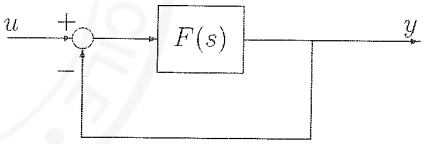
Domanda Scritta di Controlli Automatici (9CFU) - 18/6/2012

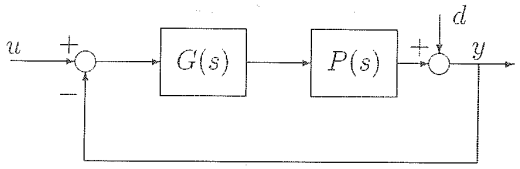
Esercizio 1 È dato il sistema di controllo:



in cui: $F(s) = \dfrac{K(s+0.1)}{s(s-p)}$. Utilizzando il criterio di Nyquist, studiare la stabilità del sistema a ciclo chiuso, per $K < 0$, $p > 0$.

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{10}{(s+5)}; \quad d(t) = \sin(\omega t).$$

Progettare $G(s)$ con la sintesi per tentativi in ω in modo che:

- $|\tilde{e}_1(t)| \leq 0.02$, essendo $\tilde{e}_1(t)$ l'errore a regime permanente per un ingresso di riferimento a rampa unitaria;
- $|\tilde{y}_d(t)| \leq 0.01$ per $\omega \leq 0.3\ rad \cdot s^{-1}$, essendo $\tilde{y}_d(t)$ la risposta a regime permanente al disturbo $d(t)$;
- $M_r \leq 2\ dB$;
- $B_3 \simeq 2.5\ Hz$.

Calcolare infine la risposta a regime permanente all'ingresso: $u(t) = (2t-3) \cdot \delta_{-1}(t)$.

$$F(s) = -\frac{0,2k}{p} \cdot \frac{(1+\frac{s}{0,1})}{s(1-\frac{s}{p})}$$ (1)

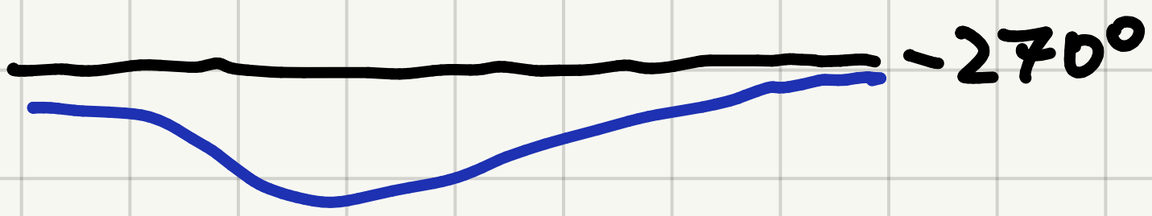
$K>0 \quad p<0$

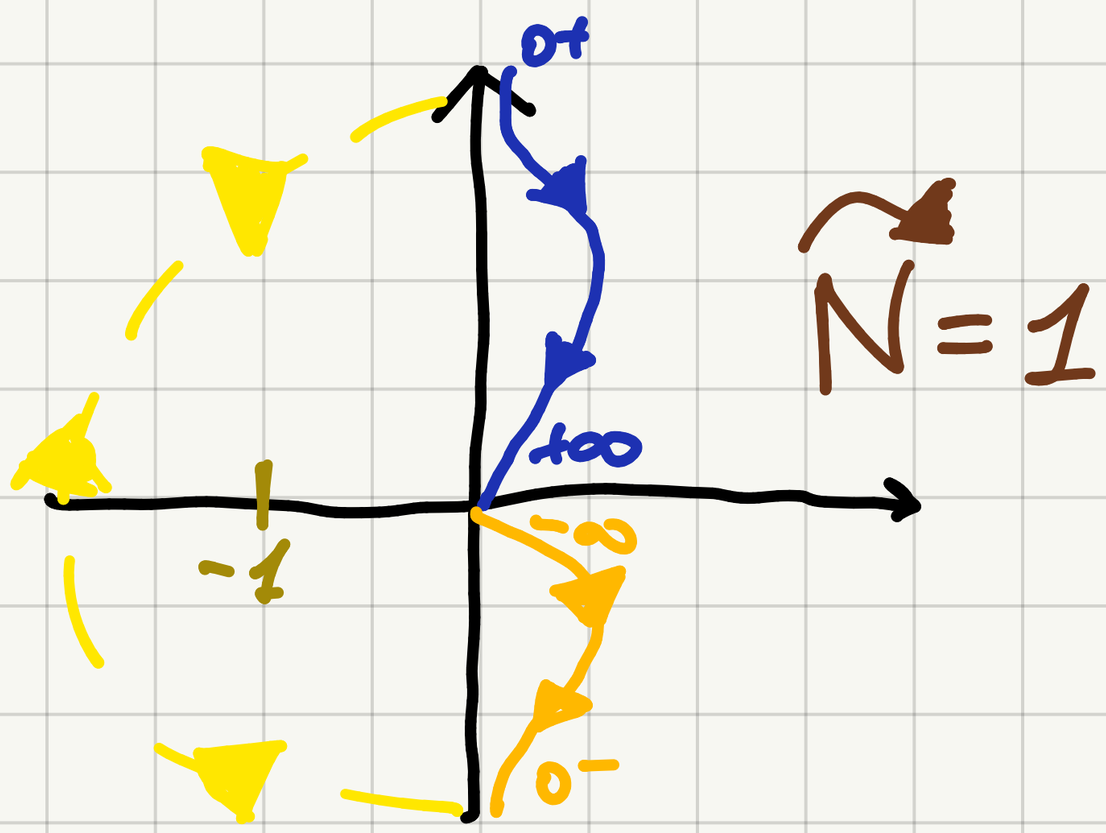
$$F(i\omega) = -\frac{0,1k}{p} \cdot \frac{(1+\frac{i\omega}{0,1})}{s(1-\frac{s}{p})}$$

$P_+ = 1$

$M(0^+)=\infty, \varphi(0^+)=-270° \qquad M(+\infty)=0, \varphi(+\infty)=0$

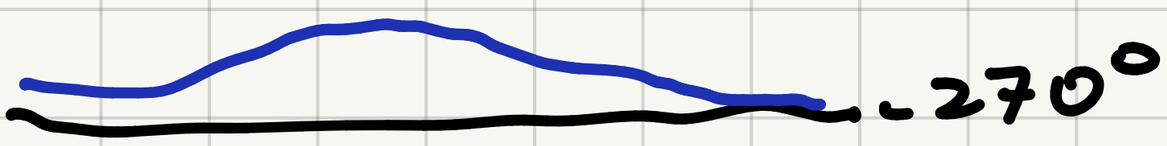
CASO 1: $|p| < 0,1$

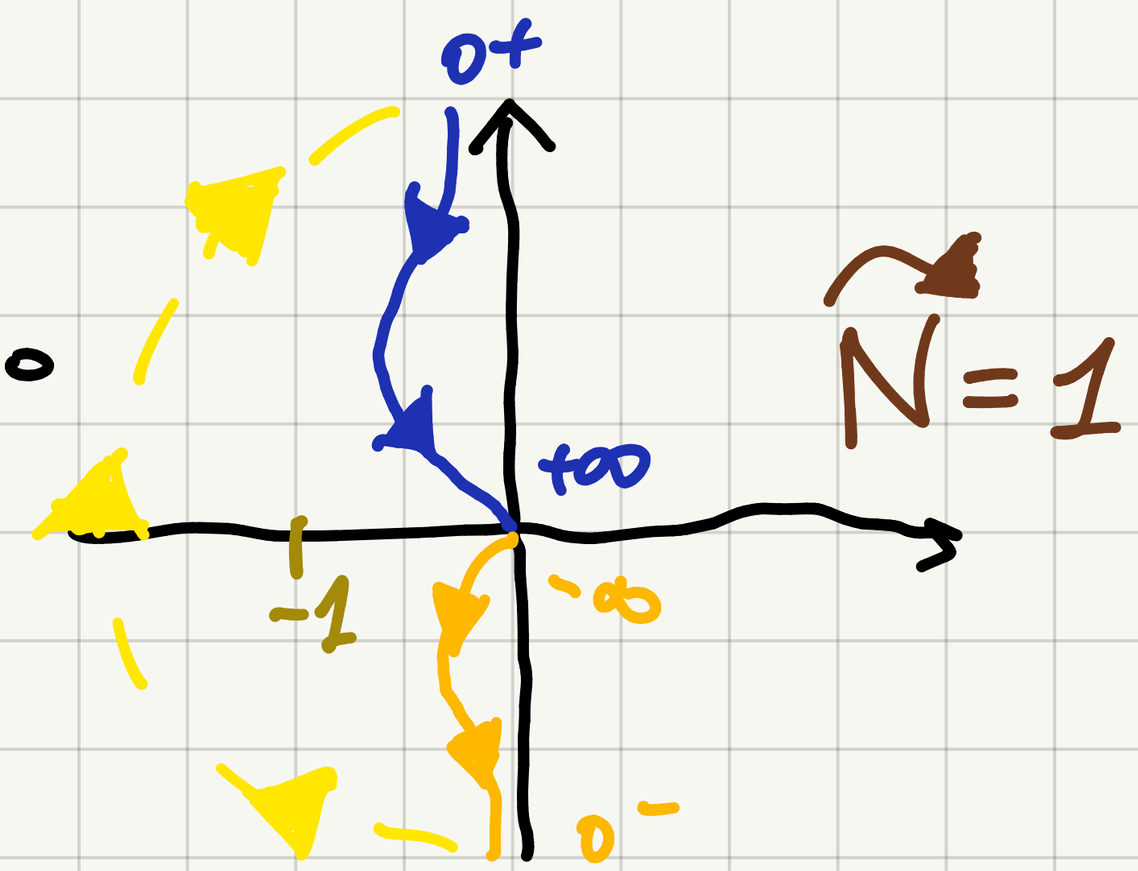




$\vec{N} \neq -P_+ \Rightarrow$ INSTABILE

CASO 2: $|p| > 0,1$





$\vec{N} \neq -P_+ \Rightarrow$ INSTABILE

$$|\tilde{e}_1(t)| = \left|\frac{K_d^2}{K_F}\right| \simeq \left|\frac{1}{K_p \cdot K_0}\right| \leq 0{,}02 \quad K_p = 2 \Rightarrow K_0 \geq 25$$

(2)

$$|\tilde{y}_{d_1}(t)| = \left|\frac{1}{1+F(i\omega)}\right| \leq 0{,}01 \Rightarrow |F(i\omega)| \geq 100 \text{ PER } \omega \leq 0{,}3 \frac{rad}{s} \hookrightarrow 40dB$$

$M_r \leq 2dB \Rightarrow M_\varphi \geq 47°$

$B_3 \simeq 2{,}5 Hz \Rightarrow \omega_c = 3 \div 5\, B_3 = 4 B_3 \simeq 10 \frac{rad}{s}$

$F(s) = R(s) \cdot G(s) \cdot P(s)$

$G(s) = \frac{25}{s}$ 1^o TENTATIVO: $R(s) = 1$

$F(i\omega) = 50 \cdot \frac{1}{i\omega(1 + \frac{i\omega}{5})}$ $50 \to 34\,dB$

PUNTI DI ROTTURA

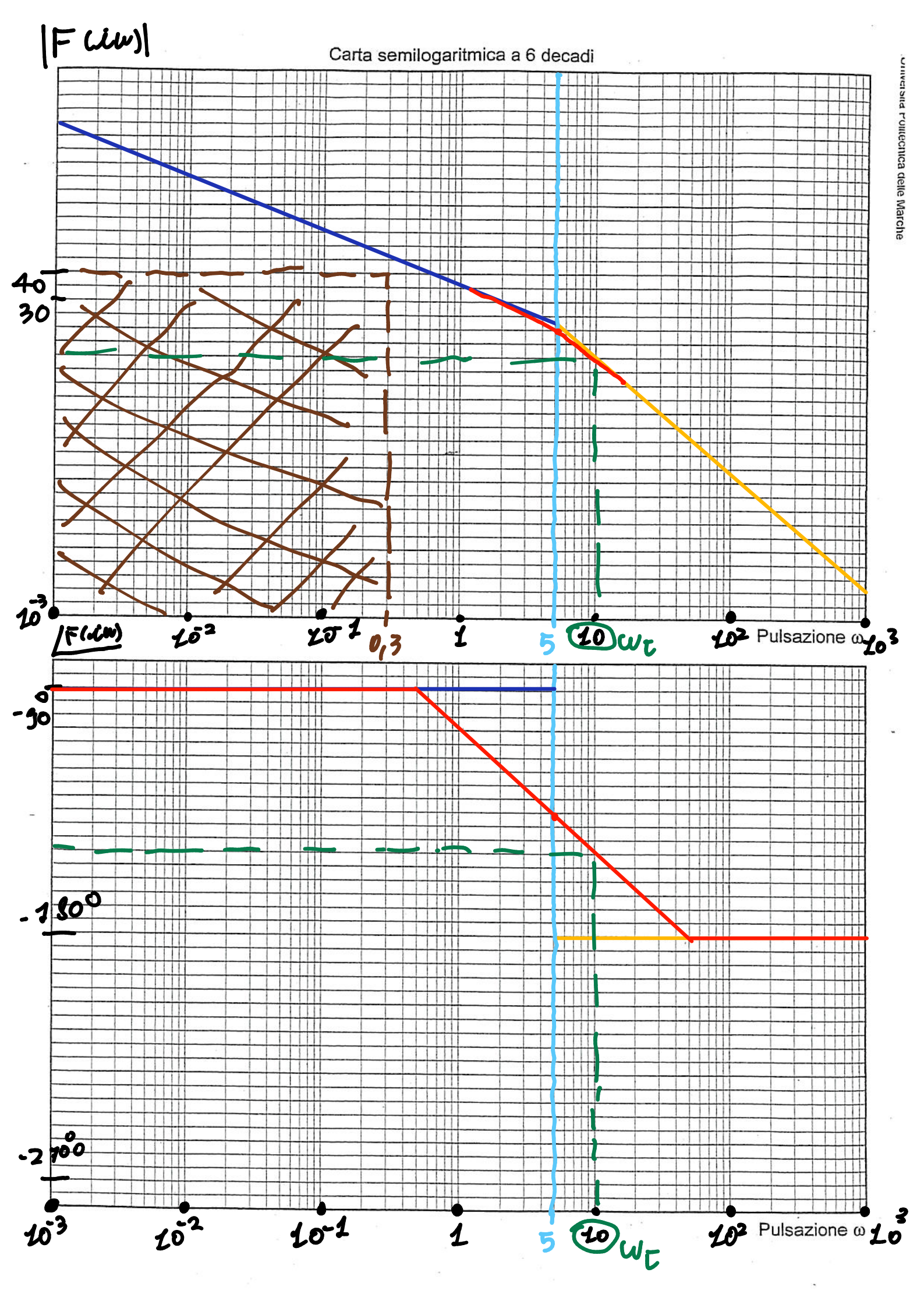
- $\omega = 0$ ● $-20dB/dec$ $-90°$ $-20dB/dec$ $-90°$
- $\omega = 5$ ● $-20dB/dec$ $-90°$ $-40dB/dec$ $-180°$

CORREZIONE MODULO

ω	$1 + \frac{i\omega}{5}$	TOT
5	$-3dB$	$-3dB$

CORREZIONE FASE

ω	$i\omega$	$1 + \frac{i\omega}{5}$	TOT
5	$-90°$	$-45°$	$-135°$


|F(jω)|
Carta semilogaritmica a 6 decadi
40
30
10^{-3}
10^{-2}
10^{-1}
0,3
1
5
10
ω_T
10^{2}
Pulsazione ω
10^{3}
-90°
-180°
-270°
10^{-3}
10^{-2}
10^{-1}
1
5
10
ω_T
10^{2}
Pulsazione ω
10^{3}

$|F(i\omega_c)| = 10\,dB$

$\angle F(i\omega_c) = -150° \Longrightarrow M_\varphi = 30°$

OBIETTIVO:

$|F(i\omega_c)| = 0$ ✗

$\Longrightarrow$ DIMINUIRE MODULO, AUMENTARE FASE

$M_\varphi \geq 47°$ ✗

$\Longrightarrow$ FUNZIONE ATTENUATRICE $R_i(s) = \dfrac{1 + \frac{s}{m_i \omega_i}}{1 + \frac{s}{m_i}}$

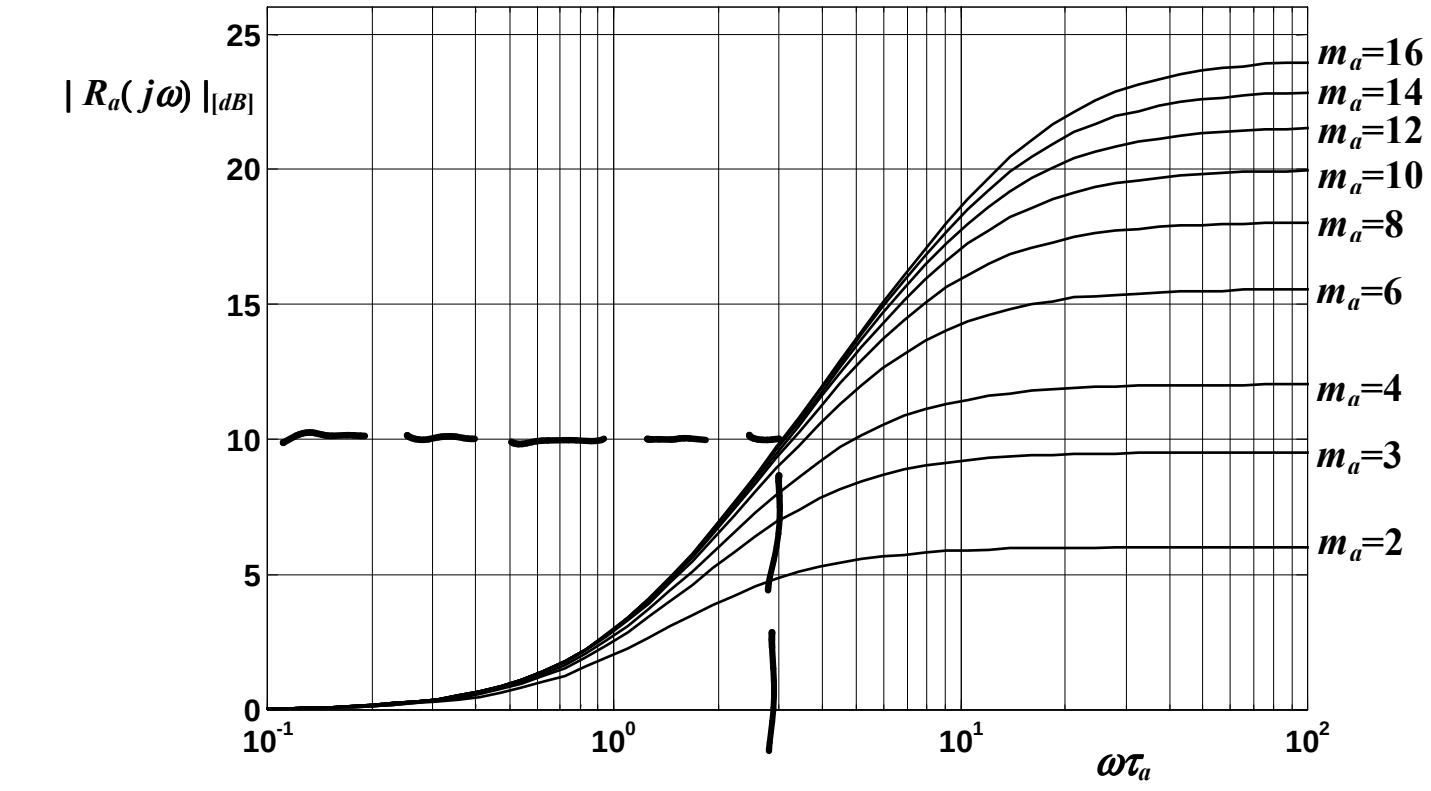
$\omega_c \tau_i = 3 \qquad m_i = 6 \qquad \omega_i = \dfrac{\omega_c}{3} = 3{,}3 \dfrac{rad}{s}$

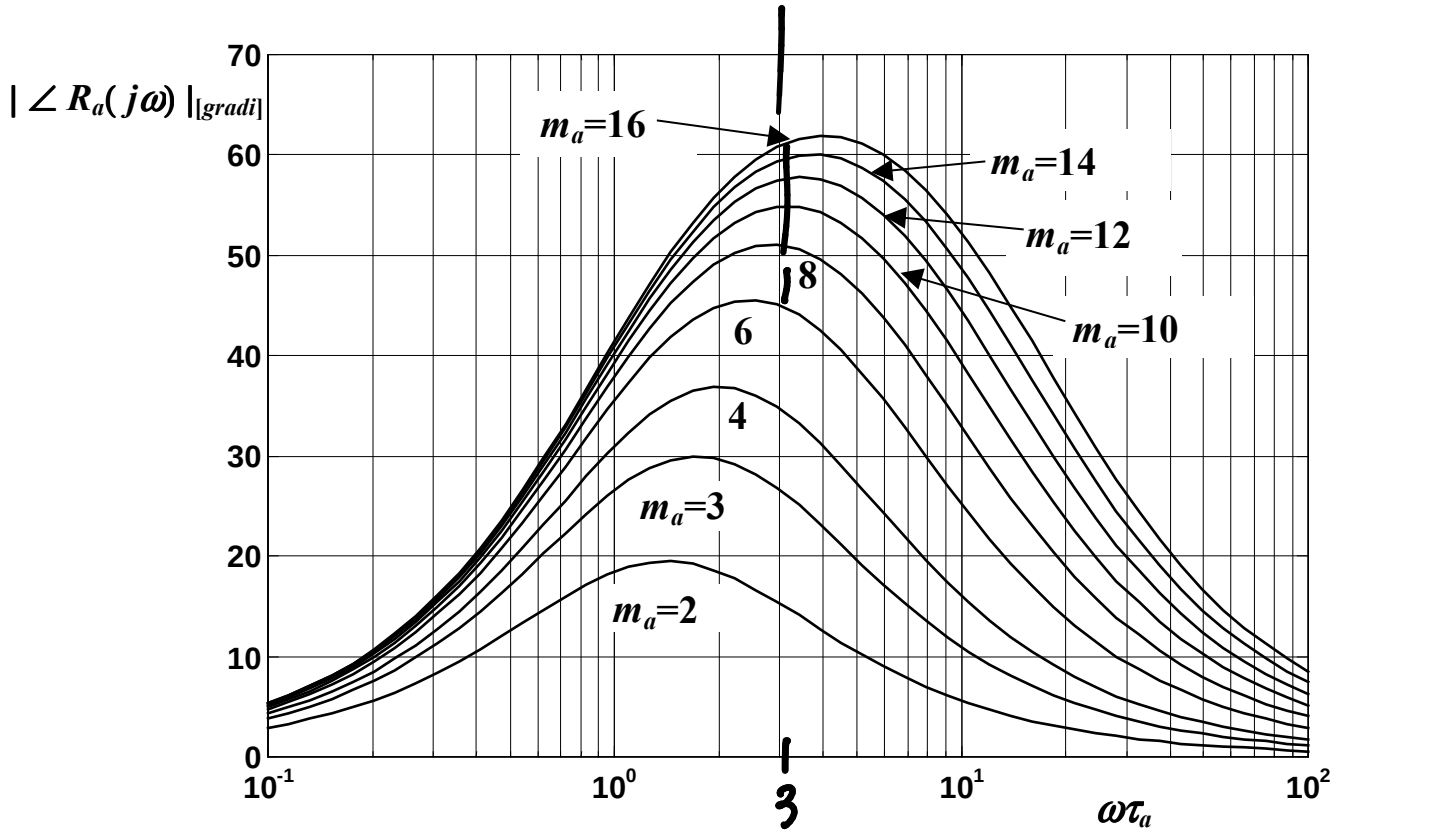
$\Longrightarrow R(s) = \dfrac{1 + \frac{s}{20}}{1 + \frac{s}{3{,}3}}$

$F(i\omega) = 50 \cdot \dfrac{1}{i\omega\left(1 + \frac{i\omega}{5}\right)} \cdot \dfrac{1 + \frac{i\omega}{20}}{1 + \frac{i\omega}{3{,}3}}$

PUNTI DI ROTTURA

• $\omega = 0$	●	$-20\,dB/dec$	$-90°$	$-20\,dB/dec$ $\quad -90°$
• $\omega = 3{,}3$	●	$-20\,dB/dec$	$-90°$	$-40\,dB/dec$ $\quad -180°$
• $\omega = 5$	●	$-20\,dB/dec$	$-90°$	$-60\,dB/dec$ $\quad -270°$
• $\omega = 20$	●	$+20\,dB/dec$	$+90°$	$-40\,dB/dec$ $\quad -180°$


| $R_a(j\omega)$ |$_{[dB]}$
25
20
15
10
5
0
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
m_a=16
m_a=14
m_a=12
m_a=10
m_a=8
m_a=6
m_a=4
m_a=3
m_a=2


| $\angle R_a(j\omega)$ |$_{[gradi]}$
70
60
50
40
30
20
10
0
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
m_a=16
m_a=14
m_a=12
m_a=10
8
6
4
m_a=3
m_a=2
3

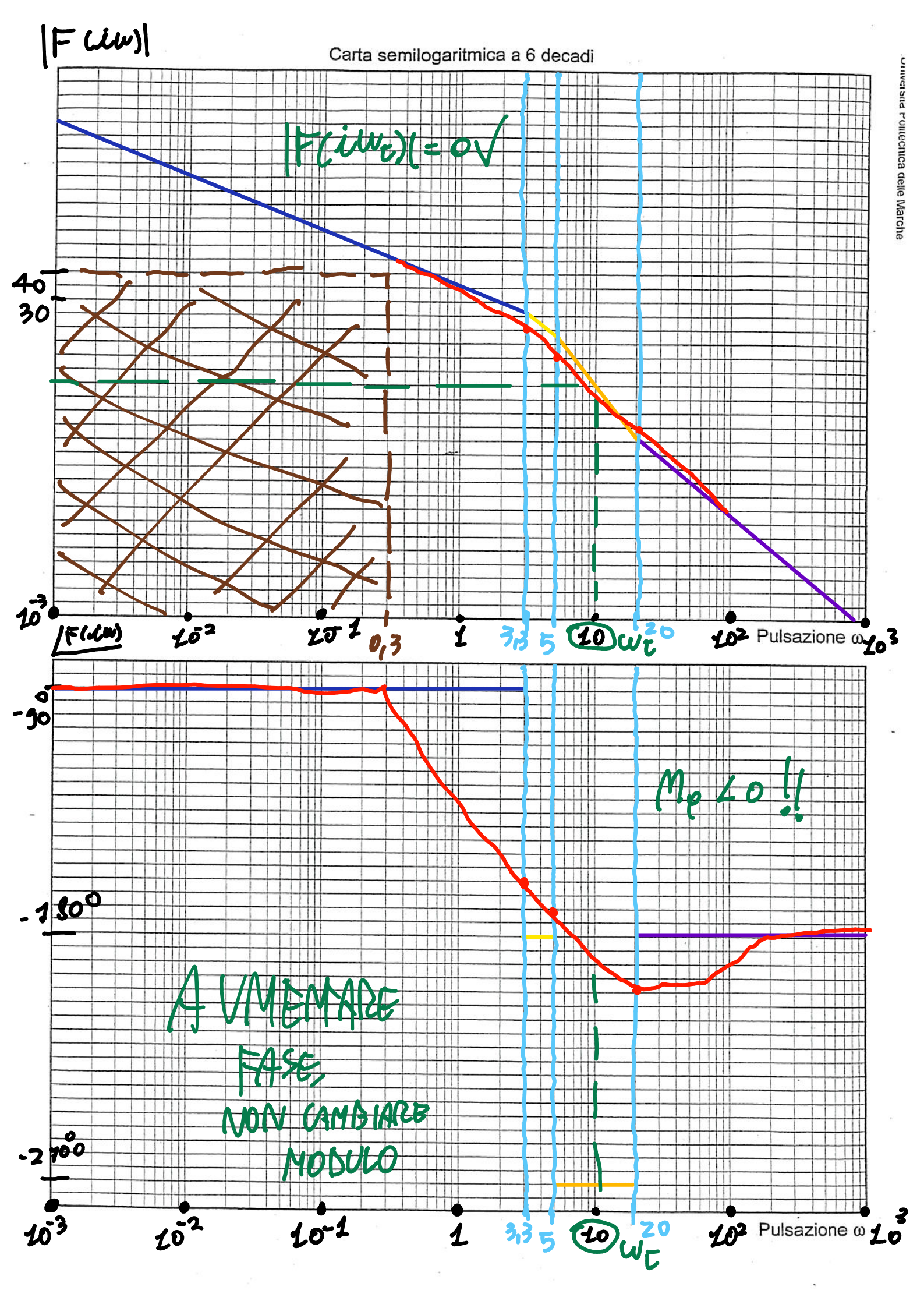
CORREZIONE MODULO

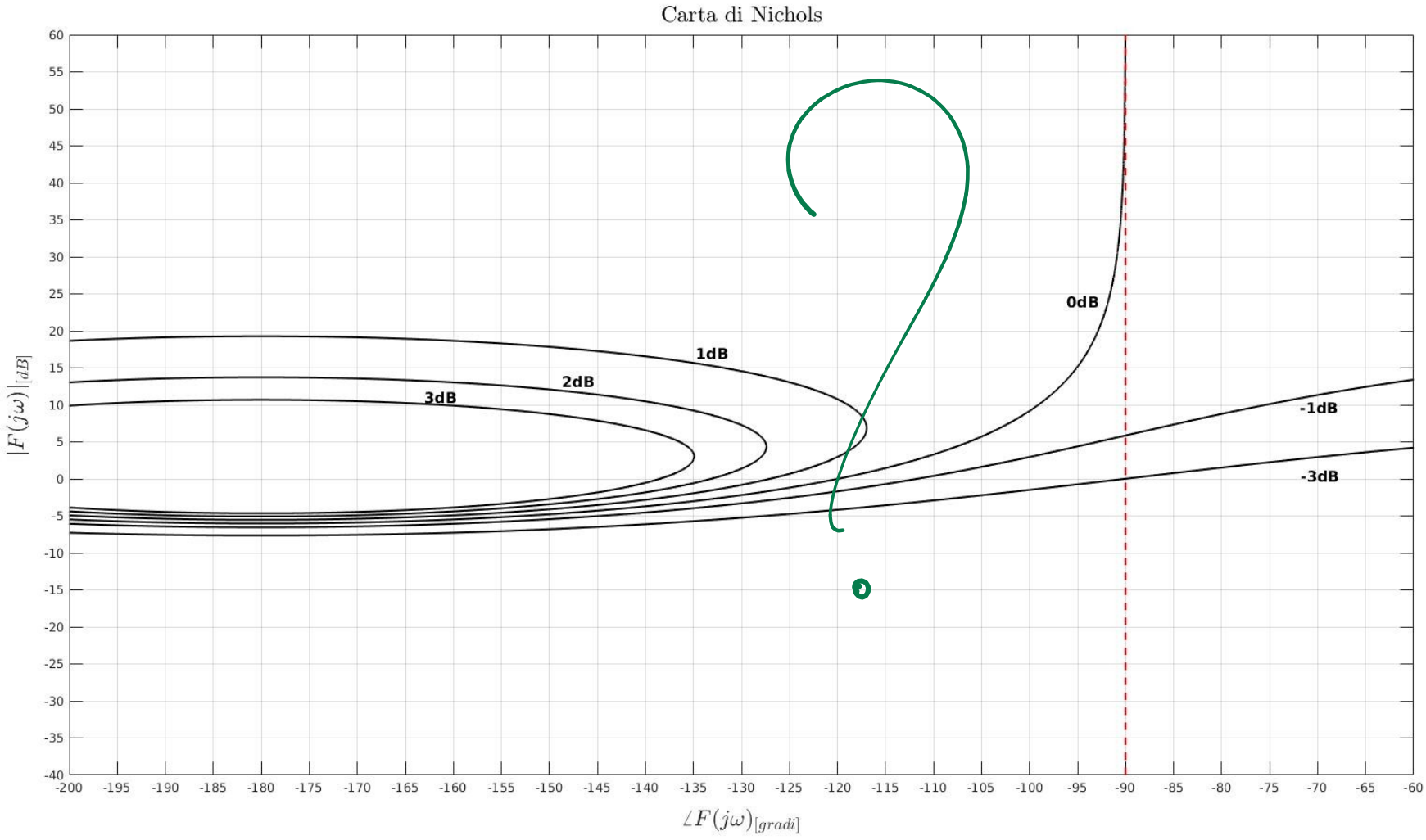
ω	$1+\frac{i\omega}{3{,}3}$	$1+\frac{i\omega}{5}$	$1+\frac{i\omega}{20}$	TOT
3,3	-3dB	-2,5dB	0	-5,5dB
5	-2,5dB	-3dB	0	-5,5dB
20	0	0	+3dB	+3dB

CORREZIONE FASE

ω	$i\omega$	$1+\frac{i\omega}{3{,}3}$	$1+\frac{i\omega}{5}$	$1+\frac{i\omega}{20}$	TOT
3,3	-90°	-45°	-35°	+10°	-160°
5	-90°	-50°	-45°	+15°	-170°
20	-90°	-80°	-75°	+45°	-200°

$$F(i\omega) = 50 \cdot \frac{1}{i\omega\left(1+\frac{i\omega}{5}\right)} \cdot \frac{1+\frac{s}{20}}{1+\frac{s}{3{,}3}}$$


|F(iω)|
Carta semilogaritmica a 6 decadi
|F(iω_t)| = 0√
40
30
10^-3
10^-2
10^-1
0,3
1
3,3
5
10
ω_t
20
10^2
Pulsazione ω
10^3
-30°
-180°
-270°
M_φ < 0 !!
A VIMEMARE FASE, NON CAMBIARE MODULO


Carta di Nichols
$|F(j\omega)|_{[dB]}$
$\angle F(j\omega)_{[gradi]}$
0dB
1dB
2dB
3dB
-1dB
-3dB

$$V(t) = (2t-3)\delta_{-1}(t) = 2(t)\delta_{-1}(t) + (-3)\delta_{-1}(t)$$

$$= 2V_1(t) - 3V_2(t)$$

- $V_1(t)$

$$\tilde{e}_{V_1}(t) = K_d V_1(t) - \tilde{y}_{V_1}(t)$$

$$\tilde{e}_{V_1}(t) = \frac{1}{K_d \cdot K_p} = \frac{1}{K_F} = \frac{1}{50}$$

$$\tilde{y}_{V_1}(t) = K_d V_1(t) - \tilde{e}_{V_1}(t) = \left(t - \frac{1}{50}\right)\delta_{-1}(t)$$

- $V_2(t)$

GRADO DI $V_2(t)$ < TIPO DI $F(s) \Rightarrow \tilde{y}_{V_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 2\left(t + \frac{1}{50}\right)\delta_{-1}(t) - 3\delta_{-1}(t)$$