QUICK REVIEW

RELIABILITY $R(t) = P(\underset{\text{FAILURE TIME}}{T} > t) = e^{-\lambda t}$ $\quad T \sim e(\lambda)$

λ = FAILURE RATE OF THE COMPONENT

$E[T] = MTTF = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{MTTF}$

MTTF → MEAN TIME TO FAILURE GENERALLY CALCULATED IN YEARS

MTTR → MEAN TIME TO REPAIR

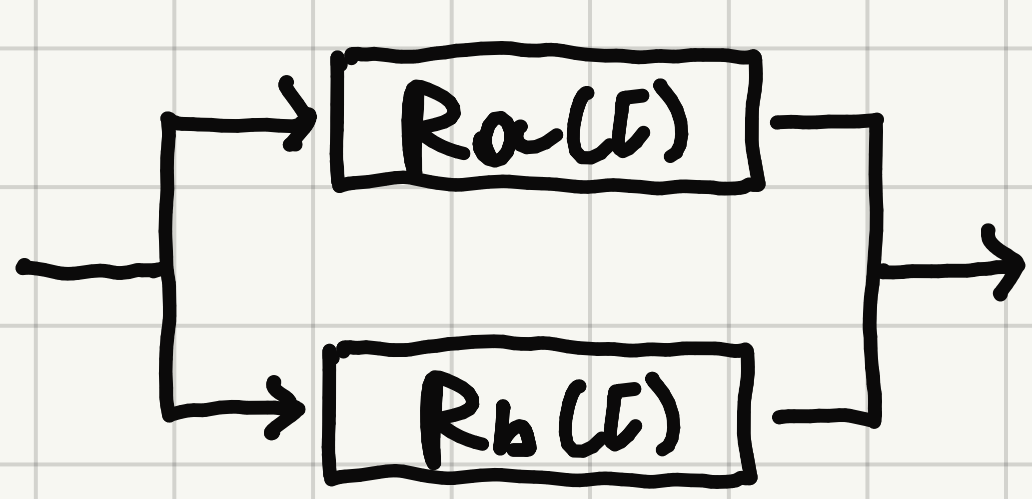
AVAILABILITY $A(t) = \frac{MTTF}{MTTF + MTTR}$

COMBINATION OF COMPONENTS

- IN SERIES $\longrightarrow \boxed{R_a(t)} \longrightarrow \boxed{R_b(t)} \longrightarrow$

$$R_s(t) = R_a(t) \cdot R_b(t)$$

- IN PALALLEL



$$R_p(t) = 1 - [(1 - R_a(t))(1 - R_b(t))]$$

Exercise session 1 - Dependability

EXERCISE 1

A heart pacemaker has a failure rate of $\lambda = 0.25 \times 10^{-8}$ per hour.

1. Which is its MTTF?
2. What is the probability that it fails during the first five years of operation?

EXERCISE 2

Consider a generic component D. Calculate the minimum integer value $MTTF_D$ to have $R_D(t) >= 0.96$ at t=5days.

EXERCISE 3

A smartphone manufacturer determines that their products have a MTTF of 59 years in normal use. Estimate how long a warranty should be set if no more than 5% of the items are to be returned for repair.

EXERCISE 4

A complex system has a failure rate of $\lambda = 0.25 \times 10^{-4}$ per hour and an MTTR = 72 hours in normal use.

1. What is its steady-state availability?
2. If MTTR is increased to 120 h, what failure rate can be tolerated without decreasing the availability of the system?

EXERCISE 5

Consider a server architecture in terms only of its main 3 components: CPU, MEMORY and Hard Drive. Consider that the components have constant failure rates of 1/64, 1/58 and 1/28 per year, respectively. Assuming that component failures are independent events,

1. Draw the RBD of the server architecture.
2. Compute the MTTF for the server.
3. Compute the reliability of the server for a 3-year mission.

① $\lambda = 0{,}25 \cdot 10^{-8}/h$ a) MTTF?

b) P(FAILURE DURING FIRST 5 YEARS)?

a) $MTTF = \frac{1}{\lambda} = 4 \cdot 10^{8}\, h = 1{,}67 \cdot 10^{7}\, d = 4{,}57 \cdot 10^{4}\, y$

b) $P(T \leq 5y) = 1 - P(T > 5) = 1 - e^{-\lambda \cdot 5y} =$

$= 1 - e^{-(0{,}25 \cdot 10^{-8}/\cancel{h})(5 \cdot 365 \cdot 24 \cancel{h})} = 0{,}0109\,\%$

② $R_D(t = 5d) \geq 0{,}96$ $MTTF_D$?

$R_D(t) = e^{-\lambda_D t} \geq 0{,}96$

$\ln(e^{-\lambda_D t}) \geq \ln(0{,}96) \rightarrow -\lambda_D t \geq \ln(0{,}96)$

$\lambda_D \leq -\frac{\ln(0{,}96)}{5d} = 8{,}1644 \cdot 10^{-3}/d$

$MTTF_D \geq 1/\lambda_D = 122{,}48\, d = 123\, d$

③ $MTTF = 59\, y$ $\hat{t} \mid R(\hat{t}) \geq 95\%$?

$\lambda = \frac{1}{MTTF} = 1{,}6949 \cdot 10^{-2}/y$

$R(\hat{t}) = e^{-\lambda \hat{t}} \geq 0{,}95$

$-\lambda \hat{t} \geq \ln(0{,}95) \rightarrow \hat{t} \leq -\frac{\ln(0{,}95)}{\lambda} = 3{,}026\, y = 3$ YEARS

④ $\lambda = 0{,}25 \cdot 10^{-4}/h$ $MTTR = 72h$ a) $A(t)$?

b) $MTTR_2 = 120h$ $\lambda_2 \mid A_2(t) = A(t)$

a) $A(t) = \frac{MTTF}{MTTF + MTTR}$ $MTTF = \frac{1}{\lambda} = 4 \cdot 10^{4}\, h$

$A(t) = \frac{40000}{40072} = 99{,}820\,\%$

b) $A_2(t) = \frac{MTTF_2}{MTTF_2 + MTTR_2} = A(t)$

$MTTF_2 = A(t)(MTTF_2 + MTTR_2)$

$MTTF_2(1 - A(t)) = A(t) \cdot MTTR_2$

$MTTF_2 = \frac{A(t)}{1 - A(t)} \cdot MTTR_2 = 6,6547 \cdot 10^{4} h$

$\lambda_2 = 1/MTTF_2 = 0,15 \cdot 10^{-4}/h$

5 $\lambda_{CPU} = \frac{1}{64}/y$ $\quad \lambda_M = \frac{1}{58}/y$ $\quad \lambda_{HD} = \frac{1}{28}/y$ INDEPENDENT EVENTS

a) RBD? b) $MTTF_S$? c) R_S(3 YEARS)?

a) → $\boxed{\text{CPU}}$ → $\boxed{\text{MEMORY}}$ → $\boxed{\text{HARD DRIVE}}$ →

b) $R_S(t) = R_{CPU}(t) \cdot R_M(t) \cdot R_{HD}(t) =$

$= e^{-\lambda_{CPU} t} \cdot e^{-\lambda_M t} \cdot e^{-\lambda_{HD} t} =$

$= e^{-(\lambda_{CPU} + \lambda_M + \lambda_{HD}) t} = e^{-6,858 \cdot 10^{-2}/y \cdot t}$

$MTTF_S = \frac{1}{\lambda_S} = 14,58 y$

c) $R_S(3 \text{ YEARS}) = e^{-6,858 \cdot 10^{-2}/y \cdot 3y} = 81,40\%$

Exercise session 2 - Dependability

EXERCISE 1

A computer system is designed to have a failure rate of one fault in 5 years in normal use. The system has no fault tolerance capabilities, so it fails upon occurrence of the first fault.

1. What is the MTTF of such a system?
2. What is the probability that the system will fail during its first year of operation?
3. The usual warranty for the system is 2 years. The vendor wishes to offer an additional insurance against failures for the first 5 years of operation at extra cost. The vendor wants to charge $20 for each 1% drop in reliability to offer such an insurance. How much should the vendor charge for such an insurance?

EXERCISE 2

A system with five modules (A, B, C, D and E) has been designed such that it operates correctly if

- modules A or B operate correctly (AND)
- modules C and D operate correctly, or module E operates correctly.

1. Draw an RBD of the system.
2. Write an expression for the reliability of the system.
3. Considering that the MTTF for modules A and B is 3412 hours, while for modules C, D and E is 1245hours, calculate the reliability value after 1 month of the system.

EXERCISE 3

A system is composed of 3 modules (A, B, and C) and it is designed to operate correctly if A and B operate correctly, or if C operates correctly. Considering that the availabilities of the modules are the following,

- $Availability_A = 0,97$
- $Availability_B = 0,92$
- $Availability_C = 0,95$

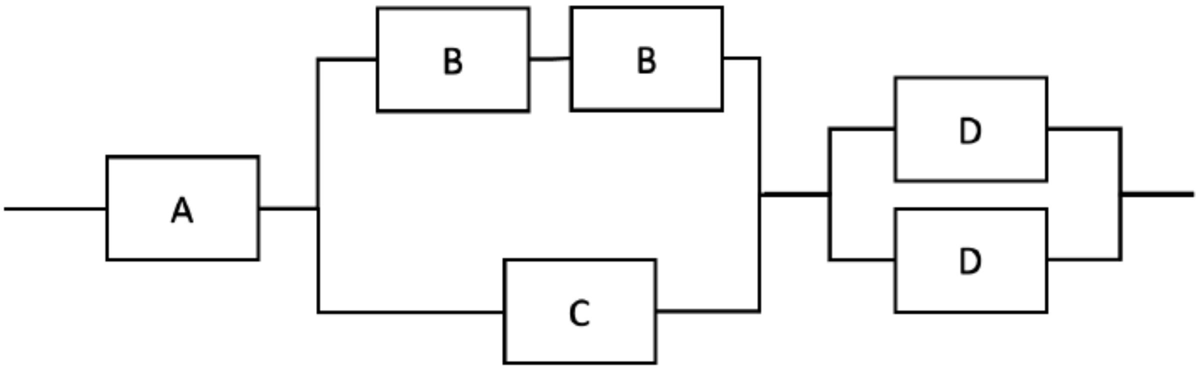
1. Draw the RBD of the system.
2. Calculate the Availability of the system.

EXERCISE 4

A system is composed by 4 non-redundant components, and it has a 1-year reliability equal to 0.92. A new version of the previous system is designed adding a novel feature and using 6 non-redundant components. What is the 1-year reliability of the new system considering that ALL the components have the same MTTF?

EXERCISE 5

A system is composed by components having the following Reliability Block Diagram:



1. Identify all the possible configurations of components that may fail although the entire system does not fail.
2. Calculate $MTTF_B$ knowing that $R_B(t)$ = 83% at time t=2 years
3. Calculate the availability of the entire system knowing that $MTTF_A = MTTF_D$ = 1 year, $MTTF_C$ = 14 months and MTTR is 21 days for all the components.

EXERCISE 6

Out of the 12 identical AC generators on the C-5 aircraft, at least 9 of them must be operating for the aircraft to complete its mission. Failures are known to follow an exponential distribution with a failure rate of 0.01 failure per hour. What is the reliability of the generator system over a 10-hour mission in case the switch is perfect?

1. $\lambda = \frac{1}{5y}$ a) MTTF? b) $P(t \leq 1\ YEAR)$?
c) $T_0 = 2y$ $T_1 = 5y$ cost $= 20\ \$/\%$ extra cost?

a) $MTTF = \frac{1}{\lambda} = 5\ YEARS$

b) $P(t \leq 1\ YEAR) = 1 - P(t > 1\ YEAR) = 1 - e^{-\lambda \cdot 1\ YEAR} = 1 - e^{-1/5} = 18{,}13\%$

c) $R(t=T_0) = e^{-\frac{1}{\lambda}\cdot T_0} = e^{-\frac{1}{5\cancel{y}}\cdot 2\cancel{y}} = e^{-\frac{2}{5}} = 67{,}03\%$

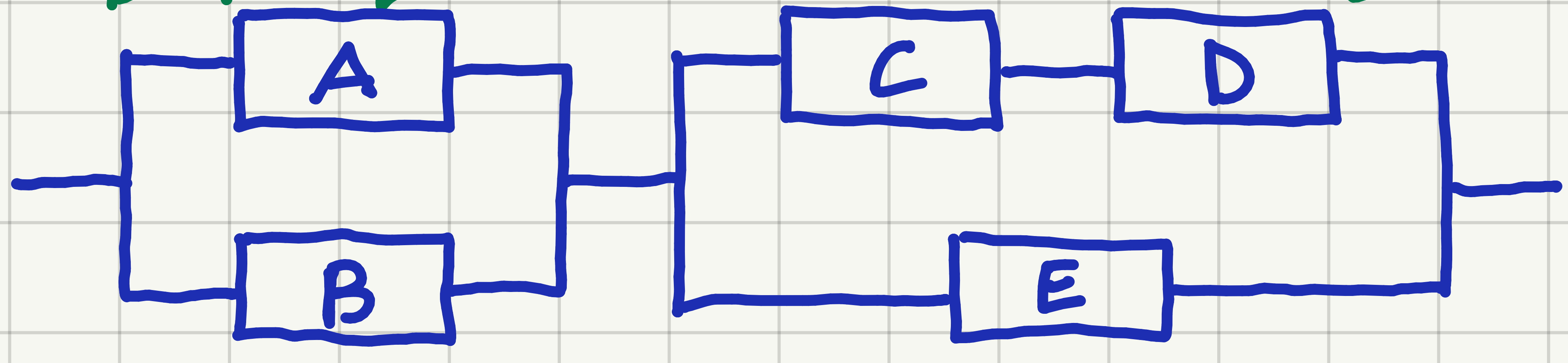
$R(t=T_1) = e^{-\frac{1}{\lambda}T_1} = e^{-\frac{1}{5\cancel{y}}\cdot 5\cancel{y}} = e^{-1} = 36{,}79\%$

$\Delta\% = R(t=T_0) - R(t=T_1) \simeq 30\%$

extra cost $= 20\ \$/\% \cdot 30\% = \$600$

2. $(A \vee B) \wedge ((C \wedge D) \vee E)$ $MTTF_A = MTTF_B = 3412h$ $MTTF_C =$
$= MTTF_D = MTTF_E = 1245h$ a) RBD? b) $R_S(t)$? c) $R(t = 1\ MONTH)$?

a)



b) $R_S(t) = R_{AB}(t) \cdot R_{CDE}(t)$

$R_{AB}(t) = 1 - [(1-R_A(t)) \cdot (1-R_B(t))]$

$R_{CDE}(t) = 1 - [(1-R_{CD}(t))(1-R_E(t))]$

$R_{CD}(t) = R_C(t) \cdot R_D(t)$ $[(1-R_A(t)) \cdot (1-R_B(t))]] \cdot [1-$

$\Rightarrow R_{CDE}(t) = 1 - [(1-R_C(t) \cdot R_D(t))(1-R_E(t))]$

$R_S(t) = [1-[(1-R_A(t)) \cdot (1-R_B(t))]] \cdot [(1-R_C(t) \cdot R_D(t))(1-R_E(t))]$

c) $T = 1\ \text{MONTH} = 30 \cdot 24h = 720h$

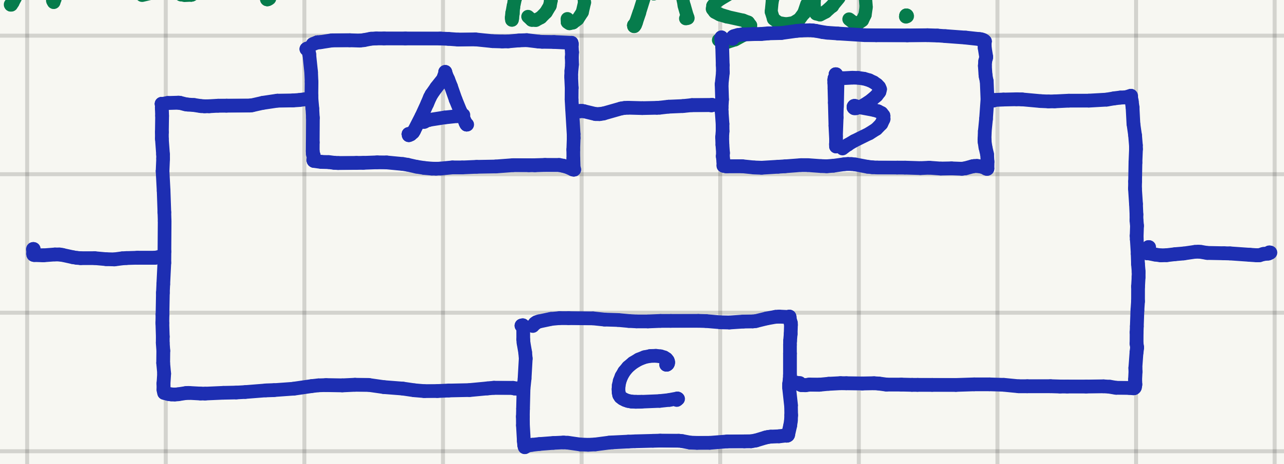
$$\lambda_A = \lambda_B = \frac{1}{3412h} \qquad \lambda_C = \lambda_D = \lambda_E = \frac{1}{1245h}$$

$$R_S(t) = \left[1-\left(1-e^{-\frac{720}{3412}}\right)^2\right]\left[1-\left(1-e^{-2\cdot\frac{720}{1245}}\right)\left(1-e^{-\frac{720}{1245}}\right)\right] = 67{,}36\%$$

3 $(A \wedge B) \vee C$ $\quad A_A(t) = 0{,}97 \quad A_B(t) = 0{,}92 \quad A_C(t) = 0{,}95$

a) RBD? $\quad$ b) $A_S(t)$?

a)

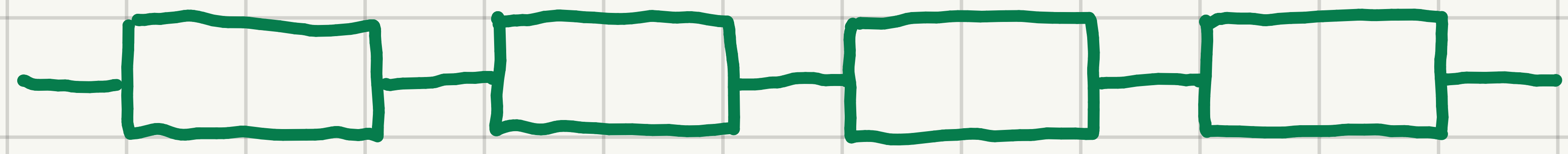


b) AVAILABILITY'S COMBINATIONS HOLD THE SAME RULES AS FOR RELIABILITY

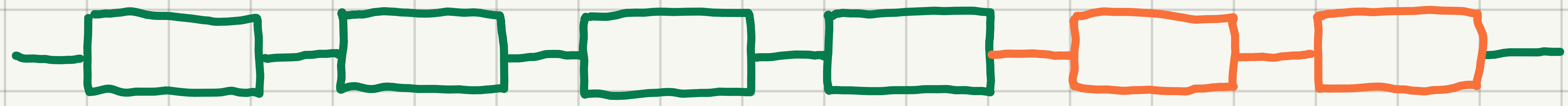
$$A_S(t) = 1 - [(1-A_{AB}(t))(1-A_C(t))]$$

$$A_{AB}(t) = A_A(t) \cdot A_B(t) = 89{,}24\% \Rightarrow A_S(t) = 99{,}462\%$$

4



$R_S(t=1y) = 0{,}92$



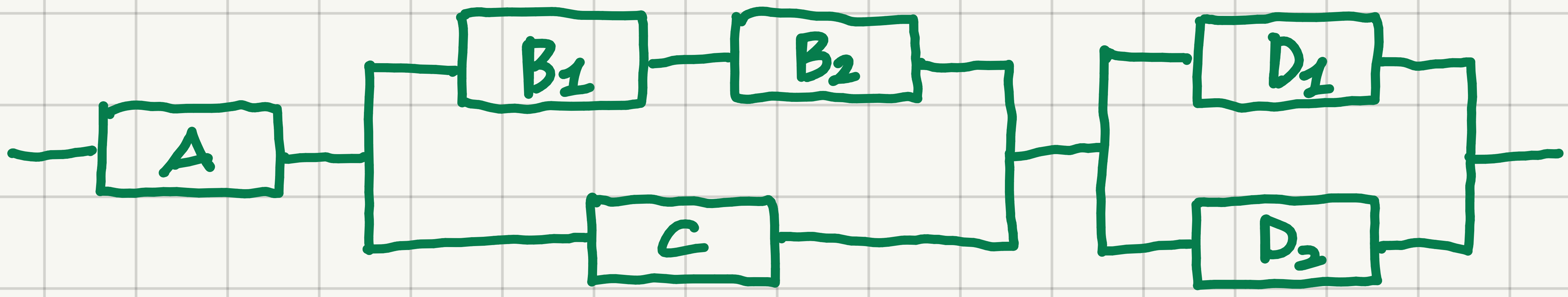
SAME MTTF FOR EACH COMPONENT $\quad R'_S(t=1y)$? $\quad$ 4 COMPONENTS IN SERIES

$$R_S(t) = e^{-4\lambda \cdot 1y} = 0{,}92 \qquad \ln\left(e^{-4\lambda\cdot 1y}\right) = \ln(0{,}92)$$

$$-4\lambda \cdot 1y = \ln(0{,}92) \rightarrow \lambda = \frac{-\ln(0{,}92)}{4y} = 2{,}08454 \cdot 10^{-2}/y$$

$$R'_S(t) = e^{-6\lambda\cdot 1y} = e^{-6\cdot 2{,}08454\cdot 10^{-2}/\cancel{y}\cdot 1\cancel{y}} = 88{,}243\%$$

⑤



a) WHICH COMPONENT(S) CAN FAIL WITHOUT MAKING THE WHOLE SYSTEM FAIL? b) $R_B(t=2\text{ YEARS})=83\%$ $MTTF_B$? c) $MTTF_A = MTTF_D = 1$ YEAR $MTTF_C = 14$ MONTHS $MTTR_{A,B,C,D} = 21$ DAYS $A_S(t)$?

a) WE HAVE TO LOOK AT THE COMPONENTS IN THE "PARALLEL PARTS": B_1, B_2, C, D_1, D_2. FOR EACH PART, AT LEAST ONE COMPONENT HAS TO NOT FAIL

$\Rightarrow$ ACCEPTABLE CONFIGURATIONS OF FAILURE:

$CD_1, CD_2, B_1D_1, B_2D_1, B_1D_2, B_2D_2, B_1B_2D_1, B_1B_2D_2$

b) $e^{-\lambda_B \cdot 2y} = 0{,}83 \rightarrow -\lambda_B \cdot 2y = \ln(0{,}83) \Rightarrow \lambda_B = 0{,}09316\,1/y$

$MTTF_B = \frac{1}{\lambda_B} = 10{,}73$ YEARS

c) $MTTF_A = MTTF_D = 1y$ $MTTF_B = 10{,}73y$ $MTTF_C = \frac{14}{12}y$

$MTTR_{A,B,C,D} = 0{,}057534y$

$$A_B(t) = \frac{MTTF_B}{MTTF_B + MTTR} = \frac{10{,}73}{10{,}73 + 0{,}057534} = 99{,}467\%$$

SIMILARLY, $A_A(t) = A_D(t) = 94{,}560\%$; $A_C(t) = 95{,}300\%$

$A_S(t) = A_A(t) \cdot A_{BBC}(t) \cdot A_{DD}(t)$

$A_{BBC}(t) = 1 - [(1 - A_{BB}(t))(1 - A_C(t))] =$

$1 - [(1 - A_B(t)^2)(1 - A_C(t))] = 99{,}950\%$

$$A_{DD}(t) = 1 - [(1 - A_D(t))(1 - A_D(t))] = 99{,}704\%$$

$$\Rightarrow A_S(t) = 94{,}233\%$$

6 $R = 12 \quad n = 9 \quad \lambda = 0{,}01/h \quad R_{sw}(t) = 1 \quad T = 10h \quad R_S(t)?$

EXACTLY i GENERATORS ARE WORKING

$$R(t = 10\,y) = e^{-\lambda \cdot 10h} = e^{-0{,}1} = 90{,}484\%$$

$$R_S(t) = R_{sw}(t) \cdot \sum_{i=n}^{R} \left[(R(t))^{i} (1 - R(t))^{n-i} \cdot \binom{R}{i} \right] =$$

$$= (90{,}484\%)^{9} \cdot (1 - 90{,}484\%)^{3} \cdot \frac{12!}{9!\,3!}$$

$$= (90{,}484\%)^{9} \cdot (1 - 90{,}484\%)^{3} \cdot \frac{\overset{2}{\cancel{12}} \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{9!}\,\cancel{6}} = 97{,}828\%$$