

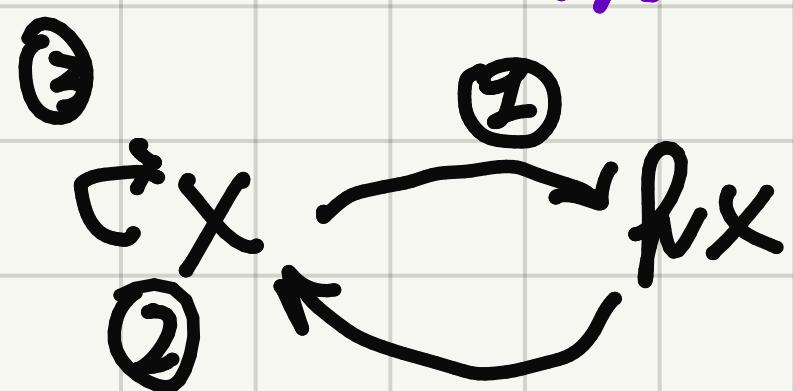
3	$\{\forall x, z (\neg R_{xz} \wedge R_{yz} \rightarrow R_{xz})\}$	ASSUMPTION
4	$\{\neg \forall x R_{xx}\}$	ASSUMPTION
5	$\{\neg R_{aa}\}$	4, δ-EXPANSION
6	$\{\exists y. R_{ay}\}$	1, γ-EXPANSION
7	$\{R_{ab}\}$	6, δ-EXPANSION
8	$\{\forall y (R_{ay} \rightarrow R_{ya})\}$	2, γ-EXPANSION
9	$\{\forall y (R_{ab} \rightarrow R_{ba})\}$	8, γ-EXPANSION
10	$\{\neg R_{ab}, R_{ba}\}$	9, β-EXPANSION
11	$\{R_{ba}\}$	7,10 RESOLUTION
12	$\{\forall y, z (R_{ay} \wedge R_{yz} \rightarrow R_{az})\}$	3, γ-EXPANSION
13	$\{\forall z (R_{ab} \wedge R_{bz} \rightarrow R_{az})\}$	12, γ-EXPANSION
14	$\{R_{ab} \wedge R_{ba} \rightarrow R_{aa}\}$	13, γ-EXPANSION
15	$\{\neg (R_{ab} \wedge R_{ba}), R_{aa}\}$	14, β-EXPANSION
16	$\{\neg R_{ab}, \neg R_{ba}, R_{aa}\}$	15, β-EXPANSION
17	$\{\neg R_{ba}, R_{aa}\}$	16,7 RESOLUTION
18	$\{R_{aa}\}$	17,11 RESOLUTION
19	\emptyset	18,5 RESOLUTION

UNIFICATION

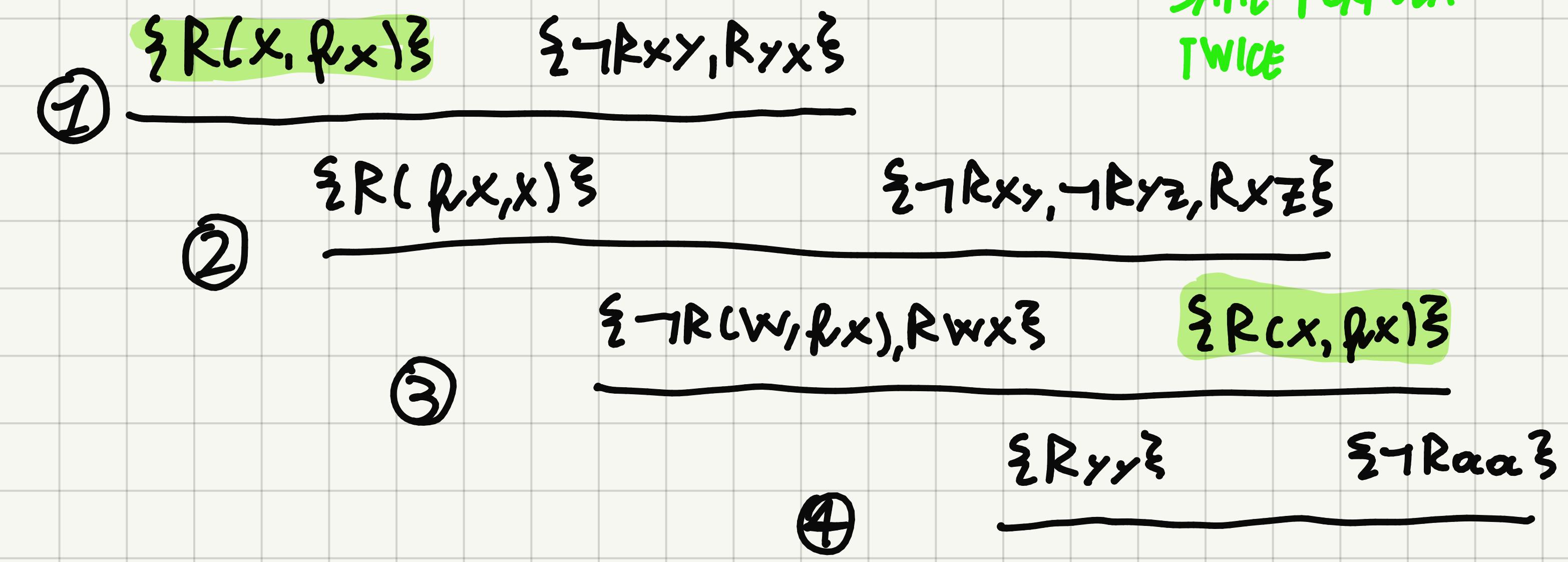
$$F = \{ \forall x \exists y. R_{xy}, \forall x y (R_{xy} \rightarrow R_{yx}), \forall x y z (R_{xy} \wedge R_{yz} \rightarrow R_{xz}), \neg \forall x R_{xx} \}$$

- $\forall x \exists y. R_{xy} \mapsto \forall x R_x f(x) \mapsto \{ \{ R(x, f(x)) \} \}$
- $\forall x y (R_{xy} \rightarrow R_{yx}) \mapsto \forall x y (\neg R_{xy} \vee R_{yx}) \mapsto \{ \{ \neg R_{xy}, R_{yx} \} \}$
- $\forall x y z (R_{xy} \wedge R_{yz} \rightarrow R_{xz}) \mapsto \forall x y z (\neg (R_{xy} \wedge R_{yz}) \vee R_{xz})$
 $\mapsto \forall x y z (\neg R_{xy} \vee \neg R_{yz} \vee R_{xz}) \mapsto \{ \{ \neg R_{xy}, \neg R_{yz}, R_{xz} \} \}$
- $\neg \forall x R_{xx} \mapsto \exists x. \neg R_{xx} \mapsto \{ \{ \neg R_{aa} \} \}$

$$CCF = \{ \{ R(x, f(x)) \}, \{ \neg R_{xy}, R_{yx} \}, \{ \neg R_{xy}, \neg R_{yz}, R_{xz} \}, \{ \neg R_{aa} \} \}$$



! WE CAN USE THE SAME FORMULA TWICE



$$\textcircled{1} \quad C_1 = \{ \{ R(x, f(x)) \} \} \quad E_1 = \{ \{ R(x, f(x)) \} \}$$

$$C_2 = \{ \{ \neg R_{wy}, R_{yw} \} \} \quad E_2 = \{ \{ \neg R_{wy} \} \}$$

$$F_1 = E_1 \cup \bar{E}_1 = \{ \{ R(x, f(x)), R_{wx} \} \}$$

$$S_1 = [x/w]$$

$$F_2 = \{ \{ R(x, f(x)), R_{xy} \} \}$$

$$S_2 = [fx/y]$$

$$F_3 = \{ R(x, f_x) \}$$

$$S = S_1 S_2$$

$$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2) S = \{ R_{yx} \left[\frac{x}{w}, \frac{f_x}{y} \right] = \{ R(f_x, x) \}$$

$$\textcircled{2} \quad C_1 = \{ R(f_x, x) \} = E_1$$

$$C_2 = \{ \neg R_{wy}, \neg R_{yz}, R_{wz} \}$$

$$E_2 = \{ \neg R_{yz} \}$$

$$F_1 = E_1 \cup \bar{E}_2 = \{ R(f_x, x), R_{yz} \}$$

$$S_1 = \left[\begin{smallmatrix} f_x \\ y \end{smallmatrix} \right]$$

$$F_2 = \{ R(f_x, x), R(f_x, z) \}$$

$$S_2 = \left[\begin{smallmatrix} x \\ z \end{smallmatrix} \right]$$

$$F_3 = \{ R(f_x, x) \}$$

$$S = S_1 S_2$$

$$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2) S = \{ \neg R_{wy}, R_{wx} \left[\frac{f_x}{y}, \frac{x}{z} \right] \}$$

$$= \{ \neg R(w, f_x), R_{wx} \}$$

$$\textcircled{3} \quad C_1 = \{ \neg R(w, f_x), R_{wx} \}$$

$$E_1 = \{ \neg R(w, f_x) \}$$

$$C_2 = \{ R(y, f_y) \} = E_2$$

$$F_1 = \bar{E}_1 \cup E_2 = \{ R(w, f_x), R(y, f_y) \}$$

$$S_1 = \left[\begin{smallmatrix} y \\ w \end{smallmatrix} \right]$$

$$F_2 = \{ R(y, f_x), R(y, f_y) \}$$

$$S_2 = \left[\begin{smallmatrix} x \\ x \end{smallmatrix} \right]$$

$$F_3 = \{ R(y, f_y) \}$$

$$S = S_1 S_2$$

$$R(C_1, C_2) = \{ R_{wx} \left[\frac{y}{w}, \frac{y}{x} \right] = \{ R_{yy} \}$$

$$\textcircled{4} \quad C_1 = \{ R_{yy} \} = E_1 \quad C_2 = \{ \neg R_{aa} \} = E_2$$

$$F_1 = \{ R_{yy}, R_{aa} \} \quad S = \left[\begin{smallmatrix} a \\ y \end{smallmatrix} \right] \quad F_2 = \{ R_{aa} \}$$

$$R(C_1, C_2) = \emptyset$$