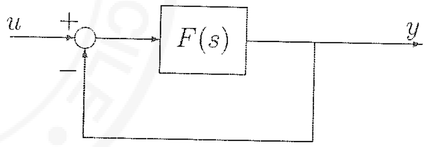
Domanda Scritta di Controlli Automatici (9CFU) - 17/9/2012

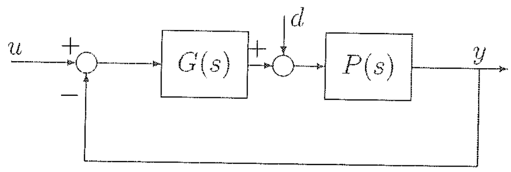
Esercizio 1 È dato il sistema di controllo:



in cui: $F(s) = \dfrac{K(s+2)}{s^2(s-p)}$. Utilizzando il criterio di Nyquist, studiare la stabilità del sistema a ciclo chiuso, per $K \in \mathbb{R}$, $K \neq 0$, $p \in \mathbb{R}$, $p \neq 0$, $p \neq -2$.

Esercizio 2

È dato il sistema di controllo:



in cui $P(s) = \dfrac{1}{(s-2)(s+4)}$; $\quad d(t) = \delta_{-1}(t)$.

Utilizzando la sintesi con il luogo delle radici, progettare $G(s)$ in modo che:

- il sistema sia astatico rispetto al disturbo $d(t)$.
- tutti i poli della funzione di trasferimento in catena chiusa abbiano parte reale minore di -3.

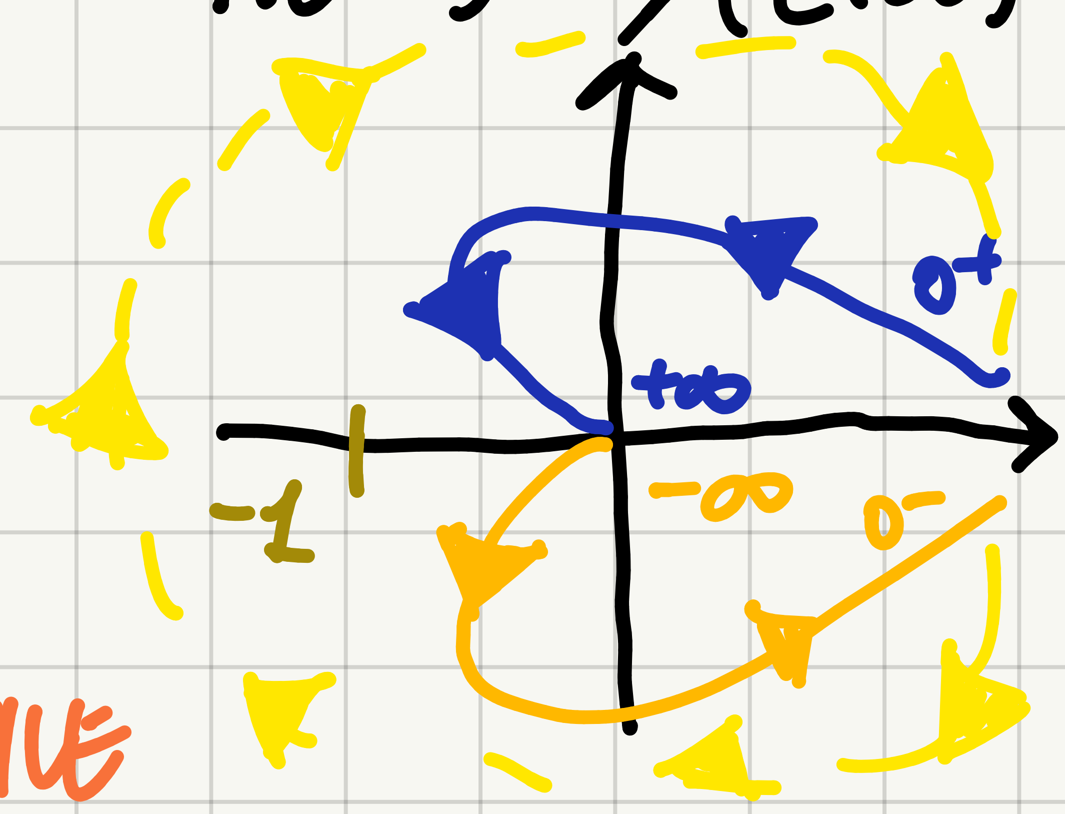
Calcolare infine la risposta a regime permanente all'ingresso $u(t) = (2t-3)\delta_{-1}(t)$.

1

$$F(s) = -\frac{2K}{P} \cdot \frac{(1+\frac{s}{2})}{s^2(1-\frac{s}{P})} \qquad F(i\omega) = -\frac{2K}{P} \cdot \frac{(1+\frac{i\omega}{2})}{(i\omega)^2(1-\frac{i\omega}{P})}$$

CASO 1: $K>0, P>0$ $\qquad P_t = 1$

$M(0^+) = \infty, \varphi(0^+) = -360°$ $\qquad M(+\infty) = 0, \varphi(+\infty) = -180°$



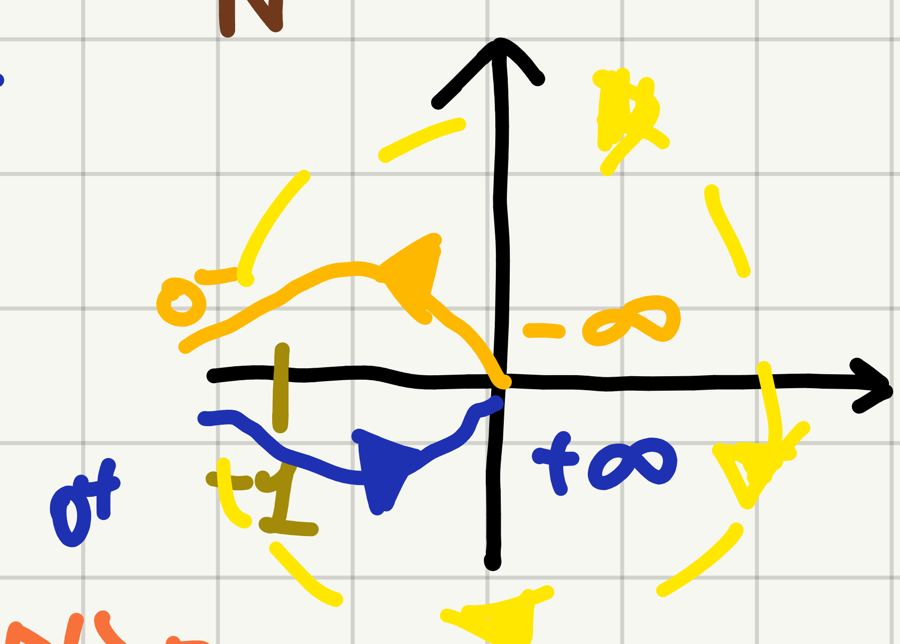
$\overset{\curvearrowright}{N} = 1$

$\overset{\curvearrowright}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

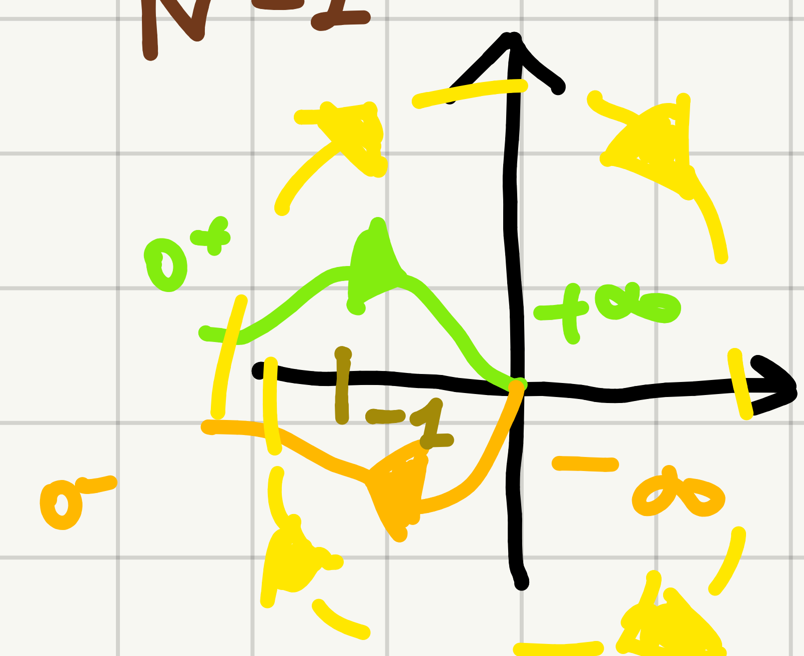
CASO 1: $K>0, P<0$ $\qquad P_t = 0$

$M(0^+) = \infty, \varphi(0^+) = -180°$ $\qquad M(+\infty) = 0, \varphi(+\infty) = -180°$

$\overset{\curvearrowright}{N} = 0$



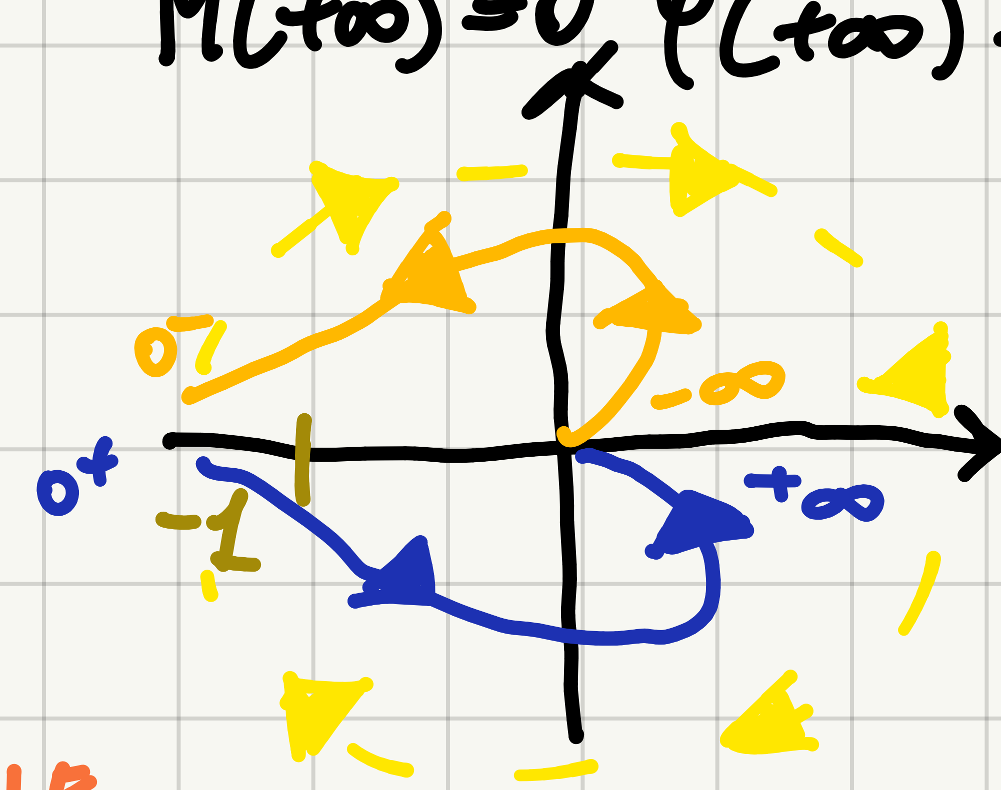
$\overset{\curvearrowright}{N} = 1$



SISTEMA STABILE PER $|P|>3$

CASO 1: $K<0, P>0$ $\qquad P_t = 1$

$M(0^+) = \infty, \varphi(0^+) = -180°$ $\qquad M(+\infty) = 0, \varphi(+\infty) = 0°$

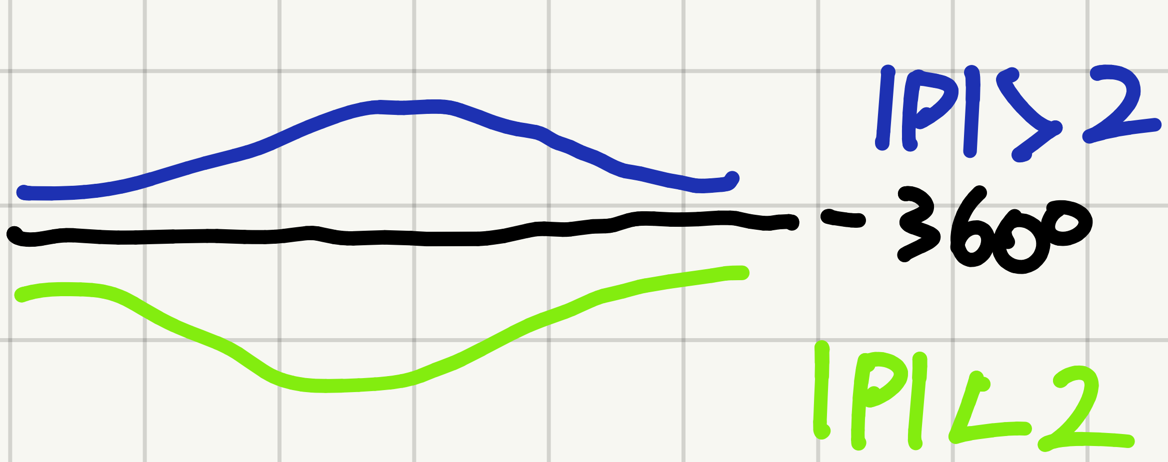


$\overset{\curvearrowright}{N} = 0$

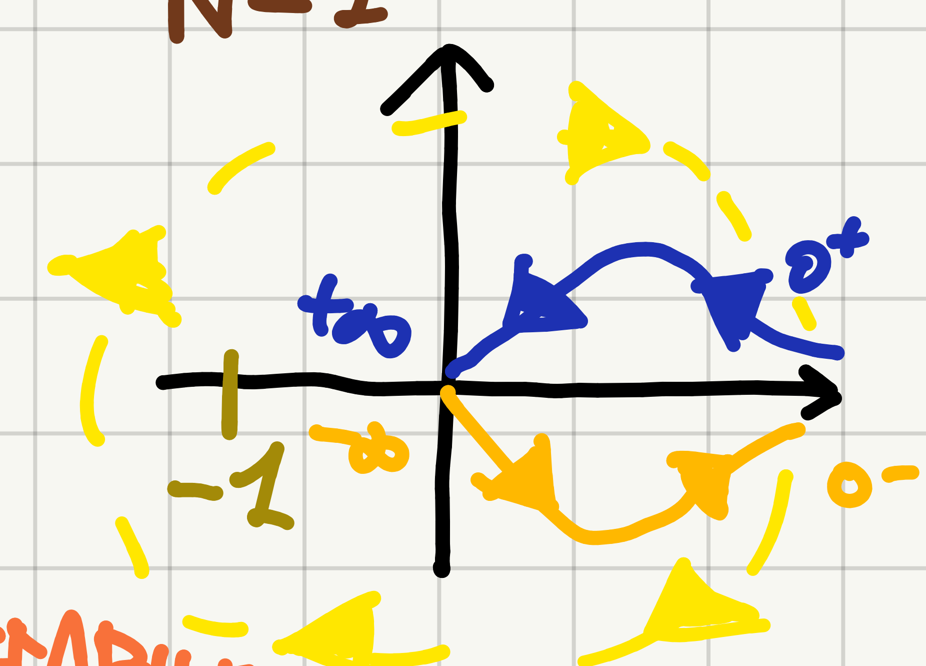
$\overset{\curvearrowright}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 1: $K<0, P<0$ $\qquad P_t = 0$

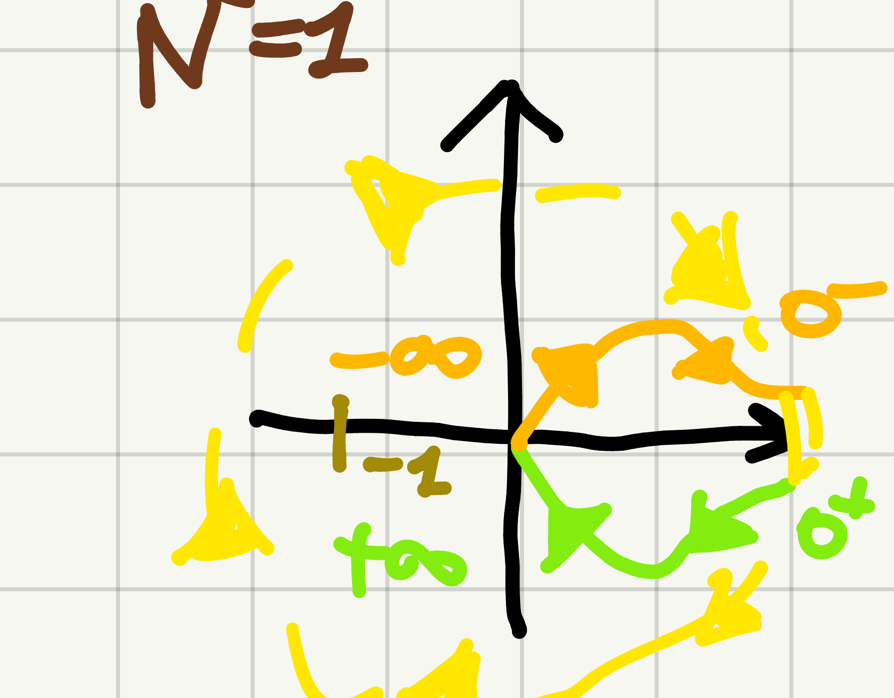
$M(0^+) = \infty, \varphi(0^+) = -360°$ $\qquad M(+\infty) = 0, \varphi(+\infty) = -360°$



$\overset{\curvearrowright}{N} = 1$



$\overset{\curvearrowright}{N} = 1$

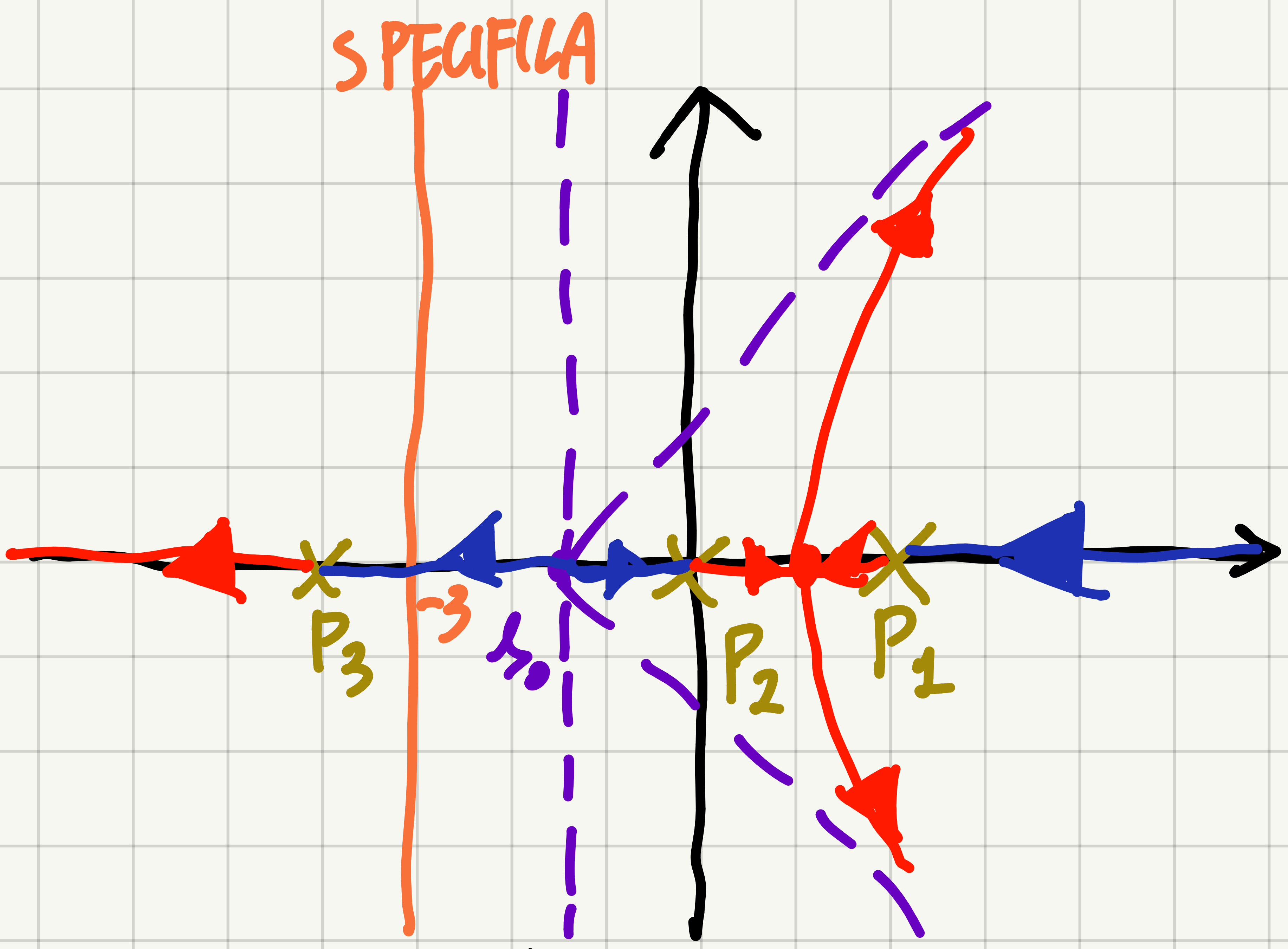


$\overset{\curvearrowright}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

ASTATISMO RISPETTO $d(t) = \delta_{-1}(t) \Rightarrow G(s) = \frac{K}{s}$

$F(s) = G(s) \cdot P(s) = \frac{1}{s(s-2)(s+4)}$ $n=3, m=0 \Rightarrow n-m=0$

$P_1 = 2,\ P_2 = 0,\ P_3 = -4$ $s_0 = \frac{\Sigma P - \Sigma Z}{n-m} = -\frac{2}{3}$



SPECIFICA NON SODDISFATTA

- PORTO $n-m=2$ $G(s) = \frac{K}{s}(s - z_1)$ $z_1 = -4$

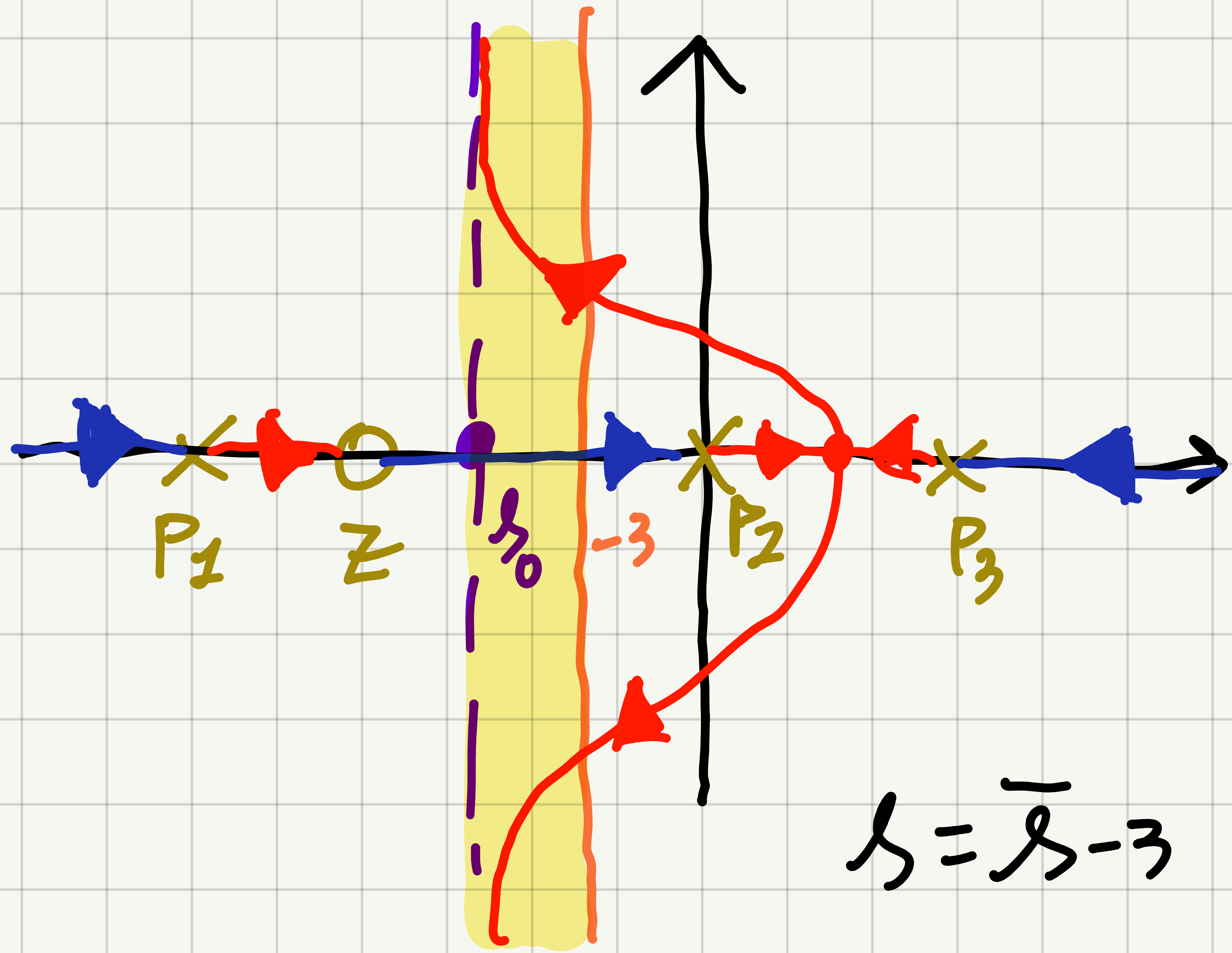
- COPPIA ZERO-POLO PER SPOSTARE s_0

$G(s) = \frac{K(s+4)}{s} \cdot \frac{(s-z_2)}{(s-p)}$ $z_2 = -6$ $s_0 = -4$

$F(s) = G(s) \cdot P(s) = \frac{K(s+6)}{s(s-p)(s-2)}$ $P_1 = P,\ P_2 = 0;\ Z = -6$
$P_3 = 2$

$-4 = \frac{2+0+P-(-6)}{2} \Rightarrow P = -16$

$$\Rightarrow G(s) = K \cdot \frac{(s+4)(s+6)}{(s+16)} \qquad F(s) = \frac{K(s+6)}{s(s+16)(s-2)}$$



$s = \bar{s} - 3$

$$f(s,K) = s(s+16)(s-2) + K(s+6)$$

$$(\bar{s}-3)(\bar{s}+13)(\bar{s}-5) + K(\bar{s}+3) = 0$$

$$\bar{s}^3 + 5\bar{s}^2 + (K-89)\bar{s} + (3K+195) = 0$$

3	1	$K-89$
2	5	$3K+195$
1	$-\frac{-2K+640}{5}$	
0	$3K+195$	

$$-\frac{1}{5}\det\begin{vmatrix} 1 & K-89 \\ 5 & 3K+195 \end{vmatrix}$$

$$\begin{cases} 2K-640>0 \\ 3K+195>0 \end{cases} \Rightarrow K > 320 \qquad \text{Scelgo } K=325$$

$$G(s) = 325 \cdot \frac{(s+4)(s+6)}{(s+16)} \qquad K_F = -\frac{975}{16}$$

$$U(t) = (2t-3)\delta_{-1}(t) = 2(t)\delta_{-1}(t) + (-3)\delta_{-1}(t)$$

$$= 2U_1(t) - 3U_2(t)$$

- $U_1(t)$

$$\tilde{e}_{U_1}(t) = K_d U_1(t) - \tilde{y}_{U_1}(t)$$

$$\tilde{e}_{U_1}(t) = \frac{1}{K_d \cdot K_p} = \frac{1}{K_F} = -\frac{975}{16}$$

$$\tilde{y}_{U_1}(t) = K_d U_1(t) - \tilde{e}_{U_1}(t) = \left(t + \frac{975}{16}\right)\delta_{-1}(t)$$

- $U_2(t)$

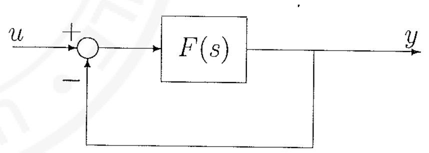
GRADO DI $U_2(t)$ $<$ TIPO DI $F(s)$ $\Rightarrow \tilde{y}_{U_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 2\left(t + \frac{975}{16}\right)\delta_{-1}(t) - 3\delta_{-1}(t)$$

Domanda Scritta di Controlli Automatici (9CFU) - 14/01/2013

Esercizio 1

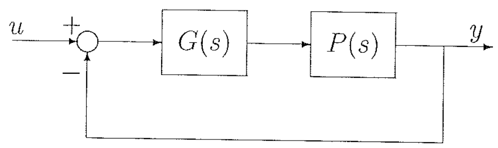
È dato il sistema in controreazione:



in cui $F(s) = \dfrac{K(s+10)}{s(s-2)(s+0.5)}$, $K \in \mathbb{R}$. Utilizzando il criterio di Nyquist, studiare la stabilità del sistema in catena chiusa al variare di $K \in \mathbb{R}$, $K \neq 0$.

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{5}{(s+2)}.$$

Progettare $G(s)$ con la sintesi per tentativi in ω in modo che:

- $|\tilde{e}_1(t)| \leq 0.08$, essendo $\tilde{e}_1(t)$ l'errore a regime permanente per un ingresso di riferimento a rampa unitaria;
- $M_r \leq 2\ dB$;
- $B_3 \simeq 2\ Hz$.

Calcolare infine la risposta a regime permanente all'ingresso: $u(t) = (2t-3) \cdot \delta_{-1}(t)$.

1

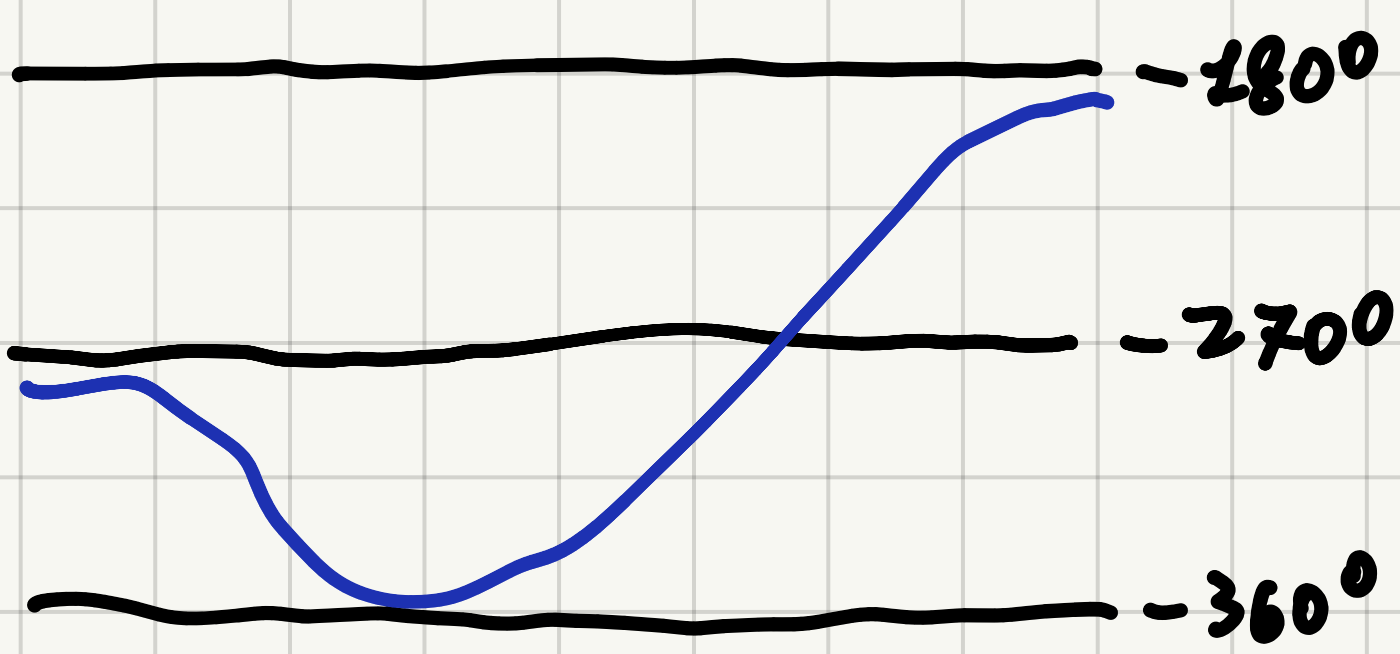
$$F(s) = -10K \cdot \frac{\left(1+\frac{s}{10}\right)}{s\left(1-\frac{s}{2}\right)\left(1+\frac{s}{0.5}\right)} \qquad F(i\omega) = -10K \cdot \frac{\left(1+\frac{i\omega}{10}\right)}{i\omega\left(1-\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{0.5}\right)}$$

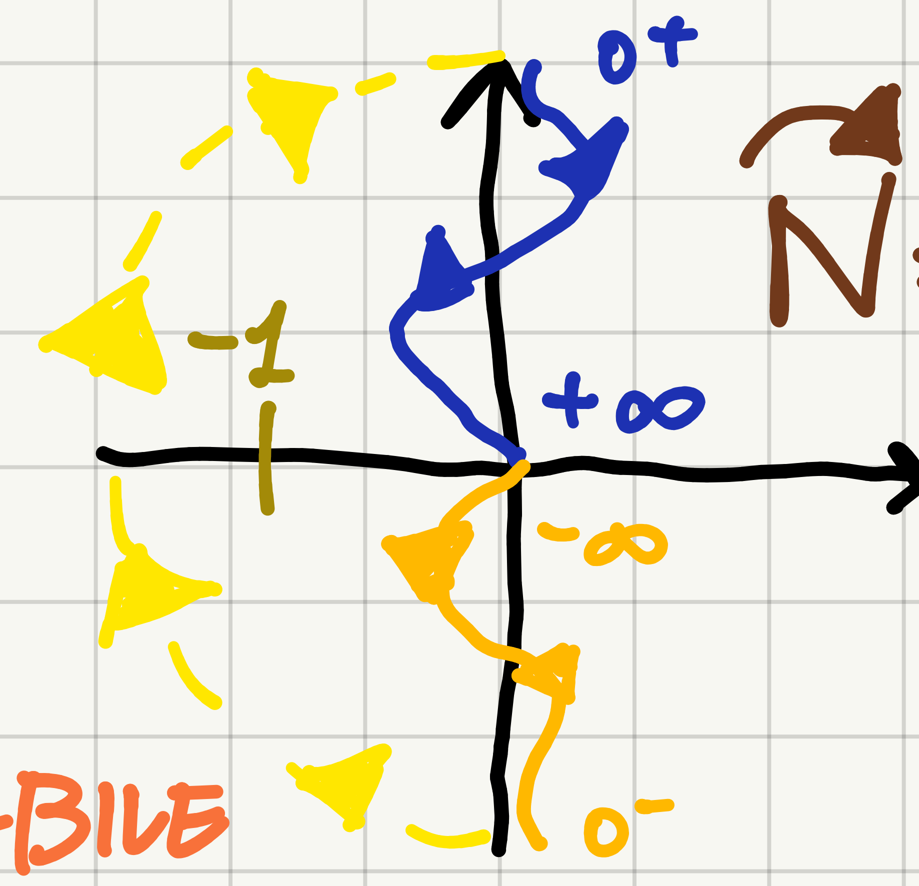
$P_+ = 1$

CASO 1: $K > 0$

$$\angle F(i\omega) = -270° - \arctan\left(\frac{\omega}{0.5}\right) + \arctan\left(\frac{\omega}{2}\right) + \arctan\left(\frac{\omega}{10}\right)$$

$M(0^+) = \infty,\ \varphi(0^+) = -270°$ $\quad$ $M(+\infty) = 0,\ \varphi(+\infty) = -180°$





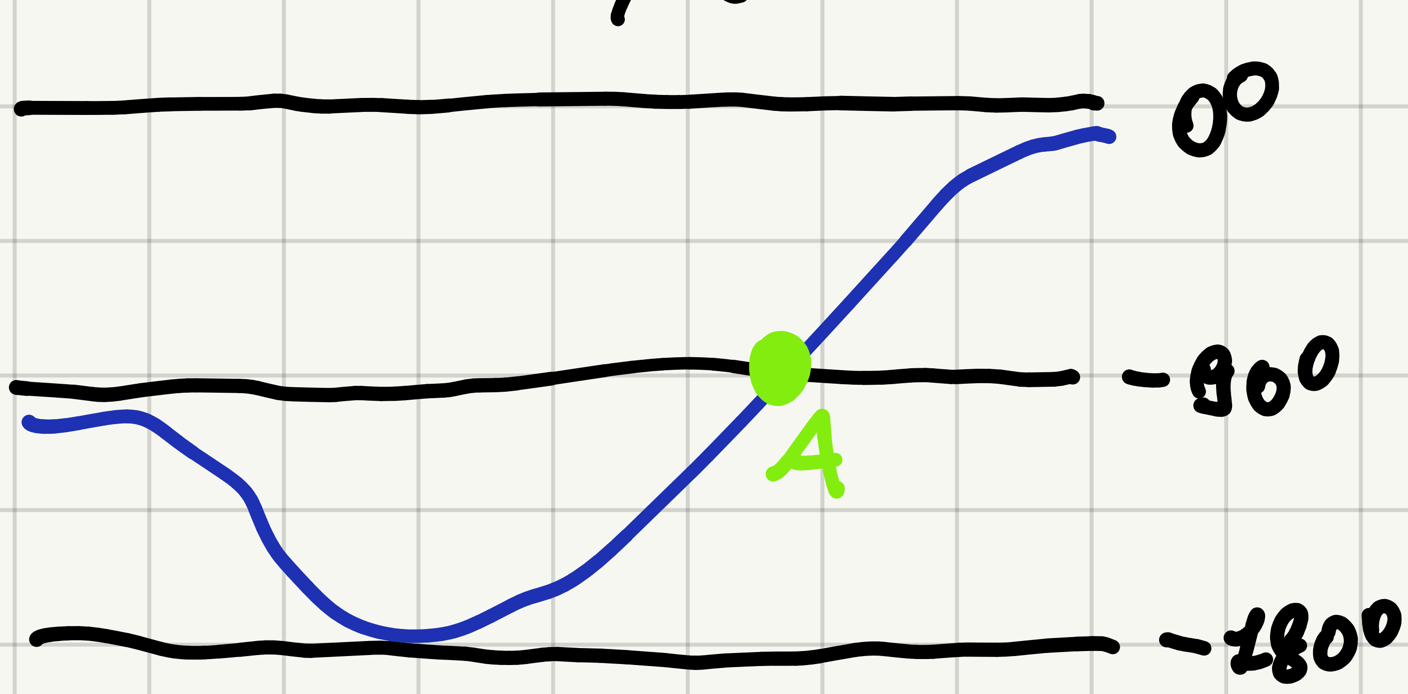
$\tilde{N} = 1$

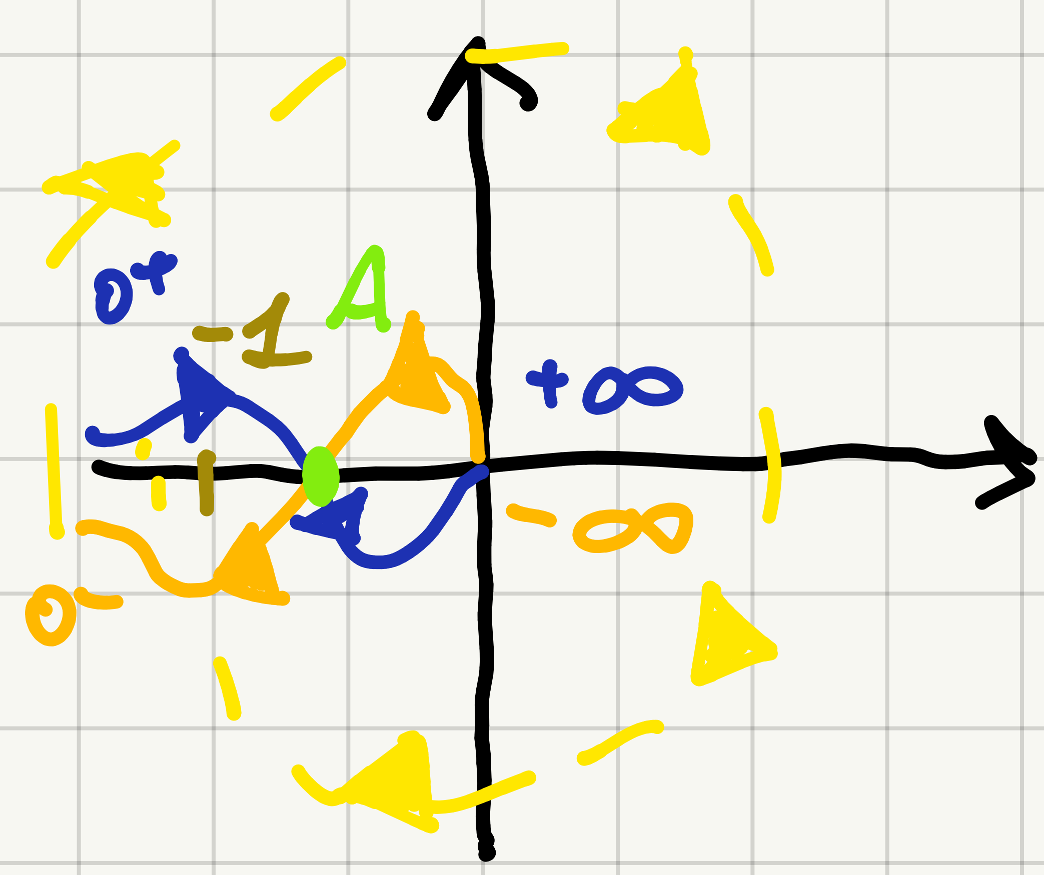
$\tilde{N} \neq -P_+ \implies$ SISTEMA INSTABILE

CASO 2: $K < 0$

$$\angle F(i\omega) = -90° - \arctan\left(\frac{\omega}{0.5}\right) + \arctan\left(\frac{\omega}{2}\right) + \arctan\left(\frac{\omega}{10}\right)$$

$M(0^+) = \infty,\ \varphi(0^+) = -90°$ $\quad$ $M(+\infty) = 0,\ \varphi(+\infty) = 0°$





$|A| < 1 \quad \tilde{N} = 1 \neq -P_+$

$|A| > 1 \quad \tilde{N} = -2 \neq -P_+$

SISTEMA INSTABILE $\forall K \in \mathbb{R}$

2

INGRESSO A RAMPA È $\delta_{-1}(t)$

$\Rightarrow |\tilde{e}_1(t)| = \left| \dfrac{K_d^2}{K_G \cdot K_P} \right| \leq 0{,}08 \quad K_P = \dfrac{5}{2} \Rightarrow K_G \geq 5$

$\Rightarrow G(s) = \dfrac{5}{s}$

SPECIFICHE UNIVOCHE

$M_r \leq 2dB \Rightarrow M_\varphi \geq 47°$

$B_3 \simeq 2Hz \Rightarrow W_t = 3 \div 5 B_3 = 4B_3 = 8 \frac{rad}{s}$

SPECIFICHE LASCHE

$F(s) = 25 \cdot \dfrac{1}{s(s+2)} \qquad F(i\omega) = \dfrac{25}{2} \cdot \dfrac{1}{i\omega(1+\frac{i\omega}{2})}$

PUNTI DI ROTTURA

- $\omega = 0$ ● $-20dB$ $-90°$ $-20dB$ $-90°$
- $\omega = 2$ ● $-20dB$ $-90°$ $-40dB$ $-180°$

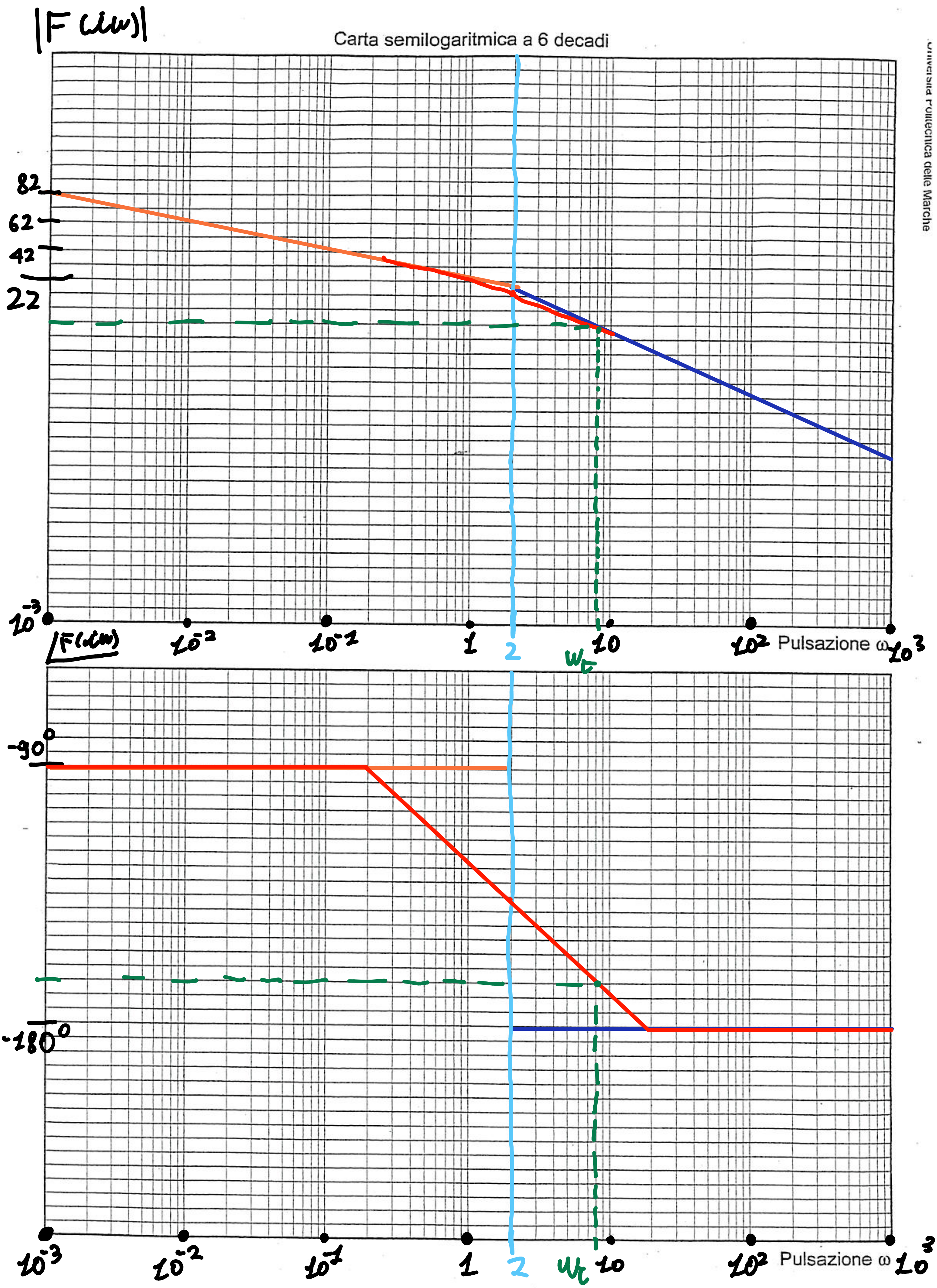
CORREZIONE MODULO

$\frac{25}{2} \rightarrow 20 \log_{10}\left(\frac{25}{2}\right) = 22dB$

ω	$1+\frac{i\omega}{2}$	TOT
2	$-3dB$	$-3dB$

CORREZIONE FASE

ω	$i\omega$	$1+\frac{i\omega}{2}$	TOT
2	$-90°$	$-45°$	$-135°$



|F(iω)|
Carta semilogaritmica a 6 decadi
Politecnica delle Marche
82
62
42
22
10^-3
10^-2
10^-1
1
2
10
ω_t
10^2
Pulsazione ω
10^3
∠F(iω)
-90°
-180°
10^-3
10^-2
10^-1
1
2
ω_t
10
10^2
Pulsazione ω
10^3

$|F(i\omega_t)| = -8\,dB$

$\angle F(i\omega_t) = -165° \Rightarrow M_\varphi = 15°$

OBIETTIVO:

- $|F(i\omega_t)| = 0$
- $M_\varphi \geq 47°$

$\Rightarrow$ AUMENTARE MODULO E FASE

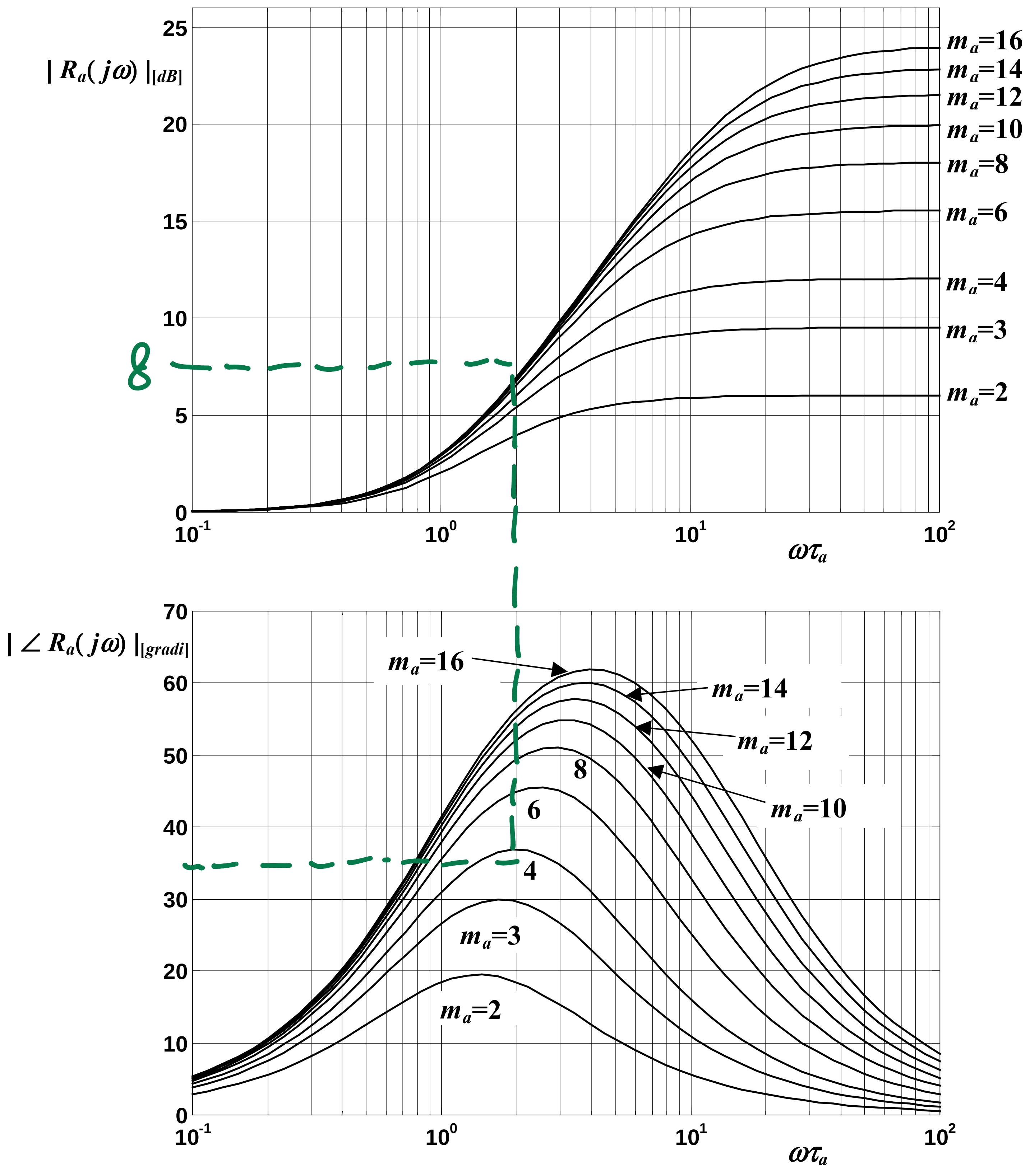
$\Rightarrow$ FUNZIONE ANTICIPATRICE $R_a(s) = \dfrac{1+\frac{s}{\omega_a}}{1+\frac{s}{m_a\omega_a}}$

$\omega_t \tau_a = 2 \Rightarrow \omega_a = \dfrac{\omega_t}{2} = 4\,\frac{rad}{s}$ $\quad m_a = 4$

$$\Rightarrow R_a(s) = \frac{1+\frac{s}{4}}{1+\frac{s}{16}}$$

$$\Rightarrow G(s) = 5 \cdot \frac{1+\frac{s}{4}}{s\left(1+\frac{s}{16}\right)}$$

$$F(i\omega) = \frac{25}{2} \cdot \frac{1}{i\omega\left(1+\frac{i\omega}{2}\right)} \cdot \frac{1+\frac{i\omega}{4}}{1+\frac{i\omega}{16}}$$



25
20
15
10
8
5
0
| $R_a(j\omega)$ |[dB]
m_a=16
m_a=14
m_a=12
m_a=10
m_a=8
m_a=6
m_a=4
m_a=3
m_a=2
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
70
60
50
40
30
20
10
0
| ∠ $R_a(j\omega)$ |[gradi]
m_a=16
m_a=14
m_a=12
m_a=10
8
6
4
m_a=3
m_a=2
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$

PUNTI DI ROTTURA

- $w=0$ 🟠 $-20dB$ $-90°$ $-20dB$ $-90°$
- $w=2$ 🔵 $-20dB$ $-90°$ $-40dB$ $-180°$
- $w=4$ 🟡 $+20dB$ $+90°$ $-20dB$ $-90°$
- $w=16$ 🟤 $-20dB$ $-90°$ $-40dB$ $-180°$

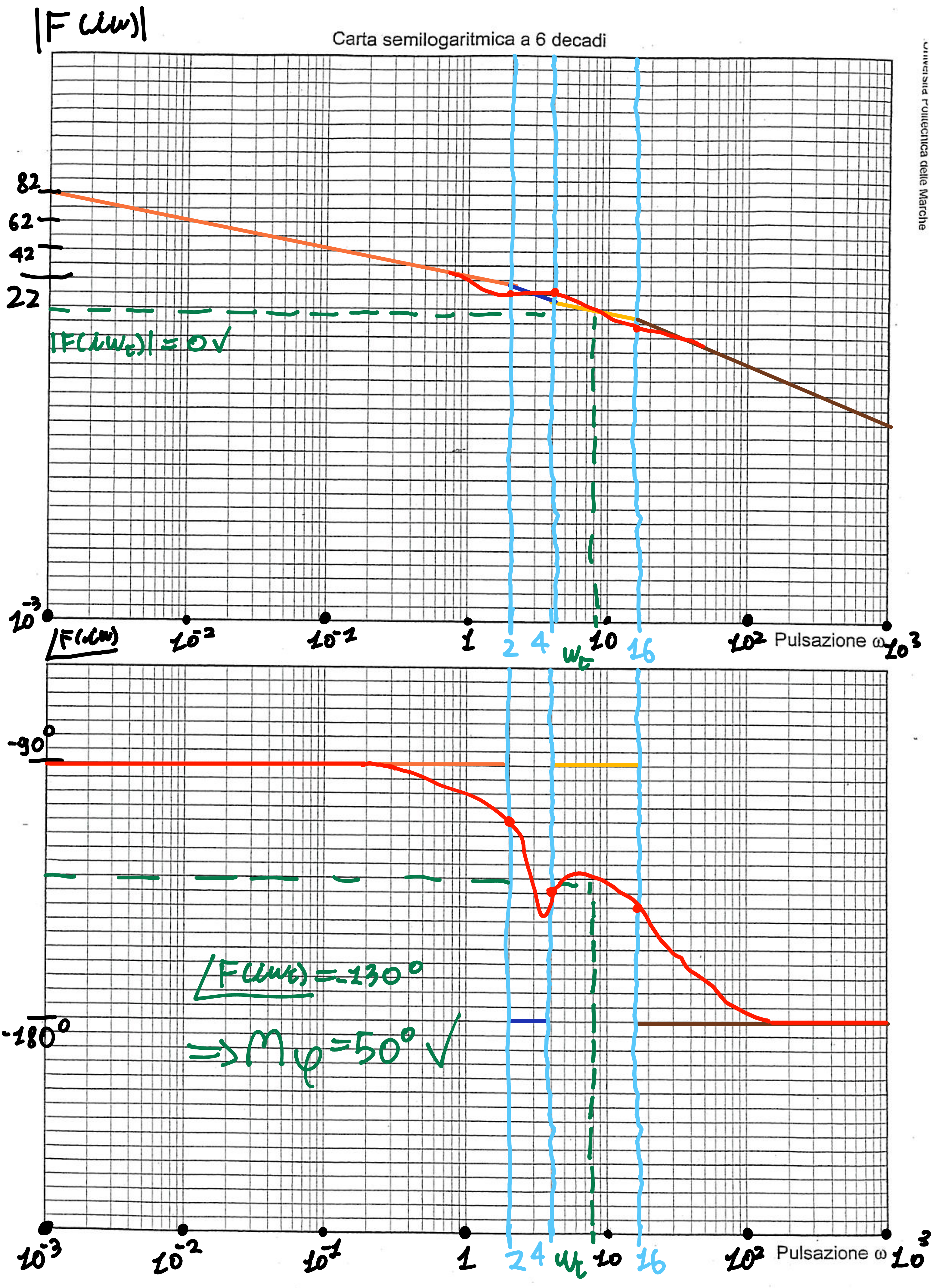
CORREZIONE MODULO

w	$1+\frac{iw}{2}$	$1+\frac{iw}{4}$	$1+\frac{iw}{16}$	TOT
2	$-3\,dB$	$+1\,dB$	0	$-2\,dB$
4	$-1\,dB$	$+3\,dB$	0	$+2\,dB$
16	0	0	$-3\,dB$	$-3\,dB$

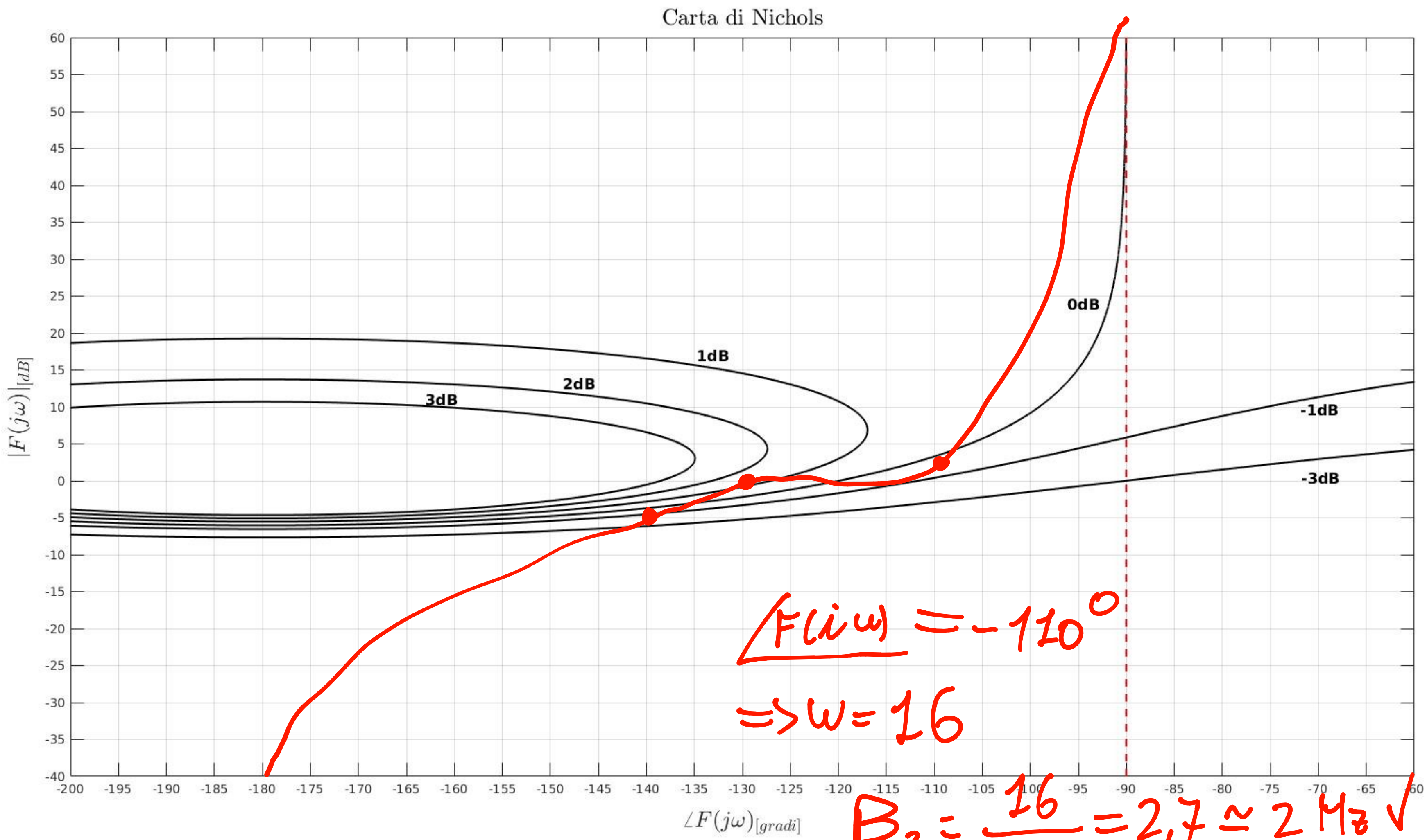
CORREZIONE FASE

w	iw	$1+\frac{iw}{2}$	$1+\frac{iw}{4}$	$1+\frac{iw}{16}$	TOT
2	$-90°$	$-45°$	$+25°$	0	$-110°$
4	$-90°$	$-75°$	$+45°$	$-15°$	$-135°$
16	$-90°$	$-80°$	$+75°$	$-45°$	$-140°$

$$F(i\omega) = \frac{25}{2} \cdot \frac{1}{i\omega\left(1+\frac{i\omega}{2}\right)} \cdot \frac{1+\frac{i\omega}{4}}{1+\frac{i\omega}{16}}$$



|F(iw)|
Carta semilogaritmica a 6 decadi
Università Politecnica delle Marche
82
62
42
22
|F(iw_t)| = 0 ✓
10^-3
10^-2
10^-1
1
2
4
w_t
10
16
10^2
Pulsazione ω
10^3
∠F(iw)
-90°
∠F(iw_t) = -130°
⇒ m_φ = 50° ✓
-180°



$\angle F(j\omega) = -110^\circ$

$\Rightarrow \omega = 16$

$B_3 = \frac{16}{2\pi} = 2{,}7 \simeq 2\ Hz$ ✓

$$u(t) = (2t-3)\delta_{-1}(t) = 2(t)\delta_{-1}(t) + (-3)\delta_{-1}(t)$$

$$= 2u_1(t) - 3u_2(t)$$

- $u_1(t)$

$$\tilde{e}_{u_1}(t) = K_d u_1(t) - \tilde{y}_{u_1}(t)$$

$$\tilde{e}_{u_1}(t) = \frac{1}{K_d \cdot K_p} = \frac{1}{K_F} = \frac{2}{25}$$

$$\tilde{y}_{u_1}(t) = K_d u_1(t) - \tilde{e}_{u_1}(t) = \left(t - \frac{2}{25}\right)\delta_{-1}(t)$$

- $u_2(t)$

GRADO DI $u_2(t)$ $<$ TIPO DI $F(s)$ $\Rightarrow \tilde{y}_{u_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 2\left(t - \frac{2}{25}\right)\delta_{-1}(t) - 3\delta_{-1}(t)$$