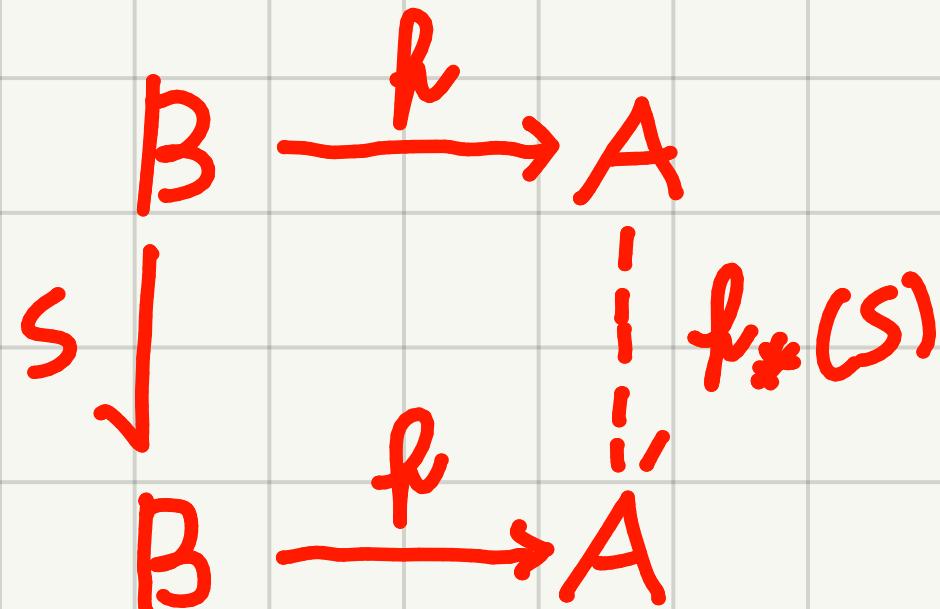
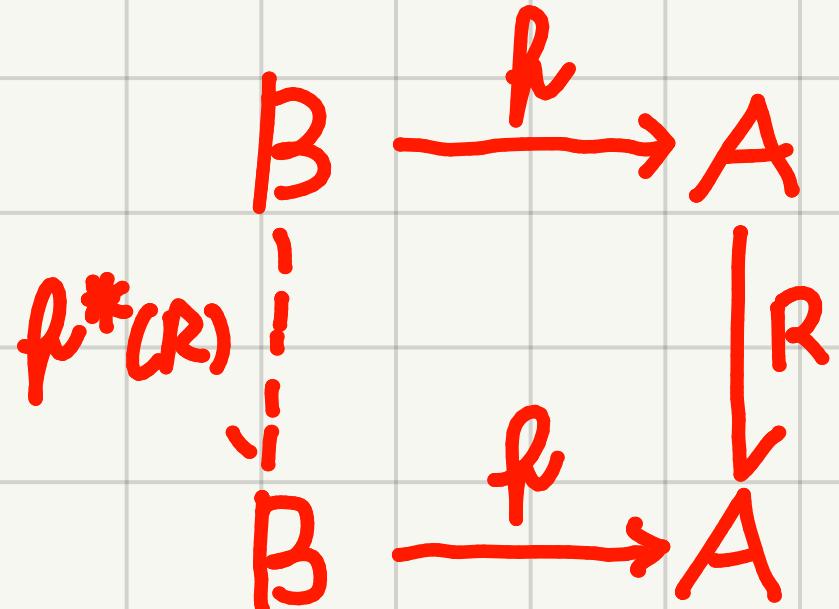


IN THE FIRST DIAGRAM BELOW, ASSUME f IS A FUNCTION AND R IS AN EQUIVALENCE RELATION ON A . PROVE THAT THE RELATION $f^*(R) := fRf^{-1}$

IS AN EQUIVALENCE RELATION ON B . CONVERSELY, ASSUME, AS IN THE



SECOND DIAGRAM, THAT WE ARE GIVEN AN EQUIVALENCE RELATION S ON B

CONTAINING THE KERNEL PAIR OF f . PROVE THAT THE RELATION $f_*(S) = f^{-1}Sf$

IS AN EQUIVALENCE RELATION ON A IF AND ONLY IF f IS SURJECTIVE

A RELATION IS EQUIVALENT IF IT IS:

- REFLEXIVE
- SYMMETRIC
- TRANSITIVE

$$aRf^{-1}b \Leftrightarrow \exists c,d (aRc \wedge cRd \wedge dR^{-1}b) \Leftrightarrow \exists c,d (aRc \wedge cRd \wedge bRd)$$

$$\Leftrightarrow f(a) = c, f(b) = d \Leftrightarrow f(a)Rf(b)$$

R) $f(a)Rf(a) \checkmark$ (R IS REFLEXIVE) AND $f(a) = c = d$

$$\Rightarrow \exists c,d (aRc \wedge cRc \wedge cR^{-1}a) \Leftrightarrow \exists c,d (aRc \wedge cRc \wedge cR^{-1}a)$$

$$\Rightarrow aRf^{-1}f(a) \checkmark$$

S) $f(a)Rf(b)$ R IS SYMMETRIC $\Rightarrow f(b)Rf(a)$. HENCE,

$$af^{\text{OP}}fb \Rightarrow bf^{\text{OP}}fa \checkmark$$

T) $f(a)Rf(b) \wedge f(b)Rf(c)$ R TRANSITIVE $\Rightarrow f(a)Rf(c)$.

HENCE, $af^{\text{OP}}fb \wedge bf^{\text{OP}}fc \Rightarrow af^{\text{OP}}fc \checkmark$

$$af^{\text{OP}}Sfb \Leftrightarrow \exists c, d (af^{\text{OP}}c \wedge cSd \wedge dfb) \Leftrightarrow \exists c, d (cf \alpha \wedge cSd \wedge dfb)$$

$$\Leftrightarrow \exists c, d (f(cc)=a \wedge cSd \wedge f(d)=b).$$

S IS AN EQUIVALENCE RELATION. f IS SURJECTIVE

R) S IS REFLEXIVE $\Rightarrow cSc \Rightarrow \exists c (f(cc)=a \wedge cSd \wedge f(d)=a)$

$$f \text{ IS SURJECTIVE} \Rightarrow |f^*(a)| > 1 \Rightarrow \exists c (f(cc)=a \wedge cSd \wedge f(d)=a)$$

$$\Leftrightarrow \exists c (cf \alpha \wedge cSd \wedge fa) \Leftrightarrow \exists c (af^{\text{OP}}c \wedge cSd \wedge fa)$$

$$\Leftrightarrow af^{\text{OP}}Sfa \checkmark$$

S) $af^{\text{OP}}Sfb \wedge bf^{\text{OP}}Sfa \Rightarrow a=b?$

$$c=h \quad d=k$$

$$\exists c, d, h, k (f(cc)=a \wedge cSd \wedge f(d)=b \wedge f(h)=a \wedge hSk \wedge$$

$$\wedge f(k)=b) \Leftrightarrow \exists c, d (f(cc)=a \wedge cSd \wedge f(d)=b \wedge dSd)$$

$$\Leftrightarrow (f(c)=a \wedge cSc \wedge f(c)=b) \Rightarrow a=b \checkmark$$

T) $af^{\text{OP}}Sfb \wedge bf^{\text{OP}}Sfc \Rightarrow af^{\text{OP}}Sfc?$

$$n=h$$

$$\exists m, n, h, k (f(cm)=a \wedge mSn \wedge f(h)=b \wedge f(h)=b \wedge hSk \wedge f(k)=c) \Rightarrow af^{\text{OP}}Sfc \checkmark$$

$$\Leftrightarrow \exists m, h, k (f(m)=a \wedge mSk \wedge hSk \wedge f(k)=c) \Leftrightarrow (af^{\text{OP}}m \wedge mSk \wedge kf(c))$$

GIVEN $A = \{1, 2, 3\}$ AND $B = \{1, 2, 3, 4, 5\}$, CONSIDER THE RELATION

$E: A \rightarrow A$ AND THE FUNCTION $P: B \rightarrow A$ DEFINED BY THE MATRICES

$$M = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \quad N = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}. \text{ PROVE THAT } E \text{ IS AN}$$

EQUIVALENCE RELATION ON A AND $E^* = PEP^{-1}$ IS AN EQUIVALENCE RELATION

ON B . MORE GENERALLY, PROVE THAT FOR ANY A AND B , IF E IS AN

EQUIVALENCE RELATION AND P A SURJECTIVE FUNCTION AS IN THE DIAGRAM

BELLOW, THEN $E^* = PEP^{-1}$ IS ALSO AN EQUIVALENCE RELATION. PROVE THAT.

IF P_E AND P_{E^*} ARE THE PROJECTION ON THE QUOTIENT SETS OF E AND E^*

RESPECTIVELY, THEN P INDUCES A BISECTION q BETWEEN THE QUOTIENTS, SO

BISECTIVE
THAT THERE EXIST A UNIQUE FUNCTION q SUCH THAT $PP_E = P_{E^*}q$ AND q IS

