Tudorache Alexandru-Theodor

grupa 142

13.06.2020

Examen geometrie și algebră liniară

1. a

2. b

3. b

4. d

5. $\Gamma: f(x) = x_1^2 - 8x_1x_2 + 7x_2^2 - 12x_1 - 6x_2 - 9 = 0$

a) $A = \begin{pmatrix} 1 & -4 \\ -4 & 7 \end{pmatrix}$ $\tilde{A} = \begin{pmatrix} 1 & -4 & -6 \\ -4 & 7 & -3 \\ -6 & -3 & -9 \end{pmatrix}$

$\delta = \det A = 7 - 16 = -9$

$\Delta = \begin{vmatrix} 1 & -4 & -6 \\ -4 & 7 & -3 \\ -6 & -3 & -9 \end{vmatrix} = -63 - 72 - 72 - 252 + 144 - 9$

$= -\cancel{135} - 324 \cancel{+135} = -324 \neq 0$ (Conică nedegenerată)

$Q: R^2 \to R$, $Q(x) = x_1^2 - 8x_1x_2 + 7x_2^2$

Pol. caract: $\lambda^2 - Tr(A)\lambda + \det A = 0$

$\Rightarrow \lambda^2 - 8\lambda - 9 = 0$

$\Delta = 64 + 36 = 100 \Rightarrow \lambda_{1,2} = \frac{8 \pm 10}{2} \begin{matrix} \nearrow 9 \\ \searrow -1 \end{matrix}$

$\cdot V_{\lambda_1} = \{ X \in \mathbb{R}^2 \mid AX = 9X \}$

$(A - 9I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -8 & -4 \\ -4 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{cases} -8x_1 - 4x_2 = 0 \\ -4x_1 - 2x_2 = 0 \end{cases} \Rightarrow -2x_1 = x_2$

$V_{\lambda_1} = \{ x_1(1,-2) \mid x_1 \in \mathbb{R} \}$

$e_1' = \frac{1}{\sqrt{5}}(1,-2)$

$\cdot V_{\lambda_2} = \{ X \in \mathbb{R}^2 \mid AX = -X \}$

$\Rightarrow (A + I_2)X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -4 \\ -4 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{cases} 2x_1 - 4x_2 = 0 \\ -4x_1 + 8x_2 = 0 \end{cases} \Rightarrow x_1 = 2x_2$

$\Rightarrow V_{\lambda_2} = \{ x_2(2,1) \mid x_2 \in \mathbb{R} \}$

$e_2' = \frac{1}{\sqrt{5}}(2,1)$

$\begin{cases} \frac{\delta f}{\delta x_1} = 0 \\ \frac{\delta f}{\delta x_2} = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 - 8x_2 - 12 = 0 \\ -8x_1 \cancel{-18x_1} + 14x_2 - 6 = 0 \end{cases} \Rightarrow \begin{cases} x_1 - 4x_2 = 6 \mid 4 \\ -4x_1 + \cancel{7}x_2 = 3 \end{cases}$

Tudorache Alexandru - Theodor, grupa 142

$$\Rightarrow \begin{cases} 4x_1 - 16x_2 = 24 \\ -4x_1 + 7x_2 = 3 \end{cases} \quad +$$

$$-9x_2 = 27 \Rightarrow x_2 = -3$$

$$x_1 = 6 + 4x_2 = 6 - 12 = -6$$

$\Rightarrow P_0(-6, -3)$ centru

$$\theta: X = X' + X_0, \quad X_0 = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$$

$$\sigma: X' = R X'', \quad R = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

$$\theta(\Gamma): X'^T A X' + \frac{\Delta}{\delta} = 0 \Rightarrow x_1^2 - 8x_1x_2 + 7x_2^2 + 36 = 0$$

$$\sigma \circ \theta(\Gamma) = \Gamma': \lambda_1 x_1^2 + \lambda_2 x_2^2 + \frac{\Delta}{\delta} = 0$$

$$\Gamma': 9x_1''^2 - x_2''^2 + 36 = 0$$

$$-\frac{x_1''^2}{4} - \frac{x_2''^2}{36} = 1$$

6. a) u_1, u_2, u_3 vect. prop. $\Leftrightarrow \exists\, v_1, v_2, v_3$ a.î.:

$$A u_1 = v_1 u_1 \Rightarrow \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = v_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow v_1 = 2$$

$$A u_2 = v_2 u_2 \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = v_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \Rightarrow v_2 = 1$$

$$A u_3 = v_3 u_3 \Rightarrow \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = v_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow v_3 = 3$$

$\Rightarrow u_1, u_2, u_3$ vect. prop.

Scanned with CamScanner

b) $\det\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \det\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = 1+1+0-0-0-1 = 1$

$\Rightarrow \{u_1, u_2, u_3\}$ SLi $\Rightarrow R$ e reper

$u_R = R_u = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$

$f(u_R) = \begin{pmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 5 \end{pmatrix}$

7. –

8. $d_1 : x_2 = x_3 = 0 \Rightarrow \frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0} = t \Rightarrow \begin{cases} x_1 = t \\ x_2 = 0 \\ x_3 = 0 \end{cases}$

$\Rightarrow A(0,0,0),\ u(1,0,0)$

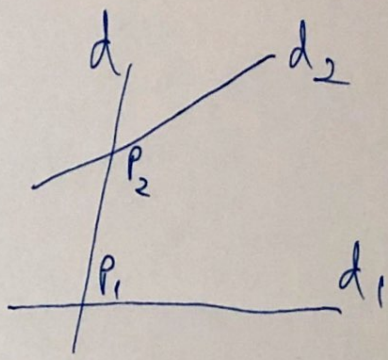
$d_2 : \begin{cases} x_2 - 1 = 0 \\ x_1 = x_3 \end{cases} \Rightarrow \frac{x_2 - 1}{0} = \frac{x_1}{1} = \frac{x_3}{1} = s \Rightarrow \begin{cases} x_1 = s \\ x_2 = 1 \\ x_3 = s \end{cases}$

$\Rightarrow B(0,1,0),\ v(1,0,1)$

$\overline{AB} = (0,1,0)$

$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0+0+0-0-1-0 = -1 \neq 0 \Rightarrow$ drepte necoplanare

Tudorache Alexandru-Theodor, grupa 142



$P_1(t,0,0) \in d_1 \cap d$

$P_2(s,1,s) \in d_2 \cap d$

$$\overline{P_1P_2} = (s-t,\ 1,\ s)$$

$$\begin{cases} \langle \overline{P_1P_2}, u \rangle = 0 \\ \langle \overline{P_1P_2}, v \rangle = 0 \end{cases} \Rightarrow \begin{cases} s-t=0 \Rightarrow s=t \\ s-t+s=0 \Rightarrow 0+s=0 \end{cases} \Rightarrow s=t=0$$

$$\Rightarrow P_1(0,0,0)\ ;\ P_2(0,1,0) \Rightarrow d = P_1P_2 : \frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{0}$$

ec. $\perp$ comune

b) $dist(d_1, d_2) = dist(P_1, P_2) = \sqrt{0^2+1^2+0^2} = 1$