

$$M = \left| \begin{array}{ccccccccc} 1 & 0 & 0 & 0 & 1 & | & 0 & 0 & 10001 \\ 0 & 1 & 1 & 0 & 0 & | & 0 & 0 & 00001 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 00000 \end{array} \right|.$$

$$R = \left| \begin{array}{ccccccccc} 0 & 0 & 1 & 0 & 0 & | & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & | & 0 & 1 & 0 \end{array} \right| \quad M^T = \left| \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \end{array} \right|.$$

$$S = MRM^{-1} = \left| \begin{array}{ccccccccc} 0 & 0 & 1 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & | & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & | & 1 & 1 & 1 \end{array} \right|.$$

$$\cdot \left| \begin{array}{ccccccccc} 0 & 0 & 1 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & | & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & | & 1 & 1 & 1 \end{array} \right| = \left| \begin{array}{ccccccccc} 1 & 0 & 1 & 0 & 1 & | & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & | & 1 & 1 & 1 \end{array} \right|.$$

$$\cdot \left| \begin{array}{ccccccccc} 0 & 0 & 1 & 0 & 0 & | & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & | & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & | & 1 & 0 & 1 \end{array} \right| = \left| \begin{array}{ccccccccc} 1 & 0 & 1 & 0 & 1 & | & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & | & 1 & 1 & 1 \end{array} \right|$$

GIVEN THE SETS $A = \{1, 2, 3, 4, 5\}$ AND $B = \{1, 2, 3\}$, CONSIDER THE FUNCTION $f: A \rightarrow B$ REPRESENTED BY THE MATRIX M_f BELOW. COMPUTE THE CANONICAL FACTORIZATION $f = pm$ OF f . PROVE THAT THE INVERSE IMAGE S OF THE ORDER RELATION $R: B \rightarrow B$ REPRESENTED BY THE MATRIX M_R BELOW ALONG f IS NOT CONTAINED IN ANY ORDER RELATION. DETERMINE THE ORDER RELATION INDUCED BY S AND SHOW THAT IT IS THE SAME OF

THE INVERSE IMAGE T OF R ALONG m

$$M_f = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}; M_R = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$K = f f^{op} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{vmatrix} P = \begin{vmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{vmatrix}$$

$$P = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{vmatrix}$$

$$m = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$S = P R P^{-1} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

1 1 0 0 0 | : 1 1 1
0 0 0 0 0 | ; 0 0 1
0 0 1 1 1 | 0 0 1 .

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} =$$

1	1	1	1	1
1	2	1	2	1
0	0	1	1	1
0	0	1	2	1
0	0	1	1	1