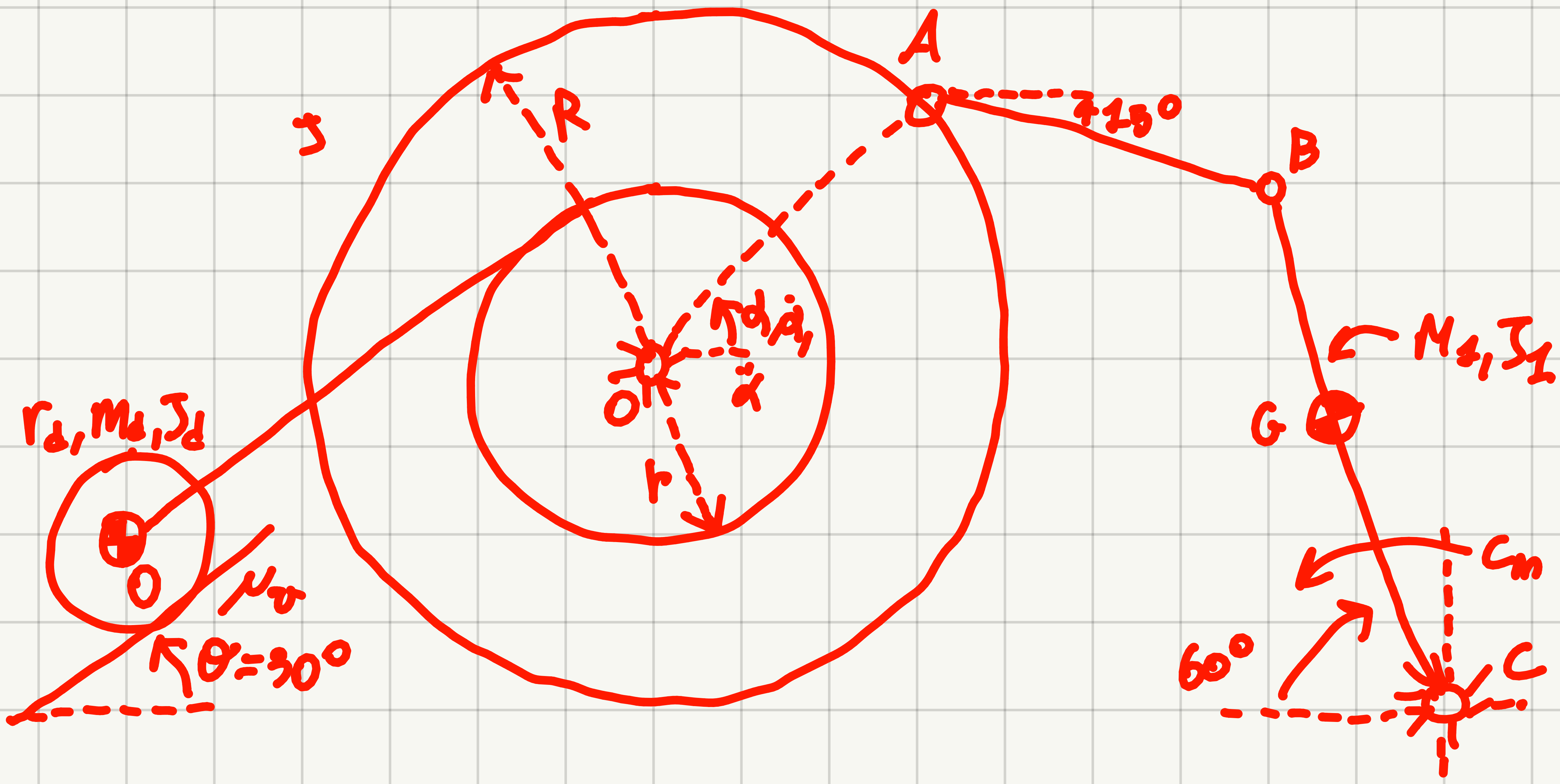
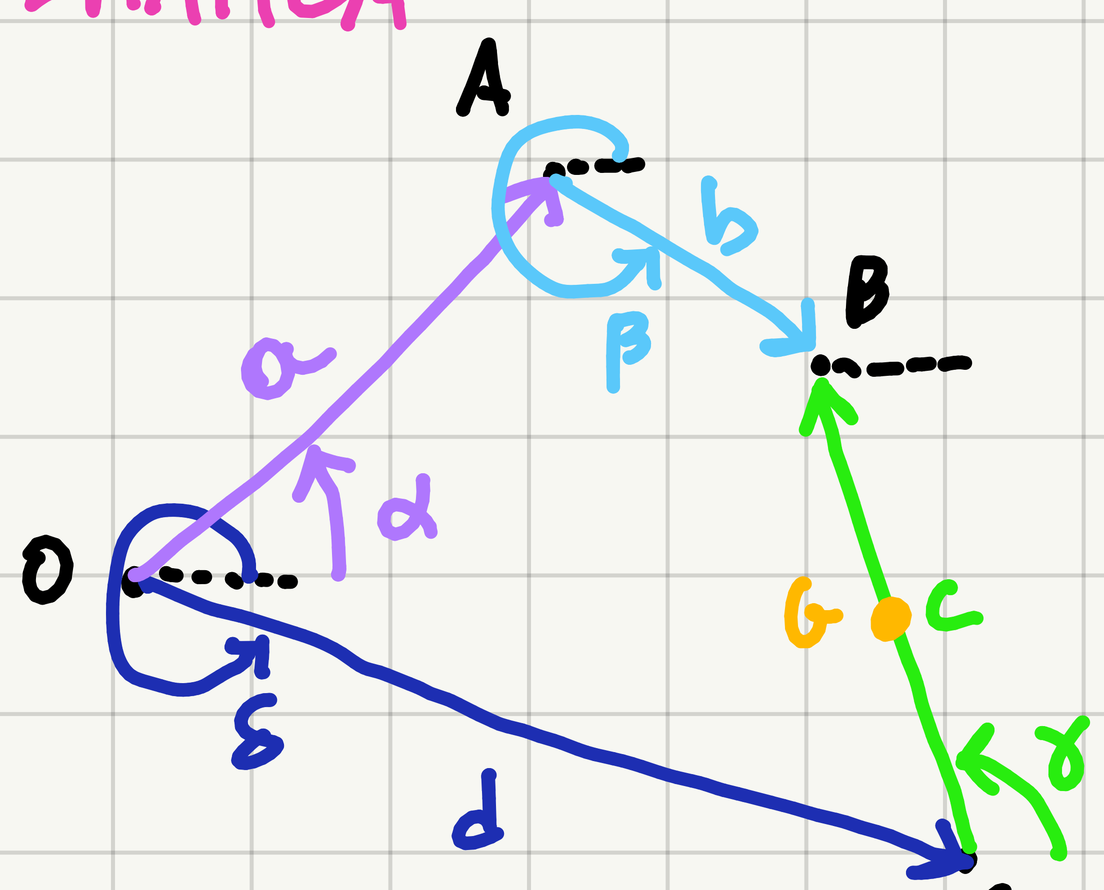


$AB = 1{,}5\,m$ $BC = 2\,m$ $R = 1\,m$ $r = 0{,}5\,m$ $r_d = 0{,}3\,m$ $\alpha = 45°$ $\dot{\alpha} = 1\ rad/s$

$\ddot{\alpha} = 1\ rad/s^2$ $J = 0{,}2\,Kg\,m^2$ $J_d = 0{,}8\,Kg\,m^2$ $J_1 = 0{,}2\,Kg\,m^2$ $M_d = 5\,Kg$

$M_1 = 2\,Kg$ $\mu_v = 0{,}05$ a) $\vec{v}_G$? $\vec{a}_G$? b) C_m?

CINEMATICA



$a = 1\,m$ $\alpha = 45°$
$\dot{\alpha} = 1\ rad/s$ $\ddot{\alpha} = 1\ rad/s^2$

$b = 1{,}5\,m$ $\beta = 360° - 15° = 345°$
$\dot{\beta}, \ddot{\beta} \neq 0$

$c = 2\,m$ $\gamma = 180° - 60° = 120°$
$\dot{\gamma}, \ddot{\gamma} \neq 0$

d FISSO δ FISSO

$$\begin{cases} a\cos\alpha + b\cos\beta = c\cos\gamma + d\cos\delta \\ a\sin\alpha + b\sin\beta = c\sin\gamma + d\sin\delta \end{cases}$$

! SI NOTI CHE ESSENDO C UNA CERNIERA, $\vec{d}$ COSTANTE

OBIETTIVI: $\dot{\gamma}, \ddot{\gamma}$

$$\vec{v}_G = \cancel{\vec{v}_C} + \vec{\dot{\gamma}} \times (G-C) \qquad \vec{a}_G = \cancel{\vec{a}_C} + \vec{\ddot{\gamma}} \times (G-C) - \dot{\gamma}^2 (G-C)$$

! SEGUENDO LA CHIUSURA DABG, PUR CALCOLANDO d E δ AVREMMO OTTENUTO, DERIVANDO, UN SISTEMA CON 4 INCOGNITE ($\dot{\beta}, \dot{\gamma}, \dot{d}, \dot{\delta}$)

$(G-C) = \overline{GC}(\cos(\gamma)\hat{\imath} + \sin(\gamma)\hat{\jmath})$ $\qquad \overline{GC} = \overline{BC}/2 = 1m$

$a = 1m \quad \alpha = 45^\circ$

$\dot{\alpha} = 1 rad/s \quad \ddot{\alpha} = 1 rad/s^2$

$b = 1{,}5m \quad \beta = 360^\circ - 15^\circ = 345$

$\dot{\beta}, \ddot{\beta} \neq 0$

$c = 2m \quad \gamma = 180^\circ - 60^\circ = 120^\circ$

$$\begin{cases} -a\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = -c\dot{\gamma}\sin\gamma \\ a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta = c\dot{\gamma}\cos\gamma \end{cases}$$

$$\begin{cases} c\dot{\gamma}\sin\gamma - b\dot{\beta}\sin\beta = a\dot{\alpha}\sin\alpha \\ c\dot{\gamma}\cos\gamma - b\dot{\beta}\cos\beta = a\dot{\alpha}\cos\alpha \end{cases}$$

$$\begin{vmatrix} c\sin\gamma & -b\sin\beta \\ c\cos\gamma & -b\cos\beta \end{vmatrix} \cdot \begin{vmatrix} \dot{\gamma} \\ \dot{\beta} \end{vmatrix} = \begin{vmatrix} a\dot{\alpha}\sin\alpha \\ a\dot{\alpha}\cos\alpha \end{vmatrix} \qquad \det A = -2{,}12$$

$$\dot{\gamma} = \frac{\det\begin{vmatrix} a\dot{\alpha}\sin\alpha & -b\sin\beta \\ a\dot{\alpha}\cos\alpha & -b\cos\beta \end{vmatrix}}{-2{,}12} = 0{,}61\, rad/s$$

$$\dot{\beta} = \frac{\det\begin{vmatrix} c\sin\gamma & a\dot{\alpha}\sin\alpha \\ c\cos\gamma & a\dot{\alpha}\cos\alpha \end{vmatrix}}{-2{,}12} = -0{,}91\, rad/s$$

$$\hat{k}\times\hat{\imath} = \det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \hat{\jmath} \qquad \hat{k}\times\hat{\jmath} = \det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\hat{\imath}$$

$$\vec{v}_G = 0{,}61\hat{k}\times(\cos(\gamma)\hat{\imath} + \sin(\gamma)\hat{\jmath}) = (-0{,}53\hat{\imath} - 0{,}31\hat{\jmath})\, m/s$$

$|\vec{v}_G| = 0{,}614\, m/s$ (SERVIRÀ PER LA PARTE DI DINAMICA)

$$\begin{cases} c\ddot{\gamma}\sin\gamma + c\dot{\gamma}^2\cos\gamma - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta = a\ddot{\alpha}\sin\alpha + a\dot{\alpha}^2\cos\alpha \\ c\ddot{\gamma}\cos\gamma - c\dot{\gamma}^2\sin\gamma - b\ddot{\beta}\cos\beta + b\dot{\beta}^2\sin\beta = a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha \end{cases}$$

$$\begin{vmatrix} c\sin\gamma & -b\sin\beta \\ c\cos\gamma & -b\cos\beta \end{vmatrix} \cdot \begin{vmatrix} \ddot{\gamma} \\ \ddot{\beta} \end{vmatrix} = \begin{vmatrix} a\ddot{\alpha}\sin\alpha + a\dot{\alpha}^2\cos\alpha - c\dot{\gamma}^2\cos\gamma + b\dot{\beta}^2\cos\beta \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + c\dot{\gamma}^2\sin\gamma - b\dot{\beta}^2\sin\beta \end{vmatrix}$$

$$\ddot{\gamma} = \frac{\det\begin{vmatrix} a\ddot{\alpha}\sin\alpha + a\dot{\alpha}^2\cos\alpha - c\dot{\gamma}^2\cos\gamma + b\dot{\beta}^2\cos\beta & -b\sin\beta \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + c\dot{\gamma}^2\sin\gamma - b\dot{\beta}^2\sin\beta & -b\cos\beta \end{vmatrix}}{-2{,}12} = 2{,}22 \ rad/s^2$$

$$\ddot{\beta} = \frac{\det\begin{vmatrix} c\sin\gamma & a\ddot{\alpha}\sin\alpha + a\dot{\alpha}^2\cos\alpha - c\dot{\gamma}^2\cos\gamma + b\dot{\beta}^2\cos\beta \\ c\cos\gamma & a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + c\dot{\gamma}^2\sin\gamma - b\dot{\beta}^2\sin\beta \end{vmatrix}}{-2{,}12} = -2{,}19 \ rad/s^2$$

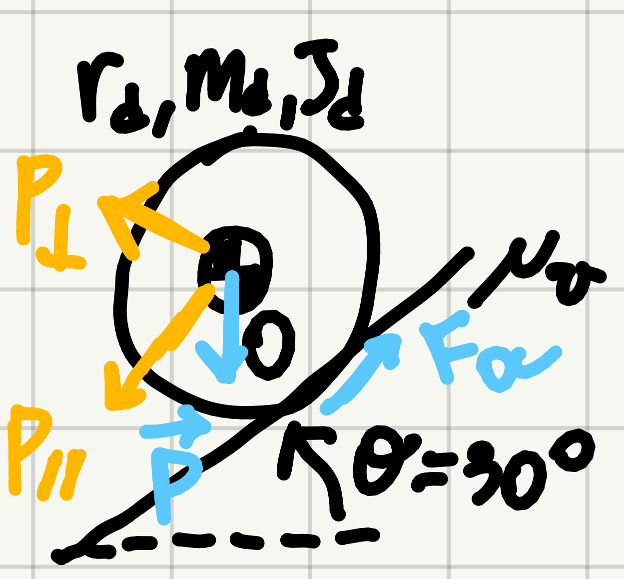
$$\vec{a}_G = 2{,}22\hat{k} \times (\cos(\gamma)\hat{i} + \sin(\gamma)\hat{j}) - 0{,}61^2(\cos(\gamma)\hat{i} + \sin(\gamma)\hat{j}) =$$

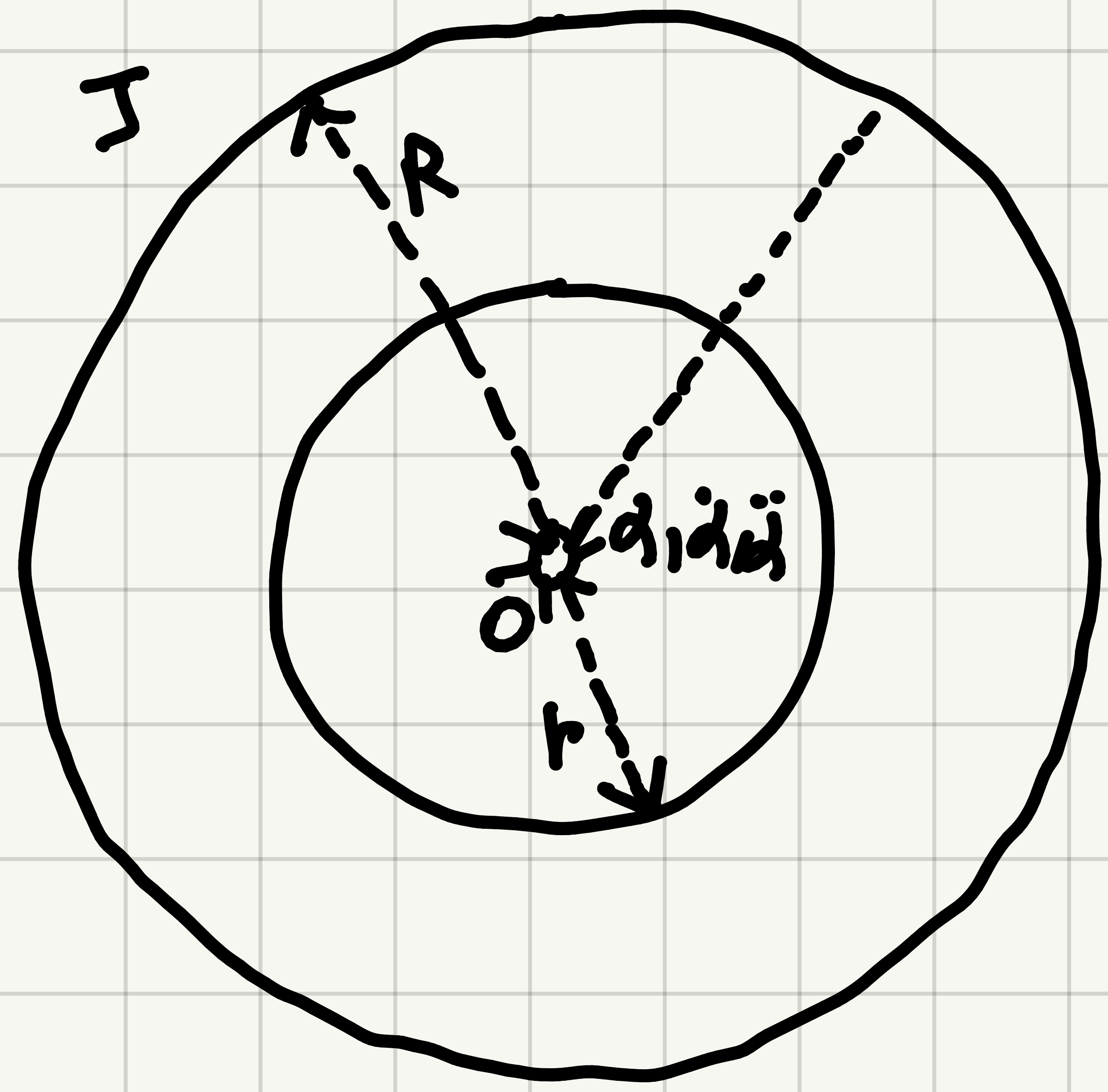
$$= (-1{,}92\hat{i} - 1{,}11\hat{j}) - (-0{,}19\hat{i} + 0{,}32\hat{j}) = (-1{,}73\hat{i} - 1{,}43\hat{j}) \ m/s^2$$

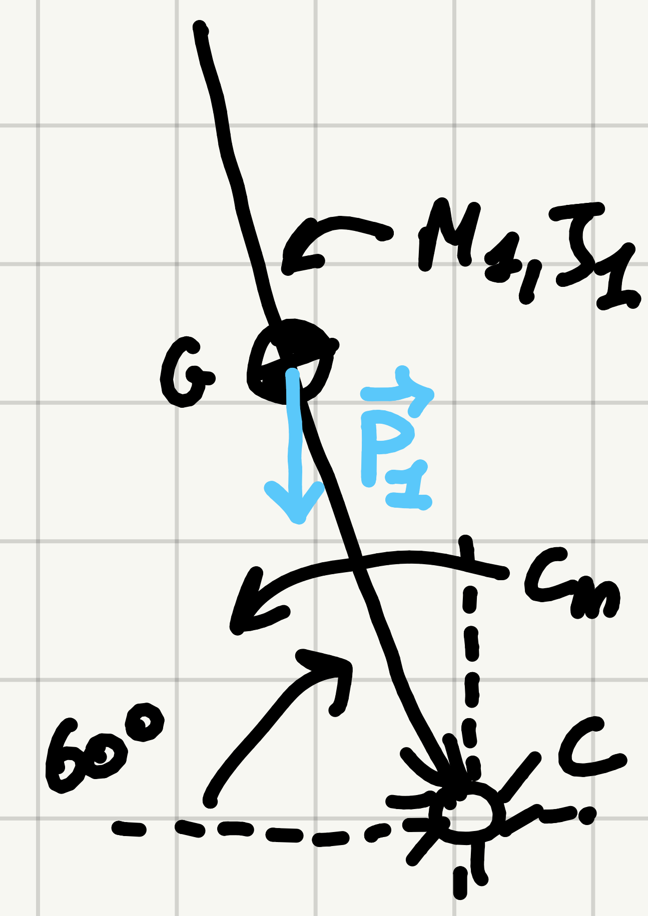
$|\vec{a}_G| = 2{,}245 \ m/s^2$ (SERVIRÀ PER LA PARTE DI DINAMICA)

DINAMICA

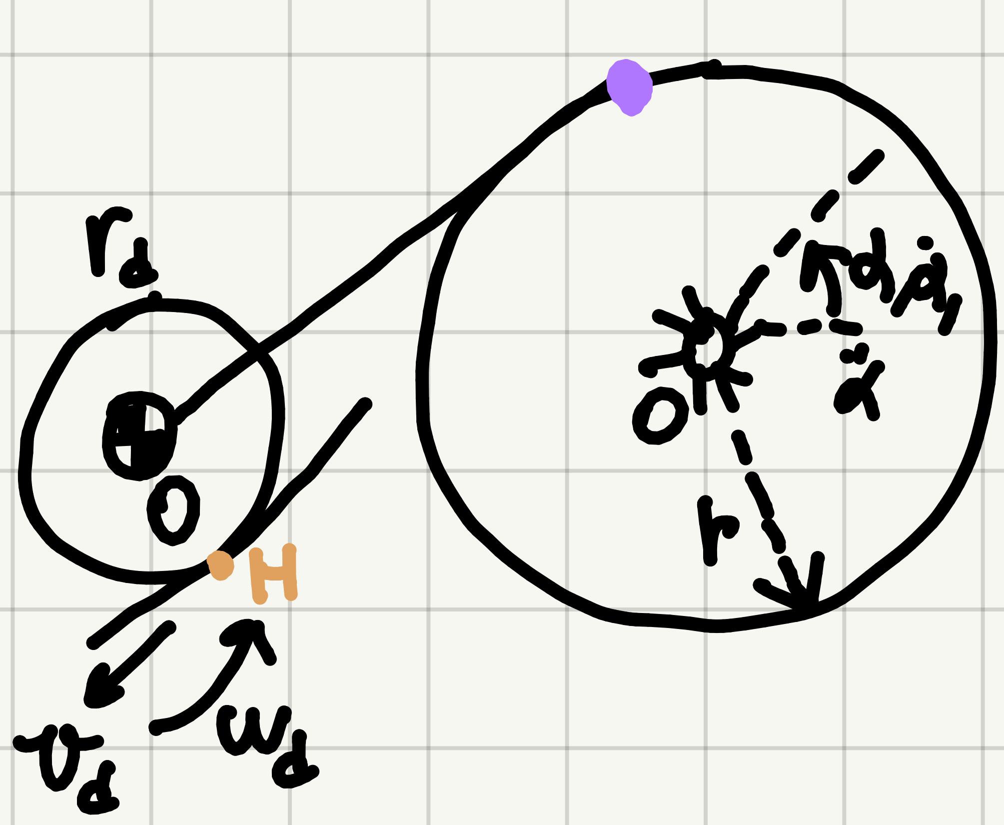
BILANCIO DI POTENZE: $\sum_{i=1}^{n} P_i = \frac{d}{dt}K$







$$K = \left(\frac{1}{2}m_d v_d^2 + \frac{1}{2}J_d\omega_d^2\right) + \left(\frac{1}{2}J\omega_{Disco}^2\right) + \left(\frac{1}{2}M_1 v_G^2 + \frac{1}{2}J_1\omega_G^2\right)$$



$\omega_d r_d = \dot{\alpha} r \Rightarrow \omega_d = \left(\frac{r}{r_d}\right)\dot{\alpha}$

$v_d = \cancel{v_H} + \omega_d r_d \Rightarrow v_d = r\dot{\alpha}$

$\omega_{Disco} = \dot{\alpha} \quad \omega_G = -\dot{\gamma}$

$$\frac{d}{dt}K = (M_d v_d a_d + J_d \omega_d \dot{\omega}_d) + (J\omega_{Disco}\dot{\omega}_{Disco}) + (M_1 v_G a_G + J_1 \omega_G \dot{\omega}_G)$$

$a_d = r\ddot{\alpha}$ $\quad \dot{\omega}_d = (\frac{r}{r_d})\ddot{\alpha}$ $\quad \dot{\omega}_{Disco} = \ddot{\alpha}$ $\quad \dot{\omega}_G = -\ddot{\gamma}$ $\quad \frac{d}{dt}K = 6{,}69\ W$

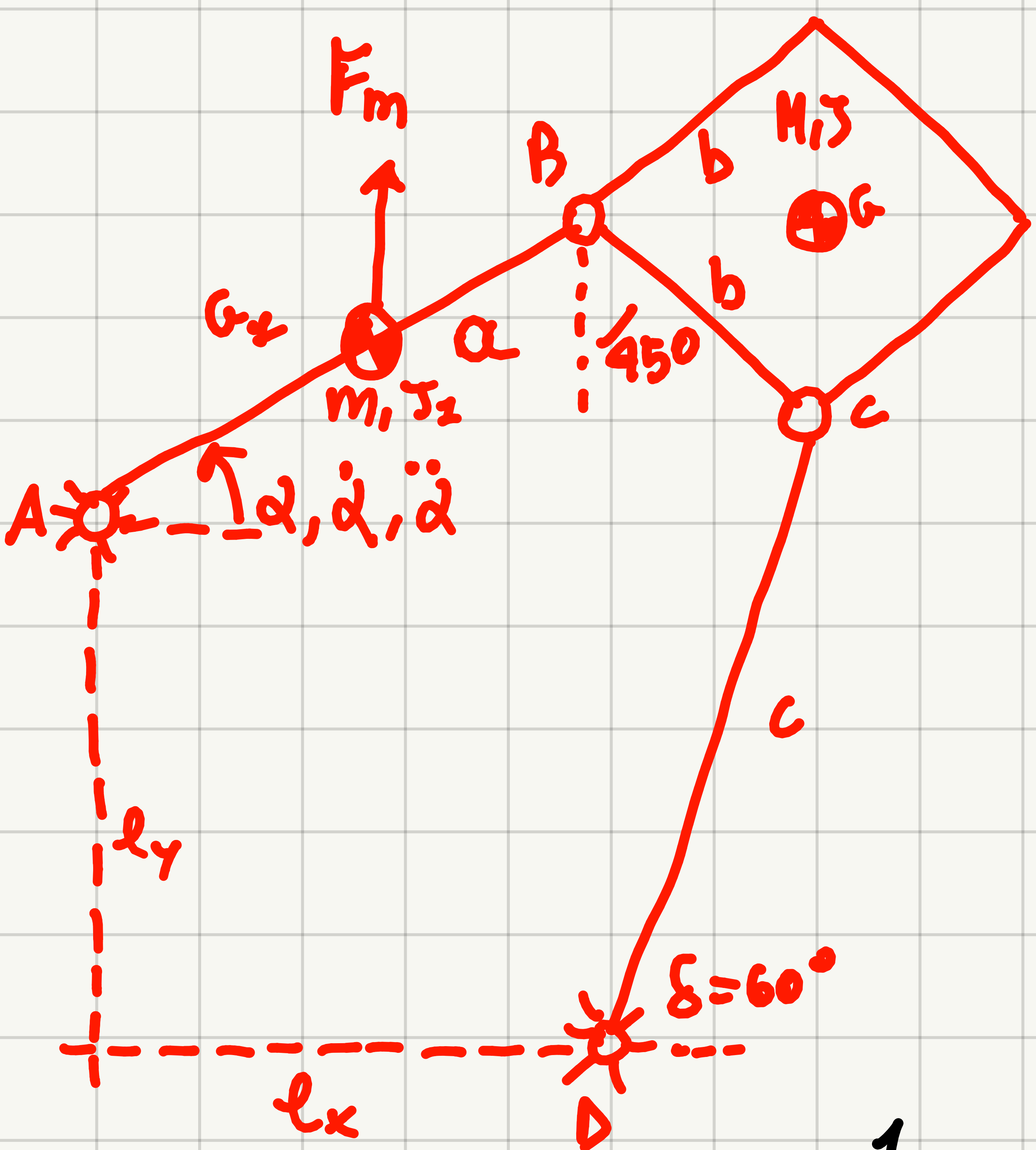
$$\sum P = (-P_{\parallel} + F_a)(-v_d) + (0) + (-P_1 v_G + C_m \dot{\gamma})$$

$$= (M_d g \dot{\alpha} r \sin(30) - \mathcal{N}_d N r_d \omega_d) + M_1 g v_G + C_m \dot{\gamma}$$

$$= M_d g \dot{\alpha} r \sin(30) - \mathcal{N}_d P_{\perp} r_d \omega_d + M_1 g v_G + C_m \dot{\gamma}$$

$$M_d g \dot{\alpha} r \sin(30) - \mathcal{N}_d M_d g r_d \omega_d \cos(30) - M_1 g v_G + C_m \dot{\gamma} = 4{,}08$$

$\Rightarrow C_m = -17{,}2N$



$\ell_x = 0{,}94\,m$ $\ell_y = 2{,}3\,m$ $a = 2\,m$

$b = 1\,m$ $c = 3\,m$ $J = 0{,}05\,kg\,m^2$

$m = 1\,kg$ $M = 2\,kg$ $\alpha = 30°$ $\dot\alpha = 1\,rad/s$

$\ddot\alpha = 1\,rad/s^2$ • $v_G, a_G, \omega_Q, \dot\omega_Q$?

• F_m? • H_A, V_A, H_D, V_D?

CINEMATICA

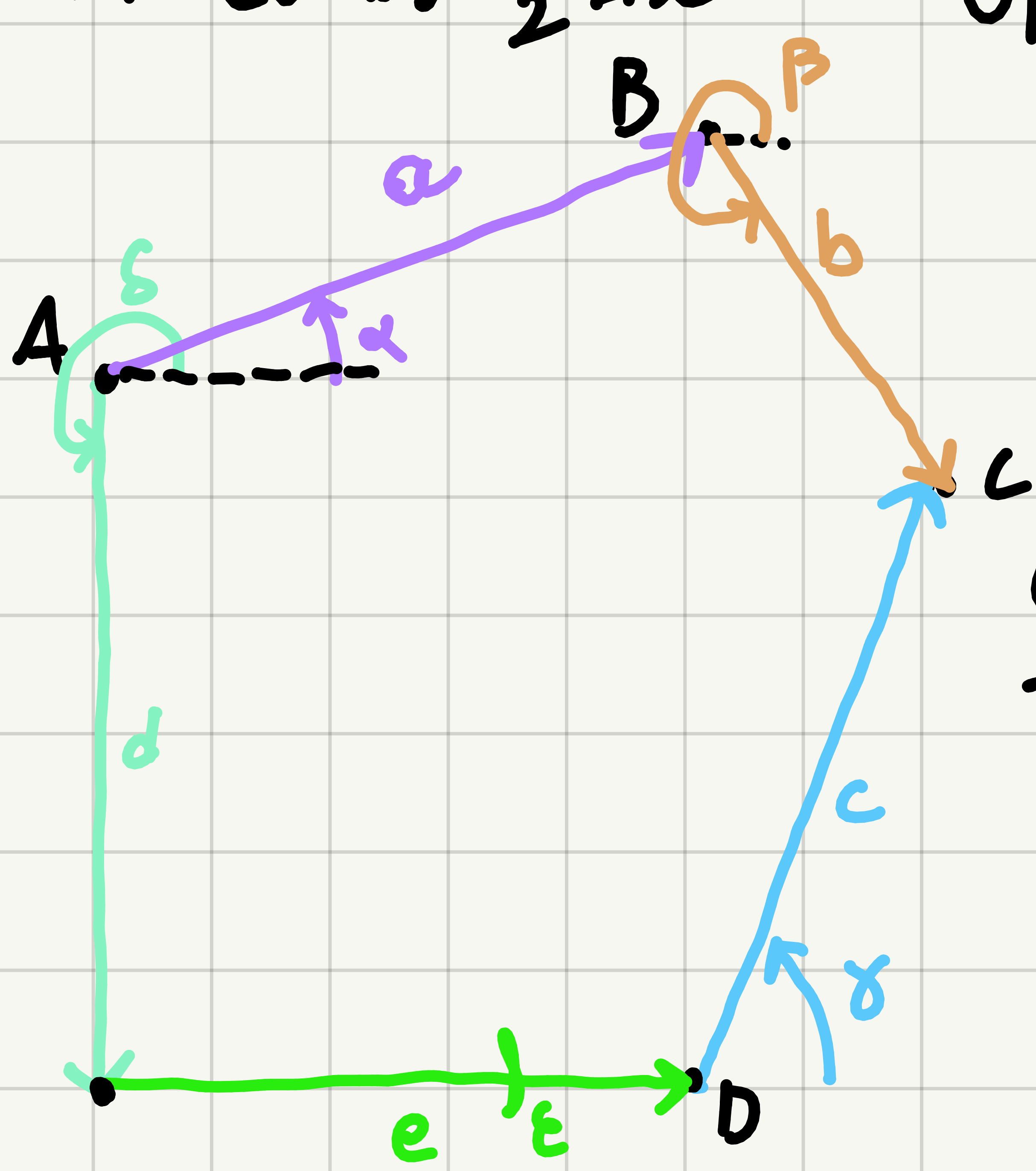
$$J_1 = \frac{1}{12} m a^2 = 0{,}67\,kg\,m^2$$

$$\vec v_G = \vec v_B + \dot\beta \times (G-B)$$

$d = L\sqrt{2}$

$$\Rightarrow (G-B) = \frac{\sqrt{2}}{2} L \hat i$$

$$\vec v_B = \cancel{\vec v_A} + \dot\alpha \hat k \times (a\cos\alpha\,\hat i + a\sin\alpha\,\hat j) =$$
$$= (-\hat i + \sqrt{3}\,\hat j)\,m/s \qquad |\vec v_B| = 2\,m/s$$



$$\vec a + \vec b = \vec c + \vec d + \vec e$$

$$\begin{cases} a\cos\alpha + b\cos\beta = c\cos\gamma + d\cos\delta + e\cos\varepsilon \\ a\sin\alpha + b\sin\beta = c\sin\gamma + d\sin\delta + e\sin\varepsilon \end{cases}$$

$a = 2m$ $\alpha = 30°$ $\dot\alpha = 1\,rad/s$
$\ddot\alpha = 1\,rad/s^2$

$$\begin{cases} -a\dot\alpha\sin\alpha - b\dot\beta\sin\beta = -c\dot\gamma\sin\gamma \\ a\dot\alpha\cos\alpha + b\dot\beta\cos\beta = c\dot\gamma\cos\gamma \end{cases}$$

$b = 1m$ $\beta = 225°$ NON FISSO

$c = 3m$ $\gamma = 60°$ NON FISSO

OBIETTIVI: $\dot\beta, \ddot\beta$

$$\begin{cases} -b\dot{\beta}\sin\beta + c\dot{\gamma}\sin\gamma = a\dot{\alpha}\sin\alpha \\ -b\dot{\beta}\cos\beta + c\dot{\gamma}\cos\gamma = a\dot{\alpha}\cos\alpha \end{cases}$$

$a = 2m$ $\quad \alpha = 30^\circ$ $\quad \dot{\alpha} = 1\, rad/s$

$\ddot{\alpha} = 1\, rad/s^2$

$b = 1m$ $\quad \beta = 225^\circ$ NON FISSO

$c = 3m$ $\quad \gamma = 60^\circ$ NON FISSO

$$\begin{vmatrix} -b\sin\beta & c\sin\gamma \\ -b\cos\beta & c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \dot{\beta} \\ \dot{\gamma} \end{vmatrix} = \begin{vmatrix} a\dot{\alpha}\sin\alpha \\ -a\dot{\alpha}\cos\alpha \end{vmatrix}$$

$$\begin{vmatrix} 0{,}7 & 2{,}6 \\ -0{,}7 & 1{,}5 \end{vmatrix} \cdot \begin{vmatrix} \dot{\beta} \\ \dot{\gamma} \end{vmatrix} = \begin{vmatrix} 1 \\ 1{,}73 \end{vmatrix} \Rightarrow \begin{cases} \dot{\beta} = -1{,}04\, rad/s \\ \dot{\gamma} = 0{,}66\, rad/s \end{cases}$$

$\Rightarrow \vec{\omega}_Q = (-1{,}04\,\hat{k})\, rad/s$

$\vec{v}_G = (-\hat{\imath} + \sqrt{3}\,\hat{\jmath}) - 1{,}04\,\hat{k} \times (\frac{\sqrt{2}}{2} \cdot 1m\,\hat{\imath}) = (-\hat{\imath} + \hat{\jmath})\, m/s$ $\quad |\vec{v}_G| = 1{,}41\, m/s$

$$\begin{cases} -b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta + c\ddot{\gamma}\sin\gamma + c\dot{\gamma}^2\cos\alpha = a\ddot{\alpha}\sin\alpha + a\dot{\alpha}^2\cos\alpha \\ -b\ddot{\beta}\cos\beta + b\dot{\beta}^2\sin\beta + c\ddot{\gamma}\cos\gamma - c\dot{\gamma}^2\sin\alpha = a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha \end{cases}$$

$$\begin{vmatrix} -b\sin\beta & c\sin\gamma \\ -b\cos\beta & c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \ddot{\beta} \\ \ddot{\gamma} \end{vmatrix} = \begin{vmatrix} a\ddot{\alpha}\sin\alpha + a\dot{\alpha}^2\cos\alpha + b\dot{\beta}^2\cos\beta - c\dot{\gamma}^2\cos\gamma \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha - b\dot{\beta}^2\sin\beta + c\dot{\gamma}^2\sin\gamma \end{vmatrix}$$

$$\begin{vmatrix} 0{,}7 & 2{,}6 \\ -0{,}7 & 1{,}5 \end{vmatrix} \cdot \begin{vmatrix} \ddot{\beta} \\ \ddot{\gamma} \end{vmatrix} = \begin{vmatrix} 2{,}82 \\ 2{,}629 \end{vmatrix} \Rightarrow \begin{cases} \ddot{\beta} = -0{,}91\, rad/s^2 \\ \ddot{\gamma} = 1{,}33\, rad/s^2 \end{cases}$$

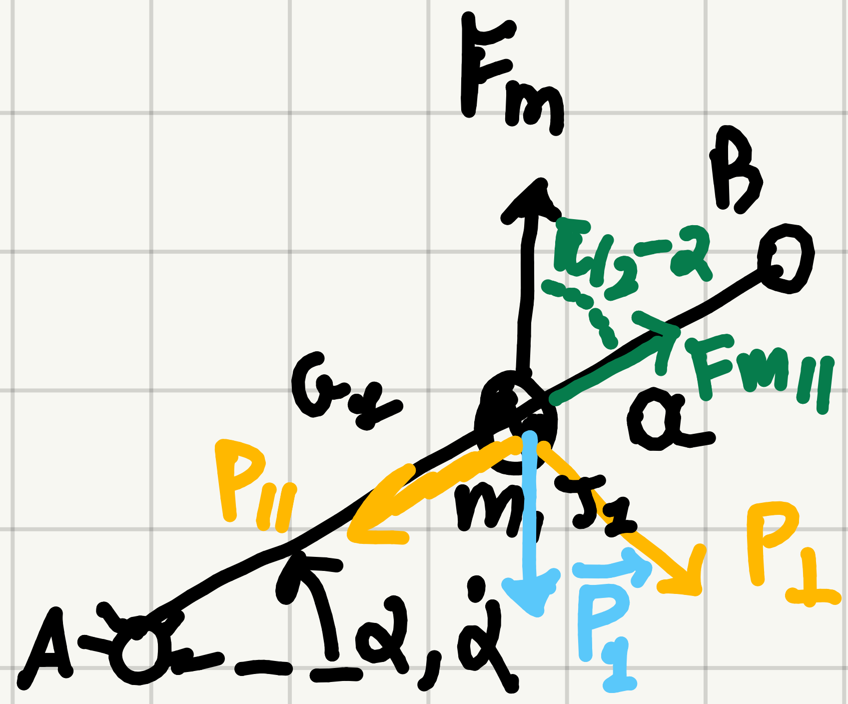
$\Rightarrow \dot{\vec{\omega}}_Q = (-0{,}91\,\hat{k})\, rad/s^2$ $\qquad \vec{a}_G = \vec{a}_B + \dot{\vec{\omega}}_Q \times (G-B) - \omega_Q^2 (G-B) =$

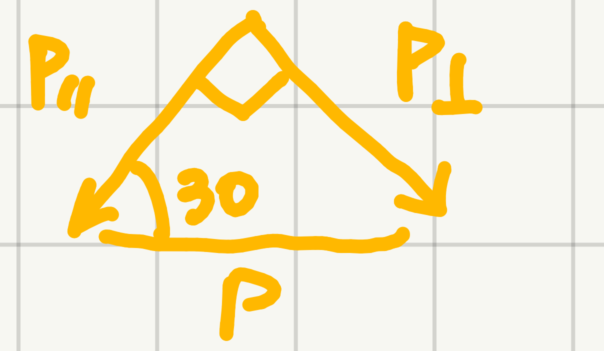
$= \cancel{\vec{a}_0} + \ddot{\alpha}\,\hat{k} \times a(\cos(\alpha)\,\hat{\imath} + \sin(\alpha)\,\hat{\jmath}) + \dot{\vec{\omega}}_Q \times (G-B) - \omega_Q^2 (G-B) =$

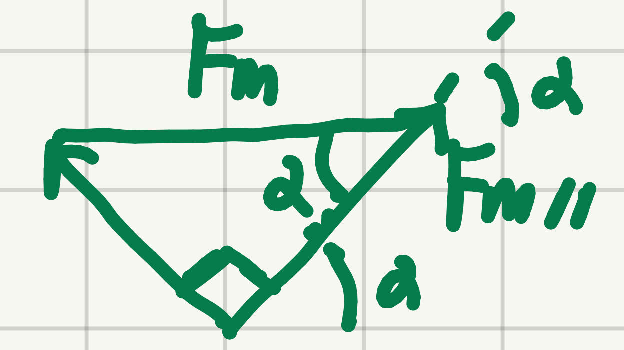
$= (\underbrace{-1{,}76\,\hat{\imath}}_{\tau} + \underbrace{1{,}09\,\hat{\jmath}}_{n})\, m/s^2$

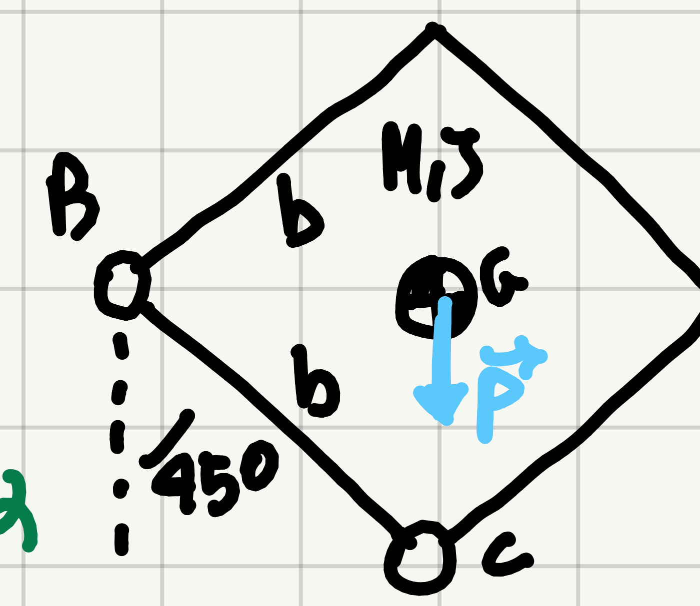
DINAMICA

BILANCIO DI POTENZE: $\sum P = \frac{d}{dt}K$







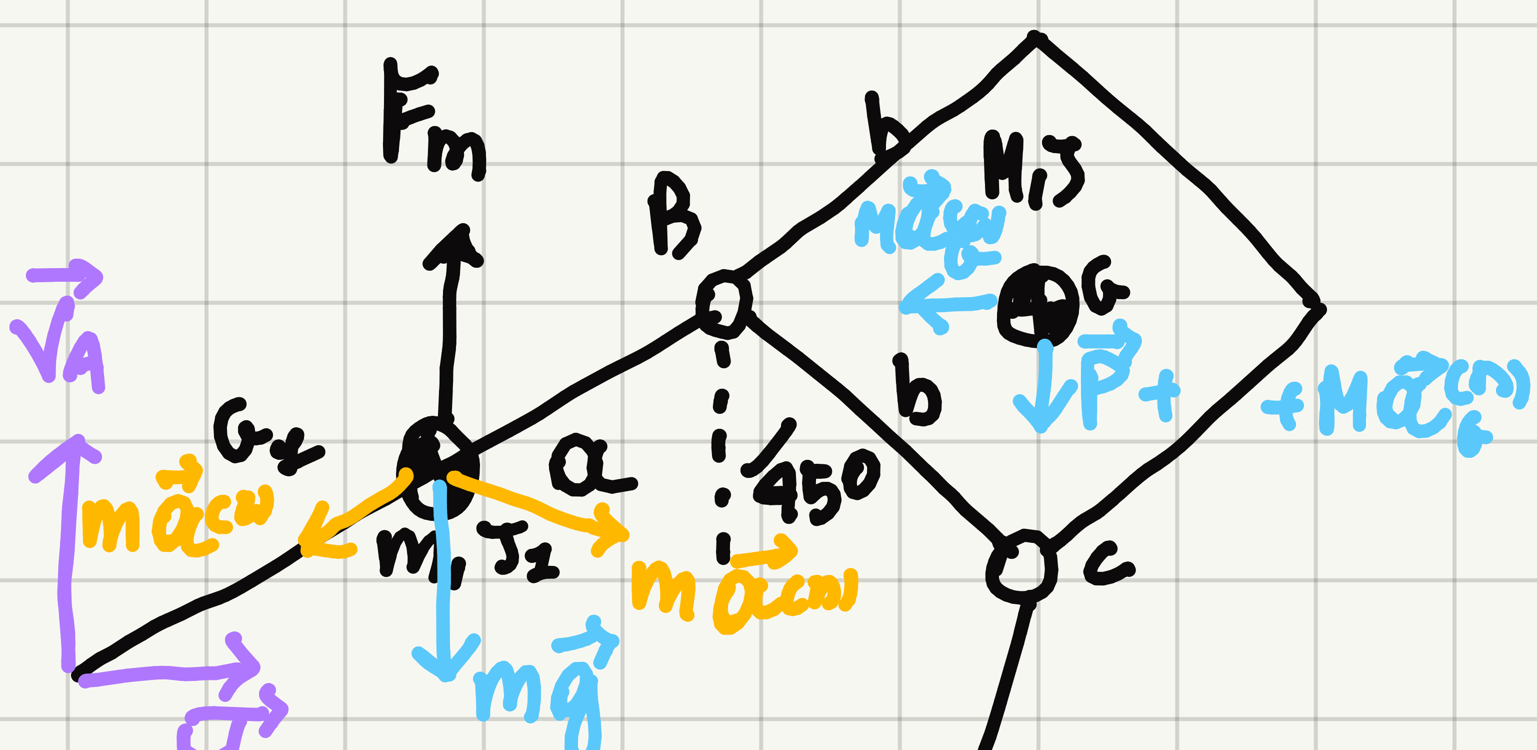


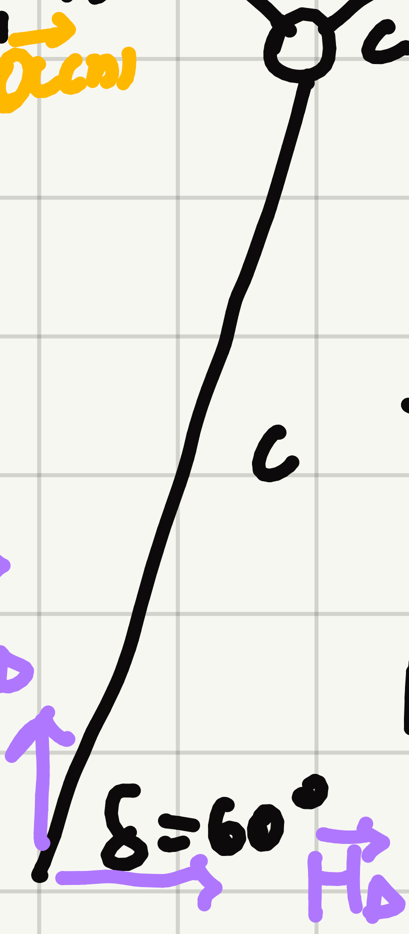
$\rightarrow P_\parallel = P_1 \cos 30$ $\quad \rightarrow F_{m\parallel} = F_m \cos\alpha$

$$\frac{d}{dt}K = (m v_B a_B^{(t)} + J_1 \dot\alpha \ddot\alpha) + (M v_G a_G^{(t)} + J_G \dot\omega_Q \omega_Q)$$

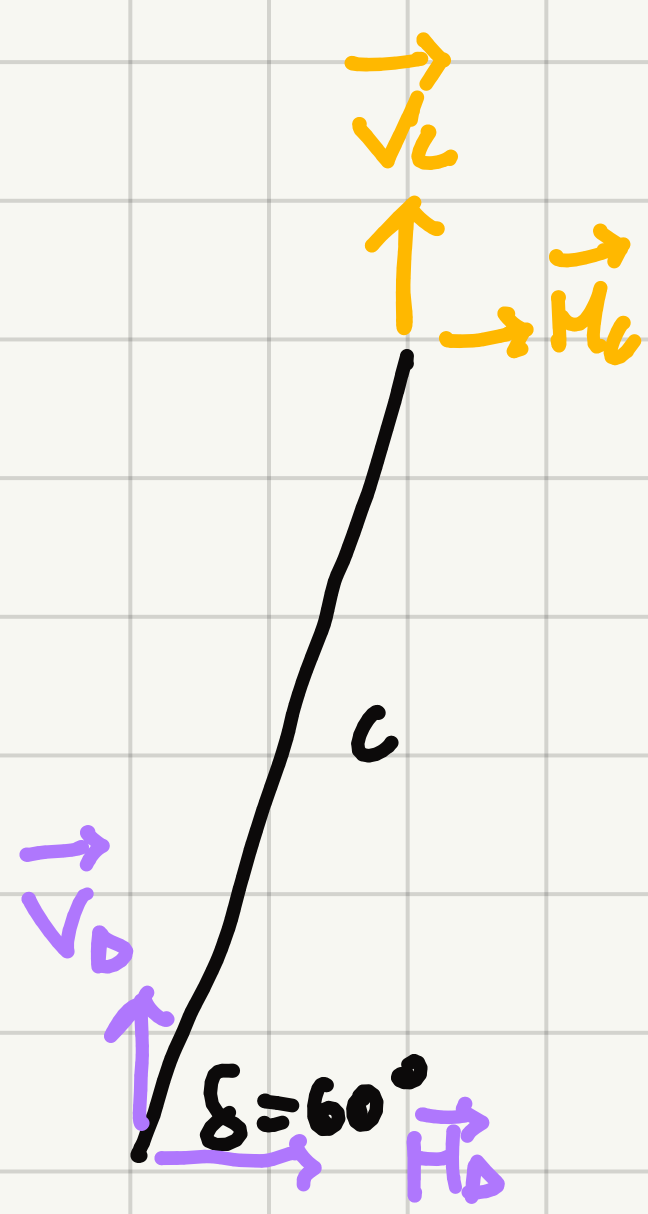
$$\sum P = (-P_{1\parallel} + F_{m\parallel}) v_B + (-P) v_G =$$

$$= (-mg\cos\alpha + F_m \cos\alpha) v_B - Mg v_G \Rightarrow F_m = 81,27\,N$$



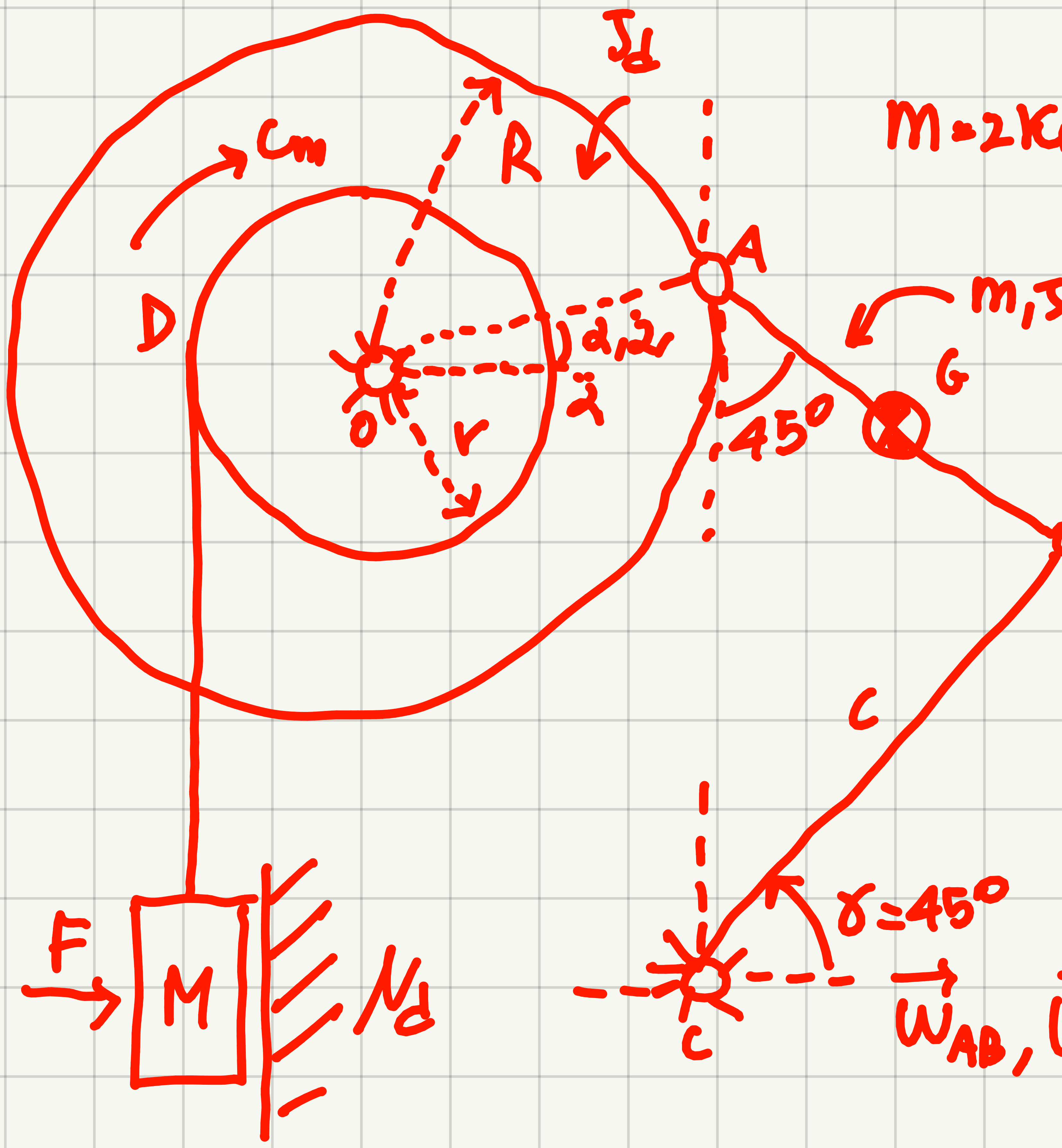


$$\begin{cases} x) \; H_A + H_D - ma\cos\alpha + ma\sin\alpha + Ma_G^{(t)} = 0 \\ y) \; V_A + V_D + F_m - mg - ma\sin\alpha - ma\cos\alpha - Mg - \\ \quad - ma_G^{(n)} = 0 \\ M_B) \; -V_A \bar{a}\cos\alpha + H_A \bar{a}\sin\alpha + (mg - F_m)\frac{a}{2}\cos\alpha + \\ \quad + ma\frac{\bar{a}}{2} - J_1\ddot\alpha = 0 \end{cases}$$



$$M_C) \quad H_D \bar{c}\sin\delta - V_D \bar{c}\cos\delta = 0$$

$$\begin{cases} H_A = -12,8\,N \\ H_D = 15,9\,N \\ V_D = 27,6\,N \\ V_A = 75,9\,N \end{cases}$$



$m = 2\,Kg$ $J = 0{,}1\,Kg\,m^2$ $M = 5\,Kg$

$J_d = 0{,}2\,Kg\,m^2$ $\alpha = 30°$

$\dot{\alpha} = -1\,rad/s$ $\ddot{\alpha} = -1\,rad/s^2$

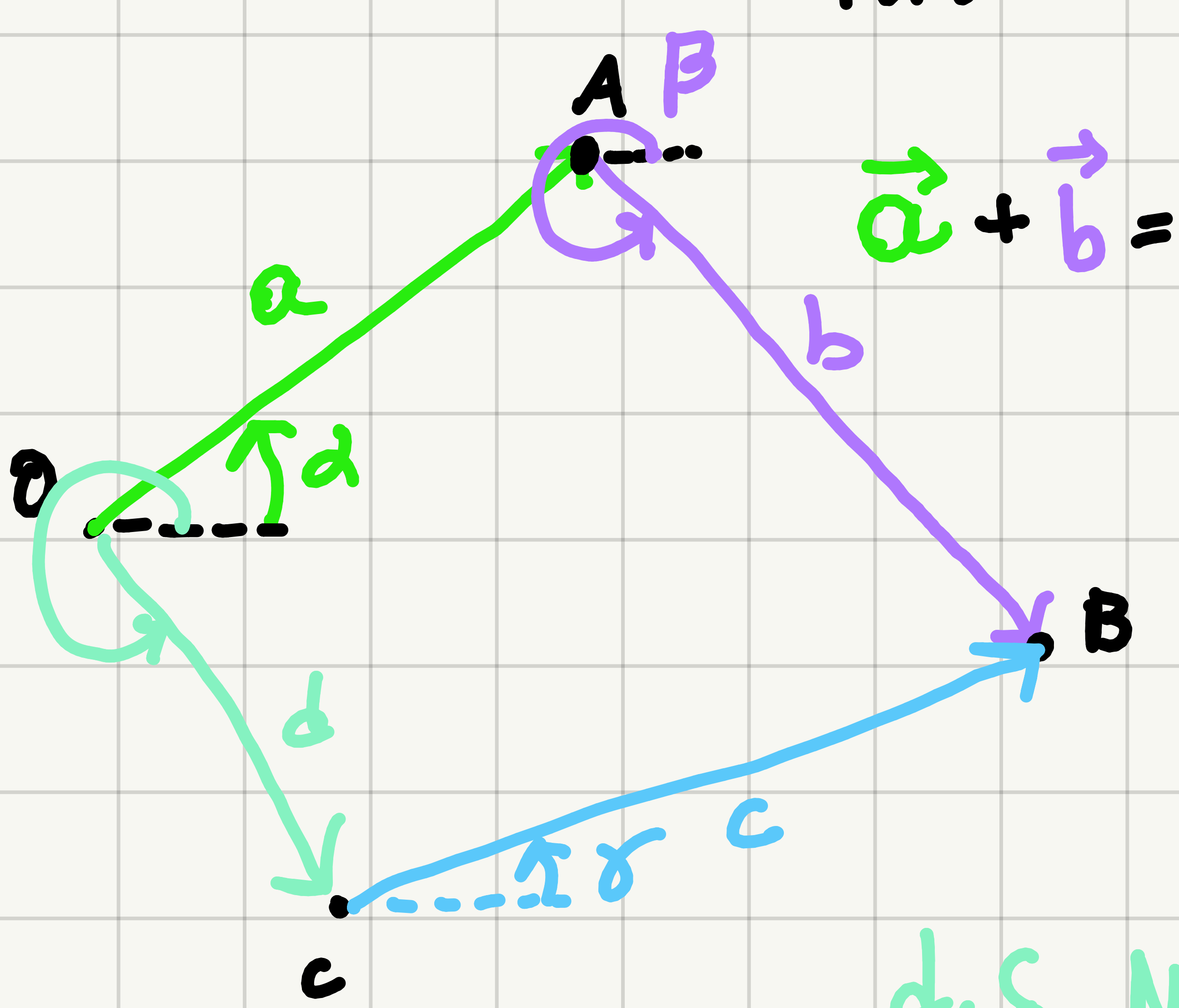
$b = 2m$ $R = 2r = 1m$ $F = 20N$

$\mu_d = 0{,}2$

$\vec{\omega}_{AB}, \vec{\dot{\omega}}_{AB}, \vec{v}_G, \vec{a}_G$? C_m?

CINEMATICA

$n = 3 \cdot 4 - (2 \cdot 4 + 2 + 1_{FUNE}) = 1$



$\vec{a} + \vec{b} = \vec{c} + \vec{d}$

$a = 1m$ $\alpha = 30°$

$\dot{\alpha} = -1\,rad/s$ $\ddot{\alpha} = -1\,rad/s^2$

$b = 2m$ $\beta = 270 + 45 = 315°$ $\dot{\beta}, \ddot{\beta} \neq 0$

$c = 2{,}5m$ $\gamma = 45°$ $\dot{\gamma}, \ddot{\gamma} \neq 0$

d, δ NON NOTI; $\dot{d}, \ddot{d}, \dot{\delta}, \ddot{\delta} = 0$

$$\begin{cases} a\cos\alpha + b\cos\beta = c\cos\gamma + d\cos\delta \\ a\sin\alpha + b\sin\beta = c\sin\gamma + d\sin\delta \end{cases}$$

$$\begin{cases} -a\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = -c\dot{\gamma}\sin\gamma + 0 \\ a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta = c\dot{\gamma}\cos\gamma + 0 \end{cases}$$

$$\begin{cases} -b\dot\beta\sin\beta + c\dot\gamma\sin\gamma = a\dot\alpha\sin\alpha \\ b\dot\beta\cos\beta - c\dot\gamma\cos\gamma = -a\dot\alpha\cos\alpha \end{cases}$$

$$\begin{vmatrix} -b\sin\beta & c\sin\gamma \\ b\cos\beta & -c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \dot\beta \\ \dot\gamma \end{vmatrix} = \begin{vmatrix} a\dot\alpha\sin\alpha \\ -a\dot\alpha\cos\alpha \end{vmatrix}$$

$\det A = -5$ $\quad \dot\beta = \dfrac{\det\begin{vmatrix} a\dot\alpha\sin\alpha & c\sin\gamma \\ -a\dot\alpha\cos\alpha & -c\cos\gamma \end{vmatrix}}{-5} = 0{,}129 \; rad/s$

$\dot\gamma = \dfrac{\det\begin{vmatrix} -b\sin\beta & a\dot\alpha\sin\alpha \\ b\cos\beta & -a\dot\alpha\cos\beta \end{vmatrix}}{-5} = -0{,}386 \; rad/s$

$\Rightarrow \vec{\omega}_{AB} = \dot\beta\hat{k} = (0{,}129\hat{k})\; rad/s$

$\vec{v}_G = \vec{v}_A + \vec{\omega}_{AB} \times (G-A)$ $\quad (G-A) = \dfrac{(A-B)}{2} = \left(\frac{2}{2}\cos(135)\hat{i} + \frac{2}{2}\sin(135)\hat{j}\right)$

$\vec{v}_A = \cancel{\vec{v}_0} + \dot\alpha\hat{k}\times(A-O) = -\hat{k}\times(R\cos(\alpha)\hat{i} + R\sin(\alpha)\hat{j})$

$\hat{k}\times\hat{i} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \hat{j}$ $\quad \hat{k}\times\hat{j} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\hat{i}$

$\Rightarrow \vec{v}_A = (0{,}5\hat{i} - 0{,}86\hat{j})\; m/s$

$\vec{v}_G = (0{,}5\hat{i} - 0{,}86\hat{j}) + 0{,}129\hat{k}\times(-0{,}707\hat{i} + 0{,}707\hat{j}) =$

$= (0{,}5\hat{i} - 0{,}86\hat{j}) + (-0{,}09\hat{i} - 0{,}09\hat{j}) = (0{,}41\hat{i} - 0{,}95\hat{j})\; m/s$

$|\vec{v}_G| = 1{,}03\; m/s$ (SERVIRÀ PER LA PARTE DI DINAMICA)

$$\begin{cases} -b\ddot\beta\sin\beta - b\dot\beta^2\cos\beta + c\ddot\gamma\sin\gamma + c\dot\gamma^2\cos\gamma = a\ddot\alpha\sin\alpha + a\dot\alpha^2\cos\alpha \\ b\ddot\beta\cos\beta - b\dot\beta^2\sin\beta - c\ddot\gamma\cos\gamma + c\dot\gamma^2\sin\gamma = -a\ddot\alpha\cos\alpha + a\dot\alpha^2\sin\alpha \end{cases}$$

$$\begin{cases} -b\dot\beta \sin\beta + c\dot\gamma \sin\gamma = a\dot\alpha \sin\alpha \\ b\dot\beta \cos\beta - c\dot\gamma \cos\gamma = -a\dot\alpha \cos\alpha \end{cases}$$

$$\begin{vmatrix} -b\sin\beta & c\sin\gamma \\ b\cos\beta & -c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \ddot\beta \\ \ddot\gamma \end{vmatrix} = \begin{vmatrix} a\ddot\alpha\sin\alpha + a\dot\alpha^2\cos\alpha + b\dot\beta^2\cos\beta - c\dot\gamma^2\cos\gamma \\ -a\ddot\alpha\cos\alpha + a\dot\alpha^2\sin\alpha + b\dot\beta^2\sin\beta - c\dot\gamma^2\sin\gamma \end{vmatrix}$$

$$\ddot\beta = \frac{\det\begin{vmatrix} a\ddot\alpha\sin\alpha + a\dot\alpha^2\cos\alpha + b\dot\beta^2\cos\beta - c\dot\gamma^2\cos\gamma & c\sin\gamma \\ -a\ddot\alpha\cos\alpha + a\dot\alpha^2\sin\alpha + b\dot\beta^2\sin\beta - c\dot\gamma^2\sin\gamma & -c\cos\gamma \end{vmatrix}}{-5} = 0{,}425\ \mathrm{rad/s^2}$$

$$\ddot\gamma = \frac{\det\begin{vmatrix} -b\sin\beta & a\ddot\alpha\sin\alpha + a\dot\alpha^2\cos\alpha + b\dot\beta^2\cos\beta - c\dot\gamma^2\cos\gamma \\ b\cos\beta & -a\ddot\alpha\cos\alpha + a\dot\alpha^2\sin\alpha + b\dot\beta^2\sin\beta - c\dot\gamma^2\sin\gamma \end{vmatrix}}{-5} = -0{,}15\ \mathrm{rad/s^2}$$

$$\Rightarrow \vec{\dot\omega}_{AB} = \ddot\beta\hat k = (0{,}425\hat k)\ \mathrm{rad/s}$$

$$\vec a_G = \vec a_A + \overbrace{\ddot\beta\hat k \times (G-A)}^{t} \overbrace{- \dot\beta^2 (G-A)}^{n}$$

$$\vec a_A = \cancel{\vec a_O} + \ddot\alpha\hat k \times (R\cos(\alpha)\hat\imath + R\sin(\alpha)\hat\jmath) - \dot\alpha^2(R\cos(\alpha)\hat\imath + R\sin(\alpha)\hat\jmath)$$

$$= (-0{,}866\hat\jmath + 0{,}5\hat\imath) - (0{,}866\hat\imath + 0{,}5\hat\jmath) = (-0{,}366\hat\imath - 0{,}166\hat\jmath)\ \mathrm{m/s^2}$$

$$\vec a_G = \vec a_A + (-0{,}3\hat\imath + 0{,}3\hat\jmath) - (-0{,}012\hat\imath + 0{,}012\hat\jmath)$$

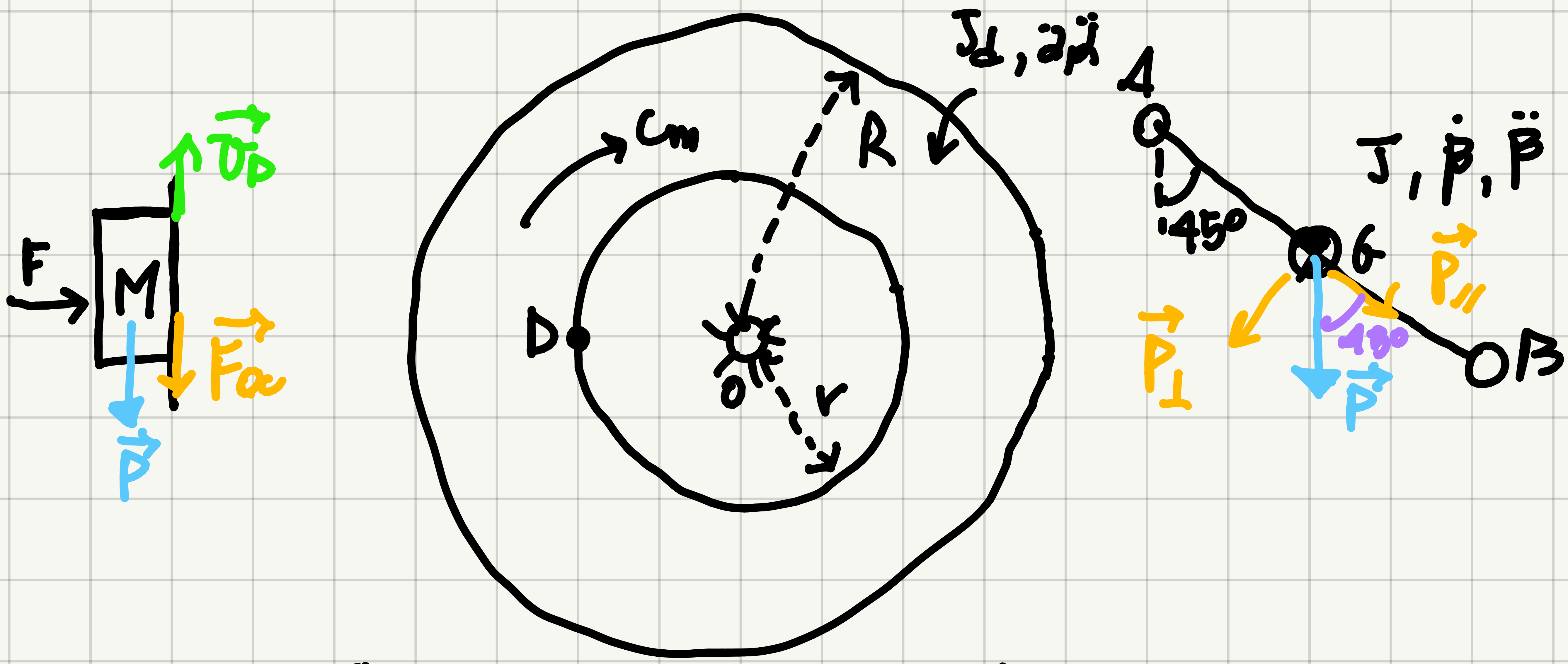
$$= \underbrace{(-0{,}666\hat\imath + 0{,}134\hat\jmath)}_{t} + \underbrace{(0{,}012\hat\imath - 0{,}012\hat\jmath)}_{n} = (-0{,}654\hat\imath + 0{,}122\hat\jmath)\ \mathrm{m/s^2}$$

$$|\vec a_G^{(a)}| = 0{,}679\ \mathrm{m/s^2}$$

(SERVIRÀ PER LA PARTE DI DINAMICA)

DINAMICA

BILANCIO DI POTENZE: $\sum P = \frac{d}{dt}K$



$$\frac{d}{dt}K = (M v_D a_D^{(t)}) + (J_d \dot{\alpha} \ddot{\alpha}) + (m v_G a_G^{(t)} + J \dot{\beta} \ddot{\beta})$$

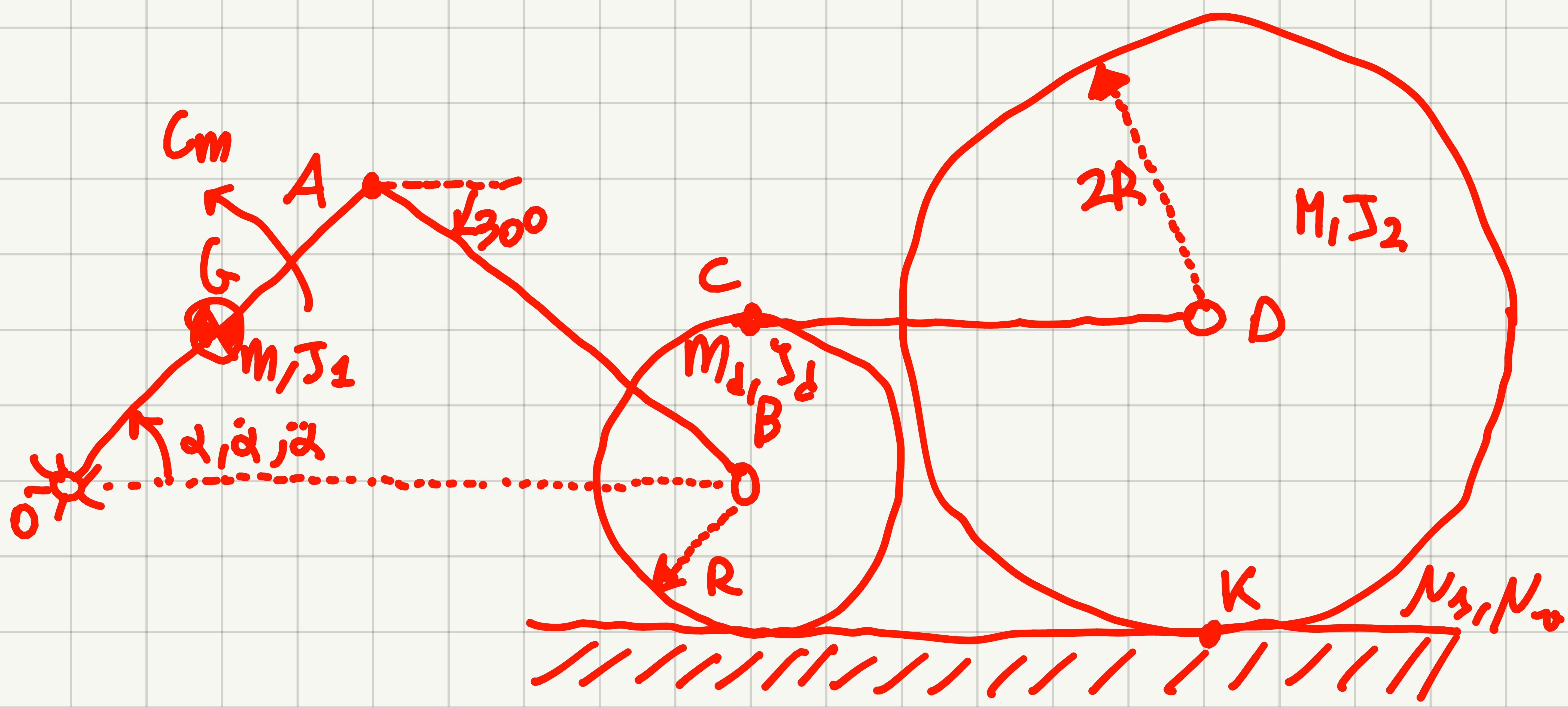
$$\sum P = ((-Mg - F_a) v_D) + (C_m \dot{\alpha}) + (-m g v_G \cos(45))$$

$$F_a = \mu_d F$$

$\vec{v}_D = \cancel{\vec{v}_0} + \dot{\alpha} \hat{k} \times (-r \hat{i}) = (0{,}5 \hat{j})\ m/s \qquad |\vec{v}_D| = 0{,}5\ m/s$

$\vec{a}_D^{(t)} = \cancel{\vec{a}_0} + \ddot{\alpha} \hat{k} \times (-r \hat{i}) = (0{,}5 \hat{j})\ m/s^2 \qquad |\vec{a}_D| = 0{,}5\ m/s^2$

$\frac{d}{dt}K = 2{,}85\ W \Rightarrow C_m = -43{,}25\ Nm$



$m = 1\,kg$ $J_1 = 0{,}1\,kg\,m^2$ $m_d = 2\,kg$ $J_d = 0{,}2\,kg\,m^2$ $M = 5\,kg$ $\alpha = 45°$

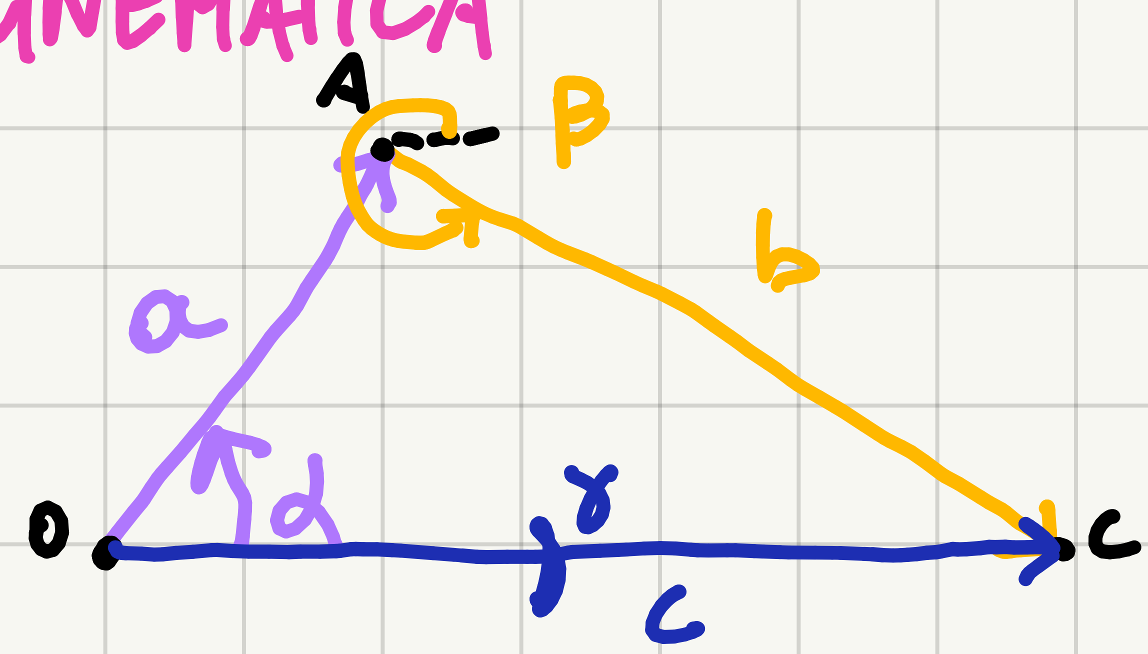
$\dot{\alpha} = 0{,}5\,rad/s$ $\ddot{\alpha} = 1\,rad/s^2$ $J_2 = 0{,}5\,kg\,m^2$ $R = 0{,}5\,m$ $OA = 1\,m$

$AB = \sqrt{2}\,m$ $N_s = 0{,}8$ $N_v = 0{,}05$ $v_D?$ $a_D?$ $C_m?$

VERIFICA DI ADERENZA IN K

$n = 3 \cdot 4 - (2 \cdot 4 + 1_D + 2 \cdot 1_{STRISCIAMENTO}) = 1$

CINEMATICA



$\vec{c} = \vec{a} + \vec{b}$

$a = 1\,m$ $\alpha = 45°$
$\dot{\alpha} = 0{,}5\,rad/s$
$\ddot{\alpha} = 1\,rad/s^2$

$b = \sqrt{2}\,m$ $\beta = 360 - 30 = 330°$ $\dot{\beta}, \ddot{\beta} \neq 0$

$c = \_$ $\gamma = 0°$ FISSO

OBIETTIVI:

$\vec{v}_B = \dot{c}\,\hat{\imath}$ $\vec{a}_B = \ddot{c}\,\hat{\imath}$

$\vec{v}_C = \vec{v}_B + \vec{\omega}_{DISCO} \times (C - B)$ FUNE INESTENSIBILE $\Rightarrow \vec{v}_D = \vec{v}_C$

$$\vec{a}_C = \underbrace{\vec{a}_B + \dot{\vec{\omega}}_{DISCO} \times (C-B)}_{\tau} \underbrace{- \omega^2_{DISCO}(C-B)}_{n}$$

FUNE INESTENSIBILE $\Rightarrow \vec{a}_D = \vec{a}_C^{(\tau)}$

$$\begin{cases} a\cos\alpha + b\cos\beta = c \\ a\sin\alpha + b\sin\beta = 0 \end{cases}$$

$$\begin{cases} -a\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = \dot{c} \\ a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta = 0 \end{cases} \rightarrow \begin{cases} \dot{c} = -0{,}558\ m/s \\ \dot{\beta} = -0{,}289\ rad/s \end{cases}$$

$\Rightarrow \vec{v}_B = (-0{,}558\,\hat{\imath})\ m/s$ $\qquad \vec{\omega}_{DISCO} = \dfrac{|\vec{v}_B|}{R}\hat{k} = (1{,}116\,\hat{k})\ rad/s$

$\rightarrow \quad \vec{v}_C = (-0{,}558\,\hat{\imath} + \omega_{DISCO}\,\hat{k}\times(0{,}5\,\hat{\jmath})) = (-0{,}558\,\hat{\imath} - 0{,}558\,\hat{\imath}) =$

$= (-1{,}116\,\hat{\imath})\ m/s \qquad \vec{v}_D = \vec{v}_C = (-1{,}116\,\hat{\imath})\ m/s$

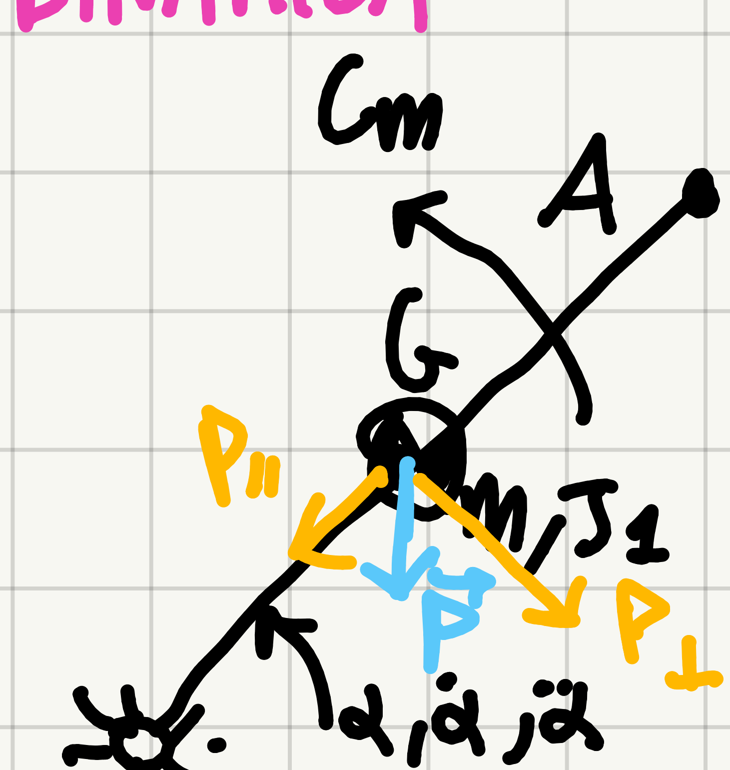
$$\begin{cases} -a\ddot{\alpha}\sin\alpha - a\dot{\alpha}^2\cos\alpha - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta = \ddot{c} \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta = 0 \end{cases}$$

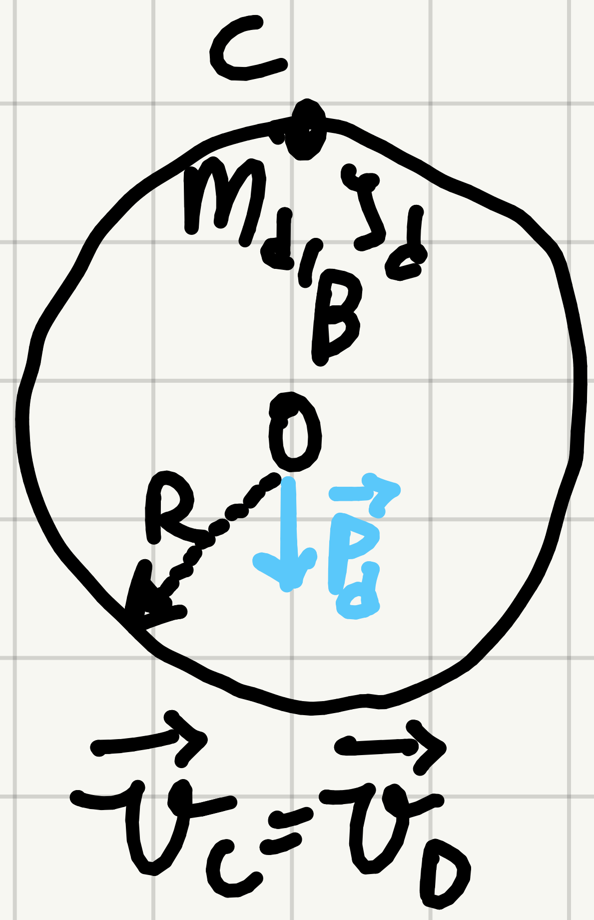
$$\rightarrow \begin{cases} \ddot{c} = -1{,}326\ m/s^2 \\ \ddot{\beta} = -0{,}481\ rad/s^2 \end{cases} \Rightarrow \vec{a}_B = (-1{,}326\,\hat{\imath})\ m/s^2$$

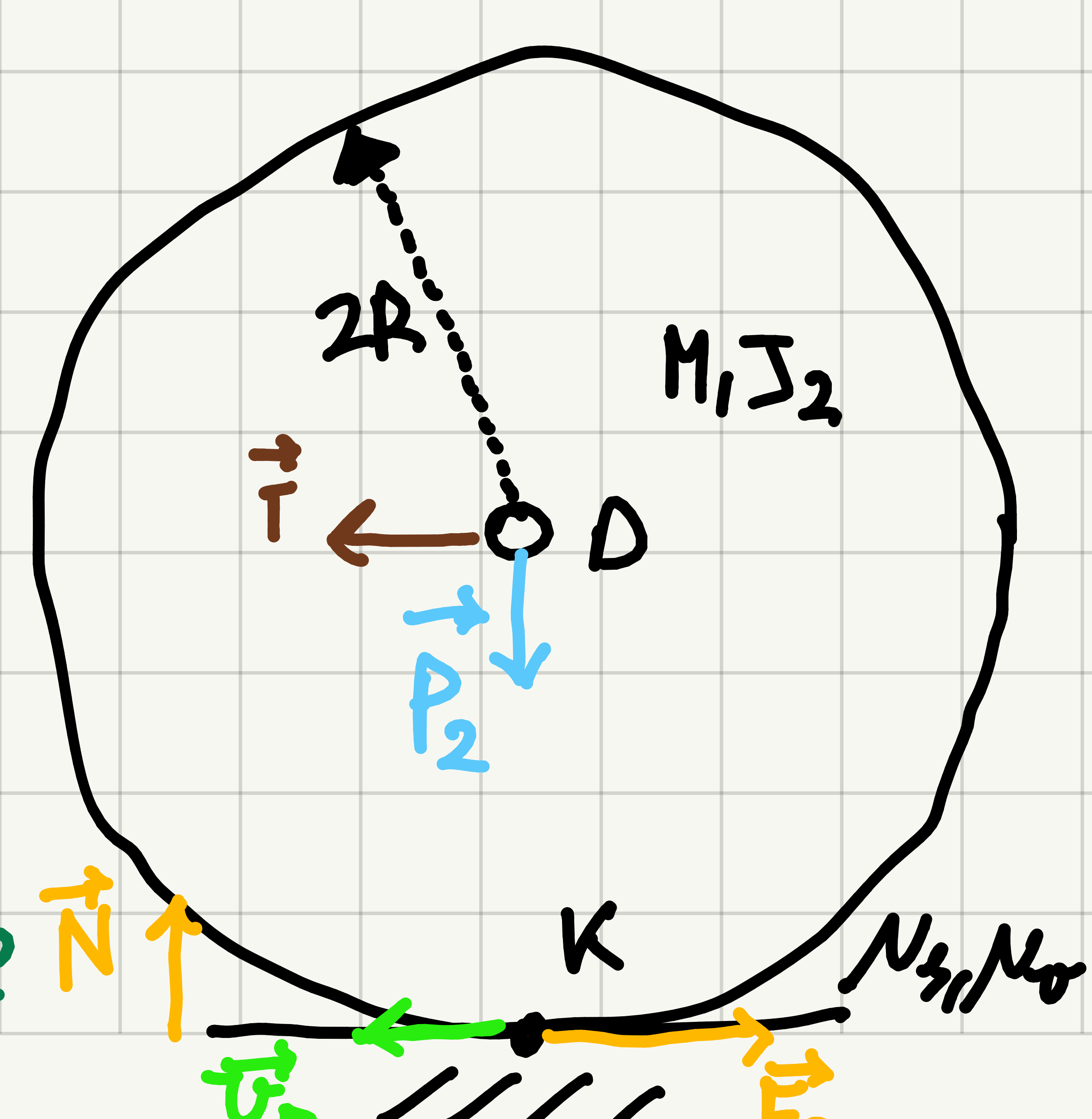
$\vec{\dot{\omega}}_{DISCO} = \dfrac{|\vec{a}_B|}{R}\hat{k} = (2{,}652\,\hat{k})\ rad/s^2$

$\vec{a}_C = (-2{,}652\,\hat{\imath} - \dot{\omega}^2(C-B))\ m/s^2 \Rightarrow \vec{a}_D = (-2{,}652\,\hat{\imath})\ m/s^2$

DINAMICA







$\omega_{DISCO}\cdot R = \omega_{DISCO_2}\cdot 2R \Rightarrow \omega_{DISCO_2} = \dfrac{\omega_{DISCO}}{2}$

BILANCIO DI POTENZE: $\sum P = \frac{d}{dt}K$

$$\frac{d}{dt}K = (m v_G a_G^{(t)} + J_1 \dot{\alpha}\ddot{\alpha}) + (m_d v_B a_B^{(t)} + J_d \omega_{DISCO}\dot{\omega}_{DISCO})$$

$$+ (M v_D a_D^{(t)} + J_2 \frac{\omega_{DISCO}}{2}\frac{\dot{\omega}_{DISCO}}{2})$$

$$\vec{v}_G = \vec{v}_O + \dot{\alpha} \times (G-O) = 0{,}5\hat{k} \times (0{,}5\cos(45)\hat{\imath} + 0{,}5\sin(45)\hat{\jmath}) =$$

$$= (-0{,}177\hat{\imath} + 0{,}177\hat{\jmath})\,m/s \quad |\vec{v}_G| = 0{,}25\,m/s$$

$$\vec{a}_G^{(t)} = \vec{a}_O + \ddot{\alpha} \times (G-O) = \hat{k} \times (0{,}5\cos(45)\hat{\imath} + 0{,}5\sin(45)\hat{\jmath}) =$$

$$= (-0{,}354\hat{\imath} + 0{,}354\hat{\jmath})\,m/s^2 \quad |\vec{a}_G^{(t)}| = 0{,}5\,m/s^2$$

$$\frac{d}{dt}K = 17{,}465\,W$$

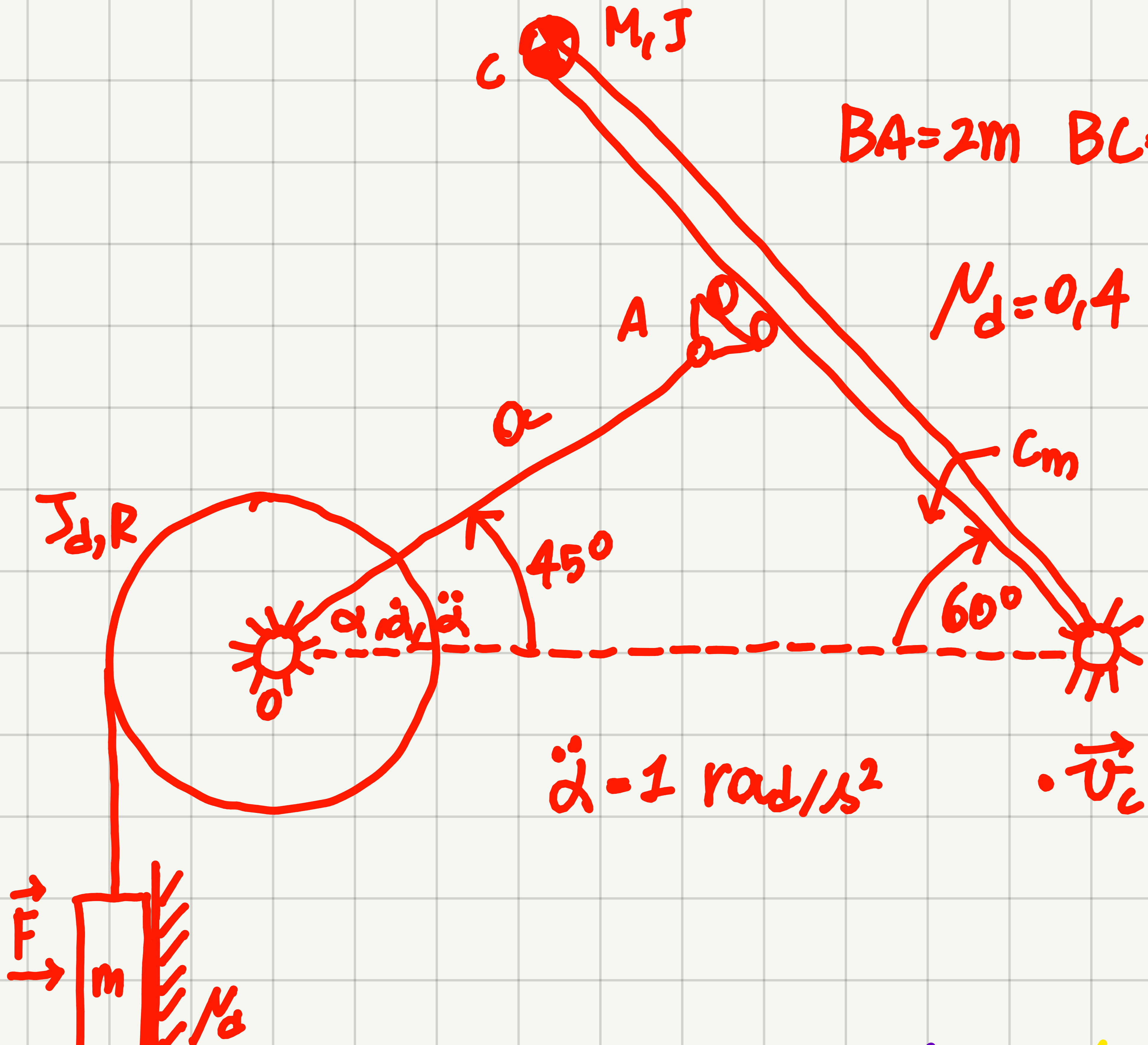
$$\sum P = (-P_{//} + C_m \dot{\alpha}) + (0) + (-F_a v)$$

$$F_a = \mu_s N = \mu_s P_2 = \mu_s M g$$

$$-mg\cos(45) + C_m \dot{\alpha} - \mu_v \frac{\omega_{DISCO}}{2} R \underbrace{N}_{Mg} = 17{,}465\,W$$

$$\Rightarrow C_m = 50{,}17\,N\cdot m \Rightarrow \vec{C}_m = (50{,}17\hat{k})\,N\cdot m$$

$T \leq F_{a\,lim}$? $\quad \cancel{M} a_D \leq \mu_s \cancel{M} g \quad 2{,}652 \leq 7{,}848$ ✓



BA = 2m BC = 4m a = 1m R = 0,5m

$\mu_d = 0{,}4$ $J = 0{,}05\,kg\,m^2$ $M = 2kg$

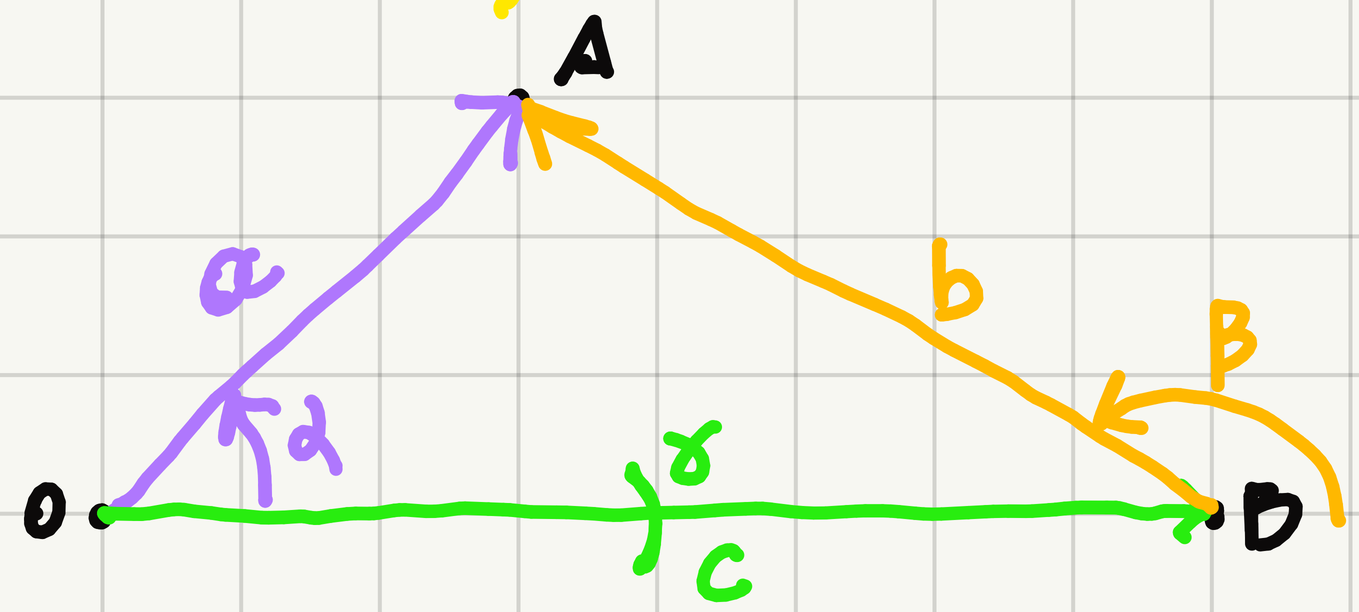
$m = 10kg$ $\alpha = 45°$

$\dot\alpha = 1\ rad/s$

$\ddot\alpha = 1\ rad/s^2$

• $\vec v_c, \vec a_c$? • C_m ?

$\vec v_C = \cancel{\vec v_B} + \omega_{BC} \times (C-B)$ $\vec a_C = \cancel{\vec a_B} + \dot{\vec\omega}_{BC} \times (C-B) - \omega_{BC}^2 (C-B)$



$\vec a = \vec b + \vec c$

$a = 1m$ $\alpha = 45°$ $\dot\alpha = 1\ rad/s$ $\ddot\alpha = 1\ rad/s^2$

$b = 2m$ $\beta = 180° - 60° = 120°$ $\dot\beta, \ddot\beta \neq 0$ $c = -$ $\gamma = 0°$ FISSO

$$\begin{cases} a\cos\alpha = b\cos\beta + c \\ a\sin\alpha = b\sin\beta \end{cases} \qquad \begin{cases} -a\dot\alpha\sin\alpha = \dot b\cos\beta - b\dot\beta\sin\beta \\ a\dot\alpha\cos\alpha = \dot b\sin\beta + b\dot\beta\cos\beta \end{cases}$$

$$\begin{vmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{vmatrix} \cdot \begin{vmatrix} \dot b \\ \dot\beta \end{vmatrix} = \begin{vmatrix} -a\dot\alpha\sin\alpha \\ a\dot\alpha\cos\alpha \end{vmatrix}$$

$$\det A = b\cos^2\beta + b\sin^2\beta = 2$$

$$\begin{vmatrix} -1/2 & -\sqrt3 \\ \sqrt3/2 & -1 \end{vmatrix} \cdot \begin{vmatrix} \dot b \\ \dot\beta \end{vmatrix} = \begin{vmatrix} -\sqrt2/2 \\ \sqrt2/2 \end{vmatrix} \qquad \dot b = \frac{\det\begin{vmatrix} -\sqrt2/2 & -\sqrt3 \\ \sqrt2/2 & -1 \end{vmatrix}}{2} = 0{,}966\ m/s$$

$$\dot\beta = \frac{\det\begin{vmatrix} -1/2 & -\sqrt2/2 \\ \sqrt3/2 & \sqrt2/2 \end{vmatrix}}{2} = 0{,}129\ rad/s^2$$

$\vec{\omega}_{BC} = \dot{\beta}\hat{k} = (0{,}129\hat{k})\ rad/s$

$$\begin{cases} -a\ddot{\alpha}\sin\alpha - a\dot{\alpha}^2\cos\alpha = \ddot{b}\cos\beta - 2\dot{b}\dot{\beta}\sin\beta - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha = \ddot{b}\sin\beta + 2\dot{b}\dot{\beta}\cos\beta + b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta \end{cases}$$

$$\begin{vmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{vmatrix} \cdot \begin{vmatrix} \ddot{b} \\ \ddot{\beta} \end{vmatrix} = \begin{vmatrix} -a\ddot{\alpha}\sin\alpha - a\dot{\alpha}^2\cos\alpha + 2\dot{b}\dot{\beta}\sin\beta + b\dot{\beta}^2\cos\beta \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha - 2\dot{b}\dot{\beta}\cos\beta + b\dot{\beta}^2\sin\beta \end{vmatrix}$$

$$\begin{vmatrix} -1/2 & -\sqrt{3} \\ -\sqrt{3}/2 & -1 \end{vmatrix} \cdot \begin{vmatrix} \ddot{b} \\ \ddot{\beta} \end{vmatrix} = \begin{vmatrix} -1{,}215 \\ 0{,}153 \end{vmatrix} \qquad \ddot{b} = \frac{\det\begin{vmatrix} -1{,}215 & -\sqrt{3} \\ 0{,}153 & -1 \end{vmatrix}}{2} = 0{,}74\ m/s^2$$

$$\ddot{\beta} = \frac{\det\begin{vmatrix} -1/2 & -1{,}215 \\ \sqrt{3}/2 & 0{,}153 \end{vmatrix}}{2} = 0{,}488\ rad/s^2$$

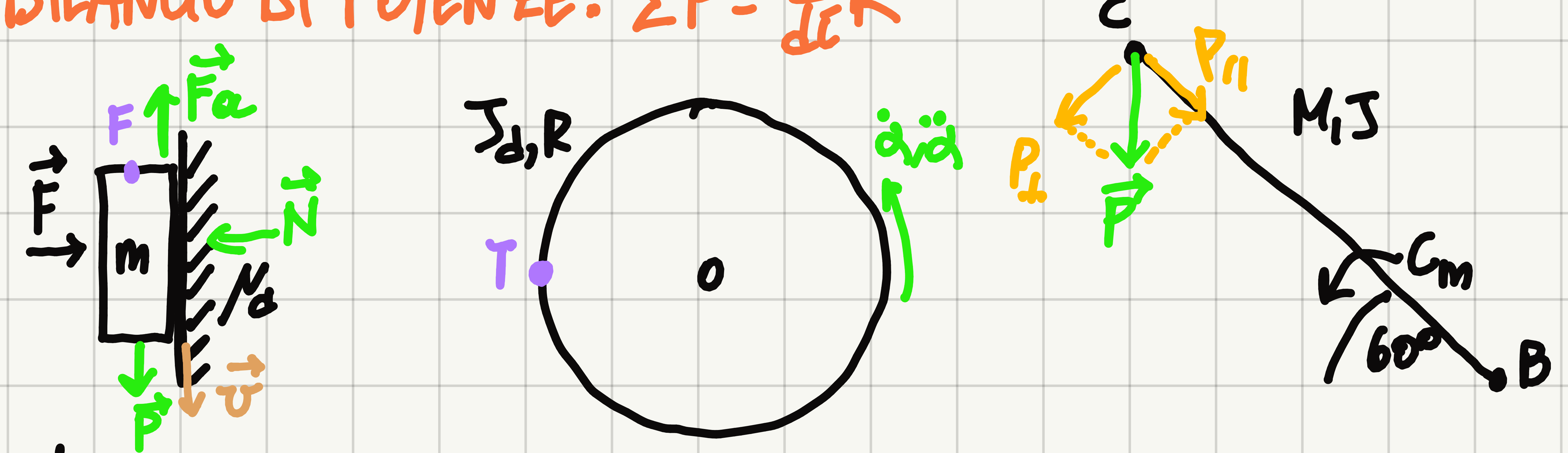
$\dot{\vec{\omega}}_{BC} = \ddot{\beta}\hat{k} = (0{,}488\hat{k})\ rad/s^2 \qquad (C-B) = (4\cos(120)\hat{\imath} + 4\sin(120)\hat{\jmath})$

$\vec{v}_C = 0{,}516\hat{k} \times (\cos(120)\hat{\imath} + \sin(120)\hat{\jmath}) = (-0{,}447\hat{\imath} - 0{,}258\hat{\jmath})\ m/s \qquad |\vec{v}_C| = 0{,}516\ m/s$

$\vec{a}_C = 1{,}952\hat{k} \times (\cos(120)\hat{\imath} + \sin(120)\hat{\jmath}) - 0{,}067(\cos(120)\hat{\imath} + \sin(120)\hat{\jmath}) =$

$= \underbrace{(-1{,}69\hat{\imath} - 0{,}98\hat{\jmath})}_{a_C^{(t)}} + \underbrace{(0{,}034\hat{\imath} - 0{,}058\hat{\jmath})}_{a_C^{(n)}} = (-1{,}66\hat{\imath} - 1{,}04\hat{\jmath}) \qquad |\vec{a}_C^{(t)}| = 1{,}95\ m/s^2$

BILANCIO DI POTENZE: $\sum P = \frac{d}{dt}K$



$\frac{d}{dt}K = (m v_F a_F^{(t)}) + (J_d \dot{\alpha}\ddot{\alpha}) + (M v_C a_C^{(t)} + J\dot{\beta}\ddot{\beta})$

$J_d = \frac{m}{2}R^2 = 1{,}25\ kgm^2$

FUNE INESTENSIBILE: $\vec{v}_T = \vec{v}_F \qquad \vec{a}_F = \vec{a}_T^{(t)}$

$$\vec{v}_T = \cancel{\vec{v}_0} + \dot{\alpha}\hat{k} \times (T-O) = \dot{\alpha}\hat{k} \times (-R\hat{\imath}) = -0{,}3\hat{\jmath} = \vec{v}_F$$

$$\vec{a}_T = \vec{a}_0 + \ddot{\alpha}\hat{k} \times (T-O) - \dot{\alpha}^2 (T-O) = \underbrace{(-0{,}5\hat{\jmath})}_{a^{(c)}} - \dot{\alpha}^2 (T-O)$$

$\Rightarrow \frac{d}{dt}K = 5{,}77\,W$ CON J_d, $4{,}52$ SENZA

$$\sum P = (mg - F_a)v_F + (0) + (C_m\dot{\beta} - P_{//}v_c)$$

$$F_a = \mu_d N = \mu_d F \qquad P_{//} = Mg\cos(60)$$

$$mgv_F - \mu_d F v_F + C_m\dot{\beta} - Mg v_c \cos(60) = 5{,}77\,W \Rightarrow C_m = -234\ Nm$$