A binary relation R has the extension property if, whenever R_{xy} and R_{xz}, then R_{yz}. Prove, using resolution, that if relation is symmetric and has the extension property, then seriality of the relation implies reflexivity. Prove, by semantical means, that the assumption on symmetry is needed

$$\forall xy(R_{xy} \to R_{yx}), \forall xyz(R_{xy} \wedge R_{xz} \to R_{yz}) \vdash \forall x \exists y R_{xy} \to \forall x R_{xx}$$

$$\Gamma = \{\forall xy(R_{xy} \to R_{yx}), \forall xyz(R_{xy} \wedge R_{xz} \to R_{yz}), \neg(\forall x \exists y R_{xy} \to \forall x R_{xx})$$

HERBRAND MODEL

STEP	FORMULA	REASON
1	$\{\forall xy(R_{xy} \to R_{yx})\}$	ASSUMPTION
2	$\{\forall xyz(R_{xy} \wedge R_{xz} \to R_{yz})\}$	ASSUMPTION
3	$\{\neg(\forall x \exists y R_{xy} \to \forall x R_{xx})\}$	ASSUMPTION
4	$\{\forall y(R_{ay} \to R_{ya})\}$	1, γ-EXPANSION
5	$\{R_{ab} \to R_{ba}\}$	4, γ-EXPANSION
6	$\{\neg R_{ab}, R_{ba}\}$	5, β-EXPANSION
7	$\{\forall yz(R_{by} \wedge R_{bz} \to R_{yz})\}$	2, γ-EXPANSION
8	$\{\forall z(R_{ba} \wedge R_{az} \to R_{az})\}$	7, γ-EXPANSION
9	$\{R_{ba} \wedge R_{ba} \to R_{aa}\}$	8, γ-EXPANSION
10	$\{\neg(R_{ba} \wedge R_{ba}), R_{aa}\}$	9, β-EXPANSION

11	$\{\neg Rba, Raa\}$	10, β-EXPANSION
12	$\{\neg(\forall x \exists y Rxy \to Raa\}$	3, γ-EXPANSION
13	$\{\neg(\exists y Ray \to Raa)\}$	12, γ-EXPANSION
14	$\{\neg(Rab \to Raa)\}$	13, δ-EXPANSION
15	$\{Rab\}$	14, α-EXPANSION
16	$\{\neg Raa\}$	14, α-EXPANSION
17	$\{Rba\}$	6,15 RESOLUTION
18	$\{Raa\}$	17,11 RESOLUTION
19	$\emptyset$	18,16 RESOLUTION

UNIFICATION

- $\{\forall xy(Rxy \to Ryx)\} \mapsto \{\neg Rxy, Ryx\}$
- $\{\forall xyz(Rxy \wedge Rxz \to Ryz)\} \mapsto \{\neg Rxy, \neg Ryz, Rxz\}$
- $\{\neg(\forall x \exists y Rxy \to \forall x Rxx)\} \mapsto \{\forall x \exists y Rxy\}, \{\neg \forall x Rxx\} \mapsto$
  $\{Rxfx\}, \{\neg Raa\}$

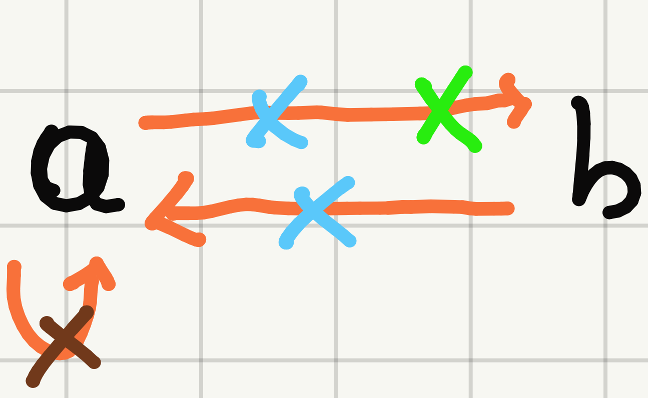
$C(F) = \{\{\neg Rxy, Ryx\}, \{\neg Rxy, \neg Ryz, Rxz\}, \{Rxfx\}, \{\neg Raa\}\}$

$$\frac{\{\neg Rxy, Ryx\} \quad \{Rxfx\}}{\{Rfx,x\} \quad \{\neg Rxy, \neg Ryz, Rxz\}}$$

$$\frac{\{Rfx,x\} \quad \{\neg Rxy, \neg Ryz, Rxz\}}{\{\neg Rxfx, Rxz\} \quad \{Rxfx\}}$$

$$\frac{\{\neg Rxfx, Rxz\} \quad \{Rxfx\}}{\{Rxz\} \quad \{\neg Raa\}}$$

$$\frac{\{Rxz\} \quad \{\neg Raa\}}{\emptyset}$$



EXTENSION
SERIALITY
REFLEXIVITY

NEED FOR SYMMETRY

3.46

Let R be a reflexive binary relation satisfying the formula $\forall uvxy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx)$. Use resolution (both Herbrand Model and Unification) to prove that R is transitive

$\forall x Rx. \forall uvxy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx) \vdash$

$\vdash \forall efg(eRf \wedge fRg \to eRg)$

$\Gamma = \{ \forall x Rx. \forall uvxy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx),$

$\neg\forall efg(eRf \wedge fRg \to eRg)\} \equiv$

$\equiv \{ \forall x Rx. \forall uvxy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx),$

$\exists efg. \neg(eRf \wedge fRg \to eRg)\}$

HERBRAND MODEL

STEP	FORMULA	REASON
1	$\{\forall x Rx\}$	Assumption
2	$\{\forall uvxy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx)\}$	Assumption
3	$\{\exists efg. \neg(eRf \wedge fRg \to eRg)\}$	Assumption
4	$\{aRa\}$	1, γ expansion
5	$\{\forall vxy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx)\}$	2, γ-expansion
6	$\{\forall xy(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx)\}$	5, γ expansion
7	$\{\forall y(xRu \wedge xRv \wedge yRu \wedge yRv \to yRx)\}$	6, γ-expansion