Homework #8

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Problem 1

Define the following measurement model:

$$y = Ax + \epsilon$$

Where:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \\ 0 & 6 \end{bmatrix} \quad y = \begin{bmatrix} 1.5377 \\ 3.6948 \\ -7.7193 \\ 7.3621 \end{bmatrix} \quad E(\epsilon\epsilon^T) = Q = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.25 \\ 0.5 & 2 & 0.25 & 1 \\ 0.5 & 0.25 & 2 & 1 \\ 0.25 & 1 & 1 & 4 \end{bmatrix}$$

Part a

In this part, we use the first two measurements to estimate our random variable x via the BLUE estimator since we don't have any prior knowledge on x but we have a probabilistic model of the error ϵ. Thus, we calculate the estimate as follows:

$$\hat{x}_a = (C_a^T Q_a^{-1} C_a)^{-1} C_a^T Q_a^{-1} y_a$$

$$cov(\hat{x}_a) = (C_a^T Q_a^{-1} C_a)^{-1}$$

Where:

$$C_a = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad y_a = \begin{bmatrix} 1.5377 \\ 3.6948 \end{bmatrix} \quad Q_a = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

Using MATLAB, we get:

$$\hat{x}_a = \begin{bmatrix} 0.6194 \\ 0.4591 \end{bmatrix} \quad cov(\hat{x}_a) = \begin{bmatrix} 4 & -2.75 \\ -2.75 & 2 \end{bmatrix}$$

Part b

Repeating the same procedure as in part a but using the first three measurements, we can write our estimate as follows:

$$\hat{x}_b = (C_b^T Q_b^{-1} C_b)^{-1} C_b^T Q_b^{-1} y_b$$

$$cov(\hat{x}_b) = (C_b^T Q_b^{-1} C_b)^{-1}$$

Where:

$$C_b = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \end{bmatrix} \quad y_b = \begin{bmatrix} 1.5377 \\ 3.6948 \\ -7.7193 \end{bmatrix} \quad Q_b = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 2 & 0.25 \\ 0.5 & 0.25 & 2 \end{bmatrix}$$

Using MATLAB, we get:

$$\hat{x}_b = \begin{bmatrix} -1.4303 \\ 1.8791 \end{bmatrix} \quad cov(\hat{x}_b) = \begin{bmatrix} 0.0679 & -0.026 \\ -0.026 & 0.1129 \end{bmatrix}$$

Part c

Finally for part c, we use all available measurements. We can write our estimate as follows:

$$\hat{x} = (C^T Q^{-1} C)^{-1} C^T Q^{-1} y$$

$$cov(\hat{x}) = (C^T Q^{-1} C)^{-1}$$

Where:

$$C = A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \\ 0 & 6 \end{bmatrix} \quad y = \begin{bmatrix} 1.5377 \\ 3.6948 \\ -7.7193 \\ 7.3621 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.25 \\ 0.5 & 2 & 0.25 & 1 \\ 0.5 & 0.25 & 2 & 1 \\ 0.25 & 1 & 1 & 4 \end{bmatrix}$$

Using MATLAB, we get:

$$\hat{x} = \begin{bmatrix} -1.2201 \\ 1.5368 \end{bmatrix} \quad cov(\hat{x}) = \begin{bmatrix} 0.0487 & 0.0054 \\ 0.0054 & 0.0618 \end{bmatrix}$$

The MATLAB code used for this problem is displayed below:

```
%% HW8 Problem 1
close all
clear all
clc
%% Initializing matrices
A = [1 2;3 4;5 0;0 6];
y = [1.5377;3.6948;-7.7193;7.3621];
Q = [1 0.5 0.5 0.25;0.5 2 0.25 1;0.5 0.25 2 1;0.25 1 1 4];
%% Part a
C_a = A(1:2,1:2);
y_a = y(1:2);
Q_a = Q(1:2,1:2);
x_blue_a = inv(C_a'*inv(Q_a)*C_a)*C_a'*inv(Q_a)*y_a;
cov_a = inv(C_a'*inv(Q_a)*C_a);
%% Part b
```

```
C_b = A(1:3,1:2);
y_b = y(1:3);
Q_b = Q(1:3,1:3);
x_blue_b = inv(C_b'*inv(Q_b)*C_b)*C_b'*inv(Q_b)*y_b;
cov_b = inv(C_b'*inv(Q_b)*C_b);
%% Part c
x_blue_c = inv(A'*inv(Q)*A)*A'*inv(Q)*y;
cov_c = inv(A'*inv(Q)*A);
```

Problem 2

Part a

Define $X_1 = [X\ Y]^T$ and $X_2 = Z$, we can find the conditional distribution of X_1 given $X_2 = z$, denoted by $f_{X_1|X_2=z}(x, y)$, as follows:

$$f_{X_1|X_2=z}(x,y) = \frac{1}{\sqrt{(2\pi)^n|\Sigma_{1|2}|}} e^{-\frac{1}{2}([x\ y]^T-\mu_{1|2})^T\Sigma_{1|2}^{-1}([x\ y]^T-\mu_{1|2})}$$

Where:

$$\mu_{1|2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(z - \mu_z)$$
$$\Sigma_{1|2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$
$$\mu_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \mu_z = 1 \quad \Sigma_{11} = \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix} \quad \Sigma_{22} = 2 \quad \Sigma_{12} = \Sigma_{21}^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad n = 2$$

Using MATLAB, we get:

$$\mu_{1|2} = \begin{bmatrix} \frac{1}{2}z - \frac{3}{2} \\ z - 1 \end{bmatrix} \quad \Sigma_{1|2} = \begin{bmatrix} 1.5 & 1 \\ 1 & 2 \end{bmatrix}$$

Part b

Define $X_1 = X|_{Z=z}$ and $X_2 = Y|_{Z=z}$, we can find the conditional distribution of X_1 given $X_2 = y|_{Z=z}$, denoted by $f_{X_1|X_2=y|_{Z=z}}(x)$, as follows:

$$f_{X_1|X_2=y|_{Z=z}}(x) = \frac{1}{\sqrt{(2\pi)^n|\Sigma_{1|2}|}} e^{-\frac{1}{2}(x-\mu_{1|2})^T\Sigma_{1|2}^{-1}(x-\mu_{1|2})}$$

Where:

$$\mu_{1|2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y|_{Z=z} - \mu_2)$$
$$\Sigma_{1|2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$
$$\mu_1 = \frac{1}{2}z - \frac{3}{2} \quad \mu_2 = z - 1 \quad \Sigma_{11} = 1.5 \quad \Sigma_{22} = 2 \quad \Sigma_{12} = \Sigma_{21} = 1 \quad n = 1$$

Using MATLAB, we get:

$$\mu_{1|2} = \frac{1}{2}y|_{Z=z} - 1 \quad \Sigma_{1|2} = 1$$

Part c

Define $X_1 = X$ and $X_2 = [Y \ Z]^T$, we can find the conditional distribution of X_1 given $X_2 = [y \ z]^T$, denoted by $f_{X_1|Y=y,Z=z}(x)$, as follows:

$$f_{X_1|Y=y,Z=z}(x) = \frac{1}{\sqrt{(2\pi)^n|\Sigma_{1|2}|}} e^{-\frac{1}{2}(x-\mu_{1|2})^T\Sigma_{1|2}^{-1}(x-\mu_{1|2})}$$

Where:

$$\mu_{1|2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}([y \ z]^T - \mu_2)$$

$$\Sigma_{1|2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

$$\mu_1 = -1 \quad \mu_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Sigma_{11} = 2 \quad \Sigma_{22} = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} \quad \Sigma_{12} = \Sigma_{21}^T = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad n = 1$$

Using MATLAB, we get:

$$\mu_{1|2} = \frac{1}{2}y|_{Z=z} - 1 \quad \Sigma_{1|2} = 1$$

Part d

We notice that we obtain the same mean vector $\mu_{1|2}$ as well as the same covariance matrix $\Sigma_{1|2}$:

$$\mu_{1|2} = \frac{1}{2}y|_{Z=z} - 1 \quad \Sigma_{1|2} = 1$$

The MATLAB code used to solve this problem is displayed below:

```
%% HW8 Problem 2
close all
clear all
clc
%% Initializing
syms z
syms x
syms y
mu = [-1;0;1];
cov = [2 2 1;2 4 2;1 2 2];
%% Part a
mu_1 = mu(1:2);
mu_z = mu(3);
cov_11 = cov(1:2,1:2);
cov_22 = cov(3,3);
cov_21 = cov(3,1:2);
cov_12 = cov_21';
mu_12 = mu_1+cov_12*inv(cov_22)*(z-mu_z);
sig_12 = cov_11-cov_12*inv(cov_22)*cov_21;
f_12 = inv(sqrt((2*pi)^2*abs(sig_12)))*exp(-1/2*(([x;y]-mu_12)'*inv(sig_12)*([x;y]-mu_12)));
%% Part b
mu_1_b = mu_12(1);
mu_2_b = mu_12(2);
cov_11_b = sig_12(1,1);
cov_22_b = sig_12(2,2);
cov_21_b = sig_12(1,2);
cov_12_b = cov_21_b';
mu_12_b = mu_1_b+cov_12_b*inv(cov_22_b)*(y-mu_2_b);
sig_12_b = cov_11_b-cov_12_b*inv(cov_22_b)*cov_21_b;
f_12_b = inv(sqrt((2*pi)*abs(sig_12_b)))*exp(-1/2*((x-mu_12_b)'*inv(sig_12_b)*(x-mu_12_b)));
%% Part c
```

```
mu_1_c = mu(1);
mu_2_c = mu(2:3);
cov_11_c = cov(1,1);
cov_22_c = cov(2:3,2:3);
cov_12_c = cov(1,2:3);
cov_21_c = cov_12_c';
mu_12_c = mu_1_c+cov_12_c*inv(cov_22_c)*([y;z]-mu_2_c);
sig_12_c = cov_11_c-cov_12_c*inv(cov_22_c)*cov_21_c;
f_12_c = inv(sqrt((2*pi)*abs(sig_12_c)))*exp(-1/2*((x-mu_12_c)'*inv(sig_12_c)*(x-mu_12_c)));
```

Problem 3

Part a

The Gram matrix for $M_k := \text{span}\{y_1, \dots, y_k\}$ is denoted as:

$$G_k = \begin{bmatrix} \langle y_1, y_1 \rangle & \langle y_1, y_2 \rangle & \cdots & \langle y_1, y_k \rangle \\ \langle y_1, y_2 \rangle & \langle y_2, y_2 \rangle & \cdots & \langle y_2, y_k \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle y_1, y_k \rangle & \langle y_1, y_2 \rangle & \cdots & \langle y_1, y_k \rangle \end{bmatrix} \quad \beta_k = \begin{bmatrix} \langle x, y_1 \rangle \\ \langle x, y_2 \rangle \\ \vdots \\ \langle x, y_k \rangle \end{bmatrix}$$

Thus:

$$G_{k+1} = \begin{bmatrix} \langle y_1, y_1 \rangle & \langle y_1, y_2 \rangle & \cdots & \langle y_1, y_{k+1} \rangle \\ \langle y_1, y_2 \rangle & \langle y_2, y_2 \rangle & \cdots & \langle y_2, y_{k+1} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle y_1, y_{k+1} \rangle & \langle y_1, y_2 \rangle & \cdots & \langle y_1, y_{k+1} \rangle \end{bmatrix} \quad \beta_{k+1} = \begin{bmatrix} \langle x, y_1 \rangle \\ \langle x, y_2 \rangle \\ \vdots \\ \langle x, y_{k+1} \rangle \end{bmatrix}$$

Since $y_{k+1} \perp M_k$, then $\langle y_{k+1}, y_i \rangle = 0 \ \forall y_i \in M_k$, thus:

$$\implies G_{k+1} = \begin{bmatrix} G_k & 0_{k\times 1} \\ 0_{1\times k} & \langle y_{k+1}, y_{k+1} \rangle \end{bmatrix}$$

Now:

$$\hat{x}_k \ := \ \operatorname*{arg\,min}_{m \in M_k} \ ||x - m||$$

Then $x - \hat{x}_k \perp M_k$ and:

$$\hat{x}_k = \alpha_1 y_1 + \dots \alpha_k y_k$$

Where:

$$G_k^T \alpha = \beta_k$$

Thus:

$$G_{k+1}^T \tilde{\alpha} = \beta_{k+1}$$

$$\implies \begin{bmatrix} G_k & 0_{k\times 1} \\ 0_{1\times k} & \langle y_{k+1}, y_{k+1} \rangle \end{bmatrix} \tilde{\alpha} = \beta_{k+1}$$

Ignoring the last row and column, we get:

$$G_k \tilde{\alpha}_k = \beta_k \implies \tilde{\alpha}_k = \alpha_k$$

Now, for the last row and column, we have:

$$\langle y_{k+1}, y_{k+1}\rangle \tilde{\alpha}_{k+1} = \langle x, y_{k+1}\rangle \implies \tilde{\alpha}_{k+1} = \frac{\langle x, y_{k+1}\rangle}{\langle y_{k+1}, y_{k+1}\rangle}$$

Since $x - \hat{x}_{k+1} \perp M_{k+1}$, then:

$$\hat{x}_{k+1} = \tilde{\alpha}_1 y_1 + \ldots + \tilde{\alpha}_k y_k + \tilde{\alpha}_{k+1} y_{k+1}$$

$$\implies \hat{x}_{k+1} = \alpha_1 y_1 + \ldots + \alpha_k y_k + \tilde{\alpha}_{k+1} y_{k+1}$$

$$\implies \hat{x}_{k+1} = \hat{x}_k + \beta y_{k+1} \quad \text{where} \quad \beta = \frac{\langle x, y_{k+1}\rangle}{\langle y_{k+1}, y_{k+1}\rangle}$$

Part b

$$\hat{y}_{k+1|K} \; := \; \underset{m \in M_k}{\arg\min} \; ||y_{k+1} - m||$$

Thus $y_{k+1} - \hat{y}_{k+1|K} \perp M_k$. Denoting $Y_{k+1} = y_{k+1} - \hat{y}_{k+1|k}$, from part a, we have:

$$\hat{x}_{k+1} = \hat{x}_k + \beta Y_{k+1}$$

$$\implies \hat{x}_{k+1} = \hat{x}_k + \beta(y_{k+1} - \hat{y}_{k+1|k})$$

Where:

$$\beta = \frac{\langle x, y_{k+1} - \hat{y}_{k+1|k}\rangle}{\langle y_{k+1} - \hat{y}_{k+1|k}, y_{k+1} - \hat{y}_{k+1|k}\rangle}$$

Problem 4

Define the following measurement model:

$$y = Cx + \epsilon$$

Where:

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \\ 0 & 6 \end{bmatrix} \quad y = \begin{bmatrix} 1.5377 \\ 3.6948 \\ -7.7193 \\ 7.3621 \end{bmatrix} \quad E(\epsilon\epsilon^T) = Q = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.25 \\ 0.5 & 2 & 0.25 & 1 \\ 0.5 & 0.25 & 2 & 1 \\ 0.25 & 1 & 1 & 4 \end{bmatrix}$$

$$E(xx^T) = P = \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix}$$

Part a

In this part, we use the first measurement to estimate our random variable x via the MVE estimator since we have some prior probabilistic model of x and we have a probabilistic model of the error ϵ. Thus, we calculate the estimate as follows:

$$\hat{x}_a = (C_a^T Q_a^{-1} C_a + P^{-1})^{-1} C_a^T Q_a^{-1} y_a$$

$$cov(\hat{x}_a) = P - P C_a^T (C_a P C_a^T + Q_a)^{-1} C_a P$$

Where:

$$C_a = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad y_a = \begin{bmatrix} 1.5377 \end{bmatrix} \quad Q_a = 1$$

Using MATLAB, we get:

$$\hat{x}_a = \begin{bmatrix} 0.3417 \\ 0.4271 \end{bmatrix} \quad cov(\hat{x}_a) = \begin{bmatrix} 0.2778 & -0.0278 \\ -0.0278 & 0.1528 \end{bmatrix}$$

Part b

Repeating the same procedure as in part a but using the first two measurements, we can write our estimate as follows:

$$\hat{x}_b = (C_b^T Q_b^{-1} C_b + P^{-1})^{-1} C_b^T Q_b^{-1} y_b$$

$$cov(\hat{x}_b) = P - P C_b^T (C_b P C_b^T + Q_b)^{-1} C_b P$$

Where:

$$C_b = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad y_b = \begin{bmatrix} 1.5377 \\ 3.6948 \end{bmatrix} \quad Q_b = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

Using MATLAB, we get:

$$\hat{x}_b = \begin{bmatrix} 0.4504 \\ 0.4963 \end{bmatrix} \quad cov(\hat{x}_b) = \begin{bmatrix} 0.1938 & -0.0812 \\ -0.0812 & 0.1188 \end{bmatrix}$$

Part c

Repeating the same procedure as in part a but using the first three measurements, we can write our estimate as follows:

$$\hat{x}_c = (C_c^T Q_c^{-1} C_c + P^{-1})^{-1} C_c^T Q_c^{-1} y_c$$

$$cov(\hat{x}_c) = P - P C_c^T (C_c P C_c^T + Q_c)^{-1} C_c P$$

Where:

$$C_c = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \end{bmatrix} \quad y_c = \begin{bmatrix} 1.5377 \\ 3.6948 \\ -7.7193 \end{bmatrix} \quad Q_c = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 2 & 0.25 \\ 0.5 & 0.25 & 2 \end{bmatrix}$$

Using MATLAB, we get:

$$\hat{x}_c = \begin{bmatrix} -1.0134 \\ 1.2402 \end{bmatrix} \quad cov(\hat{x}_c) = \begin{bmatrix} 0.0545 & -0.0105 \\ -0.0105 & 0.0828 \end{bmatrix}$$

Part d

Finally for part c, we use all available measurements. We can write our estimate as follows:

$$\hat{x} = (C^T Q^{-1} C + P^{-1})^{-1} C^T Q^{-1} y$$

$$cov(\hat{x}) = P - PC^T (CPC^T + Q)^{-1} CP$$

Where:

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \\ 0 & 6 \end{bmatrix} \quad y = \begin{bmatrix} 1.5377 \\ 3.6948 \\ -7.7193 \\ 7.3621 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.25 \\ 0.5 & 2 & 0.25 & 1 \\ 0.5 & 0.25 & 2 & 1 \\ 0.25 & 1 & 1 & 4 \end{bmatrix}$$

Using MATLAB, we get:

$$\hat{x} = \begin{bmatrix} -1.0296 \\ 1.2667 \end{bmatrix} \quad cov(\hat{x}) = \begin{bmatrix} 0.0437 & 0.0072 \\ 0.0072 & 0.0538 \end{bmatrix}$$

The MATLAB code to solve this part is displayed below:

```
%% HW8 Problem 4
close all
clear all
clc
%% Initializing matrices
C = [1 2;3 4;5 0;0 6];
y = [1.5377;3.6948;-7.7193;7.3621];
Q = [1 0.5 0.5 0.25;0.5 2 0.25 1;0.5 0.25 2 1;0.25 1 1 4];
P = [0.5 0.25; 0.25 0.5];
%% Part a
C_a = C(1,:);
y_a = y(1);
Q_a = Q(1);
x_mve_a = inv(C_a'*inv(Q_a)*C_a+inv(P))*C_a'*inv(Q_a)*y_a;
cov_a = P-P*C_a'*inv(C_a*P*C_a'+Q_a)*C_a*P;
%% Part b
C_b = C(1:2,:);
y_b = y(1:2);
Q_b = Q(1:2,1:2);
x_mve_b = inv(C_b'*inv(Q_b)*C_b+inv(P))*C_b'*inv(Q_b)*y_b;
cov_b = P-P*C_b'*inv(C_b*P*C_b'+Q_b)*C_b*P;
%% Part c
C_c = C(1:3,:);
y_c = y(1:3);
Q_c = Q(1:3,1:3);
x_mve_c = inv(C_c'*inv(Q_c)*C_c+inv(P))*C_c'*inv(Q_c)*y_c;
cov_c = P-P*C_c'*inv(C_c*P*C_c'+Q_c)*C_c*P;
%% Part d
x_mve = inv(C'*inv(Q)*C+inv(P))*C'*inv(Q)*y;
cov = P-P*C'*inv(C*P*C'+Q)*C*P;
```

Problem 5

Part a

Assuming we do not have any prior knowledge on x and ϵ, we can use the weighted least squares algorithm to compute an estimate $\hat{x}$ of the following overdetermined system:

$$y = Cx$$

$$\hat{x} = (C^T S C)^{-1} C^T S y \tag{1}$$

Using MATLAB, we find for $S = I$:

$$\hat{x} = \begin{bmatrix} -1.3169 \\ 1.4368 \end{bmatrix}$$

Part b

Applying the BLUE algorithm for all measurements:

$$\hat{x} = (C^T Q^{-1} C)^{-1} C^T Q^{-1} y \tag{2}$$

Using MATLAB with $Q = I$, we get:

$$\hat{x} = \begin{bmatrix} -1.3169 \\ 1.4368 \end{bmatrix}$$

Part c

Applying the MVE algorithm for all measurements:

$$\hat{x} = (C^T Q^{-1} C + P^{-1})^{-1} C^T Q^{-1} y \tag{3}$$

Using MATLAB with $Q = I$ and $P = 100I$, we get:

$$\hat{x} = \begin{bmatrix} -1.3163 \\ 1.4365 \end{bmatrix}$$

Now, using MATLAB with $Q = I$ and $P = 1e6I$, we get:

$$\hat{x} = \begin{bmatrix} -1.3169 \\ 1.4368 \end{bmatrix}$$

Part d

We notice that the results using WLS, BLUE and MVE with $P = 1e6I$ are very similar. Looking at equations 1,2 and 3, we can explain this similarity by first comparing equations 1 and 2. In fact, BLUE is similar to WLS when the weight matrix S is equal to the inverse of the error covariance matrix Q. Thus, for $Q^{-1} = I = S$, we obtain the same results. Now looking at equations 2 and 3, we notice that for a large P, the BLUE and MVE algorithms are similar. In fact, when the covariance matrix of the probabilistic random variable x is very large, this means that we basically do not have a good knowledge of the behavior of x and thus we cannot find an appropriate probabilistic model for it. This is the same as in BLUE where we assume that we don't have nay prior knowledge on x. Finally, we notice the discripancy in the results when P is not large.

The MATLAB code to solve this part is displayed below:

```
%% HW8 Problem 5
close all
clear all
clc
%% Initializing matrices
C = [1 2;3 4;5 0;0 6];
y = [1.5377;3.6948;-7.7193;7.3621];
Q = eye(4);
P_1 = 1e2*eye(2);
P_2 = 1e6*eye(2);
S=eye(size(C,1));
%% Part a
x_wls = inv(C'*S*C)*C'*S*y;
%% Part b
x_blue = inv(C'*inv(Q)*C)*C'*inv(Q)*y;
%% Part c
x_mve_1 = inv(C'*inv(Q)*C+inv(P_1))*C'*inv(Q)*y;
x_mve_2 = inv(C'*inv(Q)*C+inv(P_2))*C'*inv(Q)*y;
```

Problem 6

In this problem, we use the MVE algorithm with non-zero means for probabilistic random variable x and the measurements y:

$$\hat{x} = \bar{x} + PC^T(CPC^T + Q)^{-1}(y - \bar{y})$$

$$cov(\hat{x}) = P - PC^T(CPC^T + Q)^{-1}CP$$

Where:

$$\bar{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \bar{\epsilon} = 0 \quad \bar{y} = C\bar{x} + \bar{\epsilon}$$

Using MATLAB, we obtain:

$$\hat{x} = \begin{bmatrix} -0.8836 \\ 1.0802 \end{bmatrix} \quad cov(\hat{x}) = \begin{bmatrix} 0.0437 & 0.0072 \\ 0.0072 & 0.0538 \end{bmatrix}$$

The MATLAB code used in this part is displayed below:

```
%% HW8 Problem 6
close all
clear all
clc
%% Initializing matrices
C = [1 2;3 4;5 0;0 6];
y = [1.5377;3.6948;-7.7193;7.3621];
Q = [1 0.5 0.5 0.25;0.5 2 0.25 1;0.5 0.25 2 1;0.25 1 1 4];
P = [0.5 0.25; 0.25 0.5];
x_mean = [1;-1];
err_mean = 0;
y_mean = C*x_mean+err_mean;
%% Solving using MVE with non-zero mean
x_mve = x_mean +P*C'*inv(C*P*C'+Q)*(y-y_mean);
cov = P-P*C'*inv(C*P*C'+Q)*C*P;
```