$$F(i\omega) = 40 \cdot \frac{1 + \frac{i\omega}{1,6}}{i\omega\left(1+\frac{i\omega}{0,2}\right)\left(1+\frac{i\omega}{10}\right)}$$

PUNTI DI ROTTURA

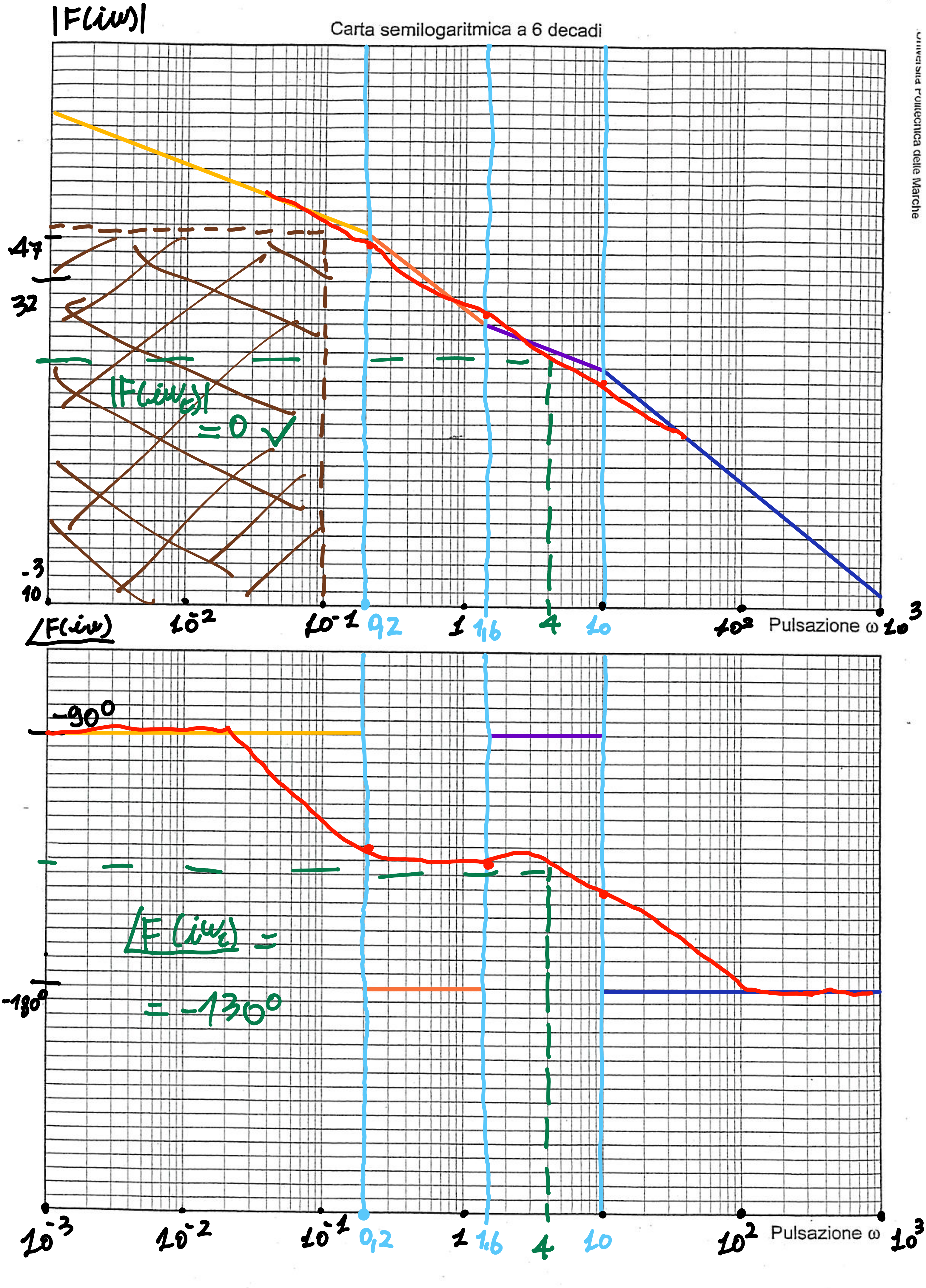
- $\omega = 0$ ● $-20dB$ $-90°$ $-20dB$ $-90°$
- $\omega = 0,2$ ● $-20dB$ $-90°$ $-40dB$ $-180°$
- $\omega = 1,6$ ● $+20dB$ $+90°$ $-20dB$ $-90°$
- $\omega = 10$ ● $-20dB$ $-90°$ $-40dB$ $-180°$

CORREZIONE MODULO

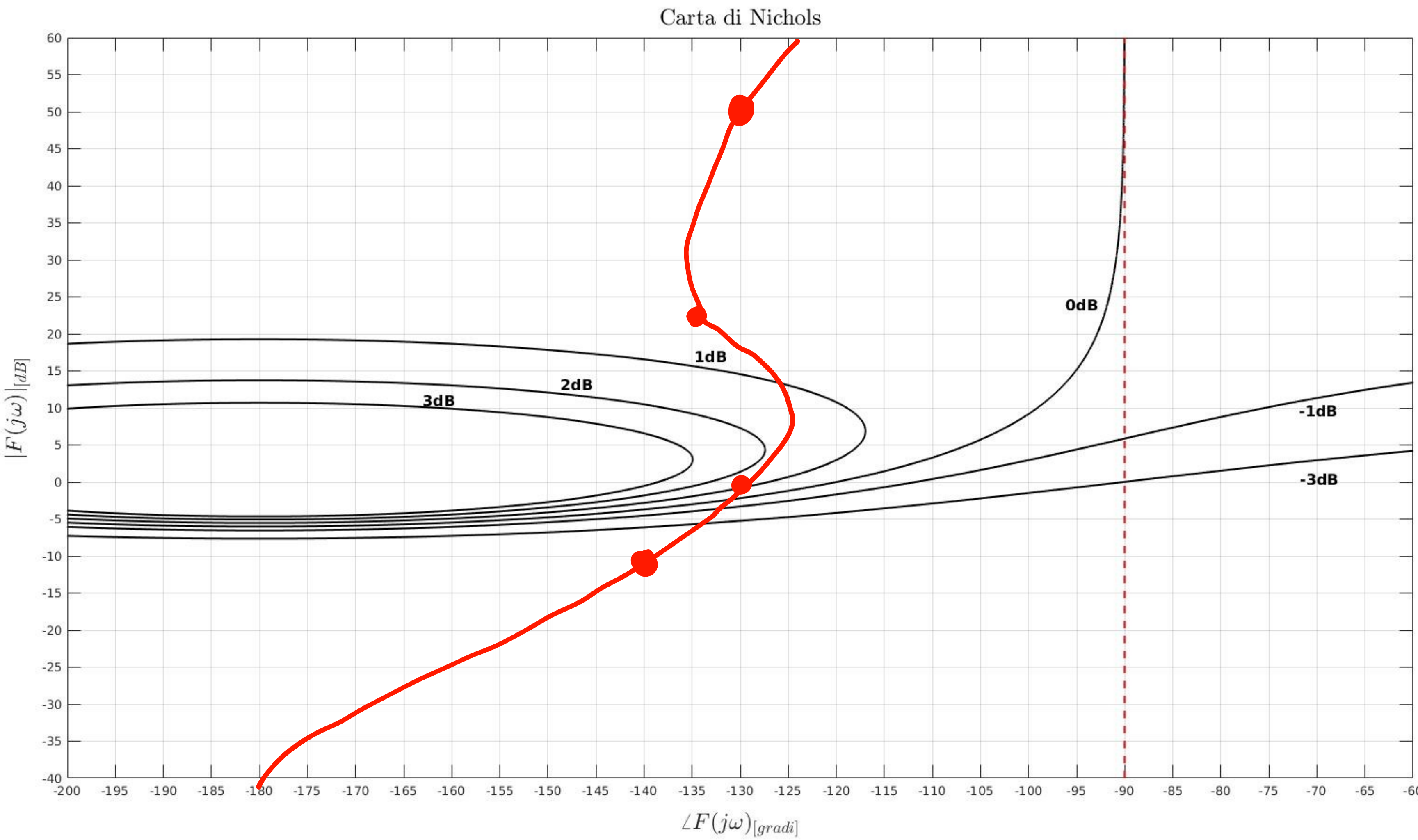
ω	$1+\frac{i\omega}{0,2}$	$1+\frac{i\omega}{1,6}$	$1+\frac{i\omega}{10}$	TOT
0,2	$-3dB$	$0°$	$0°$	$-3dB$
1,6	$0°$	$+3dB$	$0°$	$+3dB$
10	$0°$	$0°$	$-3dB$	$-3dB$

CORREZIONE FASE

ω	$i\omega$	$1+\frac{i\omega}{0,2}$	$1+\frac{i\omega}{1,6}$	$1+\frac{i\omega}{10}$	TOT
0,2	$-90°$	$-45°$	$+5°$	$0°$	$-130°$
1,6	$-90°$	$-80°$	$+45°$	$-10°$	$-135°$
10	$-90°$	$-90°$	$+80°$	$-45°$	$-145°$



|F(iω)|
Carta semilogaritmica a 6 decadi
Università Politecnica delle Marche
47
32
-3
10
|F(iω_t)| = 0 ✓
10^-2
10^-1
0,2
1
1,6
4
10
10^2
Pulsazione ω
10^3
∠F(iω)
-90°
∠F(iω_t) =
= -130°
-180°
10^-3
10^-2
10^-1
0,2
1
1,6
4
10
10^2
Pulsazione ω
10^3

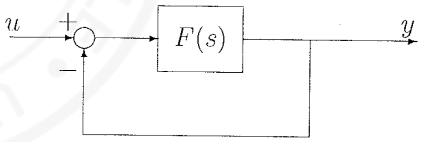


Carta di Nichols
$|F(j\omega)|_{[dB]}$
$\angle F(j\omega)_{[gradi]}$
0dB
1dB
2dB
3dB
-1dB
-3dB

Domanda Scritta di Controlli Automatici (9CFU) - 22/7/2013

Esercizio 1

È dato il sistema in controreazione:

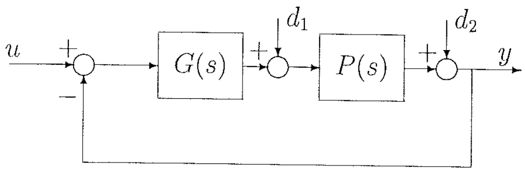


in cui $F(s) = \dfrac{K(s+5)(s+2)^2}{(s-1)(s+4)(s^2+12s+40)}$, $K \in \mathbb{R}$.

- Tracciare il luogo positivo delle radici;
- tracciare il luogo negativo delle radici;
- determinare per quali valori di K il sistema a ciclo chiuso è asintoticamente stabile;
- determinare per quali valori di K tutti i poli del sistema in catena chiusa hanno parte reale minore di -1.
- per $K = 6$, esiste la risposta a regime permanente ad un ingresso sinusoidale?

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{1}{s(s+8)}; \quad d_1(t) = \delta_{-1}(t); \quad d_2(t) = \sin(\omega t).$$

Progettare $G(s)$ con la sintesi per tentativi in ω in modo che:

- $|\tilde{y}_{d1}(t)| \leq 0.05$, essendo $\tilde{y}_{d1}(t)$ la risposta a regime permanente al disturbo $d_1(t)$;
- $|\tilde{y}_{d2}(t)| \leq 0.08$ per $\omega \leq 0.6\ rad \cdot s^{-1}$, essendo $\tilde{y}_{d2}(t)$ la risposta a regime permanente al disturbo $d_2(t)$;
- $K_G \leq 20$;
- $M_r \leq 3\ dB$;
- $B_3 \simeq 2.5\ Hz$.

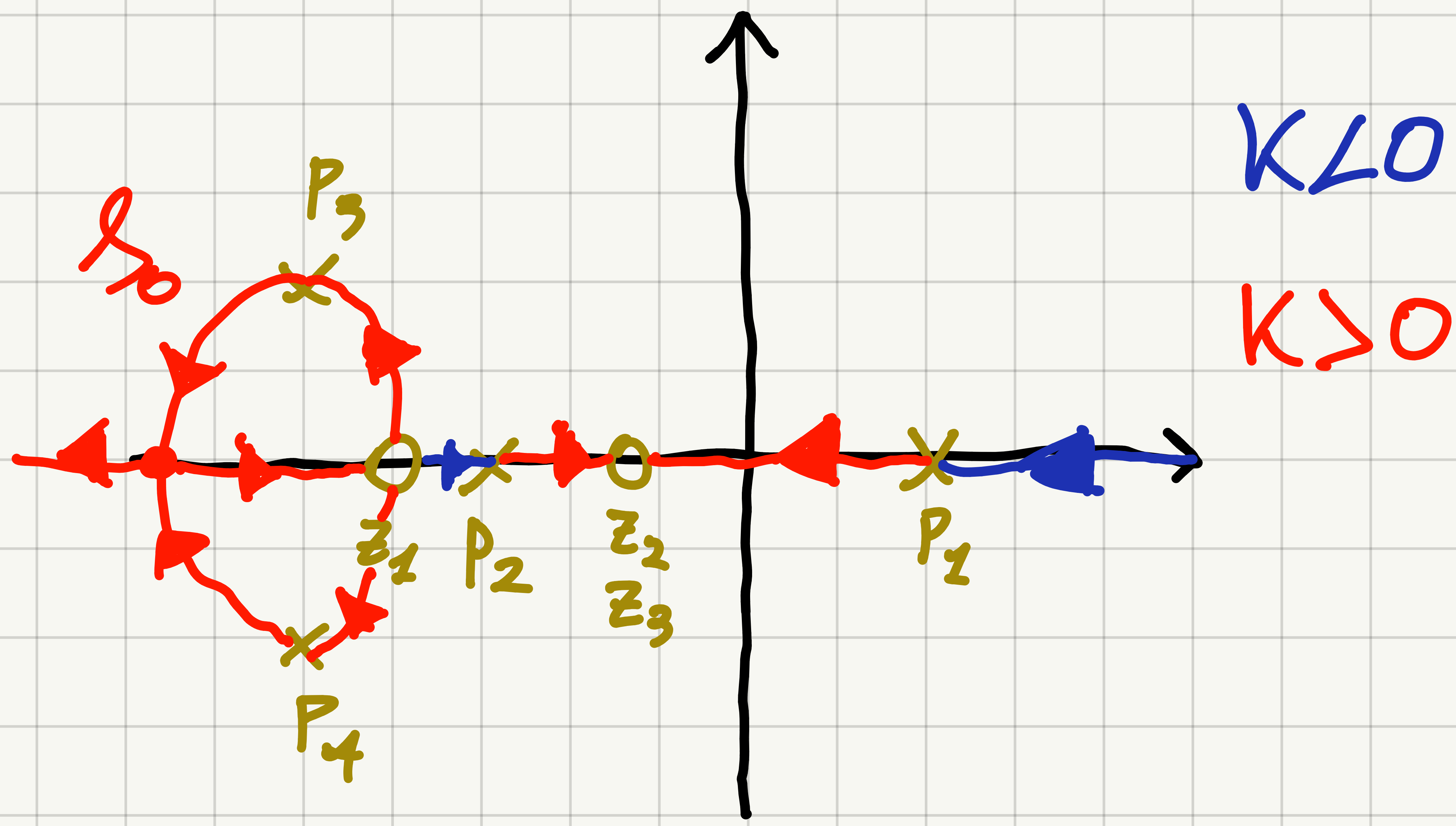
Discretizzare infine il controllore ottenuto, scegliendo opportunamente il periodo di campionamento.

1

$$F(s)=\frac{K(s+5)(s+2)^2}{(s-1)(s+4)(s^2+12s+40)} \qquad K\in\mathbb{R}$$

$n=4,\ m=3 \Rightarrow n-m=1$

$z_1=-5,\ z_2=z_3=-2;\ p_1=1,\ p_2=-4,\ p_3=-6+2i,\ p_4=-6-2i$



$$f(s,K)=(s-1)(s+4)(s^2+12s+40)+K(s+5)(s+2)^2\Big|_{s=0}=0$$

$-160+20K=0 \qquad K=8$

SISTEMA STABILE $\forall K>8$

$\Rightarrow \nexists$ RISPOSTA A REGIME PERMANENTE

PER $K=6$

$$|Y_{d1}(\omega)| = \left| \frac{K_d}{K_G} \right| \le 0{,}05 \Rightarrow K_G \ge 20$$ SPECIFICHE UNIVOCHE

②

$$\begin{cases} K_G \ge 20 \\ K_G \le 20 \end{cases} \Longrightarrow K_G = 20 \Rightarrow G(s) = 20$$

$$|Y_{d2}(t)| = \left| \frac{1}{1+F(i\omega)} \right| \le 0{,}08 \Rightarrow |F(i\omega)| \ge 12{,}5 \text{ PER } \omega \le 0{,}6 \ \frac{rad}{s}$$

$$\rightarrow 20\log_{10}(12{,}5) = 22\,dB$$

$M_r \le 3\,dB \Rightarrow M_\varphi \ge 42°$ SPECIFICHE LASCHE

$B_3 \simeq 2{,}5\,Hz \Rightarrow \omega_t = 3 \div 5 B_3 = 4B_3 = 10 \ \frac{rad}{s}$

$$F(s) = G(s) \cdot P(s) = \frac{20}{s(s+8)}$$

$$F(i\omega) = 2{,}5 \cdot \frac{1}{i\omega(1+\frac{i\omega}{8})} \qquad 2{,}5 \rightarrow 20\log_{10}(2{,}5) = 8\,dB$$

PUNTI DI ROTTURA:

- $\omega = 0$ ● $-20\,dB$ $-90°$ $-20\,dB$ $-90°$
- $\omega = 8$ ● $-20\,dB$ $-90°$ $-40\,dB$ $-180°$

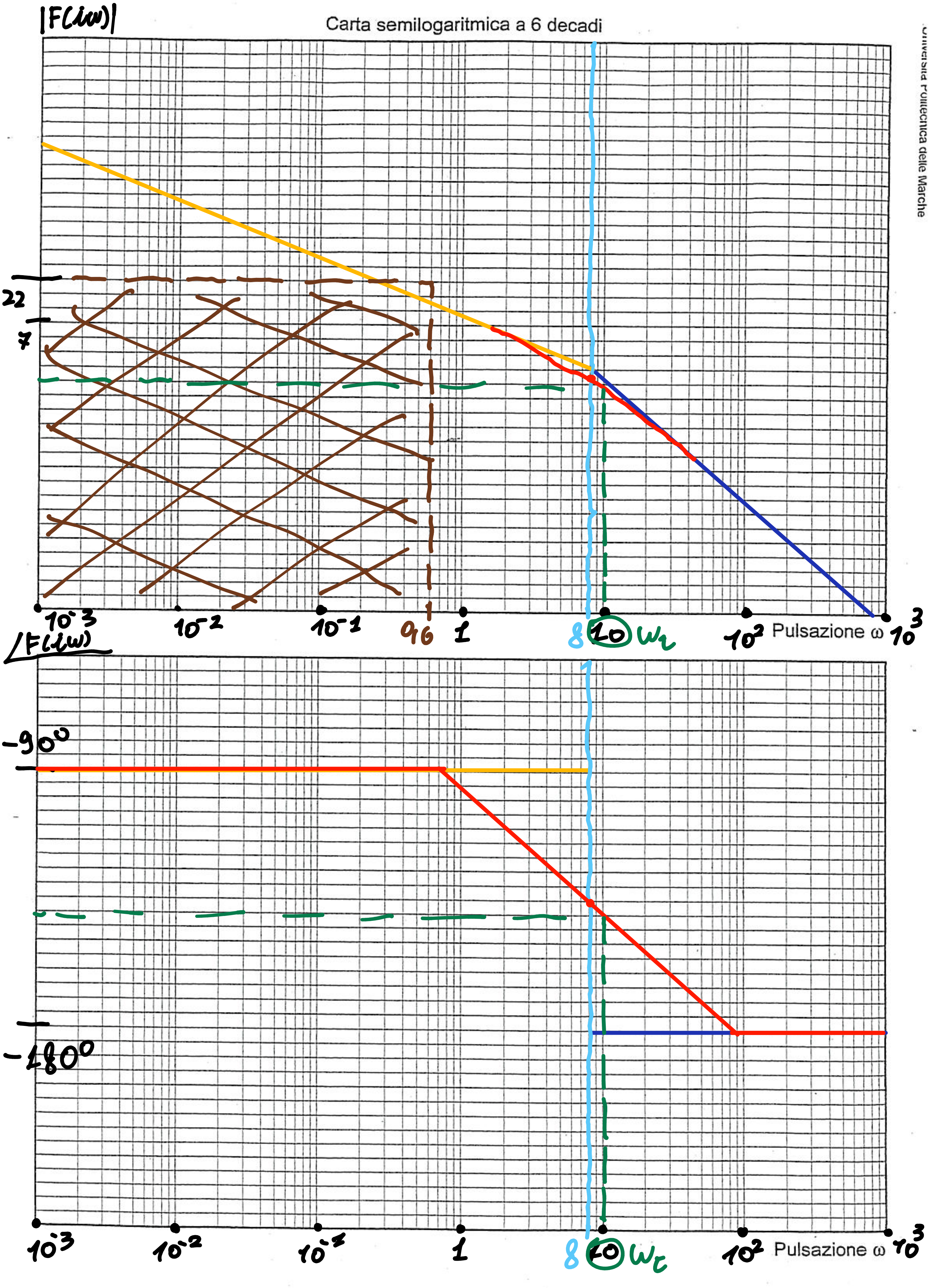
CORREZIONE MODULO:

ω	$1+\frac{i\omega}{8}$	TOT
8	$-3\,dB$	$-3\,dB$

CORREZIONE FASE:

ω	$i\omega$	$1+\frac{i\omega}{8}$	TOT
8	$-90°$	$-45°$	$-135°$

Carta semilogaritmica a 6 decadi



$|F(i\omega_c)| = -13\,dB$

$\angle F(i\omega_c) = -140° \Longrightarrow M_\varphi = 40°$

OBIETTIVO:

- $|F(i\omega_c)| = 0$
- $M_\varphi \geq 42°$ $\Longrightarrow$ AUMENTARE MODULO, AUMENTARE FASE

$\Longrightarrow$ FUNZIONE ANTICIPATRICE $R_a(s) = \dfrac{1 + \frac{s}{\omega_a}}{1 + \frac{s}{m_a \omega_a}}$

$m_a = 6 \qquad \omega_c \tau_a = 200 \Longrightarrow \omega_a = 0,1$

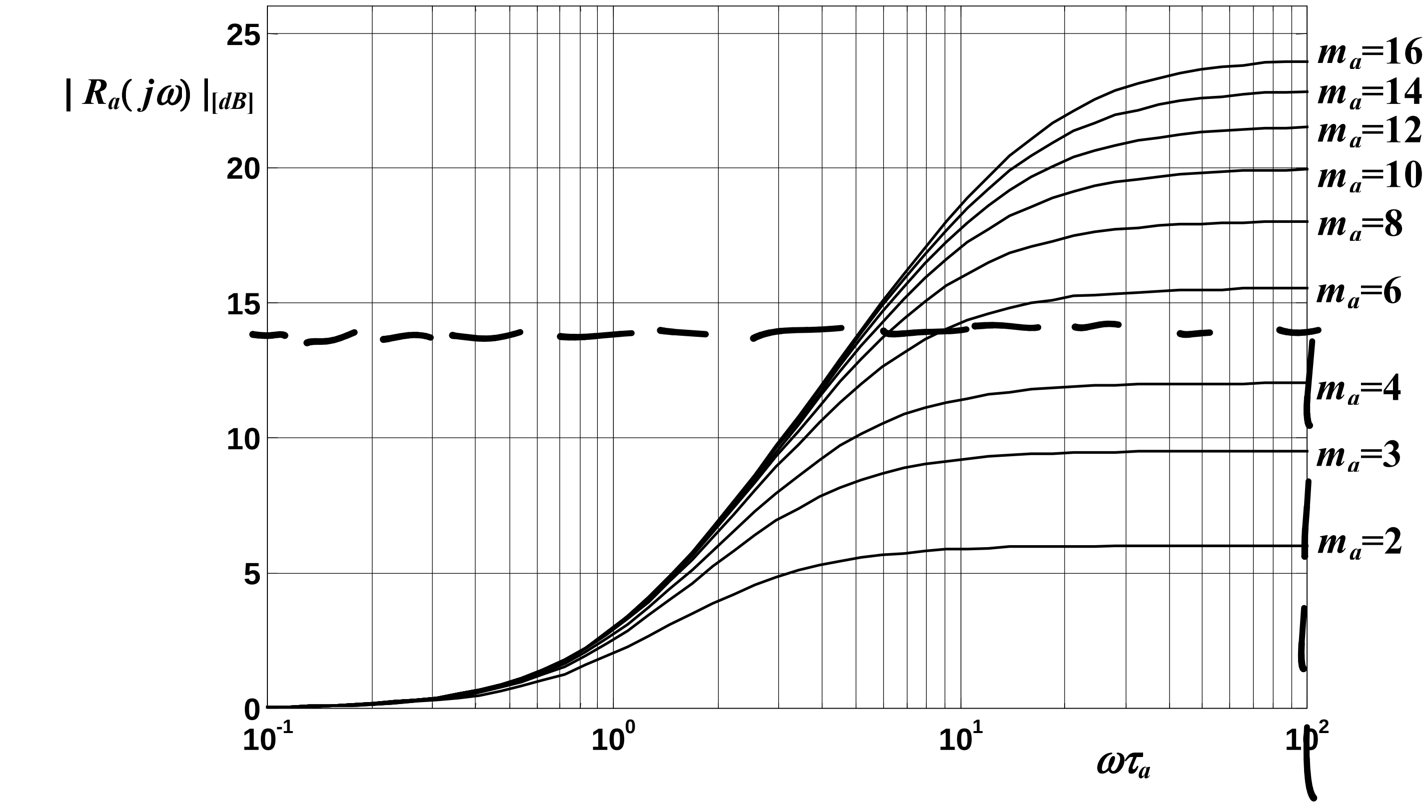
$R_a(s) = \dfrac{1 + \frac{s}{0,1}}{1 + \frac{s}{0,6}} \Longrightarrow G(s) = 20 \cdot \dfrac{1 + \frac{s}{0,1}}{1 + \frac{s}{0,6}}$

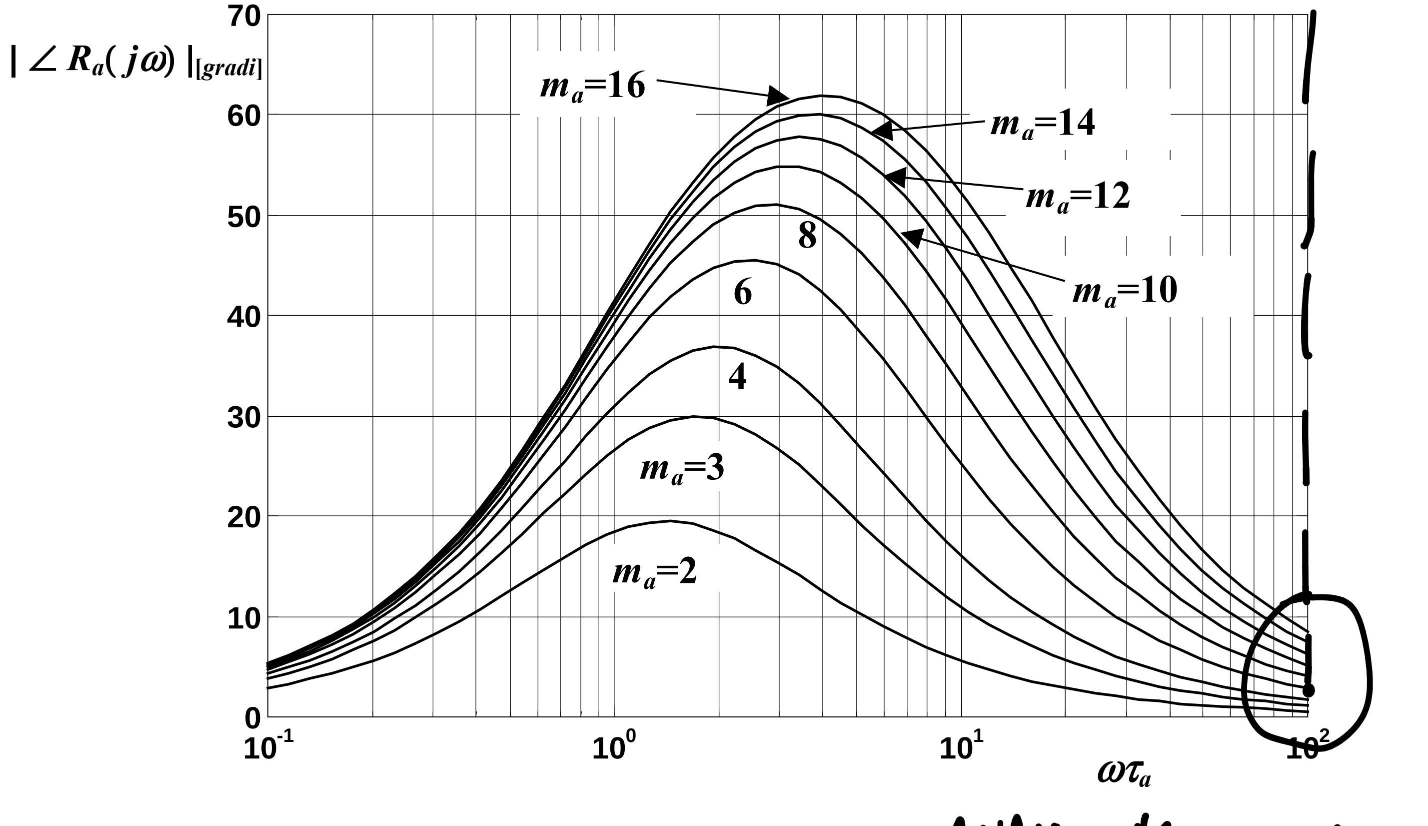
$\Longrightarrow F(s) = G(s) \cdot P(s)$

$F(i\omega) = 2,5 \cdot \dfrac{1}{i\omega\left(1 + \frac{i\omega}{8}\right)} \cdot \dfrac{1 + \frac{i\omega}{0,1}}{1 + \frac{i\omega}{0,6}}$

PUNTI DI ROTTURA:

• $\omega = 0$	$-20\,dB$	$-90°$	$-20\,dB$	$-90°$
• $\omega = 0,1$	$+20\,dB$	$+90°$	0	0
• $\omega = 0,6$	$-20\,dB$	$-90°$	$-20\,dB$	$-90°$
• $\omega = 8$	$-20\,dB$	$-90°$	$-4\,dB$	$-180°$





AUMENTO MINIMO

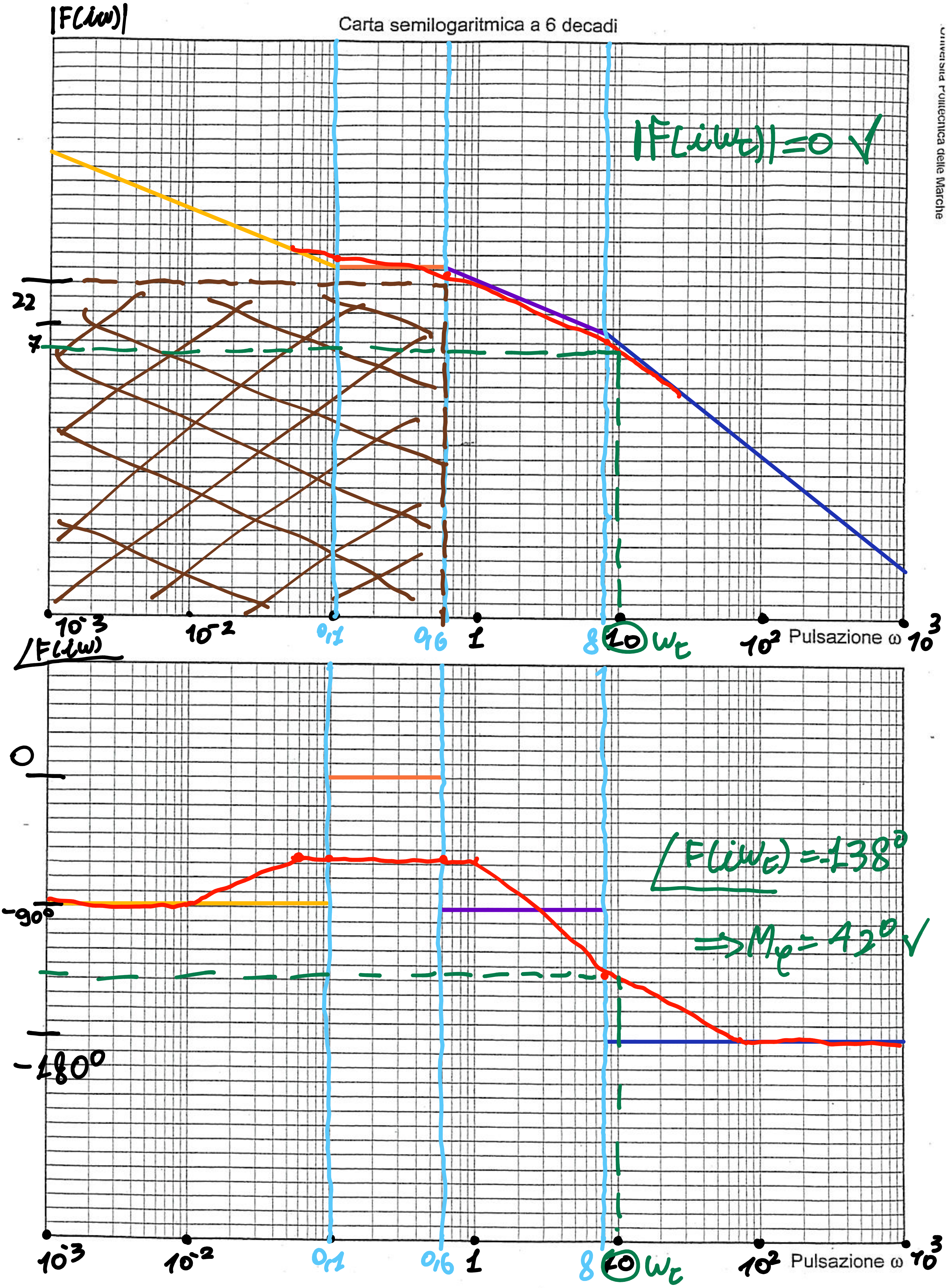
CORREZIONE MODULO:

ω	$1+\frac{i\omega}{0,1}$	$1+\frac{i\omega}{96}$	$1+\frac{i\omega}{8}$	TOT
0,1	+3dB	0	0	+3dB
0,6	0	-3dB	0	-3dB
8	0	0	-3dB	-3dB

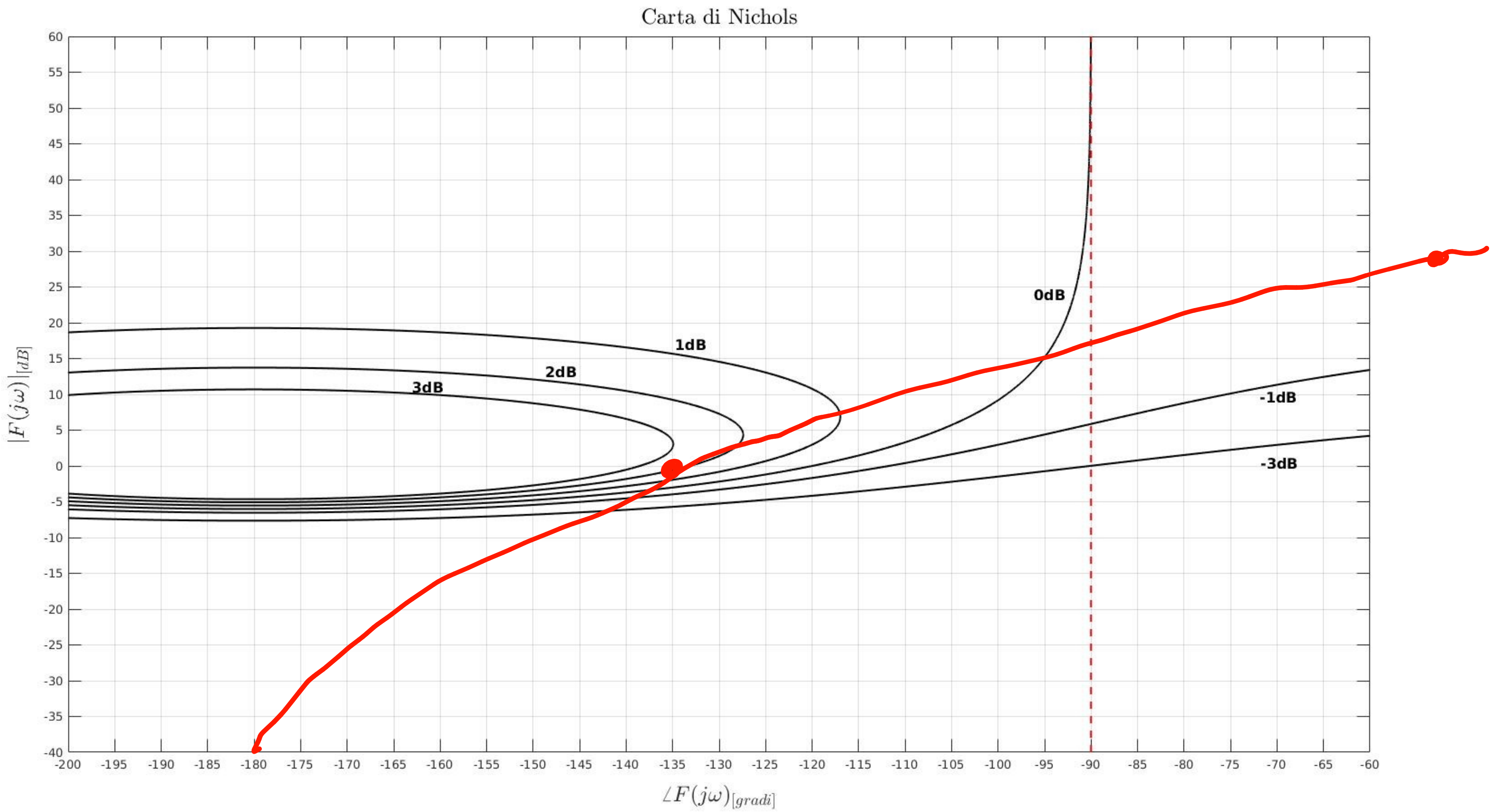
CORREZIONE FASE:

ω	$i\omega$	$1+\frac{i\omega}{0,1}$	$1+\frac{i\omega}{96}$	$1+\frac{i\omega}{8}$	TOT
0,1	$-90°$	$+45°$	$-10°$	0	$-55°$
0,6	$-90°$	$+80°$	$-45°$	0	$-55°$
8	$-90°$	$+90°$	$-90°$	$-45°$	$-135°$

$$F(i\omega)=2,5\cdot\frac{1}{i\omega\left(1+\frac{i\omega}{8}\right)}\cdot\frac{1+\frac{i\omega}{0,1}}{1+\frac{i\omega}{96}}$$



|F(iω)|
Carta semilogaritmica a 6 decadi
Università Politecnica delle Marche
|F(iω_t)|=0 ✓
22
7
10^-3
10^-2
0,1
0,6
1
8
10
ω_t
10^2
Pulsazione ω
10^3
∠F(iω)
0
-90°
-180°
∠F(iω_t)=-138°
⇒ M_φ = 42° ✓

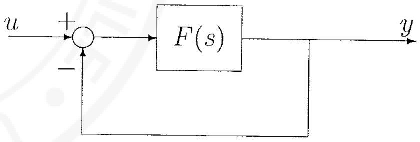


Carta di Nichols
0dB
1dB
2dB
3dB
-1dB
-3dB
60
55
50
45
40
35
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-200
-195
-190
-185
-180
-175
-170
-165
-160
-155
-150
-145
-140
-135
-130
-125
-120
-115
-110
-105
-100
-95
-90
-85
-80
-75
-70
-65
-60
$|F(j\omega)|_{[dB]}$
$\angle F(j\omega)_{[gradi]}$

Domanda Scritta di Controlli Automatici (9CFU) - 16/9/2013

Esercizio 1

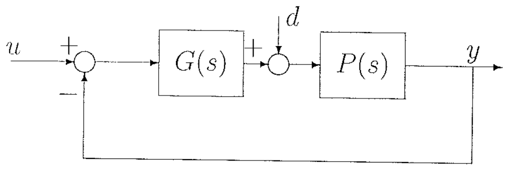
È dato il sistema di controllo:



in cui: $F(s) = \dfrac{K(s+3)}{s^2(s-p)}$. Utilizzando il criterio di Nyquist, studiare la stabilità del sistema a ciclo chiuso, per $K \in \mathbb{R}$, $K \neq 0$ e $p \in \mathbb{R}$, $p \neq 0$, $p \neq -3$.

Esercizio 2

È dato il sistema di controllo:



in cui $P(s) = \dfrac{s+5}{s(s+2)(s+4)}$; $\quad d(t) = \delta_{-1}(t)$.

Utilizzando la sintesi con il luogo delle radici, progettare $G(s)$ in modo che:

- il sistema sia astatico rispetto al disturbo $d(t)$.
- tutti i poli della funzione di trasferimento in catena chiusa abbiano parte reale minore di -1.

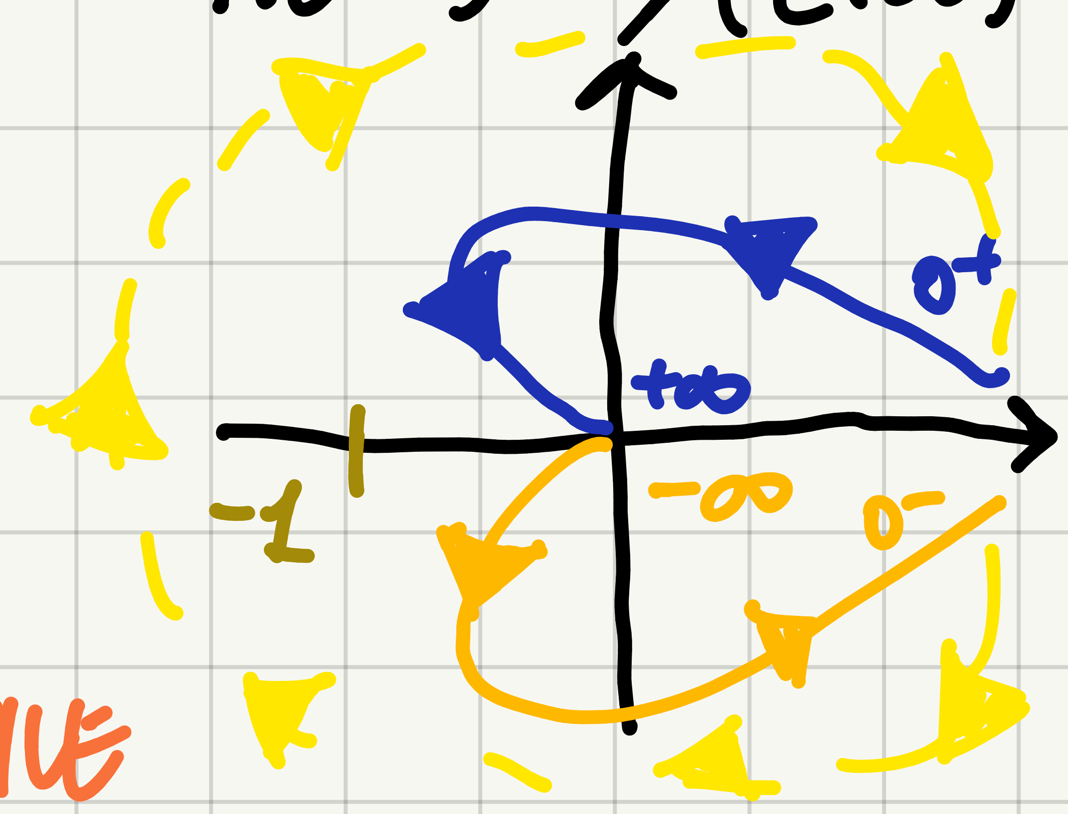
Calcolare infine la risposta a regime permanente all'ingresso $u(t) = (2t - 6)\delta_{-1}(t)$.

1

$$F(s) = -\frac{3K}{p}\cdot\frac{\left(1+\frac{s}{3}\right)}{s^2\left(1-\frac{s}{p}\right)} \qquad F(i\omega) = -\frac{3K}{p}\cdot\frac{\left(1+\frac{i\omega}{3}\right)}{(i\omega)^2\left(1-\frac{i\omega}{p}\right)}$$

CASO 1: $K>0,\ P>0$ $\quad P_t = 1$

$M(0^+) = \infty,\ \varphi(0^+) = -360^\circ \qquad M(+\infty) = 0,\ \varphi(+\infty) = -180^\circ$

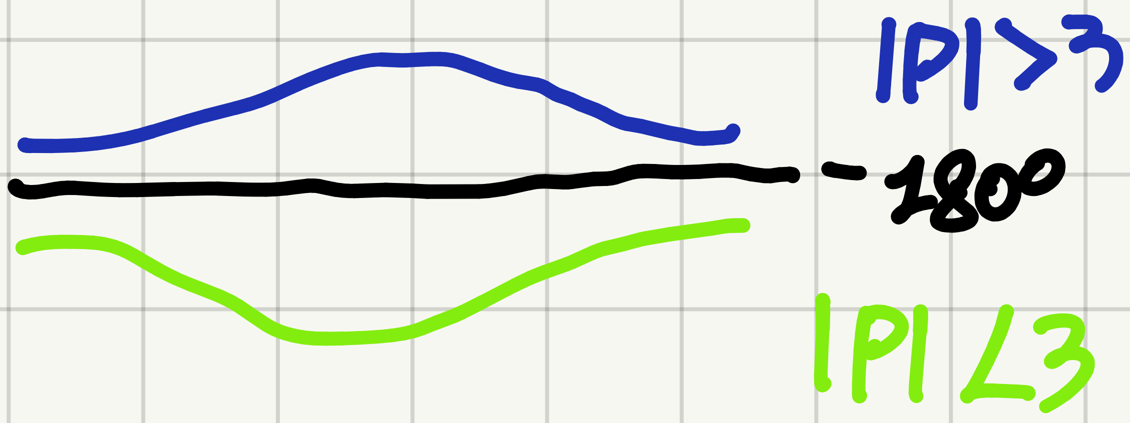


$\overset{\curvearrowright}{N} = 1$

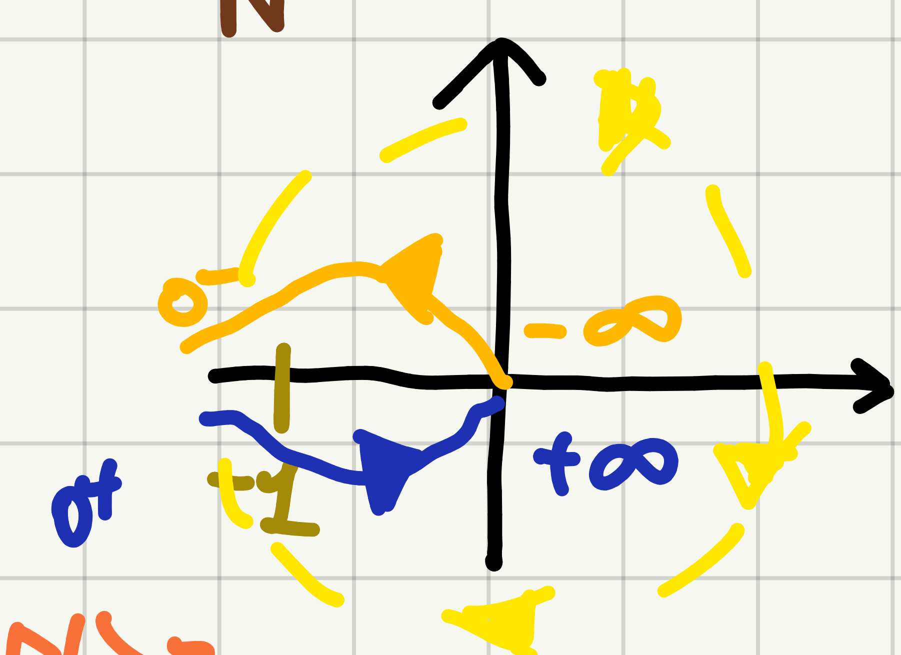
$\overset{\curvearrowright}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 1: $K>0,\ P<0$ $\quad P_t = 0$

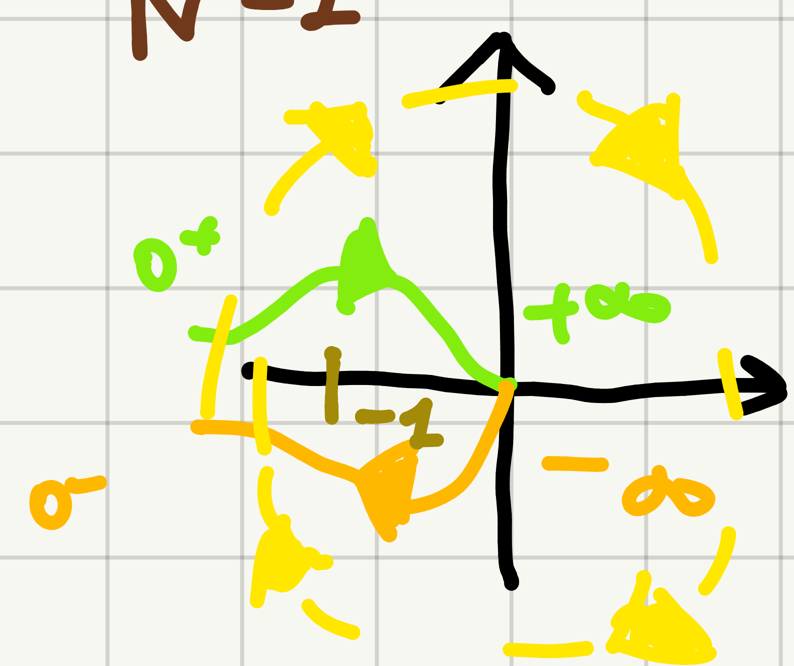
$M(0^+) = \infty,\ \varphi(0^+) = -180^\circ \qquad M(+\infty) = 0,\ \varphi(+\infty) = -180^\circ$



$\overset{\curvearrowright}{N} = 0$



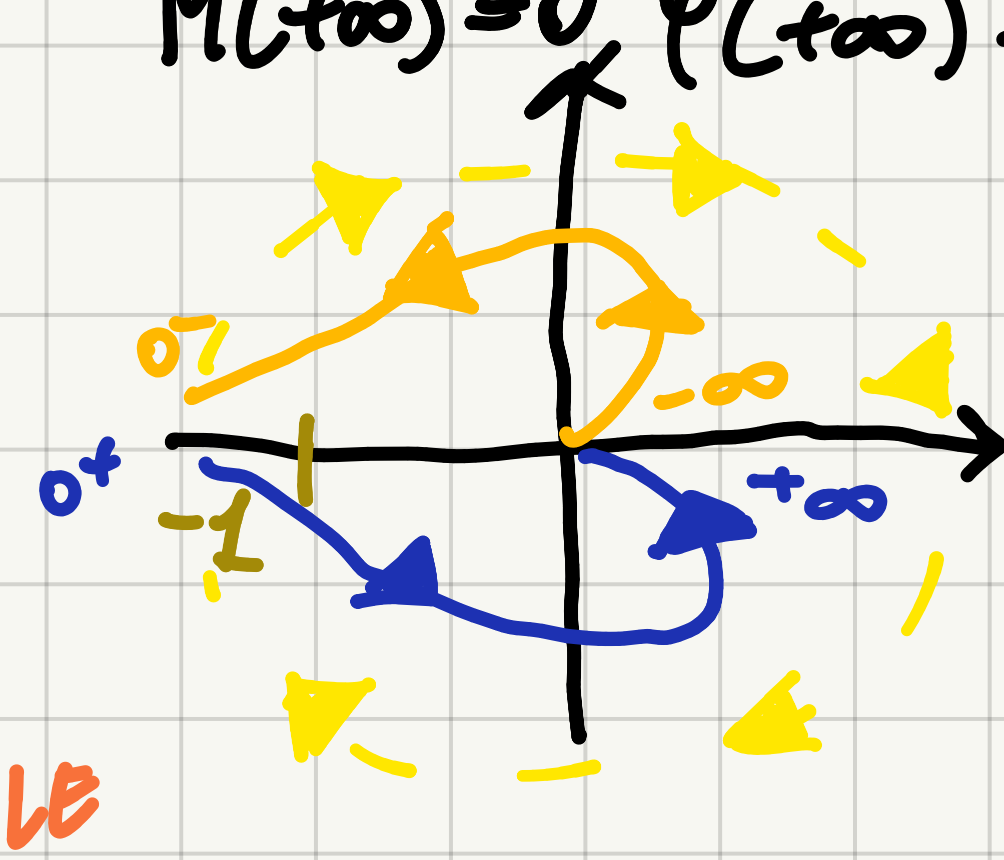
$\overset{\curvearrowright}{N} = 1$



SISTEMA STABILE PER $|P|>3$

CASO 1: $K<0,\ P>0$ $\quad P_t = 1$

$M(0^+) = \infty,\ \varphi(0^+) = -180^\circ \qquad M(+\infty) = 0,\ \varphi(+\infty) = 0^\circ$

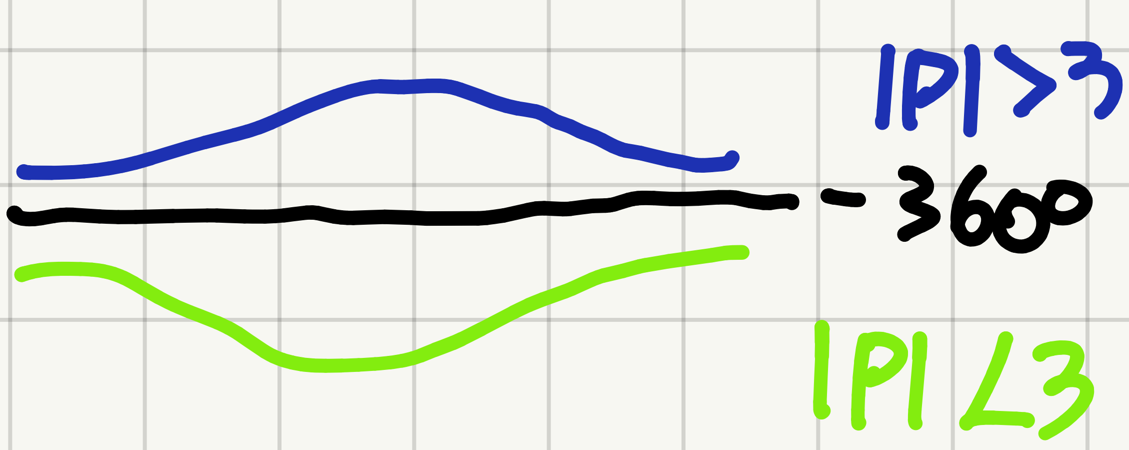


$\overset{\curvearrowright}{N} = 0$

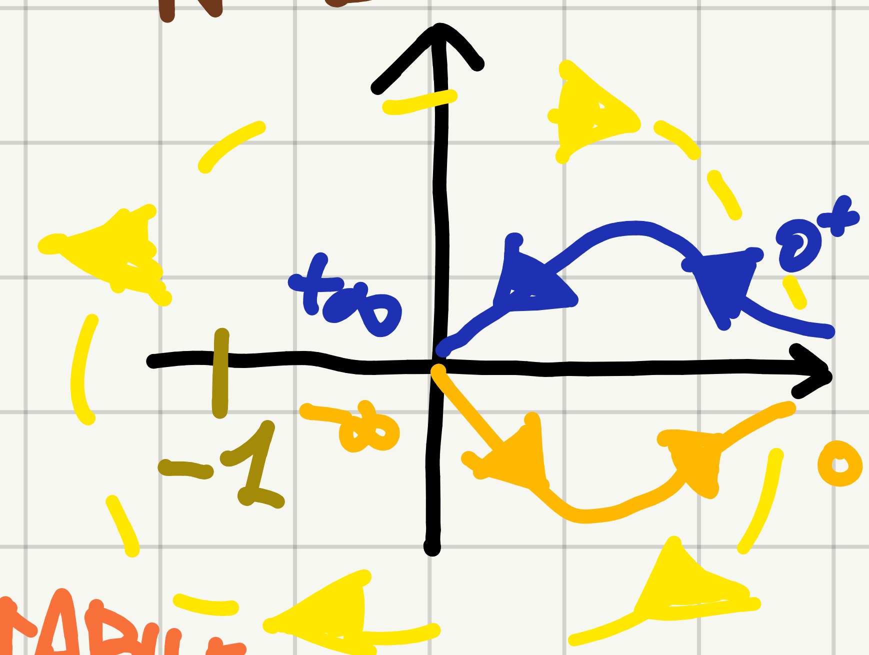
$\overset{\curvearrowright}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 1: $K<0,\ P<0$ $\quad P_t = 0$

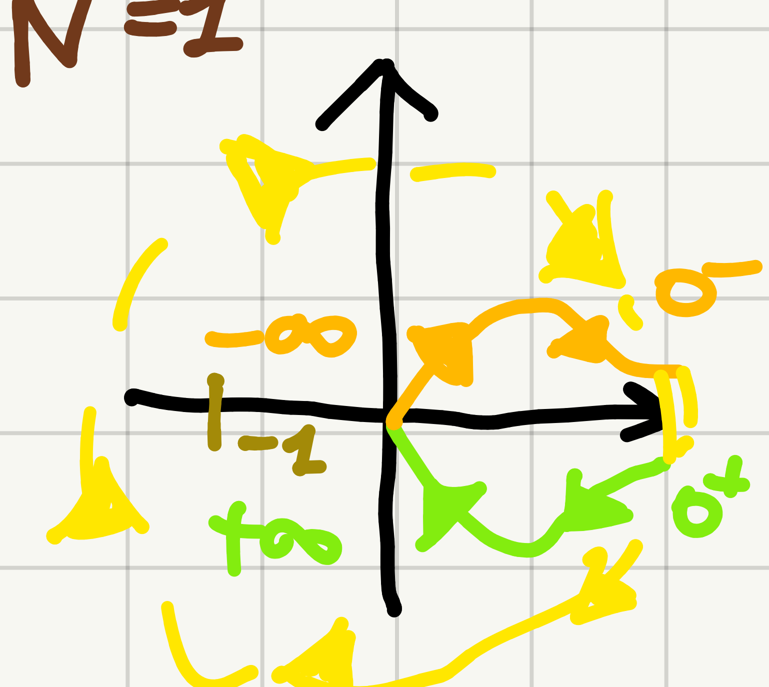
$M(0^+) = \infty,\ \varphi(0^+) = -360^\circ \qquad M(+\infty) = 0,\ \varphi(+\infty) = -360^\circ$



$\overset{\curvearrowright}{N} = 1$



$\overset{\curvearrowright}{N} = 1$



$\overset{\curvearrowright}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

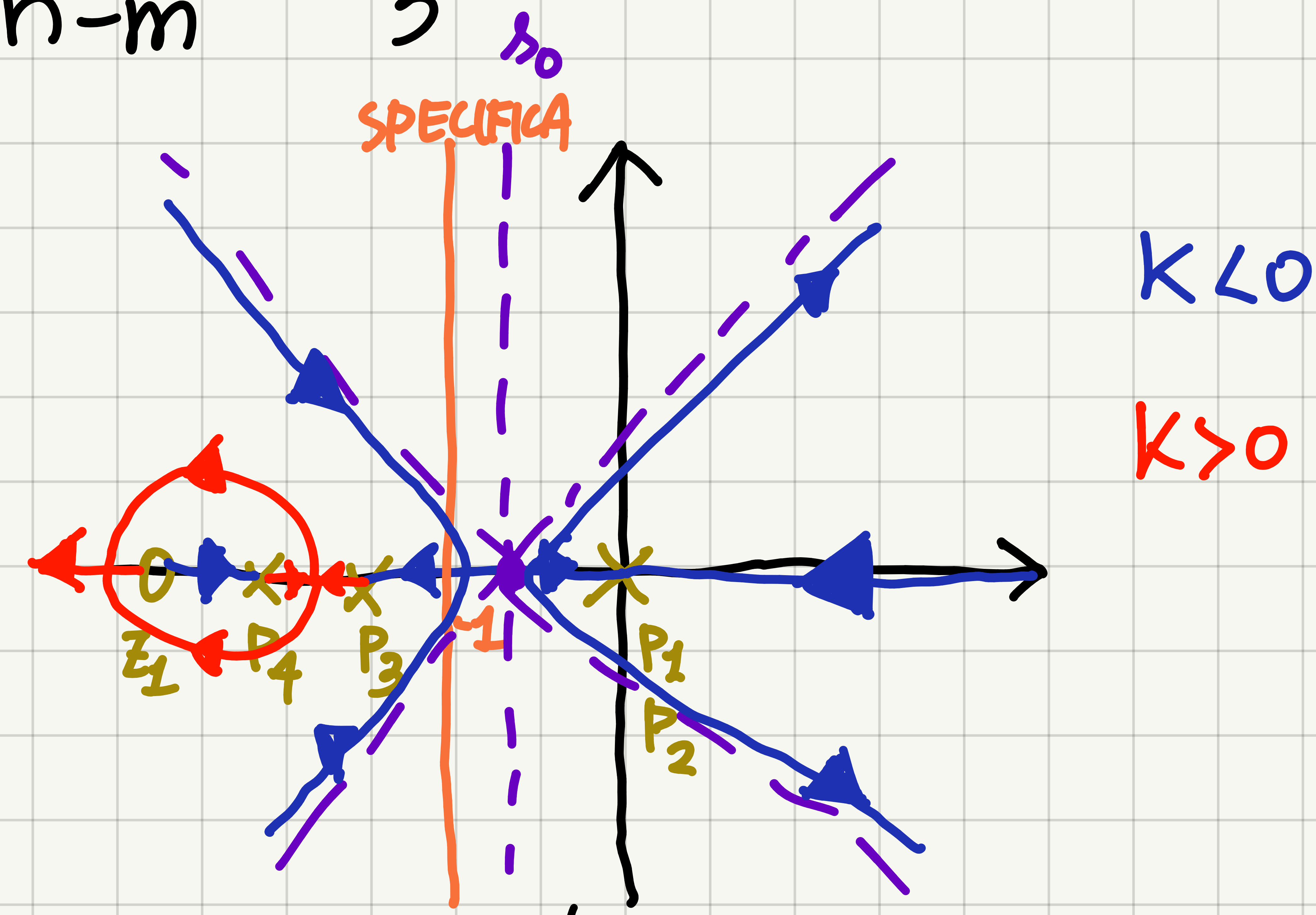
②

ASTATISMO RISPETTO DISTURBO DI GRADO $1 \Rightarrow G(s) = \frac{K}{s}$

$$F(s) = G(s) \cdot P(s) = K \cdot \frac{(s+5)}{s^2(s+2)(s+4)}$$

$n=4, m=1 \Rightarrow n-m=3$ $\quad P_1=P_2=0, P_3=-2, P_4=-4; Z_1=-5$

$$s_0 = \frac{\Sigma P - \Sigma Z}{n-m} = -\frac{1}{3}$$



SPECIFICA NON SODDISFATTA

- INTRODUCO UNO ZERO IN MODO CHE $n-m=2$

$G(s) = \frac{K}{s} \cdot (s - Z_2)$ $\quad Z_2 = -4$

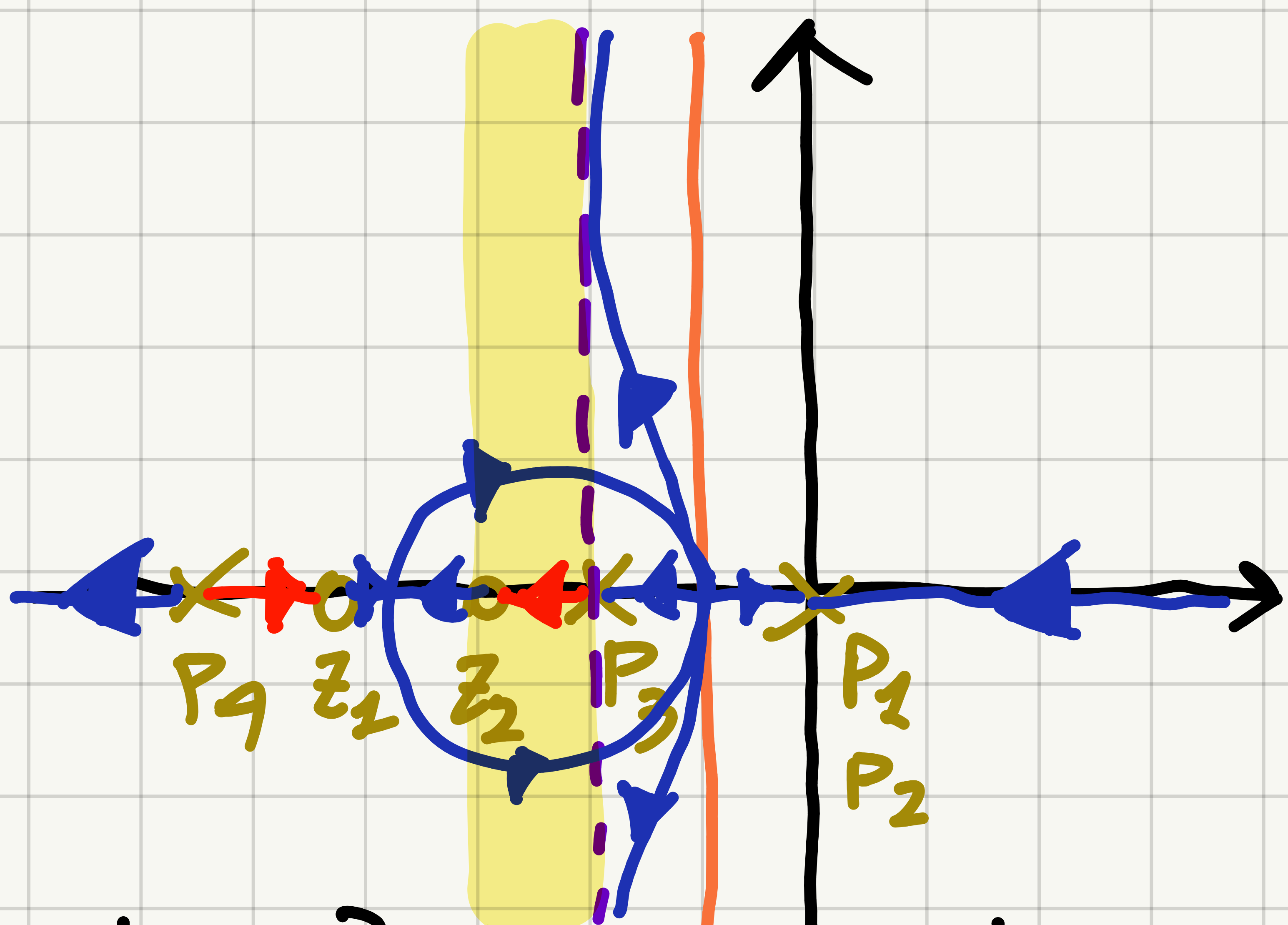
- INTRODUCO UNA COPPIA POLO-ZERO PER SPOSTARE s_0

$G(s) = \frac{K}{s} \cdot \frac{(s-Z_2)(s-Z_3)}{(s-P)}$ $\quad Z_3 = -3$ (IL POLO È IN $s_0 = -2 \Rightarrow$ NON POSSO SEMPLIFICARE $Z_3 = -2$)

$$F(s) = K \cdot \frac{(s+5)(s+3)}{s^2(s+2)(s-p)} \qquad n=4,\ m=2 \implies n-m=2$$

$$p_1=p_2=0,\ p_3=-2,\ p_4=p\ ;\ z_1=-5,\ z_2=-3$$

$$s_0=-2 \qquad -2=\frac{\Sigma p-\Sigma z}{2} \implies p=10$$



$$f(s,K)=s^2(s+2)(s+10)+K(s+5)(s+3)$$

$$\bar{s}=s+1 \implies s=\bar{s}-1$$

$$(\bar{s}-1)^2(\bar{s}+1)(\bar{s}+9)+K(\bar{s}+4)(\bar{s}+2)=0$$

$$\bar{s}^4+8\bar{s}^3+(K-10)\bar{s}^2+(6K-8)\bar{s}+(8K+9)=0$$

5	1	$K-10$	$8K+9$
4	0	8	$6K-8$
3	?	?	?
2			
1			
0			

$\implies$ SISTEMA STABILE PER K

$$U(t) = (2t-6)\delta_{-1}(t) = 2(t)\delta_{-1}(t) + (-6)\delta_{-1}(t)$$

$$= 2U_1(t) - 6U_2(t)$$

- $U_1(t)$

$$\tilde{e}_{U_1}(t) = K_d U_1(t) - \tilde{y}_{U_1}(t)$$

$$\tilde{e}_{U_1}(t) = \frac{1}{K_G \cdot K_P} = \frac{1}{K_F} = -\frac{3}{4}$$

$$\tilde{y}_{U_1}(t) = K_d U_1(t) - \tilde{e}_{U_1}(t) = \left(t + \frac{3}{4}\right)\delta_{-1}(t)$$

- $U_2(t)$

GRADO DI $U_2(t) <$ TIPO DI $F(s) \Rightarrow \tilde{y}_{U_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 2\left(t + \frac{3}{4}\right)\delta_{-1}(t) - 6\delta_{-1}(t)$$