EXTRA: WHAT IF I USE SEMANTIC TREE TO PROVE VALIDITY?

$$\forall x\,(R, fx),\ \forall x.\neg R(fx, x),\ \forall x.\neg(R(x,x) \leftrightarrow R(fx, fx))$$

$$|$$

$$R(a, fa), \neg R(fa, a), \neg(Raa \leftrightarrow R(fa, fa)), \ldots$$

$$|$$

$$R(a, fa), \neg R(fa, a), \neg(Raa \leftrightarrow R(fa, fa)), R(fa, ffa), \neg(fa, ffa), \neg R(ffa, fa), \ldots$$

$$|$$

$ffa = f^2(a)$

$$\ldots\ldots$$

WE WILL OBTAIN A SITUATION LIKE $a \to f(a) \to f^2(a) \to f^3(a) \to f^4(a) \to \ldots$

BUT WHEN WILL I FINISH? :-) L'ORTO BIO

PROVE THAT THE FOLLOWING SET OF FORMULAS ARE SATISFIABLE

a) $\{\forall x \exists y Rxy, \forall xy \exists z (Rxy \to Ryz \wedge Rzx), \forall y \neg Rxx\}$

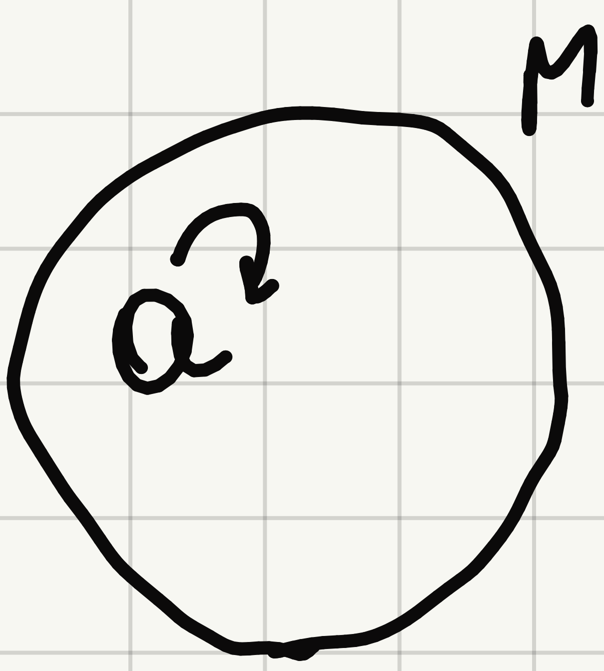
EXAMPLE OF SERIAL RELATION

- $\forall xy (Rxy \to \exists z (Ryz \wedge Rzy))$

$x \to y \quad \longrightarrow \quad y \to z \to x$

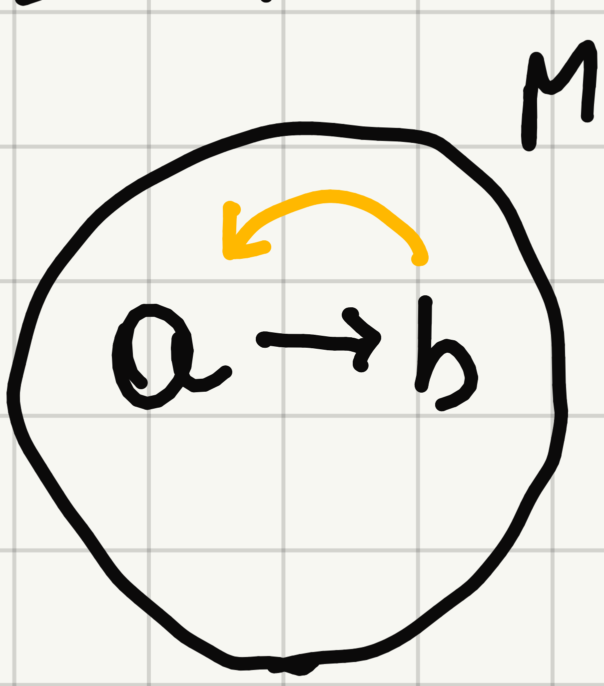
! IT IS POSSIBLE THAT $z = y$ OR $z = x$

1st TRY $\quad M = \{a\}$



✓
✓
✗

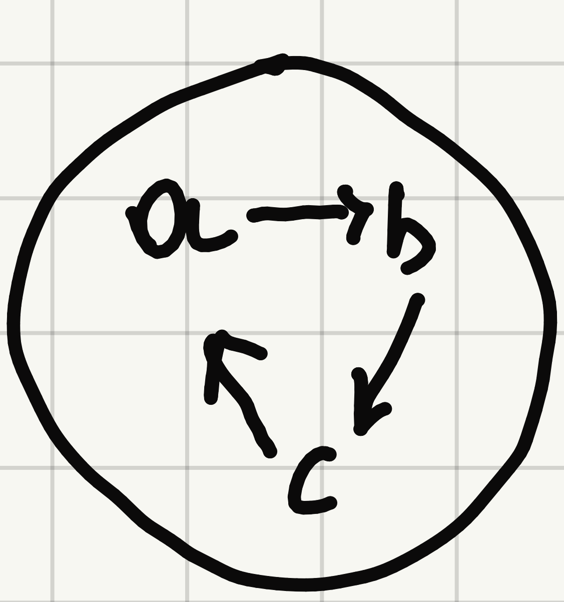
2nd TRY



✓
✗ $\quad z = a \longrightarrow \underline{aRa}$
✗

✓
✓
✓

3rd TRY



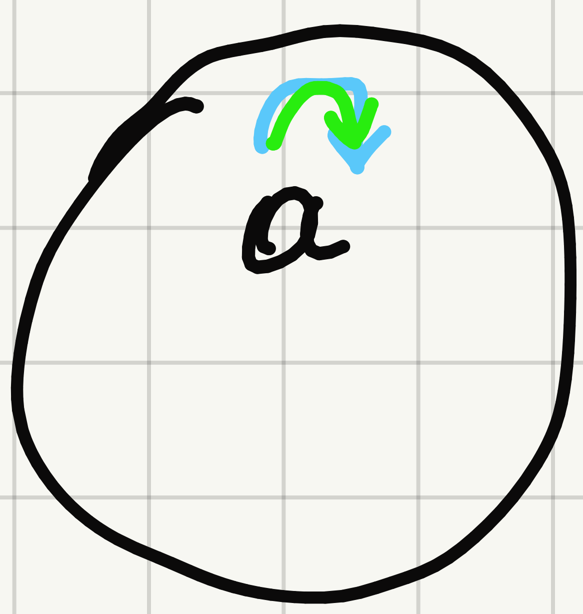
$x=a$	$x=a$	$x=b$	$x=b$	$x=c$	$x=c$
$y=b$	$y=c$	$y=a$	$y=c$	$y=a$	$y=b$
$z=c$	$z=b$	$z=c$	$z=a$	$z=b$	$z=a$
✓✓✓			✓✓✓	✓✓✓	

$\Longrightarrow$ SATISFIABLE SET

b) $\{\exists x \forall y\, Ryx, \forall x \exists y\, Ryx, \forall x (Rxx \to \exists y (\neg Ryy \wedge \neg Rxy))\}$

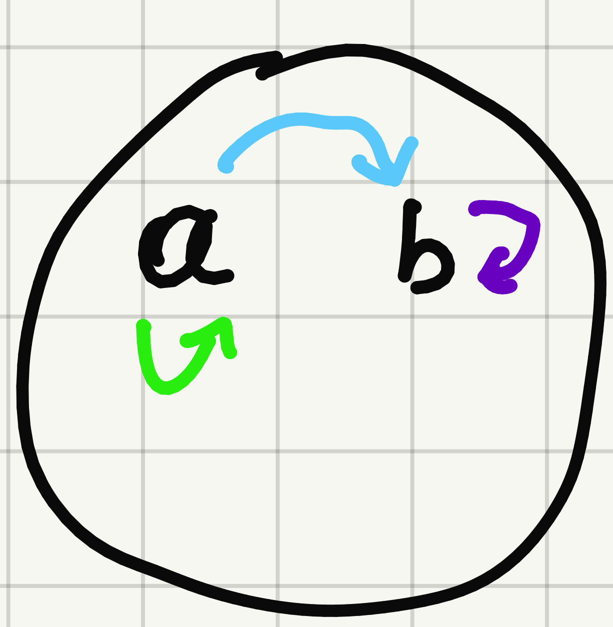
- $\exists x(\forall y(yRx)\}$
- $\forall x(\exists y(yRx)\}$
- $\forall x(xRx \to (\exists y(\neg(yRy)) \wedge \exists y(\neg(xRy)\}$

- $M=\{a\}$



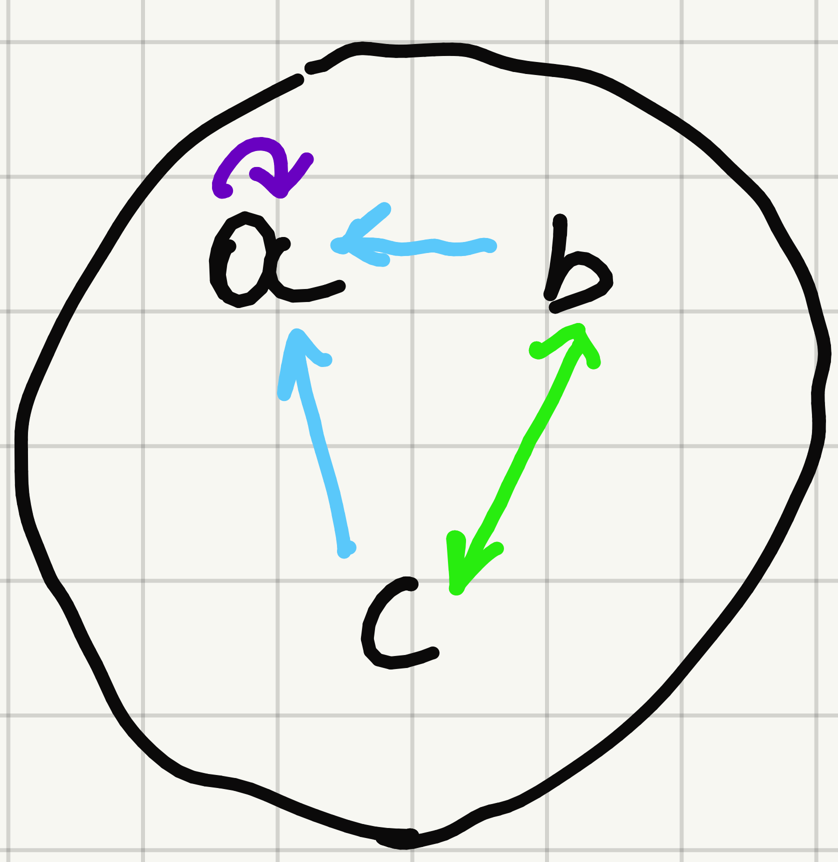
✓
✓
× $(aRa \to \ldots\ldots$ $\neg aRa$

- $M=\{a,b\}$



$y=a$	$y=a$
$x=b$	$x=a$
✓	
✓	
✓	×

- $M=\{a,b,c\}$



$x=a$, $y=a$	$x=a$, $y=b$	$y=a$, $y=c$
$x=b$, $y=a$	$x=b$, $y=b$	$x=b$, $y=c$
$x=c$, $y=a$	$x=c$, $y=b$	$x=c$, $y=c$