$$= \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{vmatrix} \quad P = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{vmatrix}$$

$$p^{op} = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$m = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$f = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

f) $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad f^{op} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}$

$$K = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \quad P = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$p^{op} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$m = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$f = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

ASSUME $R: X \to Y$ IS ANY RELATION. REGARD R AS A SUBSET OF $X \times Y$ AND CONSIDER THE PROJECTIONS p AND q FROM R TO X AND Y RESPECTIVELY, DEFINED BY THE FORMULAS $p(x,y)=x$ AND $q(x,y)=y$.
PROVE THAT $R = p^{op}q$

$$\begin{tikzcd} & R \arrow[dl, "p"'] \arrow[dr, "q"] & \\ X & & Y \end{tikzcd}$$

$$\begin{tikzcd} & R \arrow[dl, "p"'] \arrow[dr, "q"] & \\ X \arrow[rr, "R"] & & Y \end{tikzcd}$$

$p(x,y)=x, \quad q(x,y)=y$

$R = p^{op}q$?

$$\begin{aligned}
x\,p^{op}q\,y &\Leftrightarrow \exists z\,(x\,p^{op}\,z \wedge z\,q\,y) \\
&\Leftrightarrow \exists z\,(z\,p\,x \wedge z\,q\,y) \\
&\Leftrightarrow \exists z\,(p(z)=x \wedge q(z)=y) \\
&\Leftrightarrow \exists z \in R\,(z=(x,y)) \\
&\Leftrightarrow x\,R\,y
\end{aligned}$$