Q8 Solution

1

Finding the set of junction points that cover all the pipe segments is similar to the Vertex Cover Problem, which is known to be NPC, hence a very small likelihood to find a fast algorithm to solve it (for all cases).

2

We can always formulate the problem as a search problem where we search for the optimal subset of junctions. suggestion below:

We consider the pipe network to be a graph $G = (V, E)$, we define $J_s \subseteq V$ to be the selected set of vertices. In addition J_s must satisfy the condition that every edge $(u, v) \in E$ is such that $u \in J_s$ or $v \in J_s$ (i.e, alle vertices are covered).

Solution 1

Objective

$Minimize\ |J_s|$ (minimize the size of the selected junctions)

Constraints

$\forall (u, v) \in E$, $u \in J_s$ or $v \in J_s$

Solution 2

An array of binary values size |E|

- $X = [x_0, x_1, ..., x_i]$
- $\forall i, x_i \in \{0, 1\}$ ($x_i = 1$ means vertex $v_i \in V$ is present in the selected set, 0 means out of the set)

Functions

- $f_1(X) = \sum_{i=0}^{|V|} x_i$ (sum of the vertices that are present)
- $f_2(X, G)$ a function that returns the edges that are covered by these vertices

Optimization problem

Minimize $f_1(X)$ and make sure that the covered edges given by $f_2(X, G)$ are equal to the edges E in the graph G.

- Minimize: $f_1(X)$
- Subject to: $f_2(X, G) = E$