Optimization 2 Solution

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The Knapsack Problem is about selecting a subset of items to maximize the total value while respecting the weight constraint of the knapsack.

Input

- Let I be the set of n items $I = 1, 2, ..., n$.
- Each item $i \in I$ has a weight w_i, a value v_i and a maximum amount max_i.
- The knapsack cannot contain more than a total weight W.

Output

Determine the amount a_i of each item $i \in I$

Questions

1. Formulate the objective function
2. Formulate the constraints

A furniture factory can produce three types of furniture: big tables, small tables, and chairs. Producing a big table costs in total (material,etc.) kr C_{bt}, it requires T_{bt} hours of work, and has a selling price of kr S_{bt}. The production cost, selling price, and working hours for the three types of products is summaries in Table 1.

Table 1

Product	Quantity	Cost of production in Kr	Selling Price in Kr	Factory working hours
Big table	X_{bt}	C_{bt}	S_{bt}	T_{bt}
Small table	X_{st}	C_{st}	S_{st}	T_{st}
Chair	X_{ch}	C_{ch}	S_{ch}	T_{ch}

Because the production costs and selling prices have varied a lot in the past months, the working hours also varied as a consequence of that. This lead to a certain frustration , so the factory office, management office, and the sales office had a meeting to discuss the matter.

Factory office:

- Understands that the business should be profitable.
- Willing to be flexible on the working hours.
- Asked the management for smarter decision to minimize the working hours at the factory.

Management Office:

- Understand that the employees are overloaded.
- Decided to set a somehow acceptable minimum profit P_{min}.

Sales Office:

- We need at least 6 big tables.
- We need at least 10 small tables.

- We need 8 chairs for every big table.
- We need 4 chairs for every small table.
- We need 1 extra chair for every table (small or big).

Task:

Formulate this problem as an optimization problem. You don´t have to solve it.

Solution:

Objective Function

$$\text{Minimize} \quad X_{bt} \cdot T_{bt} + X_{st} \cdot T_{st} + X_{ch} \cdot T_{ch} \tag{1}$$

Constraints

Profit

$$X_{bt} \cdot (S_{bt} - C_{bt}) + X_{st} \cdot (S_{st} - C_{bst}) + X_{ch} \cdot (S_{ch} - C_{ch}) \geq P_{min} \tag{2}$$

Sales

$$X_{bt} \geq 6 \tag{3}$$

$$X_{st} \geq 10 \tag{4}$$

$$X_{ch} \geq 9 \cdot X_{bt} + 5 \cdot X_{st} \tag{5}$$