

$$= \left| \begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array} \right| P_2 = \left| \begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array} \right|$$

$$\rho^{\infty} = \left| \begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right| = m = \left| \begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right| = \left| \begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 1 & 0 \end{array} \right|$$

$$f = \left| \begin{array}{cc|cc|cc} 1 & 0 & | & 0 & 0 & 1 & | \\ 0 & 1 & | & 0 & 0 & 1 & | \\ 1 & 0 & | & 0 & 1 & 0 & | \\ 1 & 1 & | & 0 & 1 & 0 & | \end{array} \right.$$

$$f_1 = \left| \begin{array}{cccc|ccccc} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & | & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & | & 0 & 1 & 0 & 0 \end{array} \right| f^{\infty} = \left| \begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \end{array} \right|$$

$$K = \left| \begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 1 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & | & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & | & 0 & 1 & 0 & 0 \end{array} \right| =$$

$$= \left| \begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 1 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & | & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 0 & 1 \end{array} \right| P_2 = \left| \begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & | & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 0 & 1 \end{array} \right|$$

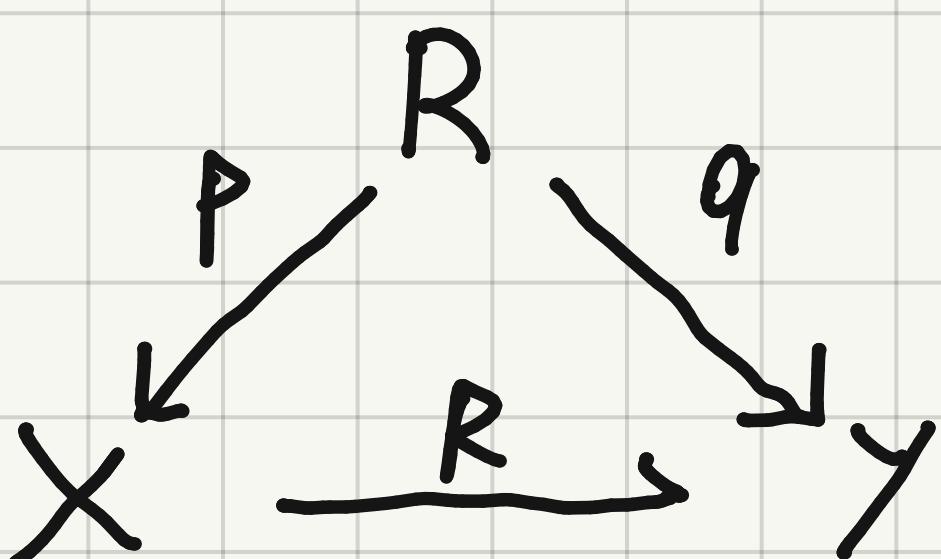
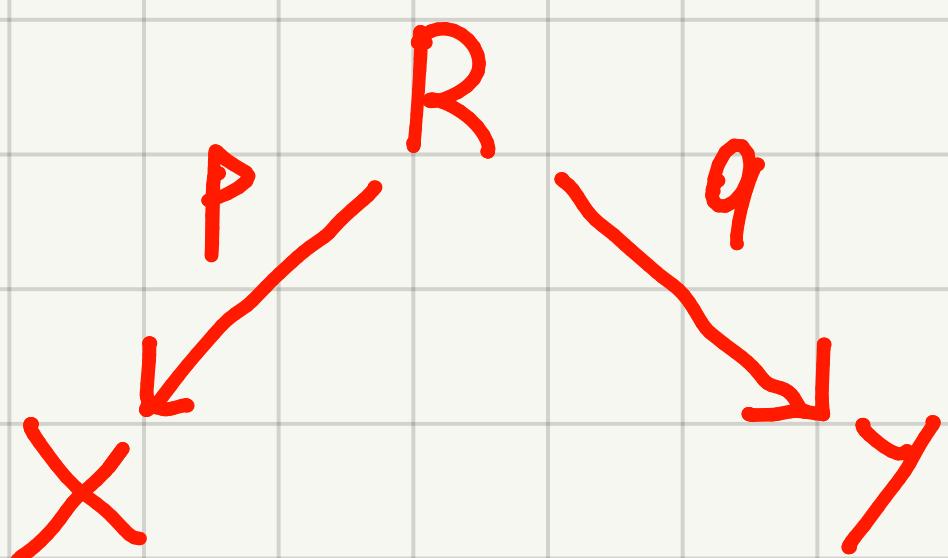
$$p_{\text{obs}} = \left| \begin{array}{cccc|c} & 1 & 0 & 0 & 0 & | \\ 1 & 0 & 0 & 1 & 1 & \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 1 & \end{array} \right|$$

$$m = \left| \begin{array}{cccc|c} & 1 & 0 & 0 & 0 & | \\ 1 & 0 & 0 & 1 & 1 & \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 1 & \end{array} \right| \cdot \left| \begin{array}{ccccc|c} & 1 & 0 & 0 & 0 & | \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & \end{array} \right| \cdot \left| \begin{array}{ccccc|c} & 0 & 0 & 0 & 0 & | \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & \end{array} \right|$$

$$= \left| \begin{array}{cccc|c} & 1 & 0 & 0 & 0 & | \\ 1 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 1 & 0 & 0 & \end{array} \right|$$

$$f = \left| \begin{array}{cccc|c} & 1 & 0 & 0 & 0 & | \\ 1 & 0 & 0 & 0 & 1 & \\ 0 & 1 & 0 & 0 & 0 & \end{array} \right| \cdot \left| \begin{array}{ccccc|c} & 1 & 0 & 0 & 0 & | \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & \end{array} \right|$$

ASSUME  $R: X \rightarrow Y$  IS ANY RELATION. REGARD  $R$  AS A SUBSET OF  $X \times Y$  AND CONSIDER THE PROJECTIONS  $P$  AND  $q$  FROM  $R$  TO  $X$  AND  $Y$  RESPECTIVELY, DEFINED BY THE FORMULAS  $p(x,y) = x$  AND  $q(x,y) = y$ . PROVE THAT  $R = p^{\text{op}} q$



$$p(x,y) = x, \quad q(x,y) = y$$

$$R = p^{\text{op}} q?$$

$$x p^{\text{op}} q y \Leftrightarrow \exists z (x p^{\text{op}} z \wedge z q y)$$

$$\Leftrightarrow \exists z (z p x \wedge z q y)$$

$$\Leftrightarrow \exists z (p(z) = x \wedge q(z) = y)$$

$$\Leftrightarrow \exists z \in R (z = (x,y))$$

$$\Leftrightarrow x R y$$