NUMERICAL METHODS

Programming Assignment 1

DUE DATE: APRIL 13, 2018

The objectives of this assignment:

1. To give you a programming experience with the numerical methods.
2. To compare various methods for finding a root.

PROBLEMS:

1. Write a method that accepts a, b, N, and TOL and carries out the Bisection procedure.
2. Write a method that accepts a, b, N, and TOL and carries out the False-Position procedure.
3. Write a method that accepts p_0, p_1, N, and TOL and carries out the Secant procedure.
4. Write a method that accepts p_0, N, and TOL and carries out Newton's procedure.

TESTING CASES:

1. Find the root of $f(x) = x^3 + 4x^2 – 10$ in the closed interval [1, 2].
   (a) Since $f(1) = –5$ and $f(2) = 14$, we can apply the Bisection method using $a = 1$, $b = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, a_n, b_n, p_n and $f(p_n)$. Output your results like the format of Table 2.1 in the textbook and check your answers.
   (b) Since $f(1) = –5$ and $f(2) = 14$, we can apply the method of False Position using $a = 1$, $b = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, a_n, b_n, p_n and $f(p_n)$. Output your results like the format of Table 2.3 in the textbook and check your answers.
   (c) Apply the Secant method using $p_0 = 1$, $p_1 = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, p_n and $f(p_n)$. Output your results like the format of Table 2.2 in the textbook and check your answers.
   (d) Apply Newton's method using $p_0 = 1$, $N = 20$, and $TOL = 0.0005$. Then print n, p_n and $f(p_n)$. Output your results like the format of Table 2.4 in the textbook and check your answers.

2. Find the root of $f(x) = e^x – 2cosx$ in the closed interval [0, 2]. Note that $f(0) * f(2) < 0$.
   (a) Apply the Bisection method using $a = 0$, $b = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, a_n, b_n, p_n and $f(p_n)$.
   (b) Apply the method of False Position using $a = 0$, $b = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, a_n, b_n, p_n and $f(p_n)$.

Cont. 2.

(c) Apply the Secant method using $p_0 = 0$, $p_1 = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, p_n and $f(p_n)$.

(d) Apply Newton's method using $p_0 = 2$, $N = 20$, and $TOL = 0.0005$. Then print n, p_n and $f(p_n)$.

Discuss your observations from parts (a) – (d).

3. The function $4x\cos(2x) - (x-2)^2 = 0$ has four roots in [0, 8]. Attempting to approximate these zeros within 10^{-5} using the Bisection, False-Position, Secant and Newton's methods. Choose your own initial for each method and run your program on this function. Discuss your observations. For example, which method converges fast?

4. Consider the function $f(x) = x^5 - 4.5x^4 + 4.55x^3 + 2.675x^2 - 3.3x - 1.3375$, find the root that lies just to the right of $x = -0.5$.

   (a) Apply the Secant method using $p_0 = -0.5$, $p_1 = -0.4975$, $N = 20$, and $TOL = 10^{-5}$. Then print n, p_n and $f(p_n)$.

   (b) Apply Newton's method using $p_0 = -0.4975$, $N = 20$, and $TOL = 10^{-5}$. Then print n, p_n and $f(p_n)$.

   Did both of the Secant and Newton's methods converge? Did they converge the roots those we desire?

5. Compare the Secant method with Newton's method for finding a root of each function below. Use the p_1 value from the Newton's method as the second starting point for the Secant method. Print out the results for both methods and discuss your observations.

   (a) $F(x) = x^3 - 3x + 1$, $p_0 = 2$.

   (b) $F(x) = x^3 - 2\sin x$, $p_0 = 1/2$.

6. Using $f(x) = x^5 - 9x^4 - x^3 + 17x^2 - 8x - 8$, *and* $p_0 = 0$, study and explain the behavior of the Newton's method. (*Hint*: The iterations are initially cyclic.)

Notes:

Besides coding, you need to write a report to analyze the testing results you obtained.