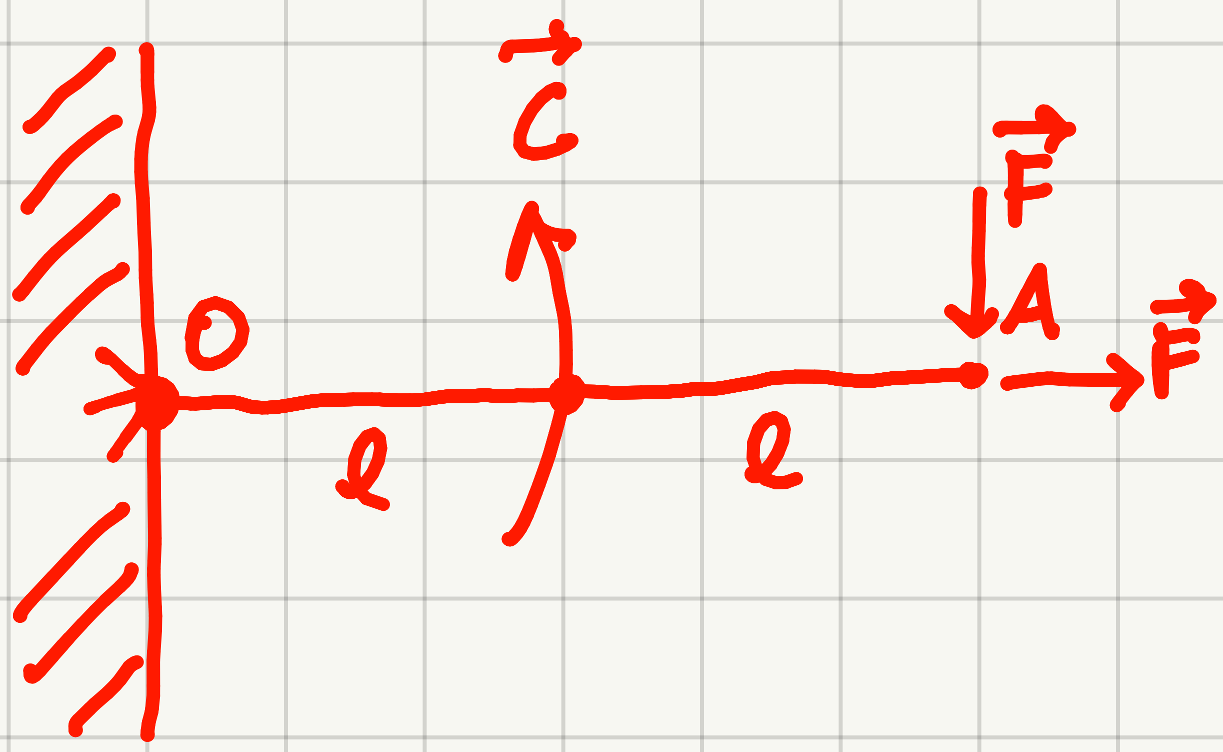
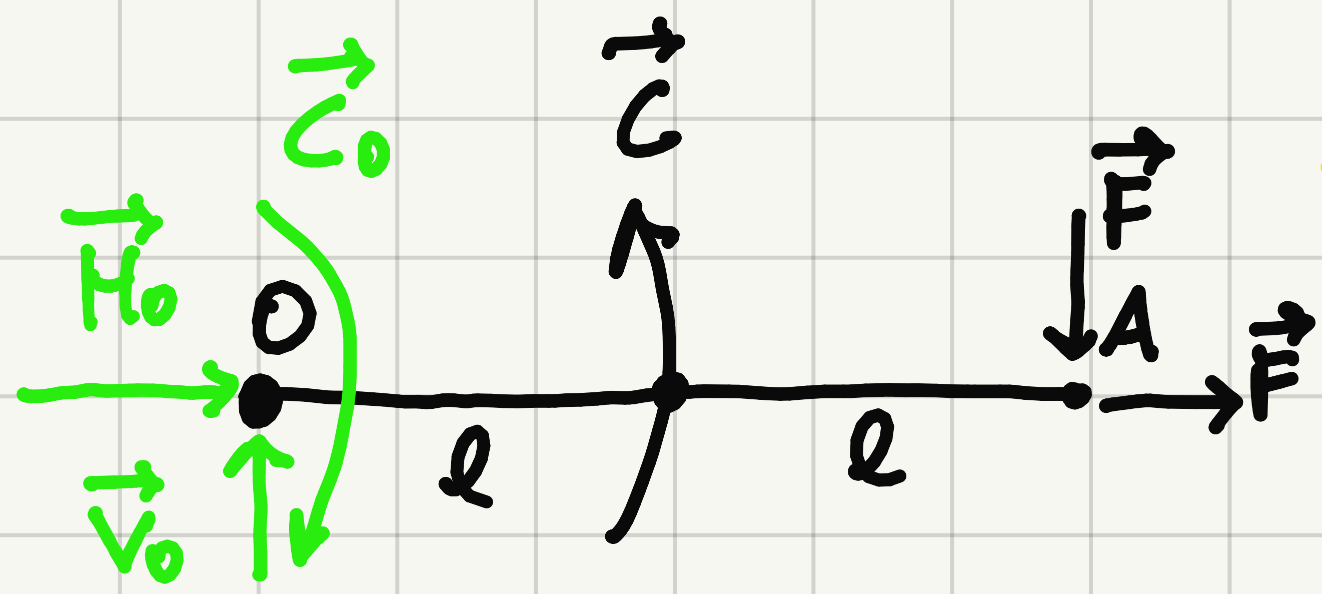
STATICA

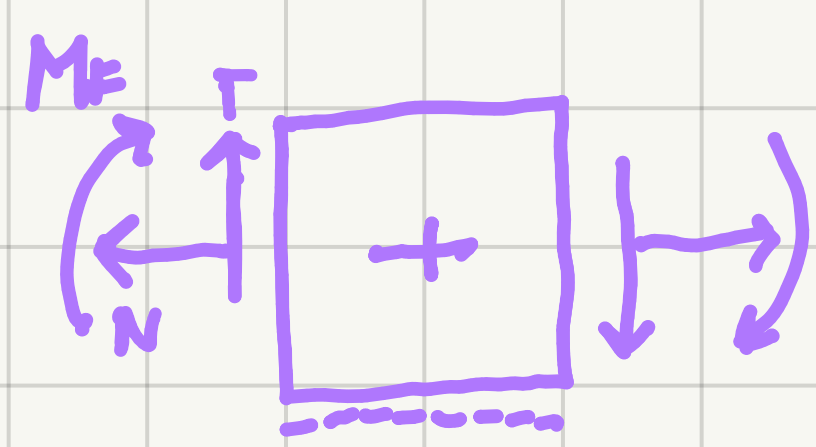
① Si consideri un'asta lineare incastrata ad una parete verticale in O



e lunga $\overline{OA}=2\ell$. In A sono applicate una forza $\vec{F}$ verso il basso ed una $\vec{F}$ lungo l'orizzontale a destra. Esterna all'asta ed una coppia antioraria applicata al baricentro $|\vec{C}|=3F\ell$.

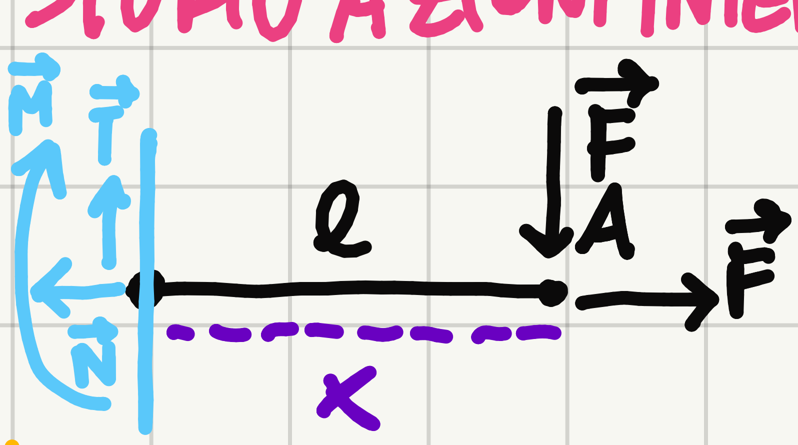
Determinare le reazioni vincolari ed il diagramma delle azioni interne





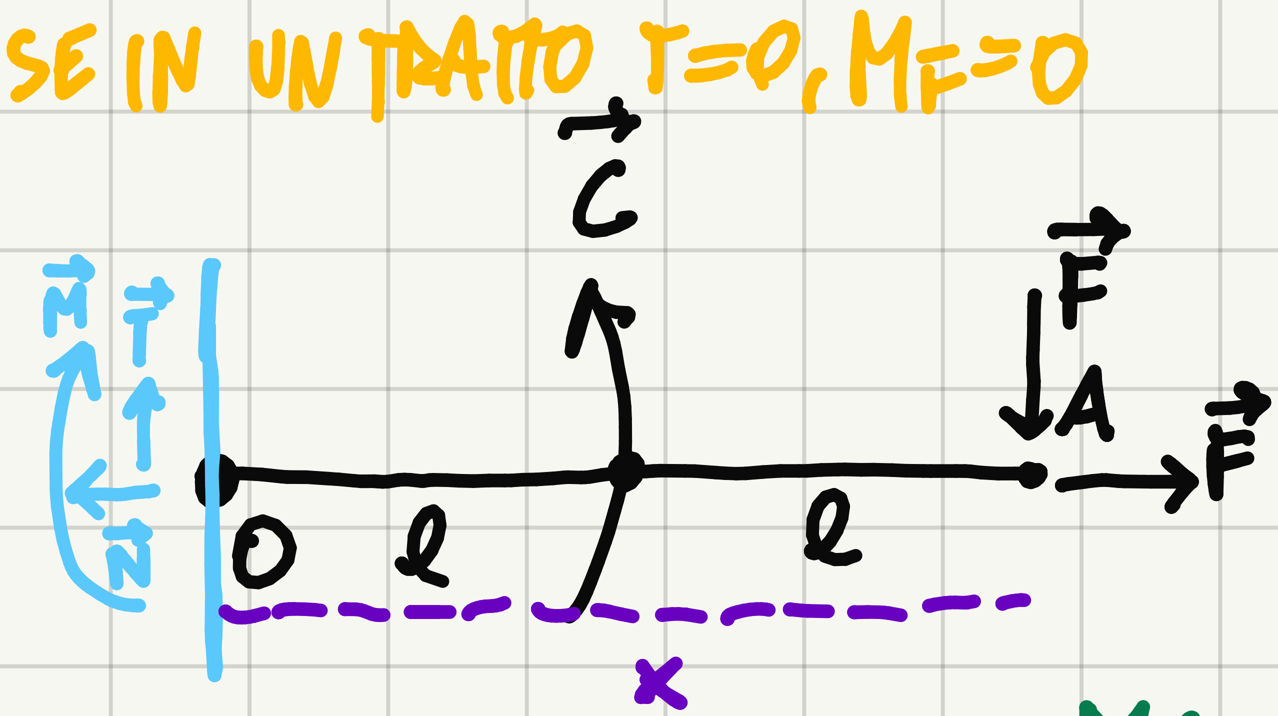
$$\begin{cases} \sum F_x=0 \\ \sum F_y=0 \\ \sum M_0=0 \end{cases} \Rightarrow \begin{cases} H_0+F=0 \\ V_0-F=0 \\ -C_0+C-F2L=0 \end{cases} \Rightarrow \begin{matrix} H_0=-F \\ V_0=F \\ C_0=F\ell \end{matrix}$$

STUDIO AZIONI INTERNE



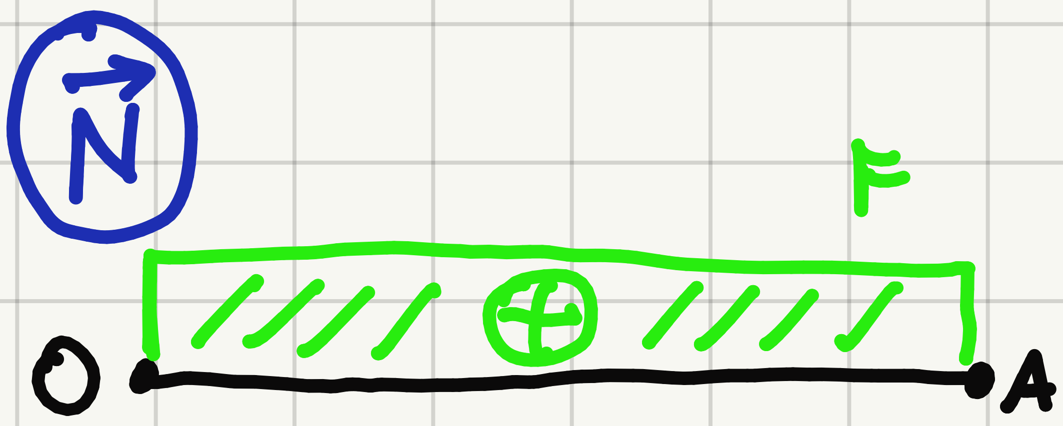
$$\begin{cases} \sum F_x=0 \\ \sum F_y=0 \\ \sum M=0 \end{cases} \rightarrow \begin{cases} N=F \\ T=F \\ -M-Fx=0 \end{cases} \quad M(0)=0 \quad M(\ell)=-F\ell$$

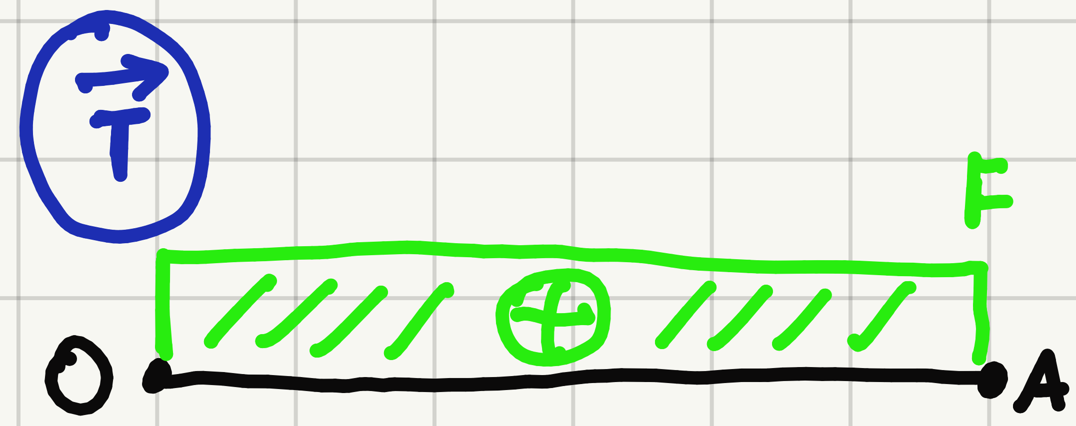
! SE IN UN TRATTO $T=0$, $M_F=0$

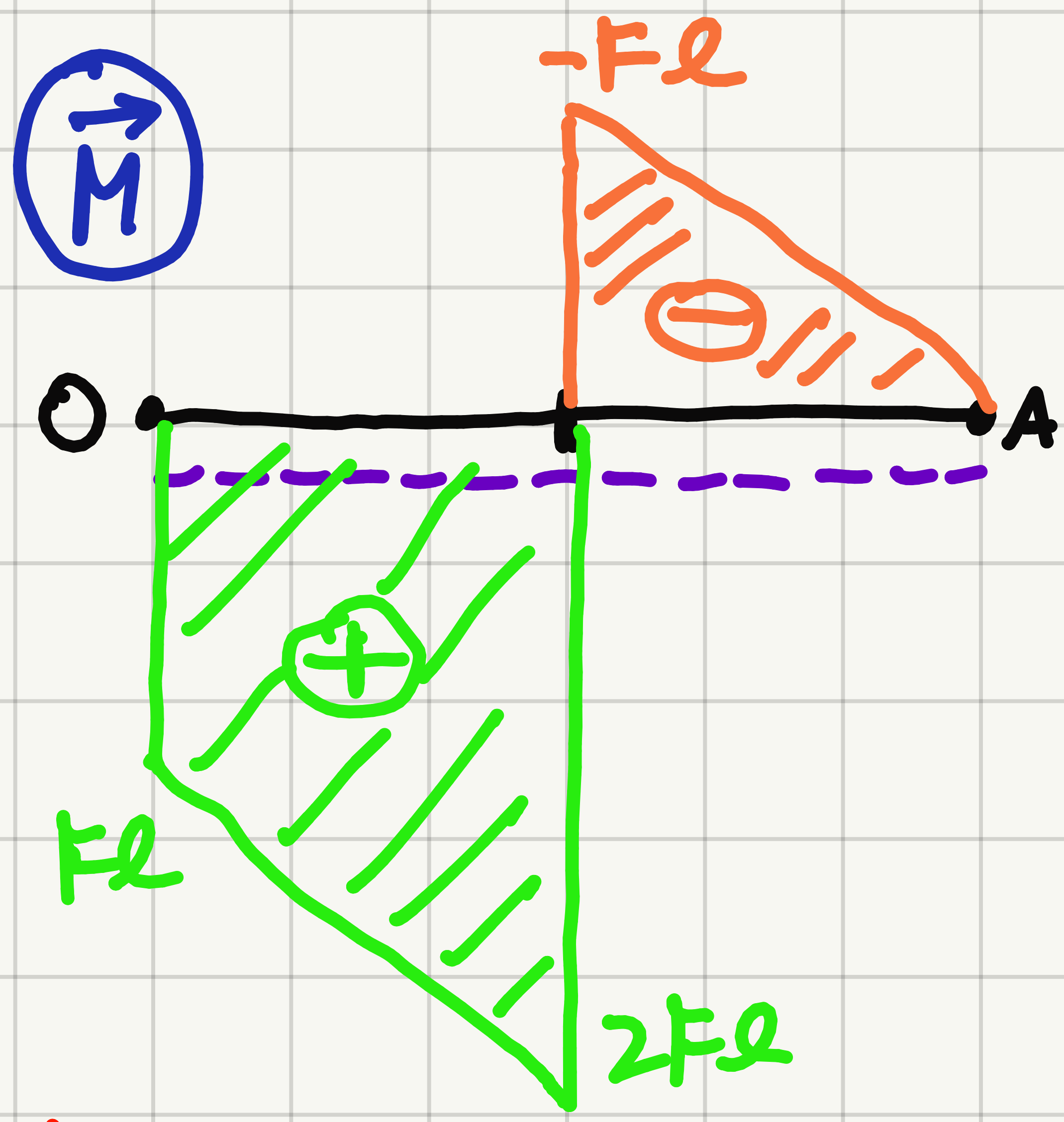


$$\begin{cases} \sum F_x=0 \\ \sum F_y=0 \\ \sum M=0 \end{cases} \rightarrow \begin{cases} N=F \\ T=F \\ -M-Fx+C=0 \end{cases}$$

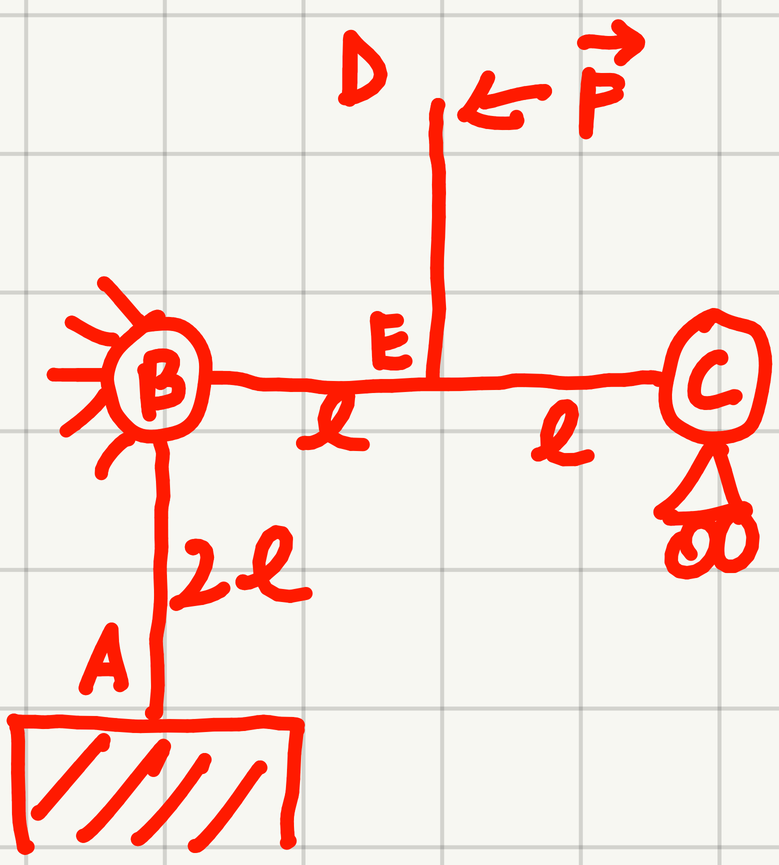
$M(\ell)=2F\ell \quad M(2\ell)=F\ell$



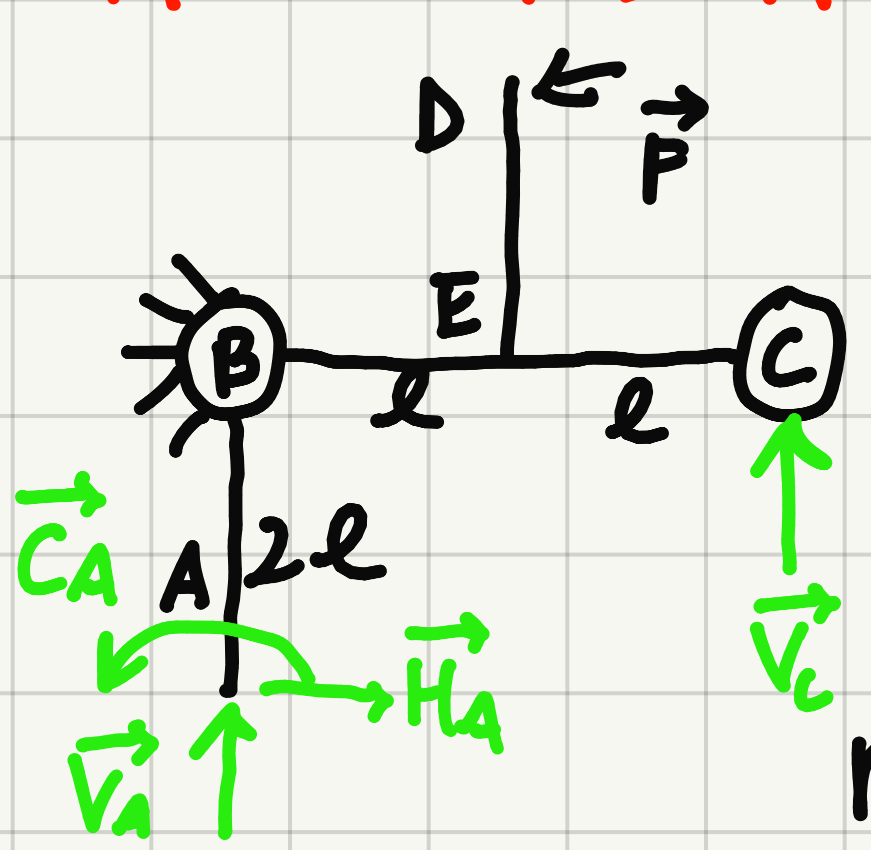




FIBRE TESE



② CONSIDERIAMO UN SISTEMA COMPOSTO DA 3 ASTE COME IN FIGURA E SIA DATA LA FORZA $\vec{F}$ NOTA. CALCOLARE LE REAZIONI VINCOLARI INTERNE ED ESTERNE E DISEGNARE I DIAGRAMMI DELLE AZIONI INTERNE

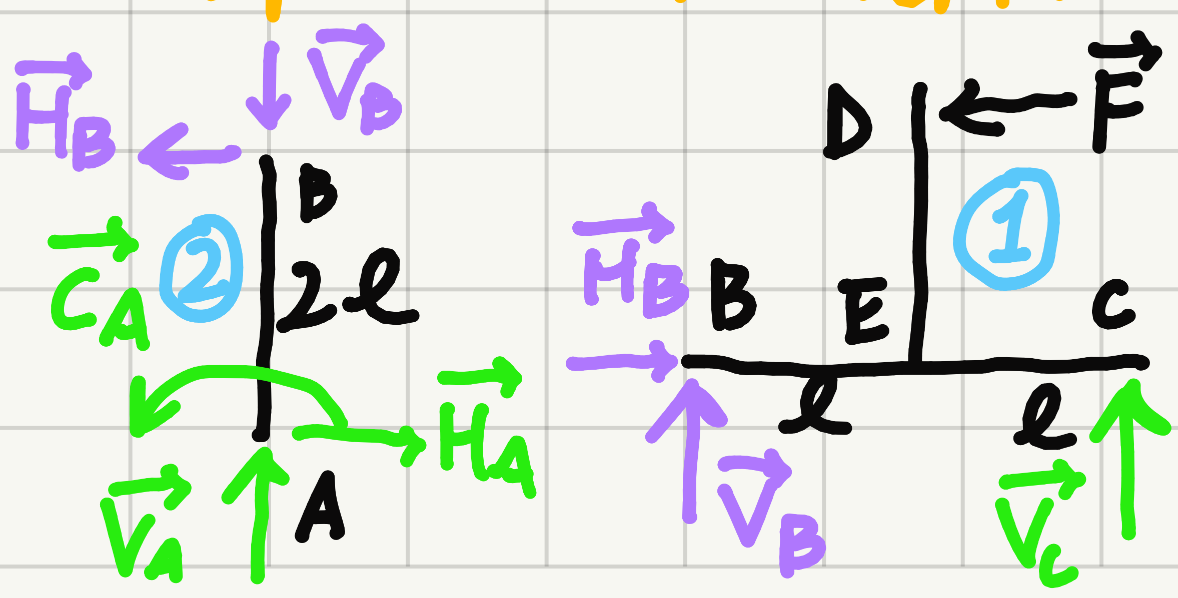


SI NOTI CHE BE, EC, DE COMPONGONO UN UNICO CORPO

$n_0=6 \quad n=2_B+1_C+3_A=0 \Rightarrow n=0$

! LE 3 EQUAZIONI SONO INSUFFICIENTI PER CALCOLARE LE 4 INCOGNITE

⇒ SPEZZO I DUE CORPI



! LE FORZE IN B, NEI DUE CORPI, SONO UGUALI IN MODULO E OPPOSTE IN SEGNO

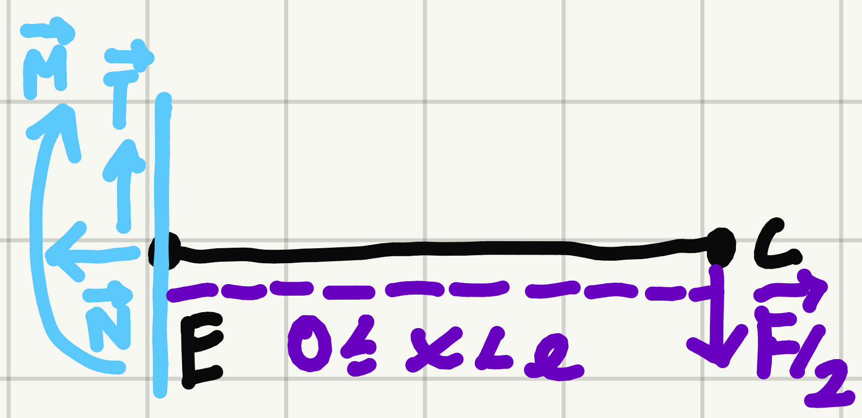
$$1 \begin{cases} H_B - F = 0 \\ V_B + V_C = 0 \\ B \quad Fl + 2l V_C = 0 \end{cases} \Rightarrow V_B = F/2 \quad V_C = -F/2$$

POSSIAMO ORA STUDIARE L'INTERO CORPO

$$B\quad\begin{cases} H_A - F = 0 \\ V_A + V_C = 0 \\ C_A + 2\ell H_A + 2\ell V_C + F\ell = 0 \end{cases} \Rightarrow H_A = F,\ V_A = F/2,\ C_A = -2F\ell$$

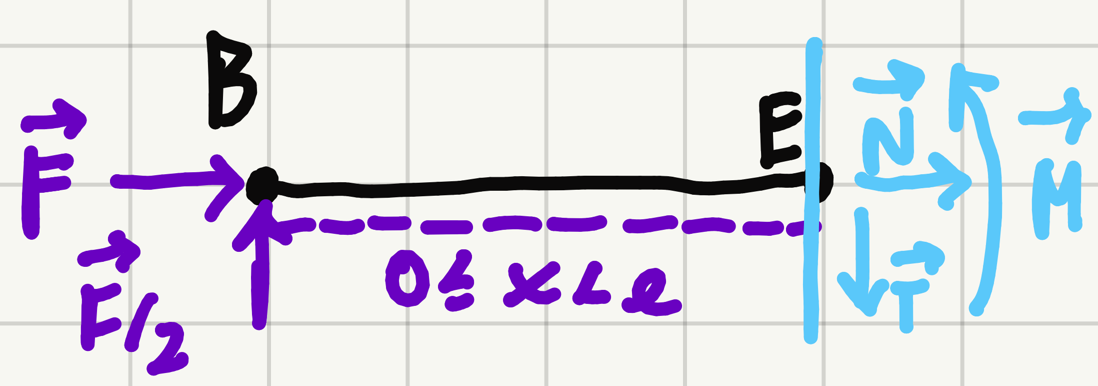
STUDIO AZIONI INTERNE

CE



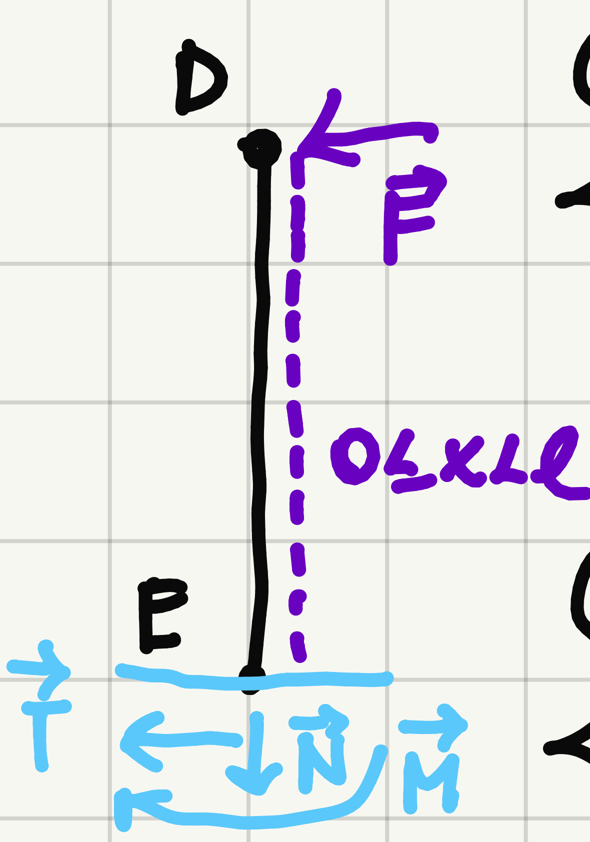
$$\begin{cases} -N = 0 \\ T - F/2 = 0 \\ -M - F/2\, x = 0 \end{cases} \Rightarrow \begin{cases} N = 0 & M(0) = 0 \\ T = F/2 & \\ M = F/2\, x & M(\ell) = -F/2\,\ell \end{cases}$$

BE



$$\begin{cases} N + F = 0 \\ F/2 - T = 0 \\ M - F/2\, x = 0 \end{cases} \Rightarrow \begin{cases} N = -F & M(0) = 0 \\ T = F/2 & \\ M = F/2\, x & M(\ell) = F/2\,\ell \end{cases}$$

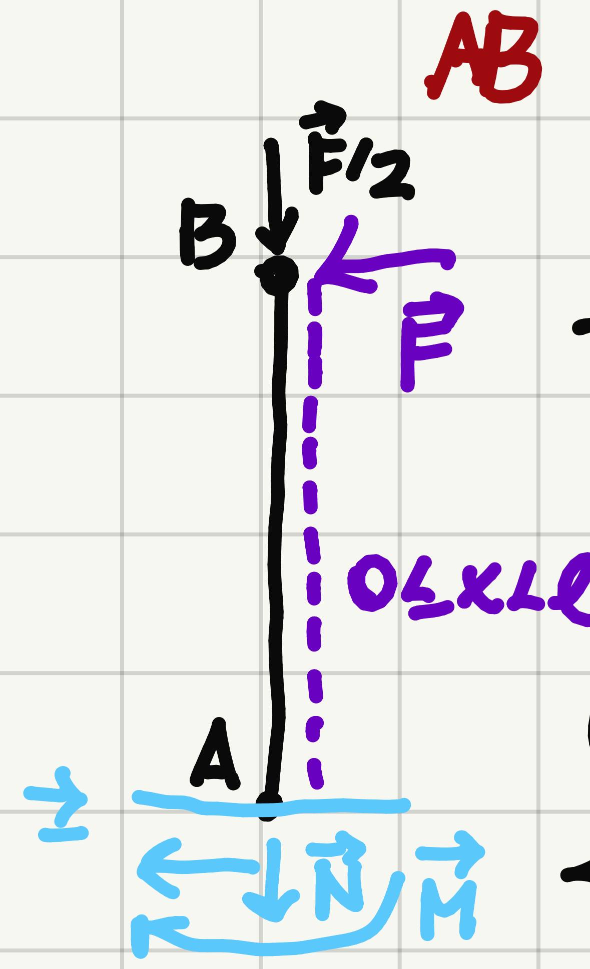
DE



$$\begin{cases} -F - T = 0 \\ -N = 0 \\ -M + Fx = 0 \end{cases}$$

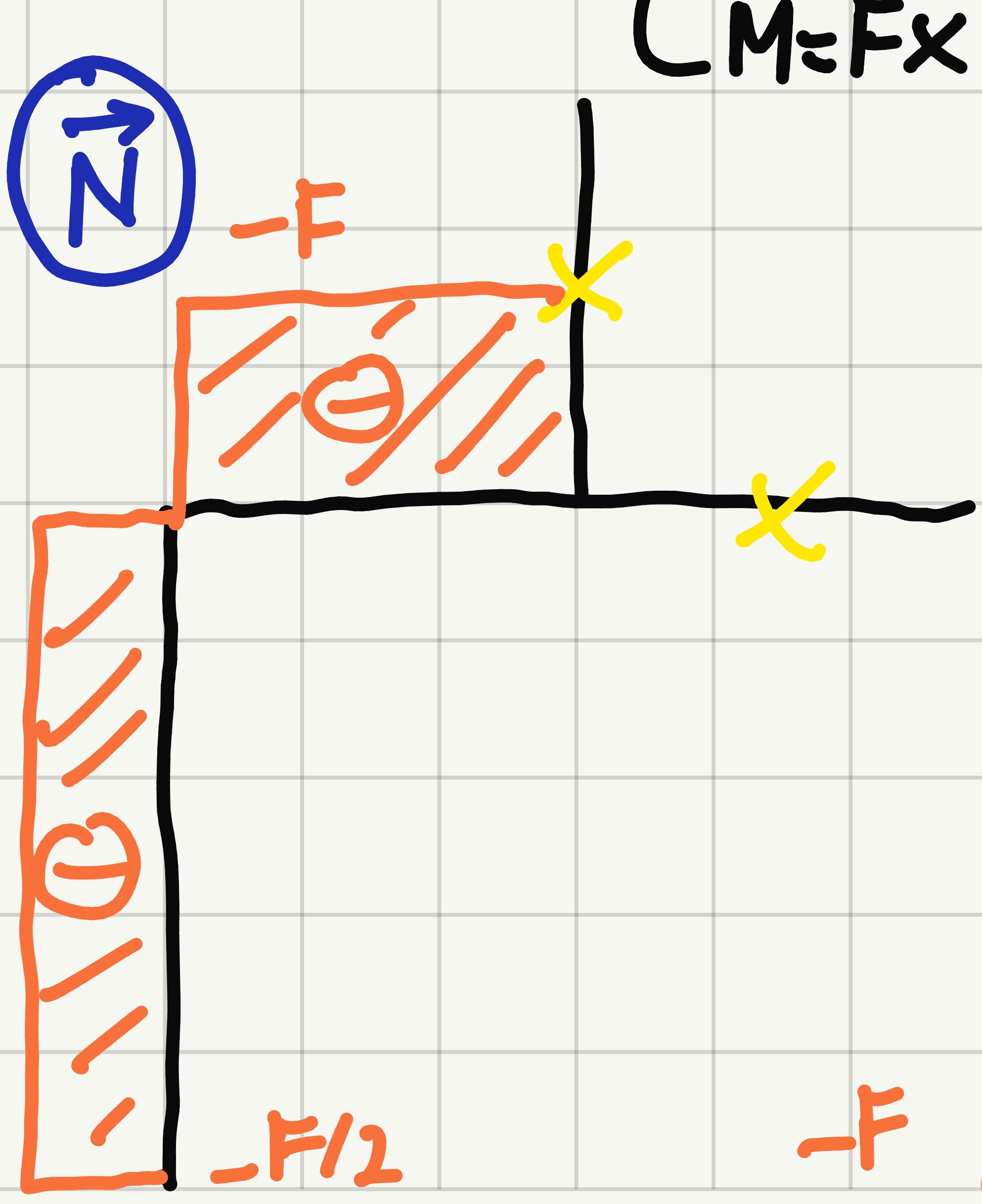
$$\begin{cases} T = -F & M(0) = 0 \\ N = 0 & \\ M = Fx & M(\ell) = F\ell \end{cases}$$

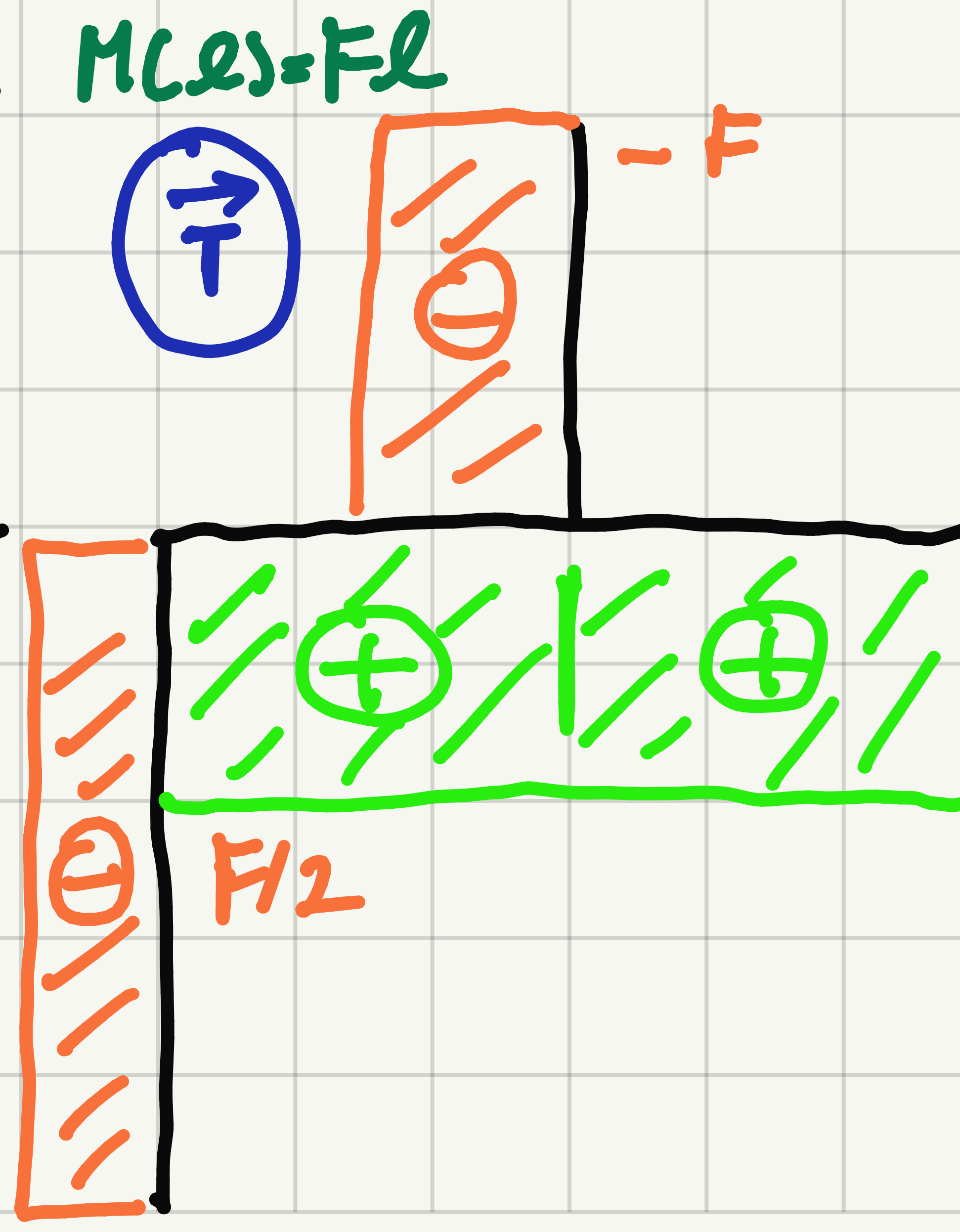
AB

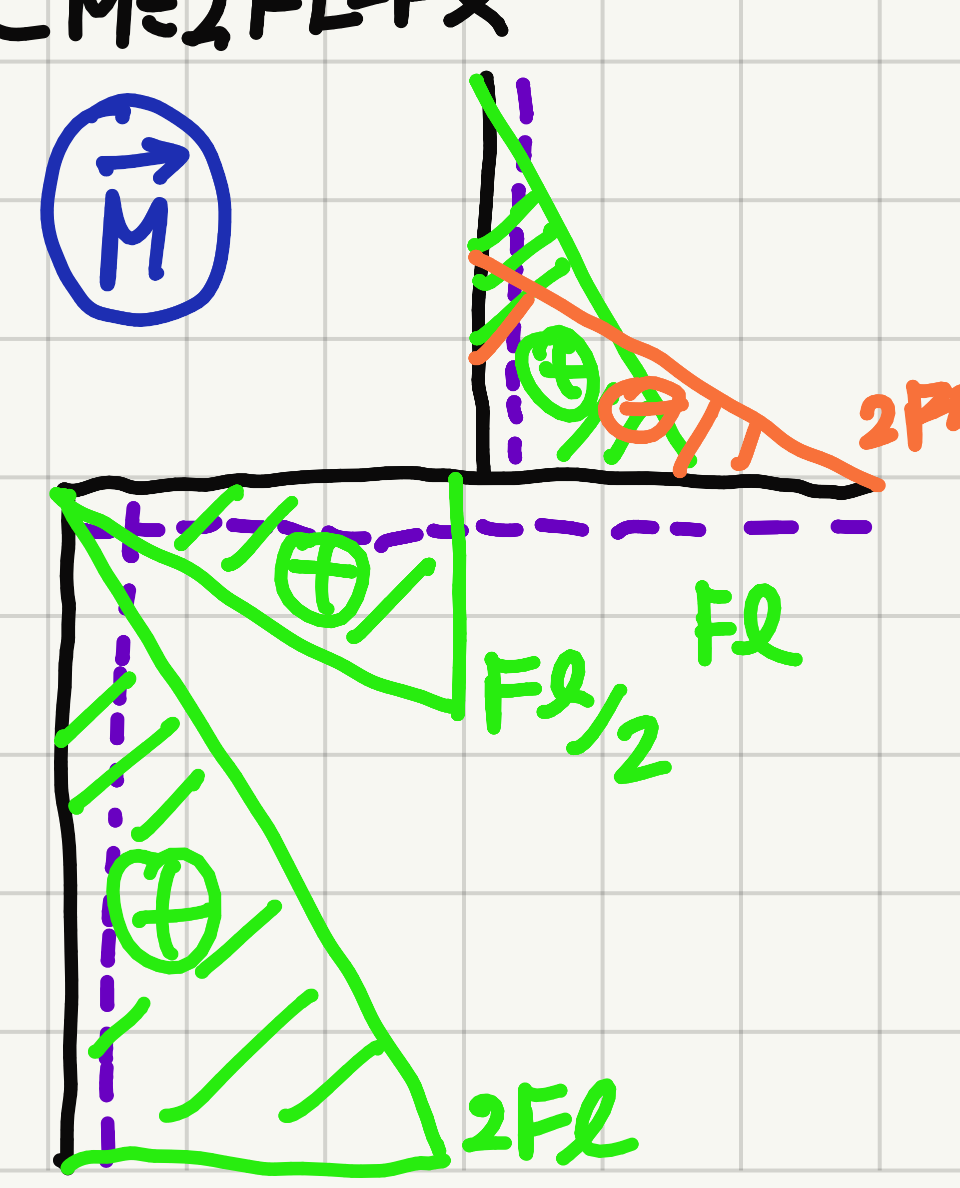


$$\begin{cases} -F - T = 0 \\ -N - F/2 = 0 \\ -M + 2FL - Fx = 0 \end{cases}$$

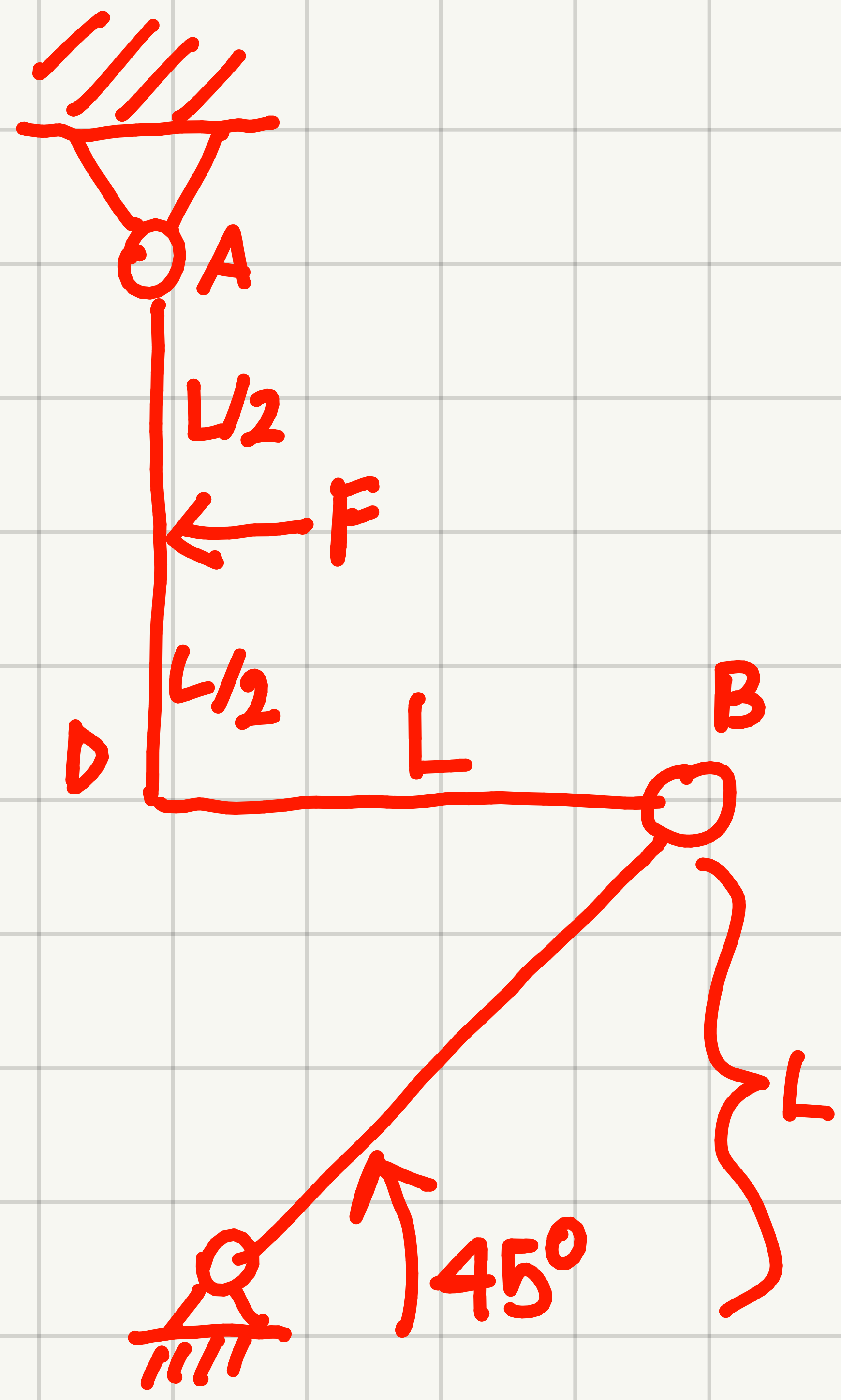
$$\begin{cases} T = -F \\ N = -F/2 \\ M = 2FL - Fx \end{cases} \quad \begin{matrix} M(0) = 2FL \\ M(2L) = 0 \end{matrix}$$







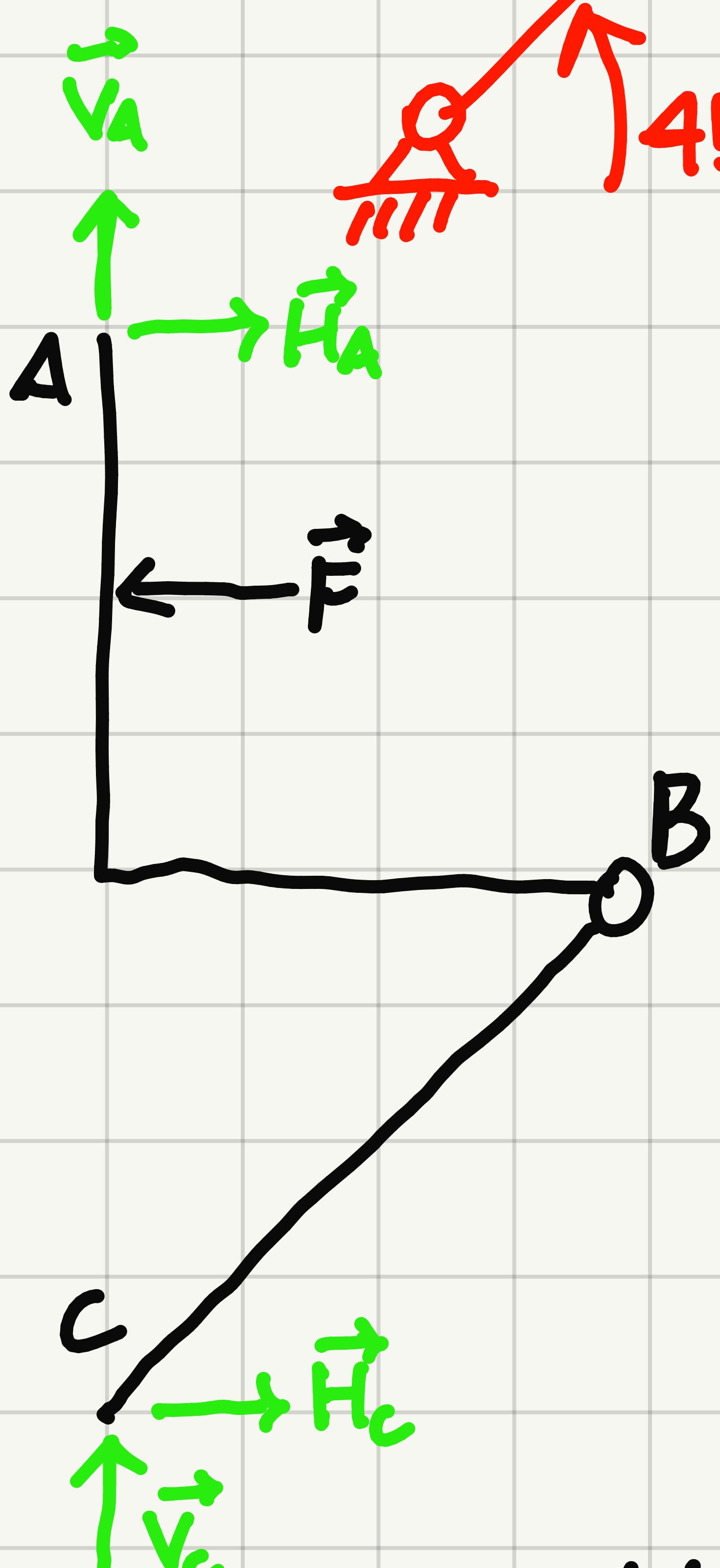
③



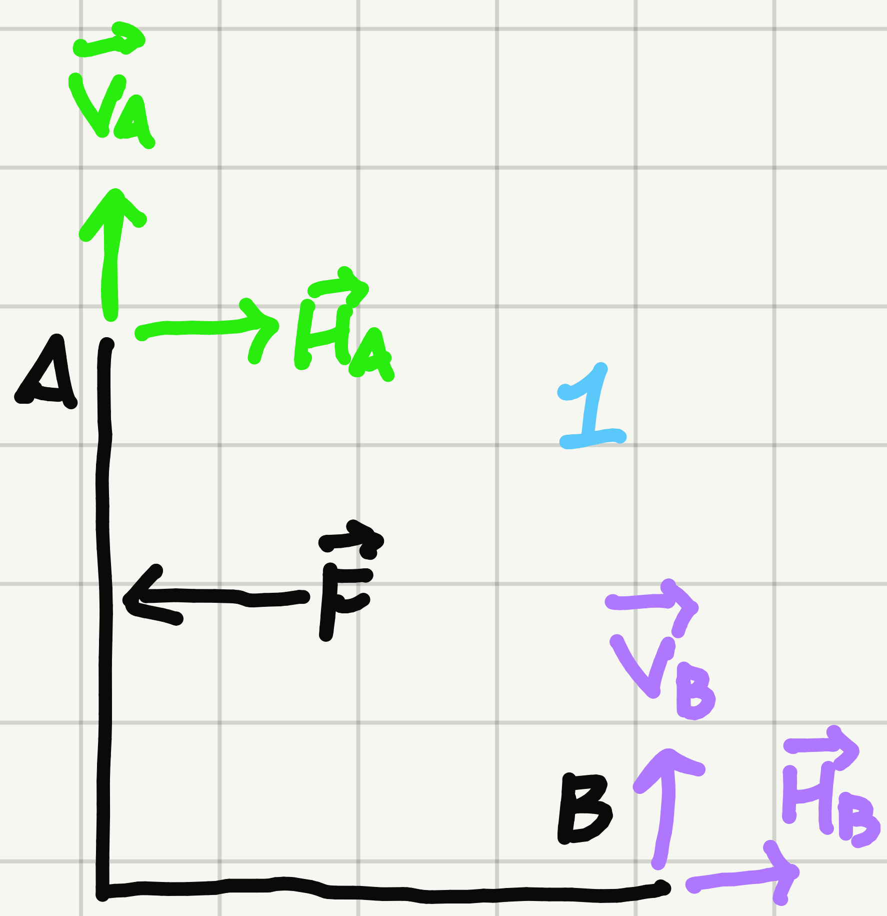
Per il sistema in figura, si calcolino i gradi di libertà, le reazioni vincolari e disegnare il diagramma delle azioni interne

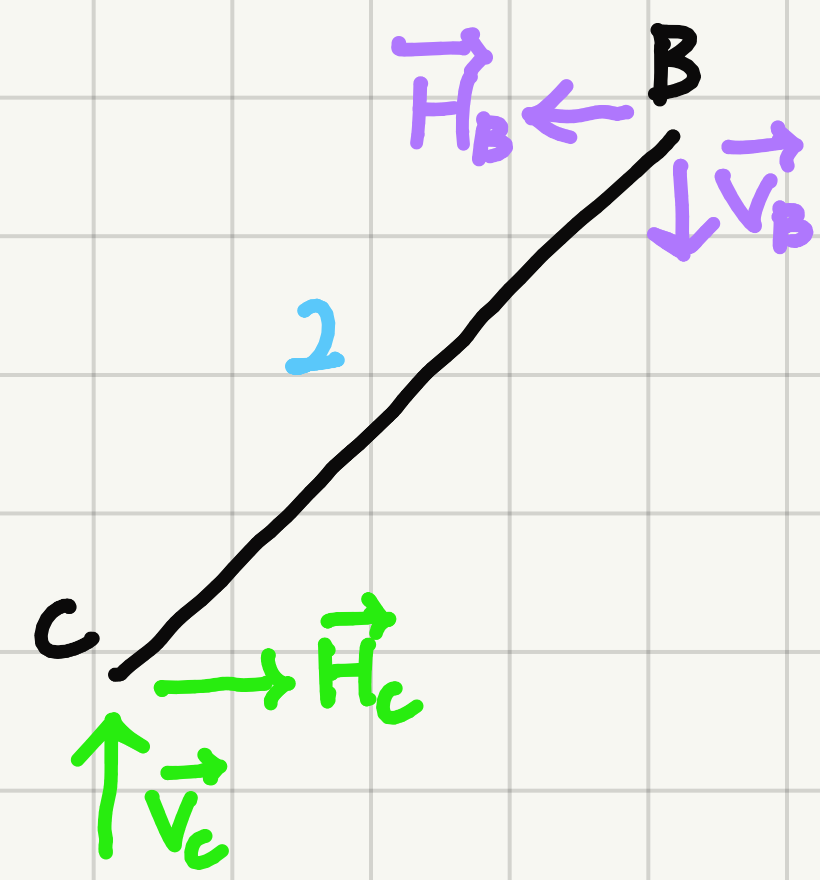
$n_0 = 3\cdot 2 = 6 \quad n_v = 2_A + 2_B + 2_C = 6$

$\Rightarrow n = 0$



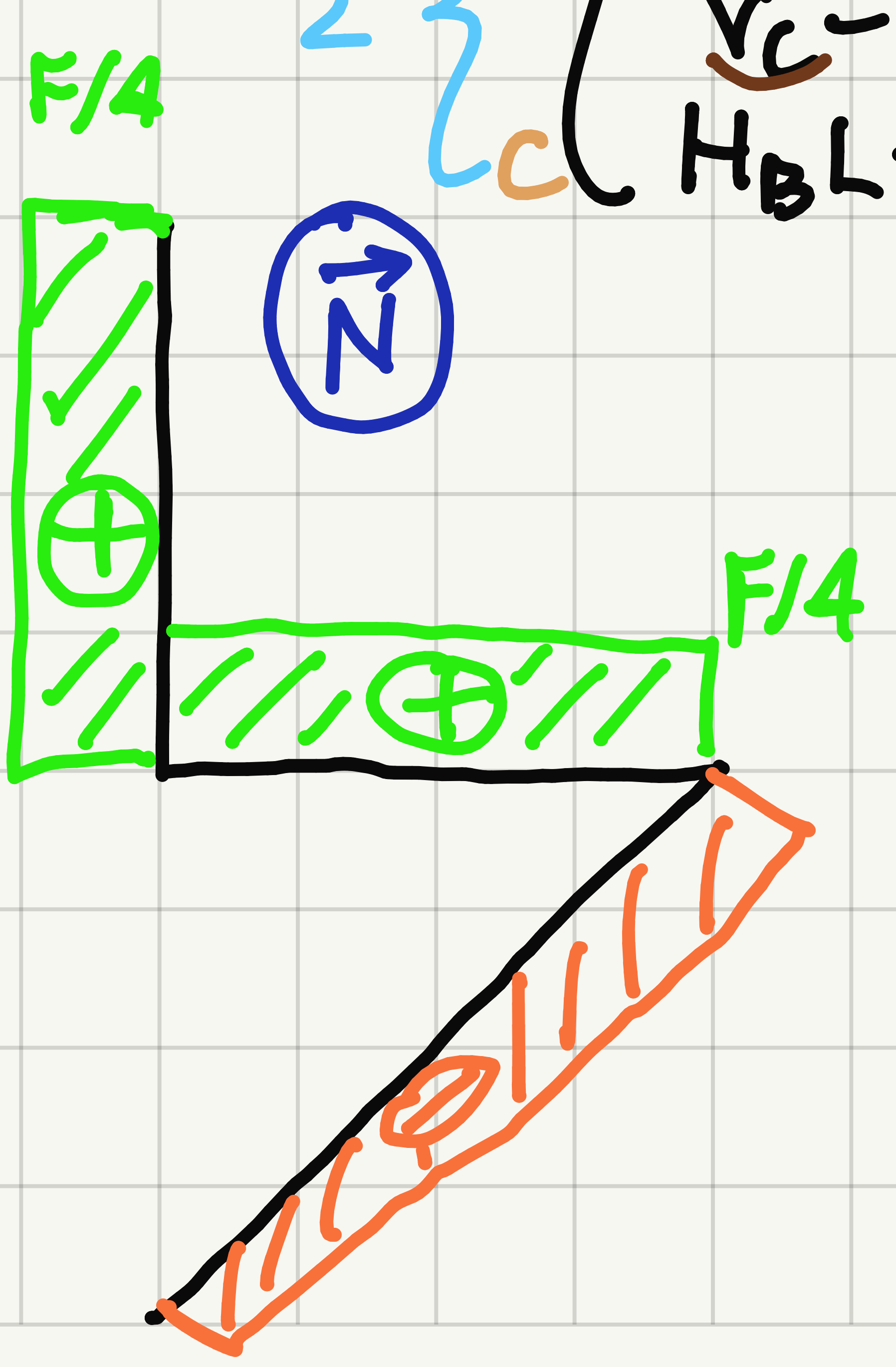
4 equazioni, 3 incognite ⇒ spezzo i due corpi

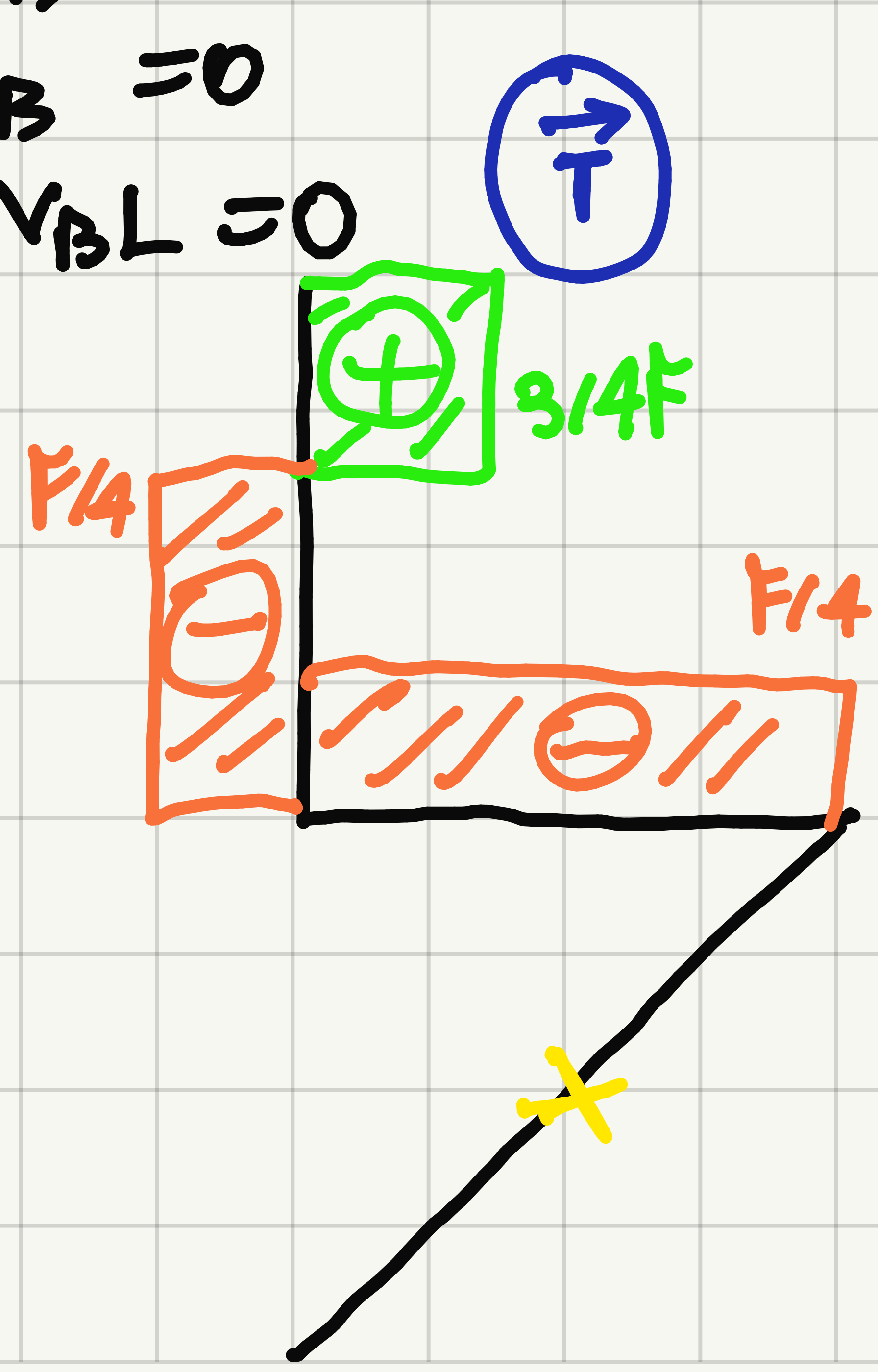


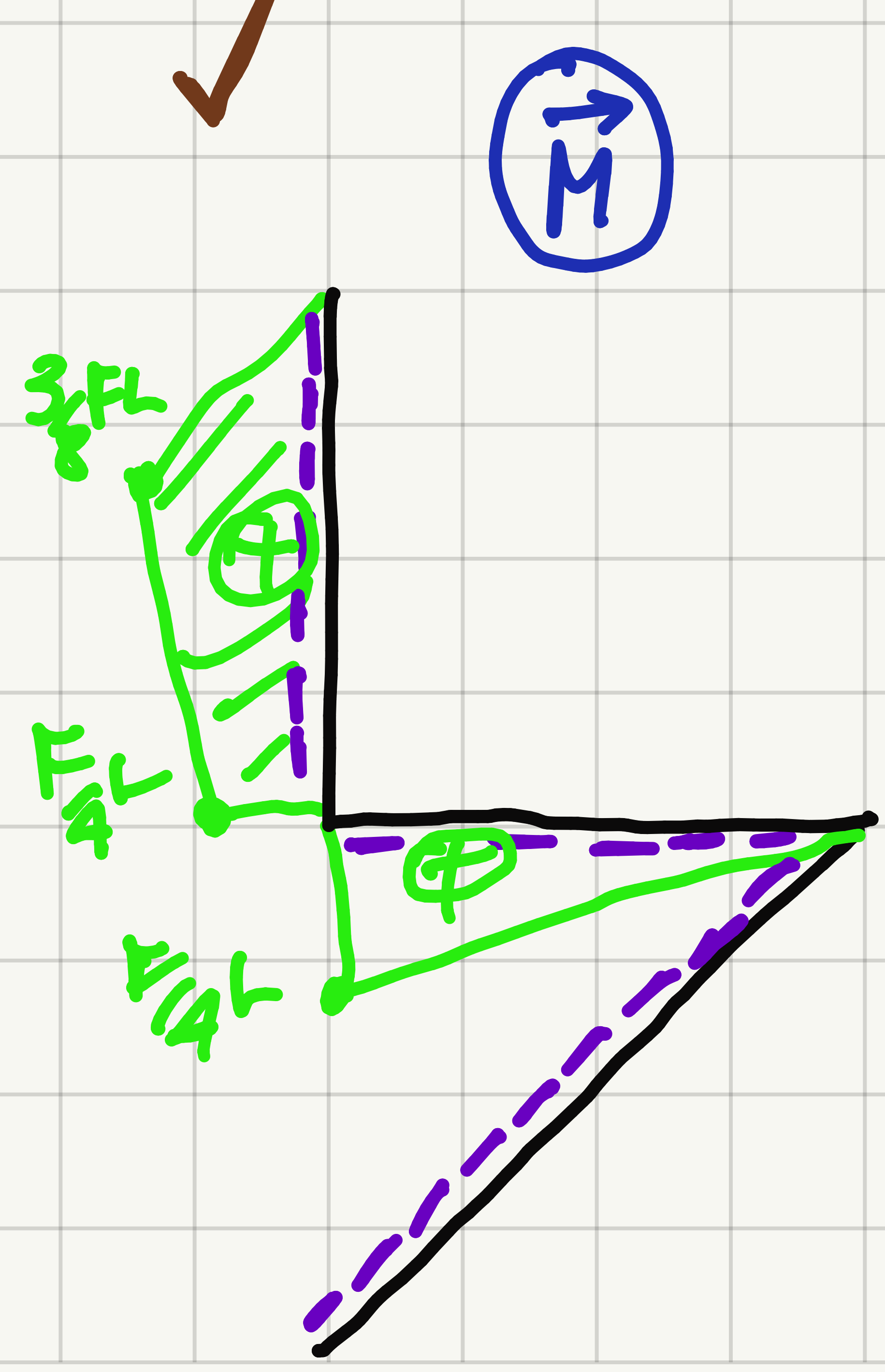


$$
\begin{cases}
H_A + H_B - F = 0 \\
V_A + V_B = 0 \\
F\,L/2 - H_A L - V_A L = 0 \quad (B) \\
H_C - H_B = 0 \\
V_C - V_B = 0 \\
H_B L - V_B L = 0 \quad (C)
\end{cases}
\qquad \begin{matrix} 1 \\ 2 \end{matrix}
$$

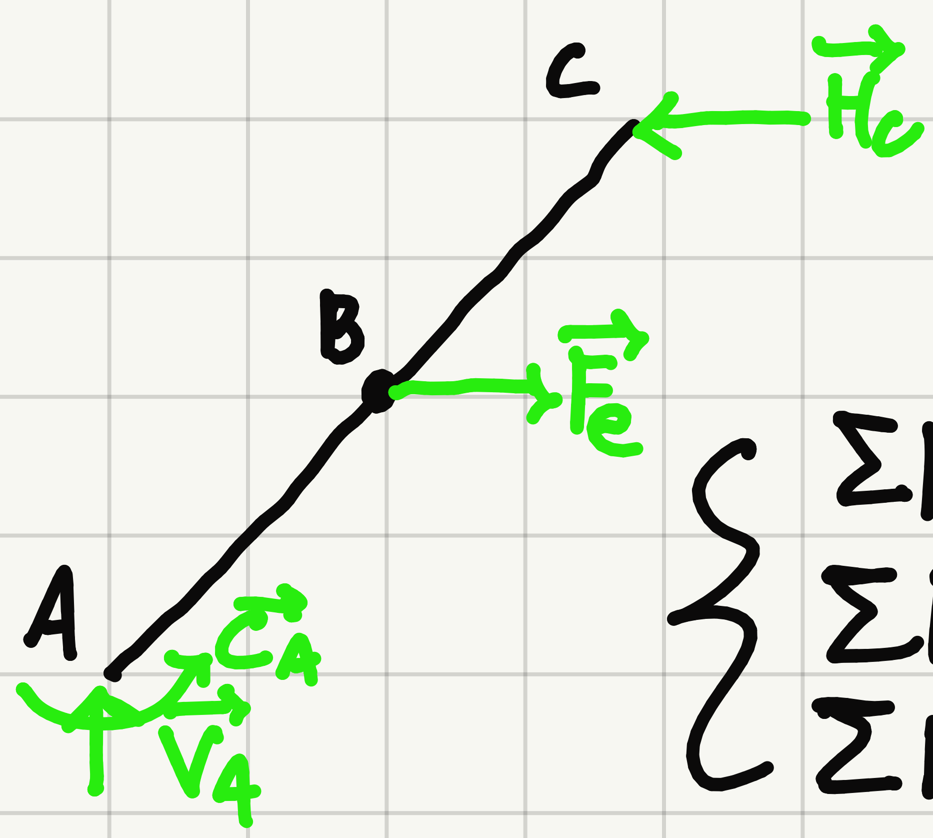
6 equazioni, 6 incognite ✓







④ CONSIDERIAMO UN'ASTA AC VINCOLATA CON UN CARRELLO SULLA VERTICALE ED UN PATTINO SULL'ORIZZONTALE. L'ASTA HA INCLINAZIONE $\theta = 60°$ E, IN B, VI È UNA MOLLA SALDATA ALLA PARETE. NOTI $AB = BC = L$ ($\Rightarrow AC = 2L$), LA COSTANTE DI ELASTICITÀ K E LA LUNGHEZZA INDEFORMATA DELLA MOLLA $\ell_0 = 0$, CALCOLARE LE REAZIONI VINCOLARI E LE AZIONI INTERNE



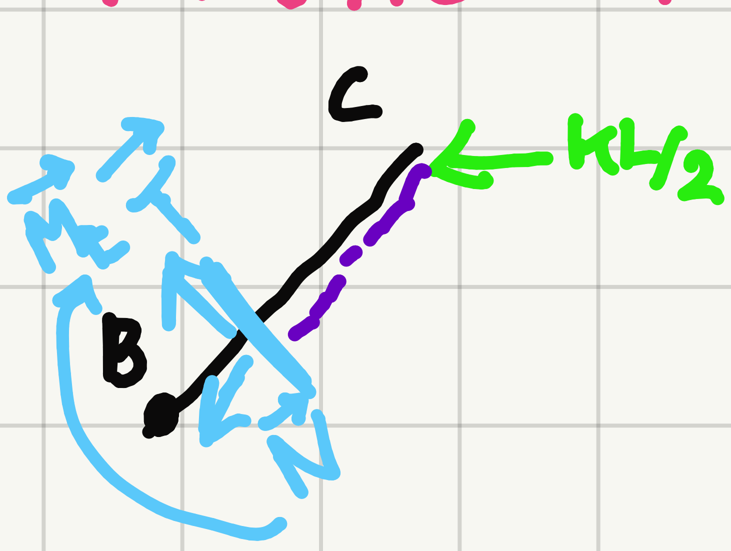
$n_0 = 3 \qquad n_v = 2_A + 1_C = 3 \Rightarrow n = 0$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma \vec{M}_A = 0 \end{cases} \qquad \begin{cases} F_e - H_c = 0 \\ V_A = 0 \\ C_A - F_e L \sin\theta + H_c 2L \sin\theta = 0 \end{cases}$$

$$\begin{cases} H_c = K L/2 \\ V_A = 0 \\ C_A = k L^2/2 \sin\theta - K L^2 \sin\theta = -K L^2/2 \cdot \sqrt{3}/2 = -\dfrac{K L^2 \sqrt{3}}{4} \end{cases} \qquad F_e = K(\ell - \ell_0) = K\ell$$

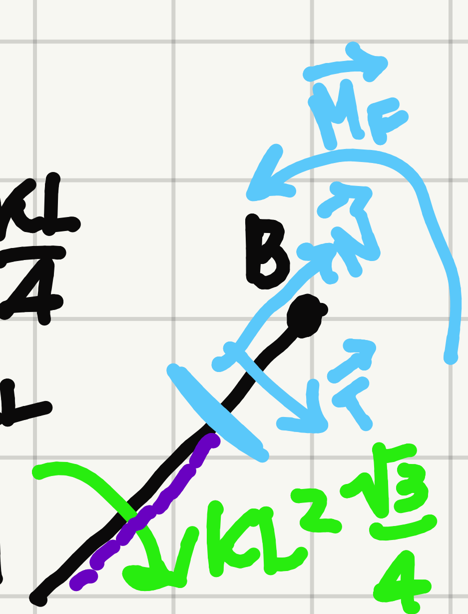
✓

STUDIO AZIONI INTERNE

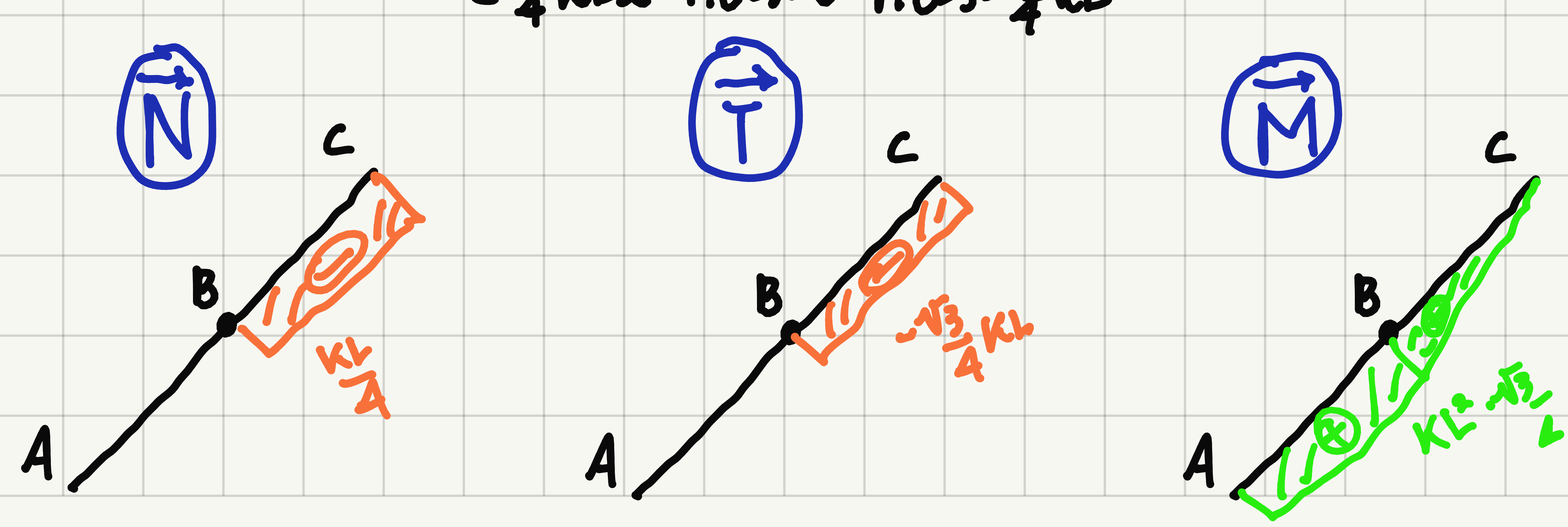


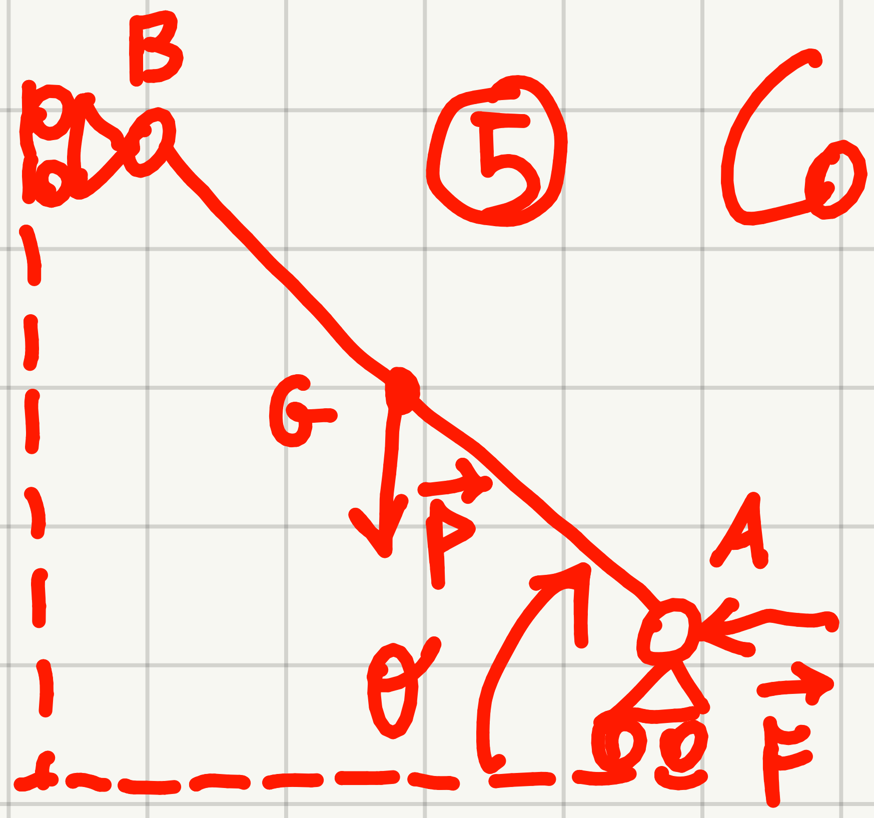
$$\begin{cases} N = -\dfrac{KL}{2} \cos\theta = -\dfrac{KL}{4} \\ T = -\dfrac{KL}{2} \sin\theta = -\dfrac{\sqrt{3}}{4} KL \\ M_F = \dfrac{KL}{2} \sin\theta \cdot x \end{cases}$$

$= \frac{\sqrt{3}}{4} KLx \quad M(0) = 0 \quad M(L) = \frac{\sqrt{3}}{4} KL^2$



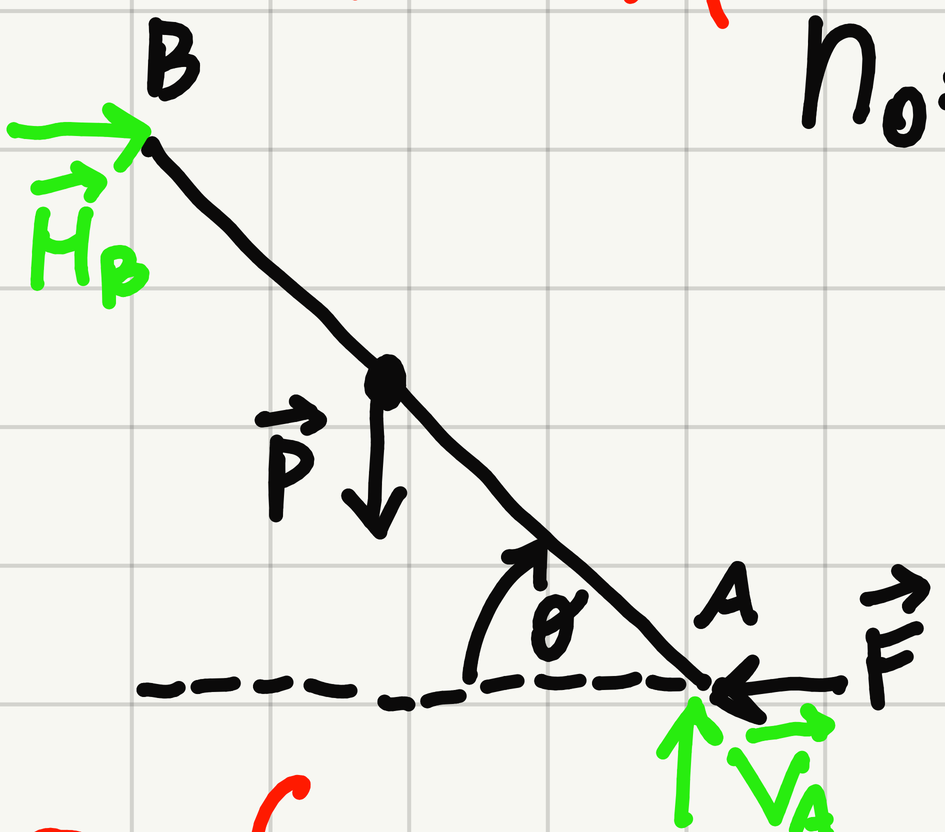
$$\begin{cases} N = 0 \\ T = 0 \\ M_F = KL^2 \dfrac{\sqrt{3}}{4} \end{cases}$$





⑤ CONSIDERARE UNA SCALA CHE SCORRE SUL PIANO VERTICALE.

AL CENTRO DELL'ASTA VI È IL BARICENTRO IN CUI È APPLICATA LA FORZA PESO. NOTA LA LUNGHEZZA DELL'ASTA $\overline{AB}=\ell$, LA MASSA m, L'INERZIA J DEL CORPO E L'ANGOLO DI EQUILIBRIO θ, DETERMINARE IL VALORE DELLA FORZA $\vec{F}$ CHE MANTENGA IL SISTEMA IN EQUILIBRIO STATICO

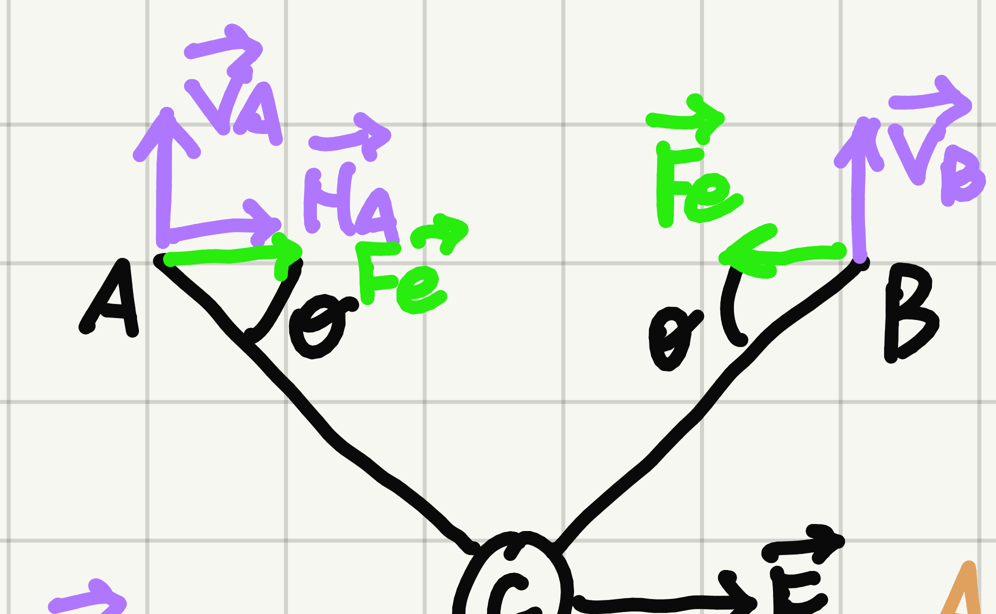


$n_0=3 \quad n_v = 1_B + 1_A + 1_F = 3 \Rightarrow n=0$

$$\begin{array}{l} \sum \vec{F}_x = 0 \\ \sum \vec{F}_y = 0 \\ \sum \vec{M}_A = 0 \end{array} \quad \begin{cases} H_B - F = 0 \rightarrow F = H_B \\ V_A - P = 0 \\ P\frac{\cancel{\ell}}{2}\cos\theta - H_B \cancel{\ell} \sin\theta = 0 \end{cases} \rightarrow F = \frac{P}{2}\cot\theta$$

⑥ SI CALCOLINO I GRADI DI LIBERTÀ E LE REAZIONI VINCOLARI DEL SEGUENTE SISTEMA COMPOSTO DA DUE ASTE DI LUNGHEZZA L E DALLA MOLLA DI LUNGHEZZA INDEFORMATA $\ell_0 = L/2$ E COSTANTE ELASTICA K E SI DISEGNINO I DIAGRAMMI DELLE AZIONI INTERNE

$n_0=6 \quad n_v = 2_A + 1_B + 2_C = 6 \Rightarrow n=1$ ACB MANOVELLISMO ORDINARIO CENTRATO + MOLLA IN AB CHE BLOCCA IL GRADO DI LIBERTÀ



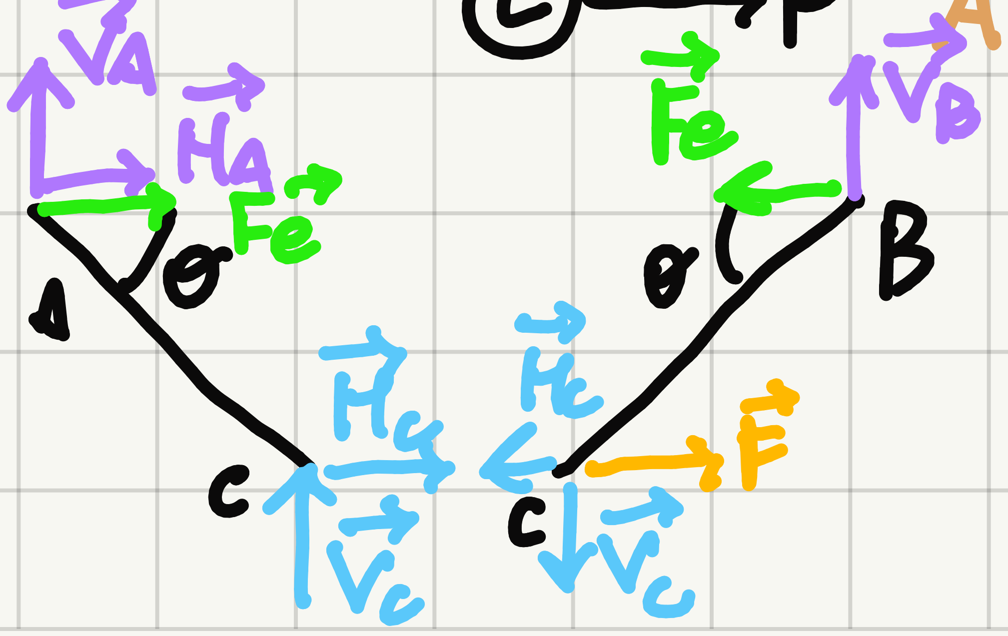
$F_e = K(\ell - \ell_0) = K(2\ell\cos\theta - \frac{L}{2})$

$$\begin{cases} H_A + F = 0 \\ V_A + V_B = 0 \\ F L \sin\theta + 2 V_B L \cos\theta = 0 \end{cases}$$

$H_A = -F$

$V_B = -\frac{F}{2}\tan\theta$

$V_A = -V_B$



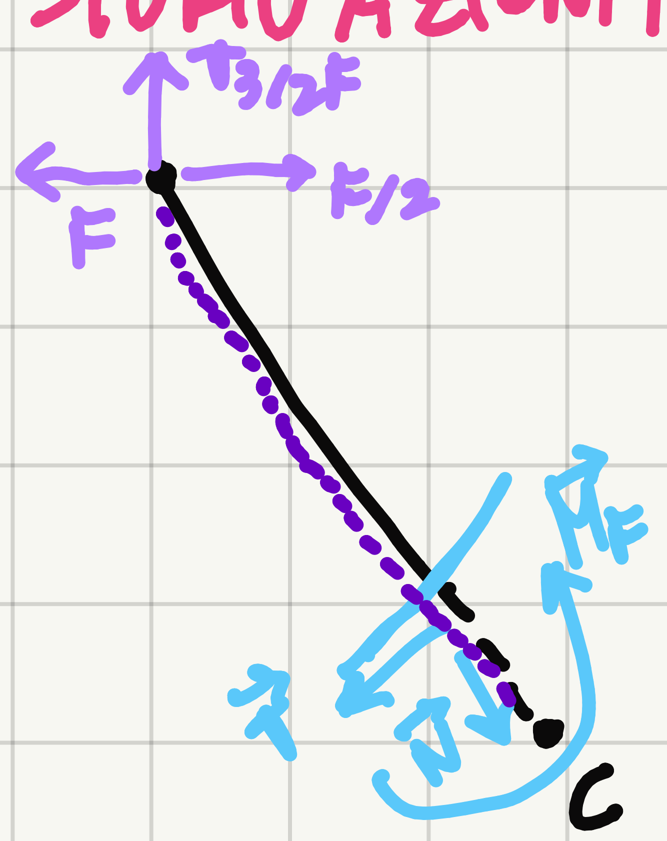
! $\vec{F}$ VA MESSA SOLO IN UNA DELLE DUE ASTE. SCELTA ARBITRARIA

$$
\begin{cases}
\begin{cases}
\end{cases} \\
\begin{cases}
-H_c + F - F_e = 0 \\
V_B - V_c = 0 \\
L F_e \sin\theta + L V_B \cos\theta = 0 \qquad K(2L\cos\theta - L/2)\cancel{\sin\theta} - \frac{F}{2}[\cancel{\cos\theta}\ \cancel{\cos\theta}] = 0
\end{cases}
\end{cases}
$$

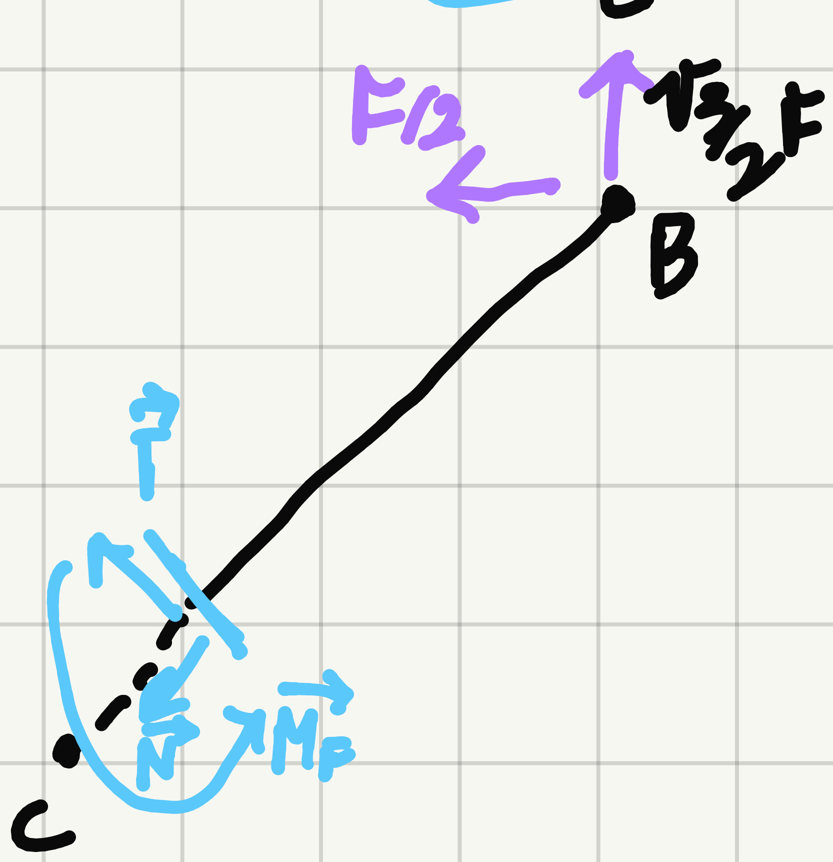
$\sin\theta$

$$\theta = \arccos\left(\frac{-KL + 2KL/2}{4KL}\right) = 60^\circ \Rightarrow V_B = -\frac{\sqrt{3}}{2}F \quad V_A = \frac{\sqrt{3}}{2}F \quad F_e = F/2$$

STUDIO AZIONI INTERNE

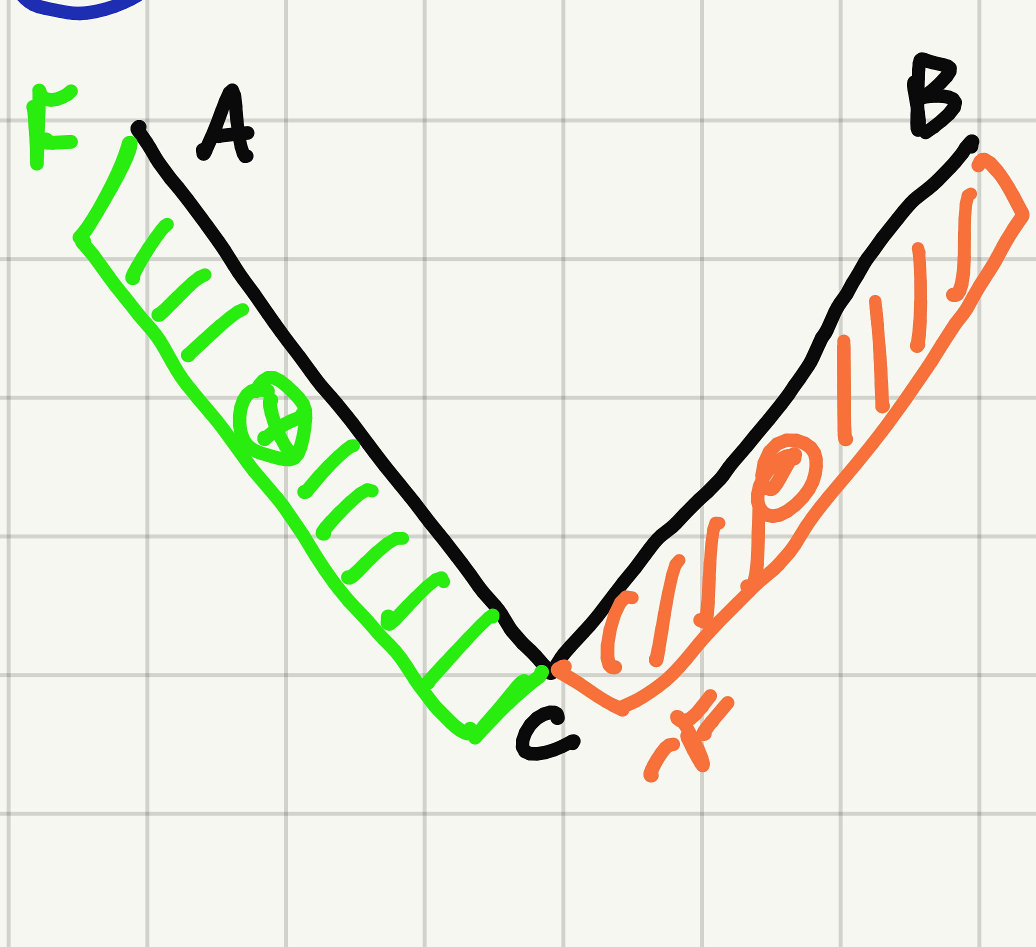


$$
\begin{cases}
N + (F/2 - F)\cos(\theta) + \frac{\sqrt{3}}{2}F\sin\theta = 0 \Rightarrow N = F \\
T = 0 \\
M_F - T x = 0 \quad \text{perché } T = 0,\ M_F = 0
\end{cases}
$$

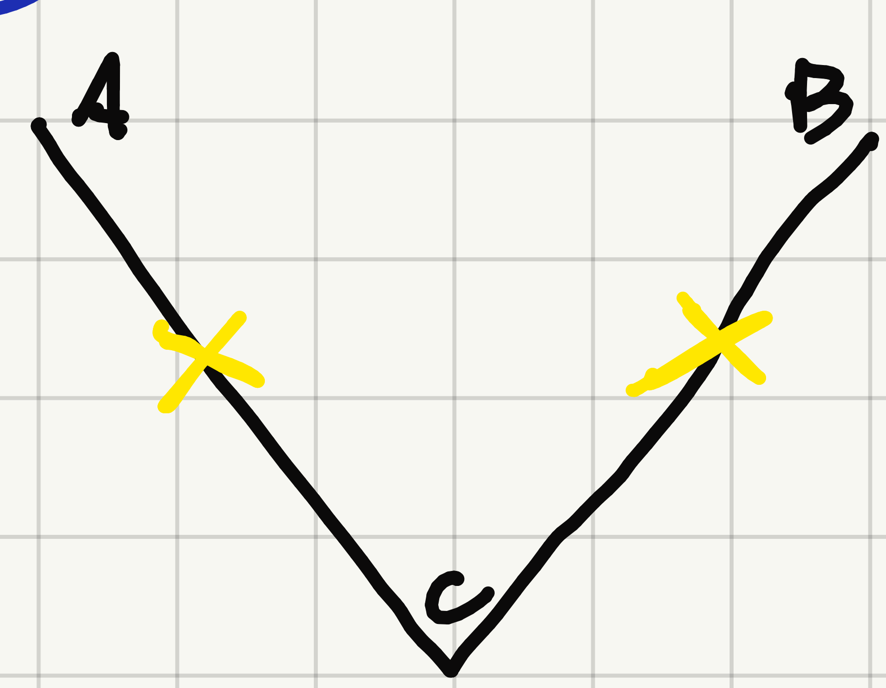


$$
\begin{cases}
N + F/2\cos\theta + \sqrt{3}/2\,F\sin\theta = 0 \rightarrow N = -F \\
T + F/2\sin\theta - \sqrt{3}/2\,F\cos\theta = 0 \rightarrow T = 0 \\
-M_F + T x = 0 \Rightarrow M_F = 0
\end{cases}
$$

$\vec{N}$



$\vec{T}$



$\vec{M}$