

USE RESOLUTION TO CHECK CONSISTENCY OF THE SET

$$S = \{(W \rightarrow X) \rightarrow \neg(Y \rightarrow Z), (X \rightarrow Y) \rightarrow \neg(Z \rightarrow W)\}$$

STEP

FORMULA

RULE

$$1 \quad \{(W \rightarrow X) \rightarrow \neg(Y \rightarrow Z)\} \quad \text{ASSUMPTION}$$

$$2 \quad \{(X \rightarrow Y) \rightarrow \neg(Z \rightarrow W)\} \quad \text{ASSUMPTION}$$

$$3 \quad \{\neg(W \rightarrow X), \neg(Y \rightarrow Z)\} \quad 1, \beta\text{-EXPANSION}$$

$$4 \quad \{W, \neg(Y \rightarrow Z)\} \quad 3, \alpha\text{-EXPANSION}$$

$$5 \quad \{\neg X, \neg(Y \rightarrow Z)\} \quad 3, \alpha\text{-EXPANSION}$$

$$6 \quad \{W, Y\} \quad 4, \alpha\text{-EXPANSION}$$

$$7 \quad \{W, \neg Z\} \quad 4, \alpha\text{-EXPANSION}$$

$$8 \quad \{\neg X, Y\} \quad 5, \alpha\text{-EXPANSION}$$

$$9 \quad \{\neg X, \neg Z\} \quad 5, \alpha\text{-EXPANSION}$$

$$10 \quad \{\neg(X \rightarrow Y), \neg(Y \rightarrow Z)\} \quad 2, \beta\text{-EXPANSION}$$

$$11 \quad \{X, \neg(Z \rightarrow W)\} \quad 10, \alpha\text{-EXPANSION}$$

$$12 \quad \{\neg Y, \neg(Z \rightarrow W)\} \quad 10, \alpha\text{-EXPANSION}$$

$$13 \quad \{X, \neg Z\} \quad 11, \alpha\text{-EXPANSION}$$

$$14 \quad \{X, W\} \quad 11, \alpha\text{-EXPANSION}$$

$$15 \quad \{\neg Y, \neg Z\} \quad 12, \alpha\text{-EXPANSION}$$

16

$\{\neg Y, W\}$

12, d. EXPANSION

17

$\{W\}$

16, 6 RESOLUTION

$\neg W$ NEVER APPEARS \Rightarrow THE SET IS CONSISTENT

ARE CORRECT

USE RESOLUTION TO CHECK WHETHER THE FOLLOWING STATEMENTS

(1) $x \rightarrow (y \wedge z) \vdash (x \rightarrow y) \wedge (x \rightarrow z)$

WE HAVE TO CHECK WHETHER THE SET $F \cup \{ \neg(x \rightarrow y), \neg(x \rightarrow z) \}$

$= \{ x \rightarrow y \wedge z, \neg((x \rightarrow y) \wedge (x \rightarrow z)) \}$ HAS A CLOSED EXPANSION

STEP	FORMULA	RULE
1	$\{ x \rightarrow y \wedge z \}$	ASSUMPTION
2	$\{ \neg((x \rightarrow y) \wedge (x \rightarrow z)) \}$	ASSUMPTION
3	$\{ \neg x, y \wedge z \}$	1, β -EXPANSION
4	$\{ \neg x, y \}$	3, d-EXPANSION
5	$\{ \neg x, z \}$	3, d-EXPANSION
6	$\{ \neg(x \rightarrow y), \neg(x \rightarrow z) \}$	2, β -EXPANSION
7	$\{ x, \neg(x \rightarrow z) \}$	6, α -EXPANSION
8	$\{ \neg y, \neg(x \rightarrow z) \}$	6, α -EXPANSION
9	$\{ x \}$	7, d-EXPANSION
10	$\{ x, \neg z \}$	7, d-EXPANSION
11	$\{ x, \neg y \}$	8, d-EXPANSION
12	$\{ \neg y, \neg z \}$	8, d-EXPANSION
13	$\{ y \}$	4, 9 RESOLUTION