

MOD550 Applied Data Analytics and Statistics for Spatial and Temporal Modeling

03 – Variogram Definition and Calculation

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Brief reminder of why we are focusing on spatial statistics

Reminder - Why Spatial Statistics?

Spatial statistics provides *quantitative* methods for:

- 1. Evaluating the *spatial continuity* and variability (heterogeneity) of spatial properties such as geological and petro-physical properties
- Interpolating properties between wells and extrapolating away from measurement points such as wells
- 3. *Integrating* different property measurements,
 - with different spatial resolution and spatial sampling,
 - and different degrees of reliability
- 4. Quantifying *uncertainty* in the spatial (e.g. reservoir) model due to sparse data sampling

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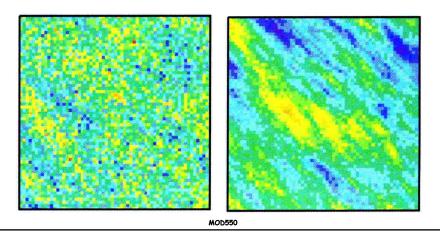
Domains where Spatial Statistics is Crucial

- 1. Environmental Science:
 - Climate change, pollution dispersion, habitat fragmentation, and biodiversity, spatial statistics help analyze and model spatial patterns of environmental phenomena
- 2. Public Health and Epidemiology:
 - Track the spread of diseases, analyze the distribution of health resources, and investigate environmental factors affecting public health
 - Identifying disease clusters and understanding the geographic determinants of health outcomes
- 3. Agriculture:
 - Analyze soil properties, crop yield data, and the spatial variability of agricultural fields to optimize inputs like fertilizers and water, thereby improving yields and reducing costs
- 4. Criminology:
 - Analyze the distribution of crime, identify hotspots, and understand the spatial dynamics of criminal activities => aids in effective policing strategies and resource allocation

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Univariate statistics (such as IDW) are not enough - need to measure and model spatial continuity

The two images below are visually quite different but have identical means and variances $(m \text{ and } s^2)$. The continuity of the images is quite different, and problems such as interpolation from a subset of the data should take this into account.



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Inverse Distance Weighting (IDW): Pros and Cons

o Pros

- Easy to understand and implement.
- Computationally Efficient
- Faster than geostatistical methods like Kriging.
- Suitable for real-time applications with moderate data sizes.
- No Assumptions About Spatial Structure
- Does not require fitting a variogram model, unlike Kriging.
- Works well when no strong spatial correlation modeling is needed.
- Good for Dense, Well-Distributed Data
- Performs well when spatial variation is smooth.

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Inverse Distance Weighting (IDW): Pros and Cons

o Cons

- Arbitrary parameter choices no systematic way to choose the best power value, leading to potential bias.
- Does not account for spatial relationships beyond simple distance weighting.
- Does not differentiate between spatial trends and random noise.
- Sensitive to data clustering
- In regions with unevenly spaced data, IDW may create unrealistic estimates.
- Overestimates influence of closely packed points and underestimates sparsely sampled regions.
- No Estimation of Uncertainty IDW does not provide confidence intervals or error estimates.
- Poor performance in anisotropic data IDW assumes isotropic spatial dependence (same influence in all directions).
- If spatial correlation changes with direction (anisotropy), IDW does not adjust accordingly.

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Anisotropy Examples | Anisotropy Examples | 3.0 | 0.0 | 0.0 | 0.0 | |-3.0 | Horizontal | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

Introduction to Kriging

o What is Kriging?

 Kriging is an advanced geostatistical interpolation method that predicts unknown values at unsampled locations based on spatial correlation.

• Why Use Kriging?

- Provides the best linear unbiased estimator (BLUE) for spatial data.
- Incorporates spatial dependence using a variogram model.
- Produces both predictions and uncertainty estimates (kriging variance).

o How It Works:

- 1. Fit a Variogram Quantifies spatial continuity.
- 2. Compute Weights Assigns optimal weights to data points based on distance and spatial correlation.
- 3. Estimate Values Generates predictions with minimal error.

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Applications of Kriging

- o Mining & Petroleum Estimating ore grades, reservoir properties
- Environmental Science Pollution mapping, climate modeling
- o Agriculture Soil property estimation, crop yield prediction

o Key Takeaway:

 Kriging optimally predicts spatial data while accounting for spatial uncertainty, making it superior to simple interpolation methods like Inverse Distance Weighting (IDW).

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How Kriging Works

- 1. Fit a Variogram Quantifies spatial continuity.
 - i. Use data to identify the experimental variogram
 - ii. Choose the model variogram to use in the Kriging calculations
- 2. Compute Weights Assigns optimal weights to data points based on distance and spatial correlation.
- 3. Estimate Values Generates predictions with minimal error.

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Semi-Variogram: A "core tool" in measuring spatial continuity and Geostatistical modeling

- o A model of the spatial continuity of variable Z is the variogram
 - it is a model of the average dis-similarity between points separated by h. Under assumptions discussed later, it is given by

$$2\gamma(h) = E \left[(Z(u) - Z(u+h))^2 \right]$$

 It can be estimated from data by the experimental <u>semi-</u>variogram, (half the variogram)

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(u_i) - z(u_{i+h})]^2$$

where

- u is a spatial location
- -h is a separation distance (vector) between u and another location
- -N(h) is the number of pairs that are separated by h
- -z is the property we are interested in (eg porosity, permeability)

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Semi-Variogram: A "core tool" in measuring spatial continuity and reservoir modeling

- o The variogram a measure of dissimilarity vs. distance.
- Calculated as ½ the average squared difference of values separated by a lag vector

 N(h)

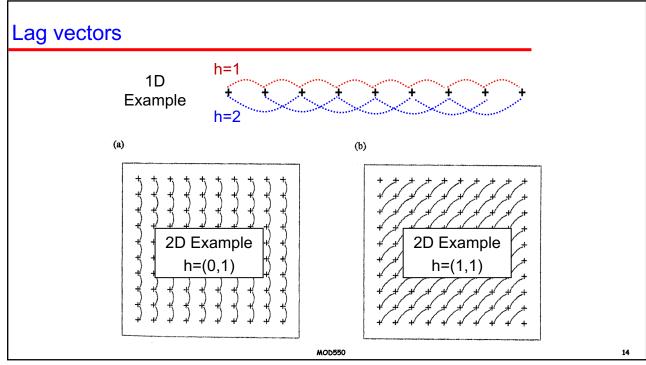
 $\hat{\gamma}_{x}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(u_i) - z(u_{i+h})]^2$

- The precise term is semivariogram (variogram if you remove the 1/2), but in practice the term variogram is used.
- The 1/2 is used so that the covariance function and variogram may be related directly: $C_x(h) = \sigma_x^2 \gamma_x(h)$
- Note the correlogram is related to the covariance function as:

$$ho_{x}(h) = \frac{C_{x}(h)}{\sigma_{x}^{2}}$$
 h-scatter plot correlation vs. lag distance

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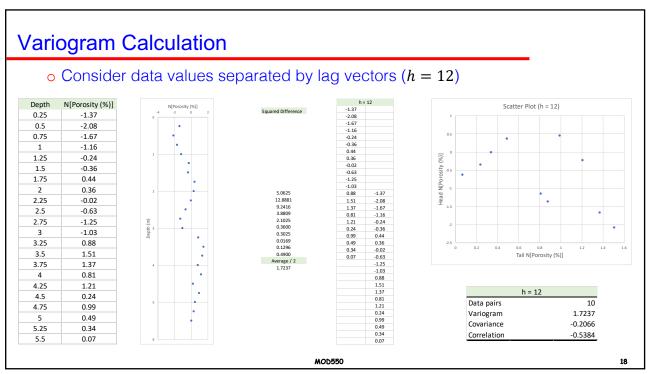
Variogram Calculation 2001 ft Lag = 6 Lag = 2Lag = 3 | Lag = 4 | Lag = 5 (ϕ_1,ϕ_2) $\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(u_i) - Z(u_{i+h})]^2$ (ϕ_1,ϕ_3) $(\varphi_1,\!\varphi_4)$ (ϕ_1,ϕ_5) $(\phi_1,\!\phi_6)$ (ϕ_1,ϕ_7) 2002 ft (ϕ_2,ϕ_3) (ϕ_2,ϕ_4) (ϕ_2,ϕ_5) (ϕ_2,ϕ_6) (ϕ_2,ϕ_7) φ₃= 6.25 2003 ft (ϕ_3,ϕ_5) (ϕ_3,ϕ_7) (ϕ_3,ϕ_4) (ϕ_3,ϕ_6) φ₄= 5.00 2004 ft (ϕ_4,ϕ_5) (ϕ_4,ϕ_6) (ϕ_4,ϕ_7) (ϕ_5,ϕ_6) (ϕ_5,ϕ_7) 2005 ft (ϕ_6,ϕ_7) $\phi_6 = 4.75$ N(3) = 4 $\gamma(3) = 3.313$ 2006 ft N(5) = 2N(6) =1 N(1) = 6 $\gamma(1)=0.845$ N(2) = 5 $\gamma(2)=2.106$ N(4) = 3φ₇= 5.00 2007 ft Lag =1 Lag =2 φ(i) φ(i+1) [φ(i)-φ(i+1)]^2 [φ(i)-φ(i+2)]^2 φ(i+2) φ(i) Covariance and Variogram 8.25 9.00 0.56 9.00 6.25 7.56 8.25 6.25 4.00 9.00 5.00 16.00 6.25 5.00 1.56 5.30 0.90 5.00 5.30 0.09 6.25 5.00 4.75 0.06 5.30 0.30 4.75 5.30 5.00 0.09 4.75 5.00 0.06 10.14 21.06 Sum/2N(L) 0.845 2.106 MOD550 15

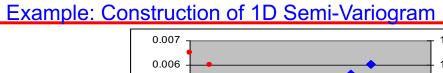
Variogram Calculation Consider N(0,1) data values separated by lag vectors (the h-values) Depth N[Porosity (%)] Scatter Plot (h = 1) Squared Difference -1.37 -1.37 0.5041 -2.08 -2.08 -1.37 0.1681 -1.67 -2.08 0.75 -1.67 0.2601 -1.67 -1.16 0.8464 Head N[Porosity (%)] -0.24 -1.16 1.25 -0.24 0.0144 -0.36 -0.24 -0.36 1.5 0.6400 0.44 -0.36 1.75 0.0064 0.36 0.44 0.36 0.1444 -0.02 0.36 2.25 -0.02 0.3721 -0.63 -0.02 2.5 -0.63 0.3844 -1.25 -0.63 0.0484 2.75 -1.25 -1.03 -1.25 3.6481 -1.03 0.88 1.51 -1.03 0.3969 3.25 0.88 0.88 0.0196 3.5 1.51 Tail N[Porosity (%)] 0.3136 1.37 0.81 1.37 0.1600 0.81 0.9409 1.21 0.81 4.25 1.21 0.5625 0.24 1.21 h = 1 4.5 0.24 0.2500 Data pairs 21 4.75 0.99 0.0225 0.49 0.99 . Variogram 0.2328 0.49 0.0729 0.34 0.49 Covariance 0.7860 5.25 0.4655 0.07 0.34 Correlation 0.7740 0.07 Average / 2 MOD550 16

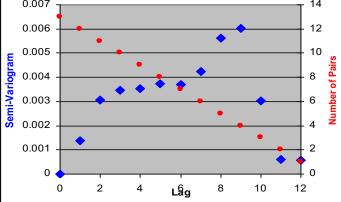
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Variogram Calculation o Consider data values separated by lag vectors (h = 2)Depth N[Porosity (%)] Scatter Plot (h = 2) Squared Difference -1.37 -2.08 -1.37 -2.08 0.0900 -1.67 -1.37 0.75 -1.67 -1.16 -2 08 -1.16 2.0449 -0.24 -1.67 Head N[Porosity (%)] 1.25 -0.24 0.6400 -0.36 0.44 -1.16 -0.24 -0.36 1.5 0.4624 1.75 0.44 0.5184 0.36 -0.36 0.36 0.2116 -0.02 -0.63 0.44 2.25 -0.02 0.9801 0.36 2.5 -0.63 1.5129 -1.25 -0.02 0.1600 -1.03 0.88 -0.63 -1.25 2.75 -1.25 4.5369 -1.03 0.88 6.4516 1.51 -1.03 3.25 0.2401 1.37 0.88 3.5 1.51 0.4900 0.81 1.51 1.37 0.0256 1.21 0.24 1.37 0.81 0.81 0.3249 4.25 1.21 0.0484 0.0625 0.99 1.21 h = 2 4.5 0.24 0.49 0.24 Data pairs 20 4.75 0.99 0.34 0.4225 0.99 . Variogram 0.5061 0.49 0.49 0.34 0.1764 0.07 Covariance 0.4606 5.25 Average / 2 Correlation 0.4925 0.07 MOD550 17

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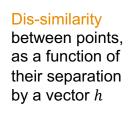


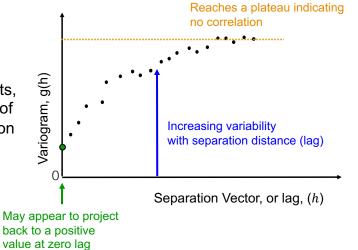
As the lag gets larger the number of pairs gets smaller and so the quality of the estimate gets worse (errors increase).

– It is unwise to go beyond a lag of around $\frac{1}{4}$ to $\frac{1}{2}$ of the size of the system.

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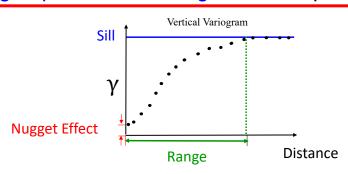






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Interpreting Experimental Variograms: Components



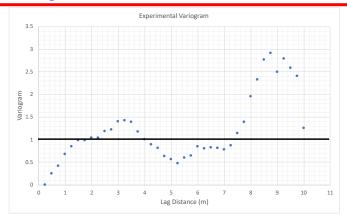
- Sill should be the variance (1.0 if the data are normal scores)
- Range = the distance at which the variogram reaches the sill
- Nugget effect = sum of geological microstructure and measurement error
 - Any error in the measurement value or the location assigned to the measurement translates to a higher nugget effect
 - Sparse data may also lead to a higher than expected nugget effect

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Experimental Variogram



- o Interpretation:
 - Cyclicity in the variogram (more later)
 - Range of about 1.5 m, no nugget effect
 - Unreliable as we go beyond 1 / 2 data extent

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Depth	Porosity (%)
0.25	-1.37
0.5	-2.08
0.75	-1.67
1	-1.16
1.25	-0.24
1.5	-0.36
1.75	0.44
2	0.36
2.25	-0.02
2.5	-0.63
2.75	-1.25
3	-1.03
3.25	0.88
3.5	1.51
3.75	1.37
4	0.81
4.25	1.21
4.5	0.24
4.75	0.99
5	0.49
5.25	0.34
5.5	0.07
5.75	-0.26
6	-0.41
6.25	-0.14
6.5	-1.44
6.75	-0.75
7	-0.78
7.25	-0.85
7.5	-0.92
7.75	-0.66
8	0.47
8.25	0.47
8.5	0.95
8.75	2.35
9	0.69
9.25	1.31
9.5	0.66
9.75	0.72
10	0.72
10	
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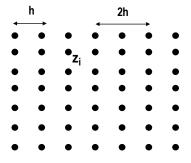
2D Variograms and the Relationship Between Covariance and Variogram under Stationarity

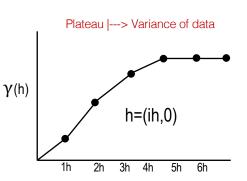
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Regularly spaced data (e.g. seismic cube):

- Lag is grid-spacing





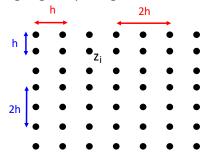
$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z_i - z_{i+h})]^2$$

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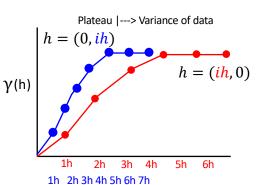
2D Semi-variogram calculation: Regularly spaced data showing **ANISOTROPY**

Regularly spaced data (e.g. seismic cube):

- Lag is grid-spacing



$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z_i - z_{i+h})]^2$$



Anisotropy: Different variograms in different directions

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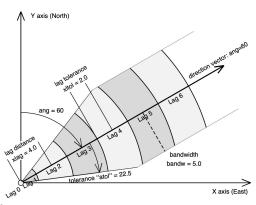
2D Semi-variogram calculation - challenges:

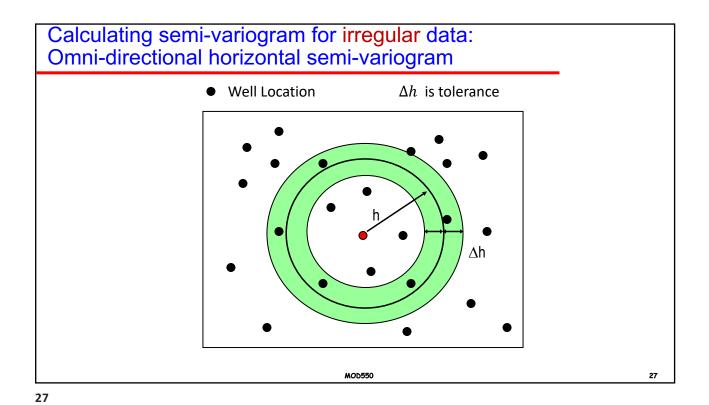
Data Scarcity

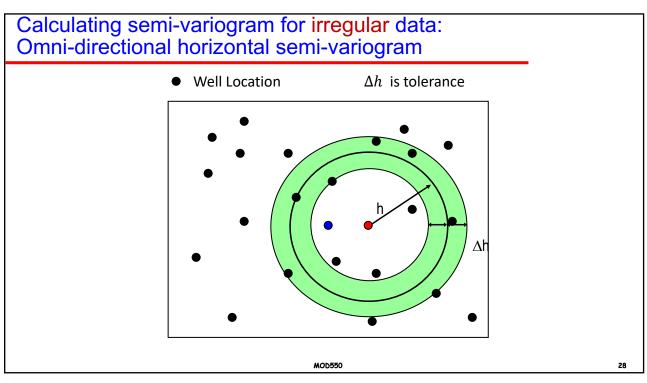
- 1. Available observation points -> small number
- 2. Equally spaces observations -> very few
- 3. Equally spaced and in the same direction?

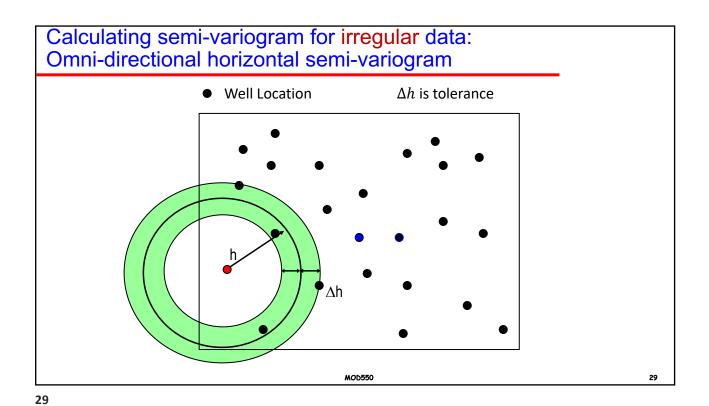
Need a method which allows us to assess spatial dissimilarity by constructing a meaningful experimental variogram despite these data scarcity challenges.

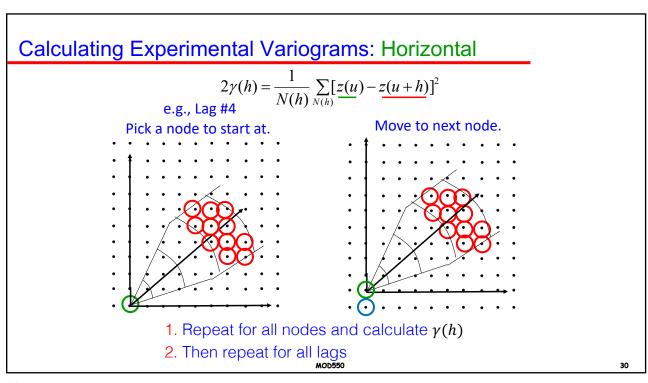
To deal with the data scarcity issues, use $h \pm \Delta h$ and $\theta \pm \Delta \theta$

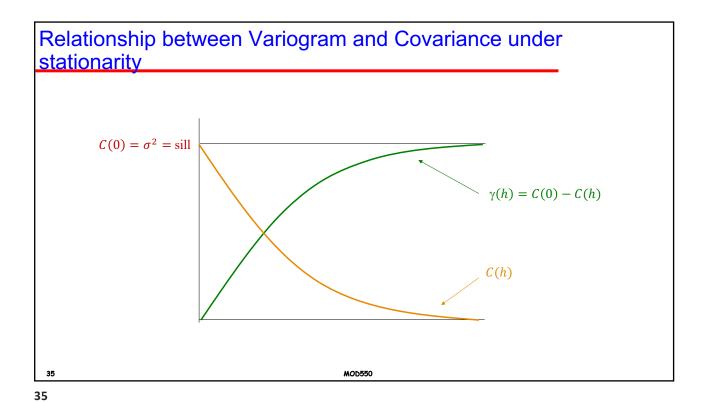


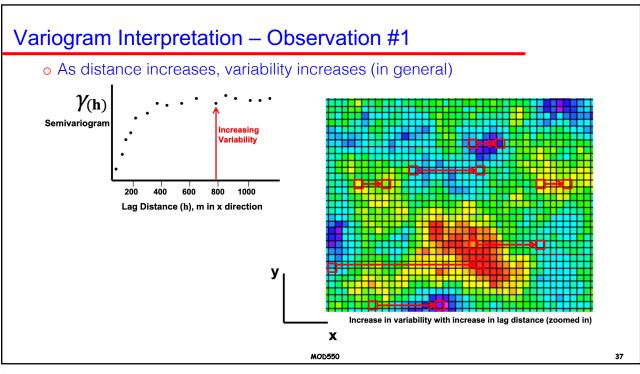






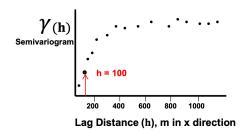






Variogram Interpretation – Observation #2

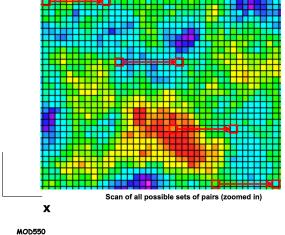
o Calculated over all possible pairs separated by lag vector, h



• The variogram:

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} (z(\mathbf{u}_{\alpha}) - z(\mathbf{u}_{\alpha} + \mathbf{h}))^{2} \quad \mathbf{y}$$

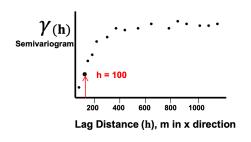
Given the number of pairs available N(h).



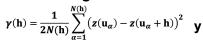
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Variogram Interpretation – Observation #3

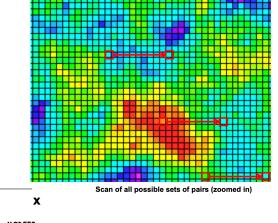
o Need to plot the sill to know the degree of correlation



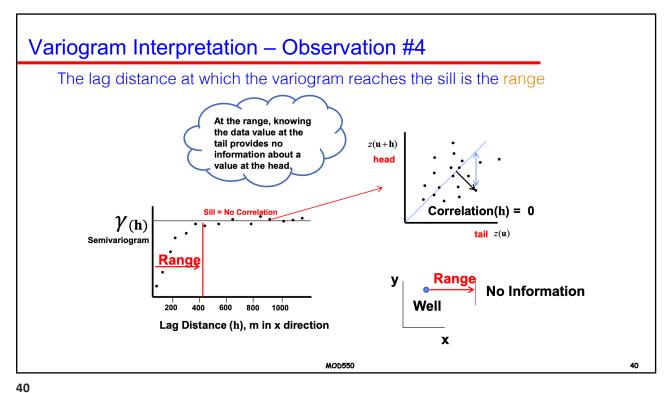
• The variogram:



Given the number of pairs available N(h).



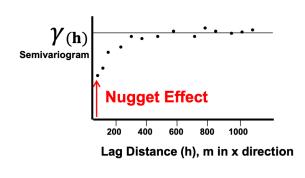
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Variogram Interpretation – Observation #5

- Sometimes there is a discontinuity in the variogram at distances less than the minimum data spacing. This is known as nugget effect:
 - the ratio of *nugget/sill*, is known as relative nugget effect (%)
 - modeled as a no correlation structure that at lags, $h > \varepsilon$, an infinitesimal distance
 - measurement error, mixing populations cause apparent nugget effect

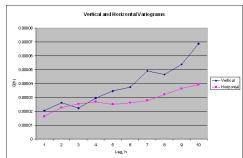


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Anisotropy

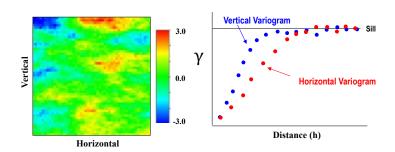
It is quite likely that the correlation range will be different in different directions





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Interpreting Experimental Variograms: Geometric Anisotropy

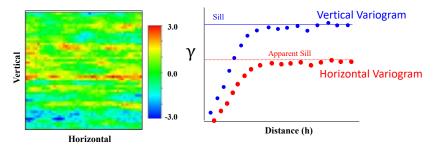


- o Variograms in different directions reach the same sill value
 - But at different ranges
- The nature of the continuity (shape of variogram) is often the same in both directions
 - Geologically justified by Walther's Law: geological variations in a vertical direction are similar to those in the horizontal (but on a different scale)

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Interpreting Experimental Variograms: Zonal Anisotropy

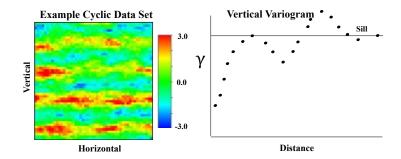


- o Compare vertical sill with horizontal sill
- When the vertical variogram reaches a higher sill:
 - likely due to additional variance from stratification/layering
- When the vertical variogram reaches a *lower* sill:
 - likely due to a significant difference in the average value in each well → horizontal variogram has additional between-well variance

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Interpreting Experimental Variograms: Cyclicity (Hole effect)



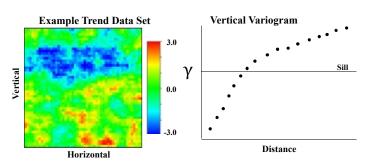
- Cyclicity may be linked to underlying geological periodicity
- o Could be due to limited data
- o Focus on the nugget effect and a reasonable estimate of the range

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- Use the data variance for the sill

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Interpreting Experimental Variograms: Trend



- o Indicates a trend (fining upward, ...) could be interpreted as a fractal
- Model to the theoretical sill;
 - the data will ensure that the trend appears in the final model
- May have to explicitly account for (remove) the trend in later simulation/modeling

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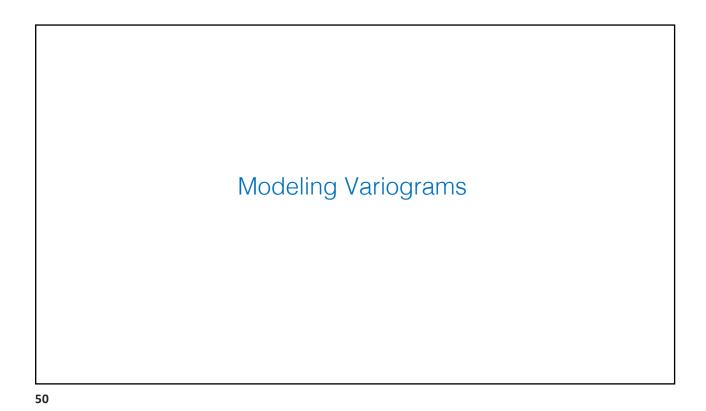
Challenges in Semi-Variogram Calculation

- o Short scale structure is most important
 - nugget due to measurement error should not be modeled

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- size of geological modeling cells
- o Vertical direction is typically well informed
 - can have artifacts due to spacing of core data
 - handle vertical trends and areal variations
- o Horizontal direction is not well informed
 - take from analog field or outcrop
 - typical horizontal vertical anisotropy ratios

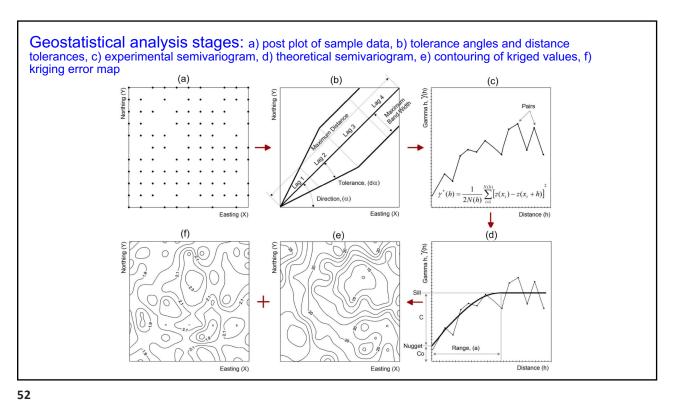
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Basics of Geostatistical Estimation

- o In any Kriging analysis, there are four major steps:
 - i. Determining an appropriate theoretical semivariogram models, used to fit experimental semivariogram and possible anisotropy,
 - ii. Performing validation methods to the semivariogram model,
 - iii. Generating kriging estimates and errors of estimates, i.e. kriging errors, for a point, zone or volume by kriging interpolation techniques,
 - Mapping the spatial distributions of the kriging estimates and kriging errors

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Variogram Modeling

- We have looked at various methods used to estimate variograms
- We will now discuss methods used to model these variograms
- o Estimation of variograms allows us to know variogram values only at the lag distances at which we compute the variogram
- o However, the primary purpose in estimating the variogram is to use this information to estimate variable values at unsampled locations
- The information about the spatial relationship is used to estimate these
- When estimating, it is not necessary that the distances between the samples and unsampled locations coincide with the distances at which we estimate the variogram

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Variogram Modeling

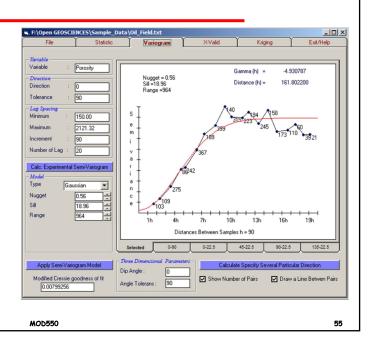
- In fact, the distances rarely coincide with the distances of the estimated variogram
- This means we have to determine the value of the variograms at distances between the lag distances at which we have estimated the variogram
- One option is simply to interpolate between the estimated values so that we know the variogram value at any lag distance
- Unfortunately, such simple approximation may create problems in the estimation procedure
- Therefore, we have to restrict ourselves to certain types of models that can be used to model the variogram

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Variogram Modeling

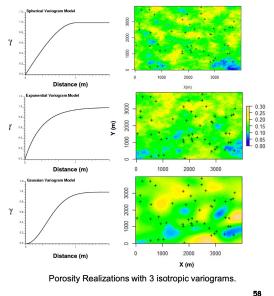
- Variogram modeling is a time consuming exercise
- Modeling variograms correctly requires practice
- The job can be made easier with interactive software



Examples of Spatial Variability for Different Variogram Models

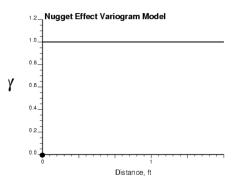
- o The three maps are remarkably similar: all three have the same 50 data, same histograms and same range of correlation, and yet their spatial variability/continuity is quite different
- o The spatial variability/continuity depends on the detailed distribution of the petrophysical attribute (ϕ, k)
- o The charts on the left are "variograms"
- Our map-making efforts should consider the spatial variability/continuity of the variable we are mapping:
 - Variability
 - Uncertainty

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Nugget Effect Variogram Model



- o The simplest model
- No spatial correlation -> a total lack of information with respect to spatial relationship MOD550

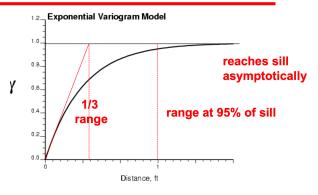
$$\gamma(h) = \begin{cases} 0, & \text{for } h = 0\\ C(0), & \text{for } h > 0 \end{cases}$$

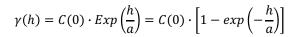
Spherical Variogram Model organized Spherical Variogram Model organized Most commonly used model organized Range = aorganized Sill = C(0) $\gamma(h) = C(0) \cdot Sph\left(\frac{h}{a}\right) = \begin{cases} C(0) \cdot \left[1.5\left(\frac{h}{a}\right) - 0.5\left(\frac{h}{a}\right)^3\right], & for h < a \\ C(0), & for h \ge a \end{cases}$

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Exponential Variogram Model

- Similar to spherical but rises more steeply and reaches the sill asymptotically
- \circ Range = a
- In practice, range defined as the distance at which the variogram reaches 95% of the sill value



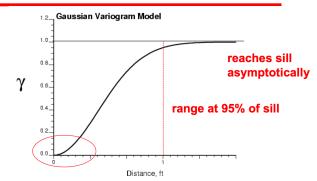


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Gaussian Variogram Model

- Implies short scale continuity; parabolic behavior at the origin, instead of linear
- o Range = a
- In practice, range defined as the distance at which the variogram reaches 95% of the sill value



$$\gamma(h) = C(0) \cdot Gauss\left(\frac{h}{a}\right) = C(0) \cdot \left[1 - exp\left(-\frac{h^2}{a^2}\right)\right]$$

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Variograms - Summary

- Spatial behavior of a regionalized variable can be observed through the semivariogram.
- For most applications in petroleum geology, sample locations are not situated on a regular grid, so a constant lag cannot be used.
- Instead, the user selects a lag size and a tolerance, usually one-half the lag size.

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- Selection of a lag size depends upon the number of samples, the dimensions of the study area, and the type and degree of continuity between wells or sample sites.
- Increasing the lag size yields more pairs at each point on the semivariogram, but the number of points decreases to the point of obscuring the appearance of the semivariogram at small distances.

Variograms - Summary

- A regionalized variable may display anisotropic behavior, such as that found on structure maps in folded areas.
- To detect anisotropy, a semivariogram is constructed for each of several distances, requiring a direction tolerance in addition to a distance tolerance.
- Geostatistical estimations require some degree of stationarity in the variable being studied; a constant semivariogram across the area is usually sufficient.
 - Even in the presence of a regional trend, stationarity may be achieved at a local level.

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Variograms - Summary

- Practical use of an observed semivariogram requires the fitting of a model that captures the main features of the plotted curve.
- Models can be classified by two characteristics: (I) their appearance at the origin, and (2) presence or absence of a sill.
- The spherical and exponential models are the most useful in oil and gas applications.
- Observed semivariograms often call for nested models, in particular one that includes a so-called nugget effect or noise component.
- Additional models that may be fitted include hole effects, which display pseudoperiodicity, and anisotropic models, which include zonal and geometric anisotropy.

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Variograms - Summary

- o Semivariograms can be fitted for one, two, three, or more dimensions.
- The complexity of fitting models in three dimensions is increased by the fact that a serious anisotropy usually exists between the horizontal directions and the vertical direction.
- As with many statistics, the semivariogram is sensitive to the distribution of the data or the presence of outliers.
- A simple transform to near-normality can markedly change and improve the semivariogram.
- The user must keep in mind that the purpose of fitting a model is to obtain a product that can be used in estimation.
- The purpose of a spatial model is for input to estimation, not to explain a natural phenomenon.

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