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The interior structure of Mercury: what we know, what we expect from BepiColombo

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Abstract

The BepiColombo mission is planned to very accurately measure the gravity field, the topography, and the tidal Love numbers of Mercury. In this paper, we review our present knowledge of the interior structure and show how the data from BepiColombo can be used to improve on our knowledge. We show that our present estimates of the core mass and volume depend mostly on our confidence in cosmochemically constrained values of the average silicate shell and core densities. The moment of inertia (MOI) C about the rotation axis will be determined very accurately from the degree 2 components of the gravity field and from measurements of the obliquity and the libration frequency of the rotation axis. The ratio C_m/C between the MOI of the solid planet to the MOI of the planet, both about the rotation axis, will additionally be obtained. If the core is liquid or if there is a liquid outer core, C_m/C will be around 0.5. In this case, C_m can be identified with the MOI of the silicate shell. If the core is solid, C_m/C will be about 1. The MOI C can be used to test and refine present models but will most likely not per se help to increase the confidence in the two-layer model beyond the present level, at least if there is a substantial inner core. C and C_m/C can be used to calculate the inner core radius and the outer core density, assuming the silicate shell density and the inner core density are given by cosmochemistry. The accuracy of the outer core density estimate depends largely on the confidence in the cosmochemical data. The inner core radius can be determined to the accuracy of the densities if the inner core radius is greater than 0.5 core radii. These values can be checked against the Love number of the planet. The higher order components of the gravity field can be used to estimate core–mantle boundary undulations and crust thickness variations. The former will dominate the gravity field at long wavelength, while the latter will dominate at short wavelengths.

1. Introduction

The BepiColombo mission scheduled for launch to Mercury in 2009 and to arrive at the planet in 2011 will provide a chance to get high-resolution gravity and topography data from Mercury and to measure the tidal Love numbers of the planet. According to the prospected mission scenario outlined in the BepiColombo System and Technology Report (ESA, 2000) the planetary orbiter, a 3 axis stabilised nadir pointing remote sensing spacecraft, will orbit the planet for a nominal mission duration of about one terrestrial year. The orbit will be nearly polar with a period of 2.32 h, a periherm altitude of 400 km, where the most accurate measurements can be taken, and an apoherm altitude of 1500 km. During the nominal mission duration the planet's surface will be covered completely once at periherm altitude.

Among the scientific goals of the mission are a better exploration of the interior structure and chemistry of the planet which in part motivates the inclusion of a radio science experiment and a laser altimeter in the strawman payload. Other science goals of the radio science experiment are related to general relativity and the time rate of change of the gravitational constant G. In this paper, we review the present knowledge of the interior structure of Mercury and show how the data expected from BepiColombo can be used to significantly improve on our knowledge. In particular, we will address the possible estimates of the core radius and density, the state of the core and the radius of a possible solid inner core. In addition, we will discuss whether or not an undulation of the core–mantle boundary could be seen in the gravity field data.

During the two encounters of the Mariner 10 spacecraft with Mercury in March 1974 and March 1975 measurements of the planet's mass and the two second degree gravitational coefficients, J_2 and C_{22}, were obtained from radio Doppler and range data. The planet's radius of 2439 ± 1 km

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was inferred from radio occultation observations. The determinations of the mass and radius gave a mean density of $5430 \pm 10\ \mathrm{kg/m^3}$, the largest density of the terrestrial planets. A combined least-squares fit to the Doppler data provided values of $J_2 = (6.0 \pm 2.0) \times 10^{-5}$ and $C_{22} = (1.0 \pm 0.5) \times 10^{-5}$ (Anderson et al., 1987 with Guiseppe Colombo, by the way). The large value of J_2 in spite of the planet's slow rotation period of 58.6462 days has been interpreted to be due to non-hydrostatic contributions to the polar oblateness of the gravity field, whereas the value of C_{22} indicates that the equatorial principal moment of inertia (MOI) are different from each other. Since a correction of J_2 for non-hydrostatic contributions to the gravity field is not possible at the present time, its value cannot be used to estimate the moment of inertia or the first moment of the density distribution, a useful quantity for interior structure models. To determine its value is one of the primary targets of most space missions to Mercury proposed to date including BepiColombo. A rough estimate from J_2 places its value at 0.34 RM^2, where M is the mass and R is the planet's (volume) average surface radius.

The interior structure of Mercury can thus at present only be modelled on the basis of the known mass of the planet and its radius. Simple models of Mercury's interior structure have been presented by e.g., BVSP (1981) and most recently by Harder and Schubert (2001). (An example of a simple two-layer model with a core and a silicate shell that we will use as a basis for our discussion in the next section is shown in Fig. 1.) In addition to the mass and radius, general perceptions about the chemistry and structure of terrestrial planets and the likely densities of prime chemical reservoirs such as a basaltic crust, a chemically more primitive mantle and an iron-rich core have been invoked.

Ground-based radar ranging data have recently been used to determine the equatorial elliptical shape of Mercury and to discuss implications for the internal structure (Anderson et al., 1996). The centre of figure (CF) has been concluded to be shifted with respect to the planet's centre of mass (CM) by 640 ± 78 m in the equatorial plane. The equatorial CF–CM offset has been interpreted to indicate a hemispheric asymmetry in crustal thickness. Assuming a representative density contrast between crust and mantle, the hemispherically averaged excess crustal thickness has been estimated to be about 13 km which is comparable to that of the Moon. An average crustal thickness of 200 ± 100 km has been calculated by comparison of Mercury's equatorial ellipticity to the gravitational coefficient C_{22} and assuming that the equatorial ellipticity is isostatically compensated due to Airy isostasy (Anderson et al., 1996).

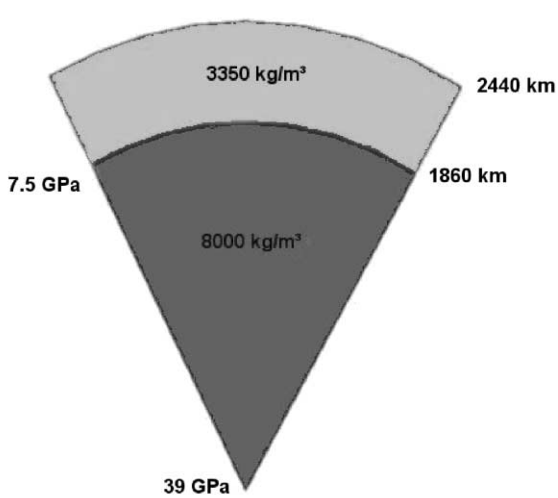

Fig. 1. Simple two-layer model of Mercury's interior structure satisfying the planetary mass and radius and using reasonable estimates of silicate shell and core densities

Mercury's average density is exceptionally large for a body of its size. Reduced to standard conditions, 0.1 GPa and room temperature, to account for self compression and temperature, it is at $5300\ \mathrm{kg/m^3}$ much larger than that of any other terrestrial planet. For example, the corresponding value for the Earth is only $4100\ \mathrm{kg/m^3}$ and that of Mars is $3800\ \mathrm{kg/m^3}$. This peculiarity indicates that the mass concentration of iron, calculated to be 560 mg/g in Mercury (Wasson, 1988), should be about twice that in the Earth, 280 mg/g, and suggests an extremely large iron core of 1800–1900 km radius that comprises roughly half of the planetary mass. If Mercury formed by condensation from the solar nebula at its present position then it should be strongly depleted in volatiles such as sulphur (e.g., Lewis, 1972, 1988; Goettel, 1988) and the core should be composed of almost pure iron. Only small amounts of a light alloying constituent in the core are then expected.

The silicate shell, the mantle and the crust, must be thin (Fig. 1), 500–700 km in total, with possibly important consequences for tectonics and volcanism. Convection in a planetary mantle depends on the ratio between the planet and core radii. For a thin shell such as the mantle of Mercury, it is expected that the convection pattern will feature a comparative large number of small scale cells (Fig. 2) since the width of the cells should be a few times the thickness of the mantle. This will tend to homogenise the tectonic patterns on the surface. Indeed, the tectonic pattern on the known part of the surface of the planet looks relatively smooth, albeit scarred by impacts, similar to that of the Moon and quite different from that of Mars. As a caveat, however, we note that there is some indication in terrestrial radar data of a possible giant volcanic dome in the unmapped area (Harmon, 1997). A confirmation of the existence of this dome, if it exists, and its subsequent mapping by high resolution imaging will be extremely valuable and will provide interesting constraints on mantle heterogeneity.

Pressure, density, temperature and elastic moduli should vary relatively little through the thin crust and mantle. A discontinuity is expected at the crust–mantle boundary, where the density would jump from around $2700\ \mathrm{kg\,m^{-3}}$ to about $3300\ \mathrm{kg\,m^{-3}}$. Major phase transition boundaries in the mantle are unlikely because of the small pressure. The pressure at the core–mantle boundary is only about 7 GPa.

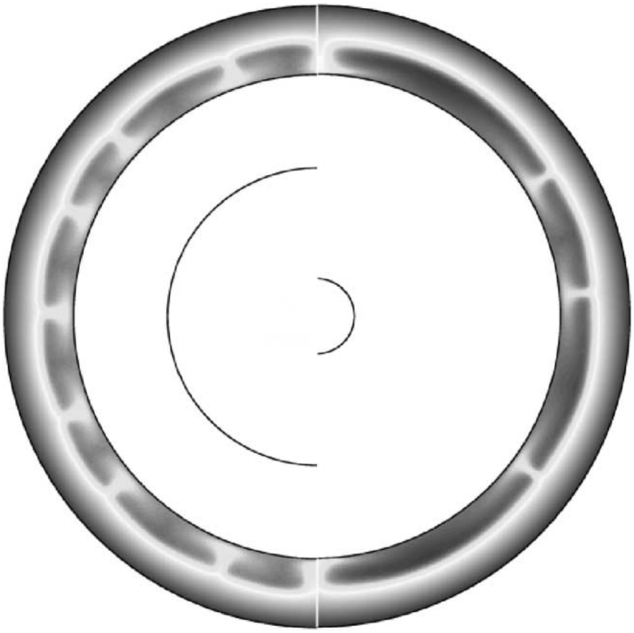

Fig. 2. Convection planform, temperature, and inner core sizes in Mercury from two thermal evolution calculations using a 2D convection code (Conzelmann, 1999). The viscosity in these calculations is temperature and pressure dependent. In the left panel, viscosity is simply temperature dependent with an activation energy of 500 kJ/mole. The concentration of sulphur in the core is 2%. In the right panel, the activation energy increases by 10% through the mantle due to increasing pressure and the sulphur concentration is 0.1%.

Chemical layering and an asthenosphere, a partially molten (upper) mantle layer, are however possible. The latter is suggested by recent thermal history calculations (Conzelmann, 1999; Conzelmann and Spohn, 1999). If detected, these layers would provide interesting clues to the accretion, differentiation, and volcanic and tectonic histories of the planet. If the above estimates of the crust thickness of 200 ± 100 km are correct then this would imply the most voluminous crust relative to the mantle among the terrestrial planets and could only be due to substantial partial melting in the mantle over its thermal history.

Perhaps the most important unknowns are related to the size, composition and physical state of the unusually large core. Since it is most likely that the magnetic field observed by Mariner is generated by a hydromagnetic dynamo or, alternatively, a thermoelectric dynamo in the core (Stevenson, 1987), at least an outer shell of the core of perhaps 500 km should be liquid. The liquid shell would have formed as a result of planetary cooling and core freezing from a hot initial state with an entirely molten core (e.g., Stevenson et al., 1983; Schubert et al., 1988). The core cannot consist of pure iron under these circumstances because it would be difficult to keep an iron core from completely freezing, as cooling models suggest. A small concentration of sulphur has been postulated (Stevenson et al., 1983) to account for a molten shell because sulphur is well known to depress the freezing point of a core alloy. Moreover, a small amount of sulphur can be reconciled with cosmochemical models that usually take Mercury's composition to be refractory because of its assumed formation close to the sun where temperatures were likely to be high in the early history of the solar nebula. Due to solid inner core growth, the sulphur concentration would become enriched in the outer core shell and the increasing depression of the freezing point would keep an outer layer liquid in spite of planet cooling. The inner core is almost pure iron in this model. Older thermal history models (e.g., Stevenson et al., 1983; Schubert et al., 1988; Spohn, 1991) based on a parameterisation of convective heat transport through the mantle invoked sulphur concentrations in the core between 1% and 5% to keep the core from freezing over the planet's lifetime. Tidal heating in the inner core has also been demonstrated to help in keeping a liquid outer core shell (Schubert et al., 1988). Recent calculations based on a more complete description of mantle convection and incorporating pressure and temperature dependent rheology (Conzelmann and Spohn, 1999) suggest that a terrestrial planet cools mostly by thickening its lithosphere (the outer rigid shell of the planet) while the deep interior stays relatively hot (Fig. 2). These models, in principle, confirm the findings of the older models but predict thicker outer core layers at the same sulphur concentration. The models have little difficulty in keeping a liquid layer in the core, even for sulphur concentrations as small as 0.1%.

The thermo-electric dynamo of Stevenson (1987) requires topographic variations of the core–mantle boundary of the order of a few kilometres in addition of a liquid outer core layer. These variations may perhaps be detected by gravity sounding of the planet.

2. Modelling the interior structure

Gravity sounding is, at present, the prime method for the exploration of the interior of Mercury and other terrestrial planets in the absence of seismic data. As we will show below, these interpretations will not be unique and self-consistent but will rely on additional information, in particular from chemistry. The BepiColombo mission will measure the components of the spherical harmonic expansion of the gravity field with an accuracy of 10^{-9} for degree $l = 2$ decreasing to 10^{-8} for $l = 20$. The cumulative error, i.e. the error in a particular component that accumulates from errors in the determination of lower order components is expected to increase from 5×10^{-4} mGal at $l = 2$ to 6×10^{-2} mGal at $l = 20$. At the nominal maximum spatial resolution of 400 km (equivalent to a half wavelength at $l = 20$), this will transfer into an accuracy of free air gravity of 0.1 mGal, or 10^{-3} mm s^{-2}. As detailed in Milani et al. (2001) there is hope that reasonable data can be expected up to a spatial resolution of $l = 25$ or 300 km. The expected accuracy in the (free-air) gravity field is about two orders of magnitude better than the Mars Global Surveyor (MGS) gravity field of Mars, currently the best gravity field of that planet. The gain in accuracy with respect to MGS is mostly due to K-band tracking of the spacecraft and an on board high accuracy accelerometer. The BepiColombo data should allow for sensible interior structure models including crust thickness variations and, perhaps, topography of the core–mantle boundary.

A laser altimeter has been proposed for the strawman payload (ESA, 2000) to complement the gravity data with accurate topography data. The accuracy of the topography data will depend on the vertical resolution of the laser altimeter. An accuracy of 1m should be attainable and would nicely complement the gravity data; the Mars Observer Laser Altimeter (MOLA) on MGS has a vertical resolution of even 50 cm. In fact, as is well known for gravity field interpretation (e.g., Tsubai, 1983), an accurate knowledge of the topography will be required to make full use of the gravity data.

The BepiColombo mission will also measure with high accuracy the libration rate and amplitude of the rotation axis and its obliquity. The latter two quantities together with the C_{20} and C_{22} coefficients of the gravity field expansion will allow an accurate determination of the MOI C and of the moment of the solid outer layer of the planet C_m both about the rotation axis (Peale, 1988; Milani et al., 2001). The expected accuracy for C/MR^2, where M is the mass and R the radius of the planet, respectively, is better than 3×10^{-4} and better than 0.05 for C_m/C (Milani et al., 2001). The ratio between C and C_m will immediately prove or disprove the existence of a fluid core. If there is a fluid core or a fluid outer core (either of the two is required for magnetic field generation), then the ratio is about 0.5, the exact value depending on the density and composition of the core and the radius of the inner core. If the core is solid, then the ratio will be 1. Since the moments of inertia are additive, the moment of inertia of the core C_c is simply the difference between C and C_m. Using C_c, the radius of the inner core can be estimated in the likely event that the core consists of a fluid outer and a solid inner core.

Since Mercury is tidally flexed by the sun, the modelling of the core radius and core structure can be augmented by considering the tidal Love number k_2. The tidal Love number measures the potential of the tidally displaced mass relative to the forcing potential. The Love number is a complex number, the amplitude of which depends on the interior structure and is a measure both of the radius of the core as well as of the radius of the inner core. The phase of k_2 depends on the dissipation rate of tidal energy or on the rate of change of elastic energy to the total elastic energy stored in the planet in one rotation cycle. The amplitude of k_2 will be measured with an accuracy of 6×10^{-4}, an accuracy that will be very useful as we will demonstrate below. The detectability of the imaginary part or phase of k_2 and its measurement with sufficient accuracy is much more difficult to predict and will depend on the Q-factor of the planet. A preliminary assessment by Milani et al. (2001) indicates that Q will have to be as low as about 20 for the phase to be measurable. This will require extraordinary large dissipation of tidal energy in the planet such as dissipation in a partially molten mantle asthenosphere.

2.1. Low order components of the gravity field and rotation data: interior structure models

For a spherically symmetric model of the interior structure, the following set of equations for the density $\rho(r)$ with suitable boundary conditions (compare Sohl and Spohn, 1997) have to be solved:

$$M = 4\pi \int_0^R \rho(r) r^2 \, dr, \tag{1}$$

$$C = \frac{8\pi}{3} \int_0^R \rho(r) r^4 \, dr, \tag{2}$$

$$C_m = \frac{8\pi}{3} \int_{R_c}^R \rho(r) r^4 \, dr, \tag{3}$$

$$C_c = \frac{8\pi}{3} \int_0^{R_c} \rho(r) r^4 \, dr. \tag{4}$$

Eqs. (1)–(4) are supplemented by an equation of state:

$$\rho(r) = \rho(P(r), T(r)), \tag{5}$$

where the pressure

$$P(r) = -\int_0^R \rho(r) g(r) \, dr, \tag{6}$$

with $g(r)$ being gravity and where the temperature $T(r)$ is given by a thermal model.

Based on our present knowledge of the mass of Mercury and cosmochemical evidence for the likely compositions and densities of the core and the silicate shell, we can estimate the masses of these major reservoirs. Assuming that the core is mostly iron, its density ρ_c should be 8000 ± 300 kg m^{-3}, where we allow for up to 5% sulphur reducing its value from that of pure iron, and for compression according to a volume averaged pressure of 14 GPa and a bulk modulus K_T of iron at that pressure of 241 GPa. (The latter value has been calculated by taking an STP value of K_T of 172 GPa and dK_T/dP of 4.95.) The silicate shell density ρ_m should be 3350 ± 250 kg m^{-3}. With these values, we get from

$$M = \tfrac{4}{3}\pi((R_P^3 - R_c^3)\rho_m + R_c^3 \rho_c) \tag{7}$$

the core radius R_c to be equal to 1860 ± 80 km, where the "error" of the core radius has been calculated from

$$\frac{\Delta R_c}{R_c} = -\frac{1}{3(\rho_c - \rho_m)} \left\{ \rho_m \frac{R^3 - R_c^3}{R_c^3} \frac{\Delta\rho_m}{\rho_m} + \rho_c \frac{\Delta\rho_c}{\rho_c} \right\} \tag{8}$$

with $\Delta\rho_c/\rho_c \approx 0.04$ and $\Delta\rho_m/\rho_m \approx 0.075$ according to the values chosen above. The negative value of $\Delta R_c/R_c$ indicates that the core radius varies in the opposite direction of variations of the densities. For instance, the core radius will decrease as the core density is increased.

The masses of the core and silicate shell relative to the planet mass are then 0.656 ± 0.06 and 0.344 ± 0.06 (or $\pm 10\%$ and 18%), respectively. In this analysis, we have ignored the errors in the planet radius and mass since these are negligible in comparison with the uncertainty in the densities.

It is not likely, that a gravity experiment at Mercury alone can give significantly better estimates in view of the ambiguity in any interpretation of the gravity field and in view of the fact that even the simplest discrete version of (1)–(6) will be an underdetermined system of algebraic equations. Improvements on the above results are possible with the help of cosmochemistry which may be able to narrow the ranges of likely densities similar to what we learned from Martian meteorites and from in situ geochemistry for Mars (Sohl and Spohn, 1997). An inclusion of the discrete version of (2) above in our estimate of the core radius and core and silicate shell masses

$$C = \tfrac{8}{15}\pi[(R_P^5 - R_c^5)\rho_m + R_c^5\rho_c] \tag{9}$$

will remove one of the unknowns from the problem (the core or the mantle density) but is not likely to improve the confidence in the model. Rather, the results may even be degraded because of the contribution of the likely core layering by a solid inner core to the moment of inertia which will result in additional errors in core density and core radius. Fig. 3 shows that the contribution to C can be substantial depending on the radius of the inner core and the core chemistry or the density difference between the outer and the inner cores. Eqs. (3) or (4) are not useful here because these equations are not linearly independent of (2). As a side we note that the situation for Mars is likely to be different because the Martian core may be entirely liquid, a suggestion made earlier (Schubert and Spohn, 1990; Spohn et al., 1998) because of the lack of a presently generated magnetic field.

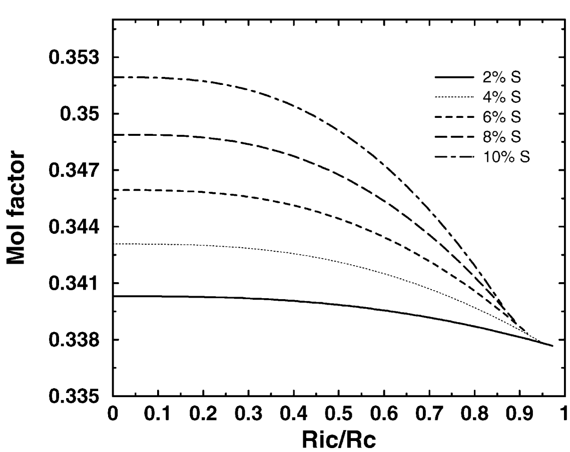

Fig. 3. Moment of inertia (MoI) factor of a three layer Mercury model as a function of inner core radius and core chemistry. The concentration of sulphur assumed in the core varies between 2% and 10%. The figure illustrates that the MoI factor will be changing noticeably with inner core radius for inner core radii larger than 20–50% of the core radius. The effect increases with increasing sulphur concentration because of the increasing density difference between the pure iron inner core and the outer core.

The values of C and C_m expected from the BepiColombo mission can, however, be used to first test the model and later to refine it by including the structure of the core. The MOI factor calculated from the above model is

$$\frac{C}{MR^2} \approx 0.3359. \tag{10}$$

The MOI factor of the core $C_c/M_cR_c^2$ is 0.4 by definition because the core is assumed to be homogeneous and the observable C_m/C is predicted to be

$$\frac{C_m}{C} \approx 0.5462 \tag{11}$$

if the core is (partly) fluid. These values can be compared with those that will be measured by the BepiColombo mission or any other Mercury mission. A significantly larger value of the planet's MoI-factor, for instance, should indicate a significantly smaller core density, caused by a much larger than expected sulphur or other light element concentration in the core or a significantly larger mantle density, for example. The hypothesis of a homogeneous core can best be tested by calculating the MoI-factor of the core from C_m/C and by comparing its value to 0.4. A significantly smaller value will indicate a substantial inner core while a slightly smaller value is likely to be due to a density increase through the core from compression. A completely fluid core would produce a tidal Love number k_2 of 0.3–0.45 (Fig. 4); a completely solid core, less likely on account of the planet's magnetic field, would produce a Love number k_2 of about 0.1. However, a completely solid core will be immediately evident from the value of C_m/C which would be about 1 in this case. The interpretation of C_m in terms of the solid silicate shell moment of inertia and the calculation of the core moment of inertia from it requires a significantly thick fluid layer in the core such that the mantle can be effectively decoupled from the core in its response to the tidal torque.

In the likely case the effort is warranted on the basis of the available data, the model can be refined by inclusion of the inner core and the radius of the inner core can be estimated with a satisfactory accuracy if its value is larger than about $0.5R_c$. This procedure will, however, again rely on independent input from cosmochemistry. We will again have to assume a silicate shell density, the value of which can be refined in the process of the modelling, and we will have to assume a value of the inner core density in order to be able to calculate the outer core density and the inner core radius. The value of the inner core radius will be useful for constraining the thermal evolution of the planet, models of which predict the radius of the inner core as a function of time by assuming that the inner core grows at the expense of the outer core due to cooling and core solidification. This perception of the inner core and allows an estimate of the inner core density which should be close to that of pure γ-iron.

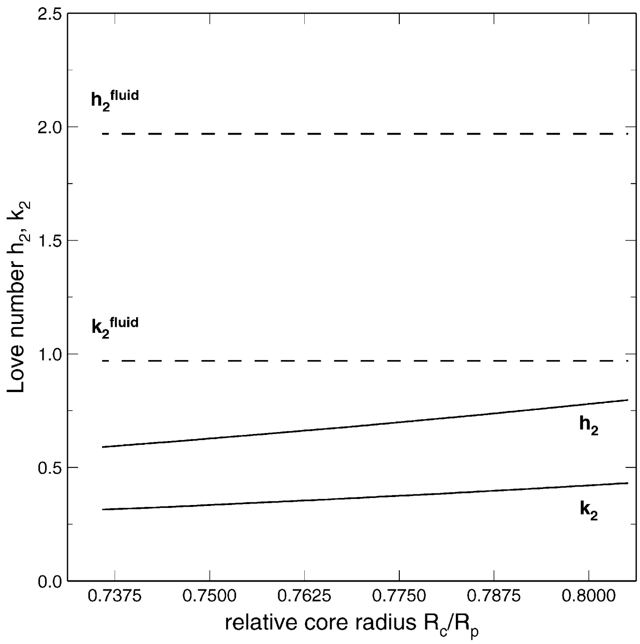

Fig. 4. Love numbers for Mercury (real part) calculated for a two layer model with a solid silicate shell and a fluid core of variable radius based on a MoI-factor of 0.3359. The core density varies between 7700 kg/m^3 producing a relative core radius of 0.805 and 8300 kg/m^3 yielding a relative core radius of 0.736. The corresponding mantle densities vary between 2950 and 3530 kg/m^3. The tidal Love number k_2 ranges between 0.3 and 0.45, while the radial Love number h_2 varies between 0.6 and 0.8. The fluid Love numbers $k_{2,\mathrm{fluid}}$ and $h_{2,\mathrm{fluid}}$ are about 1 and 2, respectively, and are related to the value of the MoI factor.

Writing the discrete versions of (1)–(4) for a three layer model with an inner core of radius R_{ic} and density ρ_{ic}, an outer core of density ρ_{oc} extending from R_{ic} to R_{c}, and a silicate shell of density ρ_{m} extending from R_{c} to R, we have

$$M = \tfrac{4}{3}\pi[(R^3 - R_{\mathrm{c}}^3)\rho_{\mathrm{m}} + (R_{\mathrm{c}}^3 - R_{\mathrm{ic}}^3)\rho_{\mathrm{oc}} + R_{\mathrm{ic}}^3\rho_{\mathrm{ic}}], \tag{12}$$

$$C = \tfrac{8}{15}\pi[(R^5 - R_{\mathrm{c}}^5)\rho_{\mathrm{m}} + (R_{\mathrm{c}}^5 - R_{\mathrm{ic}}^5)\rho_{\mathrm{oc}} + R_{\mathrm{ic}}^5\rho_{\mathrm{ic}}], \tag{13}$$

$$C_{\mathrm{m}} = \tfrac{8}{15}\pi(R^5 - R_{\mathrm{c}}^5)\rho_{\mathrm{m}}, \tag{14}$$

$$C_{\mathrm{c}} = \tfrac{8}{15}\pi[(R_{\mathrm{c}}^5 - R_{\mathrm{ic}}^5)\rho_{\mathrm{oc}} + R_{\mathrm{ic}}^5\rho_{\mathrm{ic}}]. \tag{15}$$

Assuming the silicate shell density, we calculate R_{c} from (14) and solve (12) and (13) for the density of the outer core and the radius of the inner core assuming a value for ρ_{ic}. Since the accuracy of C_{m}/C will be not much better than 5%, the accuracy of the latter has to be included in the error analysis. With the assumed accuracy of ρ_{m} (± 250 kg m^{-3} as above), we get a formal accuracy R_{c} of about 3%. The accuracy of the estimates of ρ_{oc} and R_{ic} are more difficult to calculate and will depend not only on the accuracy of the input parameters but also on the values of ρ_{oc} and R_{ic}.

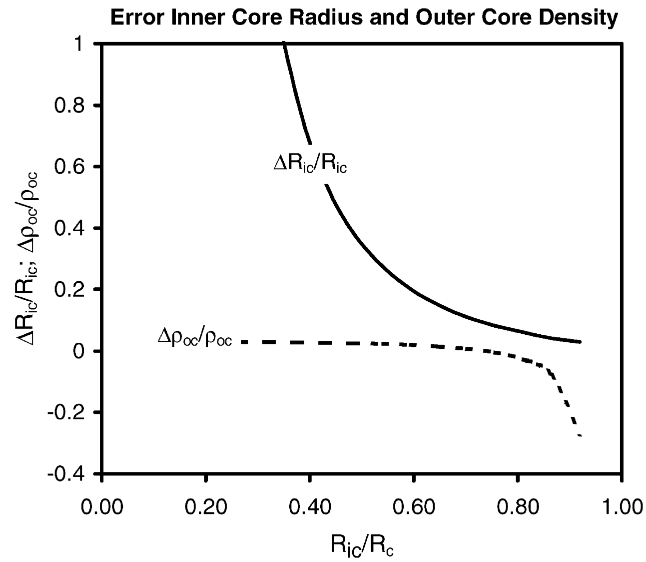

Fig. 5. Errors of inner core radius $\Delta R_{\mathrm{ic}}/R_{\mathrm{ic}}$ (solid line) and outer core density $\Delta\rho_{\mathrm{oc}}/\rho_{\mathrm{oc}}$ (dashed line) as calculated from (16) and (17).

The set of equations to be solved is

$$\frac{\mathrm{d}\rho_{\mathrm{oc}}}{\rho_{\mathrm{oc}}} + 3\left(\frac{\rho_{\mathrm{ic}}}{\rho_{\mathrm{oc}}} - 1\right)\frac{1}{(R_{\mathrm{c}}^3/R_{\mathrm{ic}}^3)}\frac{\mathrm{d}R_{\mathrm{ic}}}{R_{\mathrm{ic}}}$$
$$= -\left\{\frac{R^3 - R_{\mathrm{c}}^3}{R_{\mathrm{c}}^3 - R_{\mathrm{i}}^3}\frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{oc}}}\frac{\mathrm{d}\rho_{\mathrm{m}}}{\rho_{\mathrm{m}}} + \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{oc}}}\frac{1}{(R_{\mathrm{c}}^3/R_{\mathrm{ic}}^3)}\frac{\mathrm{d}\rho_{\mathrm{ic}}}{\rho_{\mathrm{ic}}}\right.$$
$$\left. -3\left(1 - \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{oc}}}\right)\frac{1}{(R_{\mathrm{ic}}^3/R_{\mathrm{c}}^3)}\frac{\mathrm{d}R_{\mathrm{c}}}{R_{\mathrm{c}}}\right\}, \tag{16}$$

$$\frac{\mathrm{d}\rho_{\mathrm{oc}}}{\rho_{\mathrm{oc}}} + 5\left(\frac{\rho_{\mathrm{ic}}}{\rho_{\mathrm{oc}}} - 1\right)\frac{1}{(R_{\mathrm{c}}^5/R_{\mathrm{ic}}^5)}\frac{\mathrm{d}R_{\mathrm{ic}}}{R_{\mathrm{ic}}}$$
$$= -\left\{\frac{R^5 - R_{\mathrm{c}}^5}{R_{\mathrm{c}}^5 - R_{\mathrm{i}}^5}\frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{oc}}}\frac{\mathrm{d}\rho_{\mathrm{m}}}{\rho_{\mathrm{m}}} + \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{oc}}}\frac{1}{(R_{\mathrm{c}}^5/R_{\mathrm{ic}}^5)}\frac{\mathrm{d}\rho_{\mathrm{ic}}}{\rho_{\mathrm{ic}}}\right.$$
$$\left. -5\left(1 - \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{oc}}}\right)\frac{1}{(R_{\mathrm{ic}}^5/R_{\mathrm{c}}^5)}\frac{\mathrm{d}R_{\mathrm{c}}}{R_{\mathrm{c}}}\right\}. \tag{17}$$

Taking representative values of $R_{\mathrm{c}} = 1850$ km, $\rho_{\mathrm{m}} = 3350$ kg/m^3, $\rho_{\mathrm{ic}} = 8500$ kg/m^3, $\mathrm{d}R_{\mathrm{c}}/R_{\mathrm{c}} = 0.05$, $\mathrm{d}\rho_{\mathrm{m}}/\rho_{\mathrm{m}} = 0.10$, and $\mathrm{d}\rho_{\mathrm{ic}}/\rho_{\mathrm{ic}} = 0.05$, we get $\mathrm{d}R_{\mathrm{ic}}/R_{\mathrm{ic}}$ and $\mathrm{d}\rho_{\mathrm{oc}}/\rho_{\mathrm{oc}}$ as functions of R_{ic} as shown in Fig. 5. The value of ρ_{oc} varies such that the mass of the core of is conserved. Fig. 5 shows that the inner core radius cannot be determined with much confidence unless its value is larger than about $0.5R_{\mathrm{c}}$. This is readily understood from the $R_{\mathrm{c}}/R_{\mathrm{ic}}$ to the third and fifths power and the $\rho_{\mathrm{ic}}/\rho_{\mathrm{oc}} - 1$ factors on the left-hand side of (16) and (17). The error in ρ_{oc} becomes large as the inner core radius tends towards the radius of the outer core which is again readily understood from an inspection of the factors involving the differences between the core radii and the ratios between the core densities. The confidence in R_{ic} for small inner core radii can certainly be improved to some extent by taking into account more accurately the compression at the centre of the core. Fig. 5 should be regarded as an albeit representative example. The solution of (16) and (17) depends quite sensitively on the chosen parameter values. As a conclusion, we can state that the inner core radius and the outer core density can be estimated satisfactorily with an accuracy mainly determined by our confidence in the cosmochemically constrained densities of the inner core and the mantle if the radius of the inner core is more than about half the core radius.

A similar conclusion can be drawn from the variation of the Love number k_2 with the radius of the inner core as shown in Fig. 6. This model has been calculated assuming a Maxwell rheology for the silicate shell and the inner core and an inviscid fluid outer core. The parameters of the model differ to some extent from our preferred choices above, but are in the ranges of the values considered. In particular, mantle and inner core densities are 3590 and 8000 kg/m^3, respectively. The outer core density varies such that the mass of the core is conserved. The outer core reaches the eutectic Fe–S composition at an inner core radius of $0.7R$. From this point on, the outer core density is kept constant at 5450 kg/m^3 and the core is assumed to further crystallise in the eutectic composition. (The eutectic composition is the composition where the solidus and the liquidus of the core alloy meet. For sub-eutectic sulphur concentrations, the core crystallises as almost pure iron and the sulphur is enriched in the fluid outer core. Once the eutectic composition is reached the remaining core liquid crystallises in that composition with no further compositional difference between the liquid and the solid phases.) For this reason, k_2 will not decrease further as R_{ic} increases. The silicate shell viscosity is assumed to be 10^{22} Pa s and the shear modulus is taken to be 7.5×10^{10} Pa in the silicate shell and in the inner core. The method of calculation has been detailed in Wieczerkowski (1999). As can be seen from the figure, the effect of the inner core on the Love number becomes noticeable only at radii larger than 0.5 core radii. For larger radii k_2 decreases by about 20% until it rapidly decreases to the value for a solid planet at $R_{ic} = R_c$.

The Love number will provide an important independent piece of data to constrain the radii of the core and the inner core. Because k_2 varies with both R_{ic} and R_c (compare Figs. 4 and 6), it cannot be used to independently calculate the two quantities. But together with the MoI data discussed above a reasonable model is possible. Incidentally, this model can be expected to be more accurate than our present models of Mars.

2.2. Higher order gravity field components: core–mantle boundary topography and crust thickness variations

The higher degree ($\ell > 2$) coefficients in the gravity field expansion data can be used to estimate the crust thickness and its variations with longitude and latitude. This estimate will depend on a sufficiently accurate knowledge of the topography (±1 m should be sufficient). The techniques for doing so are well developed (e.g., Wieczorek and Phillips, 1998) and have recently been applied to the Moon using Clementine data (Konopliv et al., 1998) and to Mars (Zuber et al., 1999). The latter authors have also modelled the thickness of the lithosphere and by assuming that the modelled thickness represents the time of loading and that the lithosphere base is an isotherm they have estimated the heat flow through the lithosphere at the time of loading. Since the formation of Martian crustal units spans a considerable time, these estimates could be turned into an estimate of the heat flow vs. time and provided some interesting constraints on the thermal evolution of the planet. Whether or not this will be possible at Mercury is difficult to say at present.

Here we will address the question of whether or not undulations of the core–mantle boundary could be estimated from the gravity data. The core–mantle boundary on Mercury is, as we have seen, likely located at comparatively shallow depth and presents a significant density jump. Undulations of the core–mantle boundary are interesting in the light of the proposal by Stevenson (1987) also discussed in Schubert et al. (1988) that the Mercurian dynamo generating the magnetic field is not a conventional hydromagnetic dynamo but rather a thermoelectric dynamo. The basis for this hypothesis is the notion that the magnetic field as we know it today is weaker than the field that would be predicted from a hydromagnetic dynamo. The model of a thermoelectric dynamo invokes undulations of the core–mantle boundary of the order of a few kilometres to provide the required lateral temperature gradients and associated electrical potential differences. The undulations are thought to be caused by the lateral pressure gradients applied to the core–mantle boundary by mantle convection currents. The wavelength of the undulations is then related to the wavelength of the convection pattern. A simple estimate puts this wavelength at twice the depth of the mantle noting that convection cells are likely to be of aspect ratio one. Fig. 2 shows that this may indeed be the case in Mercury but the figure also illustrates the well known fact that the aspect ratio may be significantly greater than one, in particular when temperature and pressure dependent rheology is included in the calculations. However, aspect ratios smaller than one have not been reported to our knowledge in planetary mantle convection modelling. Taking the mantle to be 500 km thick, the wavelength should then be 1000 km or greater.

We use a simple but well established model to estimate the gravity anomalies that may be produced by undulations of the core–mantle boundary. We assume a sphere of radius R_c coated with a periodic mass distribution. The surface mass density is assumed to be given by

$$\sigma = \Delta\rho w S_\ell, \tag{18}$$

where $\Delta\rho$ is the density difference across the boundary, w is the amplitude of the undulation, and S_ℓ is a spherical harmonic of degree l. The amplitude of the gravity anomaly produced by this mass distribution at the planet surface

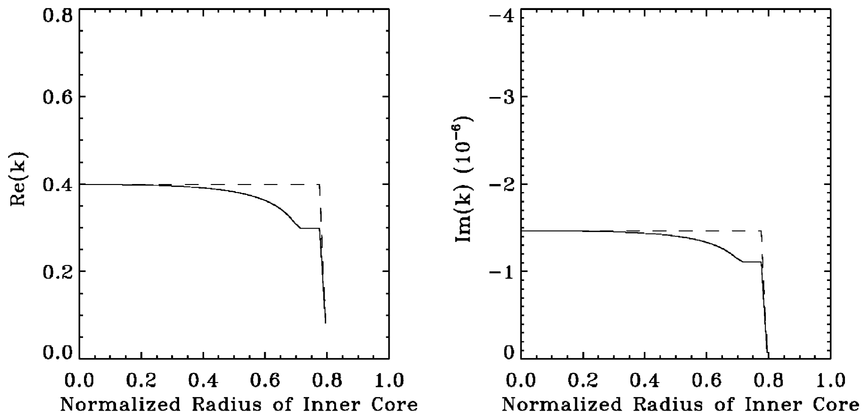

Fig. 6. Love number k_2 (real and imaginary parts) calculated for a three layer model of Mercury with a solid inner core, a liquid outer core and a silicate shell. The silicate shell and inner core densities are 3590 and 8000 kg/m^3, respectively. The outer core density varies such that the mass of the core is conserved. Its minimum allowed density is the eutectic Fe–S density of 5450 kg/m^3. The silicate shell viscosity is 10^{22} Pa s and the shear modulus is 7.5×10^{10} Pa in the silicate shell and in the inner core. The outer core is inviscid.

(assumed spherical, for simplicity) is

$$\Delta g = 4\pi\sigma G \left(\frac{\ell+1}{2\ell+1}\right)\left(\frac{R_c}{R}\right)^{\ell+2}, \tag{19}$$

where G is the gravitational constant. For calculating the effect of crust thickness variations, R_c has to be replaced with $(R - D_{cr})$, where D_{cr} is the crust thickness, and $\Delta\rho$ by the appropriate density difference at the crust–mantle boundary. At the periapsis of the spacecraft, R needs to be replaced with $R + p$, where p is periapsis height.

Fig. 7 shows the results of these calculations for core–mantle boundary undulations of 5 km magnitude for wavelength between 700 and 3000 km in comparison with the gravity anomaly produced by crust thickness variations of the same amplitude. It is easily seen that the anomalies produced by core–mantle boundary undulations decrease more rapidly with decreasing wavelength.

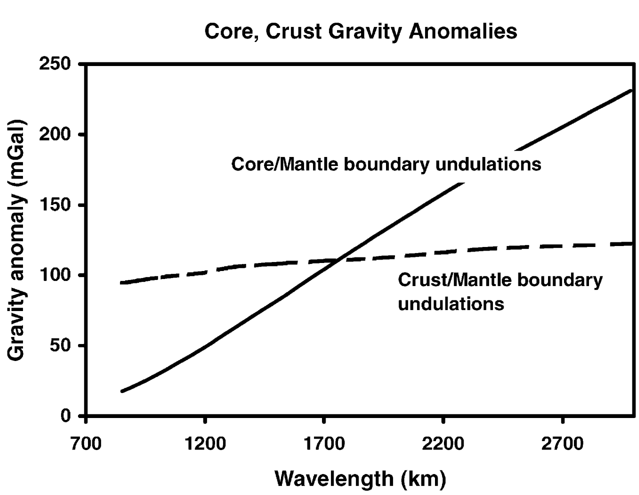

Fig. 7. Gravity anomalies as functions of wavelength predicted from a mass coating model for undulations of the core–mantle (CMB) and the crust–mantle (CrMB) boundaries. The amplitudes of the undulations are 5 km, The density difference assumed across the CMB is 3450 kg/m^3; it is 600 kg/m^3 across the CrMB.

The anomalies are well above the detection limit of the BepiColombo gravity experiments of at least 0.1 mGal. However, the core–mantle boundary undulations are hidden by the crust–mantle boundary undulations for wavelength below about 1700 km. The crossing will, of course, shift with the amplitude of the undulations if those are unequal but to the favour of detecting the core–mantle boundary undulations only if the latter are greater than the former which is perhaps unlikely. A wavelength of 1700 km corresponds to an aspect ratio of about 2. Aspect ratios of 3, however, are possible judging from the model in Fig. 2. Unfortunately, the models derived from the higher order gravity components will be subject to the non-uniqueness of all gravity field interpretations. That is, there will be no easy way of telling from the data whether or not these should be interpreted in terms of crust mantle boundary or core–mantle boundary undulations or both.

3. Discussion and conclusions

We have discussed the use of the gravity data expected from the BepiColombo mission to improve on our knowledge about the interior structure and composition of Mercury. The confidence we can have in our present models of Mercury depends mainly on the confidence we have in the values of the silicate shell density and core average densities. Cosmochemical considerations are essential here. For instance, it almost goes without saying that the core should be composed mostly of iron. Moreover, the proximity of the planet to the sun suggests that the core contains only a very small concentration of volatiles. This reasoning can however, be misleading if Mercury was not formed at its present orbital distance but at a distance farther from the sun, a possibility suggested by Wetherill (1980) on the basis of his accretion calculations.

It is important to note that even after BepiColombo, or any comparable mission to Mercury, we will still have to rely on cosmochemical estimates in our construction of interior models. And the accuracy of these models will continue to depend mainly on the confidence in these latter estimates. It is interesting to see that for a planet for which we have good reason to believe that its core is partly frozen, the knowledge of the MoI-factor will in fact not help much in increasing the accuracy of simple two- or three-layer models but will actually increase the error if the constraints on density from cosmochemistry are traded against the MoI-factor. These considerations may not apply to Mars, where we have reason to believe that the core is entirely liquid, judging from the absence of a present magnetic field and where the core is smaller.

Still, we will be particularly fortunate at Mercury because it will be possible to accurately determine not only the moment of inertia of the planet about the rotational axis but also the moments of inertia of the silicate shell and the core (about the rotational axis) should the latter be liquid at least in its outer shell. The ratio between the moments of inertia of the solid planet and that of the entire planet will immediately tell whether or not this hypothesis is correct.

The inner core radius can then be determined with the help of the core MoI-factor but with reasonable confidence only if its radius is substantial (> 0.5 core radii). For smaller inner core sizes the error in the inner core size will be larger than the core size itself. Thermal evolution models do in fact suggest that the inner core radius is large. The outer core density, however, is relatively well determined unless the core is almost frozen. Together with the core radius and the inner core density, estimates of the outer core density and inner core radius will allow a serious estimate of the chemical composition of the core and a thus tests of the chemical models that enter into the modelling.

The models we have discussed in this paper are simple two- and three constant density layer models. It is possible to refine the models by numerically solving Eqs. (1) through (7) for a multi-layer model assuming a thermal structure and a suitable equation of state such as the Murnaghan equation along the lines described most recently in Sohl and Spohn (1997) for Mars. This has been done previously for Mercury by e.g., the Basaltic Volcanism Project (BVSP, 1981). At present a calculation of such a model will not add much to our confidence in the sizes of the major reservoirs. With the gravity data and perhaps improved composition data available, a finely discretised model may be helpful.

The BepiColombo mission will likely provide us with the possibility to model the interior structure of the core, the crust–mantle boundary, the lithosphere thickness, perhaps over time, and perhaps the topography of the core–mantle boundary. With the help of these data, we will be able not only to constrain models of the Mercurian dynamo to generate the magnetic field but also models of the formation of the planet and its early evolution. A better assessment of interior structure that would be less dependent, although not completely independent, of rock properties from other fields can only be provided by seismology.

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