AE4893 (2023) – PHYSICS OF PLANETARY INTERIORS

Assignment 3 – From Planets with Love

What do tides reveal about the interior of planets?

Responsible instructors
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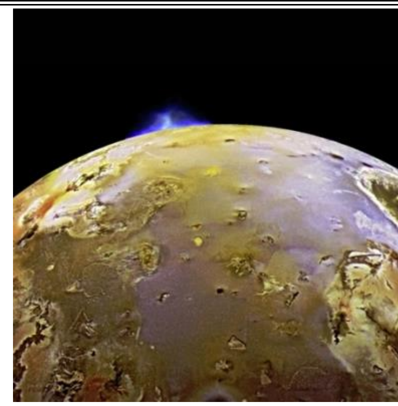
Group Size: **2** Students (one report)
Due: 17:30h, Friday 28th June, 2023
Estimated time: 25 hours
Page limit: 6 pages (excluding cover & appendices)



General introduction/background

The deformation of a planet due to tides depends on its interior properties. Because of this, observations of the tidal response can be used to learn new things about planetary interiors.
Over the years, the tidal response of various Solar System bodies has been measured using Doppler tracking and altimetry (*1*). Several authors have used these measurements to infer the interior properties of these bodies (*2*–*5*) (inverse problem). To do so, a forward model able to map interior properties to tidal response is necessary (*6*). Such a model can also be used to assess the sensitivity of the tidal response to variations in interior parameters, which provides insight into which parameters can be best constrained from observations.
In this assignment you will:

- use a state-of-the-art forward model to compute the tides of a planet of your choice (you can continue with the body you chose in for assignment one or pick a new one)
- assess the sensitivity of the tidal response to various interior parameters,
- discuss how observations of the tidal response can be used to constrain the interior.

Assignment description

You will compute the tidal response of a planet model of increasing complexity. You will first consider a one-layer and then a multi-layered model to study the sensitivity of the tidal response to interior properties.

A *Code* You will use the LOV3D software (*7*), written in Matlab. LOV3D is a software package that employs a spectral approach to compute the tidal response of a N-layer body. Given an interior model, LOV3D computes the Love numbers. An interior model consists of a series of layers with density, shear and bulk moduli and viscosity and is specified in the variable *Interior_Model*. The Love numbers are computed using function *get_Love* and stored in the variable *Love_Spectra*. While LOV3D can compute the tidal response of bodies whose properties vary with radius, latitude, and longitude, for this assignment you will assume that interior properties only vary with radius. A user-manual of the code can be found in the */docs* directory and various examples are also provided in /tests. To use the code:

1. Download the code from github. If you have git installed you can clone the repository by typing the following command: `git clone` https://github.com/mroviranavarro/LOV3D_multi
2. Carefully read the README.md file and the user manual
3. Chose a script from /tests that resembles your case. For rocky bodies you can use Test_Io_Multi_Layer_Spherically_Symmetric.mlx and for icy moons with subsurface oceans you can employ Test_Europa_Titan_Spherically_Symmetric.mlx. Open the script in MATLAB and run it as a test. If you do not have a recent version of MATLAB installed in your laptop, you can run the code online using Open in MATLAB Online

***B** Literature* Find a paper that considers the tides of the planet you will be considering for this assignment. Read the paper and note the interior parameters that the authors employ. If there is no literature for the planet you chose, consider choosing a different object.

***C** Data* The tidal response has only been measured for some of the bodies of the Solar, see (*1*). If you consider one of these bodies, you can use this data in **6)**

Deliverable

Your report should contain (answers to) the following:

***Model 1:** Homogenous planet*

1) (**Hypothesis/Physical Model/Numerical Model, 5%**) Build a one-layer model of your body. As constraints you should consider the mean density of the body and its radius. From literature (B), find a reasonable value for the mechanical properties (shear & bulk moduli and viscosity) for the one-layer model of the planet you are considering. In case the body is made of various elements (e.g., rock and ice), you can consider an average. If you cannot find an estimate for the shear modulus you can assume $\mu_{ice} = 3.3GPa, \eta_{ice} = 10^{18}Pa\ s\ , \mu_{rock} = 70GPa, \eta_{rock} = 10^{20}Pa\ s$. For the bulk modulus you can consider the body is incompressible and use a bulk modulus much higher than the shear modulus (e.g., $K = 10^5\mu$).
2) (**Verification, 20%**) Compute the tidal response (h and k Love numbers) of the body for a degree two tide and the relevant tidal frequency. The degree, order and period of the tide can be set using the input variable Forcing (Forcing.n, Forcing.m, Forcing.Td). Consider:
   a. an elastic model (remove or comment Interior_Model.eta);
   b. a viscoelastic model;
   c. an ideal fluid (this can be approximated by considering $\omega \rightarrow 0$).
   Compare the radial displacement and gravitational Love numbers (h and k) computed with LOV3D with those computed using analytical expressions (see lecture notes)
   For this task, you can use the script Test_One_Layer_Spherically_Symmeytric.mlx as starting point.
3) (**Analysis**, **25%)** Assess the sensitivity of the tidal response to the mechanical properties of the layer.

a. Starting with the viscoelastic model of **2b**, compute and plot the k Love numbers for a range of values of shear modulus and viscosity (imaginary and real component). You can consider the range $\mu = (0.1{,}10)\,\mu_0$, $\eta = (10^{-5}, 10^{5})\eta_0$. As the viscosity can vary over orders of magnitude use a logscale. For plotting you can consider using matlab *pcolor* and/or *contour* function. If you prefer, you can run the code in Matlab and plot the data in Python.

b. Explain how the tidal response depends on the shear modulus and viscosity.

You will share and discuss these results on June 13th.

Model 2: *Multi-layered planet*

4) (**Hypothesis/Physical Model/Numerical Model, 10%**) Build a multi-layered model of your planet. You can use the models you developed in assignment 1 but keep the number of layers limited to no more than 5. If you decide to use M2 or M3 from assignment 1, you can take the average density of the layer for the interior model. As in **1)**, use literature to find reasonable values for the mechanical properties. If you cannot find them, use the values suggested in **1)**.
5) (**Analysis, 10%)** Compare the Love numbers of the multi-layered model and the uniform model. Briefly discuss if the one-layer model captures the tidal response of your body.
6) **(Analysis/Communication, 30%)** Different parameters can have similar effects in the tidal response of the body. Investigate the sensitivity of the Love numbers to variations in interior parameters by
   a. varying one parameter while keeping the others constant and assess how much the Love number varies.
   b. varying the two parameters you suspect the model is most sensitive to.

   For this part, experiment to see to which parameters your model is most sensitive to. Using your results, discuss the challenges of constraining interior properties from tidal observations.

Prerequisites for report to be assigned a grade:

Please check if report includes:

- your name
- study number
- statement of how many hours you approximately spent on the assignment.
- proper referencing of literature

References

1. V. Lainey, Quantification of tidal parameters from Solar System data. *Celest Mech Dyn Astron* **126**, 145–156 (2016).
2. S. Padovan, J. L. Margot, S. A. Hauck, W. B. Moore, S. C. Solomon, The tides of Mercury and possible implications for its interior structure. *J Geophys Res Planets* **119**, 850–866 (2014).

3. I. Matsuyama, F. Nimmo, J. T. Keane, N. H. Chan, G. J. Taylor, M. A. Wieczorek, W. S. Kiefer, J. G. Williams, GRAIL, LLR, and LOLA constraints on the interior structure of the Moon. *Geophys Res Lett* **43**, 8365–8375 (2016).
4. C. Sotin, K. Kalousová, G. Tobie, Titan's Interior Structure and Dynamics After the Cassini-Huygens Mission. *The Annual Review of Earth and Planetary Sciences is online at earth.annualreviews.org* **49**, 579–607 (2021).
5. C. W. Hamilton, C. D. Beggan, S. Still, M. Beuthe, R. M. C. Lopes, D. A. Williams, J. Radebaugh, W. Wright, Spatial distribution of volcanoes on Io: Implications for tidal heating and magma ascent. *Earth Planet Sci Lett* **361**, 272–286 (2013).
6. R. Sabadini, B. Vermeersen, G. Cambiotti, "Global Dynamics of the Earth Second Edition."
7. M. Rovira-Navarro, I. Matsuyama, A. Berne, A Spectral Method to Compute the Tides of Laterally-Heterogeneous Bodies. *arXiv e-prints* **arXiv:2311.15710** (2023).