NON VALIDITY: VERIFY IF $v \not\models \varphi$ IS SATISFIABLE

$v \models \neg\varphi \iff v \models \neg(\forall x (Rx \lor Sx) \to \forall x Rx \lor \forall x Sx)$

$\iff v \models \forall x (Rx \lor Sx)$ AND $v \models \neg(\forall x Rx \lor \forall x Sx)$

$\iff \forall a \in M, v[\frac{a}{x}] \models Rx \lor Sx$ AND $v \not\models \forall x Rx$ AND $v \not\models \forall x Sx$

$\iff \forall a \in M, v[\frac{a}{x}] \models Rx \lor Sx$ AND $\exists b \in M \mid v[\frac{b}{x}] \not\models Rx$

AND $\exists c \in M \mid v[\frac{c}{y}] \not\models Sx$

$\iff \forall a \in M (v[\frac{a}{x}] \models Rx$ OR $v[\frac{a}{x}] \models Sx)$ AND

$\exists b \in M \mid v[\frac{b}{x}] \not\models Rx$ AND $\exists c \in M \mid v[\frac{c}{y}] \not\models Sx$

$\iff \forall a \in M, (a \in [[R]]$ OR $a \in [[S]])$

AND $\exists b \mid b \notin [[R]]$ AND $\exists c \mid c \notin [[S]]$

$c \notin [[S]]$

$\iff \forall a \in M (a \in [[R]] \cup [[S]])\ \exists b, c \in M \mid b \notin [[R]]$

$\iff M = [[R]] \cup [[S]], \exists b, c \in M, b \notin [[R]], c \notin [[S]]$

SATISFIABLE FOR $[[R]] = \{c\}, [[S]] = \{b\} \Rightarrow$ φ IS NOT VALID

USING SEMANTIC TREE

$\neg(\forall x(Rx \vee Sx) \to \forall x Rx \vee \forall x Sx)$

| α-FORMULA

$\forall x(Rx \vee Sx), \neg(\forall x Rx \vee \forall x Sx)$

| α-FORMULA

$\forall x(Rx \vee Sx), \neg\forall x Rx, \neg\forall x Sx$

| δ-FORMULA

$\neg Ra, \neg\forall x Sx, Ra \vee Sa, \forall x(Rx \vee Sx)$

$\neg Ra, \neg\forall x Sx, Sa, \forall x(Rx \vee Sx)$ ⋮

| // EVERY TIME, WE ADD ANOTHER ELEMENT WE ADD NEW VARIABLES

$\neg Ra, \neg Sb, Sa, Rb \vee Sb, \forall x(Rx \vee Sx)$

$\neg Ra, \neg Sb, Sa, Rb$ ⋮

SATISFIABLE LEAF

$M = \{a, b\}, [[R]] = \{b\}, [[S]] = \{a\} \to M \models \varphi \neg\varphi$

$\neg\varphi$ IS SATISFIABLE $\Rightarrow$ φ IS NOT VALID

PROVE THAT THE FOLLOWING FORMULA IS SATISFIABLE BUT NOT VALID:

$(\forall x. Ry \to \forall x. Sx) \to \forall x\, (Rx \to Sx)$

① SATISFIABILITY: NOT FIND STRUCTURE THAT DOESN'T SATISFY THE PREMISES

$\neg(\forall x. Rx \to \forall x. Sx) \equiv \forall x. Rx \wedge \exists x. \neg Sx$

- $(\forall x. Ry \to \forall x. Sx) \to \forall x\, (Rx \to Sx)$ — β-FORMULA
  - $\neg(\forall x. Rx \to \forall x. Sx)$ — α-FORMULA
    - $\forall x. Rx, \neg(\forall x. Sx)$ — α-FORMULA
      - $Ra, \forall x Rx, \exists x. \neg Sx$ — δ-FORMULA
        - $Ra, \neg Sa$ SATISFIABLE
  - $\forall x\, (Rx \to Sx)$ — β-FORMULA
    - $\neg(\forall x. Rx)$
      - $\neg Ra$
    - $\forall x. Sx$
      - Sa

- $M = \{a\}$ $[[R]] = \{a\}$ $[[S]] = \emptyset$
- $M = \{a\}$ $[[R]] = \emptyset$ $[[S]] =$ ANY SUBSET OF M
- $M = \{a\}$ $[[R]] =$ ANY SUBSET OF M $[[S]] = \emptyset$