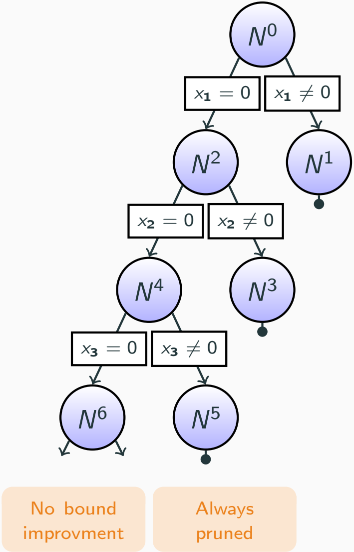

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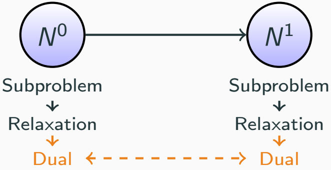

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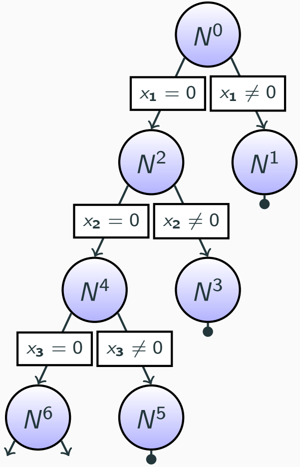

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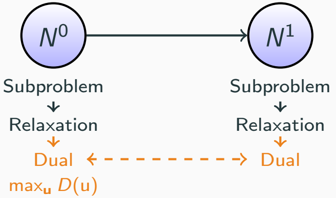

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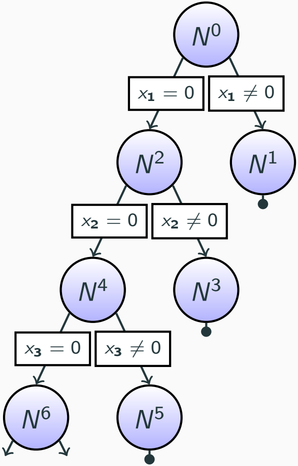

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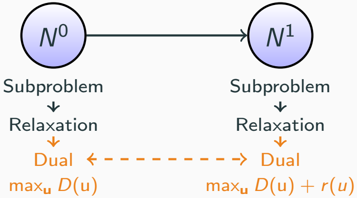

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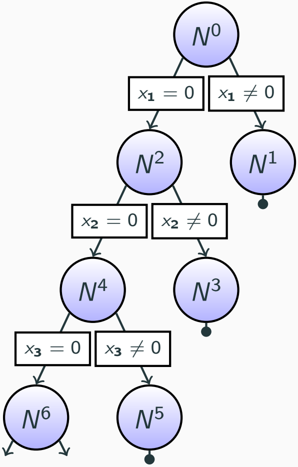

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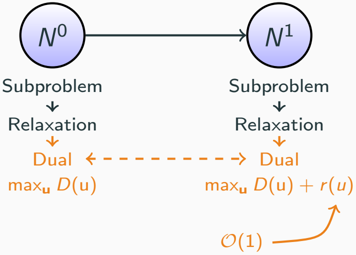

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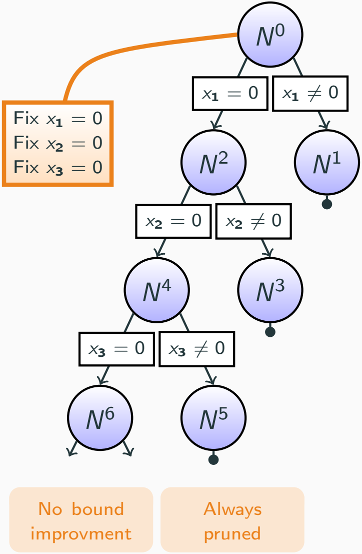

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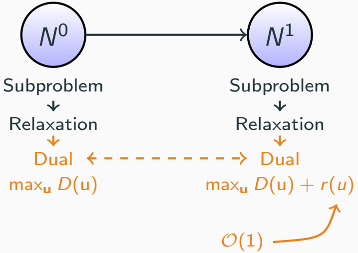

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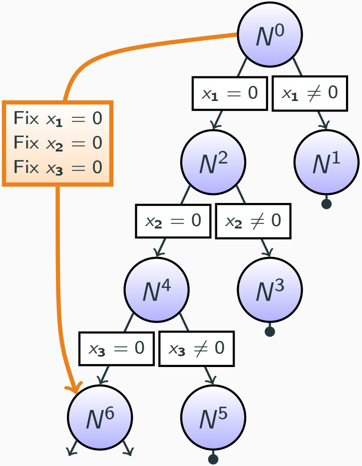

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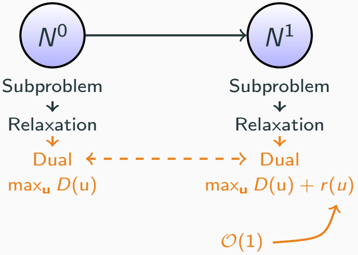

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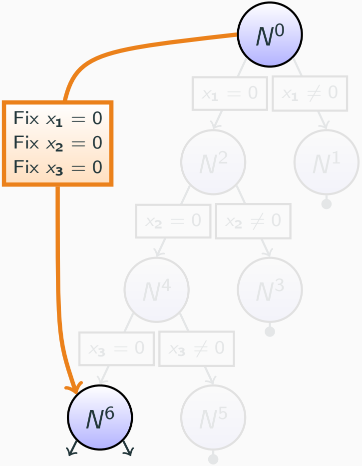

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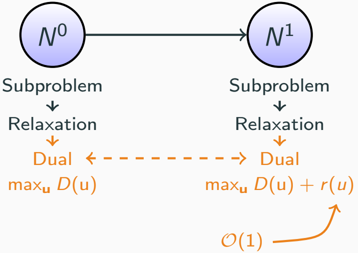

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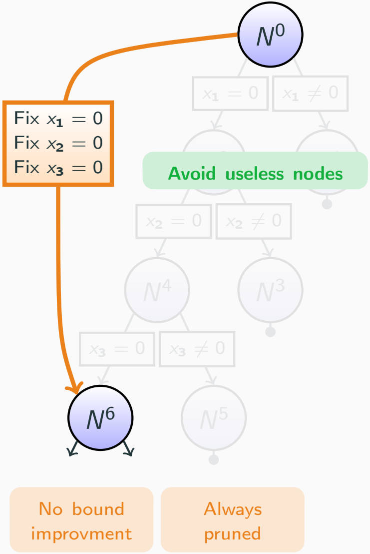

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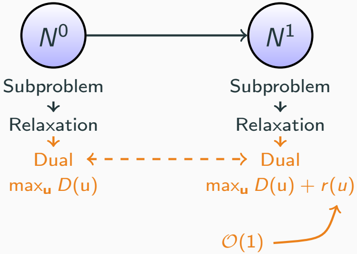

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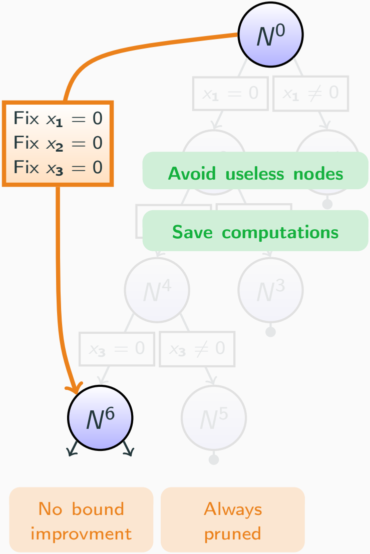

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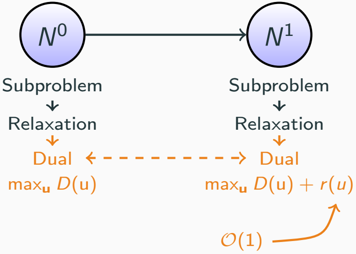

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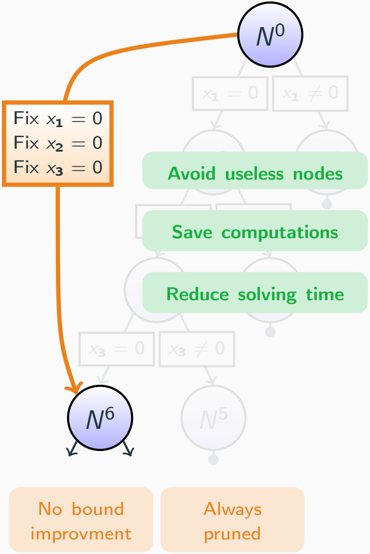

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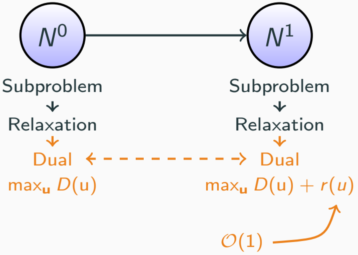
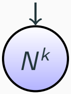


Variable fixing

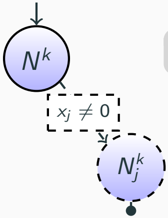


Variable fixing

N^k

Identify an index j

Variable fixing



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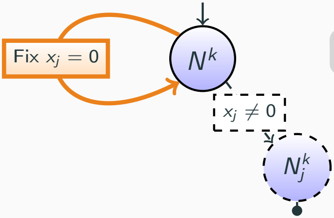

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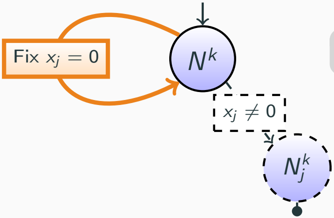

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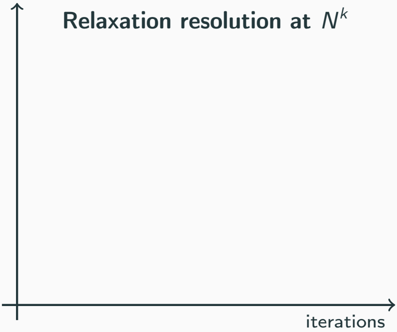
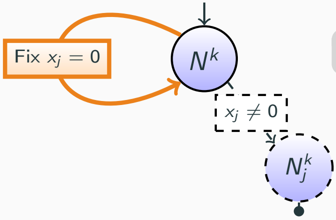


Ingredients

Variable fixing



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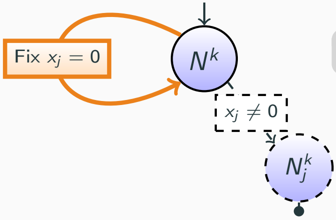
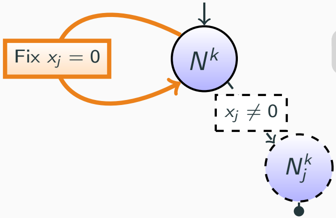


Identify an index j

Ingredients

- Already-computed quantities

Variable fixing

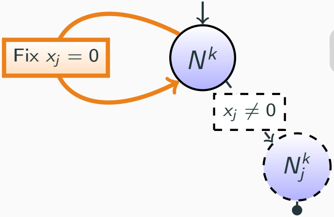


Identify an index j

Ingredients

- Already-computed quantities
- Cost-free dual evaluation

Variable fixing



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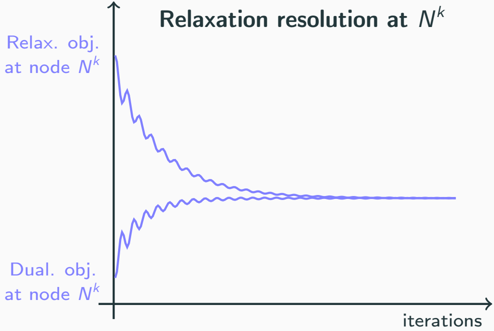

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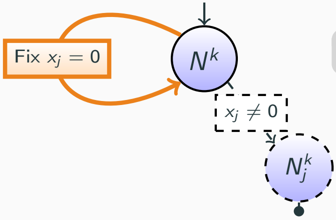

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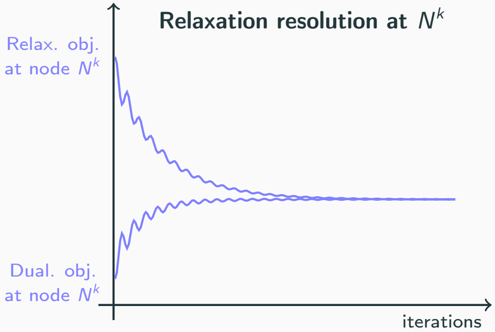
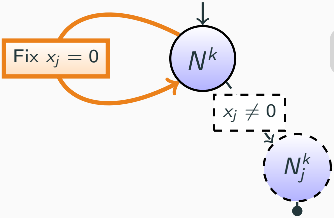


Ingredients

- Already-computed quantities
- Cost-free dual evaluation
- Dual link between nodes

Variable fixing



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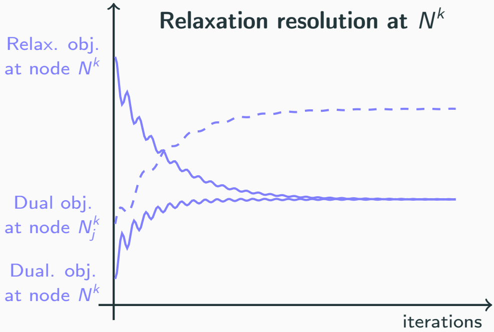

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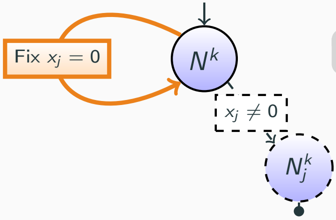

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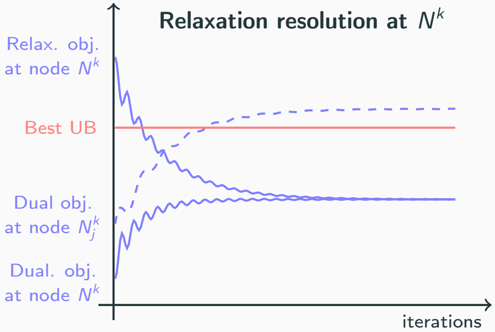

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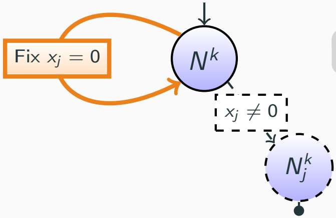

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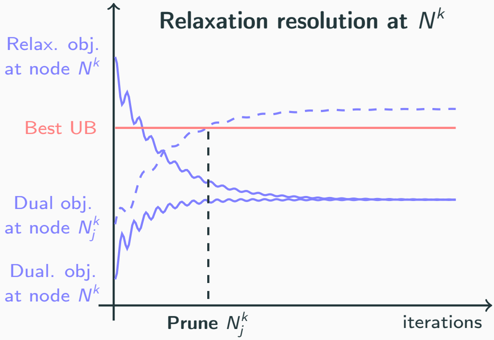

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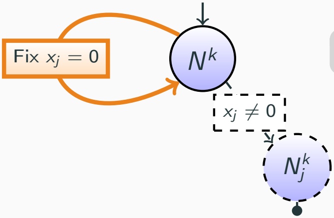

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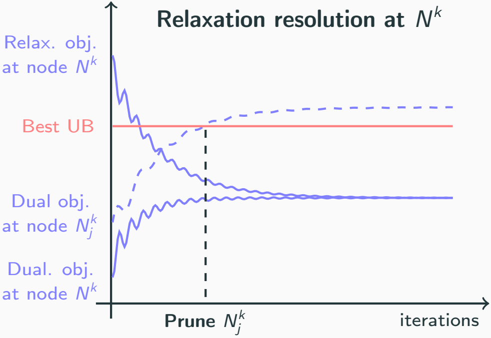

Ingredients

- Already-computed quantities
- Cost-free dual evaluation
- Dual link between nodes
- Can fix **multiple** variables

Numerical illustrations

$$\min_{x} \; F(Ax) + \lambda \|x\|_0 + G(x)$$

Numerical illustrations

$$\min_{x}\ F(Ax) + \lambda\|x\|_0 + G(x)$$

Dataset : Sparse linear regression

Numerical illustrations

$$\min_x \; F(Ax) + \lambda\|x\|_0 + G(x)$$

Dataset : Sparse linear regression

F(Ax) : Mean squared error

Numerical illustrations

$$\min_x \; F(Ax) + \lambda \|x\|_0 + G(x)$$

Dataset : Sparse linear regression
F(Ax) : Mean squared error
G(x) : Constraint $\boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u}$

Numerical illustrations

$\min_x \; F(Ax) + \lambda\|x\|_0 + G(x)$

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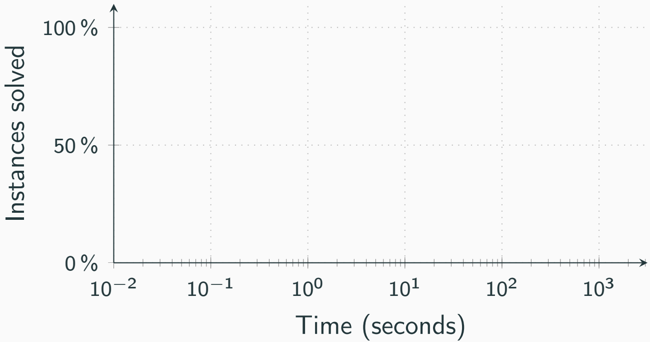

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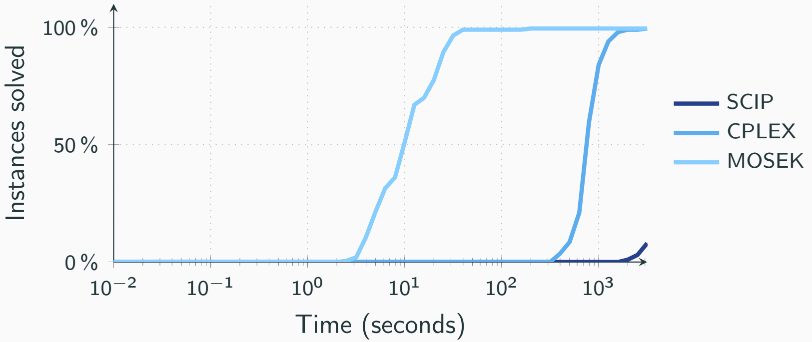

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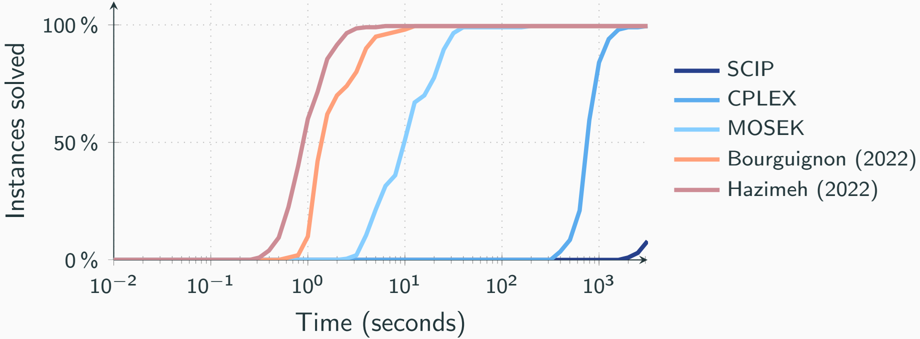

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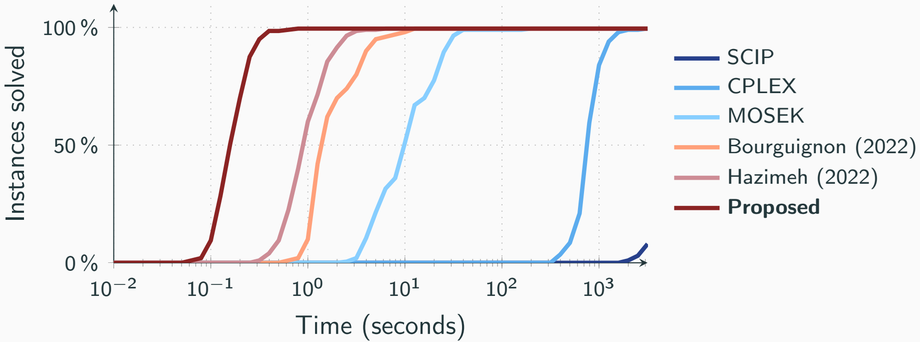

Numerical illustrations

Highlights

Highlights

- Gains up to $\times 10^5$ against MIP solvers

Numerical illustrations

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- Gains up to $\times 10^5$ against MIP solvers
- Gains up to $\times 10^2$ against tailored BnB

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- Scales up to dimension $\approx 10^5$ in minutes

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TheoGuyard/**El0ps.jl**

An Exact L0-penalized Problem Solver.

Highlights

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Efficient BnB

💪

TheoGuyard/**El0ps.jl**

An Exact L0-penalized Problem Solver.

Numerical illustrations

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Efficient BnB 💪

Generic framework 💪

TheoGuyard/El0ps.jl

An Exact L0-penalized Problem Solver.

Numerical illustrations

Highlights

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Efficient BnB 💪 → **TheoGuyard/El0ps.jl** ← Generic framework 💪

An Exact L0-penalized Problem Solver.

↑ 🥳

Works with any F and G
under mild assumptions !

Question time !

Working hypotheses

Loss function

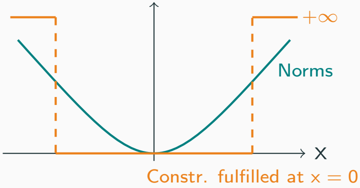
Assumptions on F :

- F is proper, convex, lower-semicontinuous
- F is differentiable with ∇F being L-Lipschitz

Modelling term

Assumptions on G :

- G is proper, convex, lower-semicontinuous
- $G = \sum_i G_i$ with $G_i \geq G_i(0) = 0$



Node problems

N^k

$\mathcal{S}_0 = \{i \mid x_i = 0\}$

$\mathcal{S}_1 = \{i \mid x_i \neq 0\}$

$\mathcal{S}_\bullet = \{i \mid x_i \text{ free}\}$

Subproblem

$$\begin{cases} \min_{\mathrm{x}} & F(\mathrm{Ax}) + \lambda\|\mathrm{x}\|_0 + G(\mathrm{x}) \\ \text{s.t.} & \mathrm{x}_{\mathcal{S}_0} = 0,\ \mathrm{x}_{\mathcal{S}_1} \neq 0 \end{cases}$$

NP-hard unless $\mathcal{S}_\bullet = \emptyset$.

Relaxation

$$\min_{\mathrm{x}} \left\{ F(\mathrm{Ax}) + H(\mathrm{x}) \right\}$$

with $H(\mathrm{x})$ being the convex envelope of $\lambda\|\mathrm{x}\|_0 + G(\mathrm{x}) + \mathbb{I}(\mathrm{x}_{\mathcal{S}_0} = 0, \mathrm{x}_{\mathcal{S}_1} \neq 0)$.

Dual

$$\max_{\mathrm{u}} \left\{ -F^\star(-\mathrm{u}) - H^\star(\mathrm{A}^{\mathrm{T}}\mathrm{u}) \right\}$$

with $H^\star(\mathrm{A}^{\mathrm{T}}\mathrm{u}) = \sum_{i \in \mathcal{S}_\bullet} [G^\star(\mathrm{a}_i^{\mathrm{T}}\mathrm{u}) - 1]_+ + \sum_{i \in \mathcal{S}_1} (G^\star(\mathrm{a}_i^{\mathrm{T}}\mathrm{u}) - 1)$