NUMERICAL METHODS

Programming Assignment 3

DUE DATE: JUNE 20, 2018

PROBLEM:

Write a program to approximate a definite integral $\int_a^b f(x)dx$ using the following composite quadrature rules:

(I) Composite Midpoint Rule
(II) Composite Trapezoidal Rule
(III) Composite Simpson's Rule

SPECIFICATION:

Assume that m is the number of basic quadrature rules used for the composition. For each composite quadrature rule, try to approximate each integral using m = 1, 2, 3, 4, 5,…, until successive approximations to the integral agree to within 10^{-6}.

OUTPUT OF THE PROBLEMS:

For each composite quadrature rule, output its final approximation and record the value of m.

TESTING CASES:

(a) $\int_0^{\pi} \sin x \, dx$

(b) $\int_0^1 e^x dx$

(c) $\int_0^1 \arctan x \, dx$

(d) $\int_{-2}^{10} (x^2 + 2x + 8) dx$

(e) $\int_0^2 \ln(1 + x^2) \, dx$

(f) $\int_{-1}^{1} \frac{\pi}{(1+x^2)^2} dx$

APPLICATION:

Use the Composite Midpoint Rule, the Composite Trapezoidal Rule and the Composite Simpson's Rule to do the following question:

A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table:

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

Question: How long is the track?

Hint:

Since the velocity, $v(t)$, is the derivative of the distance function, $s(t)$, the total distance traveled in the 84 second intervals $s(84) = \int_0^{84} v(t)dt$. However, we do not have an explicit representation for the velocity, only its values at each 6-second interval. We can appropriate the distance by doing numerical integration on the velocity to give an approximate length of the track.

DISCUSSION:

In each testing case, compare the results of the Composite Midpoint Rule, the Composite Trapezoidal Rule and the Composite Simpson's Rule. Turn in a short summary of your results.

APPENDIX: Formulas

(I) Composite Midpoint rule:

$$\int_a^b f(x)dx \cong 2h\sum_{j=0}^{n/2} f(x_{2j}),$$

where $h = \frac{b-a}{n+2}$, with an even integer n, and

$a = x_{-1} < x_0 < \ldots < x_{n+1} = b$, with $x_j = a + (j+1)*h$ for each $j = -1, 0, 1, \ldots, n+1$.

Note: Let m = the number of basic Midpoint rules used for the composition.

Then $m = (n + 2) / 2$.

(II) Composite Trapezoidal rule:

$$\int_a^b f(x)dx \cong \frac{h}{2}[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j)],$$

where $h = \frac{b-a}{n}$, and

$a = x_0 < x_1 < \ldots < x_n = b$, with $x_j = a + j*h$ for each $j = 0, 1, \ldots, n$.

Note: Let m = the number of basic Trapezoidal rules used for the composition.

Then $m = n$.

(III) Composite Simpson's rule:

$$\int_a^b f(x)dx \cong \frac{h}{3}[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)],$$

where $h = \frac{b-a}{n}$, with an even integer n, and

$a = x_0 < x_1 < \ldots < x_n = b$, with $x_j = a + j*h$ for each $j = 0, 1, \ldots, n$.

Note: Let m = the number of basic Simpson's rules used for the composition.

Then $m = n / 2$.