g) $R \subseteq S \to R^{op} \subseteq S^{op}$

$$\left.\begin{array}{l} R \subseteq S \to aRb \subseteq aSb \\ bR^{op}a = aRb \\ bS^{op}a = aSb \end{array}\right\} \to bR^{op}a \subseteq bS^{op}a$$

$$\downarrow$$

$$R^{op} \subseteq S^{op}$$

h) $R^{opop} = R$

$$((aRb)^{op})^{op} = (bR^{op}a)^{op} = aRb \to R^{opop} = R$$

ASSUME $A = \{1,2,3,4\}$. DETERMINE WHICH OF THE RELATIONS $R: A \to A$ GIVEN ARE EQUIVALENT RELATIONS

Q1) $M(R) = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}$

- REFLEXIVITY $\Leftrightarrow I \subseteq R \Leftrightarrow M(I) \leq M(R)$

  IN TERMS OF MATRIX, THE DIAGONAL MUST HAVE ALL 1

  $\Rightarrow$ IT IS REFLEXIVE ✓

- SYMMETRY $\Leftrightarrow R^{op} \subseteq R \Leftrightarrow M(R)^T \leq M(R)$

$$M(R)^T = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \subseteq M(R)$$

$\Rightarrow$ IT IS SYMMETRIC ✓ IN PARTICULAR, $M(R)^T = M(R)$

- TRANSITIVITY $\Leftrightarrow R^2 \subseteq R \Leftrightarrow M(R^2) \leq M(R)$

$$M(R^2) = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ & & & \end{vmatrix}$$

$M(R^2)_{14} > M(R)_{14}$

$\Rightarrow$ WE CAN ALREADY STOP HERE AND CONCLUDE THAT R IS NOT TRANSITIVE AND THEREFORE IT IS NOT AN EQUIVALENCE RELATION

b) $M(R) = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix}$

- REFLEXIVITY $\Leftrightarrow I \subseteq R \Leftrightarrow M(I) \leq M(R)$

IT IS SUFFICES TO NOT THAT THE DIAGONAL CONTAINS ONLY "1"

$\Rightarrow$ IT IS REFLEXIVE ✓

- SYMMETRY $\Leftrightarrow R^{OP} \subseteq R \Leftrightarrow M(R)^T \leq M(R)$

$$M(R)^T = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \leq M(R)$$

$\Rightarrow$ IT IS SYMMETRIC ✓

- TRANSITIVITY $\Leftrightarrow R^2 \subseteq R \Leftrightarrow M(R^2) \leq M(R)$

$$M(R) = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \cdot$$

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} \leq M(R)$$

$\Rightarrow$ IT IS TRANSITIVE ✓

$\Rightarrow$ R IS AN EQUIVALENTE RELATION