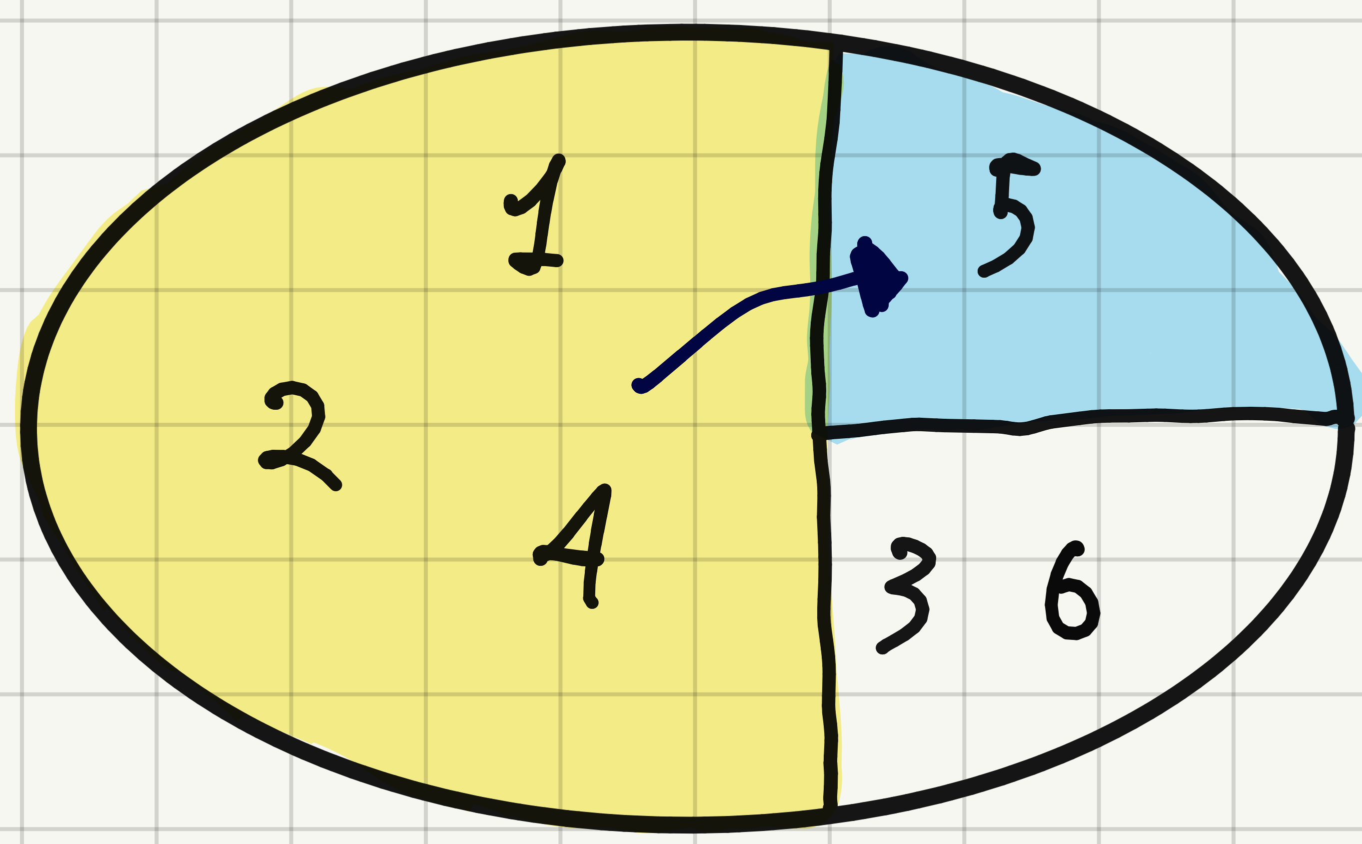
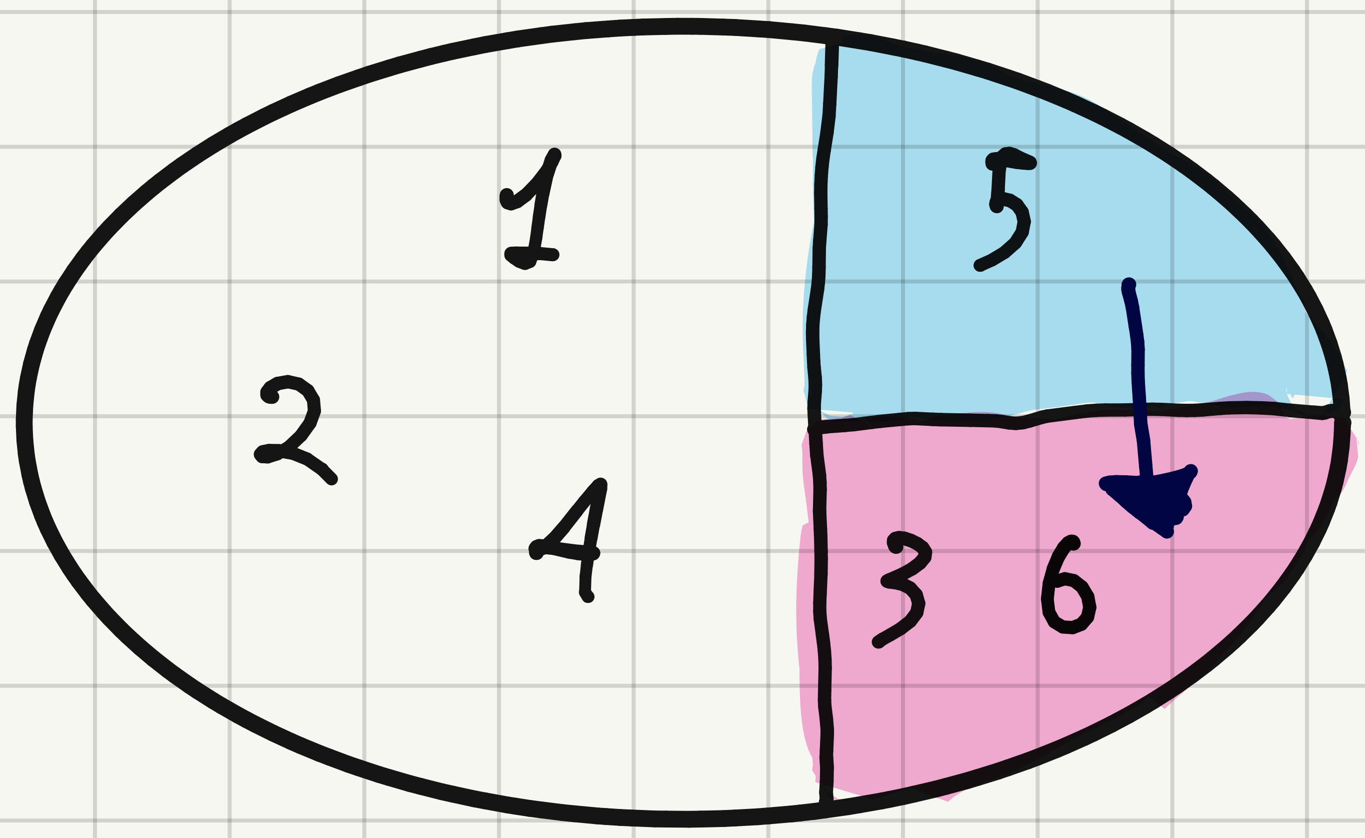
$$M(E_2) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$



$$M(E_3) = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$



$$M(E_4) = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

OBVIOUSLY, DO NOT FORGET THE TOTAL RELATION

$$M(E_4) = T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

LET $R: X \to X$ BE A REFLEXIVE AND TRANSITIVE RELATION.
PROVE THAT THE RELATION E DEFINED BY THE FORMULA

$xEy :\Leftrightarrow xRy \wedge yRx$ IS AN EQUIVALENCE RELATIONS

$R: X \to X$ $\quad xEy :\Leftrightarrow xRy \wedge yRx$ $\quad E$ EQ. REL.?

E IS AN EQUIVALENCE RELATION IF IT IS:

- REFLEXIVE

xEx? $\quad x \in R \Rightarrow xRx \equiv xRx \wedge xRx$ $\quad$ R REFLEXIVE

$\Rightarrow xEx$ ✓

- SYMMETRIC

$xEy \Rightarrow yEx$? $\quad xEy \Rightarrow xRy \wedge yRx$

R SYMMETRIC $\quad \equiv yRx \wedge xRy$

$\Rightarrow yEx$ ✓

- TRANSITIVE

$xEy \wedge yEz \Rightarrow xEz$?

$xEy \wedge yEz \Rightarrow xRy \wedge yRx \wedge yRz \wedge zRy$

$\equiv xRy \wedge yRz \wedge zRy \wedge yRz$

R TRANSITIVE $\quad \equiv xRz \wedge zRy$

$\equiv xEz$ ✓