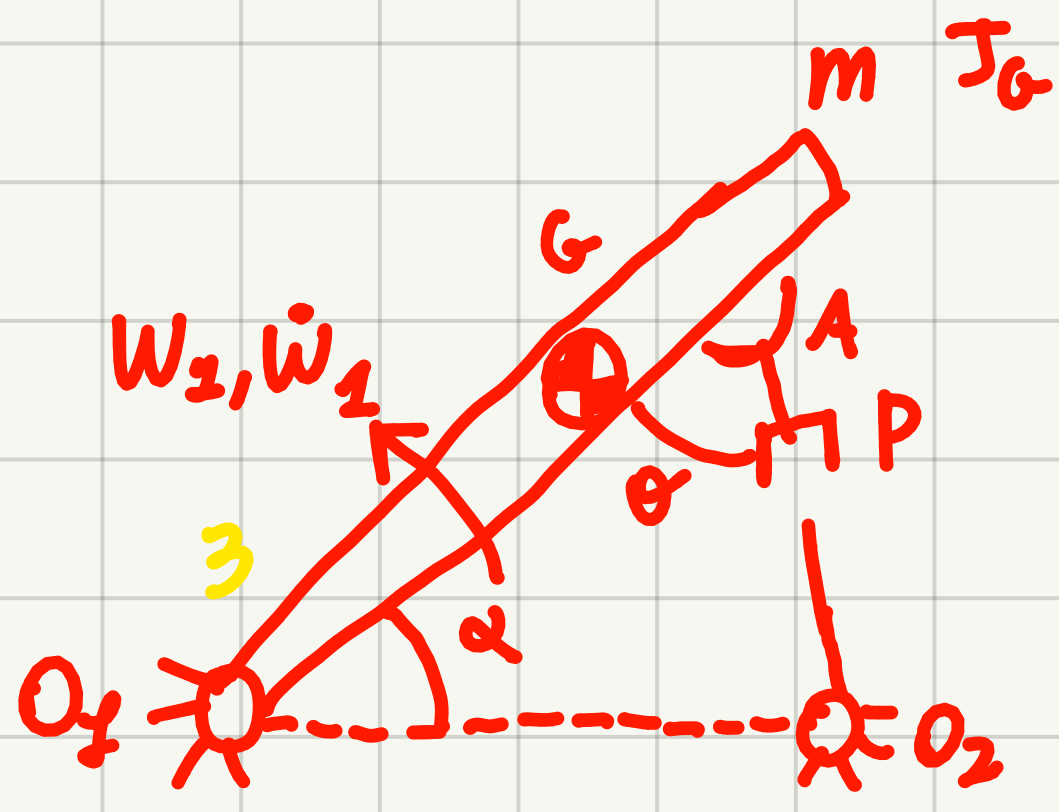
DINAMICA

① CONSIDERARE IL SEGUENTE SISTEMA. CON LE INFORMAZIONI NOTE, DETERMINARE VELOCITÀ E ACCELERAZIONE DEL PUNTO P E LE FORZE PER GARANTIRE IL MOTO

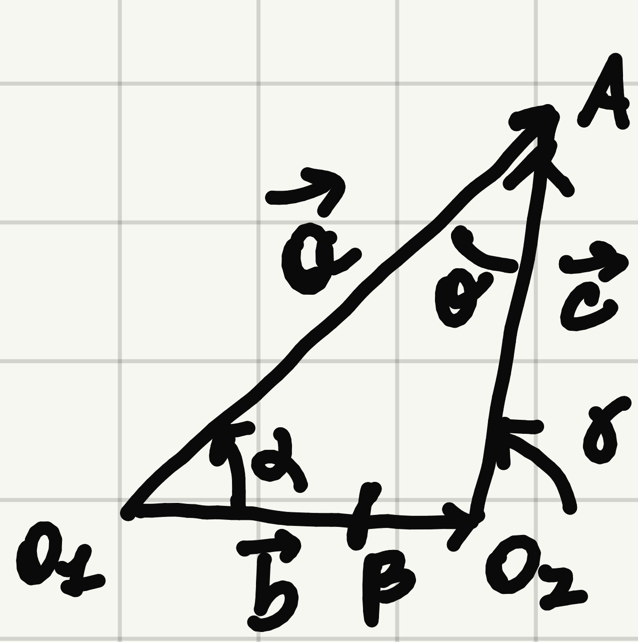


$m = 30\,kg \quad J_G = 2{,}5\,kgm^2$

$\overline{O_1G} = 1m \quad \overline{O_1A} = 1{,}5m \quad \theta = 5^\circ \quad \alpha = 30^\circ \quad \omega_1 = 0{,}5\,rad/s \quad \dot{\omega}_1 = 0{,}5\,rad/s^2$

$$n = 3\cdot 3 - (2_{O_1} + 2_{O_2} + 2_A + 2_P) = 1$$

$\vec{v}_P$ E $\vec{a}_P$ CON LA CINEMATICA



b COSTANTE

$\beta = 0$ COSTANTE

$$\begin{cases} a\cos\alpha = b\cos\beta + c\cos\gamma \\ a\sin\alpha = b\sin\beta + c\sin\gamma \end{cases}$$

$\gamma = 180 - (180 - \alpha - \theta) = 35^\circ \quad \Rightarrow \quad b = 0{,}228\,m \quad c = 1{,}308\,m$

$$\begin{cases} -a\dot{\alpha}\sin\alpha = \dot{c}\sin\gamma - c\dot{\gamma}\sin\gamma \\ a\dot{\alpha}\cos\alpha = \dot{c}\sin\alpha + c\dot{\gamma}\cos\alpha \end{cases}$$

$$\begin{vmatrix} \cos\gamma & -c\sin\gamma \\ \sin\gamma & c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \dot{c} \\ \dot{\gamma} \end{vmatrix} = \begin{vmatrix} -a\dot{\alpha}\sin\alpha \\ a\dot{\alpha}\cos\alpha \end{vmatrix} \qquad \det A = c$$

$$\dot{c} = \frac{1}{c}\cdot \det \begin{vmatrix} -a\dot{\alpha}\sin\alpha & -c\sin\gamma \\ a\dot{\alpha}\cos\alpha & c\cos\gamma \end{vmatrix} = 0{,}065\,m/s$$

$$\dot\gamma = \frac{1}{c}\cdot \det\begin{vmatrix}\cos\gamma & -a\dot\alpha\sin\alpha \\ \sin\gamma & a\dot\alpha\cos\alpha\end{vmatrix} = 0{,}5712\ rad/s \qquad \dot c = |\vec v_p|$$

$$\begin{cases} -a\ddot\alpha\sin\gamma - a\dot\alpha^2\cos\alpha = \ddot c\cos\gamma - \dot c\dot\gamma\sin\gamma - \dot c\dot\gamma\sin\gamma - c\ddot\gamma\sin\gamma - c\dot\gamma^2\cos\gamma \\ a\ddot\alpha\cos\alpha - a\dot\alpha^2\sin\alpha = \ddot c\sin\gamma + \dot c\dot\gamma\cos\gamma + \dot c\dot\gamma\cos\gamma + c\ddot\gamma\cos\gamma - c\dot\gamma^2\sin\gamma \end{cases}$$

DA NOTARE I TERMINI CHE SI SOMMANO. SE SI NOTA SI ANNULLINO, CI SONO STATI ERRORI DI CALCOLO

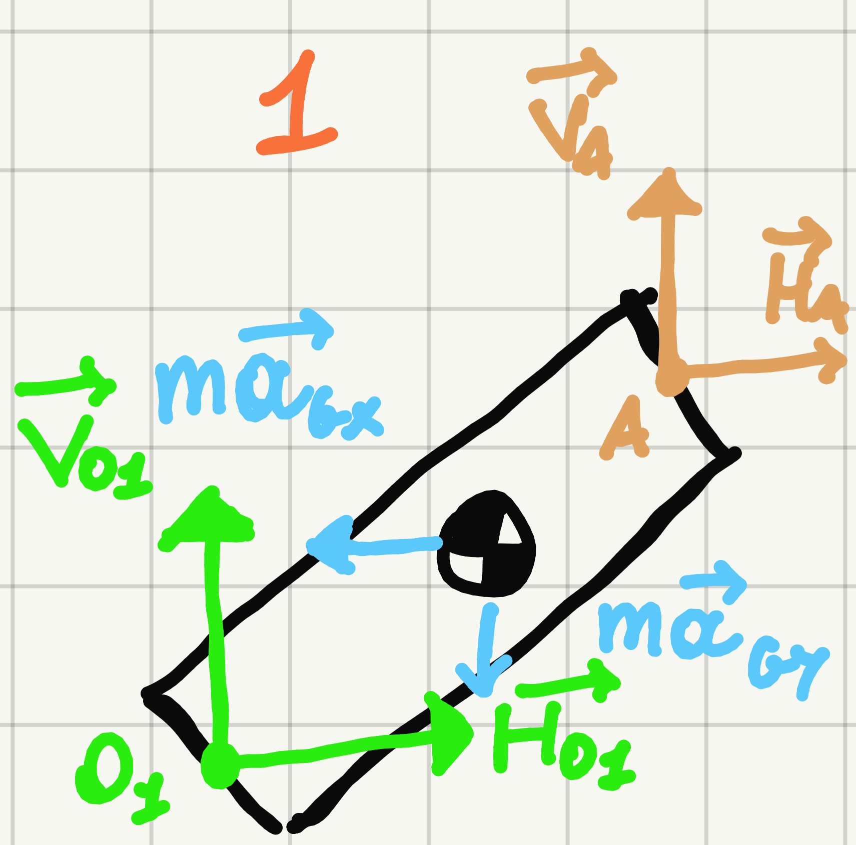
$$\rightarrow \begin{cases} \ddot c = 0{,}119\ m/s^2 \\ \ddot\gamma = 0{,}54\ rad/s^2 \end{cases} \qquad \ddot c = |\vec a_p|$$

DINAMICA (D'ALEMBERT)

TENERE IN CONSIDERAZIONE REAZIONI VINCOLARI, FORZE ESTERNE E FORZE D'INERZIA ($\vec F_{in} = -m\vec a$; $\vec C_{in} = -J_G\dot{\vec\omega}$)

SUCCESSIVAMENTE, STUDIARE I SISTEMI DI FORZE PER OGNI CORPO

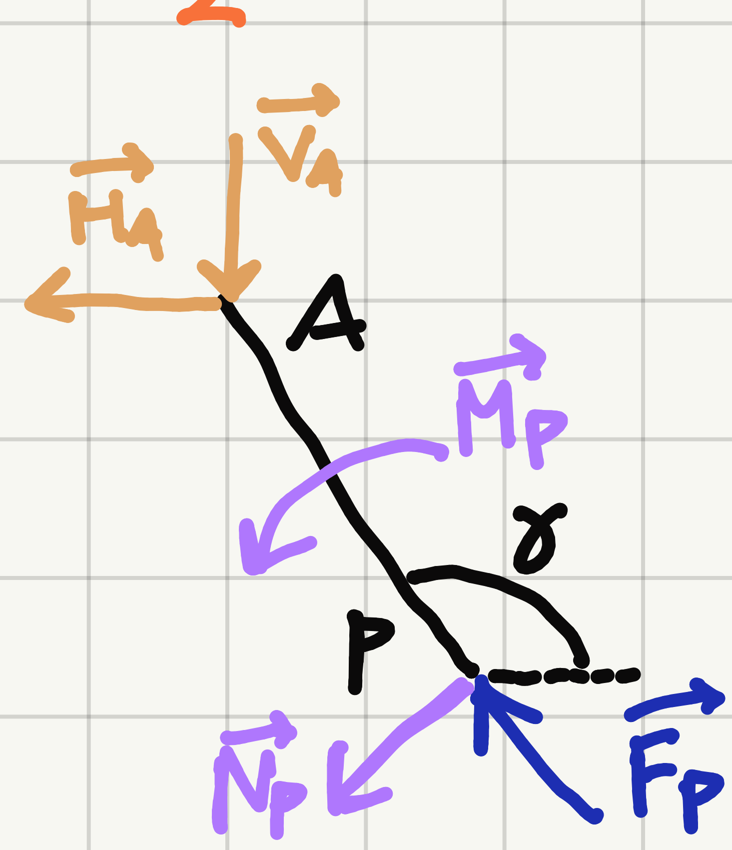
1



NEL BARICENTRO, SUPPORRE FORZE COI VERSI OPPOSTI ALLE CONVENZIONI

$$\begin{array}{l} x \\ y \\ M_{O_1} \end{array}\begin{cases} H_{O1} + H_A - m a_{Gx} = 0 \\ V_{O1} + V_A - m a_{Gy} = 0 \\ -J_G\dot\omega_1 + m a_{Gy}\cdot\overline{O_1G}\cos(\pi-\alpha) + \\ \quad + m a_{Gx}\overline{O_1G}\sin\alpha + V_A\cdot\overline{O_1A}\cos\alpha - H_A\cdot\overline{O_1A}\sin\alpha = 0 \end{cases}$$

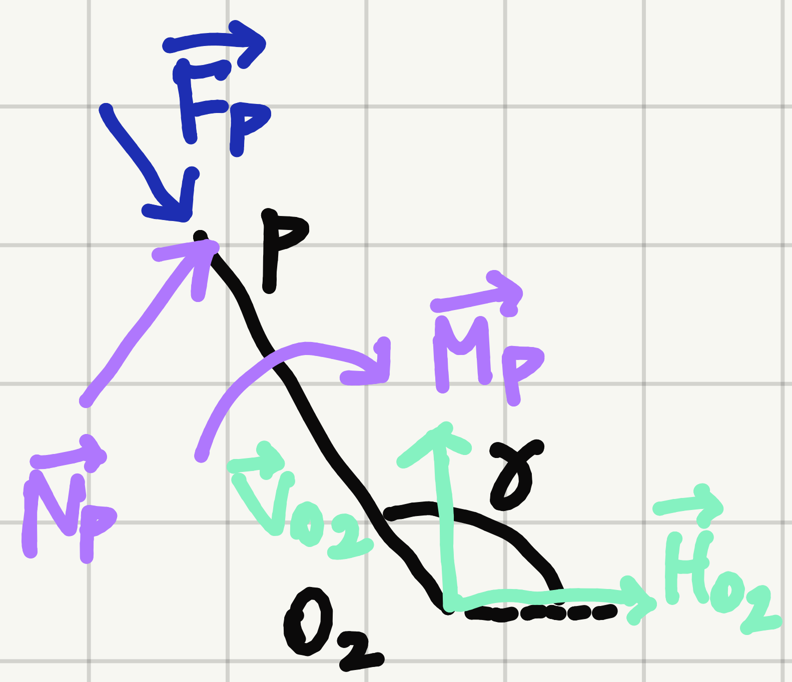
2



$$\begin{cases} -H_A + F_p\cos\gamma - N_p\sin\gamma = 0 \\ -V_A + F_p\sin\gamma + N_p\cos\gamma = 0 \\ M_p - N_p AP = 0 \end{cases}$$

! IPOTESI $\gamma \le \pi/2$

3



$$\begin{cases} H_{O_2} - F_P\cos\gamma + N_P\sin\gamma = 0 \\ V_{O_2} - F_P\sin\gamma - N_P\cos\gamma = 0 \\ -M_P - N_P\overline{O_2P} = 0 \end{cases}$$

OTTENIAMO UN SISTEMA FINALE DI 9 EQUAZIONI IN 9 INCOGNITE

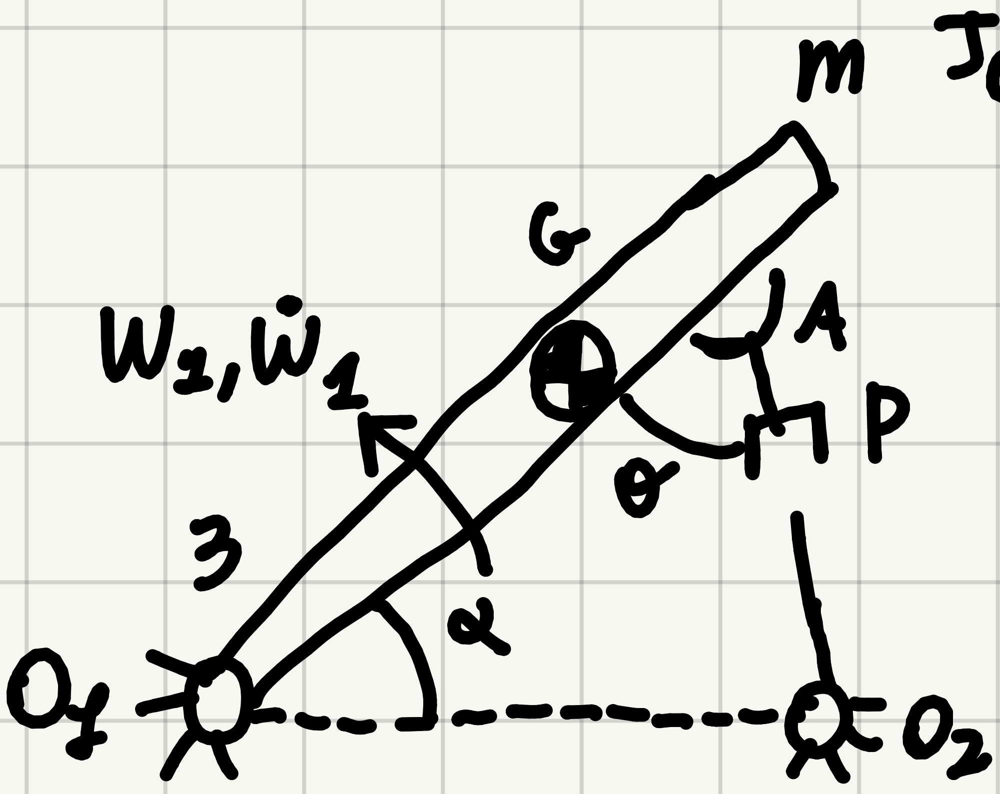
INIZIAMO OSSERVANDO CHE, ESSENDO P ANCORATA A TERRA, $M_P = N_P = 0$

DA 2 $\begin{cases} F_P\cos\gamma = H_A \\ F_P\sin\gamma = V_A \end{cases}$ $\gamma = 35°$, COME VISTO IN PRECEDENZA

$$\Rightarrow J_G\dot{\omega}_1 - m a_{Gy}\overline{O_1G}\cos\alpha + m a_{Gx}\overline{O_1G}\sin\alpha + F_P\overline{O_1A}\sin\gamma\cos\alpha -$$

$$-F_P\overline{O_1A}\cos\gamma\sin\alpha = 0 \Rightarrow F_P = 124{,}23\,N$$

DINAMICA (ENERGIA)



$$\frac{d}{dt}K = m(\vec{a}_G \cdot \vec{v}_G) + J_G(\vec{\dot{\omega}}_1 \cdot \vec{\omega}_1)$$

$$= m(a_{Gx}\hat{i} + a_{Gy}\hat{j}) \cdot (v_{Gx}\hat{i} + v_{Gy}\hat{j}) +$$

$$+ J_G(\dot{\omega}_1\hat{k}) \cdot (\omega_1\hat{k}) = m a_{Gx} v_{Gx} + m a_{Gy} v_{Gy} +$$

$$+ J\dot{\omega}_1\omega_1$$

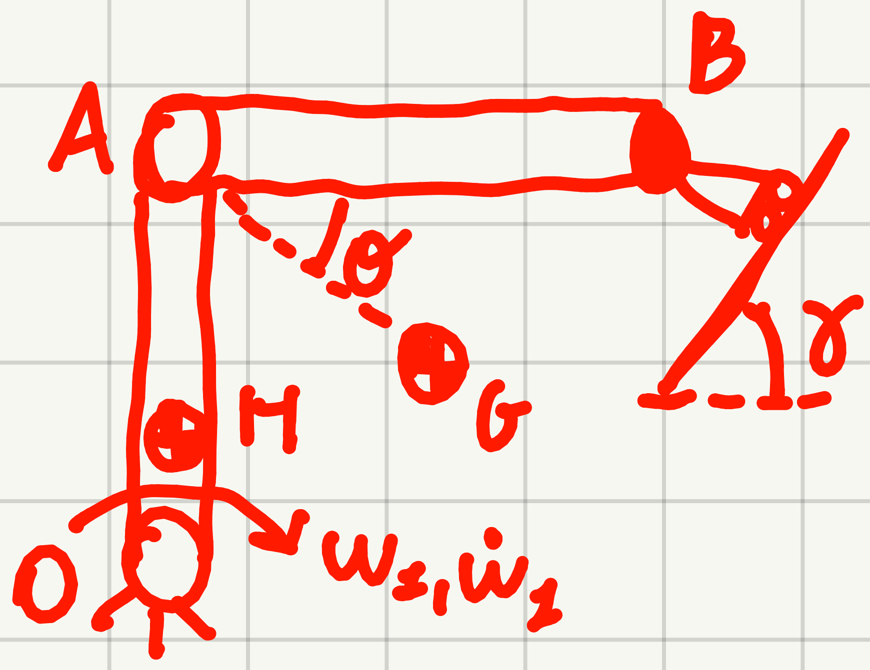
$$\sum P_{ATTIVE} = \vec{F}_P \cdot \vec{v}_P = F_P v_P$$

! I VINCOLI, AVENDO VELOCITÀ NULLA, NON GENERANO POTENZA

$$\sum P_{ATTIVE} = \frac{d}{dt}K \rightarrow F_P v_P = m a_{Gx} v_{Gx} + m a_{Gy} v_{Gy} + J_G\dot{\omega}_1\omega_1$$

$$\Rightarrow F_P = 124{,}23\,N$$

② CONSIDERARE IL SEGUENTE SISTEMA. CON LE INFORMAZIONI NOTE, DETERMINARE VELOCITÀ E ACCELERAZIONE DEL PUNTO B E LA COPPIA IN OA PER GARANTIRE IL MOTO

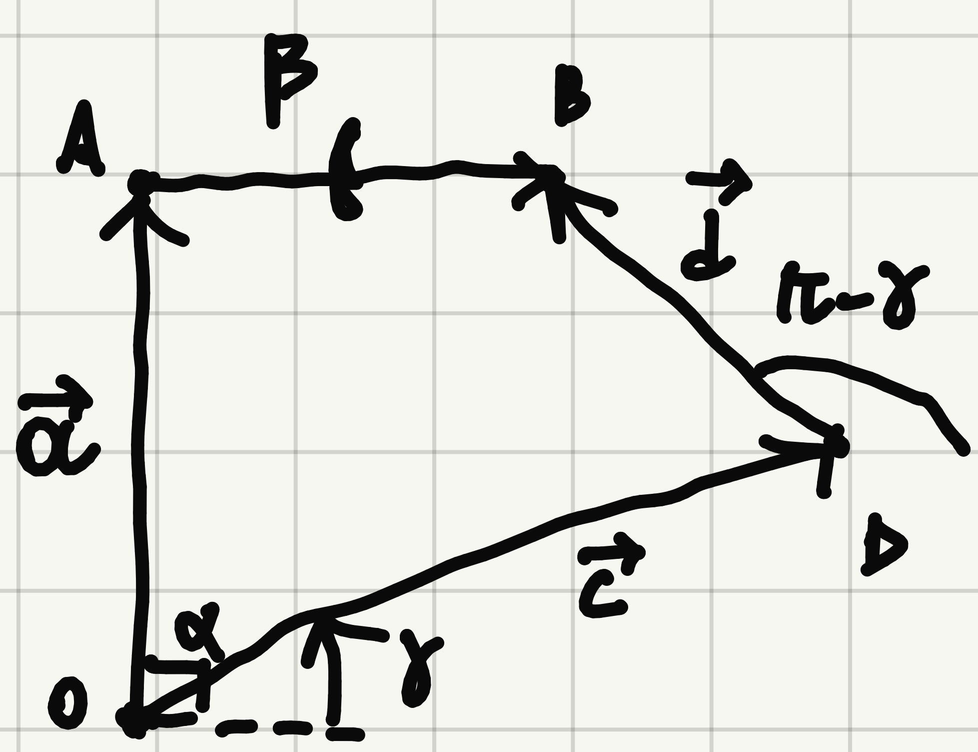


$OA = 0{,}45\,m$ $AB = 0{,}43\,m$ $AG = 0{,}1\,m$ $OH = 0{,}15\,m$ $\omega_1 = 0{,}5\,rad/s$

$\dot{\omega}_1 = 1{,}2\,rad/s^2$ $m_B = 40\,kg$ $m_{OA} = 10\,kg$ $J_{OA} = 0{,}3\,kg \cdot m^2$ $J_{AB} = 0{,}25\,kg \cdot m^2$

$n = 3 \cdot 2 - (2_O + 2_A + 1_B) = 1$

CINEMATICA



a, b, d, γ FISSI

c, α, β NON FISSI

RISOLVENDO, SI OTTENGONO I RISULTATI SEGUENTI

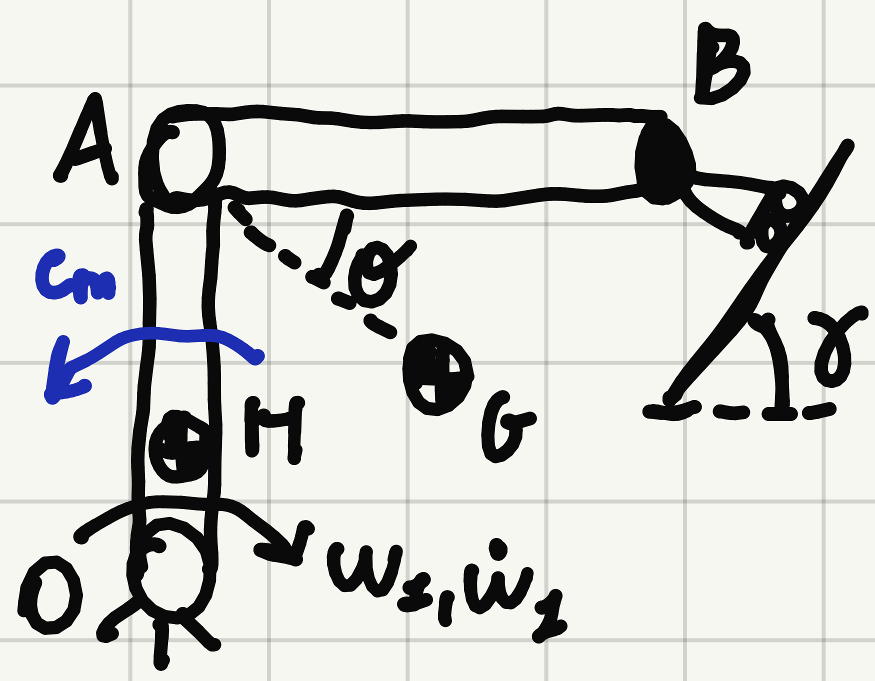
$$\begin{cases} \dot{\beta} = 0{,}3\,rad/s \\ \dot{c} = 0{,}26\,m/s \end{cases} \qquad |\vec{v}_B| = \dot{c}$$

$$\begin{cases} \ddot{\beta} = 0{,}93\,rad/s^2 \\ \ddot{c} = 0{,}58\,m/s^2 \end{cases} \qquad |\vec{a}_B| = \ddot{c}$$

$$\vec{a}_G = \vec{a}_A + \vec{\dot{\omega}}_{AB} \times (G-A) - \omega_{AB}^2 (G-A)$$

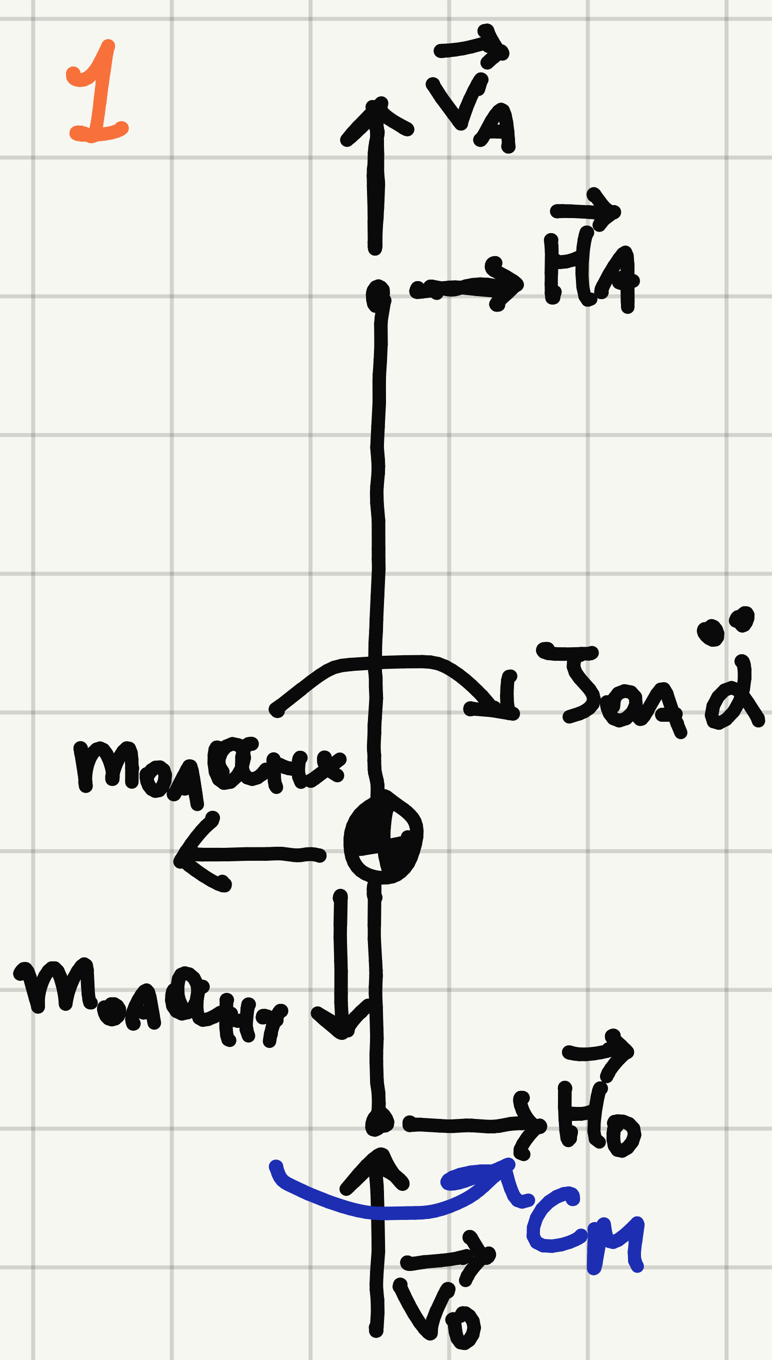
$$\vec{a}_A = \vec{a}_O + \vec{\dot{\omega}} \times (A-O) - \omega^2 (A-O)$$

DINAMICA (D'ALEMBERT)



VOGLIAMO CHE L'ASTA OA (E QUINDI OB) NON CAMBI LA SUA INCLINAZIONE

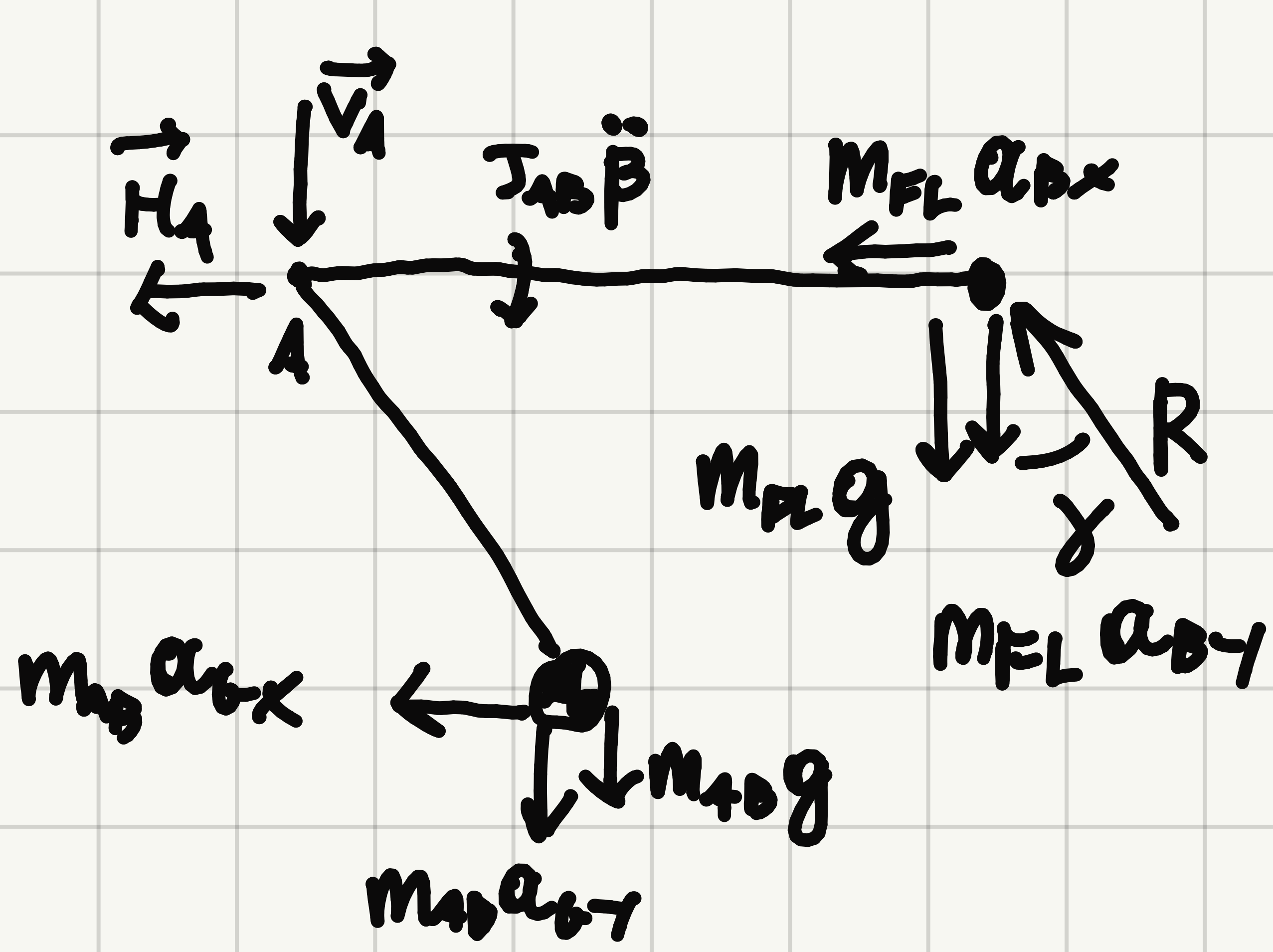
1



$$\begin{cases} H_A + H_O - m_{OA}\, a_{Hx} = 0 \\ V_O + V_A - m_{OA}\, a_{Hy} = 0 \\ C_m - J_{OA}\ddot{\alpha} + m_{OA}\, a_{Hx}\, \overline{OH} - H_A\, \overline{OA} = 0 \end{cases}$$

INO

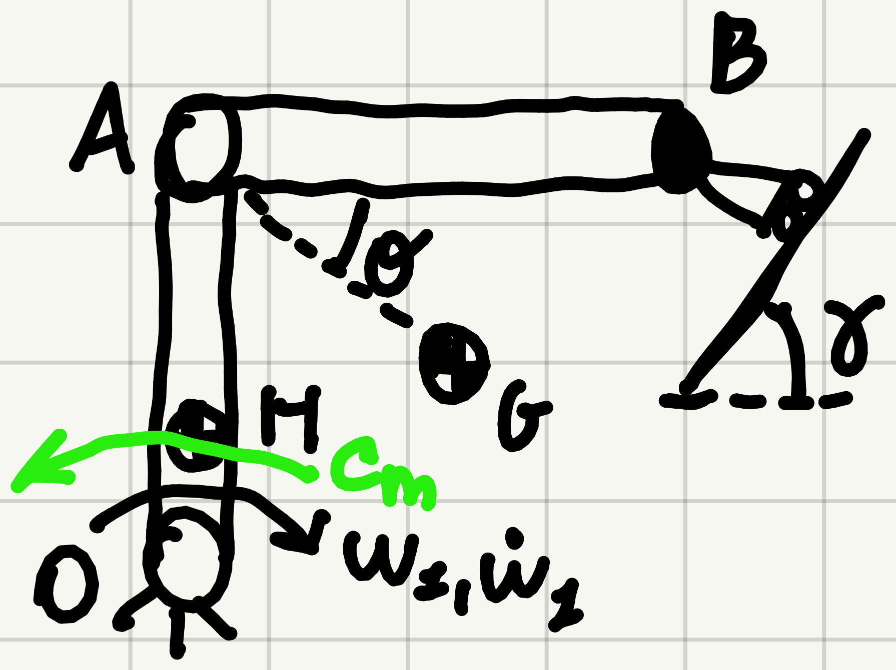
2



$$\begin{cases} -H_A - R\sin\gamma - m_{AB}\, a_{Gx} - m_{FL}\, a_{Bx} = 0 \\ -V_A + R\cos\gamma - m_{AB}(g + a_{Gy}) - m_{FL}(g + a_{By}) = 0 \\ -J_{AB}\ddot{\beta} - m_{AB}\, a_{Gx}\, \overline{AG}\sin\theta - m_{AB}\,(a_{Gy} + g)\,\overline{AG}\cos\theta - \\ \quad - m_{FL}\,(g + a_{By})\,\overline{AB} + R\,\overline{AB}\cos\gamma = 0 \end{cases}$$

$$\Rightarrow C_M = -118{,}8\ \mathrm{N\cdot m} \Rightarrow \vec{C_M} = -118{,}8\,\hat{k}\ \mathrm{N\cdot m}$$

DINAMICA (ENERGIA)



$$\frac{d}{dt}K = \frac{d}{dt}K^{(OA)} + \frac{d}{dt}K^{(AB)} + \frac{d}{dt}K^{(B)}$$

$$\frac{d}{dt}K^{(OA)} = m_{OA}(\vec{a}_H \cdot \vec{v}_H) + J_{OA}(\vec{\omega}_{OA} \cdot \vec{\omega}_{OA})$$

$$= m_{OA}\, a_{Hx} v_{Hx} + m_{OA}\, a_{Hy} v_{Hy} + J_{OA}\ddot\alpha\dot\alpha \qquad \ddot\alpha = -\dot\omega_1,\ \dot\alpha = -\omega_1$$

Analogamente, $\frac{d}{dt}K^{(AB)} = m_{AB}\, a_{Gx} v_{Gx} + m_{AB}\, a_{Gy} v_{Gy} + J_{OA}\ddot\beta\dot\beta$

$\frac{d}{dt}K^{(B)} = m_{FL}(\vec{a}_B \cdot \vec{v}_B) + \cancel{J(\vec{\omega}_B \cdot \vec{\omega}_B)}$ MASSA PUNTIFORME => NO ROTAZIONE

$$= m_{FL}\, a_B v_B$$

$$\sum P_{attive} = P^{(OA)} + P^{(AB)} + P^{(B)} = (m_{OA}\vec{g}\cdot\vec{v}_H + \vec{C}_m \cdot \vec{\omega}_{OA}) +$$

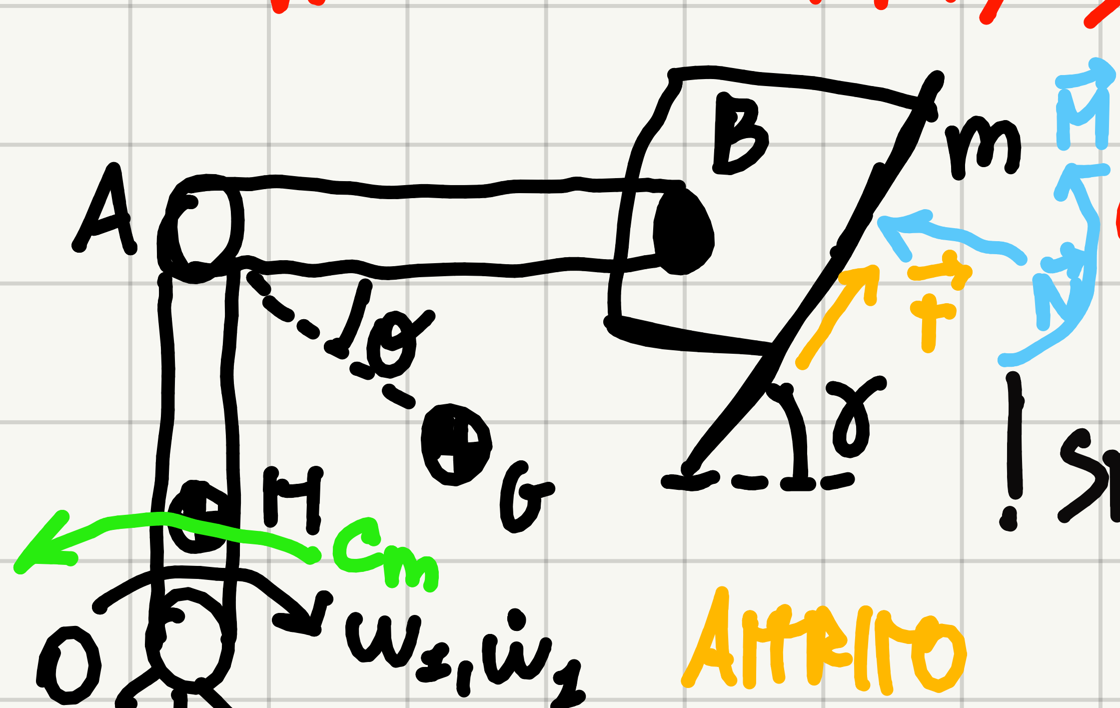
$$+(m_{AB}\vec{g}\cdot\vec{v}_G) + (m_{FL}\vec{g}\cdot\vec{v}_B) = (m_{OA}(g\hat{j}\cdot(-v_{Hx}\hat{i} + v_{Hy}\hat{j})) +$$

$$+ C_m\hat{k}\cdot\dot\alpha\hat{k}) + m_{AB}(g\hat{j}\cdot(v_{Gx}\hat{i} + v_{Gy}\hat{j})) + m_{FL}(g\hat{j}\cdot(v_{Bx}\hat{i} +$$

$$+ v_{By}\hat{j})) = -m_{OA}\, g\, v_{Hy} - m_{AB}\, g\, v_{Gy} - m_{FL}\, g\, v_{By} + C_m\dot\alpha$$

$\sum P_{attive} = \frac{d}{dt}K \rightarrow C_M = -118{,}8\ N\cdot m \Rightarrow \vec{C}_M = -118{,}8\hat{k}\ N\cdot m$

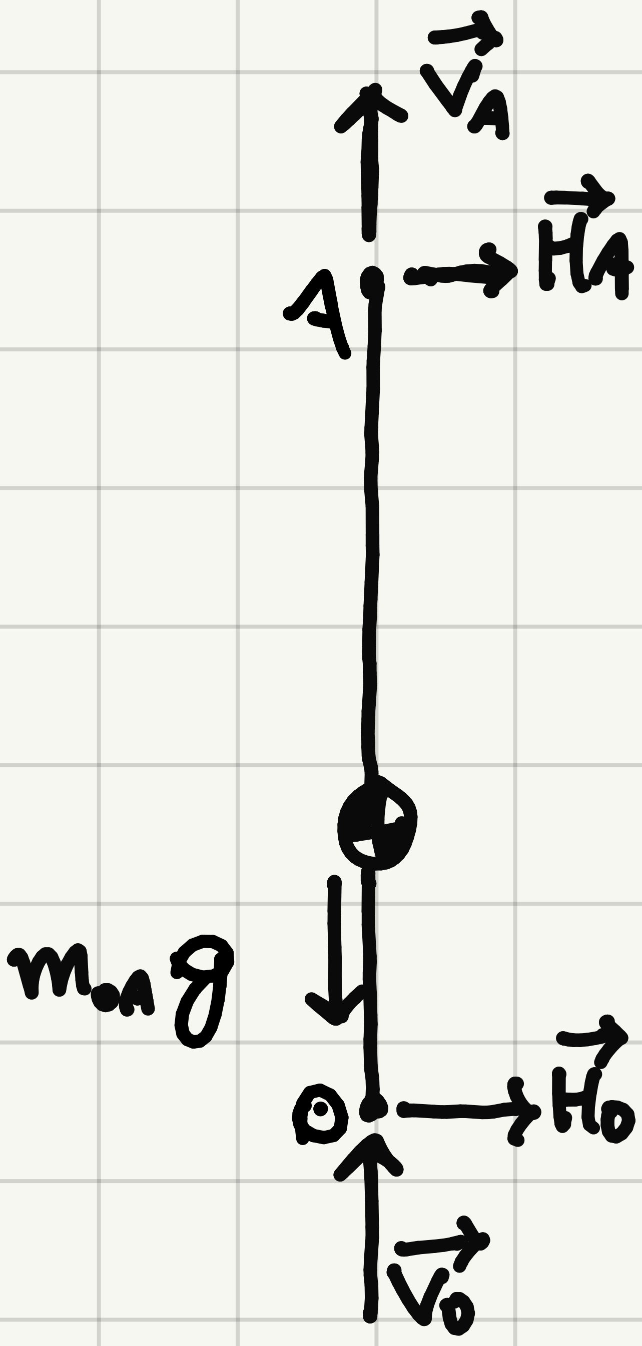
- PUNTO EXTRA: VERIFICA DI ADERENZA. SOSTITUIRE IL CARRELLO B CON UNA MASSA DI μ_s (MASSA-PIANO) PER CUI $m = m_B$ E VERIFICARE CHE SIA RISPETTATA LA STATICA



! STATICA => POICHÈ $v, \omega = 0$, C_{in} E F_{in} NON RIENTRANO NEI CALCOLI

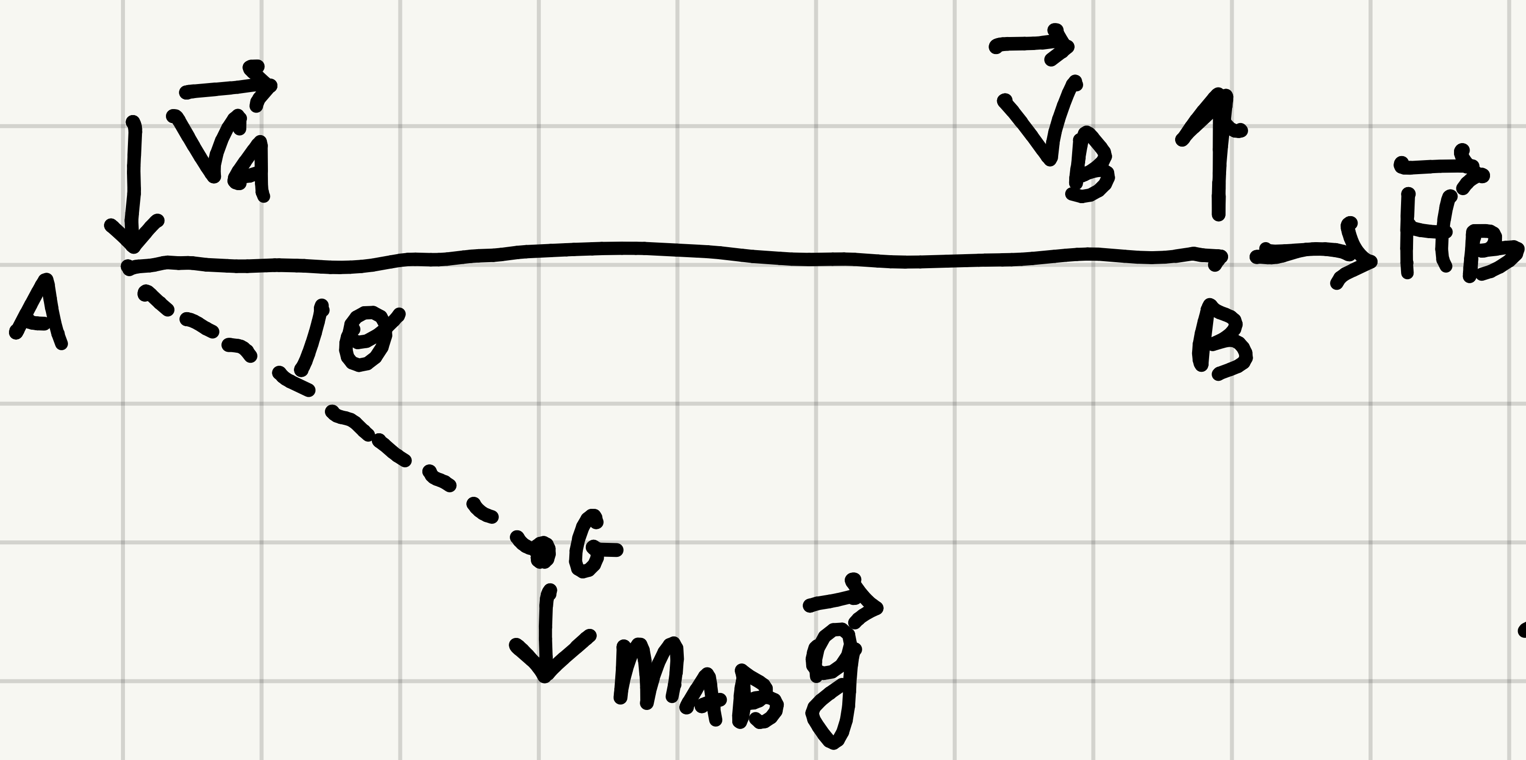
ATTRITO

VINCOLI DI PATTINO



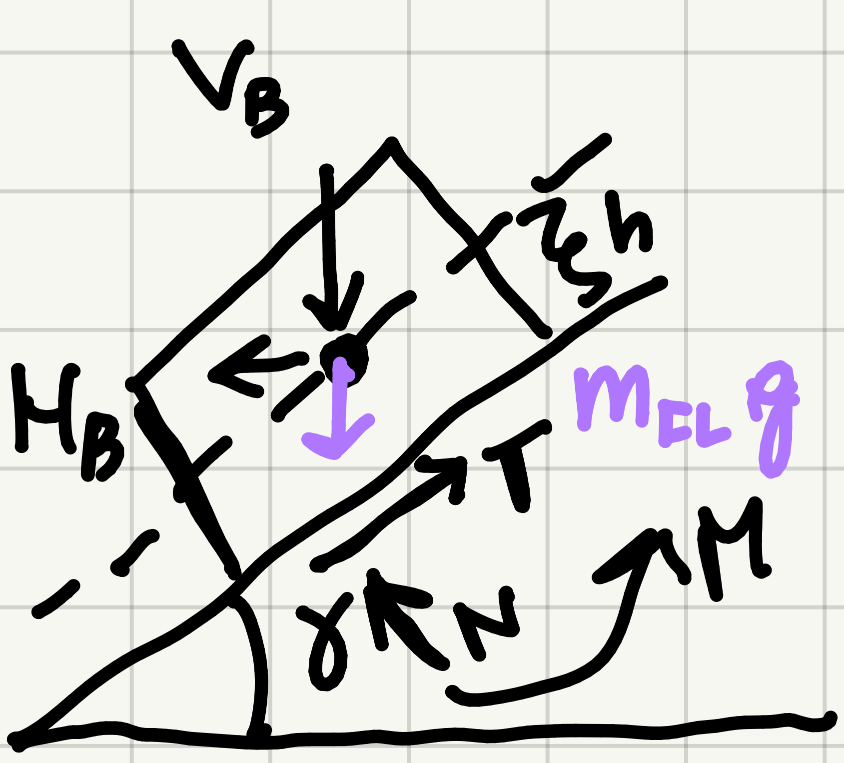
$$\begin{cases} H_0 + H_A = 0 \\ V_A + V_0 - m_{OA}\, g = 0 \\ O:\; -H_A\, OA = 0 \quad \Rightarrow H_A = 0 \end{cases}$$

$$\begin{cases} H_0 = 0 \\ V_1 + V_0 - m_{OA}\, g = 0 \\ H_A = 0 \end{cases}$$



$$\begin{cases} H_B = 0 \\ -V_A + V_B - m_{AB}\, g = 0 \\ A:\; V_B \overline{AB} - m_{AB}\, g\, \overline{AG} \cdot \cos\theta = 0 \end{cases}$$

$$\begin{cases} V_A = m_{AB}\, g \left(\dfrac{AG}{AB} \cos\theta - 1 \right) \\ V_B = m_{AB}\, g \cdot \dfrac{AG}{AB} \cos\theta \end{cases}$$



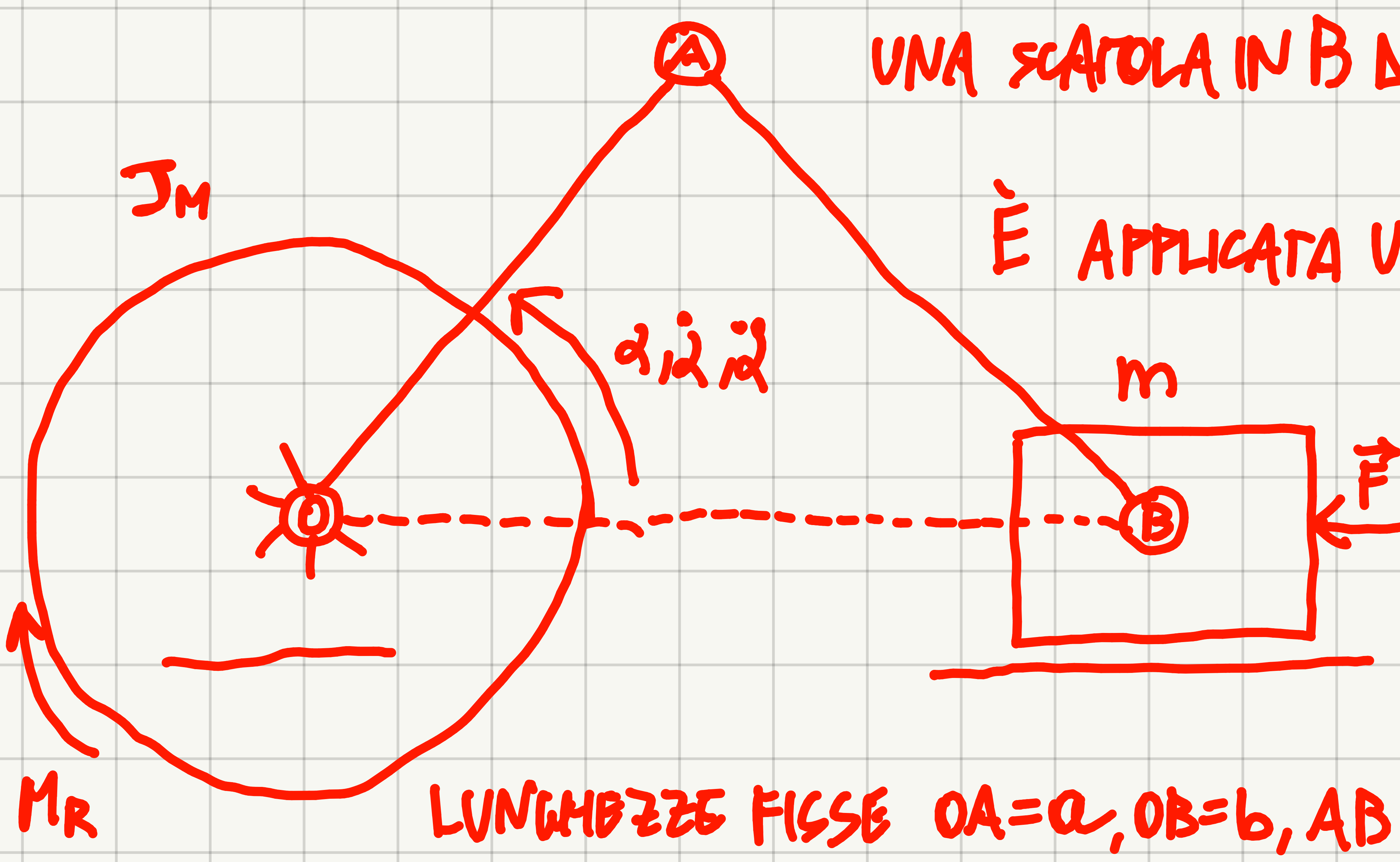
$$\begin{cases} \Sigma F_{\parallel\gamma} = 0 \\ \Sigma F_{\perp\gamma} = 0 \\ \Sigma M_B = 0 \end{cases} \qquad \begin{cases} T - (m_{FL}\, g + V_B) \sin\gamma = 0 \\ N - (m_{FL}\, g + V_B) \cos\gamma = 0 \\ \cancel{M + Fh} = 0 \end{cases}$$

IRRILEVANTE PER L'ESERCIZIO

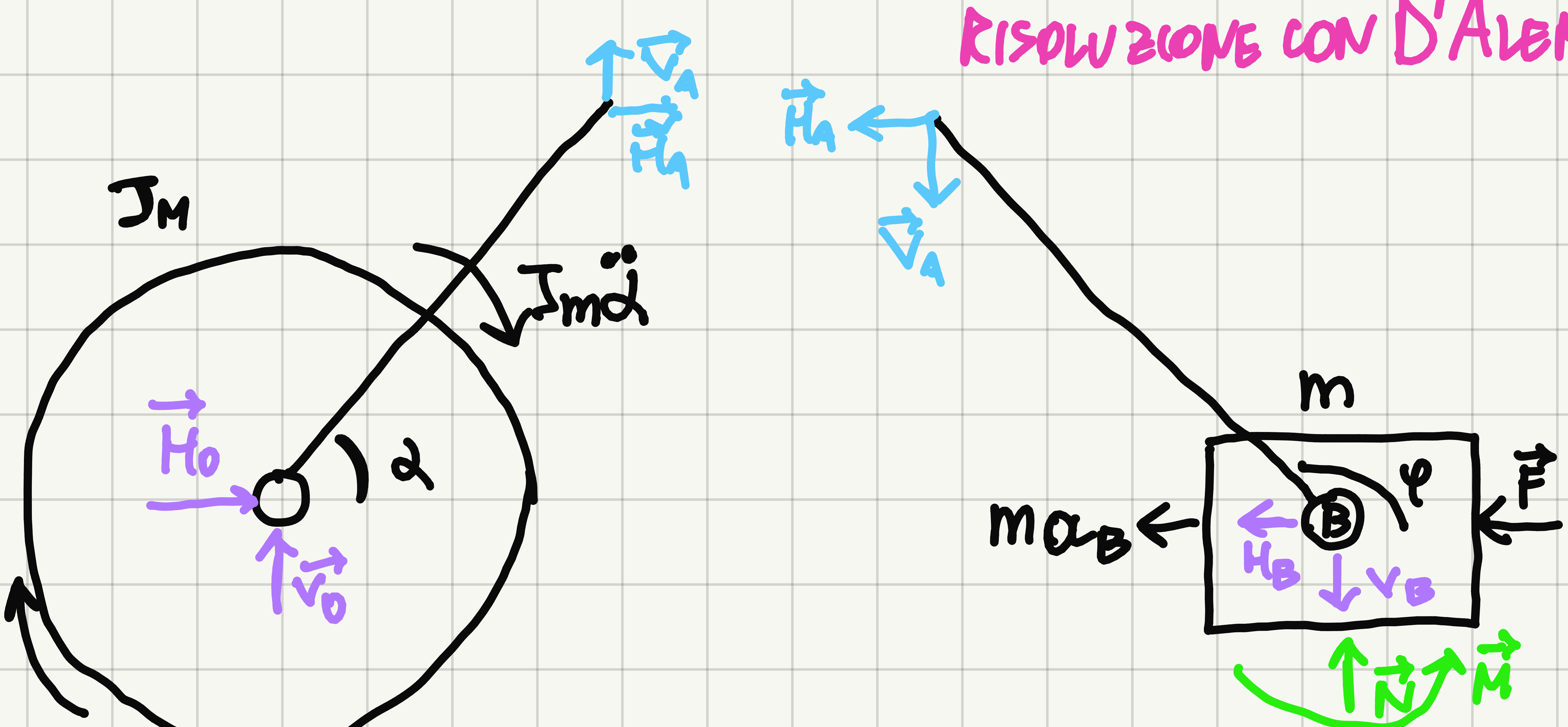
$$\begin{cases} T = 201 N \\ N = 349 N \end{cases}$$

$T_{lim} = \mu_s \cdot |\vec{N}| = 279 \qquad T \leq T_{lim}$ ✓ $\Rightarrow$ ADERENZA VERIFICATA

Consideriamo un manovellismo composto da un disco saldato all'asta OA e una scatola in B di massa m. Al sistema è applicata una coppia M_R e la forza $\vec{F}$. Siano noti i valori di $\alpha, \dot{\alpha}, \ddot{\alpha}$, le lunghezze fisse $OA = a$, $OB = b$, $AB = c$. Supponendo trascurabili le masse delle aste, calcolare il momento M_R applicato al disco in modo che venga garantito il moto assegnato.



Risoluzione con D'Alembert

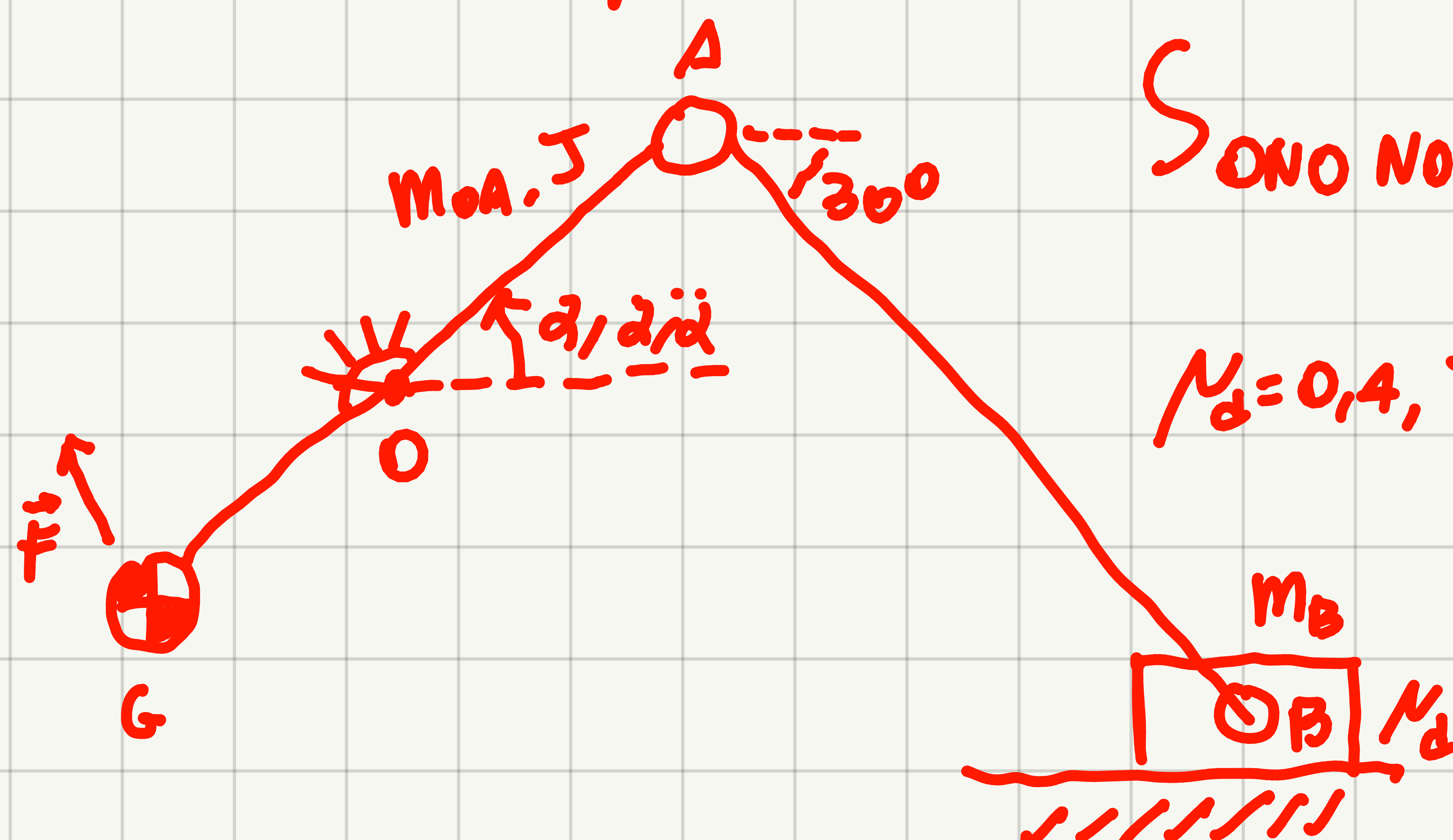


φ da teorema dei seni $\frac{c}{\sin\alpha} = \frac{a}{\sin(\pi-\varphi)}$

$$
\begin{cases}
H_0 + H_A = 0 \\
V_0 + V_A = 0 \\
M_R + J_M \ddot{\alpha} + H_A a \sin\alpha - V_A a \cos\alpha = 0 \\
-H_A - H_B = 0 \\
-V_A - V_B = 0 \\
V_B b \cos(\pi - \varphi) + H_B b \\
-m a_B - H_B - F = 0 \\
N - V_B = 0 \\
M = 0
\end{cases}
$$

SI CONSIDERI IL SEGUENTE SISTEMA COMPOSTO DA:

- ASTA OA DOTATA DI MASSA m_{OA} E CON BARICENTRO IN G
- ASTA AB DI MASSA TRASCURABILE
- BLOCCO IN B DI MASSA m_B CHE SCORRE SU UN PIANO E SOGGETTA AD ATTRITO DINAMICO μ_d



SONO NOTI $OA = 1m$, $AB = 2m$, $OG = 0{,}5m$

$\mu_d = 0{,}4$, $J = 0{,}05\ Kg \cdot m^2$, $m_{OA} = 2Kg$

$m_B = 10Kg$, $\alpha = 60°$,

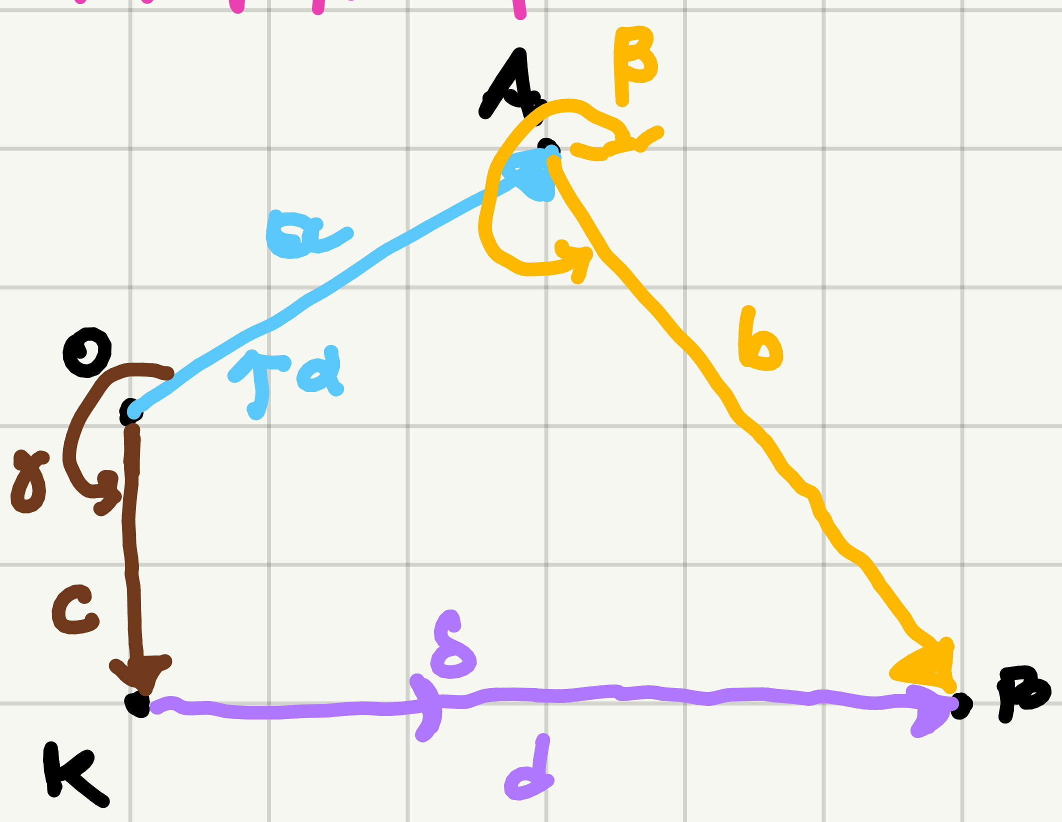
$\dot{\alpha} = 1\ rad/s$, $\ddot{\alpha} = 1\ rad/s^2$

SI CALCOLINO: $\vec{v}_B, \vec{a}_B, \vec{F}$

ESSENDO IN MOTO, CI ASPETTIAMO $n \geq 1$

$n = (3 \cdot 3) - (2_O + 2_A + 2_B + 2_{PATTINO}) = 1$ ✓

CINEMATICA



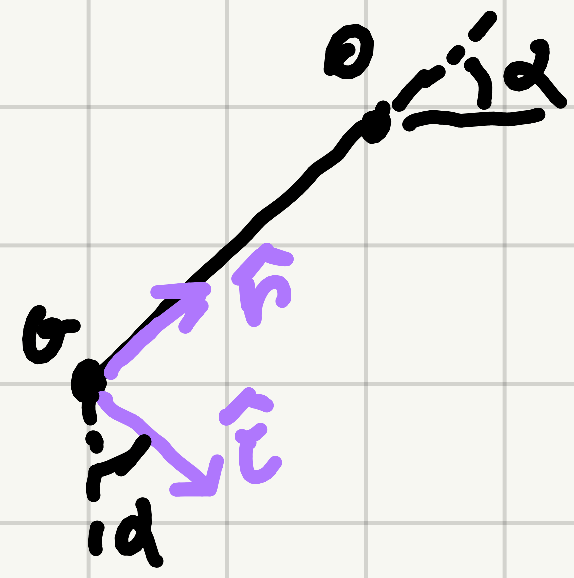
$\vec{a} + \vec{b} = \vec{c} + \vec{d}$

- a NOTO $\quad \alpha, \dot{\alpha}, \ddot{\alpha}$ NOTO
- b NOTO $\quad \beta = 360 - 30 = 330°$
  $\dot{\beta} \neq 0$; $\ddot{\beta} \neq 0$
- c ? $\quad \gamma = 270°$ FISSO
- d ? $\quad \delta = 0°$ FISSO

$$\begin{cases} a\cos\alpha + b\cos\beta = d \\ a\sin\alpha + b\sin\beta = -c \end{cases} \Rightarrow \begin{cases} d = 2{,}23m \\ c = 0{,}134m \end{cases}$$

$$\vec{v} \begin{cases} -a\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = \dot{d} \\ a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta = 0 \end{cases} \rightarrow \begin{cases} \dot{d} = -1{,}155\ m/s \\ \dot{\beta} = -0{,}289\ rad/s \end{cases} \Rightarrow \vec{v}_B = -1{,}155\vec{i}\ m/s$$

ANTICIPIAMO, PER LA PARTE DI DINAMICA, IL CALCOLO DI $\vec{v}_G$



$$\vec{v}_G = \vec{v}_O + \dot{\alpha}\,\overline{OG}\,\hat{t} = 0{,}5\hat{t}\ \text{m/s}$$

$$= 0{,}5\,\text{m/s}\,(-\cos\alpha\,\hat{j} + \sin\alpha\,\hat{i}) = (0{,}433\hat{i} - 0{,}25\hat{j})\,\text{m/s}$$

$$\vec{a}\begin{cases} -a\ddot{\alpha}\sin\alpha - a\dot{\alpha}^2\cos\alpha - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta = \ddot{d} \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta = 0 \end{cases}$$

$$\begin{cases} \ddot{d} = -1{,}3476\,\text{m/s}^2 \\ \ddot{\beta} = 0{,}1631\,\text{rad/s} \end{cases} \Rightarrow \vec{a}_B = -1{,}3476\hat{i}\ \text{m/s}^2$$

$$\vec{a}_G = \ddot{\alpha}\,\overline{OG}\,\hat{t} + \dot{\alpha}^2\,\overline{OG}\,\hat{n} = (0{,}5\hat{t} + 0{,}5\hat{n})\,\text{m/s}^2$$

DINAMICA

$$\frac{d}{dt}K = \frac{d}{dt}K^{(OA)} + \frac{d}{dt}K^{(B)}$$

- $$\frac{d}{dt}K^{(OA)} = m_{OA}(\vec{a}_G\cdot\vec{v}_G) + J(\dot{\vec{\omega}}\cdot\vec{\omega})$$
  $$= m_{OA}\,a_{Gt}\,v_{Gt} + \underbrace{m_{OA}\,a_{Gn}\,v_{Gn}}_{0} + J\dot{\alpha}\ddot{\alpha}$$

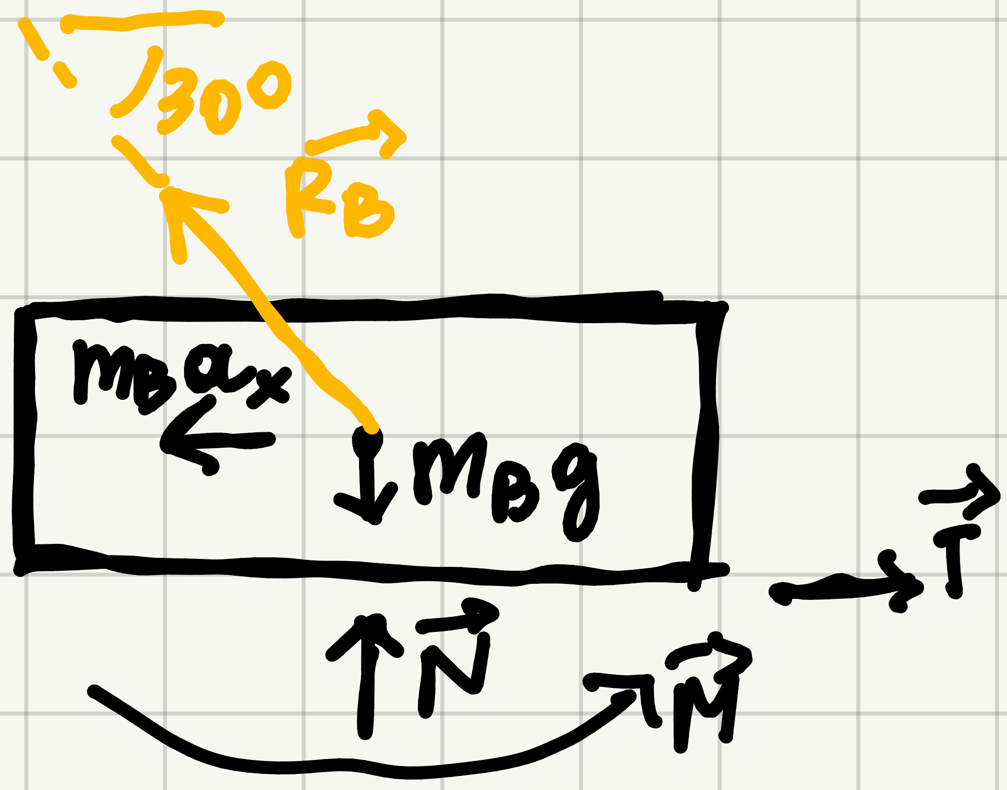
- $$\frac{d}{dt}K^{(B)} = m_B(\vec{a}_B\cdot\vec{v}_B) = m_B\,a_B\,v_B$$

NON SI MUOVE LUNGO LA VERTICALE

$$\sum P_{ATTIVE} = P^{(AG)} + \cancel{P^{(PESO)}} + P^{(F)} + P^{(ATTRITO)} =$$

$$= -m_{OA}\,g\,v_{Gy} - F a_{Gy} - T|v_B|$$

$$\vec{T} = -\mu_d N\frac{\vec{v}_{rel}}{|\vec{v}_{rel}|} \Rightarrow |\vec{T}| = -\mu_d N$$



ASTA SCARICA $\Rightarrow$ R_B INCLINATA DI 30°

$\Downarrow$

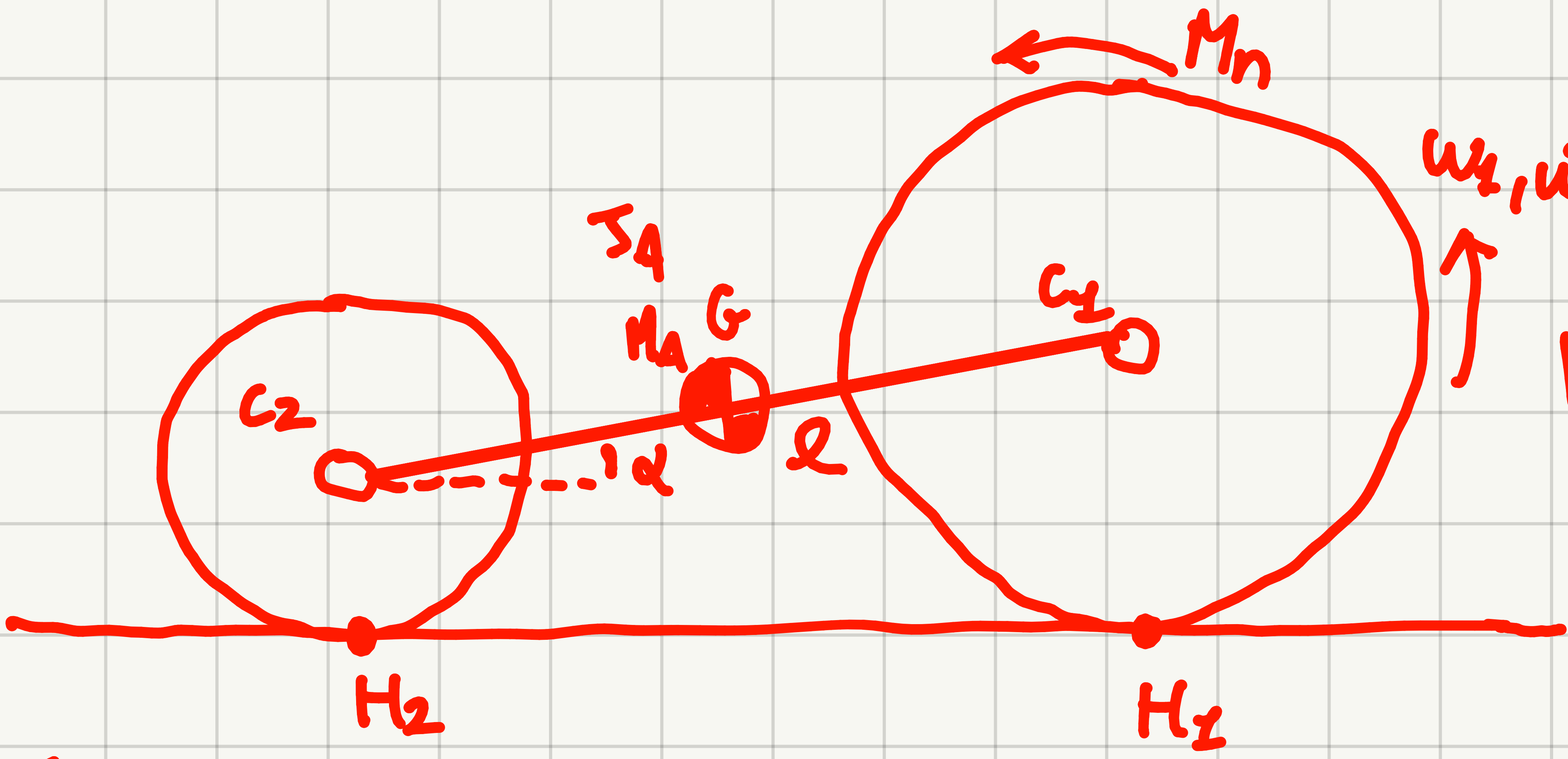
L'ASTA AB HA $m \approx 0$

$$\begin{cases} T - m_B a_{Bx} - R_B \cos 30 = 0 \\ N - m_B g + R_B \sin 30 = 0 \\ T = \mu_d N \end{cases} \Rightarrow \begin{cases} N = 73{,}374\,N \\ T = 29{,}354\,N \\ R_B = \text{QUALCOSA DI IRRILEVANTE} \end{cases}$$

$$\sum P_{ATTIVE} = \frac{d}{dt} K \qquad \Rightarrow F = -90{,}21\,N$$

$$\Rightarrow m_{OA} a_{Gt} v_{Gt} + J \ddot{\alpha} \dot{\alpha} + m_B a_B v_B = - m_{OA} g v_{Gy} - F v_{Gy} - T v_B$$

SI CONSIDERI IL SISTEMA RAFFIGURANTE UN TRATTORE IN MOVIMENTO, OSSERVANDO IN PARTICOLARE LE DIVERSE GRANDEZZE DELLE RUOTE



$R_1 = 0{,}3\,m$ $\quad R_2 = 0{,}2\,m$

$M_1 = 2\,Kg$ $\quad M_2 = 1{,}5\,Kg$

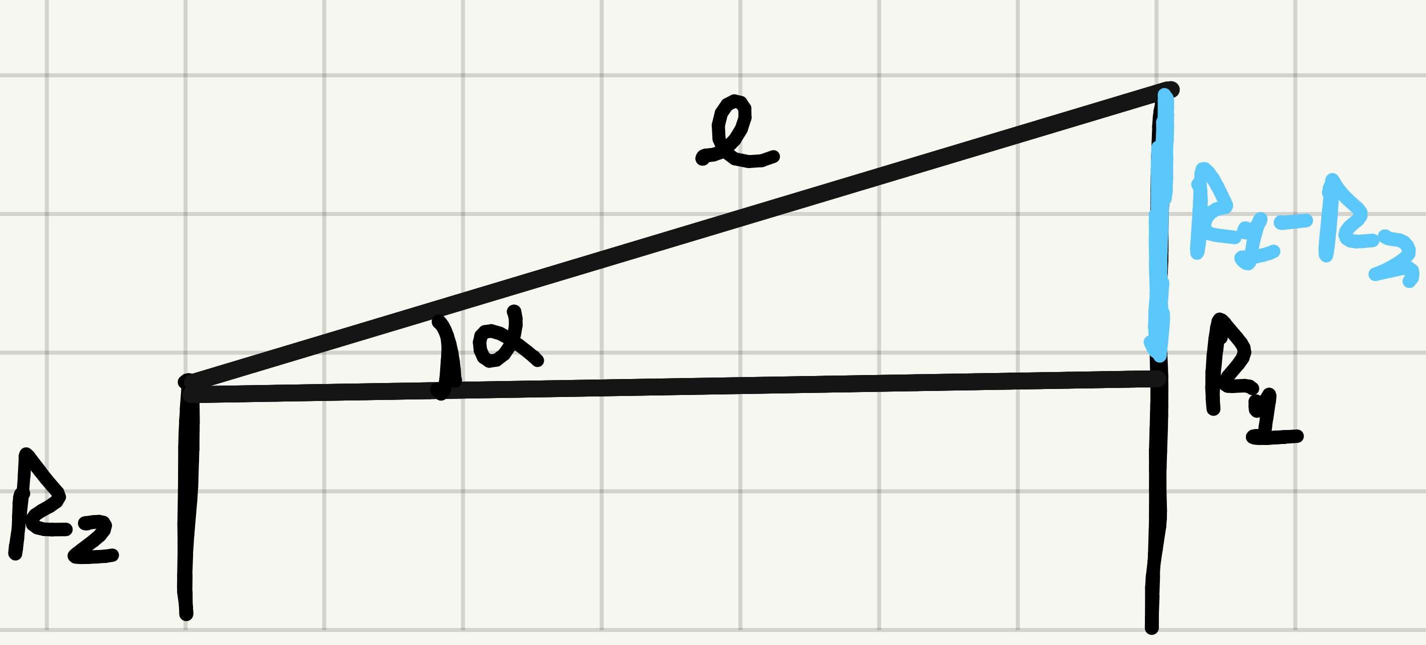
$M_A = 0{,}5\,Kg$ $\quad J_A = 0{,}8\,Kgm^2$

$J_1 = 0{,}5\,Kgm^2$ $\quad J_2 = 0{,}3\,Kgm^2$ $\quad \omega_1 = 1{,}5\,rad/s$ $\quad \dot{\omega}_1 = 0{,}3\,rad/s^2$ $\quad \alpha = 5°$ $\quad \mu_s = 0{,}8$

$\mu_d = 0{,}5$ $\quad \mu_v = 0{,}03$

CALCOLARE M_n E H_{H_2} IN CASISTICHE INDICATE

$$n = 3 \cdot 3 - (2_{C_2} + 2_{C_1} + 2_{H_2} + 2_{H_1}) = 1$$



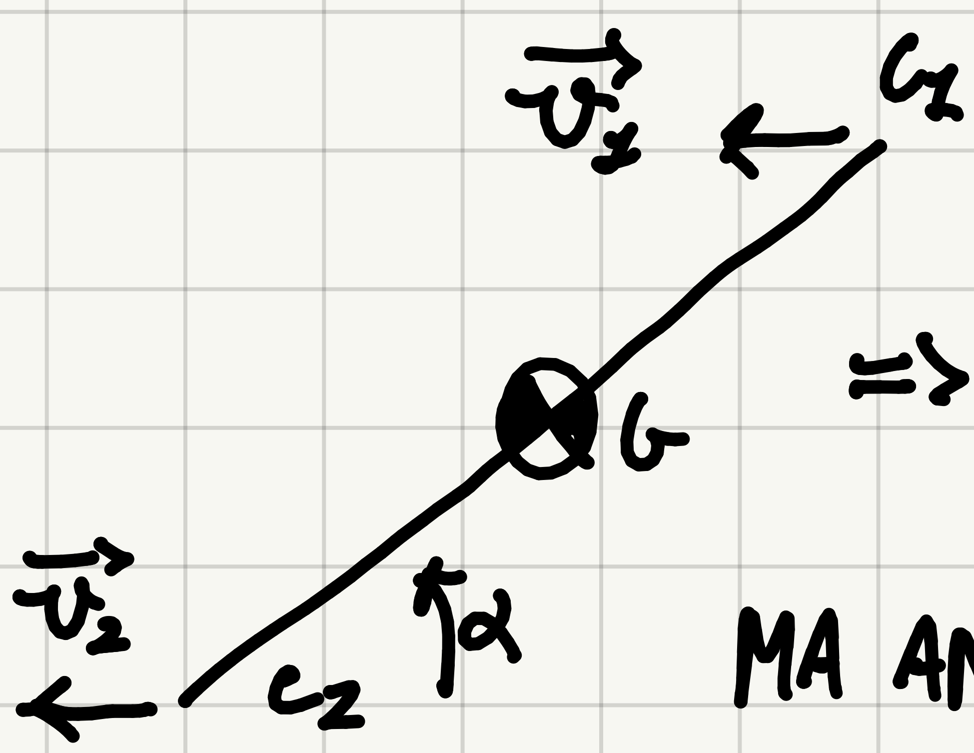
$$R_1 - R_2 = l \sin \alpha$$

$$\Rightarrow l = \frac{R_1 - R_2}{\sin \alpha}$$

$\vec{v}_1 = \cancel{\vec{v}_{H_1}} + \vec{\omega}_1 \times (C_1 - H_1) = \omega_1 \hat{k} \cdot R_1 \hat{j} = -\omega_1 R_1 \hat{i}$

! NON POSSO APPLICARE RIVALS SUI PUNTI DI CONTATTO, MA VALE SEMPRE CHE $\vec{a} = \frac{d}{dt}\vec{v}$

$\vec{a}_1 = \frac{d}{dt}\vec{v}_1 = \frac{d}{dt}(-\omega_1 R_1 \hat{i}) = -\dot{\omega}_1 R_1 \hat{i}$



CIR $\to +\infty \Rightarrow$ ASTA IN MOTO ROTAZIONALE

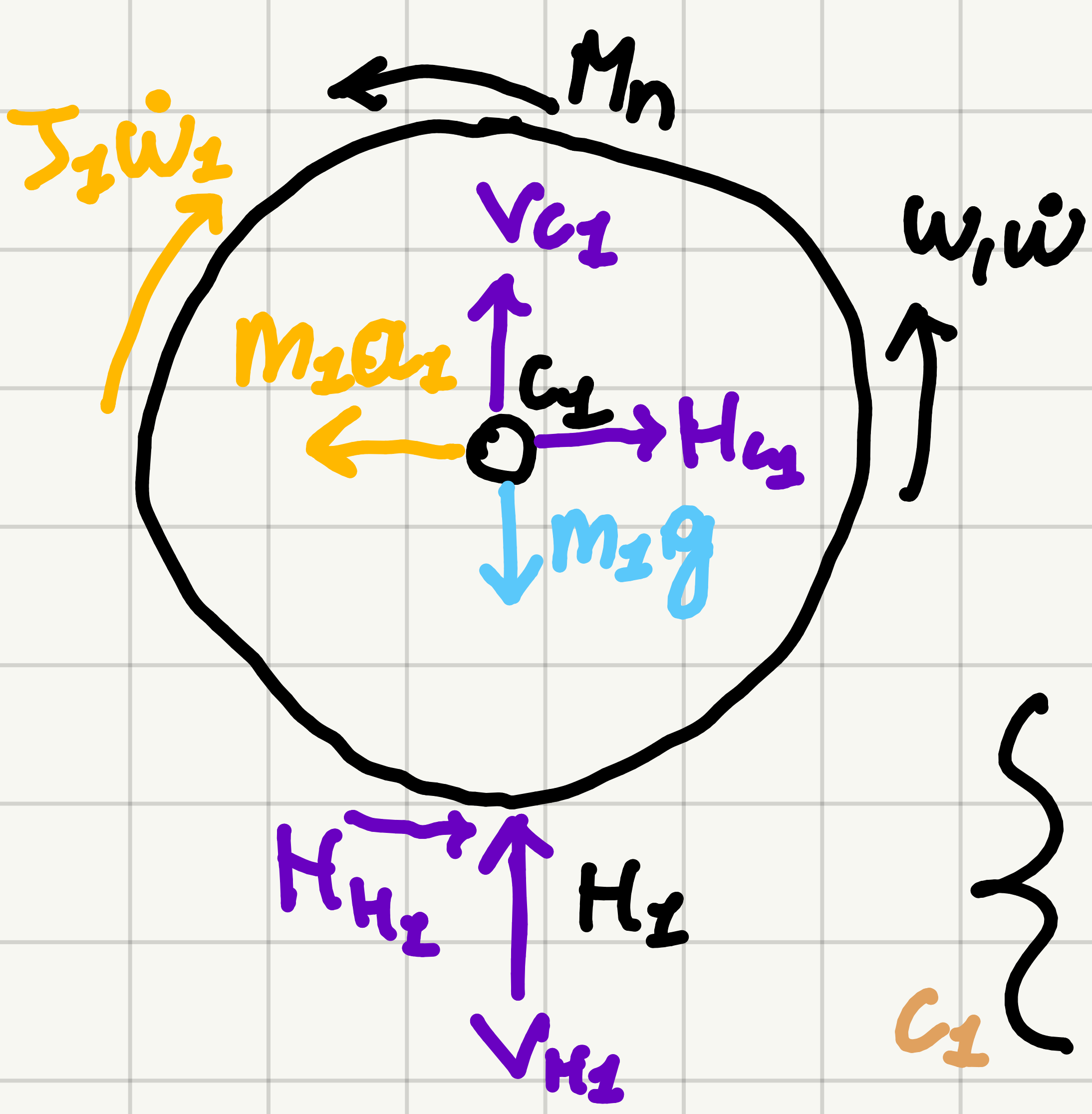
$\Rightarrow \omega_A = 0,\ \dot{\omega}_A = 0 \Rightarrow \vec{v}_G = \vec{v}_1,\ \vec{a}_G = \vec{a}_1$

MA ANCHE $\vec{v}_2 = \vec{v}_1 = -\omega_1 R \hat{i}$

INOLTRE, $\vec{v}_2 = \cancel{\vec{v}_{H_2}} + \omega_2 \hat{k} \times (C_2 - H_2) \Rightarrow \omega_2 = \frac{R_2}{R_1}\omega_1 \to \dot{\omega}_2 = \frac{R_2}{R_1}\dot{\omega}_1$

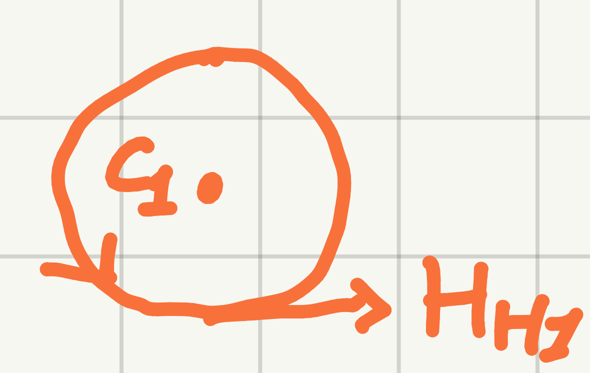
INTUITIVAMENTE, A PARITÀ DI v, SE R AUMENTA ω DIMINUISCE

CASO 1: $N_v = 0$

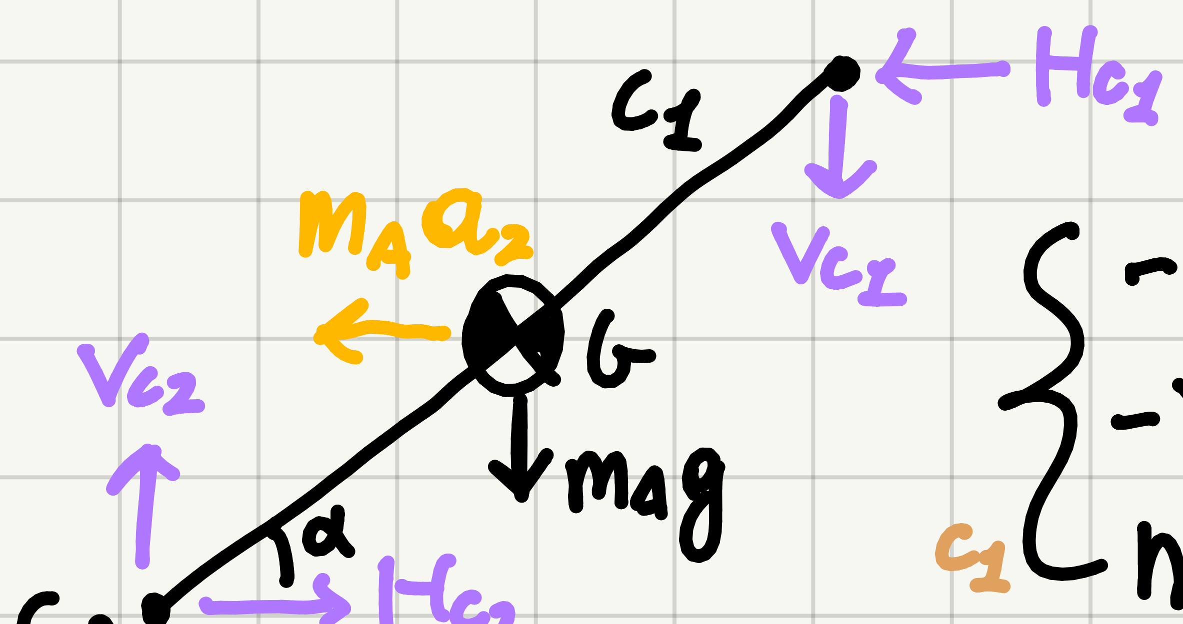


DISEGNO FORZE E COPPIE DI INERZIA OPPOSTE ALLA CONVENZIONE

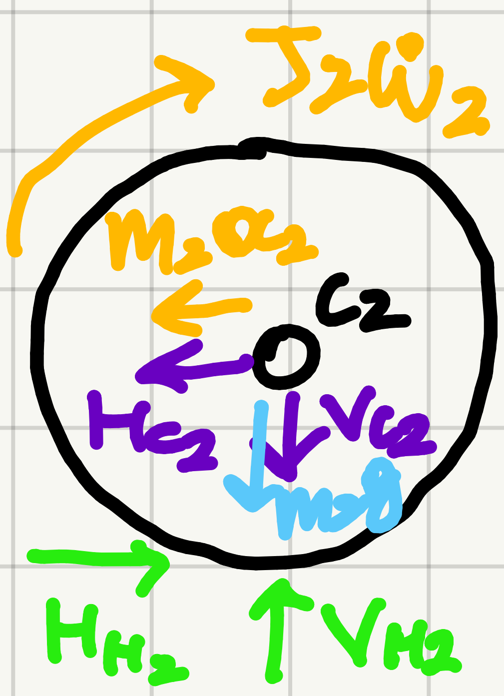
$$\begin{cases} -m_1 a_1 + H_{C_1} + H_{H_1} = 0 \\ V_{C_1} + V_{H_1} - m_1 g = 0 \\ C_1:\ M_n - J_1\dot{\omega}_1 + H_{H_1} R_1 = 0 \end{cases}$$



! 5 INCOGNITE... SERVONO ALTRE EQUAZIONI ⇒ USO GLI ALTRI CORPI



$$\begin{cases} -H_{C_1} - H_{C_2} - m_A a_2 = 0 \\ -V_{C_1} + V_{C_2} - m_A g = 0 \\ C_1:\ m_A g \frac{l}{2}\cos\alpha - m_A a_2 \frac{l}{2}\sin\alpha - V_{C_2} l\cos\alpha + H_{C_2} l \sin\alpha = 0 \end{cases}$$

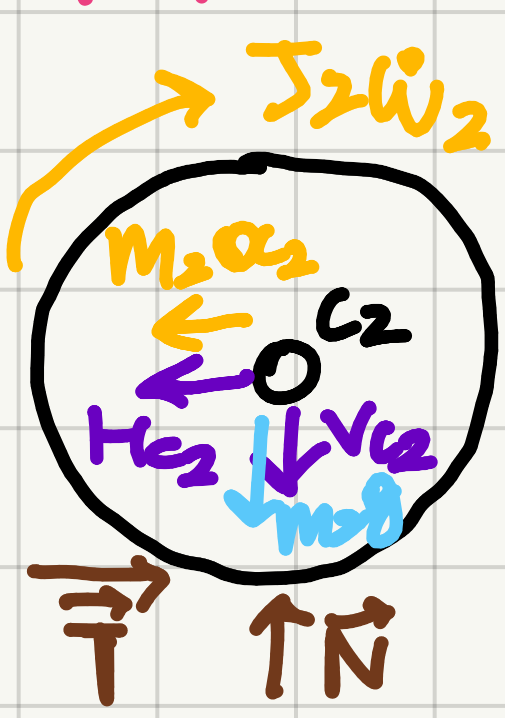


$$C_2\;\begin{cases} -H_{c_2} - m_2 a_2 + H_{H_2} = 0 \\ -V_{c_2} - m_2 g + V_{H_2} = 0 \\ -J_2\dot{\omega}_2 + H_{H_2} R_2 = 0 \end{cases}$$

9 EQUAZIONI
9 INCOGNITE ✓

$$\Rightarrow H_{H_2} = \frac{J\dot{\omega}_2}{R_2} = 1{,}125\,N \qquad M_n = 0{,}768\,N\cdot m$$

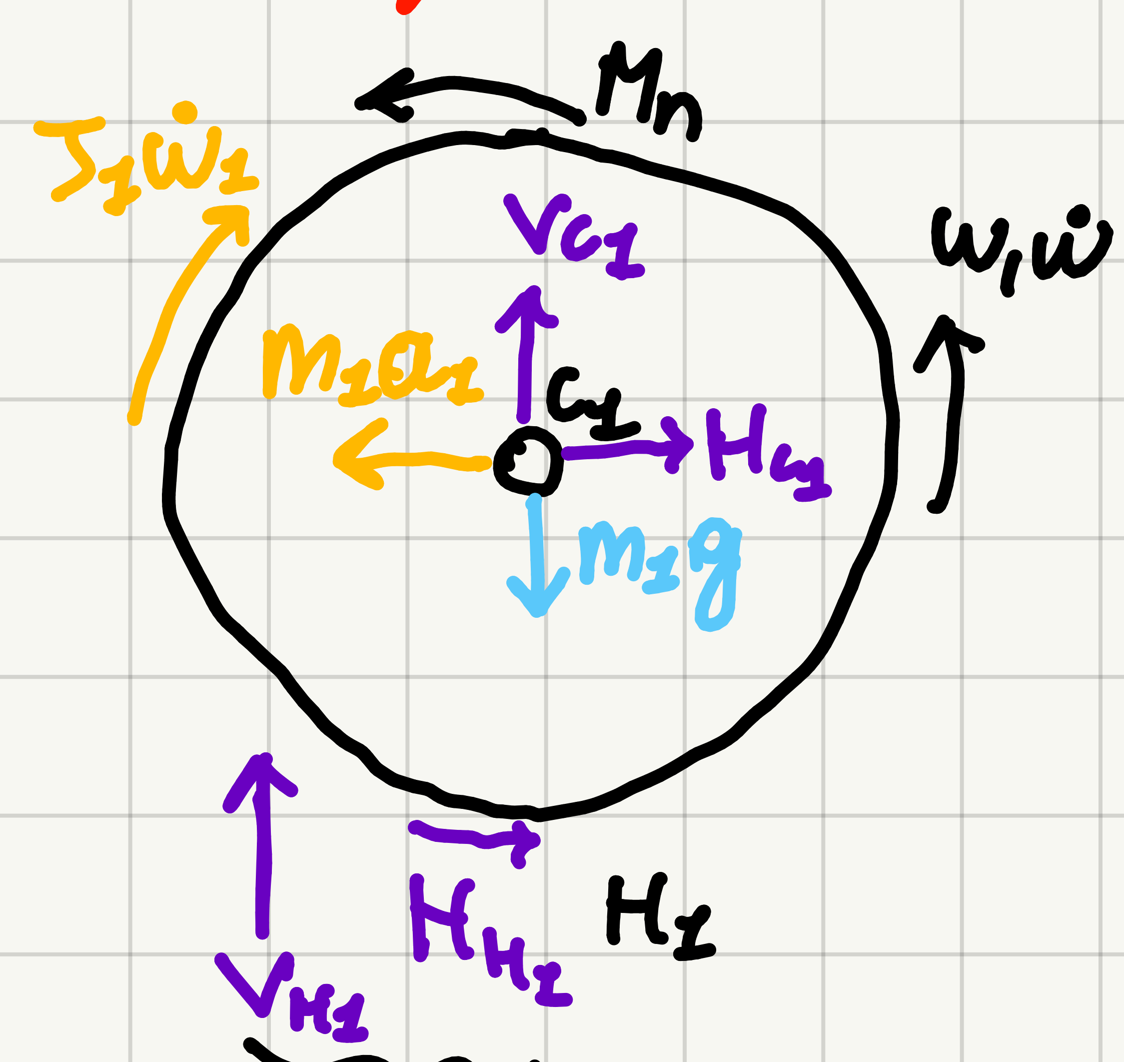
VERIFICA DI ADERENZA: $T \leq T_{lim}$?



SI NOTI CHE $T = H_{H_2}$ E $N = V_{H_2}$

$$\Rightarrow \begin{cases} T = 1{,}125\,N \\ N = 17{,}29\,N \end{cases} \qquad T_{lim} = \mu_s \cdot N = 13{,}38 \Rightarrow T \leq T_{lim} \checkmark$$

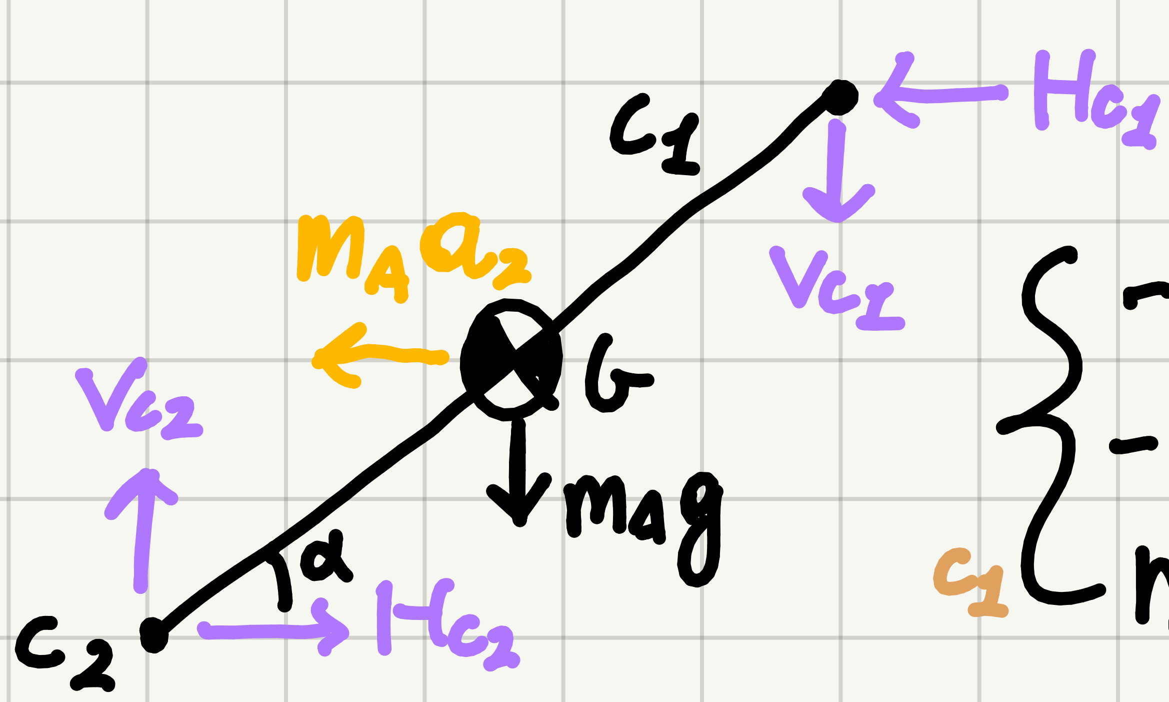
CASO 1: $\mu_v = 0{,}03$



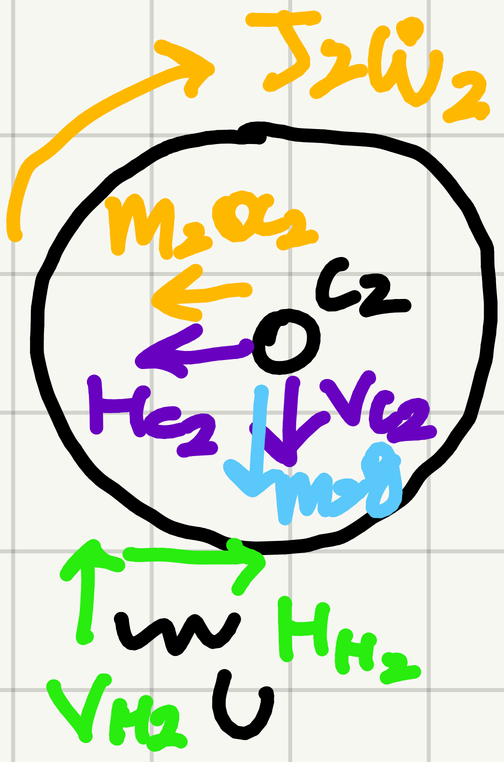
$$C_1\;\begin{cases} -m_1 a_1 + H_{c_1} + H_{H_1} = 0 \\ V_{c_1} + V_{H_1} - m_1 g = 0 \\ M_n - J_1\dot{\omega}_1 + H_{H_1} R_1 - V_{H_1} U = 0 \end{cases}$$

$$U = \mu_v R_1$$

U V_{H_1} SPOSTATO A SINISTRA PER GENERARE UN MOTO IN SENSO ORARIO E QUINDI FARE RESISTENZA



$$C_1\;\begin{cases} -H_{c_1} - H_{c_2} - m_A a_2 = 0 \\ -V_{c_1} + V_{c_2} - m_A g = 0 \\ m_A g \frac{l}{2}\cos\alpha - m_A a_2 \frac{l}{2}\sin\alpha - V_{c_2}\, l\cos\alpha \\ \quad + H_{c_2}\, l \sin\alpha = 0 \end{cases}$$



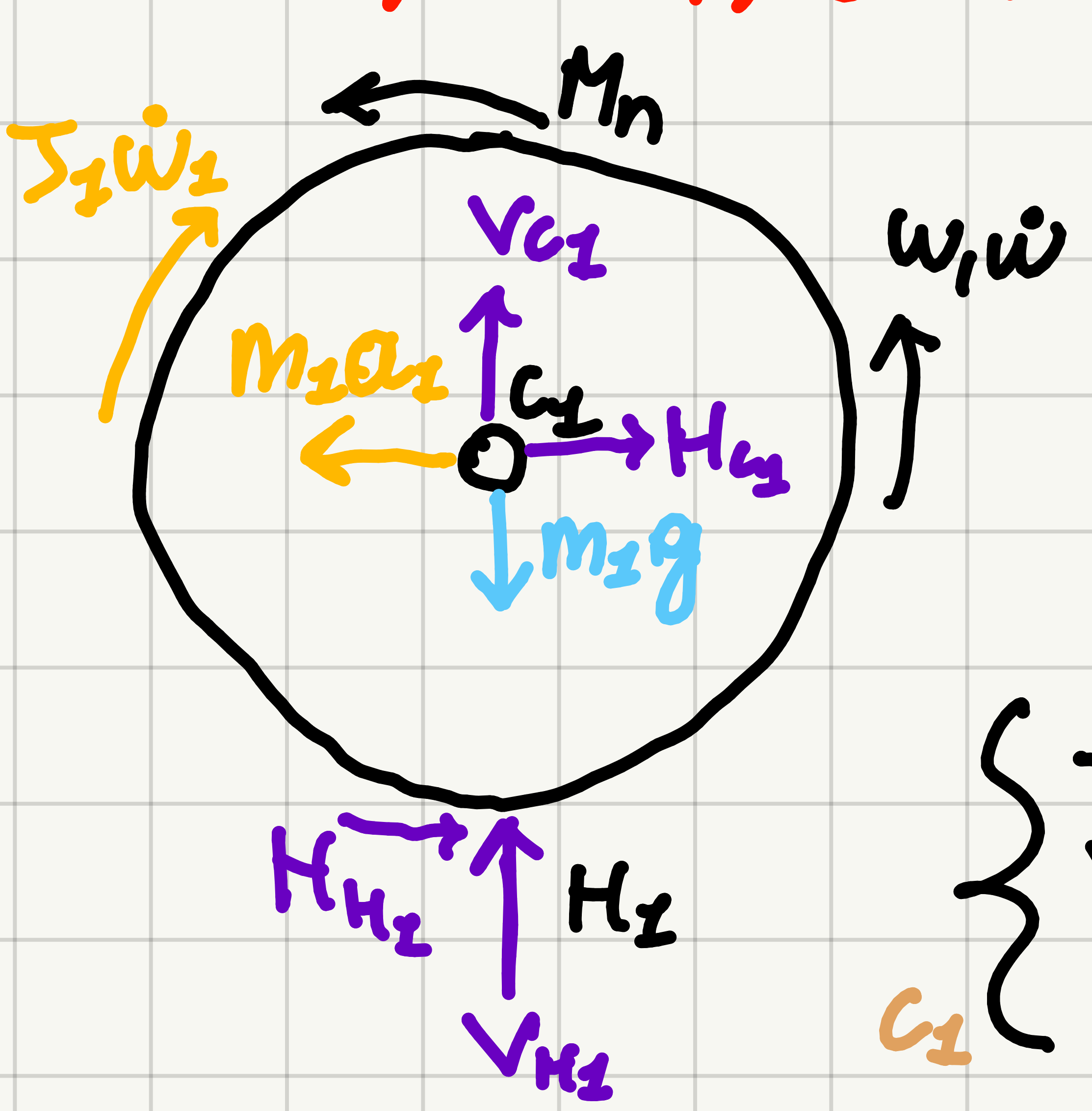
$$\begin{cases} -H_{c_2} - m_2 a_2 + H_{H_2} = 0 \\ -V_{c_2} - m_2 g + V_{H_2} = 0 \\ -J_2\dot{\omega}_2 + H_{H_2} R_2 - V_{H_2} u = 0 \end{cases}$$

$u = \mu_v R_2 \Rightarrow M_n = 1{,}12\,Nm \quad H_{H_2} = 1{,}64\,N$

SI NOTI L'ASPETTATO AUMENTO DI M_n DOVUTO AI NUOVI CONTRIBUTI

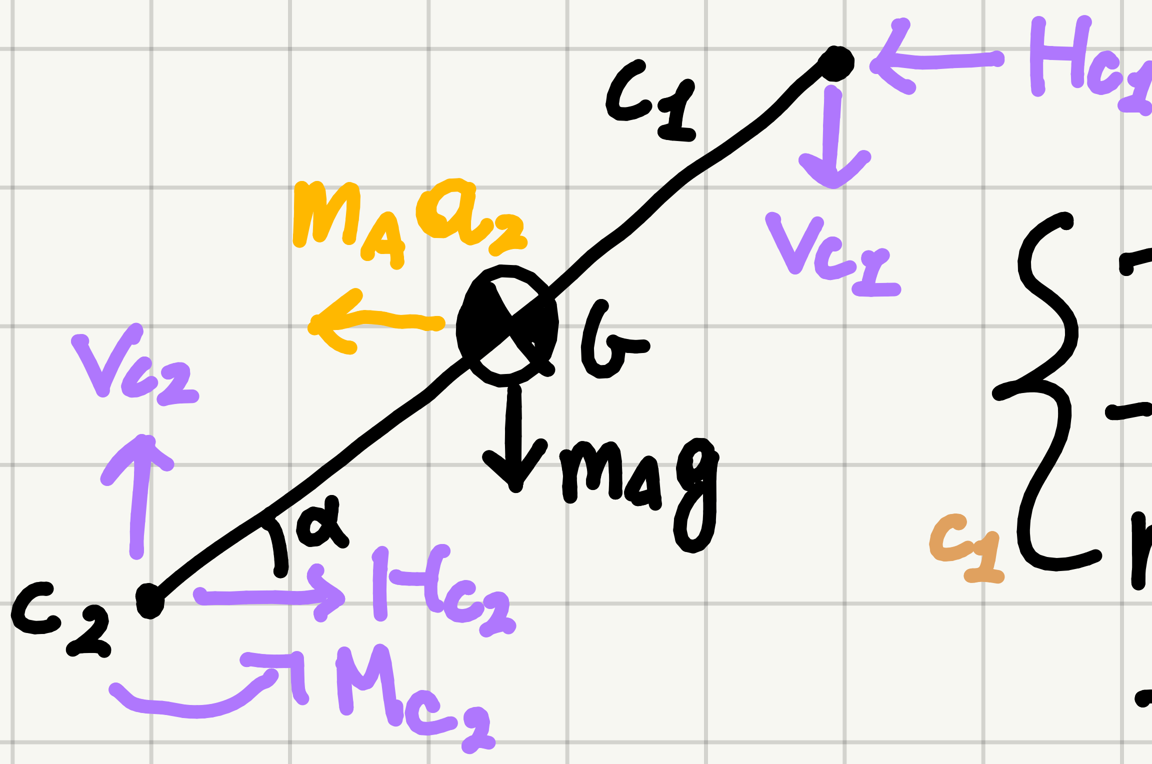
CASO 3: $\mu_v = 0$, $\mu_d = 0{,}5$ SOLO SU D_2

IMPROVVISAMENTE, D_2 SI BLOCCA



DISEGNO FORZE E COPPIE DI INERZIA OPPOSTE ALLA CONVENZIONE

$$\begin{cases} -m_1 a_1 + H_{c_1} + H_{H_1} = 0 \\ V_{c_1} + V_{H_1} - m_1 g = 0 \\ C_1:\ M_n - J_1\dot{\omega}_1 + H_{H_1} R_1 = 0 \end{cases}$$



$$\begin{cases} -H_{c_1} - H_{c_2} - m_A a_2 = 0 \\ -V_{c_1} + V_{c_2} - m_A g = 0 \\ C_1:\ m_A g \frac{l}{2}\cos\alpha - m_A a_2 \frac{l}{2}\sin\alpha - V_{c_2} l\cos\alpha \\ \quad + H_{c_2} l \sin\alpha + M_c = 0 \end{cases}$$

! ORA ABBIAMO 9 EQUAZIONI E 10 INCOGNITE => LA 10MA EQUAZIONE È DATA DAL MODELLO DI ATTRITO DINAMICO

$$\begin{cases} H_{H_2} - H_{c_2} - m_2 a_2 = 0 \\ -V_{c_2} - m_2 g + V_{H_2} = 0 \\ -M_{c_2} + H_{H_2} R_2 = 0 \end{cases}$$

! SI NOTI CHE, NON ESSENDO PIÙ IN ROTAZIONE, $C_{in} = 0$

$H_{H_2} = \mu_d \cdot |V_{H_2}|$

RIPETIAMO I TRE ESERCIZI COL BILANCIAMENTO DI POTENZE

CASO 1

$$\frac{d}{dt}K = \frac{d}{dt}K^{(D_1)} + \frac{d}{dt}K^{(ASTA)} + \frac{d}{dt}K^{(D_2)}$$

- $\frac{d}{dt}K^{(D_1)} = m_1 \vec{a}_1 \cdot \vec{v}_1 + J_1 \dot{\vec{\omega}} \cdot \vec{\omega} = m_1(-R_1\dot{\omega}_1 \hat{i}) \cdot (-R_1\omega_1 \hat{i}) + J_1(\dot{\omega}_1\hat{k}) \cdot$
  $\cdot (\omega_1 \hat{k}) = m_1 R_1^2 \dot{\omega}_1 \omega_1 + J_1 \dot{\omega}_1 \omega_1$
- $\frac{d}{dt}K^{(ASTA)} = m_a(\vec{a}_G \cdot \vec{v}_G) = m_a R_1^2 \dot{\omega}_1 \omega_1$
- $\frac{d}{dt}K^{(D_2)} = m_2 \vec{a}_2 \cdot \vec{v}_2 + J_1 \dot{\vec{\omega}}_2 \cdot \omega_2 = m_2(-R_2\dot{\omega}_2 \hat{i}) \cdot (-R_2\omega_2\hat{i}) +$
  $+ J_1(\dot{\omega}_2\hat{k})(\omega_2\hat{k}) = m_2 R_2^2 \dot{\omega}_2 \omega_2 + J_2 \dot{\omega}_2 \omega_2$

MOTO ⊥ FORZE PESO

$$\sum P_{ATTIVE} = \cancel{P_{PESO}^{(D_1)}} + \cancel{P_{PESO}^{(ASTA)}} + \cancel{P_{PESO}^{(D_2)}} + P_{MOTO} = M_n \cdot \omega_2$$

$$\frac{d}{dt}K = \sum P_{ATTIVE} \Rightarrow M_n = 0{,}768\ N \cdot m$$

PER RISOLVERE H_{H_2} OCCORRE PER FORZA USARE L'EQUILIBRIO DINAMICO

CASO 2

PER L'ENERGIA CINETICA ABBIAMO LE STESSE EQUAZIONI DEL CASO 1

$$\frac{d}{dt}K = \frac{d}{dt}K^{(D_1)} + \frac{d}{dt}K^{(ASTA)} + \frac{d}{dt}K^{(D_2)}$$

- $\frac{d}{dt}K^{(D1)} = m_1 \vec{a}_1 \cdot \vec{v}_1 + J_1 \dot{\vec{\omega}} \cdot \vec{\omega} = m_1(-R_1\dot{\omega}_1 \hat{i}) \cdot (-R_1\omega_1 \hat{i}) + J_1(\dot{\omega}_1 \hat{k}) \cdot$

  $\cdot (\omega_1 \hat{k}) = m_1 R_1^2 \dot{\omega}_1 \omega_1 + J_1 \dot{\omega}_1 \omega_1$

- $\frac{d}{dt}K^{(CASSA)} = m_c(\vec{a}_G \cdot \vec{v}_G) = m_c R_1^2 \dot{\omega}_1 \omega_1$

- $\frac{d}{dt}K^{(D2)} = m_2 \vec{a}_2 \cdot \vec{v}_2 + J_2 \dot{\vec{\omega}}_2 \cdot \omega_2 = m_2(-R_2\dot{\omega}_2 \hat{i}) \cdot (-R_2\omega_2 \hat{i}) +$

  $+ J_2(\dot{\omega}_2 \hat{k})(\omega_2 \hat{k}) = m_2 R_2^2 \dot{\omega}_2 \omega_2 + J_2 \dot{\omega}_2 \omega_2$

MOTO ⊥ FORZE PESO

$\sum P_{ATTIVE} = \cancel{P_{PESO}^{(D1)}} + \cancel{P_{PESO}^{(CASSA)}} + \cancel{P_{PESO}^{(D2)}} + P_{MOTO} + P_{RESISTENZA}^{(D1)} + P_{RESISTENZA}^{(D2)}$

$= M_n \omega_1 - \mu_v R_1 |v_{H_1} \omega_1| - \mu_v R_2 |v_{H_2} \omega_2|$

ESSENDO L'ATTRITO DISSIPATIVO, PONGO IL SEGNO "–" DAVANTI. PER EVITARE DI FAR CASINO COI SEGNI, USO I MODULI

$\frac{d}{dt}K = \sum P_{ATTIVE}$ MA HO 3 INCOGNITE ⇒ SERVE L'EQUILIBRIO DINAMICO

CASO 3

- $\frac{d}{dt}K^{(D1)} = m_1 \vec{a}_1 \cdot \vec{v}_1 + J_1 \dot{\vec{\omega}} \cdot \vec{\omega} = m_1(-R_1\dot{\omega}_1 \hat{i}) \cdot (-R_1\omega_1 \hat{i}) + J_1(\dot{\omega}_1 \hat{k}) \cdot$

  $\cdot (\omega_1 \hat{k}) = m_1 R_1^2 \dot{\omega}_1 \omega_1 + J_1 \dot{\omega}_1 \omega_1$

- $\frac{d}{dt}K^{(CASSA)} = m_c(\vec{a}_G \cdot \vec{v}_G) = m_c R_1^2 \dot{\omega}_1 \omega_1$

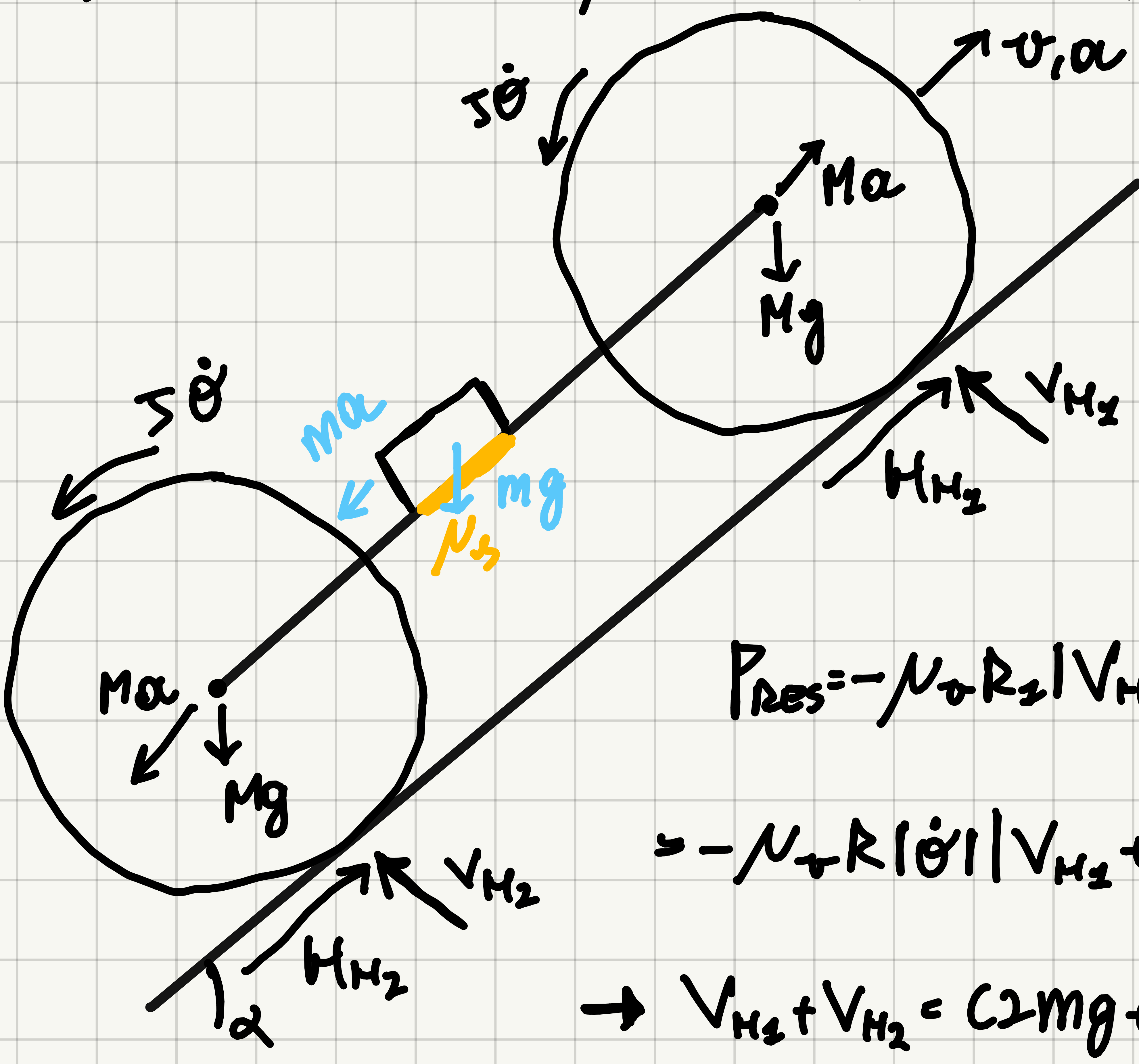
- $\frac{d}{dt}K^{(D2)} = m_2 \vec{a}_2 \cdot \vec{v}_2 = m_2 R_2^2 \dot{\omega}_2 \omega_2$

MOTO ⊥ FORZE PESO

$\sum P_{ATTIVE} = \cancel{P_{PESO}^{(D1)}} + \cancel{P_{PESO}^{(CASSA)}} + \cancel{P_{PESO}^{(D2)}} + P_{MOTO} + P_{RESISTENZA}^{(D2)} = M_n \omega_1 - \mu_v R_2 |v_{H_2} \omega_2|$

INFO EXTRA SU CONDIZIONI DI ADERENZA

1) Ruote uguali; moto di puro rotolamento

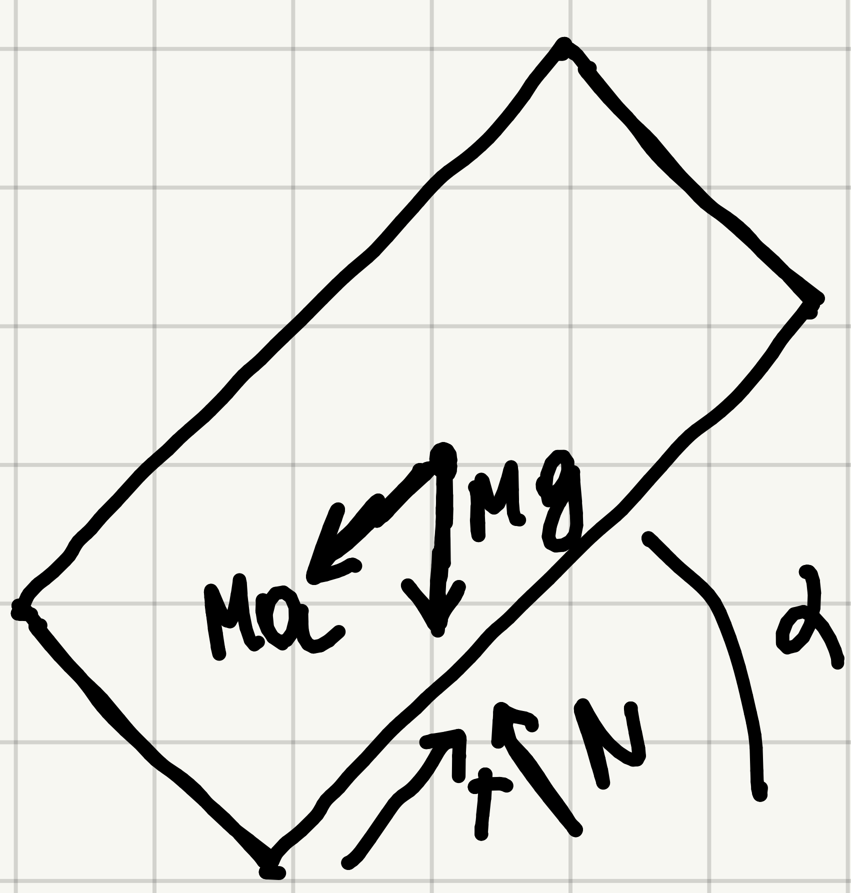


$$P_{RES} = -\mu_v R_1 |V_{H_1} \cdot \omega_1| - \mu_v R_2 |V_{H_2} \cdot \omega_2| =$$

$$= -\mu_v R |\dot{\theta}| |V_{H_1} + V_{H_2}|$$

$$\rightarrow V_{H_1} + V_{H_2} = (2mg + Mg)\cos\alpha$$

2) Calcolo a_{lim}



$$\begin{cases} T - Ma - Mg\sin\alpha = 0 \\ -Mg\cos\alpha + N = 0 \\ T_{lim} = \mu_s N \end{cases}$$

In condizioni limite, $T = T_{lim}$

$$\Rightarrow T_{lim} - Ma_{lim} - Mg\sin\alpha = 0 \Rightarrow a_{lim} = \frac{T_{lim} - Mg\sin\alpha}{M}$$