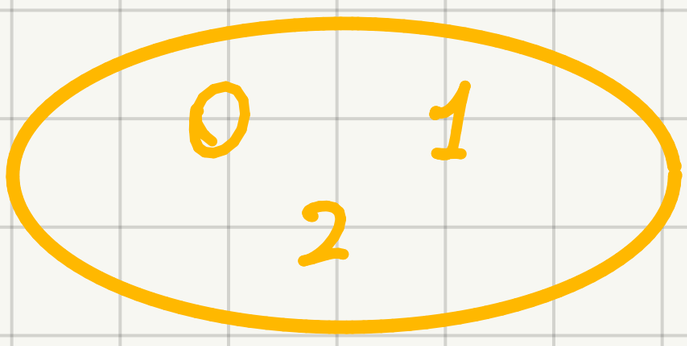
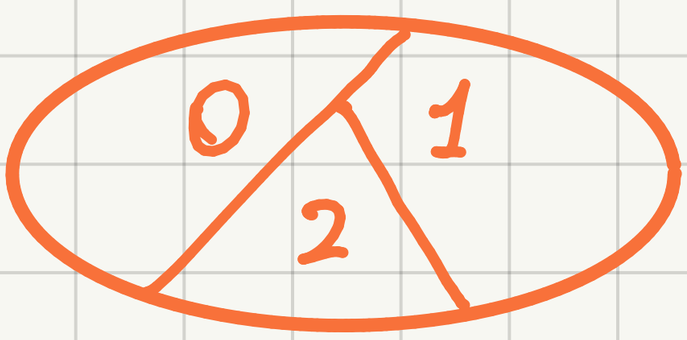
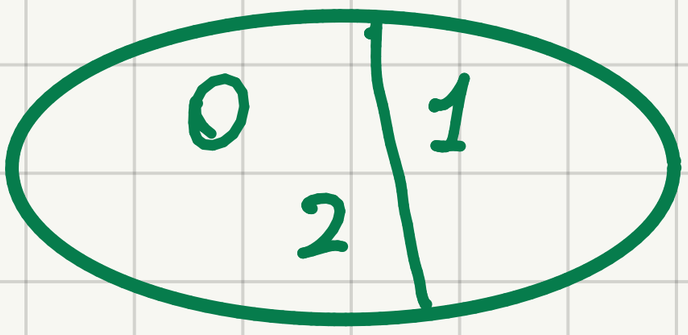
DETERMINE ALL EQUIVALENCE RELATIONS ON A SET WITH THREE ELEMENTS

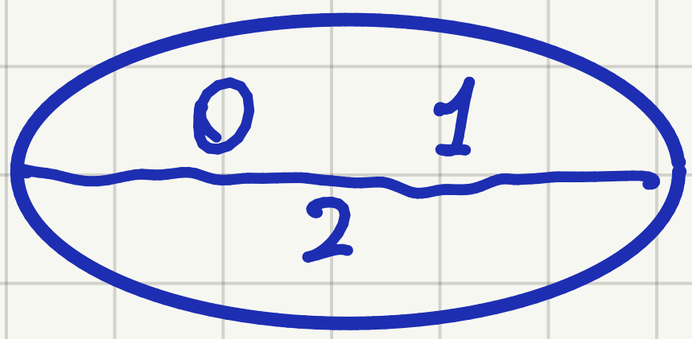
1ST) DRAW THE GRAPH

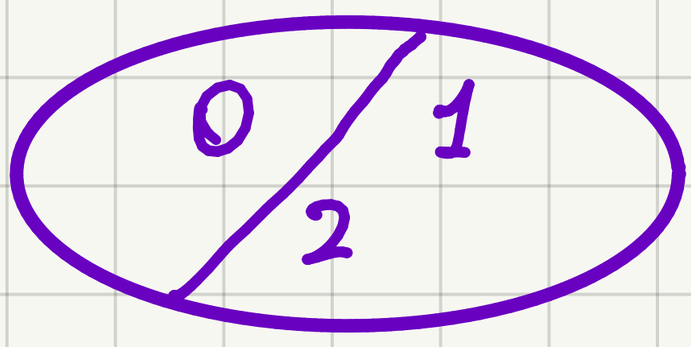


2ND) PARTITIONINGS









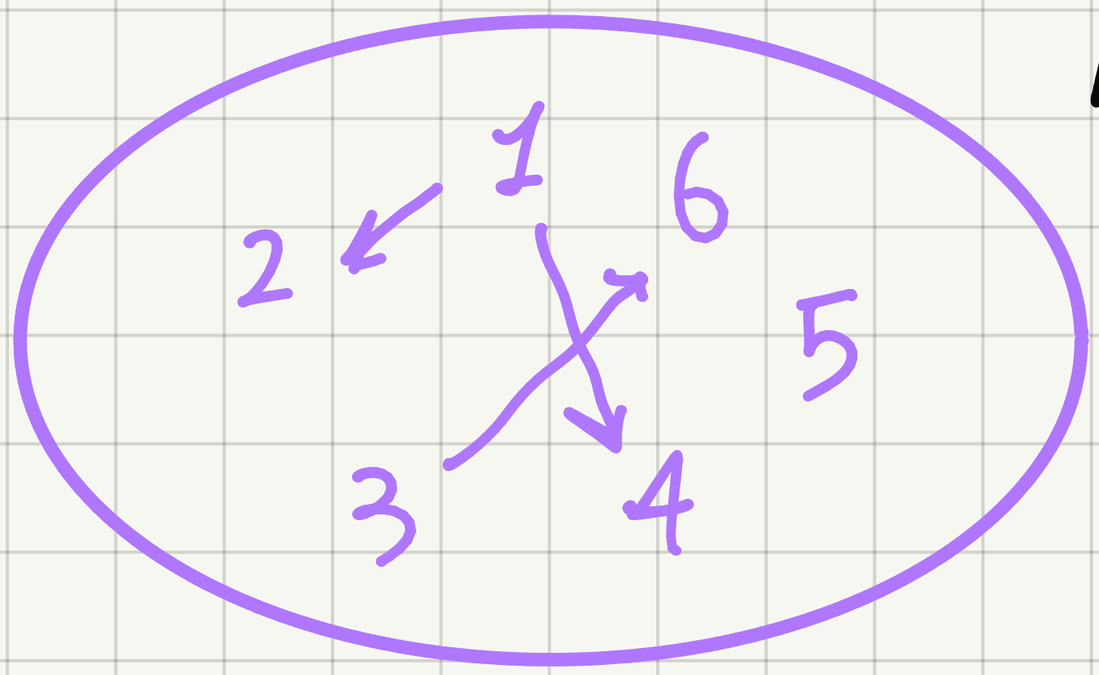
$$M(E_1)=\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad M(E_2)=\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$M(E_3)=\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad M(E_4)=\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$M(E_5)=T=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

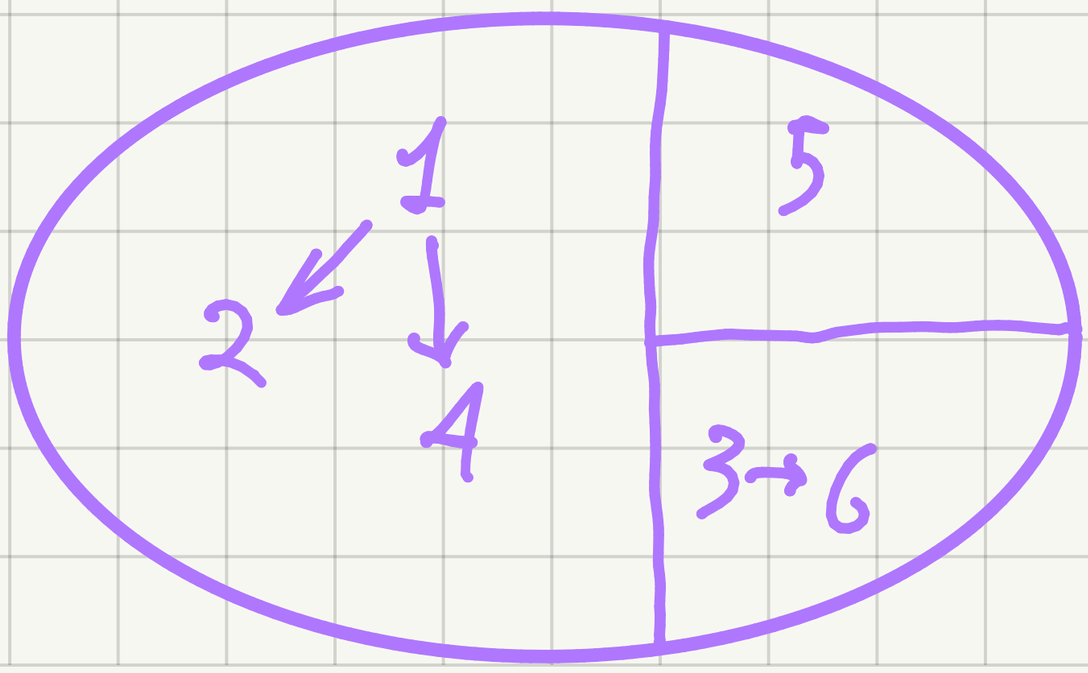
$$M(R) = \left|\begin{array}{cccccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right| \quad \left|\begin{array}{l} A = \{1,2,3,4,5,6\} \\ \text{FIND ALL} \\ \text{EQUIVALENCE} \\ \text{RELATIONSE} \\ \text{CONTAINING} \\ R: A \to A \end{array}\right.$$

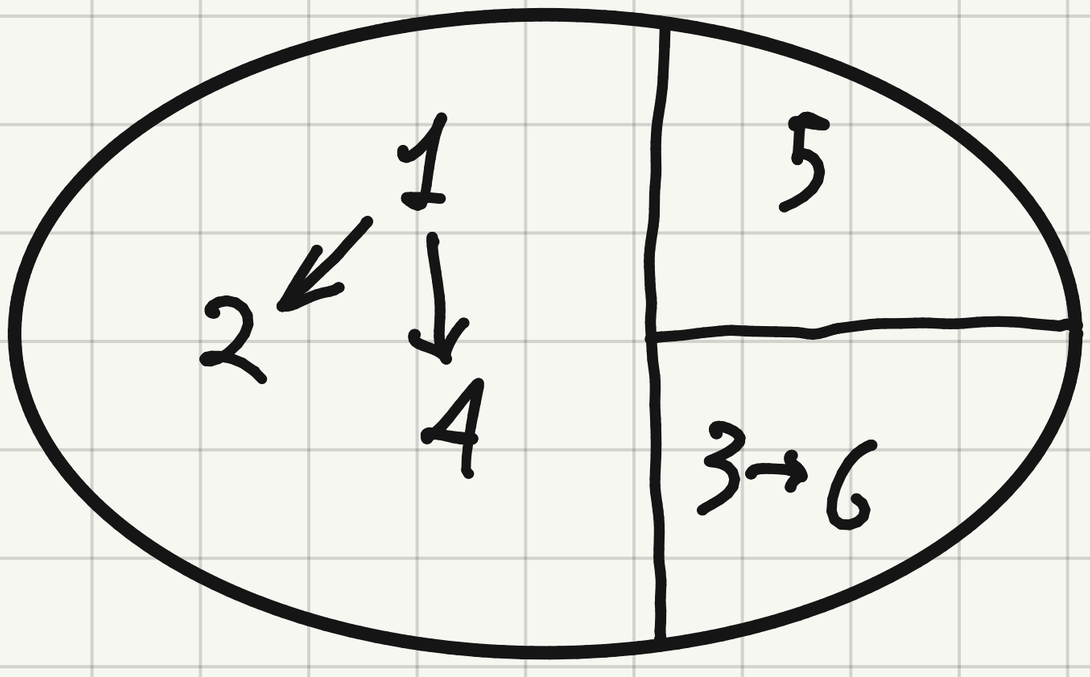
FIRST, WE DRAW THE GRAPH



AS WE CAN SEE, R IS NOT AN EQUIVALENCE RELATION

WE CAN FIND THE EQUIVALENCE CLOSURE OF R AS $R^t = \bigcup_{i \in I} S^i$, WHERE $S = R \cup I \cup R^{op}$. FOR A 6×6, CLEARLY IT IS NOT A GOOD IDEA. AN EASIER SOLUTION COMES BY LOOKING AT THE GRAPH AND ELEMENTS THAT ARE CONNECTED REGARDLESS FROM THE DIRECTION





$$M(E_1) = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

$[1] = [2] = [4]$

$= \{1, 2, 4\}$

$[3] = [6] = \{3, 6\}$

OTHER EQUIVALENCE RELATION ARE OBTAINED BY APPLYING PARTITIONING ASSOCIATED WITH EQUIVALENCE