

RESOLUTION

$$\{\neg x \Rightarrow y \vee z, \neg y \Rightarrow \neg z, \neg x \Rightarrow \neg y, \neg x\}$$

STEP	FORMULA	RULE
1	$\{\neg x \Rightarrow y \vee z\}$	ASSUMPTION
2	$\{\neg y \Rightarrow \neg z\}$	ASSUMPTION
3	$\{\neg x \Rightarrow \neg y\}$	ASSUMPTION
4	$\{\neg x\}$	ASSUMPTION
5	$\{x, y \vee z\}$	\perp, β -EXPANSION
6	$\{x, y, z\}$	\perp, β -EXPANSION
7	$\{y, \neg z\}$	$2, \beta$ -EXPANSION
8	$\{x, \neg y\}$	$3, \beta$ -EXPANSION
9	$\{y, z\}$	4, 6 RESOLUTION
10	$\{\neg y\}$	4, 8 RESOLUTION
11	$\{y\}$	7, 9 RESOLUTION
12	\perp	10, 11 RESOLUTION

$F \vee \{\neg \varphi\}$ HAS A CLOSED EXPANSION $\Rightarrow F \vdash \varphi$

EXERCISE: TRANSLATE THE FOLLOWING ASSERTION IN PROPOSITIONAL LOGIC. THEN.

VERIFY IT USING BOTH SEMANTIC AND RESOLUTION CALCULUS

1) LA MACCHINA DI GIOVANNI È ROSSA O DECAPOTABILE

2) SE LA MACCHINA DI GIOVANNI È ROSSA, È UN'ALFA ROMEO

3) LA MACCHINA DI GIOVANNI NON È UN'ALFA ROMEO

4) LA MACCHINA DI GIOVANNI È DECAPOTABILE

$1, 2, 3 \models 4?$

$X = \text{LA MACCHINA DI GIOVANNI È ROSSA}$ $Y = \text{MACCHINA DECAPOTABILE}$

$Z = \text{LA MACCHINA DI GIOVANNI È UN'ALFA ROMEO}$

$X \vee Y, X \Rightarrow Z, \neg Z \models Y$

SEMANTIC

X	Y	Z	F_1	F_2	F_3	φ
0	0	0	0	1	1	0
0	0	1	0	1	0	0
0	1	0	1	1	1	1
0	1	1	1	1	0	1
1	0	0	1	0	1	0
1	0	1	1	1	0	0
1	1	0	1	0	1	1
1	1	1	1	1	0	1

F_3 SATISFIES ALL PREMISES AND CONCLUSION $\Rightarrow F \models \varphi$

RESOLUTION

$\{\neg x \vee y, x \Rightarrow z, \neg z, \neg y\}$

STEP	FORMULA	RULE
1	$\{\neg x \vee y\}$	ASSUMPTION
2	$\{x \Rightarrow z\}$	ASSUMPTION
3	$\{\neg z\}$	ASSUMPTION
4	$\{\neg y\}$	ASSUMPTION
5	$\{x, y\}$	1, P-EXPANSION
6	$\{\neg x, z\}$	2, P-EXPANSION
7	$\{x\}$	4,5 RESOLUTION
8	$\{\neg x\}$	3,6 RESOLUTION
9	\perp	7,8 RESOLUTION

$F \cup \{\neg \psi\}$ HAS A CLOSED EXPANSION \Rightarrow THE STATEMENT IS

CORRECT $\Rightarrow F \vdash \psi$