

46

525

15,8 RESOLUTION

17

+

16,90 RESOLUTION

THE SET HAS A CLOSED EXPANSION \Rightarrow THE STATEMENT IS CORRECT

(EXAM-TYPE EXERCISES)

PROVE THE FOLLOWING CLAIMS, BOTH SEMANTICALLY (YOU CAN USE

EITHER TRUTH TABLE OR SEMANTIC TREE, AS YOU PREFER) AND

APPLYING RESOLUTION CALCULUS

$$\text{DC)} \quad \overbrace{X \wedge Y \rightarrow Z, (X \rightarrow Z) \leftrightarrow (Y \rightarrow Z)}^{F_1} \vdash \overbrace{X \vee Y \rightarrow Z}^{F_2} \quad \overbrace{\varphi}^{\psi}$$

SEMANTICAL

X	Y	Z	$X \wedge Y$	F_1	$X \rightarrow Z$	$Y \rightarrow Z$	F_2	$X \vee Y$	φ
0	0	0	0	1	1	1	1	0	1
0	0	1	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0	1	0
0	1	1	0	1	1	1	1	1	1
1	0	0	0	1	0	1	0	1	0
1	0	1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1	1	0
1	1	1	1	1	1	1	1	1	1

$F \models \varphi \checkmark$

RESOLUTION

PROVE $F \cup \neg \varphi \models$ HAS A CLOSED EXPANSION

$$\{ x \wedge y \rightarrow z, (x \rightarrow z) \leftrightarrow (y \rightarrow z), \neg(x \vee y \rightarrow z) \}$$

STEP

FORMULA

RULE

1	$\{ x \wedge y \rightarrow z \}$	ASSUMPTION
2	$\{ (x \rightarrow z) \leftrightarrow (y \rightarrow z) \}$	ASSUMPTION
3	$\{ \neg(x \vee y \rightarrow z) \}$	ASSUMPTION
4	$\{ \neg(x \wedge y), z \}$	1, β -EXPANSION
5	$\{ \neg x, \neg y, z \}$	4, α -EXPANSION
6	$\{ (x \rightarrow z) \rightarrow (y \rightarrow z) \}$	2, BIIMPLICATION
7	$\{ (y \rightarrow z) \rightarrow (x \rightarrow z) \}$	2, BIIMPLICATION
8	$\{ (x \rightarrow z), \neg(y \rightarrow z) \}$	7, β -EXPANSION
9	$\{ \neg x, z, \neg(y \rightarrow z) \}$	8, β -EXPANSION
10	$\{ \neg x, y, z \}$	9, α -EXPANSION
11	$\{ \neg x, z, \neg z \}$	9, α -EXPANSION
12	$\{ \neg \neg(x \rightarrow z), (y \rightarrow z) \}$	6, β -EXPANSION
13	$\{ \neg \neg(x \rightarrow z), \neg y, z \}$	12, β -EXPANSION
14	$\{ x, \neg y, z \}$	13, α -EXPANSION
15	$\{ \neg y, z, \neg z \}$	13, α -EXPANSION