

# PROPOSITIONAL LOGIC

FORMULAS

CONSTRUCT PARSING TREE AND DETERMINE THE SUBFORMULAS OF THE FOLLOWING

Q)  $X \vee y \rightarrow \neg X$

FORMATION SEQUENCE

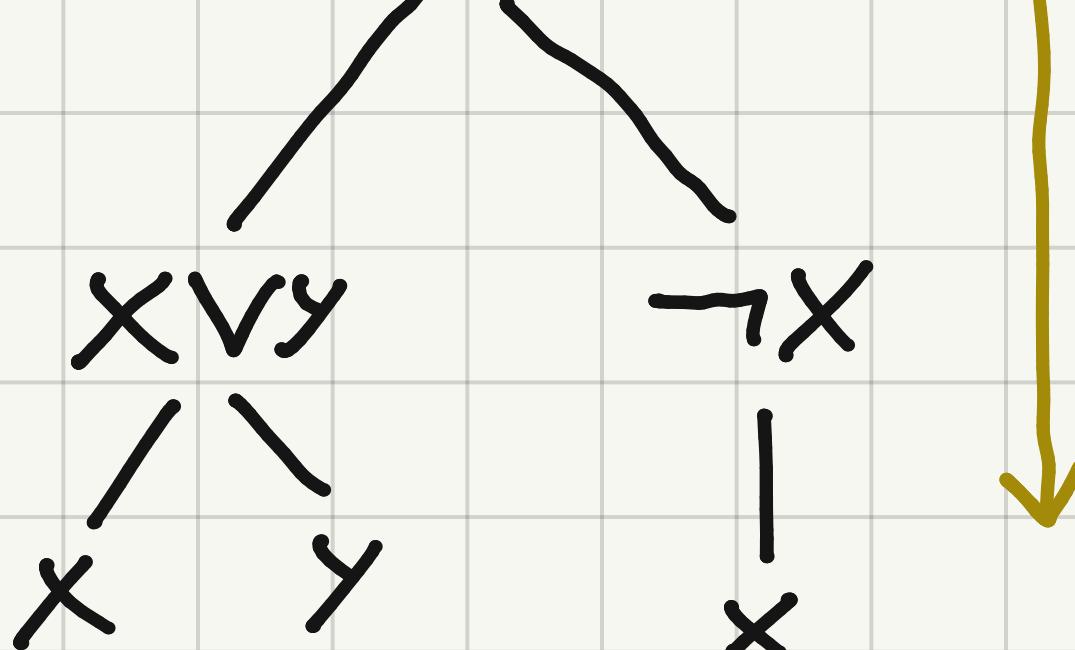
STEP	FORMULA	REASON
1	$X$	VARIABLE
2	$y$	VARIABLE
3	$\neg X$	1. NEGATION
4	$X \vee y$	1,2, DISJUNCTION
5	$X \vee y \rightarrow \neg X$	3,4, IMPLICATION

FORMATION TREE

(ADVICE: FROM

$X \vee y \rightarrow \neg X$

TOP TO BOTTOM)



SUBFORMULAS

$$\begin{aligned}
 S(\varphi) &= \{\varphi\} \cup \bigcup_{i=1}^n S(\varphi_i) = \{X \vee y \rightarrow \neg X\} \cup (S(X \vee y) \cup S(\neg X)) \\
 &= \{X \vee y \rightarrow \neg X\} \cup S(X \vee y) \cup S(\neg X) = \\
 &= \{X \vee y \rightarrow \neg X, X \vee y, \neg X\} \cup S(X) \cup S(Y) \cup S(X) \\
 &= \{X \vee y \rightarrow \neg X, X \vee y, \neg X, X, Y\}
 \end{aligned}$$

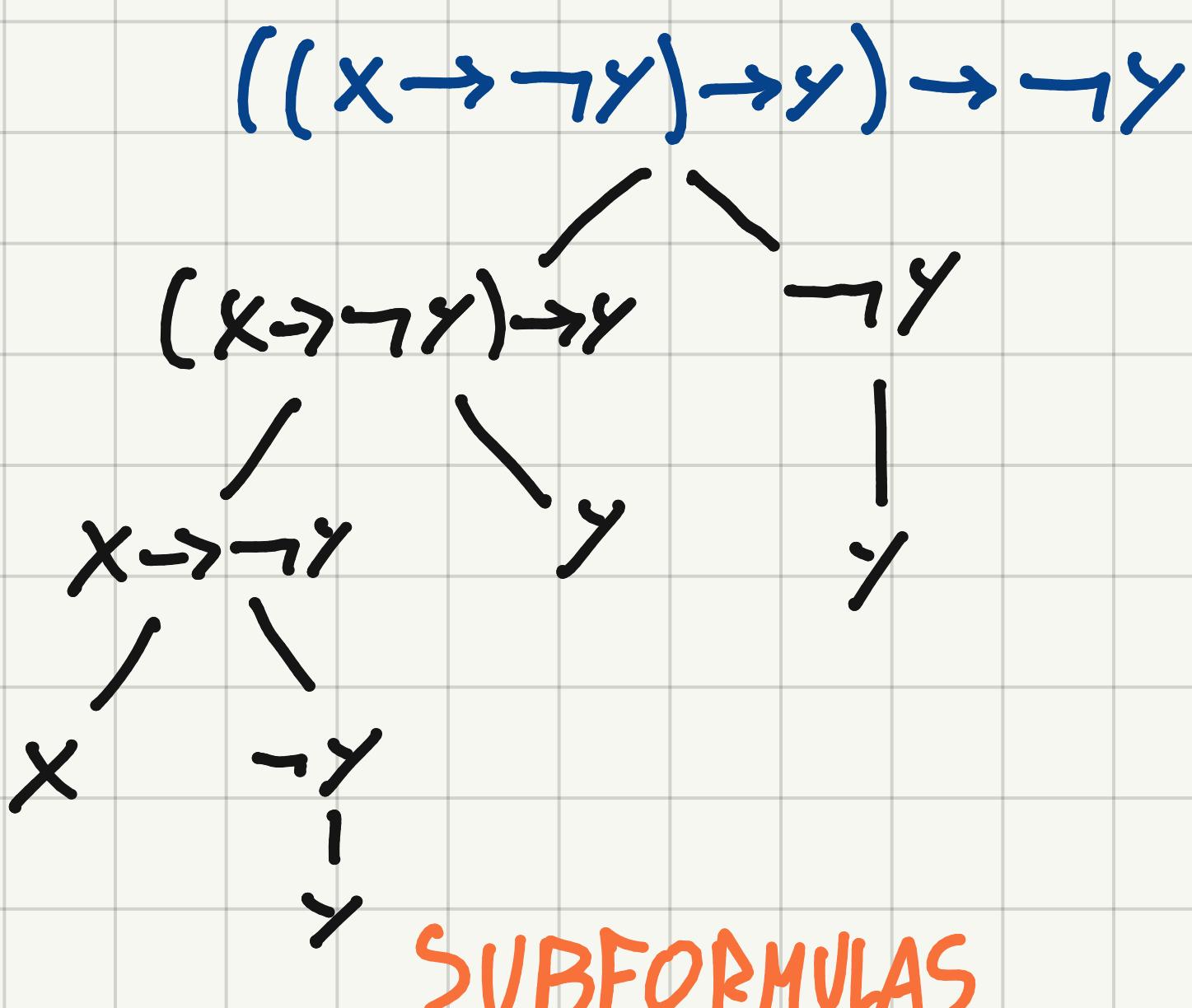
NOTICE: THE SUBFORMULAS CORRESPOND WITH THE STEPS ON THE F.S.

$$b) ((x \rightarrow \neg y) \rightarrow y) \rightarrow \neg y$$

### FORMATION SEQUENCE

STEP	FORMULA	REASON
1	$x$	VARIABLE
2	$y$	VARIABLE
3	$\neg y$	2, NEGATION
4	$x \rightarrow \neg y$	1, 3, IMPLICATION
5	$(x \rightarrow \neg y) \rightarrow y$	4, 2, IMPLICATION
6	$((x \rightarrow \neg y) \rightarrow y) \rightarrow \neg y$	5, 3, IMPLICATION

### FORMATION TREE



$$\begin{aligned}
 h(\varphi) &= \{\varphi\} \cup \bigcup_{i=1}^n h(\varphi_i) = \{(x \rightarrow \neg y) \rightarrow y\} \cup \{h((x \rightarrow \neg y) \rightarrow y)\} \\
 &= \{((x \rightarrow \neg y) \rightarrow y) \rightarrow \neg y\} \cup h((x \rightarrow \neg y) \rightarrow y) \cup h(\neg y) \\
 &= \{((x \rightarrow \neg y) \rightarrow y) \rightarrow \neg y, (x \rightarrow \neg y) \rightarrow y, \neg y\} \cup h(x \rightarrow \neg y) \cup h(y) \\
 &\quad \cup \{x, \neg y\} \\
 &= \{((x \rightarrow \neg y) \rightarrow y) \rightarrow \neg y, (x \rightarrow \neg y) \rightarrow y, \neg y, x \rightarrow \neg y\} \cup h(x) \cup h(y) \\
 &= \{((x \rightarrow \neg y) \rightarrow y) \rightarrow \neg y, (x \rightarrow \neg y) \rightarrow y, \neg y, x \rightarrow \neg y, x\}
 \end{aligned}$$

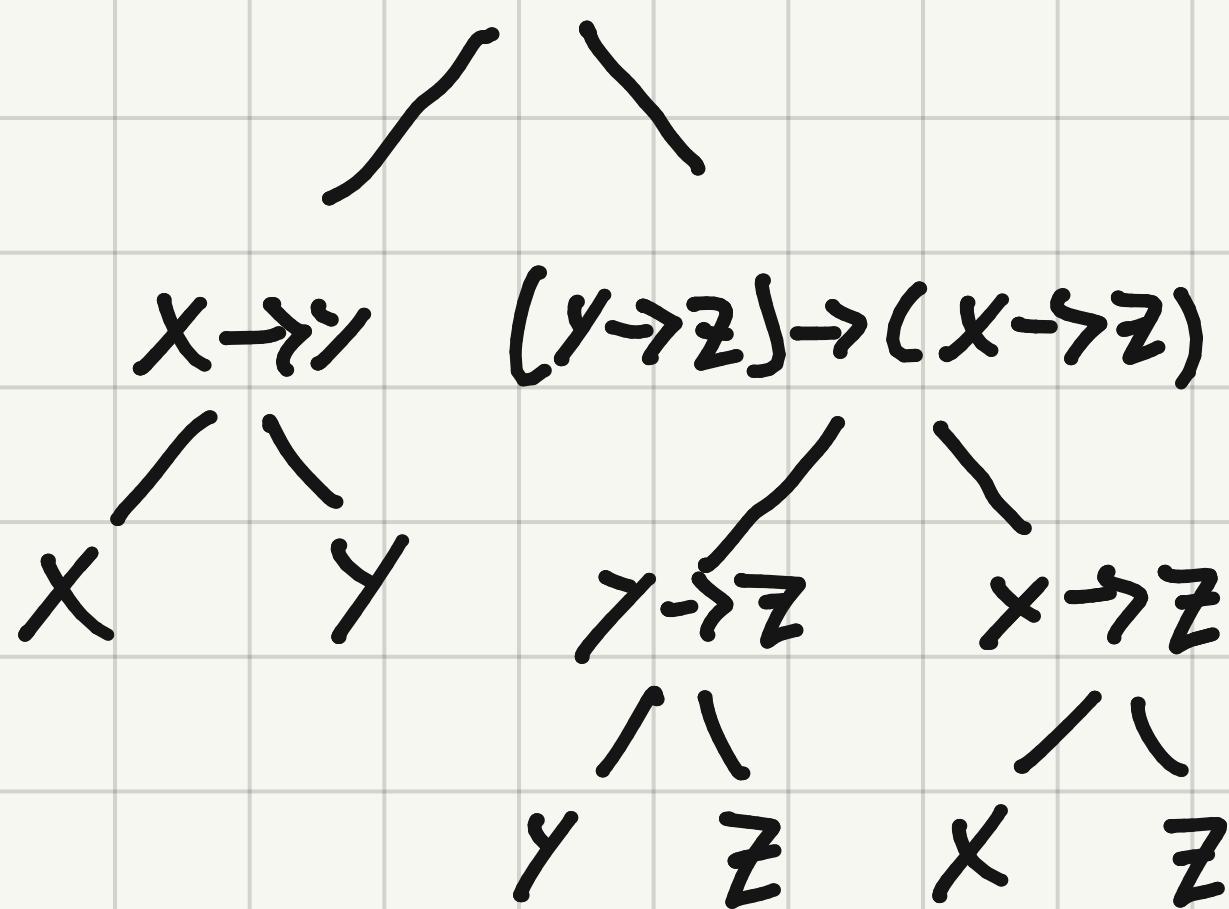
$$C) (X \rightarrow Y) \rightarrow ((Y \rightarrow Z) \rightarrow (X \rightarrow Z))$$

### FORMATION SEQUENCE

STEP	FORMULA	REASON
1	X	VARIABLE
2	Y	VARIABLE
3	Z	VARIABLE
4	$X \rightarrow Y$	1, 2, IMPLICATION
5	$Y \rightarrow Z$	2, 3, IMPLICATION
6	$X \rightarrow Z$	1, 3, IMPLICATION
7	$(Y \rightarrow Z) \rightarrow (X \rightarrow Z)$	5, 6, IMPLICATION
8	$(X \rightarrow Y) \rightarrow ((Y \rightarrow Z) \rightarrow (X \rightarrow Z))$	4, 7, IMPLICATION

### FORMATION TREE

$$(X \rightarrow Y) \rightarrow ((Y \rightarrow Z) \rightarrow (X \rightarrow Z))$$



### SUBFORMULAS

$$\Delta(\varphi) = \{\varphi\} \cup \bigcup_{i=1}^n \Delta(\varphi_i) = \{\varphi\} \cup \Delta(X \rightarrow Y) \cup \Delta((Y \rightarrow Z) \rightarrow (X \rightarrow Z)) =$$

$$= \{\varphi, X \rightarrow Y, (Y \rightarrow Z) \rightarrow (X \rightarrow Z)\} \cup \Delta(x) \cup \Delta(y) \cup \Delta(z) \cup \Delta(x \rightarrow z) =$$

$$= \{\varphi, X \rightarrow Y, (Y \rightarrow Z) \rightarrow (X \rightarrow Z), X, Y, Y \rightarrow Z, X \rightarrow Z\} \cup \Delta(y) \cup \Delta(z) \cup \Delta(x) \cup \Delta(z) =$$

$$= \{y \rightarrow z, (y \rightarrow z) \rightarrow (y \rightarrow z), X \rightarrow Y, (Y \rightarrow Z) \rightarrow (X \rightarrow Z), X, Y, Y \rightarrow Z, X \rightarrow Z, Z\}$$