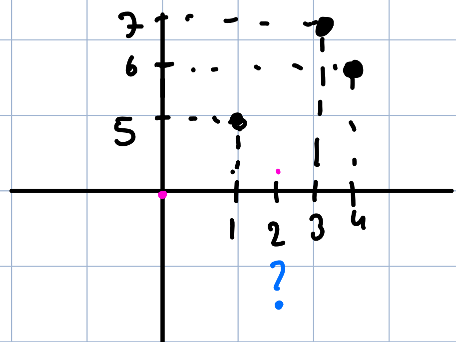
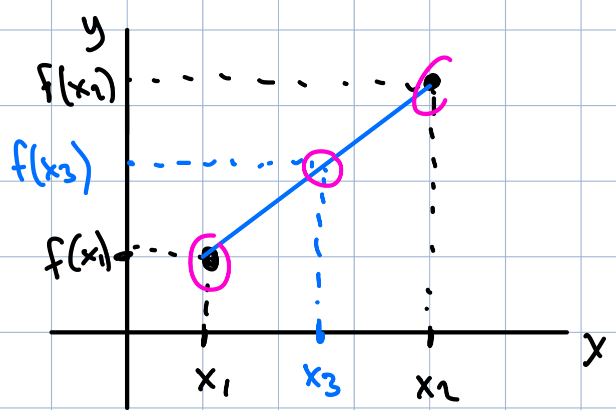
Interpolasyon

Bir veri grubunda eksik verileri, veri grubundaki diğer veriler ile elde edilmesi:



1) Linear Interpolasyon



$x_1, f(x_1)$ ve $x_2, f(x_2)$ biliniyor

$x_1 < x_3 < x_2 \longrightarrow f(x_3) = ?$

1. yol: 2 noktadan doğru denklemi

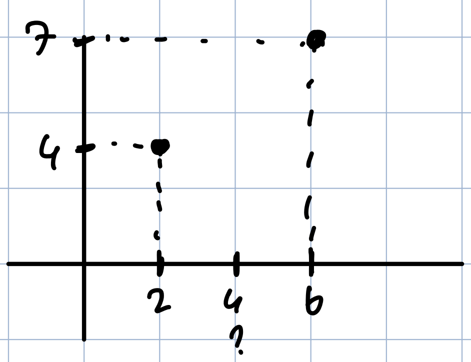
$$y - y_0 = m.(x - x_0)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

2. yol: Eğim üzerinden

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_3) - f(x_1)}{x_3 - x_1}$$

Örnek: Linear interpolasyon ile $x=4$ için $y=?$ $(2,4)$ ve $(6,7)$ noktaları ile:



$$\frac{7-4}{6-2} = \frac{f(4)-4}{4-2}$$

$$\frac{3}{4} = \frac{y-4}{2} \quad 2y - 8 = 3 \quad y = 5{,}5$$

Örnek 2:

x	$f(x) = \log(x)$
4	0,60206
4,5	0,6532125
5,5	0,7403627
6	0,7781513

a) $\log(5) = ?$ using $x=4$ ve $x=6$

$$\frac{\log(6)-\log(4)}{6-4} = \frac{\log(5)-\log(4)}{5-4}$$

$$\log(5) = 0,69010565 \mathbin{/\!/}$$

b) $\log(5) = ?$ $x=4,5$ $x=5,5$ için bul

$$\log(5) = 0,6967876$$

c) Bağıl hata a veya b için bul $\log(5)$ gerçek $= 0,69897$

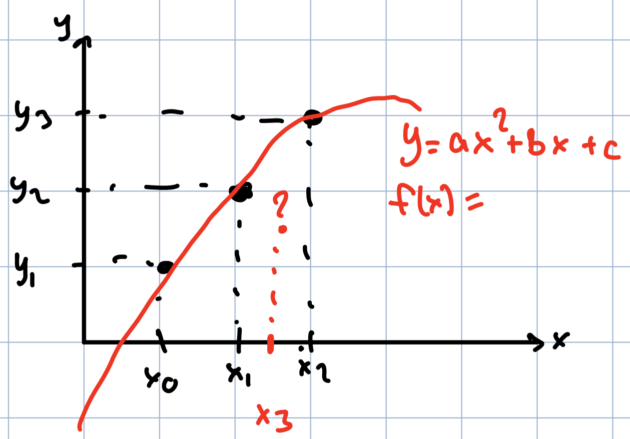
relative error

$$\frac{\text{Gerçek} - \text{Yaklaşık}}{\text{Gerçek}} = \frac{0,69897 - \underset{a}{0,69010565}}{0,69897}$$

$$\cong \overset{\times 100}{0,013} = \%1,3 \text{ hata} \mathbin{/\!/}$$

Quadratic Interpolation

* 3 nokta gereklidir



$(x_0, y_0), (x_1, y_1), (x_2, y_2)$

$f(x_3) = ?$

1. yol:

$$\left.\begin{aligned} ax_0^2 + bx_0 + c &= y_0 \\ ax_1^2 + bx_1 + c &= y_1 \\ ax_2^2 + bx_2 + c &= y_2 \end{aligned}\right\}$$

3 bilinmeyenli denklem sistemini çözüp a, b, c bulunur.

$f(x) = ax^2 + bx + c$

$f(x_3) \nearrow$

* 2. yol

$$\boxed{f(x) = b_0 + b_1(x - x_0) + b_2 \cdot (x - x_0)(x - x_1)}$$

$b_0 = f(x_0)$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$x_0 \quad x_1 \quad x_2$ yi küçükten büyüğe olacak şekilde seç

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Örnek: $(1,-2)$, $(2,-1)$ ve $(3,4)$ Quadratic ile $x=2{,}5$ $f(x)=?$

x_0 $\quad$ x_1 $\quad$ x_2

$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0).(x-x_1)$$

$$= \overset{-2}{b_0} + \overset{1}{b_1}(x-1) + \overset{2}{b_2}(x-1).(x-2)$$

$$b_0 = f(x_0) = -2$$

$$b_1 = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{-1-(-2)}{2-1} = 1 = b_1$$

$$b_2 = \frac{\dfrac{f(x_2)-f(x_1)}{x_2-x_1} - \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0} = \frac{\frac{5}{1}-1}{2} = 2$$

$$f(2{,}5) = -2 + 1(x-1) + 2(x-1).(x-2)$$

$$-2 + 1{,}5 + (1{,}5) = 1$$

örnek 2:

	x	$f(x) = \log(x)$
	4	0,6020600
x_0	4,5	0,6532125
x_1	5,5	0,7403627
x_2	6	0,7781513

$f(5) = ?$ $\overset{x_0}{x = 4,5}$, $\overset{x_1}{x = 5,5}$ ve $\overset{x_2}{x = 6}$

Quadratic ile bulup bagil hata hesaplayın. $\log(5)$ real $= 0,69897$

$f(5) = ?$

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = 0,6532125$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_2} = \frac{0,0871502}{1}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} = \frac{0,0755772 - 0,0871502}{1,5}$$

$$-0,0077153$$

$$f(5) = 0,6532125 + 0,0871502(0,5) - 0,0077153(0,5)(-0,5)$$

$$\boxed{f(5) = 0,698716425}$$

$$\text{Hata} = \frac{\text{Gerçek} - \text{bulunan}}{\text{gerçek}} = 0,00036 \approx \%0,03$$

Newton Interpolasyon Polinomu

Consider the data given in the table:

x	-5	-1	0	2
$f(x)$	-2	6	1	3

Construct the data the Newton's interpolating polynomial of the maximum possible order to approximate $f(x)$ at $x=1$

x_i	y_i	1. bölünmüş farklar	2. bölünmüş farklar	3. farklar
-5	-2			
		$\frac{6-(-2)}{-1-(-5)} = 2$		
-1	6		$\frac{-5-2}{0-(-5)} = -\frac{7}{5}$	
		$\frac{1-6}{0-(-1)} = -5$		$\frac{2-\left(-\frac{7}{5}\right)}{2-(-5)} = \frac{17}{35}$
0	1		$\frac{1-(-5)}{2-(-5)} = 2$	
		$\frac{3-1}{2-0} = 1$		
2	3			

Newton Interpolasyon formülü: tablo eleman sayısı $= n$ $n-1$ polinom üretilir.

$$P_3(x) = -2 + 2.(x-(-5)) + \left(\frac{-7}{5}\right).(x-(-5)).(x-(-1)) + \frac{17}{35}.(x-(-5))(x-(-1))(x-0)$$

$$f(1) \approx P_3(1) = -2 + 2.6 - \frac{7}{5}.6.2 + \frac{17}{35}.6.2.1 = -2+12-\frac{84}{5}+\frac{12.17}{35}$$

$$= -0.97$$

x	y	1	2	3
-5	-2			
		$\frac{6--2}{-1--5} = 2$		
-1	6		$\frac{-5-2}{0--5} = -\frac{7}{5}$	
		$\frac{1-6}{0+1} = -5$		$\frac{2--\frac{7}{5}}{2--5} = \frac{17}{35}$
0	1		$\frac{1--5}{2--1} = 2$	
		$\frac{3-1}{2-0} = 1$		
2	3			

$$P_3(x) = -2 + 2.(x-(-5)) + -\frac{7}{5}.(x-(-5))(x--1) + -\frac{17}{35}(x--5)(x--1)(x)$$

Lagrange Interpolasyonu

linear

1. derece $(n=1)$ = doğrusal interpolasyon

2. derece $(n=2)$ = eğrisel (quadratic)

quadratic

$\left(\vdots \quad \vdots \quad \vdots \quad \vdots \right)$ $n =$ nokta -1

① $$f_n(x) = \sum_{i=0}^{n} L_i(x) \cdot f(x_i)$$

Lagrange budur

② $$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$\prod_{k=1}^{5} (k+1) = \boxed{3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}$$

$n = 1$ için lagrange

$$f_1(x) = \sum_{i=0}^{1} L_i(x) \cdot f(x_i) = L_0(x) \cdot f(x_0) + L_1(x) \cdot f(x_1)$$

$$L_0(x) = \prod_{\substack{j=0 \\ (j \neq 0)}}^{1} \frac{x - x_j}{x_0 - x_j} = \frac{x - x_1}{x_0 - x_1}$$

$$L_1(x) = \prod_{\substack{j=0 \\ j \neq 1}}^{1} \frac{x - x_j}{x_1 - x_j} = \frac{x - x_0}{x_1 - x_0}$$

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} \cdot f(x_0) + \frac{x - x_0}{x_1 - x_0} \cdot f(x_1)$$

— o —

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$n=2$ için Lagrange

$$f_2(x) = \sum_{i=0}^{2} L_i(x) . f(x_i) = L_0(x) . f(x_0) + L_1(x) . f(x_1) + L_2(x) . f(x_2)$$

$$L_0(x) = \prod_{i \neq 0}^{2} \frac{x - x_j}{x_i - x_j} = \overbrace{\frac{(x - x_1)}{(x_0 - x_1)} \cdot \frac{(x - x_2)}{(x_0 - x_2)}}^{n=0}$$

$$L_1(x) = \prod_{\substack{j=0 \\ j \neq 1}}^{2} \frac{x - x_j}{x_i - x_j} = \overbrace{\frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2}}^{n=1}$$

$$L_2(x) = \prod_{\substack{j=0 \\ j \neq 2}}^{2} \frac{x - x_j}{x_i - x_j} = \overbrace{\frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1}}^{n=2}$$

$$f_2(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} . f(x_0) + \frac{x - x_0}{x_1 - x_0} . \frac{x_1 - x_2}{x_1 - x_2} . f(x_1) + \ldots$$

o

Taktik

$f(x_i)$ hangisi seçili ise $\dfrac{x - x_i \text{ hariç}}{x_i - x_i \text{ hariç}}$

$$f_3(x) = \frac{x - (x_0 \text{ haricindekiler})}{\underset{x_i}{x_0} - (x_0 \text{ haricindekiler})} . f(x_0) + \frac{x - (x_1 \text{ harici})}{\underset{x_i}{x_1} - (x_1 \text{ harici})} . f(x_1)$$

$$\frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} . f(x_0)$$

Örnek:

x	f(x)
1	4,75
2	4
3	5,25
5	19,75
6	36

f(4) lagrange ile a) 1. derece için $(x_0=3,\ x_1=5)$

b) 2. için $(x_0=2,\ x_1=3,\ x_2=5)$

c) 3. için $(x_0=2,\ x_1=3,\ x_2=5,\ x_3=6)$

a) $$f_n(x)=\sum_{i=0}^{n} L_i(x)f(x_i) \qquad L_i(x)=\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{x-x_j}{x_0-x_j}$$

$n=1$ için $(x_0=3,\ x_1=5)$

$$f_1(x)=\frac{x-x_1}{x_0-x_1}\cdot f_0(x)+\frac{x-x_0}{x_1-x_0}\cdot f(x_1)=\frac{x-5}{3-5}\cdot 5{,}25+\frac{x-3}{5-3}\cdot 19{,}75$$

$$f_1(4)=\frac{1}{2}\cdot 5{,}25+\frac{1}{2}\cdot 19{,}75=\frac{1}{2}(5{,}25+19{,}75)=12{,}5$$

b) $n=2$ için $(x_0=2,\ x_1=3,\ x_2=5)$

$$f_2(x)=f(x_0)\cdot\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}+f(x_1)\cdot\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$
$$+f(x_2)\cdot\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$f_2(x)=\frac{(x-3)(x-5)}{(2-3)(2-5)}\cdot f(2)+\frac{(x-2)(x-5)}{(3-2)(3-5)}\cdot f(3)+\frac{(x-2)(x-3)}{(x_2-x_0)(x_2-x_1)}\cdot f(5)=\underline{10{,}5}$$

Chapter 7 Eng

$\sec = \frac{1}{\cos}$

$\sin x = \cos x \, dx$

$\cos x = -\sin x \, dx$

$\tan x = \sec^2 x \, dx$

$\ln x = \frac{1}{x} \, dx$

$e^x = e^x \, dx$

Numerical Integration

$\int_a^b f(x)\,dx \approx$ Yaklaşık hesaplama fikirlerinin bütününe sayısal integrasyon denir.

Newton Cotes

$$\int_a^b f(x)\,dx \cong \int_a^b f_n(x)\,dx$$

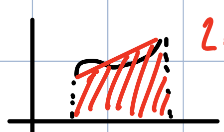
polinom $f_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \dots + a_nx^n$

Sayısal integrasyon çeşitleri

Doğru uydurma

$f_n(x) = a_0 + a_1x$

Trapezodial Rule

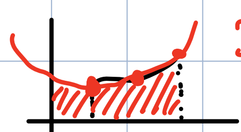


2 nokta

Parabol Uydurma

$f_n(x) = a_0 + a_1x + a_2x^2$

Simpson 1/3 rule

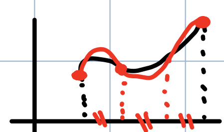


3 nokta

Kübik Polinom uydurma

$f_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

Simpsone 3/8 rule

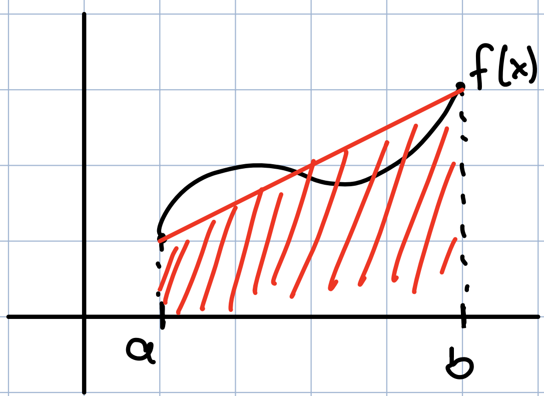


4 nokta

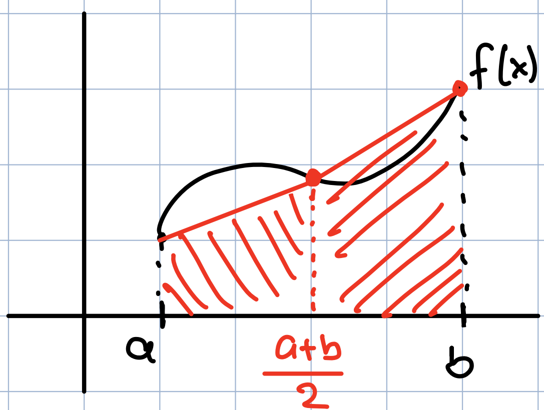
Trapezoidal Rule (yamuk)

$$\int_a^b f(x)\,dx \cong \int_a^b f_1(x)\,dx$$

→ 1. derece polinom



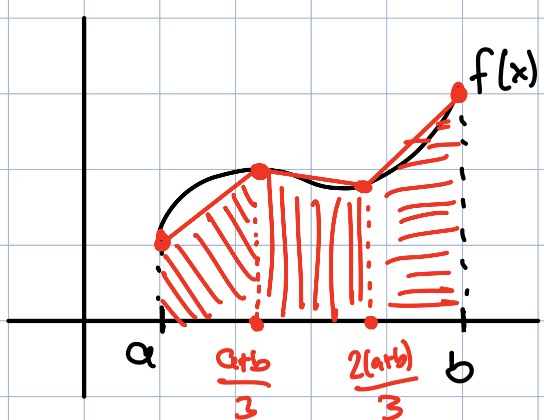
$$= (b-a) \cdot \frac{f(a)+f(b)}{2}$$ (n = 1 için)



(n = 2 için)

$$\int_a^{\frac{a+b}{2}} f(x)\,dx + \int_{\frac{a+b}{2}}^{b} f(x)\,dx = \left(\frac{a+b}{2} - a\right) \cdot \frac{f(a) + f\left(\frac{a+b}{2}\right)}{2}$$

$$+ \left(b - \left(\frac{a+b}{2}\right)\right) \cdot \frac{f\left(\frac{a+b}{2}\right) + f(b)}{2}$$



n = 3 için

örnek $\int_1^2 \left(x+\frac{1}{x}\right)^2 dx$

a) analitik hesaplayın

b) Trapezodial ile $n=1$ ve $n=2$ ile hesaplayınız.

a) $$\int_1^2 x^2+2+\frac{1}{x^2}\,dx = \frac{x^3}{3}+2x+\left(-\frac{1}{x}\right)\Bigg|_1^2 = \left(\frac{8}{3}+4-\cancel{\frac{1}{2}}\right)-\left(\frac{1}{3}+2-\cancel{\frac{1}{2}}\right)$$

$$\frac{7}{3}+2=\frac{13}{3}$$

$$=4{,}8333$$

b) $n=1$ için

$$\int_1^2 \left(x+\frac{1}{x}\right)^2 dx = (b-a).\frac{f(a)+f(b)}{2} = (2-1)\,\frac{f(1)+f(2)}{2}$$

$$f(x)=\left(x+\frac{1}{x}\right)^2$$

$$=1.\ \frac{4+\frac{5}{24}}{2}=\frac{41}{8}$$

$5{,}125$

Hata $=\dfrac{\text{Gerçek}-\text{Hesaplanan}}{\text{Gerçek}}$

Kural

$$\boxed{(b-a).\frac{f(a)+f(b)}{2}}$$

$n=2$ için

$$\int_1^{1,5}\left(x+\frac{1}{x}\right)^2 dx+\int_{1,5}^{2}\left(x+\frac{1}{x}\right)^2 dx = \underbrace{(1{,}5-1)\,\frac{f(1)+f(1{,}5)}{2}}_{2{,}1736111}+\underbrace{(2-1{,}5).\frac{f(1{,}5)+f(2)}{2}}_{2{,}736111}$$

$\cong 4{,}9097\ldots$

örnek:

$$\int_0^4 xe^{2x}\,dx$$ gerçek sonuç ve trapezius $n=4$ bulup hata hesaplayın.

LAPTÜ

$u = x$ $\quad \int dv = \int e^{2x}\,dx$

$du = dx$ $\quad v = \dfrac{e^{2x}}{2}$

$$\boxed{u.v - \int v\,du} = x.\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx$$

$$x.\frac{e^{2x}}{2} - \frac{e^{2x}}{4}\Big|_0^4$$

$$= \frac{7e^8}{4} + \frac{1}{4} = 5216{,}926477$$

$n: 4$

$$\int_0^1 xe^{2x}\,dx + \int_1^2 \ldots + \int_2^3 \ldots + \int_3^4 xe^{2x}\,dx$$

$$(b-a)\,\frac{f(a).f(b)}{2}$$

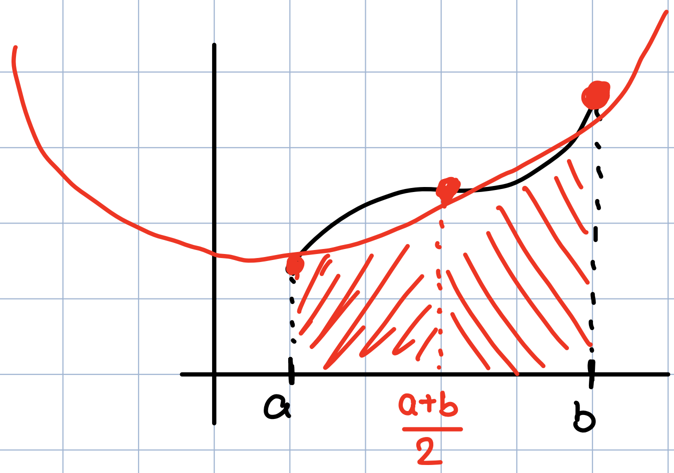
$= 7288{,}797711$ yaklaşık sonuç

$$\text{Relative Error} = \frac{G - Y}{G}$$

1/3 Simpson Rule

$n=1$ için

$$\int_a^b f(x)dx \cong \int_0^b f_2(x)dx \cong (b-a).\frac{f(a)+4.f\left(\frac{a+b}{2}\right)+f(b)}{6}$$



$n=2$ için $\int_a^{\frac{a+b}{2}} f(x)dx + \int_{\frac{a+b}{2}}^b f(x)dx$

kural uygulanır — kural uygulanır

Soru:

$$\int_{-1}^{3} (x^3+1)\,dx$$

a) analitik

b) simpson $\frac{1}{3}$ $n=1$ ve $n=2$ ye göre

a) $\left.\frac{x^4}{4}+x\right|_{-1}^{3} = 23{,}25-(-0{,}75) = 24$

b) $n=1$ $\int_{-1}^{3} x^3+1\,dx = (b-a)\,\frac{f(a)+4f\left(\frac{a+b}{2}\right)+f(b)}{6}$

$$= 4.\frac{f(-1)+4f(1)+f(3)}{6}$$

$$= 4.\frac{0+8+28}{6} = 24$$

Örnek:

$$\int_4^{16} \sqrt{x}\, dx$$ analitik? 1/3 rule? error? $n=1$, $n=2$

a) $$\left.\frac{2x^{3/2}}{3}\right|_4^{16} = 42{,}6666 - 5{,}3333 = \underline{37{,}3333}$$

b) $n=1$

$$= (b-a).\frac{f(a)+4f(\frac{a+b}{2})+f(b)}{6} = \cancel{12}^{2}.\frac{f(4)+4f(10)+f(16)}{\cancel{6}}$$

$$2(2+4\sqrt{10}+4)$$
$$= 12+8\sqrt{10}$$
$$= \underline{37{,}29822}$$

$n=2$

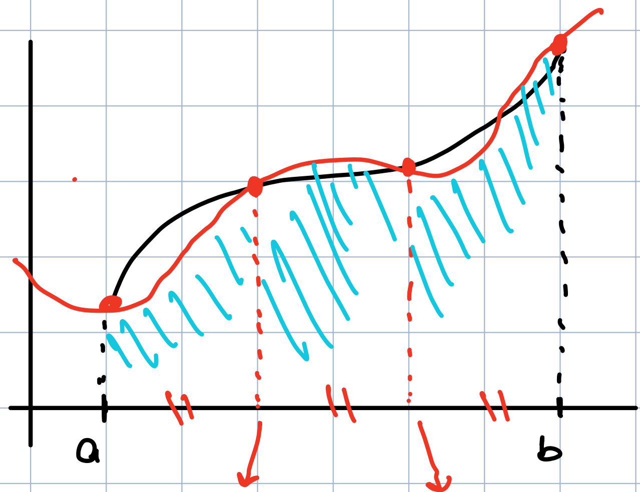
$$\int_4^{10} \quad \int_{10}^{16}$$

$$\frac{\cancel{(10-4)}.f(4)+4.f(7)+f(10)}{\cancel{6}} + \cancel{(16-10)}\frac{(f(10)+4f(13)+f(16)}{\cancel{6}}$$

$$= \underline{37{,}3297}$$

Simpsom 3/8 Kuralı

$$\int_a^b f(x)\,dx \cong \int_a^b f_3(x)\,dx \overset{n=1}{=} (b-a)\cdot\frac{f(a)+3f(x_1)+3f(x_2)+f(b)}{8}$$



$x_1 = a + \frac{b-a}{3}$ $\qquad$ $x_2 = a + 2\,\frac{b-a}{3}$

$n=2$ için $\displaystyle\int_a^{\frac{a+b}{2}} f(x)\,dx + \int_{\frac{a+b}{2}}^{b} f(x)\,dx$

örnek:

$$\int_0^6 \frac{1}{1+x^4}\,dx$$

Simpsom 3/8 ile $n=1$ ve $n=2$ için yap

$n=1 \Rightarrow$

$$\int_0^6 \frac{1}{1+x^4}\,dx = (\overset{6}{b}-\overset{0}{a})\cdot\frac{f(\overset{0}{a})+3f(x_1)+3f(x_2)+F(\overset{6}{b})}{8}$$

$\frac{6-0}{3} = 2$

$x_1 = 0 + 2 = 2$

$x_2 = 0 + 2.2 = 4$

$$\frac{6\cdot\left(1+3.\frac{1}{17}+3.\frac{1}{257}+\frac{1}{1297}\right)}{8} \cong 0{,}8917$$

$n=2$ için $\Rightarrow$

$$\int_0^3 \frac{1}{1+x^4}\,dx + \int_3^6 \frac{1}{1+x^4}\,dx$$

$$\frac{3.\left(f(0)+3f(1)+3f(2)+f(3)\right)}{8} + \frac{3\left(f(3)+3f(4)+3.f(5)+f(6)\right)}{8} \cong 1{,}0192$$

Newton Cotes

Trapezius Rule

$$\int_a^b f(x)\,dx = \int_a^b f_1(x)\,dx \Rightarrow (b-a)\cdot\frac{f(a)+f(b)}{2}$$

Simpson 1/3 Rule

$$\int_a^b f(x)\,dx = \int_a^b f_2(x)\,dx \Rightarrow (b-a)\cdot\frac{f(a)+4f\left(\frac{a+b}{2}\right)\cdot f(b)}{6}$$

Simpson 3/8 Rule

$$\int_a^b f(x)\,dx = \int_a^b f_3(x)\,dx \Rightarrow (b-a)\cdot\frac{f(a)+3f(x_1)+3f(x_2)+f(b)}{8}$$

Quadratic Rule

$$f = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

$$b_0 = x_0$$

$$b_1 = \frac{f(x_2)-f(x_0)}{x_2-x_0}$$

$$b_2 = \frac{\dfrac{f(x_2)-f(x_1)}{x_2-x_1} - \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$$

Linear Interpolation

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(x_3)-f(x_1)}{x_3-x_1}$$

Newton Interpolasyon

(nokta-1 işlem vardır)

x	y	1.	2.	3.
x_0	y_0			
		$\left\{\frac{y_1-y_0}{x_1-x_0}\right\}$ a		
x_1	y_1		$\frac{b-a}{x_2-x_0}$ A	
		$\left\{\frac{y_2-y_1}{x_2-x_1}\right\}$ b		$\frac{B-A}{x_3-x_0}$ C
x_2	y_2		$\frac{c-b}{x_3-x_1}$ B	
		$\left\{\frac{y_3-y_2}{x_3-x_2}\right\}$ c		
x_3	y_3			

$$f(x) = y_0 + a\cdot(x-x_0) + A(x-x_1)(x-x_0) + C(x-x_2)(x-x_1)(x-x_0)$$