Approximate Bayesian Computation (ABC) Algorithm

Bayes' theorem relates the conditional probability (or density) of a particular parameter value θ given the observations y_{obs} to the likelihood of the observation given the parameter θ. The formula of Bayes' theorem is:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)}$$

where $P(\theta|y)$ is the posterior probability, $P(\theta)$ is the prior belief on θ, and $P(y|\theta)$ is the likelihood function.

There are cases where the likelihood function in untractable, due to the existence of latent variables.

The ABC Rejection Algorithm is a common method where one can approximate the likelihood function by simulations, the outcomes of which are compared with the observed data. More specifically, with the ABC rejection algorithm, a set of parameter points is first sampled from the prior distribution. Given a sampled parameter point θ, a dataset $\hat{D}$ is then simulated under the statistical model M specified by θ. If the generated $\hat{D}$ is too different from the observed data D, the sampled parameter value is discarded. In precise terms, $\hat{D}$ is accepted with tolerance $\varepsilon \geq 0$ if:

$$d(\hat{D}, D) \leq \varepsilon$$

Algorithm for ABC:

1. Draw θ from the prior.
2. Generate the simulated data $g(\theta, x) = \mathcal{N}(\theta_1 x_1 + \theta_2 x_2, \sigma^2)$.
3. Calculate the distance $d(g(\theta, x), y)$.
4. Accept the samples if $d(g(\theta, x)) < \varepsilon$ (threshold).