USING SEMANTIC TREE

$(x \land y \to z) \to ((x \to z) \land (y \to z))$

β-EXPANSION

$\neg(x \land y \to z)$ $\quad$ $(x \to z) \land (y \to z)$

α-FORMULA

$x \to z, \; y \to z$

2 FORMULAS TO BE SATISFIED SIMULTANEOUSLY

β-FORMULA

$\neg x, y \to z$ $\quad$ $z, y \to z$

β-FORMULA

$\neg x, \neg y$ $\quad$ $\neg x, z$

THE EVALUATION $v(x) = v(y) = 0, v(z)$ SATISFIES THE LEFT-MOST AND SO THE NODE φ. HENCE, φ IS SATISFIABLE.

$\neg((x \land y \to z) \to ((x \to z) \land (y \to z)))$

α-EXPANSION

$x \land y \to z, \; \neg((x \to z) \land (y \to z))$

β-EXPANSION

$x \land y \to z, \neg(x \to z)$ $\quad$ $x \land y \to z, \neg(y \to z)$

α-EXPANSION

$x \land y \to z, x, \neg z$

β-FORMULA

$\neg(x \land y), x, \neg z$ $\quad$ $z, x, \neg z$ UNSATISFIABLE

β-FORMULA

$\neg x, x, \neg z$ $\quad$ $\neg y, x, \neg z$ → $\neg\varphi$ SATISFIABLE FOR $v(x) = 1, v(y) = v(z) = 0$

=> ∃ VALUATION SUCH THAT φ IS SATISFIABLE BUT NOT VALID

PROVE THE FOLLOWING EQUIVALENCES

a) $x \vee (y \to z) \equiv x \vee y \to x \vee z$

1^{st} WAY: USING TRUTH TABLE

x	y	z	$y \to z$	φ_1	$x \vee y$	$x \vee z$	φ_2
0	0	0	1	1	0	0	1
0	0	1	1	1	0	1	1
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

$\varphi_1 \equiv \varphi_2$

2^{ND} WAY: BOOLEAN RULES

φ_1: $x \vee \overbrace{(y \to z)}^{\beta\text{-FORMULA}} \equiv x \vee (\neg y \vee z) \equiv x \vee \neg y \vee z$

φ_2: $\overbrace{x \vee y \to x \vee z}^{\beta\text{-FORMULA}} \equiv \neg(x \vee y) \vee (x \vee z)$

$\equiv (\neg x \wedge \neg y) \vee (x \vee z) \overset{\text{DISTRIBUTIVITY}}{\equiv} (\overbrace{\neg x \vee x}^{T} \vee z) \wedge (\neg y \vee x \vee z)$

$\equiv T \wedge (\neg y \vee x \vee z) \equiv \neg y \vee x \vee z = x \vee \neg y \vee z$

$\varphi_1 \equiv \varphi_2$

b) $X \vee (Y \leftrightarrow Z) \equiv X \vee Y \leftrightarrow X \vee Z$

TRUTH TABLE

X	Y	Z	$Y \leftrightarrow Z$	φ_1	$X \vee Y$	$X \vee Z$	φ_2
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

$\varphi_1 \equiv \varphi_2$

ALGEBRA RULES

φ_1 : $X \vee (Y \leftrightarrow Z) \equiv$

$X \vee ((Y \rightarrow Z) \wedge (Z \rightarrow Y)) \equiv$

$(X \vee (Y \rightarrow Z)) \wedge (X \vee (Z \rightarrow Y)) \equiv$

$((X \vee Y) \rightarrow (X \vee Z)) \wedge ((X \vee Z) \rightarrow (X \vee Y)) \equiv$

$X \vee Y \leftrightarrow X \vee Z \equiv \varphi_2$