c) $\forall x((Rx \to Sx) \to Tx) \models \forall x(Rx \to (Sx \to Tx))$

$F \cup \{\neg\varphi\} = \{\forall x((Rx \to Sx) \to Tx), \neg(\forall x(Rx \to (Sx \to Tx)))\}$

$\forall x((Rx \to Sx) \to Tx), \neg(\forall x(Rx \to (Sx \to Tx)))$

| α-EXPANSION

$\forall x((Rx \to Sx) \to Tx), \forall x Rx, \neg(\forall x(Sx \to Tx))$

| α-EXPANSION

$\forall x((Rx \to Sx) \to Tx), \forall x Rx, \forall x Sx, \neg\forall x Tx$

/ \ β-EXPANSION

$\neg(\forall x(Rx \to Sx)), \forall x Rx, \forall x Sx, \neg\forall x Tx$

|

$\forall x Rx, \neg\forall x Sx, \forall x Rx, \forall x Sx, \neg\forall x Tx$

|

$Ra, \neg Sa, Ra, Sa, \neg Ta$

$\forall x Tx, \forall x Rx, \forall x Sx, \exists x \neg Tx$

|

$Ta, Ra, Sa, \neg Ta$

BOTH UNSATISFIABLE LEAFS $\Rightarrow F \cup \{\neg\varphi\}$ UNSATISFIABLE

$\Rightarrow F \models \varphi$

USE RESOLUTION TO PROVE THAT THE FOLLOWING SET OF FORMULAS ARE UNSATISFIABLE

a) $\{\exists x \forall y (Ryx \leftrightarrow \neg Ryy)\}$

HERBRAND MODEL

| STEP | FORMULA | RULE |
|---|---|---|
| 1 | $\{\exists x \forall y (Ryx \leftrightarrow \neg Ryy)\}$ | ASSUMPTION |
| 2 | $\{\forall y (Rya \leftrightarrow \neg Ryy)\}$ | 1, δ-EXPANSION |
| 3 | $\{Raa \leftrightarrow \neg Raa\}$ | 2, γ-EXPANSION |
| 4 | $\{Raa \rightarrow \neg Raa\}$ | 3, α- EXPANSION |
| 5 | $\{\neg Raa \rightarrow Raa\}$ | 3, α- EXPANSION |
| 6 | $\{\neg Raa\}$ | 4, β- EXPANSION |
| 7 | $\{\neg\neg Raa\}$ | 4, β- EXPANSION |
| 8 | $\{Raa\}$ | 7, DOUBLE NEGATION |
| 9 | $\{\neg\neg Raa\}$ | 5, β- EXPANSION |
| 10 | $\{Raa\}$ | 9, DOUBLE NEGATION |
| 11 | $\{\neg Raa\}$ | 5, β- EXPANSION |
| 12 | $\emptyset$ | 6, 10 RESOLUTION |

# UNIFICATION

$\exists x \forall y (Ryx \leftrightarrow \neg Ryy)$ ① WE SKOLEMIZE THE FORMULA

$\mapsto \forall y(Rya \leftrightarrow \neg Ryy)$

$\mapsto \forall y((Rya \to \neg Ryy) \wedge (\neg Ryy \to Rya))$

$\mapsto \forall y((\neg Rya \vee \neg Ryy) \wedge (\neg\neg Ryy \vee Rya))$

$\mapsto \forall y((\neg Rya \vee \neg Ryy) \wedge (Ryy \vee Rya))$

$\mapsto \{\{\neg Rya, \neg Ryy\}, \{Ryy, Rya\}\}$

② WE CAN DERIVE THE EMPTY CLAUSE USING

$C = \{\{\neg Rya, \neg Ryy\}, \{Ryy, Rya\}\}$

$C_1 = \{\neg Rya, \neg Ryy\}$

$C_2 = \{Ryy, Rya\} \mapsto C_2\underline{[x/y]} = \{Rxx, Rxa\}$

ROBINSON ALGORITHM CANNOT UNIFY TWO FORMULAS IF THEY HAVE CLAUSES WITH THE SAME VARIABLE

$F_1 = \{\neg Rya, \neg Ryy\}$ $E_1 = \{\neg Rya, \neg Ryy\}$

$F_2 = \{Rxx, Rxa\}$ $E_2 = \{Rxx, Rxa\}$ $S_4 = [a/y]$

$F_3 = \overline{E}_1 \cup E_2 = \{Ry\underline{a}, R\underline{y}y, Rxx, Rxa\}$ $y \to$ VARIABLE, $a$ FIRST SYMBOL OF $A$ NOT CONTAINING $y$

$F_4 = E_3 S_4 = \{R\underline{a}a, R\underline{x}x, Rxa\}$ $S_5 = [a/x]$

$F_5 = \{Raa\}$ $S = S_4 S_5$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = (\emptyset \cup \emptyset)S = \emptyset S = \emptyset$