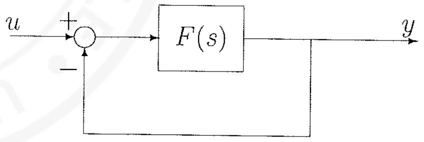
Domanda Scritta di Controlli Automatici (9CFU) - 11/11/2013

Esercizio 1

È dato il sistema in controreazione:

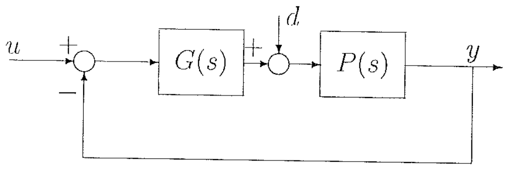


in cui $F(s) = \dfrac{K(s+1)(s^2+8s+20)}{(s-2)(s+3)(s+5)(s+10)}$, $K \in \mathbb{R}$.

- Tracciare il luogo positivo delle radici;
- tracciare il luogo negativo delle radici;
- determinare per quali valori di K il sistema a ciclo chiuso è asintoticamente stabile;
- per $K = 6$, esiste la risposta a regime permanente ad un ingresso sinusoidale?

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{72(s+1)}{s(s+6)^2}; \quad d(t) = \delta_{-1}(t).$$

Progettare $G(s)$ con la sintesi per tentativi in ω in modo che:

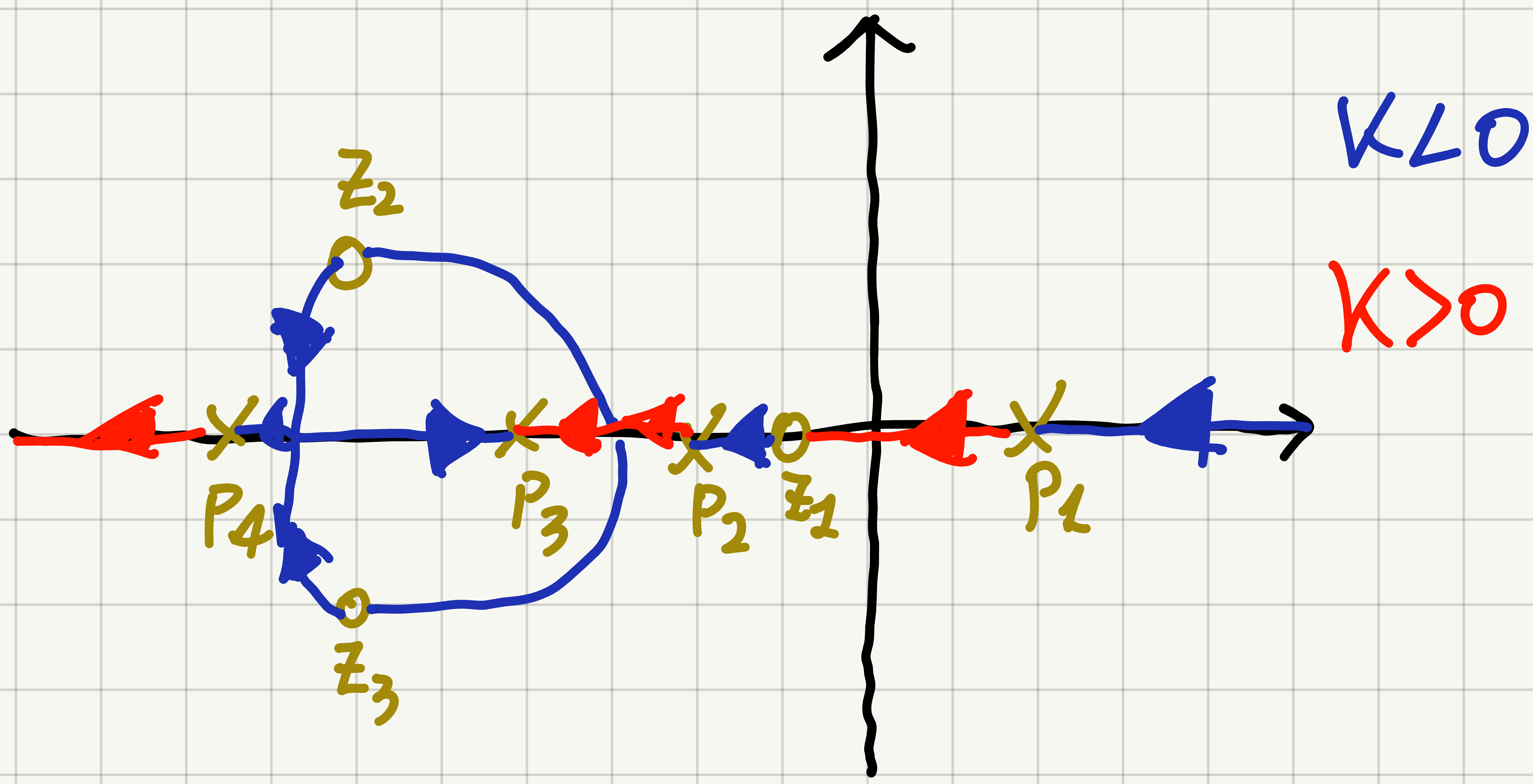
- $|\tilde{y}_d(t)| \leq 0.05$, essendo $\tilde{y}_d(t)$ la risposta a regime permanente al disturbo $d(t)$;
- $M_r \leq 2\ dB$;
- $B_3 \simeq 2.5\ Hz$.

Discretizzare infine il controllore ottenuto, scegliendo opportunamente il periodo di campionamento.

1

$$F(s) = \frac{K \cdot (s+1)(s^2+8s+20)}{(s-2)(s+3)(s+5)(s+10)} \qquad n=4, m=3 \Rightarrow n-m=1$$

$P_1=2,\ P_2=-3,\ P_3=-5,\ P_4=-10;\ Z_1=-1,\ Z_2=-8+2i,\ Z_3=-8-2i$



$$f(s,K) = (s-2)(s+3)(s+5)(s+10) + K(s+1)(s^2+8s+20)$$

$$f(s,K)\Big|_{s=0} = 0$$

$$-300 + 20K = 0 \qquad K = 15$$

$\Rightarrow$ SISTEMA STABILE $\forall\ K>15$

$\Rightarrow \nexists$ RISPOSTA A REGIME PERMANENTE

PER $K=6$

2

$$|\tilde{Y}_{d_1}(t)| = \left|\frac{K_d}{K_G}\right| \le 0{,}05 \Rightarrow K_G \ge 20$$

$F(s)$ ASTATICO RISPETTO DISTURBO $\delta_{-1}(t)$

$$\Longrightarrow F(s) = G(s)\cdot P(s) = \frac{\hat{F}(s)}{s} \quad . \text{ POLO IN } P(s) \Rightarrow G(s) = K$$

$$F(s) = G(s)\cdot P(s) = 40\cdot\frac{(1+s)}{s\left(1+\frac{s}{6}\right)^2}$$

$$M_r \le 2dB \Rightarrow M_\varphi \ge 47^\circ$$

$$\omega_t = 3\div 5\, B_3 = 4 B_3 = 10 \frac{rad}{s}$$

$$F(i\omega) = 40\frac{(1+i\omega)}{(i\omega)\left(1+\frac{i\omega}{6}\right)^2} \qquad 40 \to 20\log_{10}(40) = 32dB$$

PUNTI DI ROTTURA:

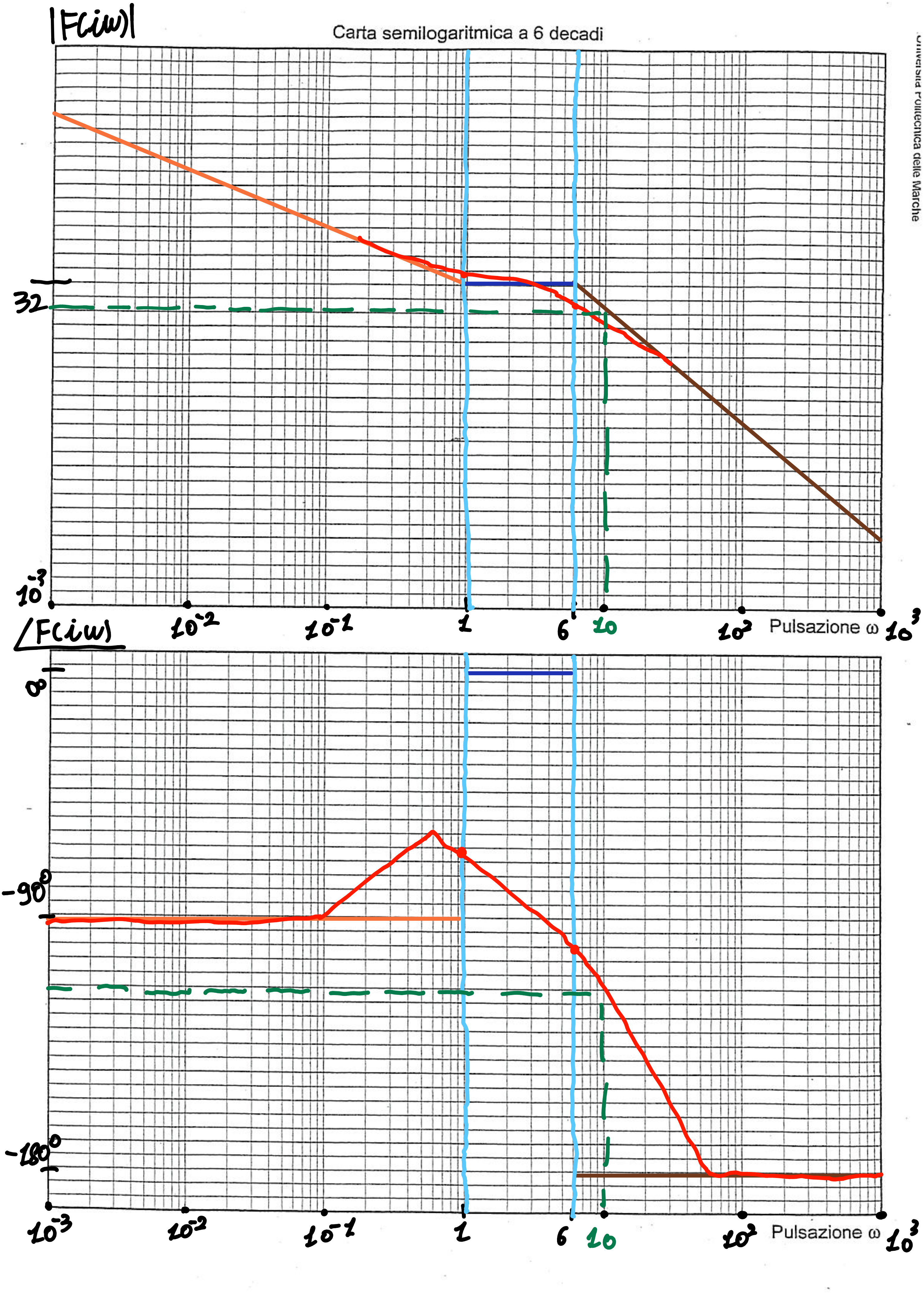
• $\omega=0$	$-20dB$	-90°	$-20dB$	-90°
• $\omega=1$	$+20dB$	$+90^\circ$	0	0
• $\omega=6$	$-40dB$	-180°	$-40dB$	-180°

CORREZIONE MODULO

ω	$1+i\omega$	$1+\frac{i\omega}{6}$	TOT
1	$+3dB$	0	$+3dB$
6	0	$-6dB$	$-6dB$

CORREZIONE FASE

ω	$i\omega$	$1+i\omega$	$1+\frac{i\omega}{6}$	TOT
1	-90°	$+45^\circ$	-20°	-65°
6	-90°	$+80^\circ$	-90°	-100°



|F(iω)|
Carta semilogaritmica a 6 decadi
Università Politecnica delle Marche
32
10^-3
10^-2
10^-1
1
6
10
10^2
Pulsazione ω
10^3
∠F(iω)
0°
-90°
-180°
10^-3
10^-2
10^-1
1
6
10
10^2
Pulsazione ω
10^3

$|F(i\omega_t)| = 23\,dB$

$\angle F(i\omega_t) = -115^\circ \Rightarrow M_\varphi = 65^\circ$

OBIETTIVO

- $|F(i\omega_t)| = 0$ ✗
- $M_\varphi \geq 47^\circ$ ✓

$\Rightarrow$ DIMINUIRE MODULO

$\Rightarrow$ FUNZIONE ATTENUATRICE $R_i(s) = \dfrac{1 + \frac{s}{m_i \omega_i}}{1 + \frac{s}{\omega_i}}$

$\omega_t \tau_i = 60 \Rightarrow \omega_i = 0{,}2 \quad m_i = 14$

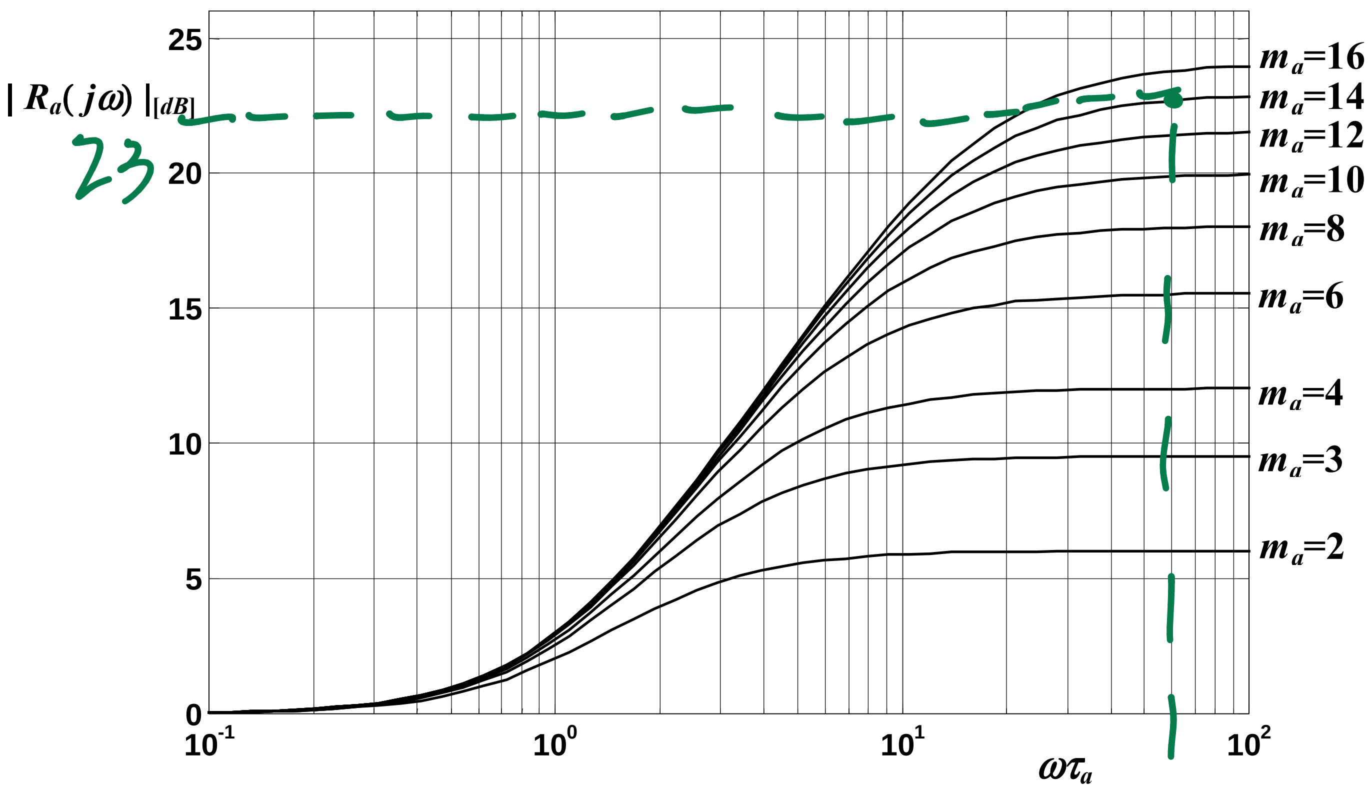
$$\Rightarrow R_i(s) = \frac{1+\frac{s}{2{,}8}}{1+\frac{s}{0{,}2}} \Rightarrow G(s) = 20 \cdot \frac{1+\frac{s}{2{,}8}}{1+\frac{s}{0{,}2}}$$

$$F(s) = 40 \cdot \frac{(1+s)}{s\left(1+\frac{s}{6}\right)^2} \cdot \frac{\left(1+\frac{s}{2{,}8}\right)}{\left(1+\frac{s}{0{,}2}\right)}$$

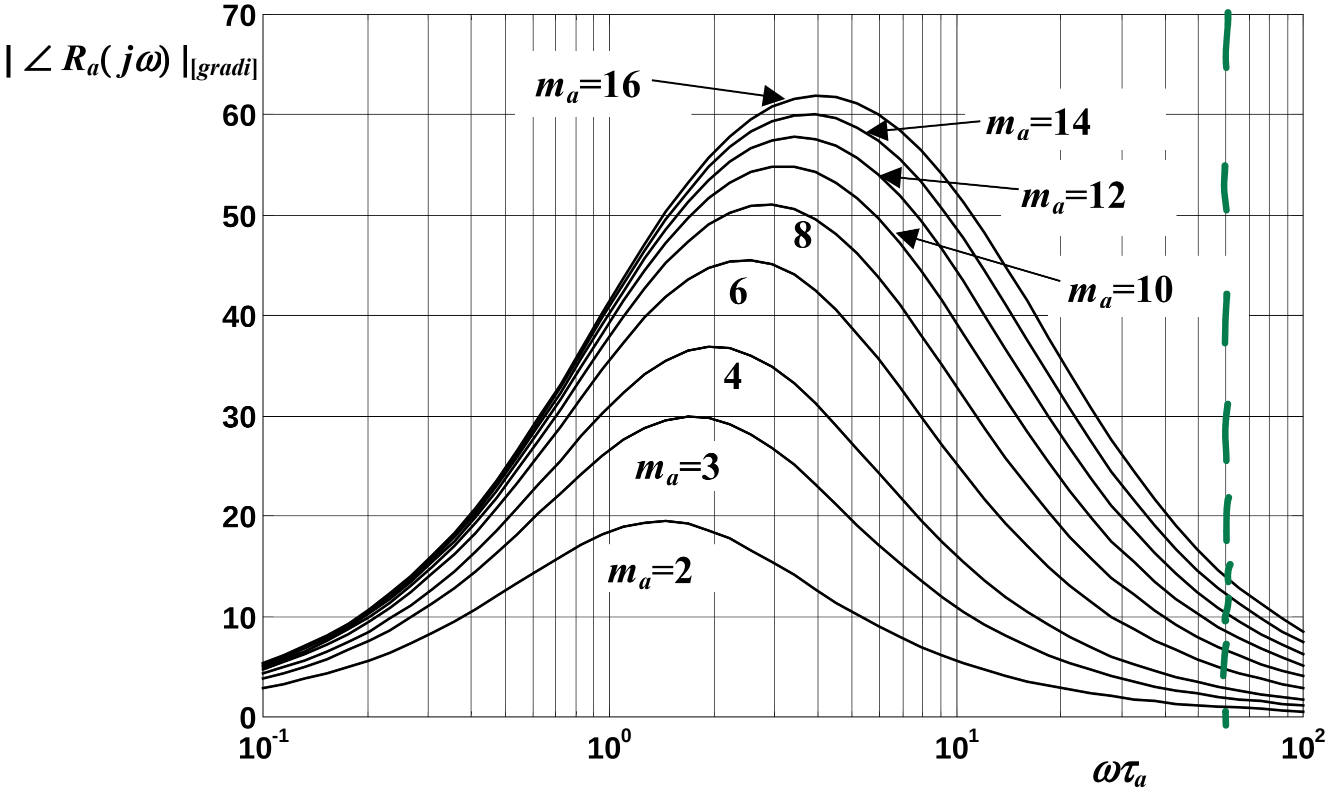
$$F(i\omega) = 40 \cdot \frac{(1+i\omega)}{i\omega\left(1+\frac{i\omega}{6}\right)^2} \cdot \frac{\left(1+\frac{i\omega}{2{,}8}\right)}{\left(1+\frac{i\omega}{0{,}2}\right)}$$

PUNTI DI ROTTURA

$\omega = 0$	$-20\,dB$	-90°	$-20\,dB$	-90°
$\omega = 0{,}2$	$-20\,dB$	-90°	$-40\,dB$	-180°
$\omega = 1$	$+20\,dB$	$+90^\circ$	$-20\,dB$	-90°
$\omega = 2{,}8$	$+20\,dB$	$+90^\circ$	0	0
$\omega = 6$	$-40\,dB$	-180°	$-40\,dB$	-180°



$|R_a(j\omega)|_{[dB]}$
23
25
20
15
10
5
0
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
$m_a=16$
$m_a=14$
$m_a=12$
$m_a=10$
$m_a=8$
$m_a=6$
$m_a=4$
$m_a=3$
$m_a=2$



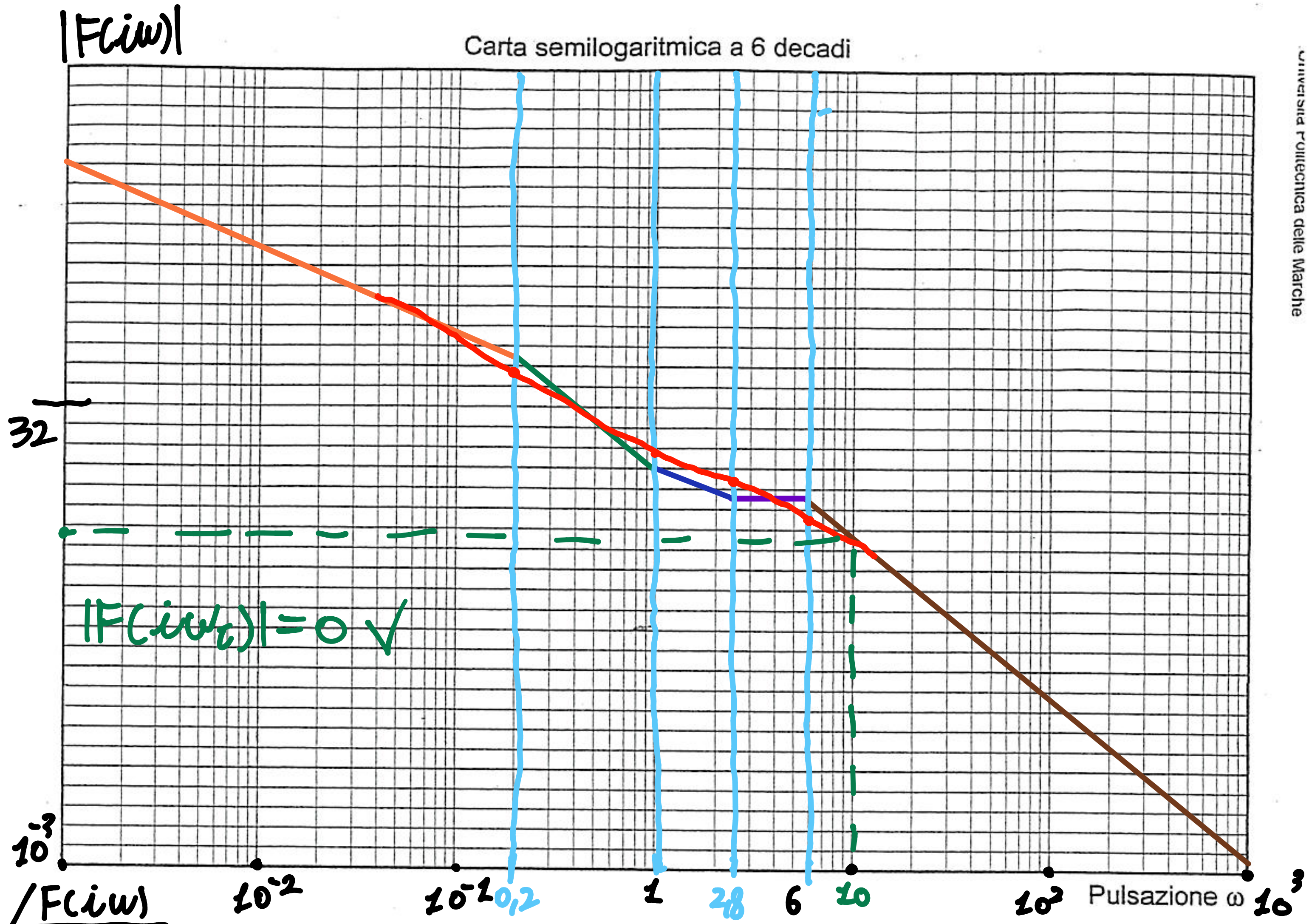
$|\angle R_a(j\omega)|_{[gradi]}$
70
60
50
40
30
20
10
0
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
$m_a=16$
$m_a=14$
$m_a=12$
$m_a=10$
8
6
4
$m_a=3$
$m_a=2$

CORREZIONE MODULO

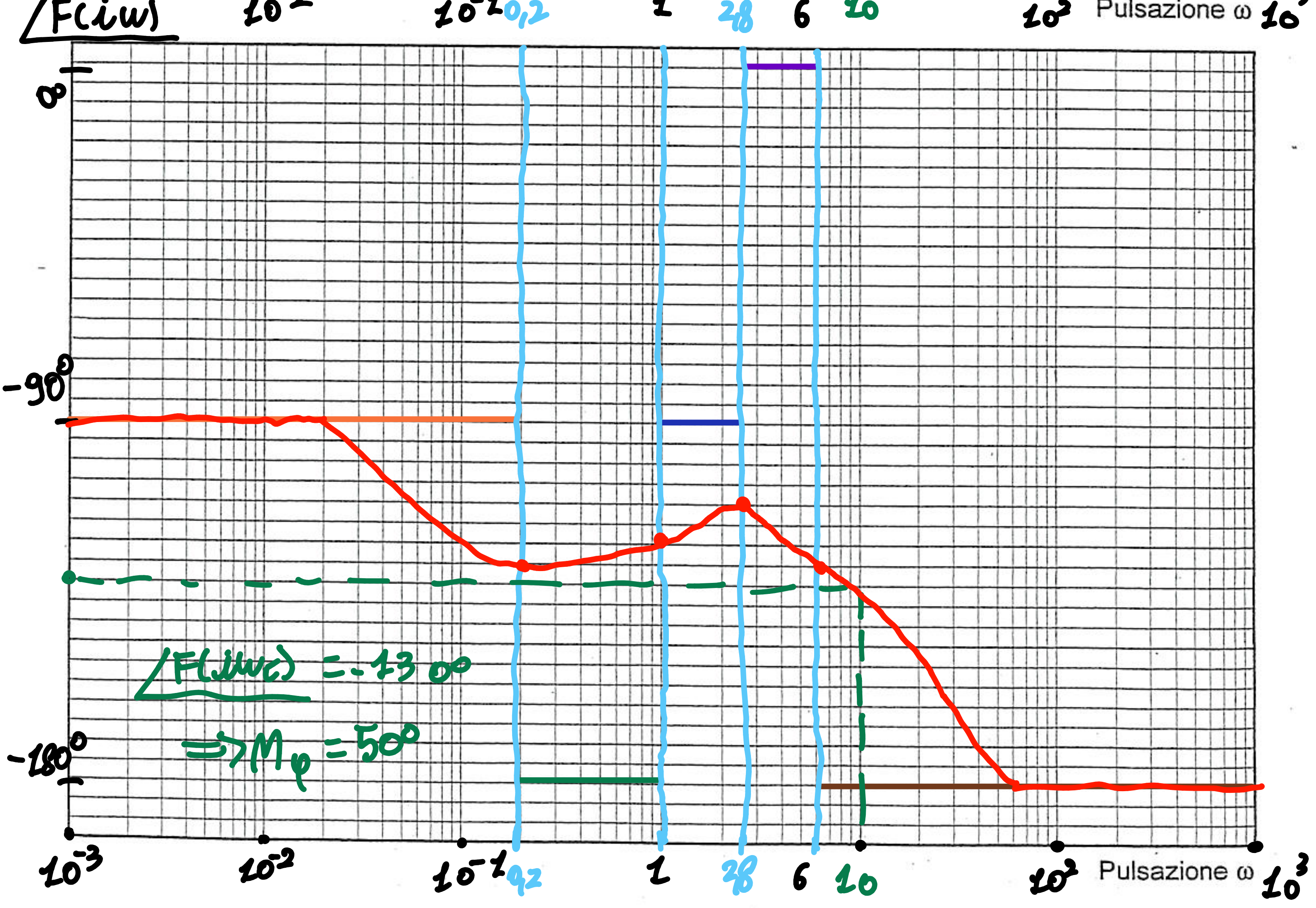
ω	$1+\frac{i\omega}{0,2}$	$1+\frac{i\omega}{1}$	$1+\frac{i\omega}{2,8}$	$1+\frac{i\omega}{6}$	TOT
0,2	−3dB	0	0	0	−3dB
1	0	+3dB	+1dB	0	+4dB
2,8	0	+1dB	+3dB	0	+4dB
6	0	0	+1dB	−6dB	−5dB

CORREZIONE FASE

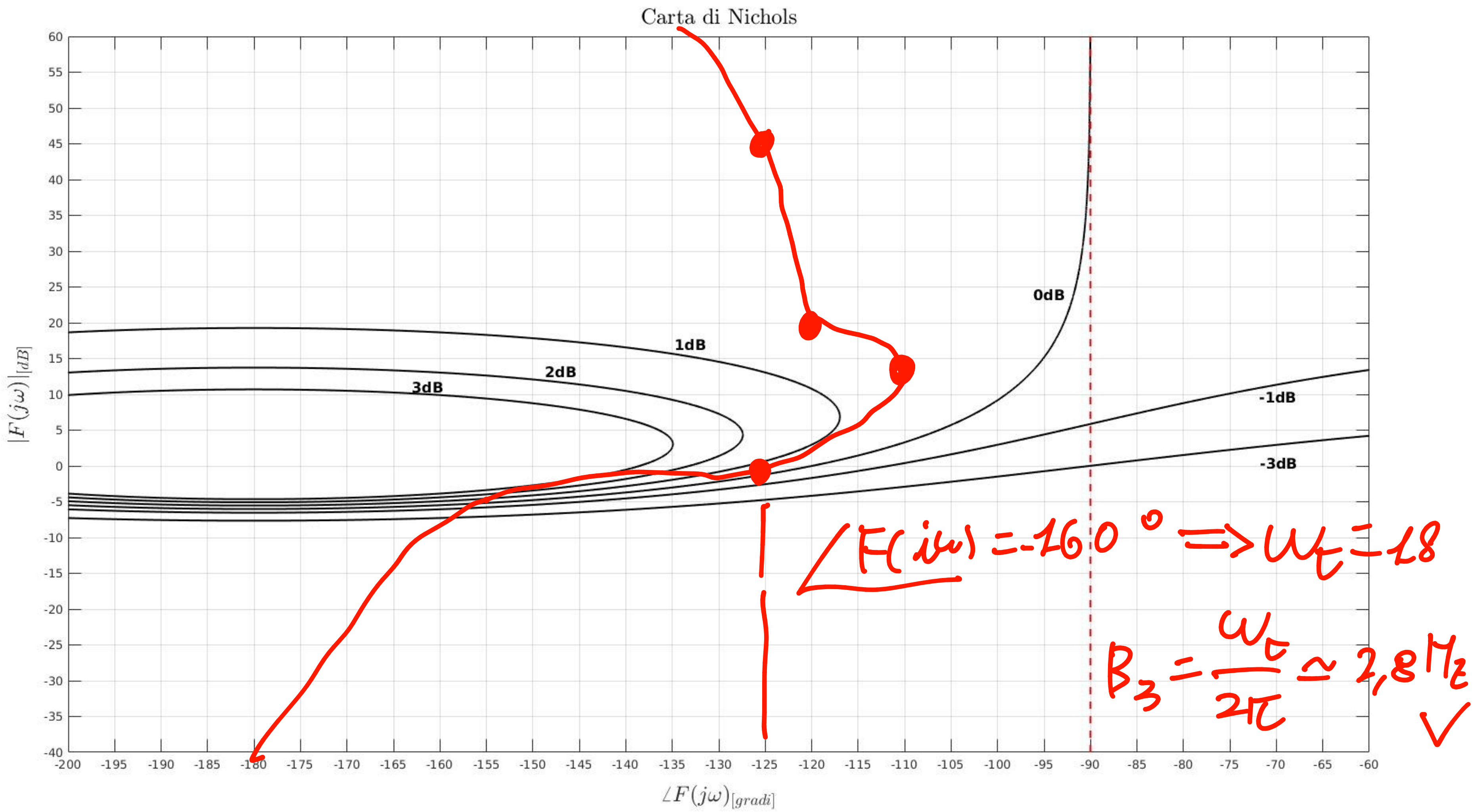
ω	$i\omega$	$1+\frac{i\omega}{0,2}$	$1+\frac{i\omega}{1}$	$1+\frac{i\omega}{2,8}$	$1+\frac{i\omega}{6}$	TOT
0,2	−90°	−45°	+10°	0	0	−125°
1	−90°	−80°	+45°	+25°	−20°	−120°
2,8	−90°	−90°	+70°	+45°	−50°	−116°
6	−90°	−90°	+80°	+65°	−90°	−125°



|F(iω)|
Carta semilogaritmica a 6 decadi
Università Politecnica delle Marche
32
10^-3
|F(iω_t)| = 0 √
10^-2
10^-1
0,2
1
2,8
6
10
10^2
Pulsazione ω
10^3



∠F(iω)
0°
-90°
-180°
∠F(jω_t) = -130°
⟹ M_φ = 50°
10^-3
10^-2
10^-1
0,2
1
2,8
6
10
10^2
Pulsazione ω
10^3



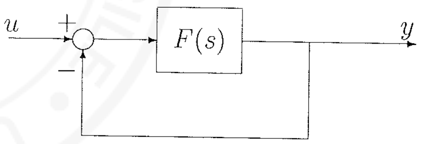
$\angle F(j\omega) = -160° \Rightarrow \omega_t = 18$

$B_3 = \frac{\omega_t}{2\pi} \simeq 2,8\ Hz$ ✓

Domanda Scritta di Controlli Automatici (9CFU) - 16/12/2013

Esercizio 1

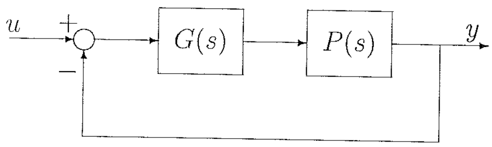
È dato il sistema di controllo:



in cui: $F(s) = \dfrac{(s-z)}{s(s+p)}$. Utilizzando il critèrio di Nyquist, studiare la stabilità del sistema a ciclo chiuso, per $z \in \mathbb{R}$, $p \in \mathbb{R}$, $z \neq 0$, $p \neq 0$, $p \neq -z$.

Esercizio 2

È dato il sistema di controllo:



in cui $P(s) = \dfrac{(s+3)(s+4)}{s(s+8)(s^2+4s+5)}$.

Utilizzando la sintesi con il luogo delle radici, progettare $G(s)$ in modo che:

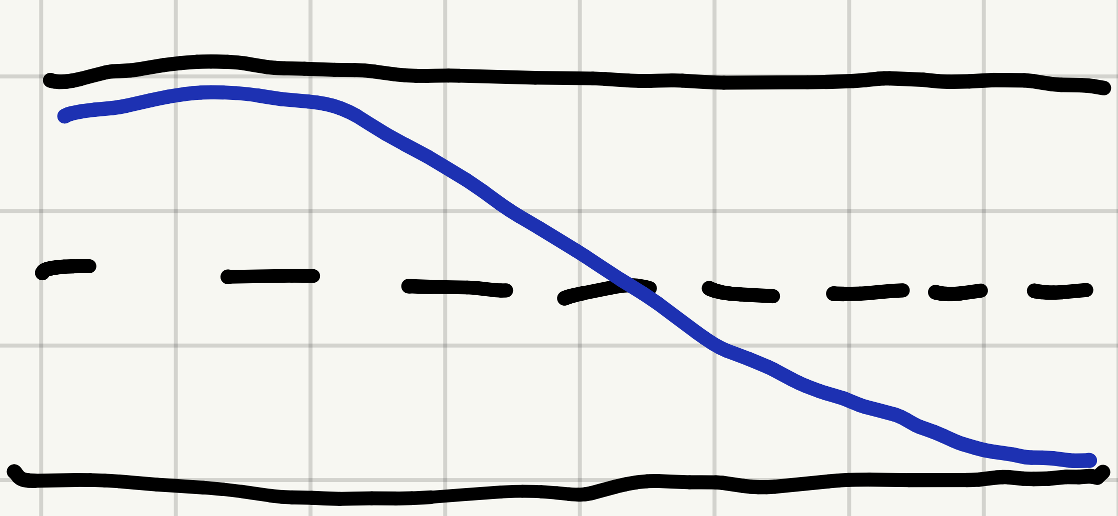
- $|\tilde{e}_2| \leq 0.05$;
- tutti i poli della funzione di trasferimento in catena chiusa abbiano parte reale minore di -1.

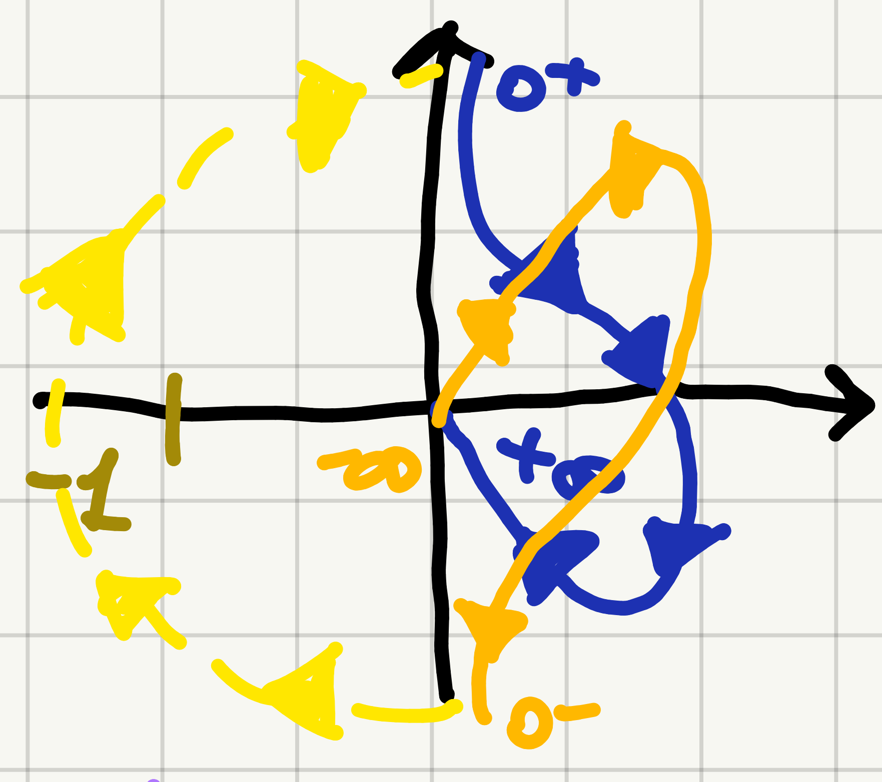
Calcolare infine la risposta a regime permanente all'ingresso $u(t) = (2t-6)\delta_{-1}(t)$.

$$F(s) = -\frac{Z}{P}\cdot\frac{\left(1-\frac{s}{Z}\right)}{s\left(1+\frac{s}{P}\right)} \qquad F(i\omega) = -\frac{Z}{P}\cdot\frac{\left(1-\frac{i\omega}{Z}\right)}{i\omega\left(1+\frac{i\omega}{P}\right)}$$

CASO 1: $Z>0,\ P>0$ $\quad P_t = 0$

$M(0^+)=\infty,\ \varphi(0^+)=-270°$ $\quad M(+\infty)=0,\ \varphi(+\infty)=-450°$



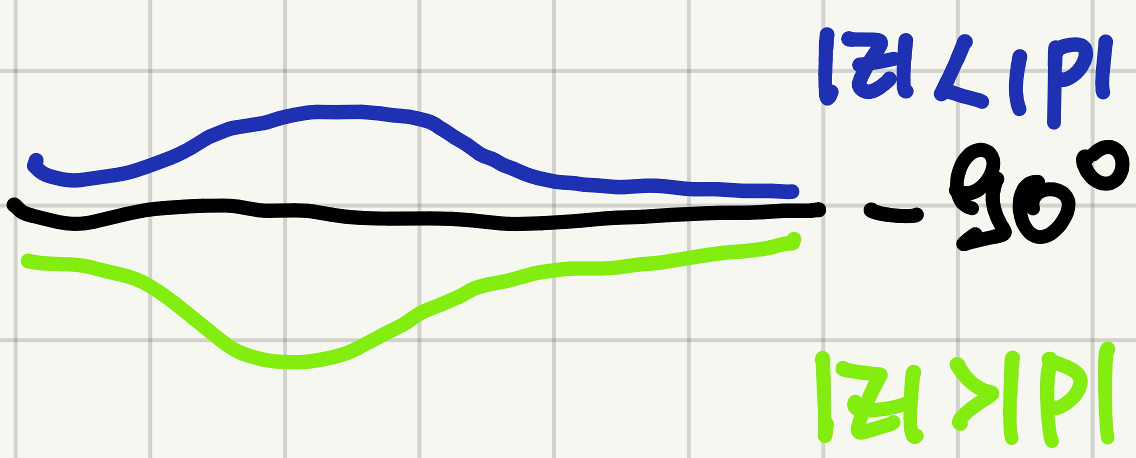


$\vec{N}=1$

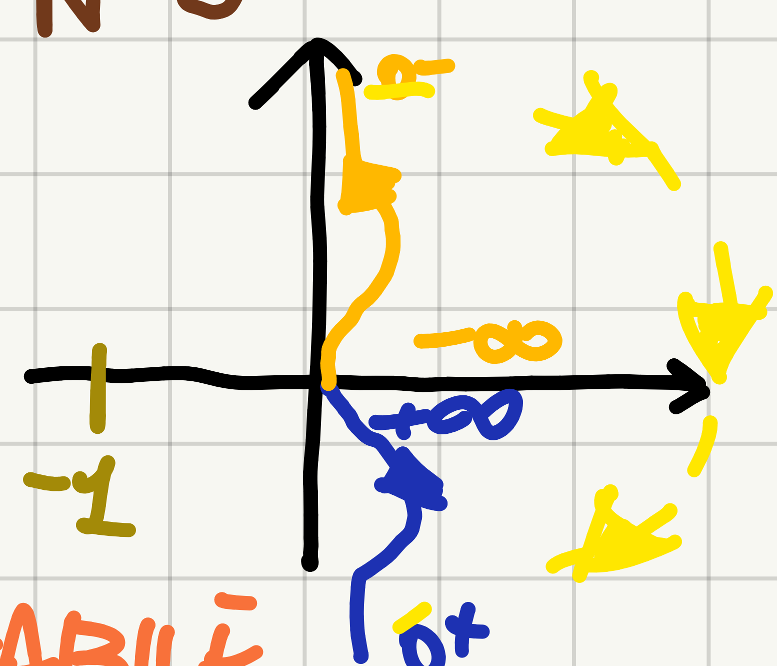
$\vec{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 2: $Z>0,\ P<0$ $\quad P_t = 1$

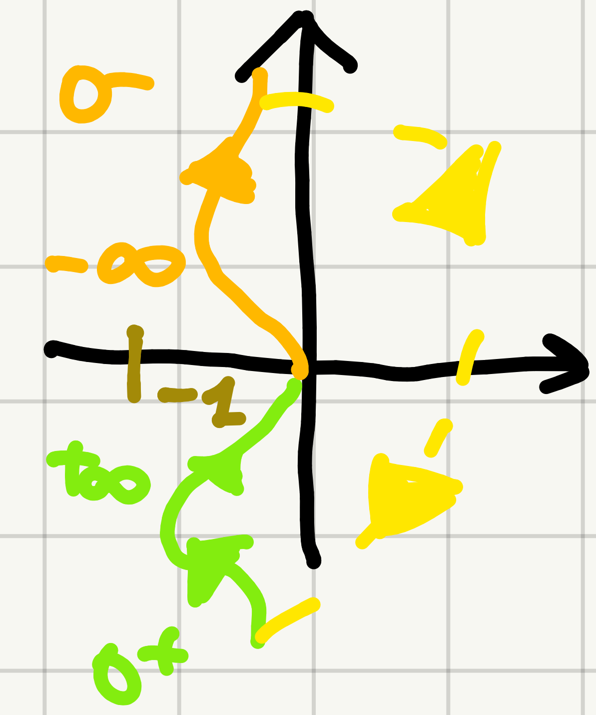
$M(0^+)=\infty,\ \varphi(0^+)=-90°$ $\quad M(+\infty)=0,\ \varphi(+\infty)=-90°$



$\vec{N}=0$



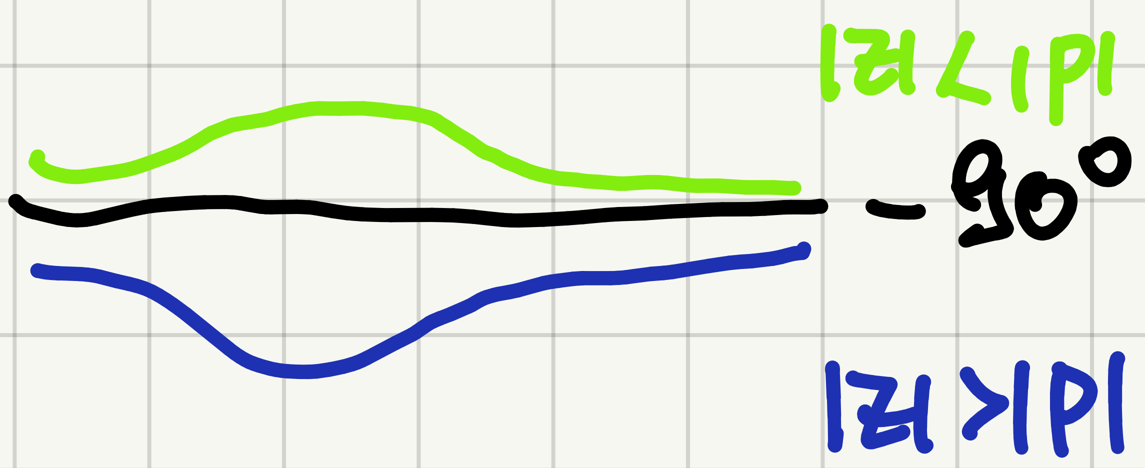
$\vec{N}=0$



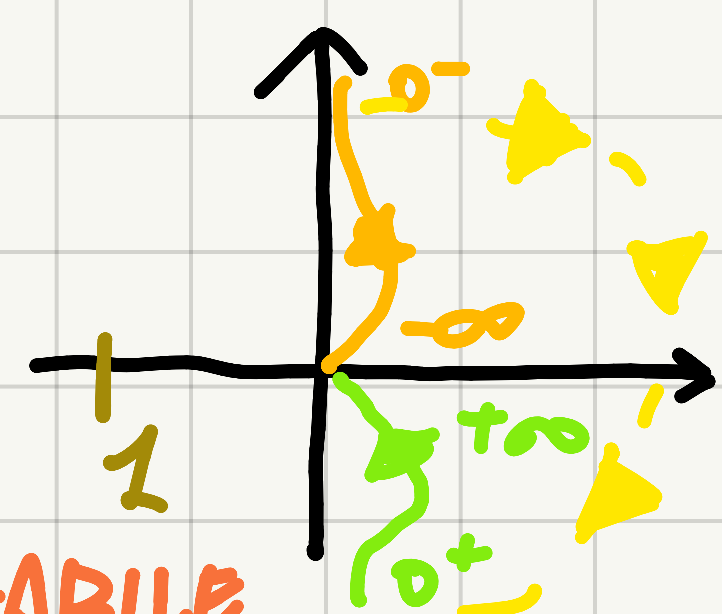
$\vec{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 3: $Z<0,\ P>0$ $\quad P_t = 0$

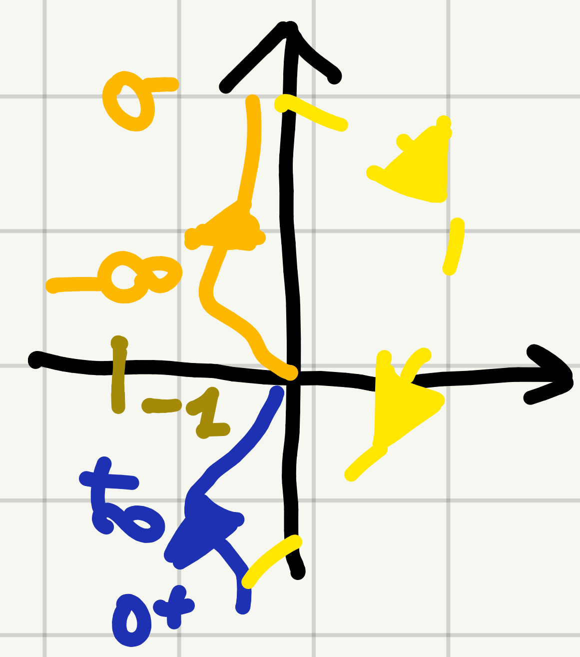
$M(0^+)=\infty,\ \varphi(0^+)=-90°$ $\quad M(+\infty)=0,\ \varphi(+\infty)=-90°$



$\vec{N}=0$



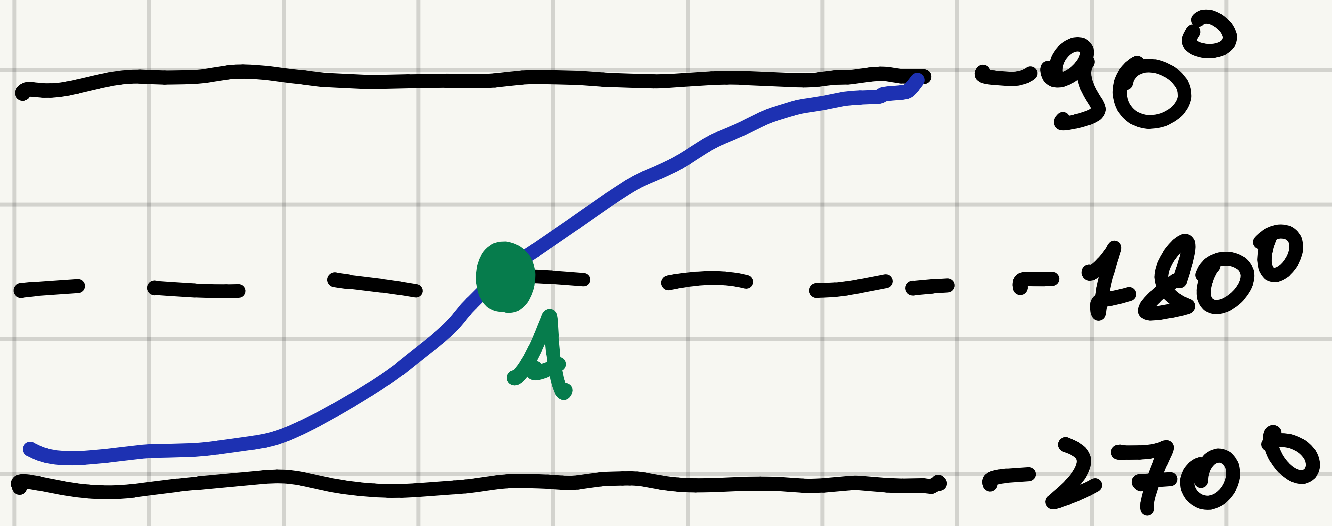
$\vec{N}=0$

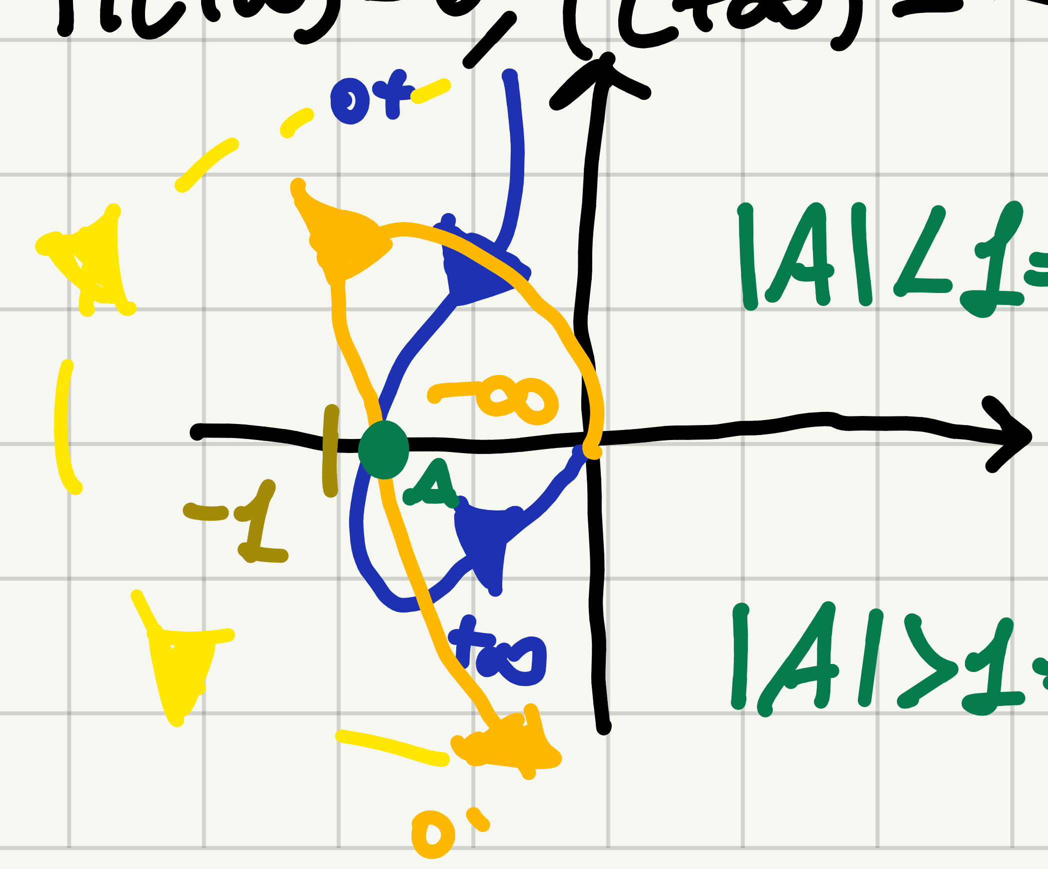


$\vec{N} = -P_t \Rightarrow$ SISTEMA STABILE

CASO 4: $Z<0,\ P<0$ $\quad P_t = 1$

$M(0^+)=\infty,\ \varphi(0^+)=-270°$ $\quad M(+\infty)=0,\ \varphi(+\infty)=-90°$





$|A|<1 \Rightarrow \vec{N}=-1$

$|A|>1 \Rightarrow \vec{N}=1$

SISTEMA STABILE PER $|A|<1$

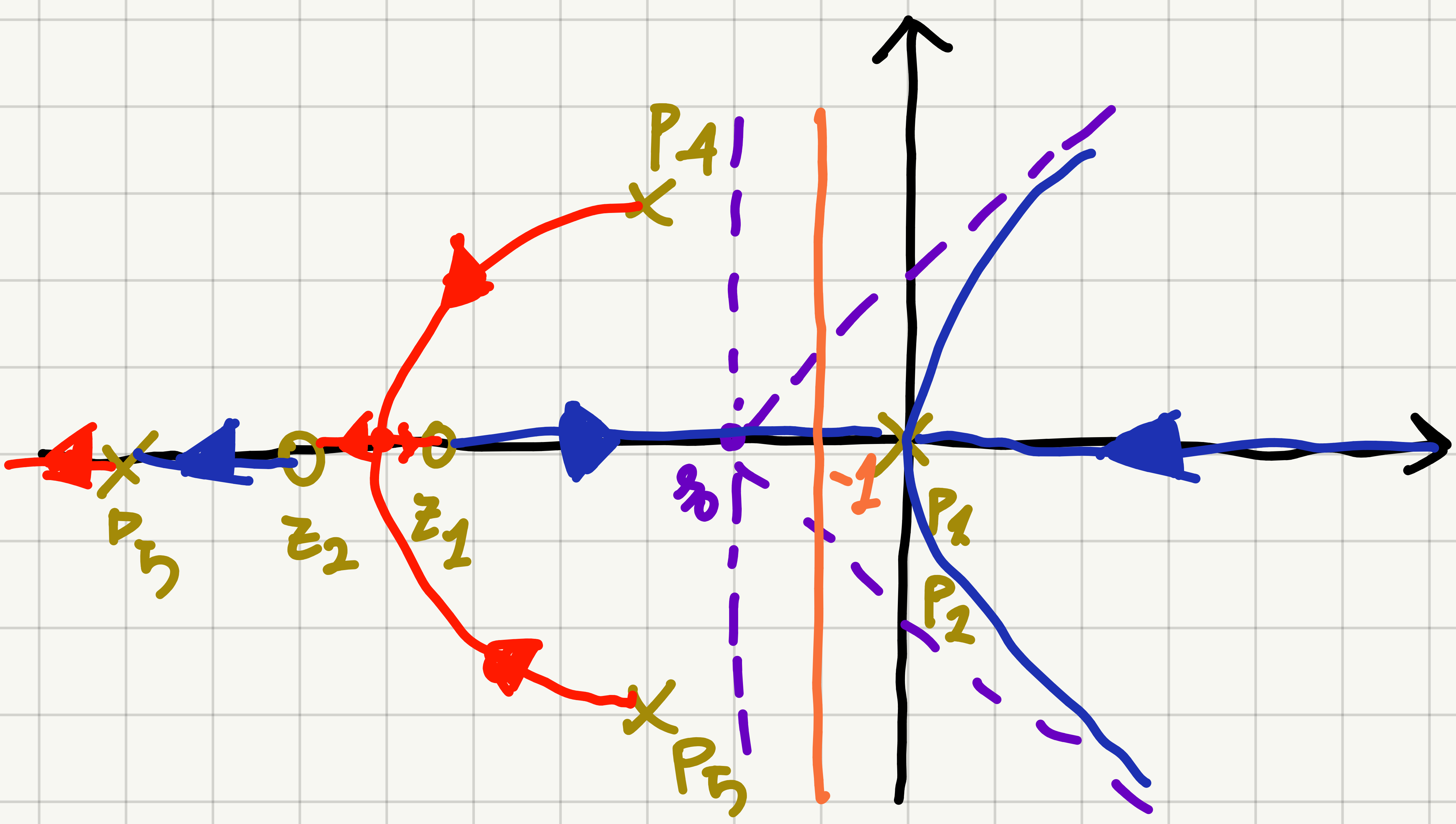
2

$$|e_2| = \left|\frac{K_d}{K_G K_p}\right| \le 0{,}05 \qquad K_p = \frac{12}{40} \Rightarrow K_G \ge 67$$

$$G(s) = \frac{K}{s} \qquad F(s) = G(s)\cdot P(s) = \frac{K\cdot(s+3)(s+4)}{s^2(s+8)(s^2+4s+5)}$$

$z_1 = -3,\ z_2 = -4;\ p_1 = p_2 = 0,\ p_3 = -8,\ p_4 = -2+i,\ p_5 = -2-i$

$n = 5,\ m = 2 \qquad n-m = 3 \qquad s_0 = \frac{\sum p - \sum z}{n-m} = -\frac{5}{3}$



SPECIFICA NON SODDISFATTA

PONGO $n-m=2$

$$G(s) = \frac{K}{s}\cdot\frac{(s-z_1)(s-z_2)}{(s-p)}$$

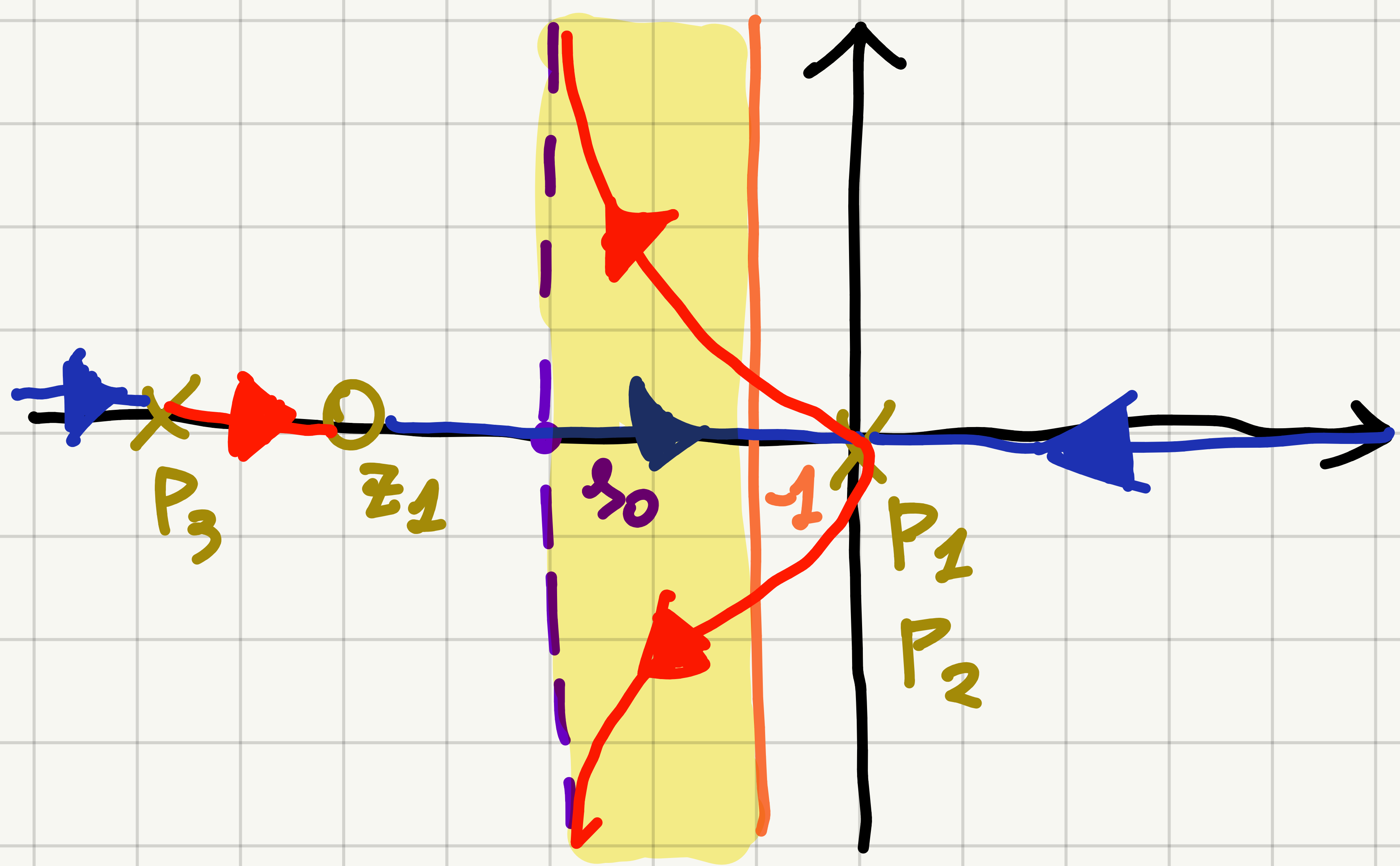
COPPIA ZERO-POLO PER SPOSTARE s_0

$(s-z_1)(s-z_2) = (s^2+4s+5) \qquad s_0 = -2$

$$\Rightarrow F(s) = \frac{K\cdot(s+3)(s+4)}{s^2(s+8)(s-p)} \qquad s_0 = \frac{-8+p+3+4}{2} \Rightarrow p = -3$$

$$F(s) = \frac{K(s+4)}{s^2(s+8)} \qquad K_G \ge 67\cdot\frac{5}{3}$$

$z_1 = -4;\ P_1 = P_2 = 0,\ P_3 = -8 \qquad s_0 = -2$



$$f(s,K) = s^2(s+8) + K(s+4) \qquad s = \bar{s} - 1$$

$$(\bar{s}-1)^2(\bar{s}+7) + K(\bar{s}+3) = 0$$

$$\bar{s}^3 + 5\bar{s}^2 + (K-13)\bar{s} + (3K+7) = 0$$

$$\begin{array}{c|cc} 3 & 1 & K-13 \\ 2 & 5 & 3K+7 \\ 1 & -\frac{71-2K}{5} & \\ 0 & 3K+7 & \end{array} \qquad \begin{cases} -\frac{71-2K}{5} > 0 \\ 3K+7 > 0 \end{cases} \Rightarrow K > 36$$

$$\Rightarrow K \geq 67 \cdot \frac{5}{3}$$

$$G(s) = 112 \cdot \frac{(s^2+4s+5)}{(s+3)} \Rightarrow F(s) = 112 \cdot \frac{(s+4)}{s^2(s+8)}$$

$$\Rightarrow K_F = 57 \cdot \frac{5}{3}$$

$$U(t) = (2t-6)\delta_{-1}(t) = 2(t)\delta_{-1}(t) + (-6)\delta_{-1}(t)$$

$$= 2U_1(t) - 6U_2(t)$$

- $U_1(t)$

$$\tilde{e}_{U_1}(t) = K_d U_1(t) - \tilde{y}_{U_1}(t)$$

$$\tilde{e}_{U_1}(t) = \frac{1}{K_d \cdot K_p} = \frac{1}{K_F} = \frac{1}{57}$$

$$\tilde{y}_{U_1}(t) = K_d U_1(t) - \tilde{e}_{U_1}(t) = \left(t - \frac{1}{57}\right)\delta_{-1}(t)$$

- $U_2(t)$

GRADO DI $U_2(t) <$ TIPO DI $F(s) \Longrightarrow \tilde{y}_{U_2}(t) = \delta_{-1}(t)$

$$\Longrightarrow \tilde{y}(t) = 2\left(t - \frac{1}{57}\right)\delta_{-1}(t) - 6\delta_{-1}(t)$$