$S \subseteq B \times C$

EXTRA: LET $A=\{1,2,3\}$; $B=\{a,c\}$; $C=\{\alpha,\beta,\gamma\}$; $R \subseteq A\times B$,

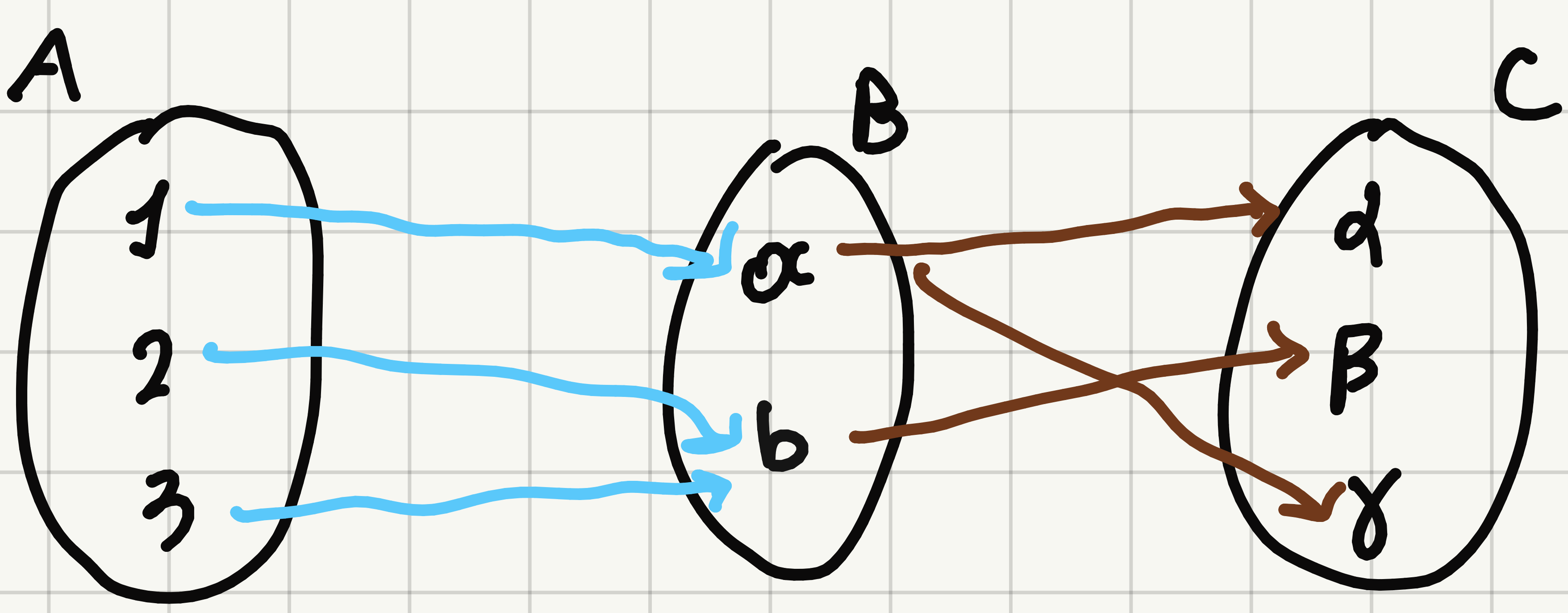
$R=\{(1,a),(2,b),(3,b)\}$ $\quad S=\{(a,\alpha),(a,\gamma),(b,\beta)\}$

1) $R \circ S \subseteq A \times C$ ?

2) $M_R, M_S, M_{R\circ S}$ ?

3) PROVE $M_{R\circ S} = M_R \cdot M_S$

1)




$\Rightarrow$ $R\circ S = \{(1,\alpha),(1,\gamma),(2,\beta),(3,\beta)\}$

2) $M_R = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{vmatrix}$ $\quad M_S = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$

$M_{R\circ S} = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix}$

3) $M_R \cdot M_S = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \doteq$

$= \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = M_{RS}$

EXTRA: LET $A = \{2,3,4\}$ AND $B = \{5,6,9\}$ AND $R \subseteq A \times B$

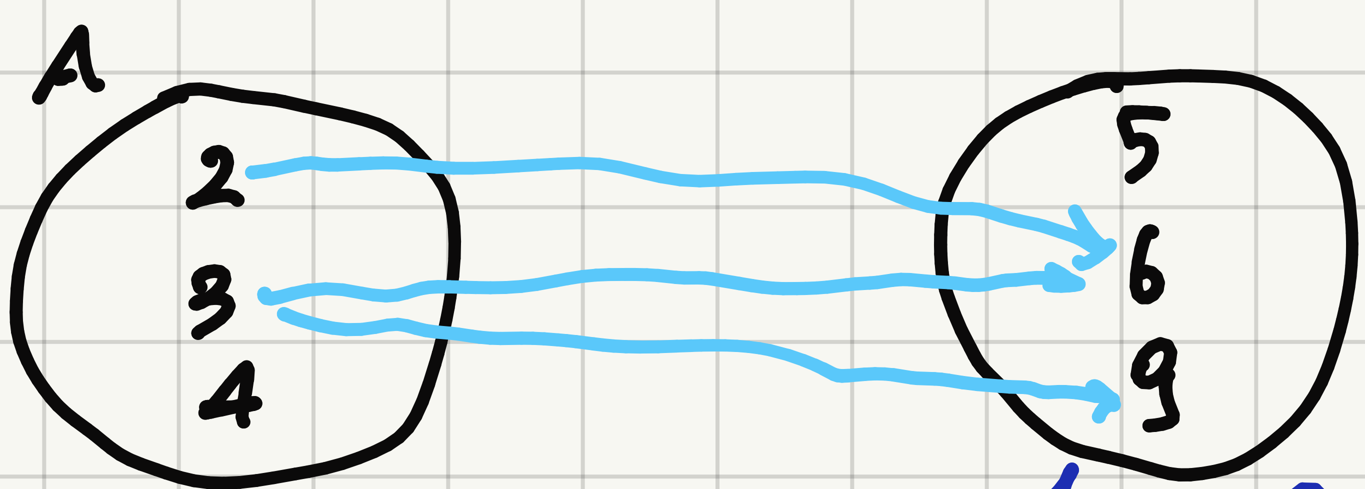
$(a,b) \in R \iff a$ DIVIDES $b$

1) $R$? REPORT GRAPH AND MATRIX

2) $R^{op} \subseteq B \times A$

3) $R \cdot R^{op}$

1) $R = \{(2,6),(3,6),(3,9)\}$




$$M_R = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

2) $R^{op} = \{(6,2),(6,3),(9,3)\}$

$$M_{R^{op}} = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

3) $$M_R \cdot M_{R^{op}} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$\Longrightarrow R \cdot R^{op} = \{(2,\overset{2}{5}),(2,\overset{3}{6}),(3,\overset{2}{5}),(3,\overset{3}{6})\}$

EXTRA: LET $A = \{a, b, c\}$; $B = \{x, y\}$; $R, S \subseteq A \times B$

$R = \{(a,x), (b,x), (a,y)\}$ $S = \{(b,x), (c,x)\}$

1) $M_R, M_S$?

2) $R \cup S$? $M_{R \cup S}$?

3) $R \cap S$? $M_{R \cap S}$?

1) $M_R = \begin{vmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{vmatrix}$ $M_S = \begin{vmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{vmatrix}$

2) $R \cup S = \{(a,x), (b,x), (c,x), (a,y)\}$

$M_{R \cup S} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{vmatrix}$

3) $R \cap S = \{(b,x)\}$ $M_{R \cap S} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{vmatrix}$