LET X BE A SET. PROVE THAT THE FOLLOWING FORMULAS HOLD FOR BINARY RELATIONS ON X

a) $R \subseteq U, S \subseteq V \Rightarrow RS \subseteq UV$

$\forall (x,y) \in X^2$

$xRSy \Rightarrow \exists z. (xRz \land zSy)$ // DEFINITION OF PRODUCT RELATION

$\Rightarrow \exists z. (xUz \land zVy)$ // $R \subseteq U, S \subseteq V$

$\Rightarrow xUVy \Rightarrow RS \subseteq UV$

b) $(R \cap S)(U \cap V) \subseteq RU \cap RV \cap SU \cap SV$

FROM a, $\underbrace{(R\cap S)}_{\subseteq R}\underbrace{(U \cap V)}_{\subseteq U} \subseteq RU$

SIMILARLY, $(R\cap S)(U\cap V) \subseteq RV$

$(R\cap S)(U\cap V) \subseteq SU$

$(R\cap S)(U\cap V) \subseteq SV$

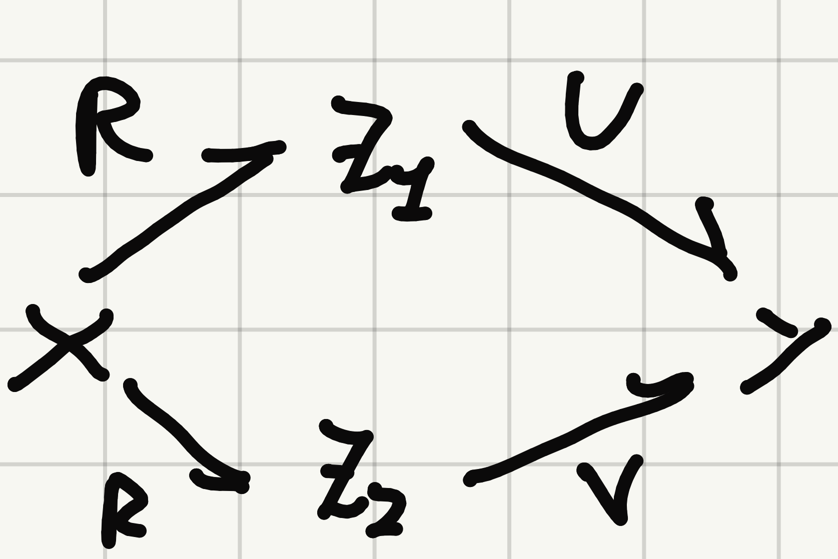
HENCE, $(R\cap S)(U\cap V) \subseteq RU \cap RV \cap SU \cap SV$

PROPER INCLUSION? $(R\cap S)(U\cap V) \subset RU \cap RV \cap SU \cap SV$

PUT $R = S$

$\rightarrow R(U\cap V) \subset RU \cap RV$?

$xR(U\cap V)y \Rightarrow \exists z. (xRz \land z(U\cap V)y)$



$\Rightarrow U \cap V = \emptyset \Rightarrow R(U\cap V) = \emptyset$

BUT $RU = \{(x,y)\} = RV$

c) $(R \cup S)(U \cup V) = RU \cup RV \cup SU \cup SV$

$(x,y) \in X^2$

$(R \cup S)(U \cup V) \Leftrightarrow \exists z. (x(R \cup S)z \wedge z(U \cup V)y)$

$\Leftrightarrow \exists z. ((xRz \cup xSz) \wedge (zUy \cup zVy))$

$\cup\, xzSVyz$

$\Leftrightarrow \exists z. (xzRUyz \cup xzRVyz \cup xzSUyz)$

$\Leftrightarrow xz(RU \cup RV \cup SU \cup SV)$

$\rightarrow$ $(R \cup S)(U \cup V) = RU \cup RV \cup SU \cup SV$

d) $(RS)^{op} = S^{op} R^{op}$

$(x,y) \in X^2$

$(RS)^{op} \Leftrightarrow \exists z. (xRz \wedge zSy)^{op}$

$\Leftrightarrow \exists z. (zR^{op}x \wedge yS^{op}z)$

$\Leftrightarrow \exists z. (yS^{op}z \wedge zR^{op}x)$

$\Leftrightarrow y(S^{op}R^{op})x$

$\rightarrow$ $(RS)^{op} = S^{op} R^{op}$

e) $(R\cap S)^{op} = R^{op} \cap S^{op}$

$(x,y) \in X^2$

$(R\cap S)^{op} \Leftrightarrow y(R\cap S)z$

$\Leftrightarrow yRz \wedge ySz$

$\Leftrightarrow R^{op} \cap S^{op}$

$\rightarrow (R\cap S)^{op} = S^{op} \cap R^{op} = R^{op} \cap S^{op}$

f) $(R\cup S)^{op} = R^{op} \cup S^{op}$

$(R\cup S)^{op} \Leftrightarrow y(R\cup S)x$

$\Leftrightarrow yRx \vee ySx$

$\Leftrightarrow xR^{op}y \vee xS^{op}y$

$\Leftrightarrow x(R^{op} \cup S^{op})y$

$\Leftrightarrow R^{op} \cup S^{op}$