ΣΥΣΤΗΜΑΤΑ ΑΝΑΜΟΝΗΣ

3η ΟΜΑΔΑ ΑΣΚΗΣΕΩΝ

ΘΟΔΩΡΗΣ ΑΡΑΠΗΣ – EL18028

16 ΜΑΙΟΥ, 2023

Ονοματεπώνυμο: Θοδωρής Αράπης	**ΑΜ:** el18028

Προσομοίωση συστήματος M/M/1/10

(1)

Παρακάτω παρουσιάζονται τα αναλυτικά trace των πρώτων 10 από τις 30 (για εξοικονόμηση χώρου) μεταβάσεις για κάθε τιμή του λ:

```
********** LAMBDA = 1 ARRIVAL PER MINUTE **********

******* TRANSITION 1 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 1

******* TRANSITION 2 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 2

******* TRANSITION 3 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 3

******* TRANSITION 4 *******
Current state: 2
The next transition is an arrival.
The total number of arrivals until now is: 4

******* TRANSITION 5 *******
Current state: 4
The next transition is a departure.
The total number of arrivals until now is: 4

******* TRANSITION 6 *******
Current state: 2
The next transition is a departure.
The total number of arrivals until now is: 4

******* TRANSITION 7 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 5

******* TRANSITION 8 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 6

******* TRANSITION 9 *******
Current state: 2
The next transition is a departure.
The total number of arrivals until now is: 6

******* TRANSITION 10 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 7
```

```
********** LAMBDA = 5 ARRIVAL PER MINUTE **********

******* TRANSITION 1 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 1

******* TRANSITION 2 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 2

******* TRANSITION 3 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 3

******* TRANSITION 4 *******
Current state: 2
The next transition is an arrival.
The total number of arrivals until now is: 4

******* TRANSITION 5 *******
Current state: 4
The next transition is a departure.
The total number of arrivals until now is: 4

******* TRANSITION 6 *******
Current state: 2
The next transition is a departure.
The total number of arrivals until now is: 4

******* TRANSITION 7 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 5

******* TRANSITION 8 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 6

******* TRANSITION 9 *******
Current state: 2
The next transition is a departure.
The total number of arrivals until now is: 6

******* TRANSITION 10 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 7
```

```
********** LAMBDA = 10 ARRIVAL PER MINUTE **********

******* TRANSITION 1 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 1

******* TRANSITION 2 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 2

******* TRANSITION 3 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 3

******* TRANSITION 4 *******
Current state: 2
The next transition is an arrival.
The total number of arrivals until now is: 4

******* TRANSITION 5 *******
Current state: 4
The next transition is a departure.
The total number of arrivals until now is: 4

******* TRANSITION 6 *******
Current state: 2
The next transition is a departure.
The total number of arrivals until now is: 4

******* TRANSITION 7 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 5

******* TRANSITION 8 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 6

******* TRANSITION 9 *******
Current state: 2
The next transition is a departure.
The total number of arrivals until now is: 6

******* TRANSITION 10 *******
Current state: 0
The next transition is an arrival.
The total number of arrivals until now is: 7
```

Για την παραγωγή των παραπάνω αποτελεσμάτων χρησιμοποιήθηκε το ακόλουθο κομμάτι κώδικα:

```
# Debugging (1)
  if transitions <= 30
    printf("\n******* TRANSITION %d *******\n", transitions);
    printf("Current state: %d\n", current_state);
    if (current_state == final_state) && (random_number < threshold)
      printf("Either a client was declined or the system reached the final state!\n");
    elseif (current_state == 0) || ((random_number < threshold) && (current_state != final_state))
      current_state = current_state + 1;
      arrival_counter = arrival_counter + 1;
      printf("The next transition is an arrival. \n");
      printf("The total number of arrivals until now is: %d\n", arrival_counter);
    else
      current_state = current_state - 1;
      printf("The next transition is a departure.\n");
      printf("The total number of arrivals until now is: %d\n", arrival_counter);
    endif
  endif
```

(2)

Απενεργοποιούμε τώρα το προηγούμενο τμήμα κώδικα και εκτελούμε την προσομοίωση για τις τιμές του $\lambda = \{1, 5, 10\}$. Ο κώδικας τις προσομοίωσης είναι ο ακόλουθος:

```
# Simulation of a M/M/1/10 system

clc;
clear all;
close all;

lambdas = [1, 5, 10];
mu = 5;
final_state = 10;
states = 0 : 1 : 10;

for l = 1 : length(lambdas)
  rand("seed", 1);
  printf("********** LAMBDA = %d ARRIVALS PER MINUTE **********\n", lambdas(l));
  arrivals = zeros(1, final_state + 1);
  total_arrivals = 0;
  index = 0;
  transitions = 0;
  current_state = 0;
  previous_mean_clients = 0;
  threshold = lambdas(l)/(lambdas(l) + mu);
  arrival_counter = 0;

  while transitions >= 0
    transitions = transitions + 1;

    if mod(transitions,1000) == 0
      index = index + 1;
      for i=1:length(arrivals)
          P(i) = arrivals(i)/total_arrivals;
      endfor

      mean_clients = 0;

      for i=1:1:length(arrivals)
        mean_clients = mean_clients + (i-1).*P(i);
      endfor

      to_plot(index) = mean_clients;

      if abs(mean_clients - previous_mean_clients) < 0.00001 || (transitions > 1000000)
        break;
      endif

      previous_mean_clients = mean_clients;

    endif

    random_number = rand(1);

    # Debugging (1)
#      if transitions <= 30
#        printf("\n******* TRANSITION %d *******\n", transitions);
#        printf("Current state: %d\n", current_state);
#       if (current_state == final_state) && (random_number < threshold)
#          printf("Either a client was declined or the system reached the final state!\n");
#        elseif (current_state == 0) || ((random_number < threshold) && (current_state != final_state))
#          current_state = current_state + 1;
#          arrival_counter = arrival_counter + 1;
#          printf("The next transition is an arrival. \n");
#          printf("The total number of arrivals until now is: %d\n", arrival_counter);
#        else
#          current_state = current_state - 1;
#          printf("The next transition is a departure.\n");
#          printf("The total number of arrivals until now is: %d\n", arrival_counter);
#        endif
#      endif


```

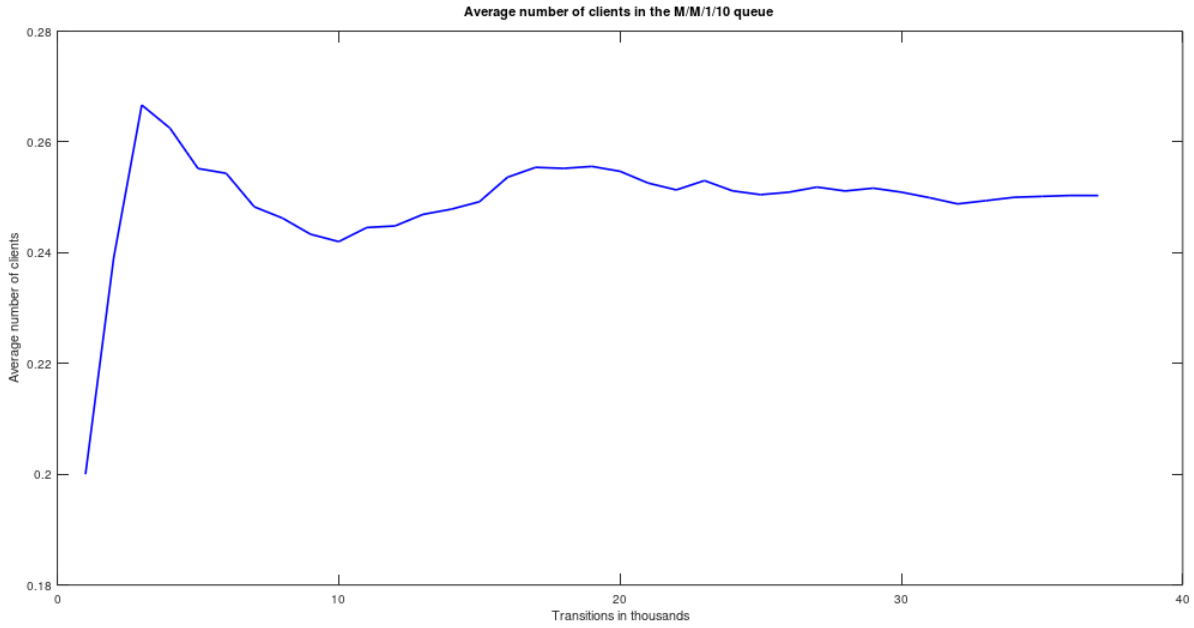
```
    if ((current_state == 0) || ((random_number < threshold) && (current_state != final_state)))   # Arrival
      total_arrivals = total_arrivals + 1;
      arrivals(current_state + 1) = arrivals(current_state + 1) + 1;
      current_state = current_state + 1;
    elseif (current_state == final_state) && (random_number < threshold) # Final state
      total_arrivals = total_arrivals + 1;
      arrivals(current_state + 1) = arrivals(current_state + 1) + 1;
    elseif random_number >= threshold          # Departure case
      current_state = current_state - 1;
    endif

  endwhile

  for i = 1 : length(arrivals)
    P_percent(i) = P(i) * 100;
    printf("The ergodic probability of the state %d is: %g%% \n", (i - 1), P_percent(i))
  endfor

  printf("The chance of rejecting a client is: %f%%\n", P_percent(final_state + 1))
  printf("The mean number of clients in the system is: %f \n", mean_clients);
  mean_delay = mean_clients/(lambdas(l)*(1 - P(final_state + 1)));
  printf("The mean delay time of a client in the system (Little Law) is: %f minutes \n", mean_delay);

  figure(l);
  plot(to_plot,"b","linewidth",1.3);
  title("Average number of clients in the M/M/1/10 queue ");
  xlabel("Transitions in thousands");
  ylabel("Average number of clients");

  figure(l + 3);
  bar(states, P, 'g', 0.4);
  title("Ergodic probabilities of each state");
  xlabel("State");
  ylabel("Probabilities");

  for i = 1 : index
     to_plot(i) = 0;
  endfor

endfor
```

Τρέχουμε λοιπόν την παραπάνω προσομοίωση, λαμβάνουμε τα εξής αποτελέσματα και δημιουργούμε τις ζητούμενες γραφικές για τις διάφορες τιμές του λ:

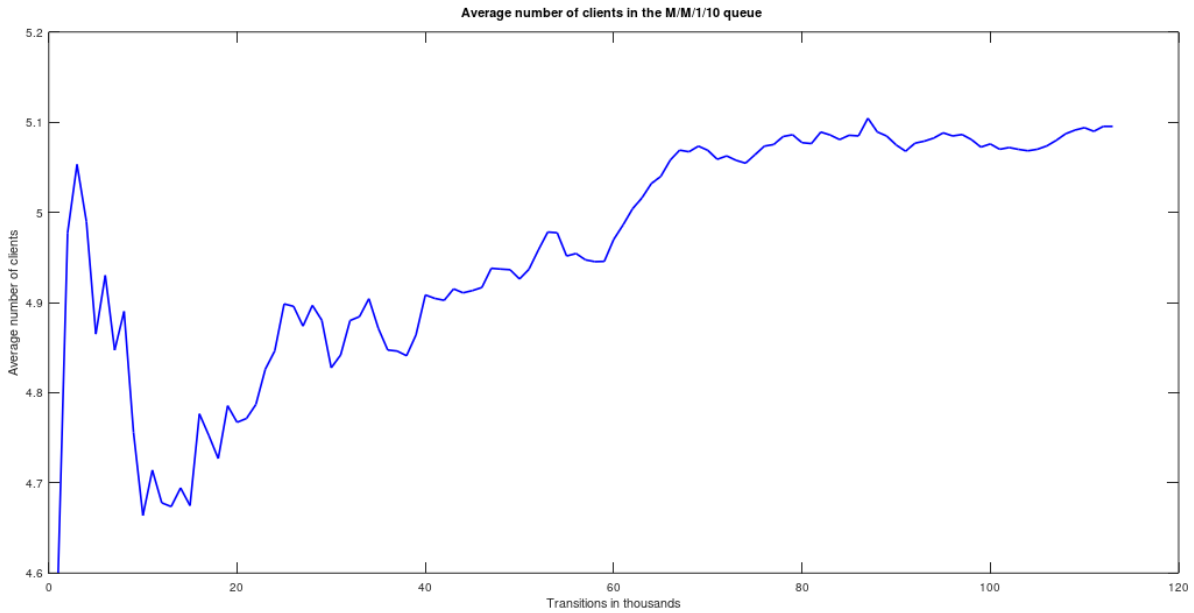
```
********** LAMBDA = 1 ARRIVALS PER MINUTE **********
The ergodic probability of the state 0 is: 80.0216%
The ergodic probability of the state 1 is: 15.9568%
The ergodic probability of the state 2 is: 3.22162%
The ergodic probability of the state 3 is: 0.621622%
The ergodic probability of the state 4 is: 0.12973%
The ergodic probability of the state 5 is: 0.0432432%
The ergodic probability of the state 6 is: 0.00540541%
The ergodic probability of the state 7 is: 0%
The ergodic probability of the state 8 is: 0%
The ergodic probability of the state 9 is: 0%
The ergodic probability of the state 10 is: 0%
The chance of rejecting a client is: 0.000000%
The mean number of clients in the system is: 0.250324
The mean delay time of a client in the system (Little Law) is: 0.250324 minutes
********** LAMBDA = 5 ARRIVALS PER MINUTE **********
The ergodic probability of the state 0 is: 8.94921%
The ergodic probability of the state 1 is: 8.78239%
The ergodic probability of the state 2 is: 8.75543%
The ergodic probability of the state 3 is: 8.50268%
The ergodic probability of the state 4 is: 8.82452%
The ergodic probability of the state 5 is: 9.24241%
The ergodic probability of the state 6 is: 9.32161%
The ergodic probability of the state 7 is: 9.25083%
The ergodic probability of the state 8 is: 9.39238%
The ergodic probability of the state 9 is: 9.38732%
The ergodic probability of the state 10 is: 9.59121%
The chance of rejecting a client is: 9.591211%
The mean number of clients in the system is: 5.095339
The mean delay time of a client in the system (Little Law) is: 1.127178 minutes
********** LAMBDA = 10 ARRIVALS PER MINUTE **********
The ergodic probability of the state 0 is: 0.0479279%
The ergodic probability of the state 1 is: 0.0853715%
The ergodic probability of the state 2 is: 0.174487%
The ergodic probability of the state 3 is: 0.38567%
The ergodic probability of the state 4 is: 0.772837%
The ergodic probability of the state 5 is: 1.54455%
The ergodic probability of the state 6 is: 3.08311%
The ergodic probability of the state 7 is: 6.19393%
The ergodic probability of the state 8 is: 12.4785%
The ergodic probability of the state 9 is: 25.0112%
The ergodic probability of the state 10 is: 50.2224%
The chance of rejecting a client is: 50.222415%
The mean number of clients in the system is: 9.014146
The mean delay time of a client in the system (Little Law) is: 1.810885 minutes
```

(a) Εργοδικές πιθανότητες που υπολογίζει η προσομοίωση

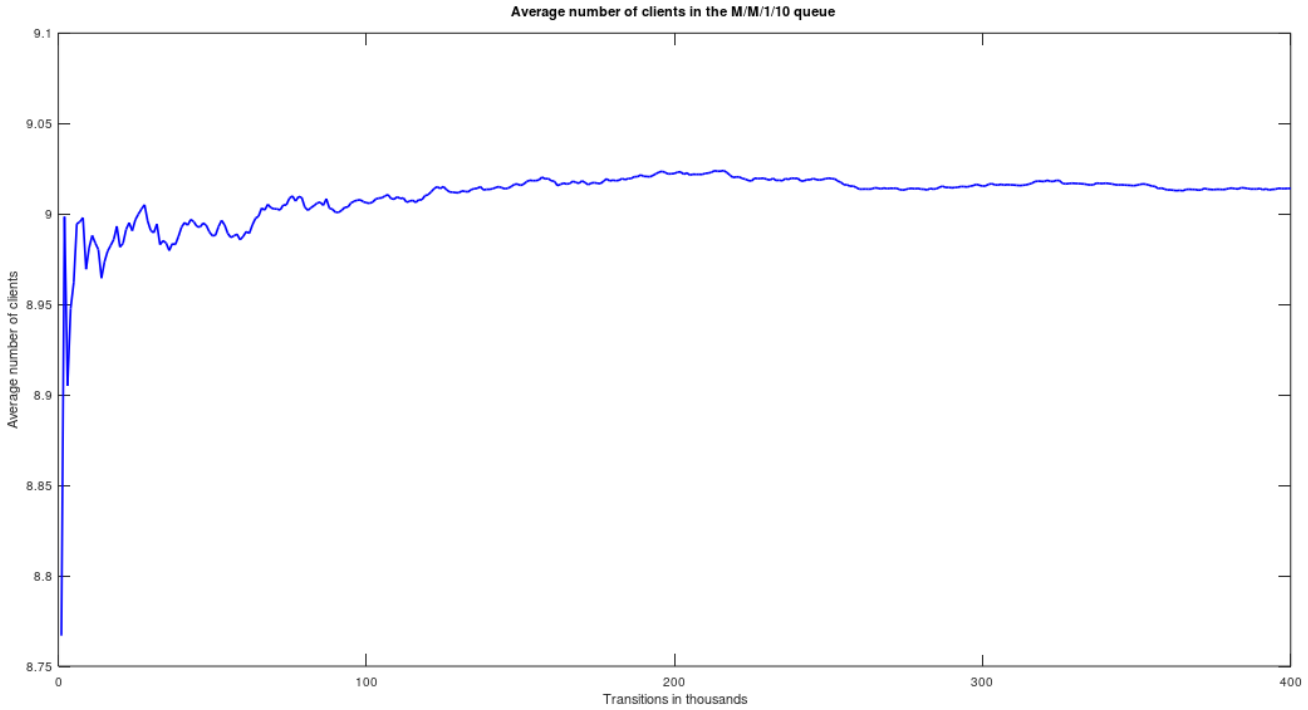
$\boldsymbol{\lambda = 1}$



$\boldsymbol{\lambda = 5}$

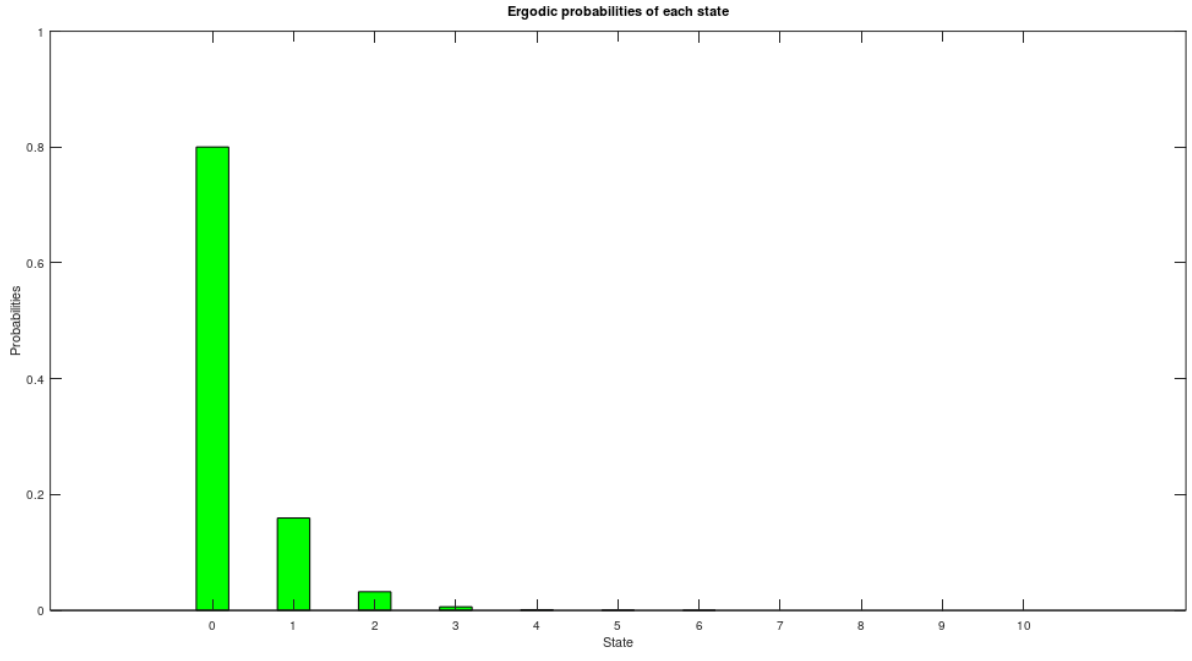


$\underline{\boldsymbol{\lambda = 10}}$

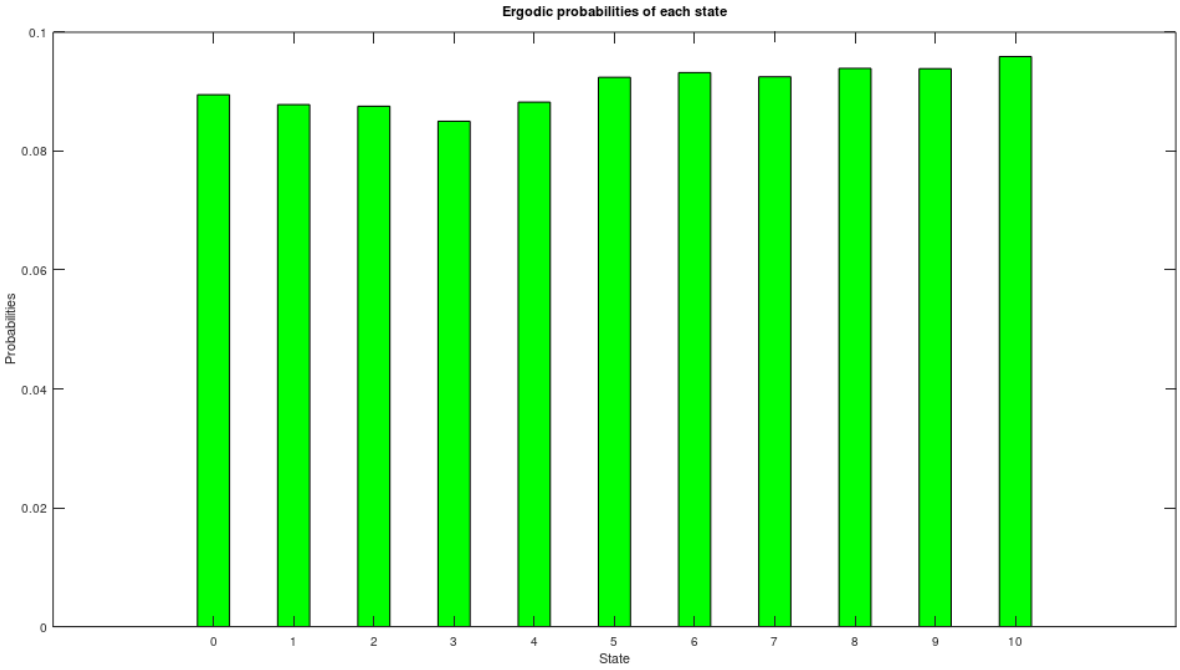


(a) **<u>Εξέλιξη του μέσου αριθμού πελατών στο σύστημα για κάθε κατάσταση</u>**

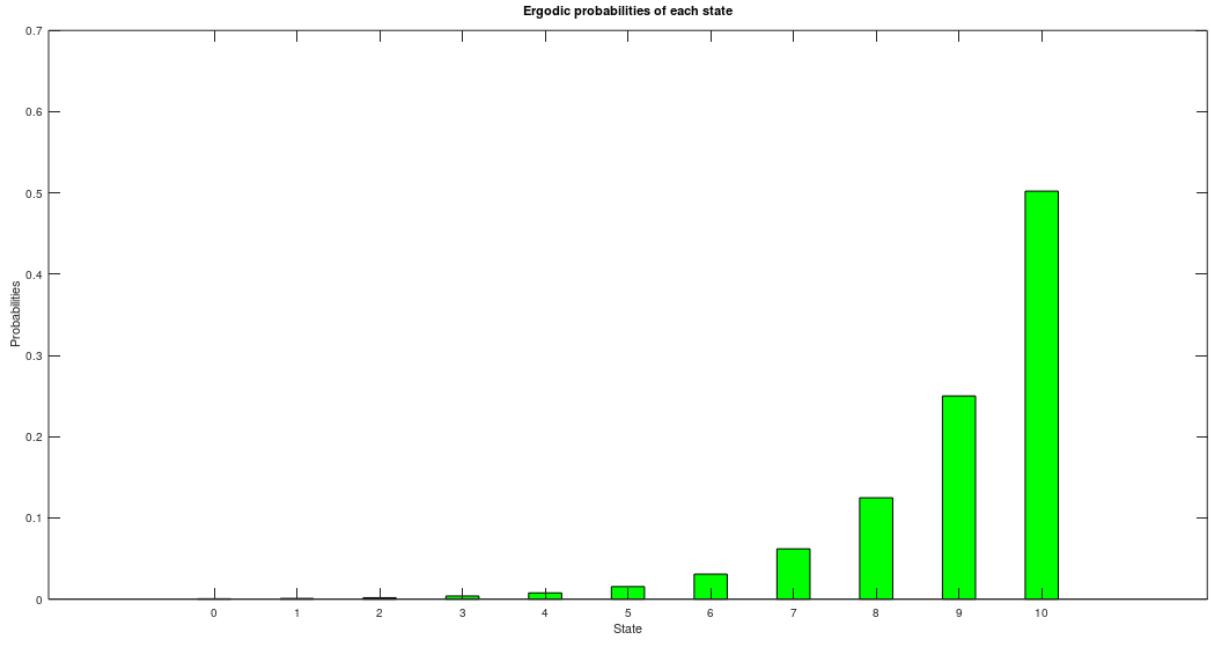
$\underline{\boldsymbol{\lambda = 1}}$



$\lambda = 5$



$\lambda = 10$



(3)

Από τις παραπάνω γραφικές παραστάσεις, εύκολα συμπεραίνουμε ότι με την αύξηση του λ αυξάνεται αντίστοιχα και ο απαιτούμενος αριθμός μεταβάσεων προκειμένου να ικανοποιηθεί το κριτήριο σύγκλισης. Πιο ειδικά, παρατηρούμε ότι για λ=1, 5, 10 τα μεταβατικά φαινόμενα εξασθενούν ύστερα από περίπου 10, 60, 100 χιλιάδες μεταβάσεις αντίστοιχα. Συνεπώς, θα μπορούσαμε να αγνοήσουμε τα παραπάνω πλήθη αρχικών μεταβάσεων με σκοπό την επίτευξη ταχύτερης σύγκλισης.

(4)

Αρκούν δύο μικρές αλλαγές στην προσομοίωση ώστε το σύστημά μας να υποστηρίζει μεταβλητούς ρυθμούς εξυπηρέτησης. Αρχικά, προκειμένου να αναπαραστήσουμε τις διάφορες τιμές του $\mu_i = \mu(i+1)$, θα δημιουργούσαμε έναν πίνακα με τις εξής τιμές:

$$m = [2\mu, 3\mu, 4\mu, 5\mu, 6\mu, 7\mu, 8\mu, 9\mu, 10\mu, 11\mu]$$

Όπου $\mu = 1\ πελάτης/sec$ και $i = \{1, 2, \dots, 10\}$.

Έπειτα, θα πρέπει να ορίσουμε το threshold ως πίνακα που θα λαμβάνει μία τιμή για κάθε μία από τις 10 καταστάσεις.