3	$\{\forall xyz(Rxy \wedge Ryz \rightarrow Rxz)\}$	ASSUMPTION
4	$\{\neg \forall x\, Rxx\}$	ASSUMPTION
5	$\{\neg Raa\}$	4, δ-EXPANSION
6	$\{\exists y.\, Ray\}$	1, δ-EXPANSION
7	$\{Rab\}$	6, δ-EXPANSION
8	$\{\forall y(Ray \rightarrow Rya)\}$	2, γ-EXPANSION
9	$\{\forall y(Rab \rightarrow Rba)\}$	8, γ-EXPANSION
10	$\{\neg Rab, Rba\}$	9, β-EXPANSION
11	$\{Rba\}$	7,10 RESOLUTION
12	$\{\forall yz(Ray \wedge Ryz \rightarrow Raz)\}$	3, γ-EXPANSION
13	$\{\forall z(Rab \wedge Rbz \rightarrow Raz)\}$	12, γ-EXPANSION
14	$\{Rab \wedge Rba \rightarrow Raa\}$	13, γ-EXPANSION
15	$\{\neg(Rab \wedge Rba), Raa\}$	14, β-EXPANSION
16	$\{\neg Rab, \neg Rba, Raa\}$	15, β-EXPANSION
17	$\{\neg Rba, Raa\}$	16,7 RESOLUTION
18	$\{Raa\}$	17,11 RESOLUTION
19	$\emptyset$	18,5 RESOLUTION

UNIFICATION

$F = \{\forall x \exists y. Rxy, \forall xy(Rxy \to Ryx), \forall xyz(Rxy \wedge Ryz \to Rxz), \neg \forall x Rxx\}$

- $\forall x \exists y. Rxy \mapsto \forall x Rx f(x) \mapsto \{\{R(x, f(x))\}\}$
- $\forall xy(Rxy \to Ryx) \mapsto \forall xy(\neg Rxy \vee Ryx) \mapsto \{\{\neg Rxy. Ryx\}\}$
- $\forall xyz(Rxy \wedge Ryz \to Rxz) \mapsto \forall xyz(\neg(Rxy \wedge Ryz) \vee Rxz)$

  $\mapsto \forall xyz(\neg Rxy \vee \neg Ryz \vee Rxz) \mapsto \{\{\neg Rxy, \neg Ryz, Rxz\}\}$
- $\neg \forall x Rxx \mapsto \exists x. \neg Rxx \mapsto \{\{\neg Raa\}\}$

$C(F) = \{\{R(x, f(x))\}, \{\neg Rxy. Ryx\}, \{\neg Rxy. \neg Ryz, Rxz\}, \{\neg Raa\}\}$

! WE CAN USE THE SAME FORMULA TWICE

① $\{R(x, fx)\}$ $\{\neg Rxy, Ryx\}$

② $\{R(fx, x)\}$ $\{\neg Rxy, \neg Ryz, Rxz\}$

③ $\{\neg R(w, fx), Rwx\}$ $\{R(x, fx)\}$

④ $\{Ryy\}$ $\{\neg Raa\}$

$\emptyset$

① $C_1 = \{R(x, fx)\}$ $E_1 = \{R(x, fx)\}$

$C_2 = \{\neg Rwy, Ryw\}$ $E_2 = \{\neg Rwy\}$

$F_1 = E_2 \cup \bar{E}_2 = \{R(x, fx), Rwy\}$ $S_1 = [x/w]$

$F_2 = \{R(x, fx), Rxy\}$ $S_2 = [fx/y]$

$F_3 = \{R(x, fx)\}$ $\quad S = S_1 S_2$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = \{R_{yw}\}[{}^{x}/_{w}, {}^{fx}/_{y}] = \{R(fx, x)\}$

② $C_1 = \{R(fx, x)\} = E_1$

$C_2 = \{\neg Rwy, \neg Ryz, Rwz\}$ $\quad E_2 = \{\neg Ryz\}$

$F_1 = E_1 \cup \bar{E}_2 = \{R(fx, x), Ryz\}$ $\quad S_1 = [{}^{fx}/_{y}]$

$F_2 = \{R(fx, x), R(fx, z)\}$ $\quad S_2 = [{}^{x}/_{z}]$

$F_3 = \{R(fx, x)\}$ $\quad S = S_1 S_2$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = \{\neg R_{wy}, R_{wz}\}[{}^{fx}/_{y}, {}^{x}/_{z}]$

$= \{\neg R(w, fx), Rwx\}$

③ $C_1 = \{\neg R(w, fx), Rwx\}$ $\quad E_1 = \{\neg R(w, fx)\}$

$C_2 = \{R(y, fy)\} = E_2$

$F_1 = \bar{E}_1 \cup E_2 = \{R(w, fx), R(y, fy)\}$ $\quad S_1 = [{}^{y}/_{w}]$

$F_2 = \{R(y, fx), R(y, fy)\}$ $\quad S_2 = [{}^{x}/_{x}]$

$F_3 = \{R(y, fy)\}$ $\quad S = S_1 S_2$

$R(C_1, C_2) = \{Rwx\}[{}^{y}/_{w}, {}^{y}/_{x}] = \{Ryy\}$

④ $C_1 = \{Ryy\} = E_1$ $\quad C_2 = \{\neg Raa\} = E_2$

$F_1 = \{Ryy, Raa\}$ $\quad S = [{}^{a}/_{y}]$ $\quad F_2 = \{Raa\}$

$R(C_1, C_2) = \emptyset$