3^{rd} TECHNIQUE (NOT ALWAYS WORKING): HORN RESOLUTION

"IT DOESN'T COME FROM HORNY, HORN WAS A MATHEMATICIAN" - LUCA MAURI, 29/11/2023

$$CCF) = \{\{R(x, f(x))\}, \{\neg Rxy, Ryx\}, \{\neg Rxy, \neg Ryz, Rxz\}, \{\neg Raa\}\}$$

A CLAUSE IS OF **HORN TYPE** IF IT HAS **AT MOST ONE POSITIVE LITERAL**

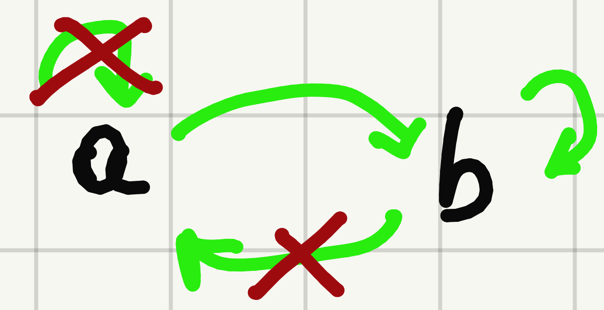
IF WE KNOW THE SET HAS A CLOSED EXPANSION, WE CAN CHOOSE ALGORITHMICALLY ONE OF THE NEGATIVE LITERAL. WE'LL ALWAYS USE THE MOST LEFT LITERAL

$$\dfrac{\dfrac{\dfrac{\dfrac{\{\neg Raa\}, \{\neg Rxy, \neg Ryz, Rxz\}}{\{\neg Ray, \neg Rya\}} \qquad \{R(x, fx)\}}{\{\neg R(fa, a)\}} \qquad \{\neg Rxy, Ryx\}}{\{\neg R(a, fa)\}} \qquad \{R(x, fx)\}}{\emptyset}$$

Use resolution to prove the following facts:

a) Every reflexive, Euclidean binary relation is an equivalence relation

- Reflexive $\forall x Rx$
- Euclidean $\forall xyz(Rxy \land Rxz \to Ryz)$
- Equivalence relation:



- Reflexive ✓ (in premises)
- Symmetric $\forall xy(Rxy \to Ryx)$
- Transitive $\forall xyz(Rxy \land Ryz \to Rxz)$

$\{\forall x(Rxx), \forall xyz(Rxy \land Rxz \to Ryz) \vdash \forall xy(Rxy \to Ryx), \forall xyz(Rxy \land Ryz \to Rxz)\}$

We first verify the first formula in the conclusions, that is

$\forall x(Rxx), \forall xyz(Rxy \land Rxz \to Ryz) \vdash \forall xy(Rxy \to Ryx)$

$\{\forall x(Rxx), \forall xyz(Rxy \land Rxz \to Ryz), \neg\forall xy(Rxy \to Ryx)\}$

Unification

- $\{\forall x(Rxx)\} \mapsto \{Rxx\}$
- $\{\forall xyz(Rxy \land Rxz \to Ryz)\} \mapsto \{\neg Rxy, \neg Rxz, Ryz\}$
- $\{\neg\forall xy(Rxy \to Ryx)\} \mapsto \{\exists xy.\neg(Rxy \to Ryx)\} \mapsto \{Rab\}, \{\neg Rba\}$

$C(F) = \{\{Rxx\}, \{\neg Rxy, \neg Rxz, Ryz\}, \{Rab\}, \{\neg Rba\}\}$

$$\dfrac{\{Rxx\} \qquad \{\neg Rxy, \neg Rxz, Ryz\}}{\{\neg Rxz, Rxz\}}\left[\tfrac{x}{y}\right] \qquad \left[\tfrac{x}{z}\right]\dfrac{\{Rxx\} \qquad \{\neg Rxy, \neg Rxz, Ryz\}}{\{\neg Rxy, Ryx\}}$$

$$\left[\tfrac{a}{x}, \tfrac{b}{z}\right]\dfrac{\{\neg Rxz, Rxz\} \qquad \{Rab\}}{\{Rab\}} \qquad \left[\tfrac{b}{x}, \tfrac{a}{y}\right]\dfrac{\{\neg Rxy, Ryx\} \qquad \{\neg Rba\}}{\{\neg Rab\}}$$

$$\dfrac{\{Rab\} \qquad \{\neg Rab\}}{\emptyset}$$

NOW WE CAN CONSIDER IT WITH THE SET OF PREMISES AND PROVE

$\{\forall x Rxx, \forall xyz(Rxy \land Rxz \to Ryz), \forall xy(Rxy \to Ryx), \neg \forall xyz(Rxy \land Ryz \to Rxz)\}$

HERBRAND

STEP	FORMULA	REASON
1	$\{\forall x Rxx\}$	ASSUMPTION
2	$\{\forall xyz(Rxy \land Rxz \to Ryz)\}$	ASSUMPTION
3	$\{\forall xy(Rxy \to Ryx)\}$	ASSUMPTION
4	$\{\neg \forall xyz(Rxy \land Ryz \to Rxz)\}$	ASSUMPTION
5	$\{Raa\}$	1, γ-EXPANSION
6	$\{\forall yz(Ray \land Raz \to Ryz)\}$	2, γ-EXPANSION
7	$\{\forall z(Rab \land Raz \to Rbz)\}$	6, γ-EXPANSION
8	$\{Rab \land Raa \to Rba\}$	7, γ-EXPANSION
9	$\{\neg(Rab \land Raa), Rba\}$	8, β-EXPANSION
10	$\{\neg Rab, \neg Raa, Rba\}$	9, β-EXPANSION