We want to provide a

The function



For a fixed t, e, and T, when graphically analyzed, is an overall decreasing function following a path similar to linear functions of the form f(x) = – (x – a) + b, except that the graph is oscillating.

Upon further graphical analysis, we can see that when removing certain terms, the graph follows a certain trend. Keeping only the sine term gives the function just the properties of a sine graph oscillating along the E-axis, where e is actually the amplitude of the oscillations. The term 2\*pi\*t/T shifts the latter graph vertically, proportional to that term’s value. The addition of –E “projects” the latter graph onto that of f(x) = –x.

Putting it all together, e determines the amplitude of the oscillations – as e increases, the amplitude increases – the negative E term in the middle projects the trigonometric function onto its polynomial (in this case, linear) equivalent, and as t increases, the x-intercept(s) of the graph increases, shifting the graph to the right.

Since we are talking about a fixed case here: T = 1, e = 0.25, and t = 0.01, 0.03, …, 0.97, 0.99, it may be useful to note that for these fixed constant cases, the solution to f(E) = 0 ranges from E = 0.0837432 for t = 0.01 to E = 6.19944 for t = 0.99, while the rest of the solutions lie within that range (0.0837432, 6.19944) approximately.

Upon doing trial and error, we discover that the value of the first term, the expression 2\*pi\*t/T is “generally in the neighborhood” of the actual zero of the function. For example, for t = 0.01, the expression is equal to 0.062831853, while the actual root is E = 0.0837432; when t = 0.99, the expression is equal to 6.220353454, while the actual root is E = 6.19944 (these are performed with T = 1, e = 0.25). This finding fits with the previously mentioned idea that as t increases, the E-intercept(s) of the graph increases, shifting the graph to the right. The graph is centered with its root near zero for when t = 0.01, and is centered on zero when the expression equals zero.

“Generally in the neighborhood” is a completely heuristic term; for our purposes, we just want to find an interval such that the endpoints evaluated at the function are opposite signs. Therefore, we employ another final heuristic measure for the endpoints needed for bisection method: the endpoints will be plus and minus 1 of the expression’s value 2\*pi\*t/T. For example, for t = 0.99, the value for a will be a = 6.2203533454 – 1, and b = 6.2203533454 + 1. This heuristic should provide two points with opposite signs, at least for fixed T = 1 and e = 0.25.