NUMERICAL METHODS

Programming Assignment 2

EARLY BIRD DEADLINE: May 23, 2018 (This assignment score * 120%)

DEADLINE: May 30, 2018

OBJECTS OF THE ASSIGNMENT:

Use nature cubic splines to approximate arbitrary functions and plot graphic curves.

PROBLEM:

Write a computer program for a natural cubic spline in the framework of the handout or the textbook, § 3.5, pp. 87 - 97.

SPECIFICATIONS:

The program should contain the following tasks:

1. Input the interpolation data for the interpolation point x_i, and $f(x_i)$, $0 \le i \le n$.
2. Find a cubic spline $S(x)$.

Remarks:

[i] Write $S(x)$ in the form:

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \text{ for } i = 0, 1, 2, ..., n - 1.$$

[ii] Follow the steps below:

Step 1. Find a_i, $0 \le i \le n$.

Step 2. Solve a tridiagonal system of linear equations for the coefficients c_i, $1 \le i \le n - 1$.

(Note that $c_0 = c_n = 0$ for the natural boundaries.)

Step 3. Find b_i, $0 \le i \le n - 1$.

Step 4. Find d_i, $0 \le i \le n - 1$.

(You may see the handout for the detail.)

3. Evaluate the value of the natural spline at a given point x.
4. Use MTALAB (or any available software) to plot the curve of a natural cubic spline.

INPUT OF THE PROBLEM:

1 n (note the value of n, not $n + 1$).

2 x_i, $i = 0, 1, 2, . . . , n$.

3 $f(x_i)$, $i = 0, 1, 2, . . . , n$.

($f(x_i)$ may be a given value y_i or evaluated from a given function $f(x)$.)

Test Cases:

Part (I) A natural cubic spline

1 Table 3.14 (textbook, pp. 95).

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
x	0.9	1.3	1.9	2.1	2.6	3.0	3.9	4.4	4.7	5.0	6.0	7.0	8.0	9.2	10.5	11.3	11.6	12.0	12.6	13.0	13.3
$f(x)$	1.3	1.5	1.85	2.1	2.6	2.7	2.4	2.15	2.05	2.1	2.25	2.3	2.25	1.95	1.4	0.9	0.7	0.6	0.5	0.4	0.25

(a) Output the coefficients of the natural cubic spline $S(x)$, a_i, b_i, c_i, d_i, $0 \le i \le 19$.

(You may check your results with Table 3.15.)

(b) Use Matlab (or any available software) to plot the graph of the natural cubic spline $S(x)$.

(You may check your graph with Figure 3.12.)

2 A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, and the distance in feet:

Time (x)	0	3	5	8	13
Distance ($f(x)$)	0	225	383	623	993

(a) Output the coefficients of the natural cubic spline $S(x)$, a_i, b_i, c_i, d_i, $0 \le i \le 3$.

(b) Use Matlab (or any available software) to plot the graph of the natural cubic spline $S(x)$.

(c) Use the natural cubic spline interpolation to predict the position of the car when $x = 10$ sec.

3 The following table details the USA population from 1910 to 1980.

Year (x)	1910	1920	1930	1940	1950	1970	1980
Population ($f(x)$)	91,972,266	105,710,620	122,775,046	131,669,275	150,697,361	203,235,298	226,547,082

(a) Output the coefficients of the natural cubic spline $S(x)$, a_i, b_i, c_i, d_i, $0 \le i \le 5$.

(b) Use Matlab (or any available software) to plot the graph of the natural cubic spline $S(x)$.

(c) Use the natural cubic spline interpolation to predict the populations in 1960.

(d) The real populations in 1960 was 179,323,175 (= $f(1960)$). Calculate the relative error:

$$\frac{|f(1960) - S(1960)|}{|f(1960)|}.$$

4 Consider Range's function that is defined as $y = f(x) = \dfrac{1}{1+25x^2}$. Collect interpolation data from this function on equally 11 points in the interval $[-2, 2]$.

(a) Output the coefficients of the natural cubic spline $S(x)$, a_i, b_i, c_i, d_i, $0 \le i \le 9$.

(b) Use Matlab (or any available software) to plot the graph of the natural cubic spline $S(x)$.

5 Repeat case 4 with more number of nodes. Thus, try a larger value of n.

Part (II) A pair of natural cubic splines

Now consider the curve C on the x-y plane given by $C : (x(t), y(t))$, $a \le t \le b$, where the values of $x(t)$ and $y(t)$ are given at the nodes $a = t_0 < t_1 < \ldots < t_n = b$. Approximate this curve by $(S_x(t), S_y(t))$, where $S_x(t)$ and $S_y(t)$ are natural cubic splines of $x(t)$ and $y(t)$, respectively, at the given nodes.

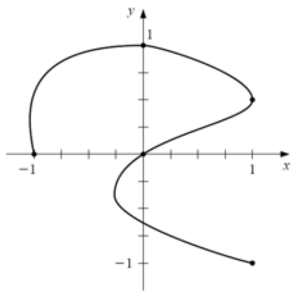
6 Use the following data to construct a pair of natural cubic splines $S_x(t)$ and $S_y(t)$.

i	0	1	2	3	4
t_i	0	0.25	0.5	0.75	1
x_i	-1	0	1	0	1
y_i	0	1	0.5	0	-1

(a) Output the coefficients of the natural cubic splines $S_x(t)$ and $S_y(t)$.

(Note that there are **two** natural cubic splines. Thus, one is $\boldsymbol{S_x(t)}$, and the other is $\boldsymbol{S_y(t)}$).

(b) Use Matlab (or any available software) to plot the pair of natural cubic splines $(S_x(t), S_y(t))$, and check your plot with the following figure.



7 Use the following data to construct a pair of natural cubic splines $S_x(t)$ and $S_y(t)$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
t_i	1	2	3	4	5	6	7	8	9	10	11	12	13
x_i	7	4	3	0	-3	-4	-7	-4	-3	0	3	4	7
y_i	0	2	5	8	5	2	0	-2	-5	-8	-5	-2	0

(a) Output the coefficients of the natural cubic splines $S_x(t)$ and $S_y(t)$.

(Note that there are **two** natural cubic splines. Thus, one is $\boldsymbol{S_x(t)}$, and the other is $\boldsymbol{S_y(t)}$).

(b) Use Matlab (or any available software) to plot the pair of natural cubic splines $(S_x(t), S_y(t))$.

8 A unit circle can be represented by the following parametric form:

$x(t) = \cos t,\ t \in [0, 2\pi]$

$y(t) = \sin t,\ t \in [0, 2\pi]$

Take $n = 81$ and choose your nodes, t_i, uniformly on the interval $[0, 2\pi]$.

(a) Output the coefficients of the natural cubic splines $S_x(t)$ and $S_y(t)$.

(Note that there are **two** natural cubic splines. Thus, one is $\boldsymbol{S_x(t)}$, and the other is $\boldsymbol{S_y(t)}$).

(b) Use Matlab (or any available software) to plot the pair of natural cubic splines $(S_x(t), S_y(t))$.

9 Repeat case 8 with more number of nodes. Thus, try a larger value of n.

WHAT YOU NEED TO DO:

1. In test cases 1 – 5
   (a) Output the coefficients of the nature cubic spline, a_i, b_i, c_i, d_i, $0 \le i \le n-1$.
   (b) Use Matlab (or any available software) to plot the obtained spline curve.
   (c) Display the estimated function value for the given x. (cases 2 and 3)
   (d) Calculate the relative error in case 3.
2. In test cases 6 – 9
   (a) Output the coefficients of the natural cubic splines $S_x(t)$ and $S_y(t)$.
   (b) Use Matlab (or any available software) to plot the obtained curve ($S_x(t)$, $S_y(t)$).
3. Report

   Describe how Matlab (or any available software) helps you to plot curves and show your resulting graphs. Discuss how the experimental results agree with the theory and everything else you consider important.