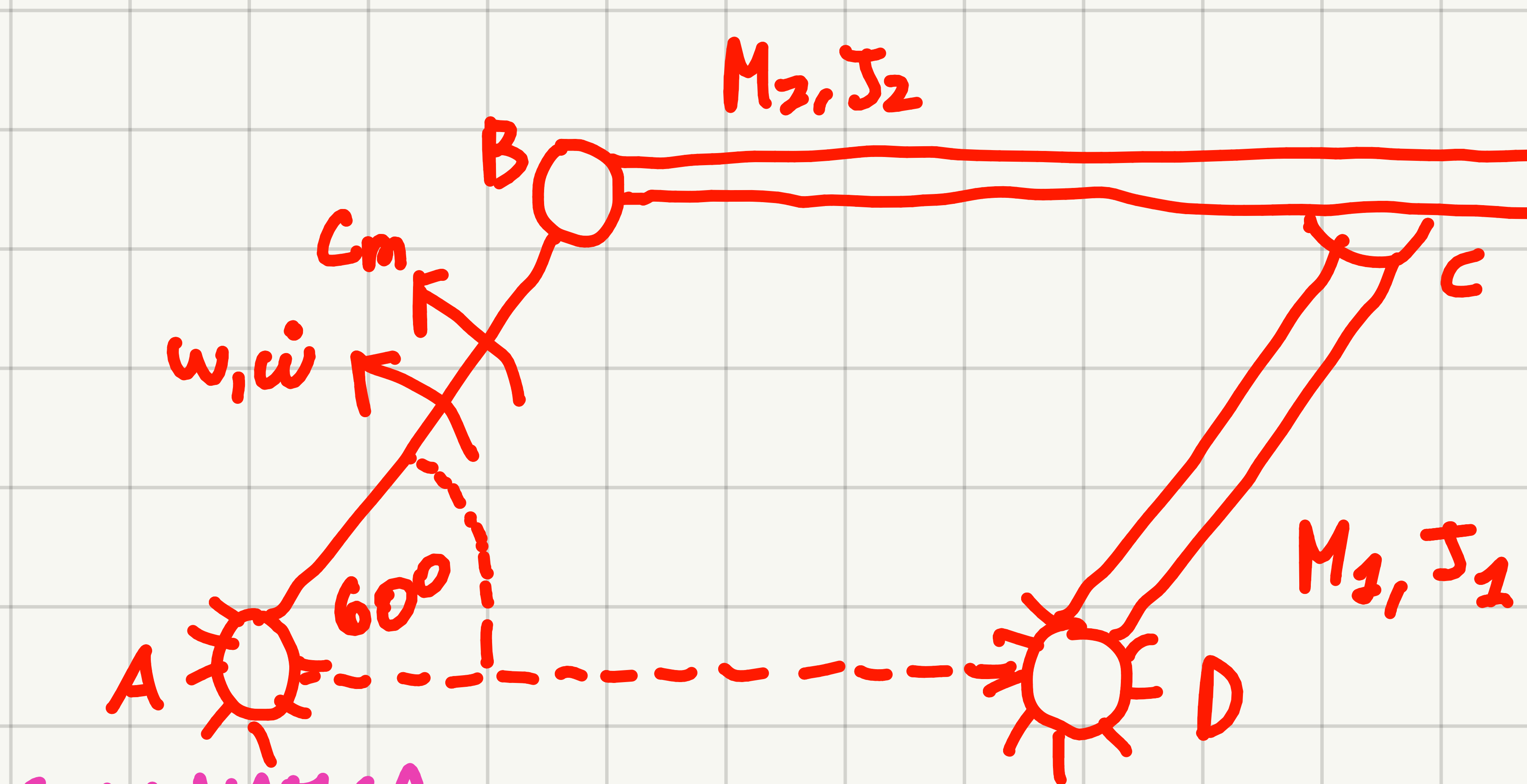


$J_2 = 1\,kg \cdot m^2$

$M_2 = 20\,kg$ $\quad J_1 = 2\,kg \cdot m^2$

$M_1 = 30\,kg$ $\quad BE = 1\,m$

$AB = CD = 0{,}5\,m$ $\quad AD = BC = 0{,}8\,m$

$F = 500\,N$ $\quad \omega = 5\,rad/s$

$\dot{\omega} = 0{,}5\,rad/s^2$

$\vec{v}_E, \vec{a}_E\,?$ $\quad C_m\,?$

CINEMATICA

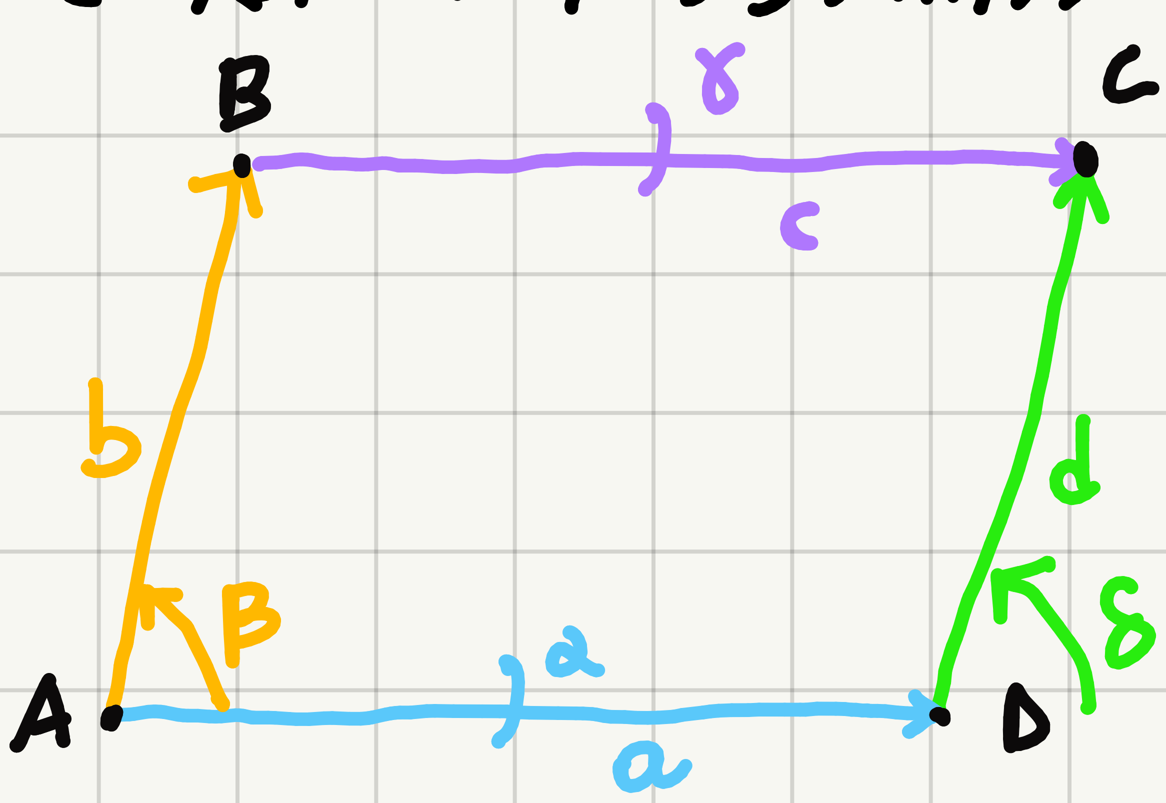
$$\vec{v}_E = \vec{v}_B + \vec{\omega}_{BE} \times (E-B)$$

$$\vec{v}_B = \cancel{\vec{v}_A} + \vec{\omega} \times (B-A) = 5\hat{k} \times (0{,}5\cos(60)\hat{i} + 0{,}5\sin(60)\hat{j}) =$$

$$= (-2{,}17\hat{i} + 1{,}25\hat{j})\,m/s \qquad (E-B) = 1\hat{i}$$

$$\hat{k} \times \hat{i} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \hat{j}$$

$$\hat{k} \times \hat{j} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\hat{i}$$



a, α FISSI

$b = 0{,}5\,m$ FISSO

$\beta = 60°$ $\quad \dot{\beta} = 5\,rad/s$ $\quad \ddot{\beta} = 0{,}5\,rad/s^2$

$c = 0{,}8\,m$ $\quad \gamma = 0°$ NON FISSO

$d = 0{,}5\,m$ FISSO $\quad \delta = 60°$

$\dot{\delta} = 5\,rad/s$ $\quad \ddot{\delta} = 0{,}5\,rad/s^2$

$$\begin{cases} a\cos\alpha + d\cos\delta = b\cos\beta + c\cos\gamma \\ a\sin\alpha + d\sin\delta = b\sin\beta + c\sin\gamma \end{cases}$$

$$\begin{cases} -d\dot{\delta}\sin\delta = -b\dot{\beta}\sin\beta - c\dot{\gamma}\sin\gamma \\ d\dot{\delta}\cos\delta = b\dot{\beta}\cos\beta + c\dot{\gamma}\cos\gamma \end{cases}$$

$$\begin{vmatrix} -b\sin\beta & -c\sin\gamma \\ b\cos\beta & c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \dot{\beta} \\ \dot{\gamma} \end{vmatrix} = \begin{vmatrix} -d\dot{\delta}\sin\delta \\ d\dot{\delta}\cos\delta \end{vmatrix}$$

$$\begin{vmatrix} -0{,}433 & 0 \\ 0{,}25 & 0{,}8 \end{vmatrix} \cdot \begin{vmatrix} \dot{\beta} \\ \dot{\gamma} \end{vmatrix} = \begin{vmatrix} -2{,}17 \\ 1{,}25 \end{vmatrix} \qquad \begin{cases} \dot{\beta} = 5\,rad/s \\ \dot{\gamma} = -0{,}0036\,rad/s \end{cases}$$

$$\vec{v}_E = (-2{,}17\hat{\imath} + 1{,}25\hat{\jmath}) - 0{,}0036\hat{\jmath} = (-2{,}17\hat{\imath} + 1{,}2464\hat{\jmath})\,m/s$$

$$|\vec{v}_E| = 2{,}5\,m/s$$

$$\vec{a}_E = \underbrace{\vec{a}_B + \vec{\dot{\omega}}_{BE} \times (E-B)}_{\tau} \underbrace{- \omega_{BE}^2 (E-B)}_{n}$$

$$\begin{cases} -d\ddot{\delta}\sin\delta - d\dot{\delta}^2\cos\delta = -b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta - c\ddot{\gamma}\sin\gamma - c\dot{\gamma}^2\cos\gamma \\ d\ddot{\delta}\cos\delta - d\dot{\delta}^2\sin\delta = b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta + c\ddot{\gamma}\cos\gamma - c\dot{\gamma}^2\sin\gamma \end{cases}$$

$$\begin{vmatrix} -b\sin\beta & -c\sin\gamma \\ b\cos\beta & c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \ddot{\beta} \\ \ddot{\gamma} \end{vmatrix} = \begin{vmatrix} -d\ddot{\delta}\sin\delta - d\dot{\delta}^2\cos\delta + b\dot{\beta}^2\cos\beta + c\dot{\gamma}^2\cos\gamma \\ d\ddot{\delta}\cos\delta - d\dot{\delta}^2\sin\delta + b\dot{\beta}^2\sin\beta + c\dot{\gamma}^2\cos\gamma \end{vmatrix}$$

$$\begin{vmatrix} -0{,}433 & 0 \\ 0{,}25 & 0{,}8 \end{vmatrix} \cdot \begin{vmatrix} \ddot{\beta} \\ \ddot{\gamma} \end{vmatrix} = \begin{vmatrix} -0{,}216 \\ 0{,}125 \end{vmatrix} \qquad \begin{cases} \ddot{\beta} = 0{,}499\,rad/s^2 \\ \ddot{\gamma} = 0{,}00036\,rad/s^2 \end{cases}$$

$$\vec{a}_B = \cancel{\vec{a}_A} + \vec{\dot{\omega}} \times (B-A) - \omega^2 (B-A) = 0{,}5\hat{k} \times (0{,}5\cos(60)\hat{\imath} +$$

$$+ 0{,}5\sin(60)\hat{\jmath}) - 25(0{,}5\cos(60)\hat{\imath} + 0{,}5\sin(60)\hat{\jmath}) =$$

$$= \underbrace{(-0{,}217\hat{\imath} + 0{,}125\hat{\jmath})}_{\tau} + \underbrace{(-6{,}25\hat{\imath} - 10{,}83\hat{\jmath})}_{n}\; m/s^2$$

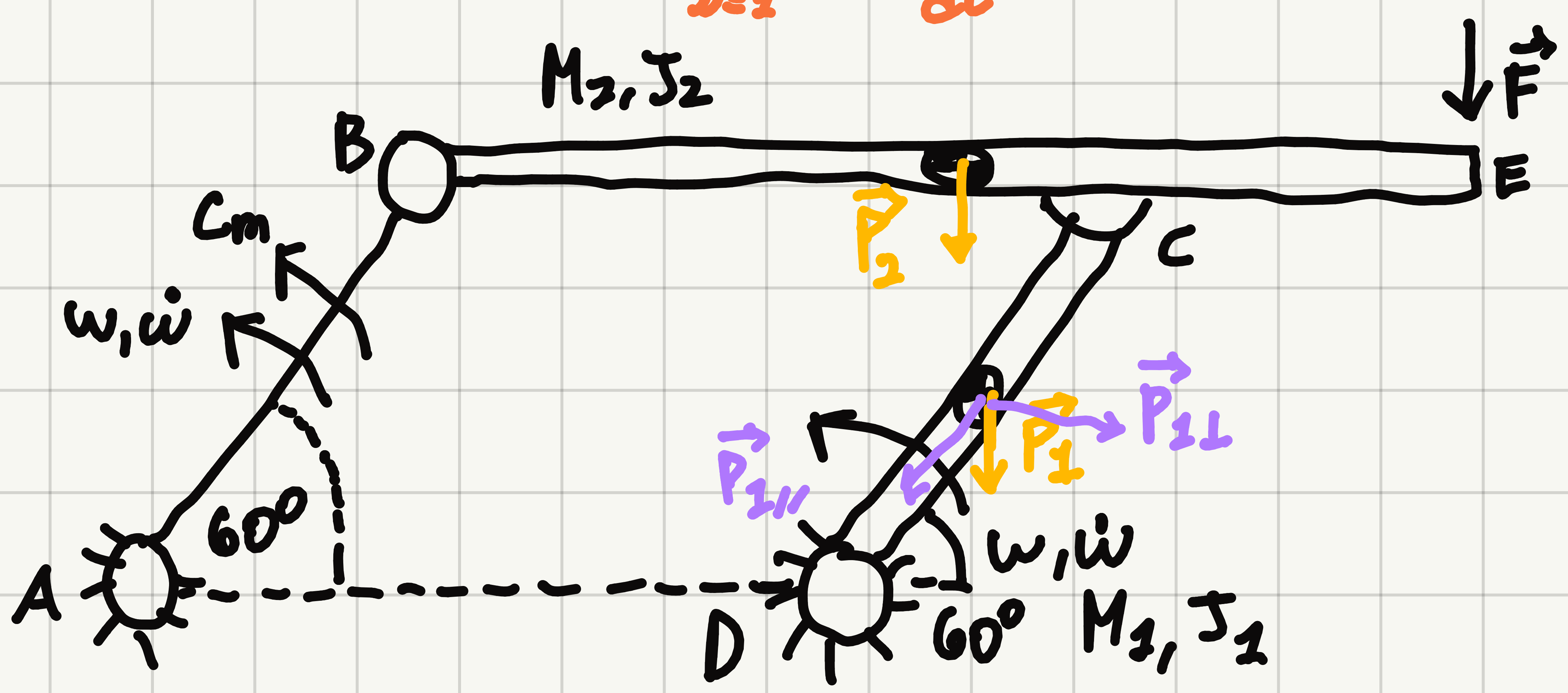
$$\vec{a}_E = (0{,}217\hat{\imath} + 0{,}125\hat{\jmath}) + 1{,}3\cdot 10^{-7}\hat{\jmath} - \dot{\gamma}^2(E-B) - (6{,}25\hat{\imath} + 10{,}83\hat{\jmath})$$

$$= \underbrace{(0{,}217\hat{\imath} + 0{,}125\hat{\jmath})}_{\tau} + \underbrace{(-6{,}25\hat{\imath} - 10{,}83\hat{\jmath})}_{n}\; m/s^2$$

$$|a_E^{(\tau)}| = 0{,}25\,m/s^2 \qquad |a_E^{(n)}| = 12{,}5\,m/s^2$$

DINAMICA

BILANCIO DI POTENZE: $\sum_{i=1}^{n} P_i = \frac{d}{dt}K$



$$\frac{d}{dt}K = (M_2 v_2 a_2^{(t)} + J_2 \dot{\gamma} \ddot{\gamma}) + (M_1 v_1 a_1 + J_1 \omega \dot{\omega})$$

$$\vec{v}_2 = \vec{v}_B + \dot{\gamma} \hat{k} \times \frac{EB}{2}(\hat{i}) = (-2{,}17\hat{i} + 1{,}25\hat{j}) - 0{,}0036 \cdot 0{,}5\hat{j} =$$

$$= (-2{,}17\hat{i} + 1{,}25\hat{j}) \quad |\vec{v}_2| = 2{,}5 \, m/s \quad |\vec{a}_2^{(t)}| = 0{,}25 \, m/s^2$$

$$\vec{a}_2^{(t)} = \vec{a}_B^{(t)} + \ddot{\gamma} \hat{k} \times \frac{EB}{2}\hat{i} = (-0{,}217\hat{i} + 0{,}125\hat{j}) \, m/s^2$$

$$\vec{v}_1 = \cancel{\vec{v}_D} + \omega \hat{k} \times \frac{(C-D)}{2} = 0{,}25\hat{k} \times (\cos(60)\hat{i} + \sin(60)\hat{j}) \quad |\vec{v}_1| = 0{,}25 \, m/s$$

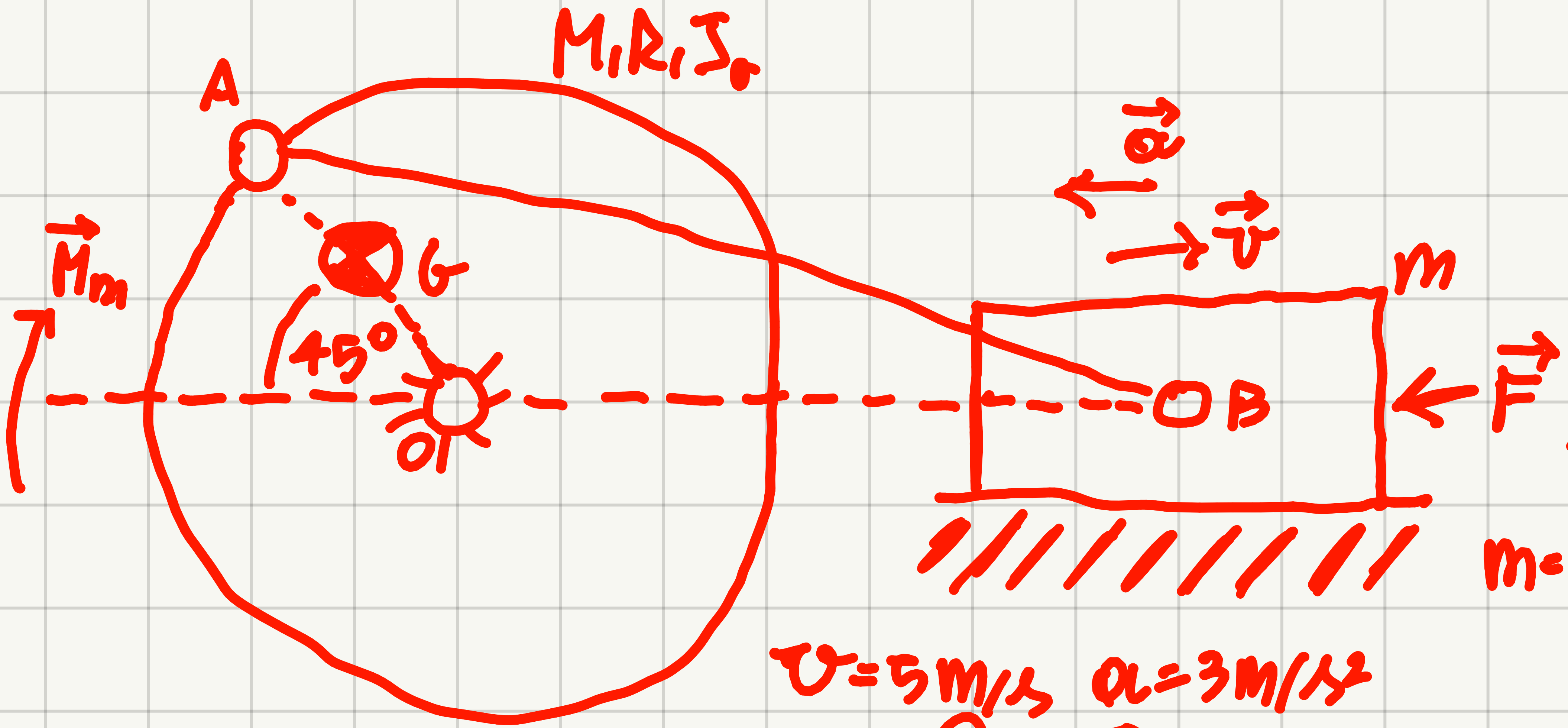
$$\vec{a}_1^{(t)} = \cancel{\vec{a}_D} + \dot{\omega} \hat{k} \times \frac{(C-D)}{2} = 0{,}25\hat{k} \times (\cos(60)\hat{i} + \sin(60)\hat{j}) \quad |\vec{a}_1^{(t)}| = 0{,}25 \, m/s^2$$

$$\frac{d}{dt}K = 19{,}38 \, W$$

$$\sum P = C_m \omega - P_{1\parallel} v_1 - P_2 v_{2y} - F v_{2y} = C_m \omega - M_1 g v_1 \cos(60) +$$

$$M_2 g v_2 + F v_2$$

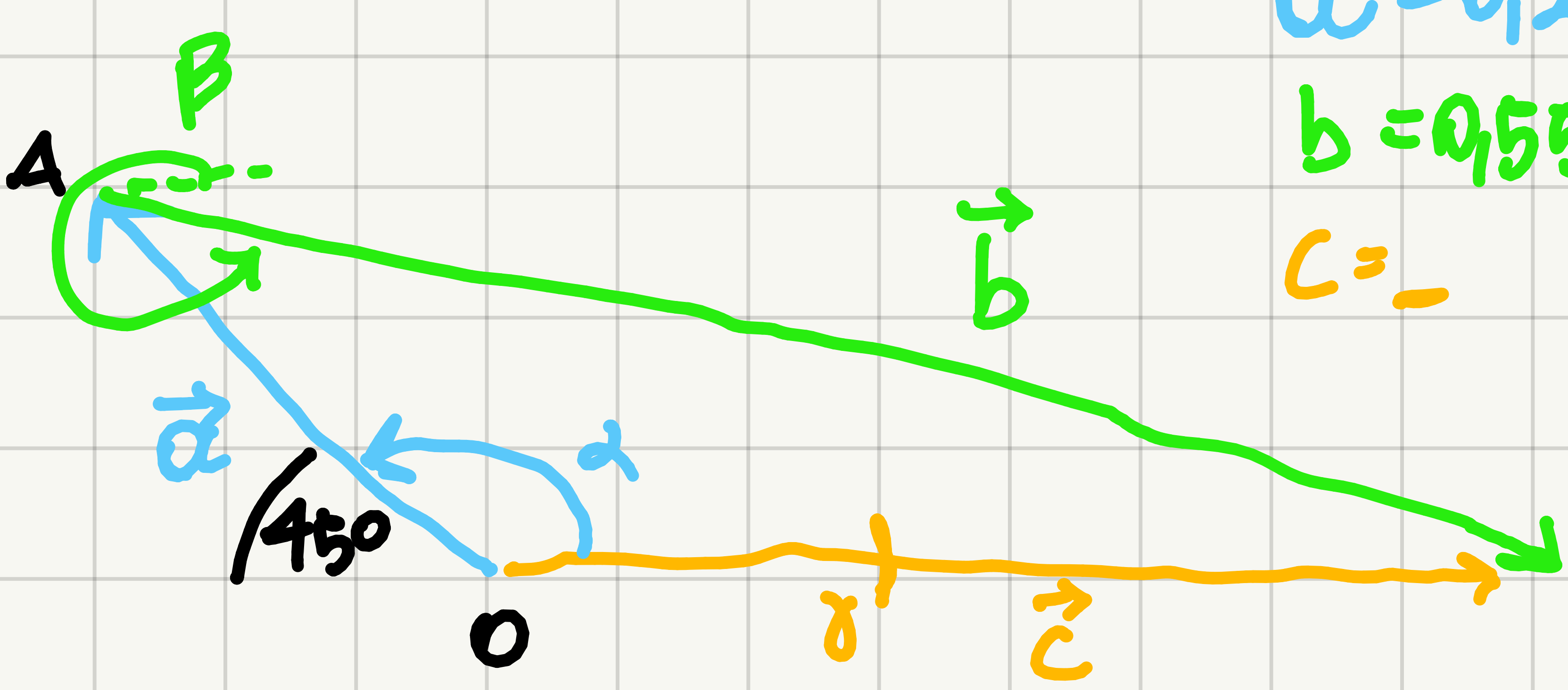
$\Rightarrow$ $C_m = 310 \, Nm$



R = 0,2 m
AB = 0,55 m
OG = 0,1 m
M = 1 kg
$J_G = 0{,}03\ \text{kg m}^2$
m = 30 kg F = 1000 N

$v = 5\ m/s$ $a = 3\ m/s^2$
$\omega, \dot{\omega}$? $\vec{M}_m$?

CINEMATICA



$a = 0{,}2\ m$ $\alpha = 135°$
$b = 0{,}55\ m$ $\beta =$
$c = \_$ $\dot{c} = 5\ m/s$ $\ddot{c} = -3\ m/s^2$ $\gamma = 0°$ FISSO

OBIETTIVI: $\omega = \dot{\alpha}$
$\dot{\omega} = \ddot{\alpha}$

$$\begin{cases} a\cos\alpha + b\cos\beta = c \\ a\sin\alpha + b\sin\beta = 0 \end{cases} \Rightarrow \begin{cases} c = 0{,}39\ m \\ \beta = -\arcsin\left(\frac{a}{b}\sin\alpha\right) = -14{,}89° = 345{,}11° \end{cases}$$

$$\begin{cases} -a\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = \dot{c} \\ a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta = 0 \end{cases}$$

$\dot{c} = v = 5\ m/s$

$$\begin{cases} -a\dot{\alpha}\sin\alpha + b\frac{a}{b}\dot{\alpha}\frac{\cos\alpha}{\cos\beta}\sin\beta = \dot{c} \\ \dot{\beta} = -\frac{a}{b}\dot{\alpha}\frac{\cos\alpha}{\cos\beta} \end{cases} \Rightarrow \begin{cases} \dot{\alpha} = -48{,}2\ rad/s \\ \dot{\beta} = -12{,}82\ rad/s \end{cases}$$

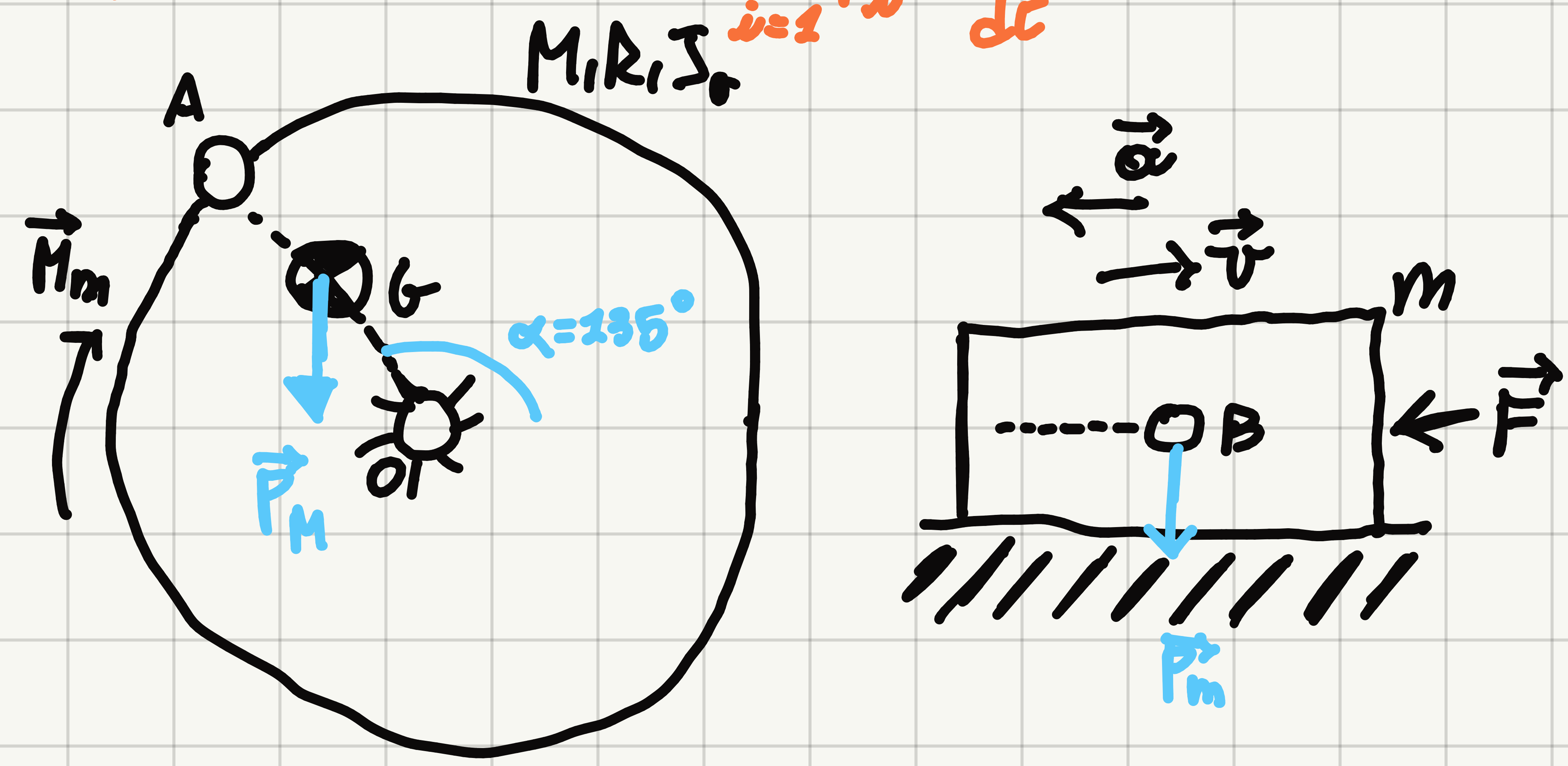
$$\begin{cases} -a\ddot{\alpha}\sin\alpha - a\dot{\alpha}^2\cos\alpha - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta = \ddot{c} \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta = 0 \end{cases}$$

$\ddot{c} = -a = -3\ m/s^2$

$$\begin{cases} \ddot{\alpha} = 3134\ rad/s^2 \\ \ddot{\beta} = -\dfrac{a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha - b\dot{\beta}^2\sin\beta}{b\cos\beta} \end{cases}$$

DINAMICA

BILANCIO DI POTENZE: $\sum_{i=1}^{n} P_i = \frac{d}{dt}K$



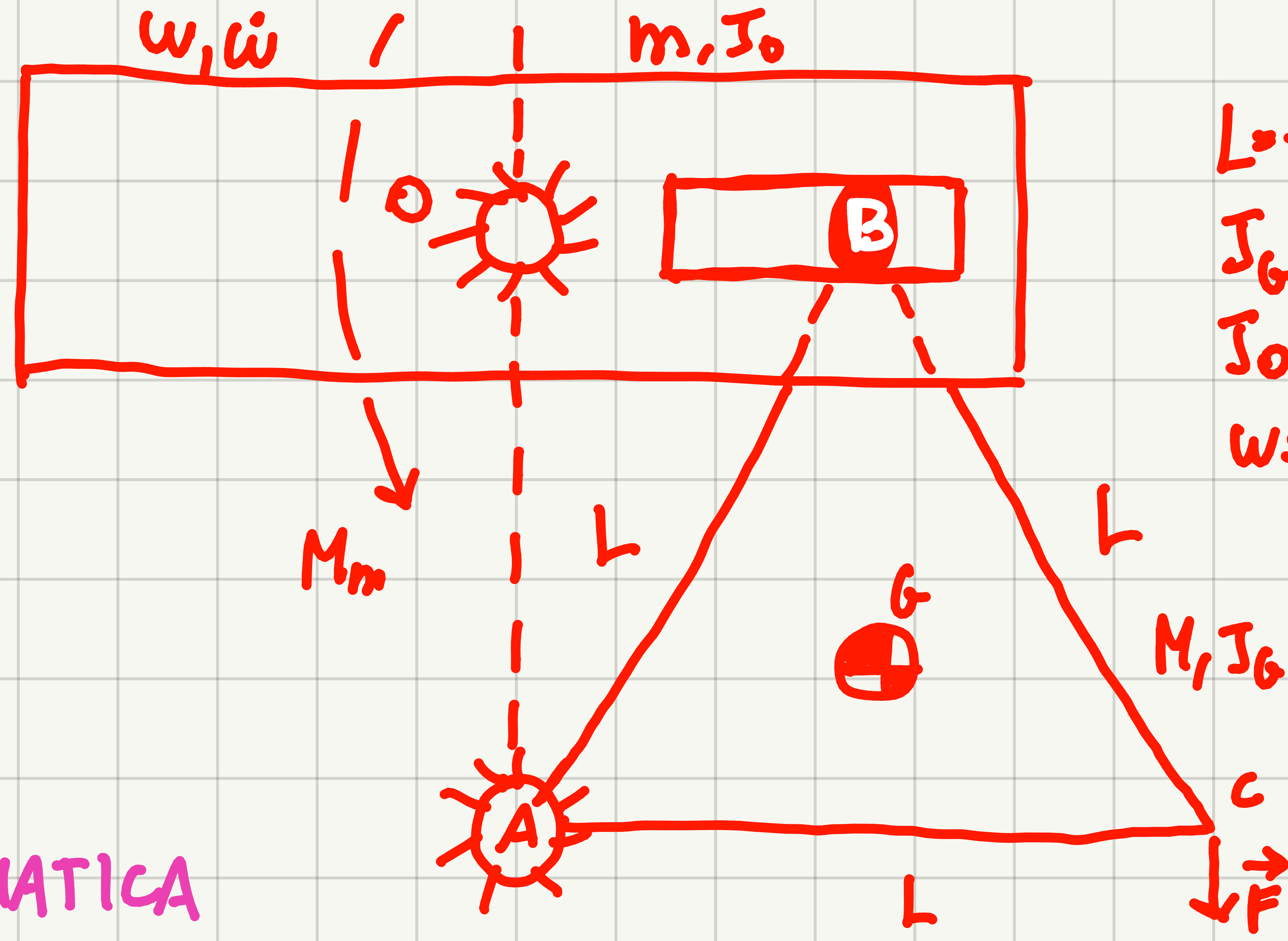
$$\frac{d}{dt}K = (M v_G a_G^{(t)} + J_G \dot{\alpha}\, \ddot{\alpha}) + (m v_B a_B)$$

$$\left.\begin{array}{l} \vec{v}_G = \cancel{\vec{v}_0} + \dot{\alpha}\hat{k} \times (G-O) \quad |\vec{v}_G| = \dot{\alpha} \cdot \overline{OG} = 4{,}82\, m/s \\ \vec{a}_G^{(t)} = \vec{a}_0^{(t)} + \ddot{\alpha}\hat{k} \times (G-O) \quad |\vec{a}_G^{(t)}| = \ddot{\alpha} \cdot \overline{OG} = 313{,}4\, m/s^2 \end{array}\right\} \frac{d}{dt}K = 6492\, W$$

$$\sum P = (P_{\parallel} v_0 - M_m \dot{\alpha}) - F v$$

$$-Mg\sin(\alpha) v_G - M_m \dot{\alpha} - F v = 6492\, W \Rightarrow M_m = -30{,}32\, Nm$$

16/07/2013



$L = 1{,}5\,m$ $M = 5\,Kg$

$J_G = 1{,}25\,Kgm^2$ $m = 3\,Kg$

$J_o = 0{,}4\,Kgm^2$ $F = 30\,N$

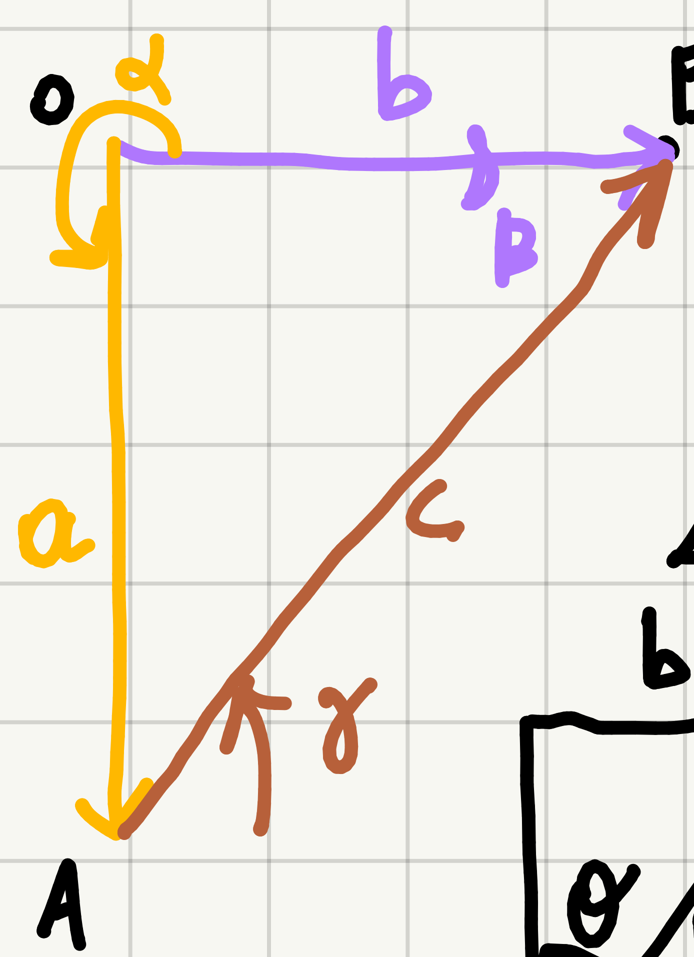
$w = 2\,rad/s$ $\dot{w} = 0{,}5\,rad/s^2$

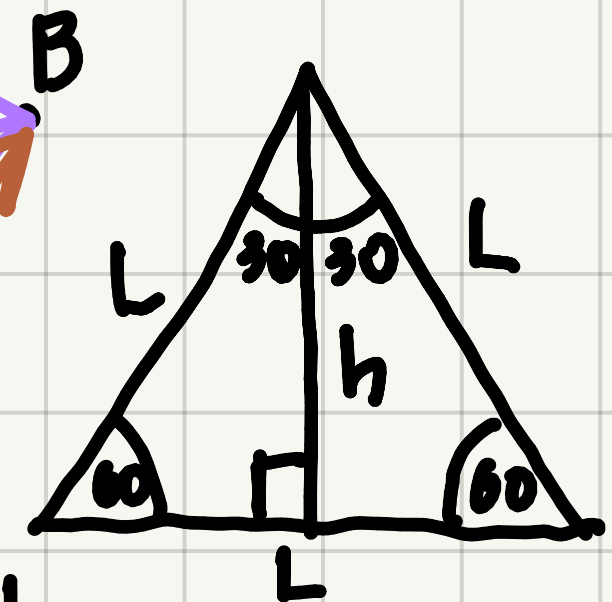
a) VELOCITÀ E ACCELERAZIONE ANGOLARE ABC?

b) a_G, a_G?

c) M_m?

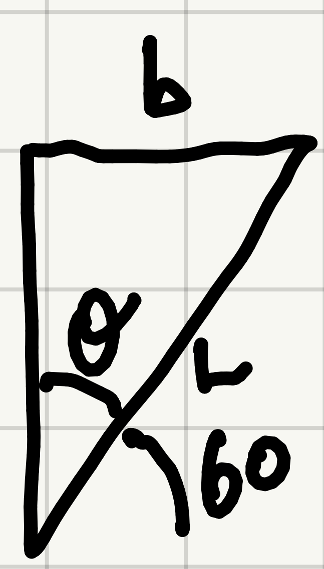
CINEMATICA





$h = \sqrt{L^2 - \left(\frac{L}{2}\right)^2} = 1{,}3\,m$

$h = L\cos(30) = 1{,}3\,m$



$\theta = 90 - 60 = 30°$ $b = L\sin\theta = 0{,}75\,m$

$a = 1{,}3\,m$ $\alpha = 270°$ FISSO $b = 0{,}75\,m$ NON FISSO $\beta = 0°$

$c = 1{,}5\,m$ $\gamma = 60°$ $\dot{\beta} = \omega$ $\ddot{\beta} = \dot{\omega}$

OBIETTIVI: $\gamma, \dot{\gamma}$

$\vec{b} = \vec{a} + \vec{c}$

$$\begin{cases} b\cos\beta = c\cos\gamma \\ b\sin\beta = -a + c\sin\gamma \end{cases}$$

$$\begin{cases} \dot{b}\cos\beta - b\dot{\beta}\sin\beta = -c\dot{\gamma}\sin\gamma \\ \dot{b}\sin\beta + b\dot{\beta}\cos\beta = c\dot{\gamma}\cos\gamma \end{cases}$$

$$\begin{cases} \dot{b}\cos\beta + c\dot{\gamma}\sin\gamma = b\dot{\beta}\sin\beta \\ \dot{b}\sin\beta - c\dot{\gamma}\cos\gamma = -b\dot{\beta}\cos\beta \end{cases}$$

$$\begin{vmatrix} \cos\beta & c\sin\gamma \\ \sin\beta & -c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \dot b \\ \dot\gamma \end{vmatrix} = \begin{vmatrix} b\dot\beta\sin\beta \\ -b\dot\beta\cos\beta \end{vmatrix}$$

$\det A = -0{,}75$ $\quad \dot b = \dfrac{\det\begin{vmatrix} 0 & c\sin\gamma \\ -b\dot\beta & -c\cos\gamma \end{vmatrix}}{\det A} = 2{,}6\,m/s$

$\dot\gamma = \dfrac{\det\begin{vmatrix} 1 & 0 \\ 0 & -b\dot\beta \end{vmatrix}}{\det A} = 2\,rad/s \Rightarrow \vec{\omega}_{ABC} = 2\hat{k}\ rad/s$

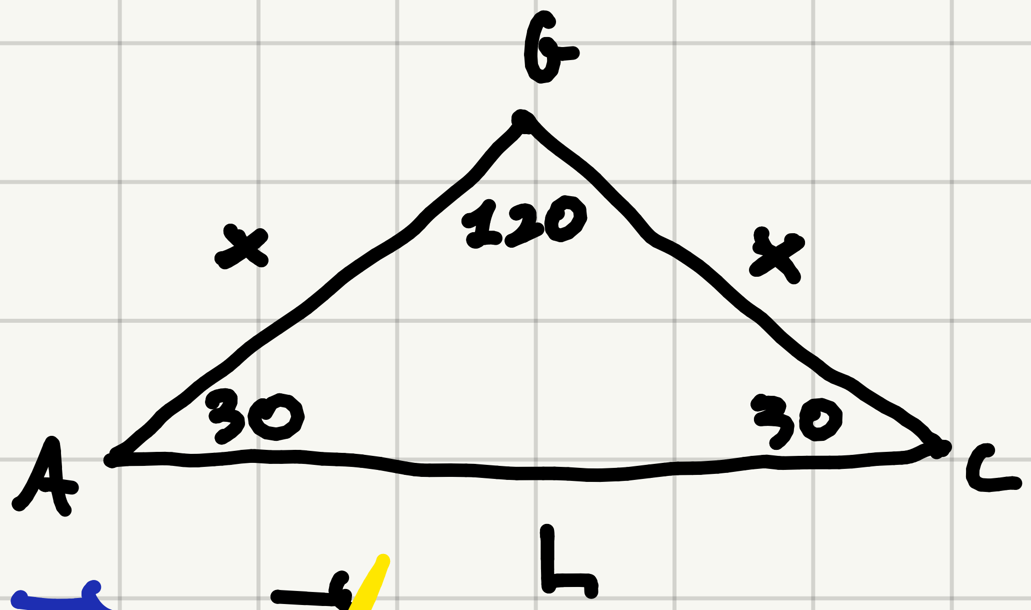
$$\begin{cases} \ddot b\cos\beta - \cancel{\dot b\dot\beta}\sin\beta + c\ddot\gamma\sin\gamma + c\dot\gamma^2\cos\gamma = \cancel{\dot b\dot\beta\sin\beta} + \cancel{b\ddot\beta\sin\beta} - b\dot\beta^2\cos\beta \\ \ddot b\sin\beta + \cancel{\dot b\dot\beta\cos\beta} - c\ddot\gamma\cos\gamma + c\dot\gamma^2\sin\gamma = \cancel{\dot b\dot\beta\cos\beta} + b\ddot\beta\cos\beta - \cancel{b\dot\beta^2}\sin\beta \end{cases}$$

$$\begin{cases} \ddot b\cos\beta - \cancel{\dot b\dot\beta\sin\beta} + c\ddot\gamma\sin\gamma + c\dot\gamma^2\cos\gamma = \cancel{\dot b\dot\beta\sin\beta} + \cancel{b\ddot\beta\sin\beta} - b\dot\beta^2\cos\beta \\ \ddot b\sin\beta + \cancel{\dot b\dot\beta\cos\beta} - c\ddot\gamma\cos\gamma - c\dot\gamma^2\sin\gamma = \cancel{\dot b\dot\beta}\cos\beta + b\ddot\beta\cos\beta - \cancel{b\dot\beta^2\sin\beta} \end{cases}$$

$$\begin{vmatrix} \cos\beta & c\sin\gamma \\ \sin\beta & -c\cos\gamma \end{vmatrix} \cdot \begin{vmatrix} \ddot b \\ \ddot\gamma \end{vmatrix} = \begin{vmatrix} -b\dot\beta^2 - c\dot\gamma\cos\gamma \\ b\ddot\beta + c\dot\gamma^2\sin\gamma \end{vmatrix}$$

$$\ddot b = \frac{\det\begin{vmatrix} -b\dot\beta^2 - c\dot\gamma^2\cos\gamma & c\sin\gamma \\ b\ddot\beta + c\dot\gamma^2\sin\gamma & -c\cos\gamma \end{vmatrix}}{-0{,}75} = 12\,m/s^2$$

$$\ddot\gamma = \frac{\det\begin{vmatrix} 1 & -b\dot\beta^2 - c\dot\gamma^2\cos\gamma \\ 0 & b\ddot\beta + c\dot\gamma^2\sin\gamma \end{vmatrix}}{-0{,}75} = -7{,}43\,rad/s^2 \Rightarrow \vec{\dot\omega}_{ABC} = -7{,}43\hat{k}\ rad/s^2$$



$$\frac{L}{\sin(120)} = \frac{x}{\sin(30)} \Rightarrow x = 0{,}87\,m$$

$$\vec{v}_G = \cancel{\vec{v}_A} + \omega_G \times (G-A) = 0 + 2\hat{k} \times (0{,}87\cos(30)\hat{i} + 0{,}87\sin(30)\hat{j})$$

$$= 1{,}5\hat{j} - 0{,}87\hat{i} \qquad |\vec{v}_G| = 1{,}73\, m/s$$

$$\vec{a}_G = \underbrace{\cancel{\vec{a}_A} + \dot{\omega} \times (G-A)}_{\tau} - \underbrace{\omega^2 (G-A)}_{n} = 7{,}43(-\hat{k}) \times (0{,}87\cos(30)\hat{i} + 0{,}87\sin(30)\hat{j}) - 4(0{,}87\cos(30)\hat{i} + 0{,}87\sin(30)\hat{j}) =$$

$$= (7{,}43 \cdot 0{,}87\sin(30) - 4 \cdot 0{,}87\cos(30))\hat{i} +$$

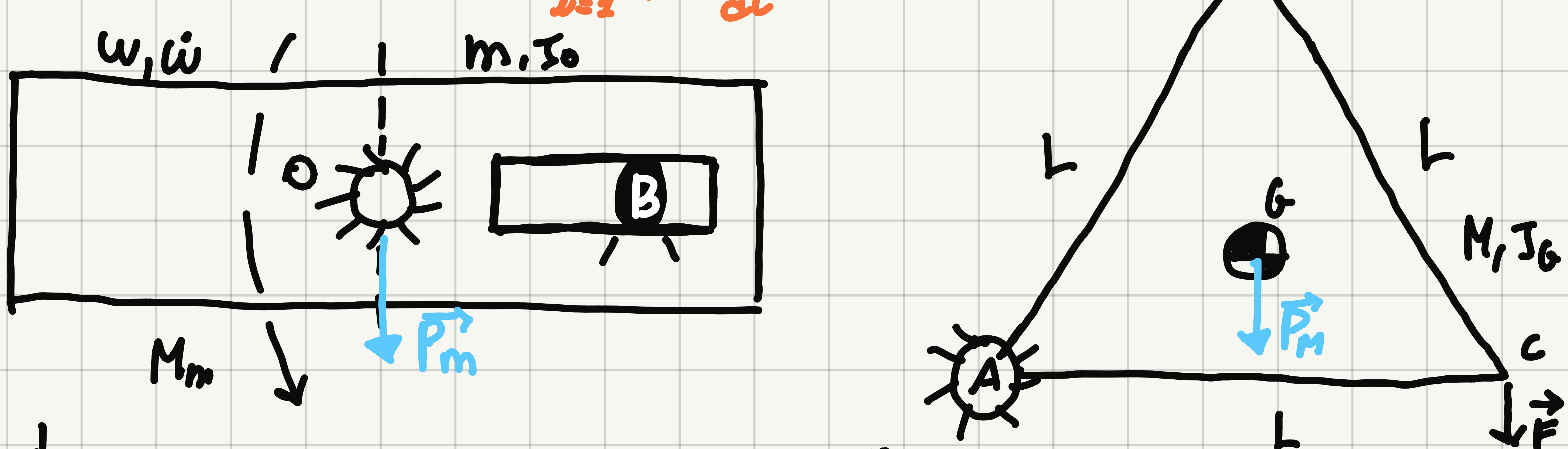
$$(-7{,}43 \cdot 0{,}87\cos(30) - 4 \cdot 0{,}87\sin(30))\hat{j} \qquad \tau$$

$$= 0{,}22\hat{i} - 7{,}34\hat{j} \qquad \vec{a}_G = (-5{,}6\,\hat{i} + 3{,}23\hat{j}) + (-3\hat{i} - 1{,}74\hat{j})$$

DINAMICA

$$|a_G^{(c)}| = 6{,}6\, m/s^2$$

BILANCIO DI POTENZE: $\sum_{i=1}^{n} P_i = \frac{d}{dt}K$

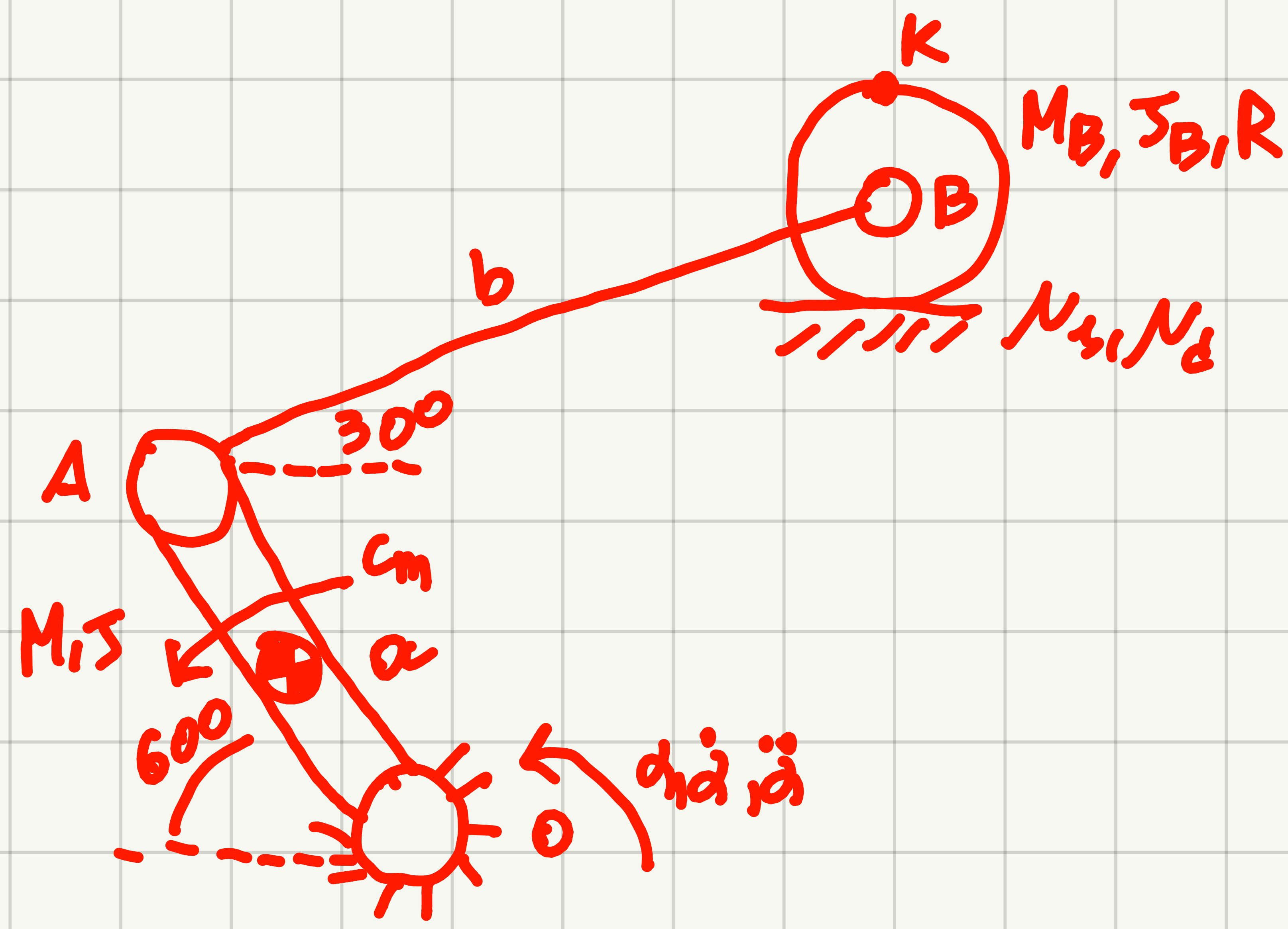


$$\frac{d}{dt}K = (J_O \dot{\omega}\omega) + (M v_G a_G^{\tau} + J_G \dot{\gamma}\ddot{\gamma}) = 74{,}85\, W$$

$$\sum P = (M_m \omega) + (M g v_G + F v_C)$$

$$\vec{v}_C = \vec{v}_A + \omega_{ABC}\hat{k} \times L\hat{i} \qquad |\vec{v}_C| = 3\, m/s$$

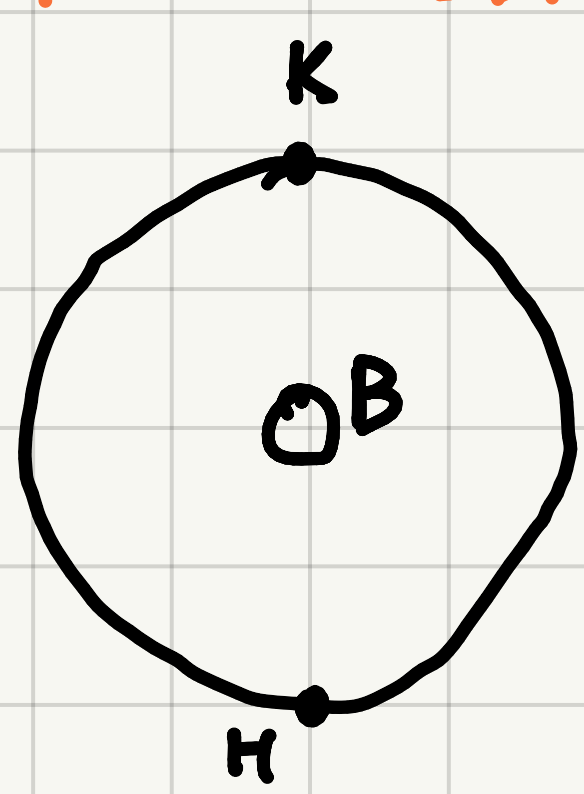
$$\rightarrow M_m = -50\, Nm$$



$a = b = 1m$ $\quad R = 0,1m$
$J = 0,5 kgm^2$ $\quad J_B = 0,1 kgm^2$
$M = 0,5 kg$ $\quad M_B = 1 kg$
$\mu_s = 0,8$ $\quad \mu_v = 0,05$
$\dot{\alpha} = 1 rad/s$ $\quad \ddot{\alpha} = 1 rad/s^2$
v_K, a_K?
C_m?

CINEMATICA

MOTO DEL DISCO CON CIR ≡ H

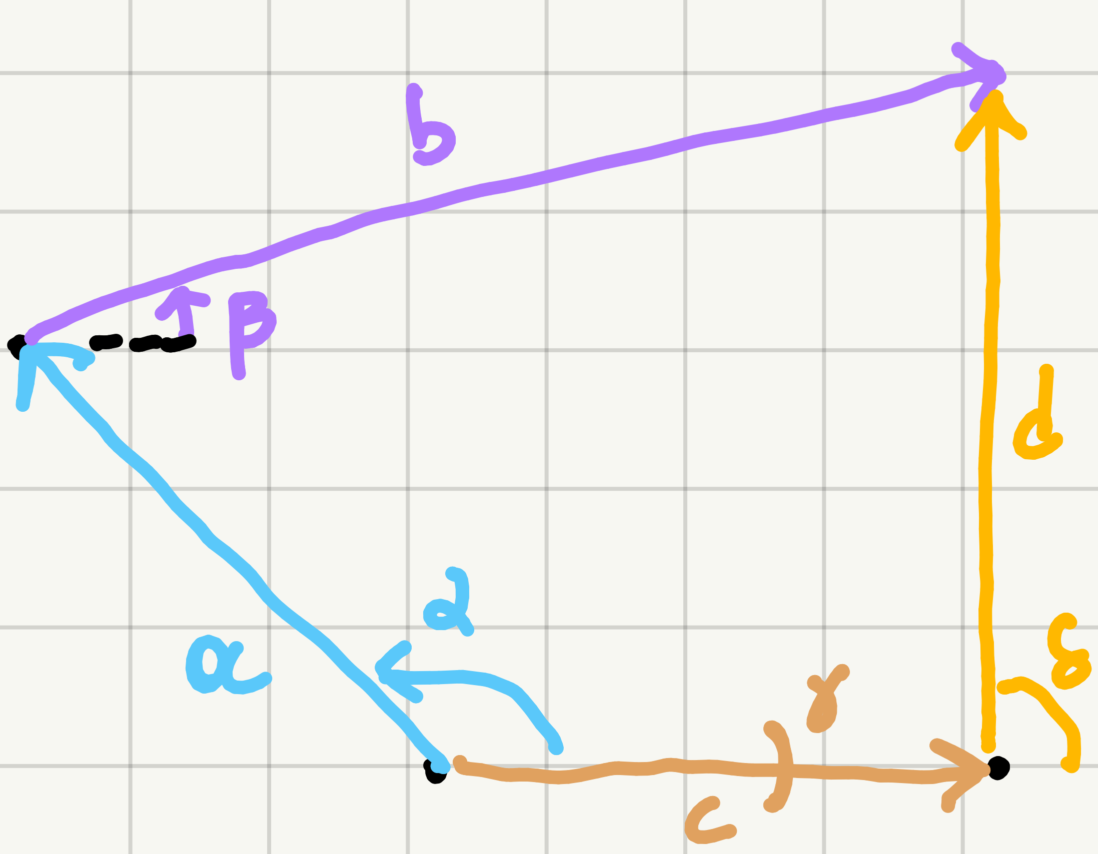


$$\vec{v}_B = \cancel{\vec{v}_H} + \vec{\omega}_d \times (B-H) \qquad \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\hat{i}$$

$$v_B \hat{i} = \omega_d \hat{k} \times R \hat{j} = \omega_d R (-\hat{i})$$

$$\Rightarrow \quad \omega_d = -\frac{v_B}{R} \quad \Rightarrow \quad \dot{\omega}_d = -\frac{a_B}{R}$$

$$\vec{v}_K = \cancel{\vec{v}_H} + \vec{\omega}_d \times (2R\hat{j}) \qquad \vec{a}_K = a_H + \dot{\vec{\omega}}_d \times (2R\hat{j}) - \omega_d^2 (2R\hat{j})$$



$a = 1m$ $\quad \alpha = 120°$ $\quad \dot{\alpha} = 1 rad/s$ $\quad \ddot{\alpha} = 1 rad/s^2$
$b = 1m$ $\quad \beta = 30°$ $\quad \dot{\beta}, \ddot{\beta}$ NON NOTI
$c = ?$ $\quad \dot{c}, \ddot{c}?$ $\quad \gamma = 0°$ FISSO
$d?$ $\quad \delta = 90°$ FISSO

$$\vec{a} + \vec{b} = \vec{c} + \vec{d}$$

$$\begin{cases} a\cos\alpha + b\cos\beta = c \\ a\sin\alpha + b\sin\beta = d \end{cases}$$

$$\begin{cases} -a\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = \dot{c} \\ a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta = 0 \end{cases} \rightarrow \begin{cases} \dot{c} = -1,15 m/s \\ \dot{\beta} = 0,5774 rad/s \end{cases}$$

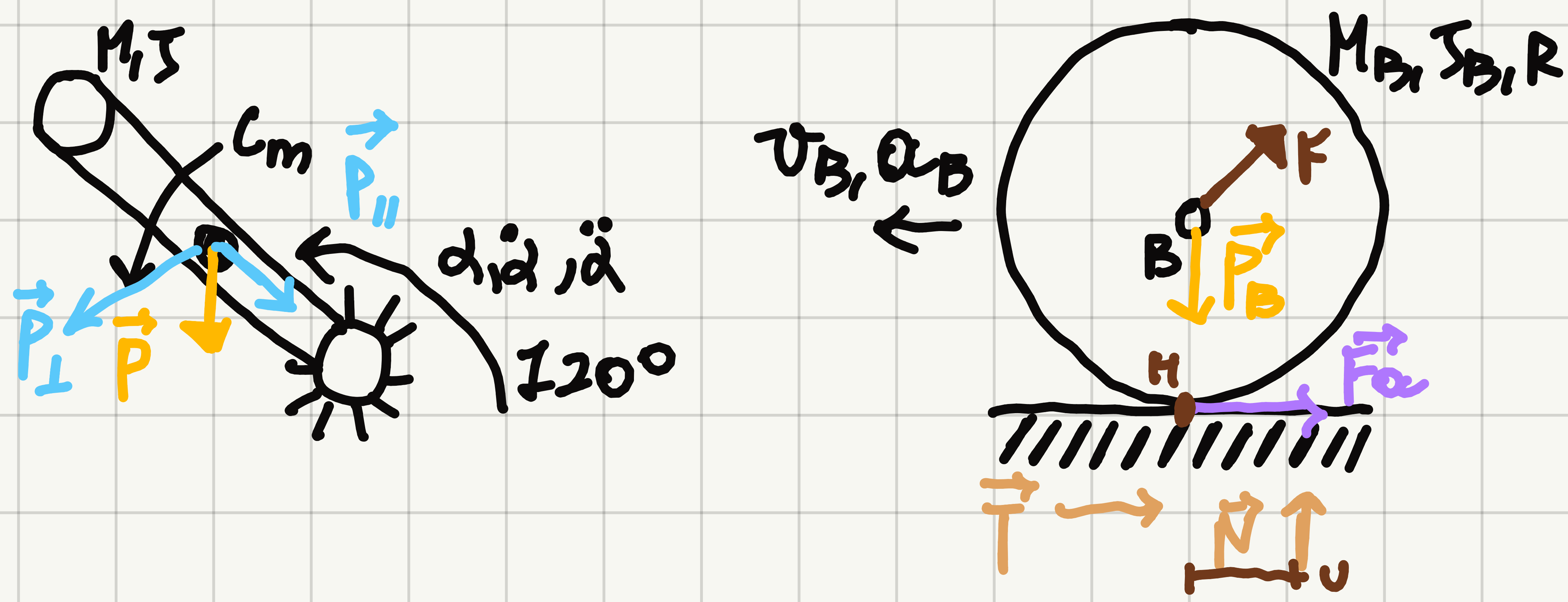
$$\begin{cases} -a\ddot{\alpha}\sin\alpha - a\dot{\alpha}^2\cos\alpha - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta = \ddot{c} \\ a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\sin\alpha + b\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta = 0 \end{cases} \rightarrow \begin{cases} \ddot{c} = -1{,}54\ m/s^2 \\ \ddot{\beta} = 1{,}77\ rad/s^2 \end{cases}$$

$$w_d = -\frac{\dot{c}}{R} = -11{,}54\ rad/s \Rightarrow \vec{v}_K = (-2{,}31\hat{i})\ m/s$$

$$\dot{w}_d = -15{,}4\ rad/s^2 \quad a_H = \ddot{c} \Rightarrow \vec{a}_K = (-3{,}08\hat{i} - 13{,}33\hat{j})\ m/s^2$$

DINAMICA

BILANCIO DI POTENZE: $\sum_{i=1}^{n} P_i = \frac{d}{dt}K$



$$\frac{d}{dt}K = (M v_G a_G^{(t)} + J\dot{\alpha}\ddot{\alpha}) + (M_B v_B a_B^{(t)} + J_B w_d \dot{w}_d)$$

$$\vec{v}_G = \cancel{\vec{v}_O} + \dot{\alpha}\hat{k} \times \frac{a}{2}(\cos(120)\hat{i} + \sin(120)\hat{j}) =$$

$$= (-0{,}125\hat{i} - 0{,}125\hat{j})\ m/s \qquad |\vec{v}_G| = 0{,}177\ m/s$$

$$\vec{a}_G = \underbrace{\vec{a}_O + \ddot{\alpha}\hat{k} \times \frac{a}{2}(\cos(120)\hat{i} + \sin(120)\hat{j})}_{a_G^{(t)}} - \dot{\alpha}^2\frac{a}{2}(\cos(120)\hat{i} +$$

$$+ \sin(120)\hat{j}) = ((-0{,}125\hat{i} - 0{,}125\hat{j}) + a_G^{(n)})\ m/s^2$$

$$|\vec{a}_G^{(t)}| = 0{,}177\ m/s^2 \qquad \frac{d}{dt}K = 20W$$

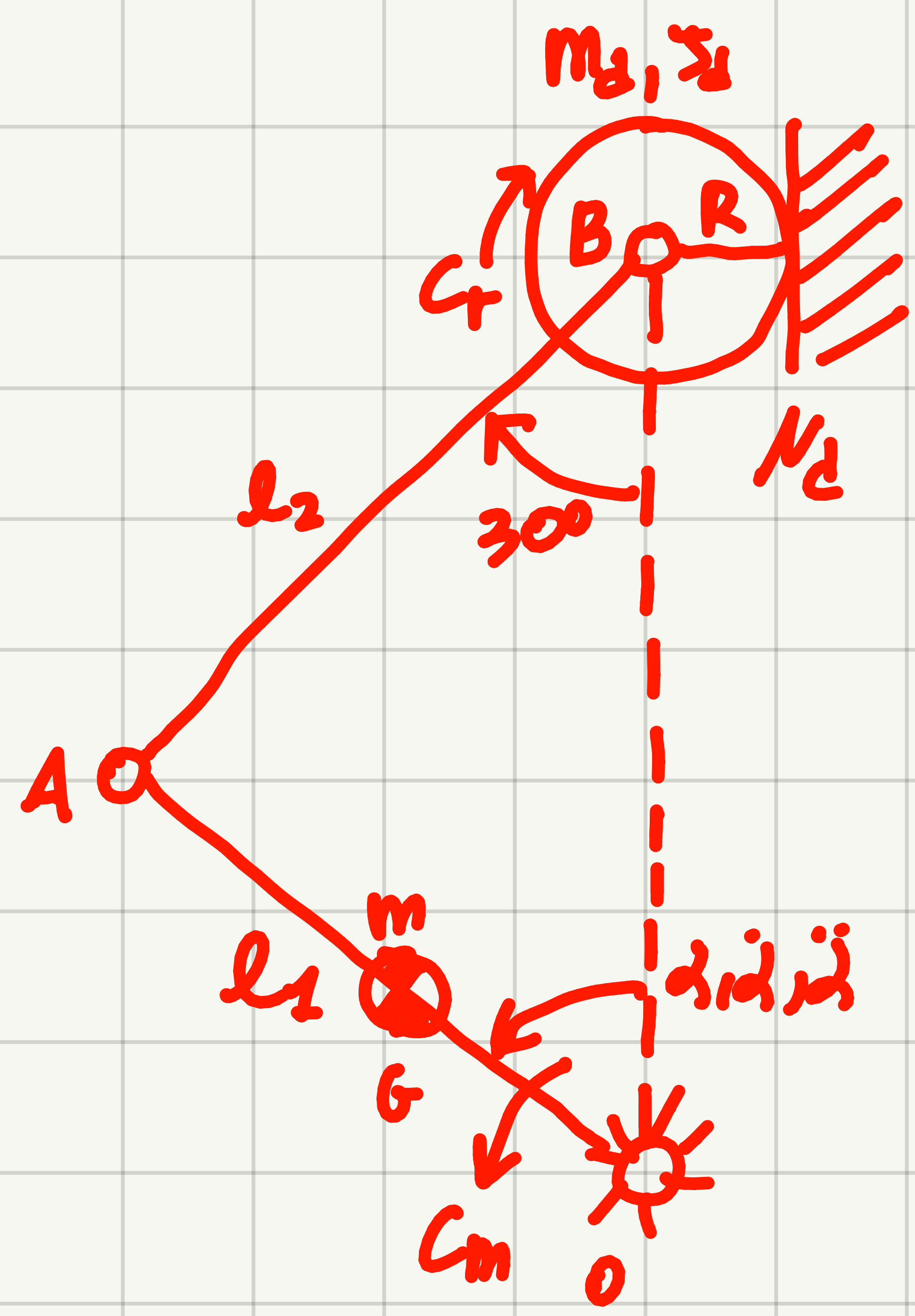
$$\sum P = (C_m\dot{\alpha} + P_{\parallel}v_G) + (-M_B v_B a_B) - (N v w)$$

$$\begin{cases} F\cos(30) + T - M_B a_B = 0 \\ F\sin(30) + N - M_B g = 0 \\ H \quad J_B \dot{\omega} - M_B a_B R + NU + F \cdot R \sin(30+90) = 0 \end{cases} \qquad U = \mu_v R$$

$$\begin{cases} // \\ F = -\frac{N}{\sin(30)} + \frac{M_B g}{\sin 30} \\ N \mu_v R - \frac{NR}{\cos(120)} + \frac{M_B g R}{\cos(120)} = M_B a_B R - J_B \dot{\omega} \end{cases} \quad U = \mu_v R \rightarrow N = 20{,}17 N$$

$$C_m \dot{\alpha} + M g v_0 \sin(120) - N \mu_v R \omega_d = 20W$$

$$\Rightarrow \quad C_m = 20{,}41 \, Nm$$



$l_1=\sqrt{2}\,m$ $\quad l_2=2\,m$ $\quad R=0{,}2\,m$ $\quad \alpha=45°$

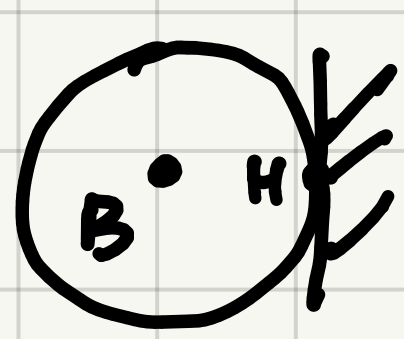
$\dot\alpha=1\,rad/s$ $\quad \ddot\alpha=1\,rad/s^2$ $\quad J_d=0{,}05\,kgm^2$

$m=2\,kg$ $\quad m_d=2\,kg$ $\quad C_r=0{,}6\,Nm$ $\quad \mu_d=0{,}05$

a) v_b, ω_{DISCO}? b) $a_B, \dot\omega_{DISCO}$? c) C_m?

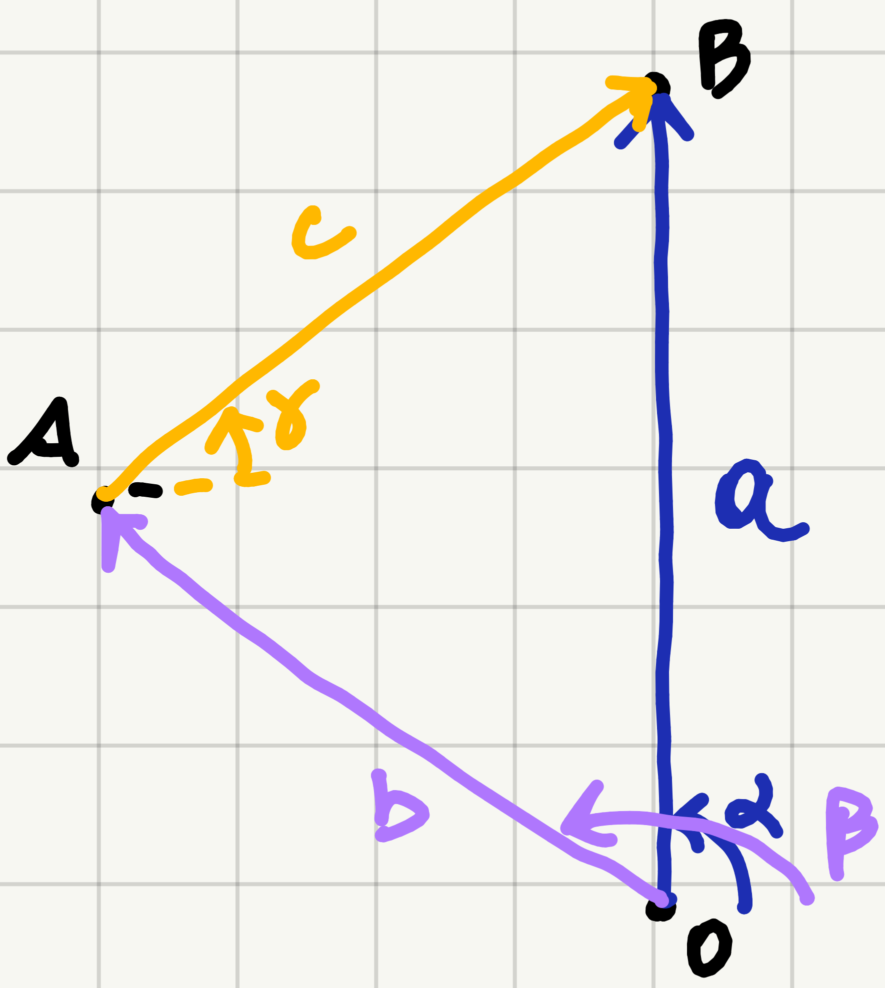
CINEMATICA

MOTO DEL DISCO



$$\vec v_B = \cancel{\vec v_H} + \vec\omega_{DISCO}\times(B-H)$$

$$v_B\,\hat j = \omega_{DISCO}\,\hat k \times (-R\,\hat i) = -\omega_{DISCO}\,R\,\hat j \rightarrow \omega_{DISCO} = \frac{-v_B}{R} \qquad \dot\omega_{DISCO} = \frac{-a_B}{R}$$



$$\vec a = \vec b + \vec c$$

$a=$ __ $\quad \alpha=90°$ FISSO

$b=1{,}41\,m$ $\quad \beta=\alpha+90°=135°$ $\quad \dot\beta=1\,rad/s$ $\quad \ddot\beta=1\,rad/s^2$

$c=2\,m$ $\quad \gamma=$ __ $\qquad$ OBIETTIVI: $\dot a, \ddot a$

$$\begin{cases} b\cos\beta + c\cos\gamma = 0 \\ b\sin\beta + c\sin\gamma = a \end{cases} \rightarrow \begin{cases} \gamma = 60° \\ a = 2{,}73\,m \end{cases}$$

$$\begin{cases} -b\dot\beta\sin\beta - c\dot\gamma\sin\gamma = 0 \\ b\dot\beta\cos\beta + c\dot\gamma\cos\gamma = \dot a \end{cases} \rightarrow \begin{cases} \dot\gamma = -0{,}58\,rad/s \\ \dot a = -1{,}58\,m/s \end{cases}$$

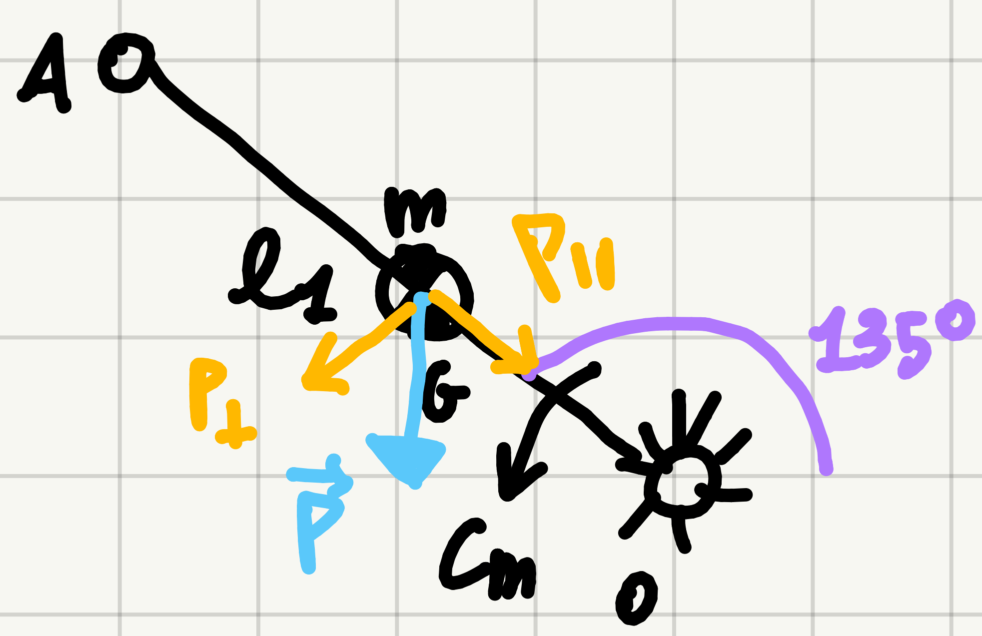
$$\Rightarrow \vec v_B = (-1{,}58\,\hat j)\,m/s \quad \Rightarrow \vec\omega_{DISCO} = (7{,}9\,\hat k)\,rad/s$$

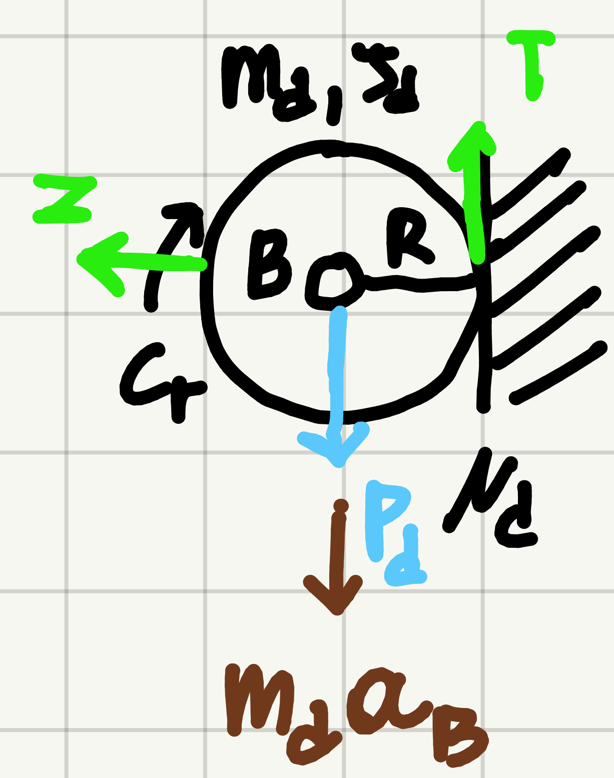
$$\begin{cases} -b\ddot\beta\sin\beta - b\dot\beta^2\cos\beta - c\ddot\gamma\sin\gamma - c\dot\gamma^2\cos\gamma = 0 \\ b\ddot\beta\cos\beta - b\dot\beta^2\sin\beta + c\ddot\gamma\cos\gamma - c\dot\gamma^2\sin\gamma = \ddot\alpha \end{cases} \rightarrow \begin{cases} \ddot\gamma = -0{,}19\ rad/s^2 \\ \ddot\alpha = -2{,}77\ m/s^2 \end{cases}$$

$\Rightarrow \vec a_B = (-2{,}77\,\hat j)\ m/s^2 = a_B^{(t)} \Rightarrow \dot{\vec\omega}_{DISCO} = 13{,}85\ rad/s^2$

DINAMICA

Bilancio di potenze: $\sum_{i=1}^{n} P_i = \frac{d}{dt}K$





$$\frac{d}{dt}K = \left(m v_G a_G^{(t)} + \frac{1}{12}m\left(\frac{l}{2}\right)^2\dot\beta\ddot\beta\right) + \left(m_d v_B a_B^{(t)} + J_d\,\omega_{DISCO}\,\dot\omega_{DISCO}\right)$$

$$\vec v_G = \cancel{\vec v_O} + \dot\beta\hat k \times \left(\frac{\sqrt2}{2}\cos\beta\,\hat i + \frac{\sqrt2}{2}\sin\beta\,\hat j\right) =$$

$$= \left(-\frac12\hat i - \frac12\hat j\right) m/s \qquad |\vec v_G| = 0{,}71\ m/s$$

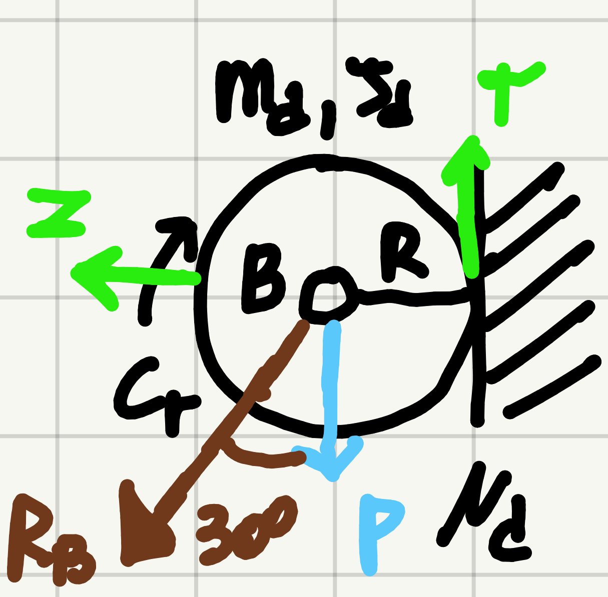
$$P = -\mu_v N R\omega$$

$$\vec a_G = \cancel{\vec a_O} + \ddot\beta\hat k \times \left(\frac{\sqrt2}{2}\cos\beta\,\hat i + \frac{\sqrt2}{2}\sin\beta\,\hat j\right) - \dot\beta^2\left(\frac{\sqrt2}{2}\cos\beta\,\hat i + \frac{\sqrt2}{2}\sin\beta\,\hat j\right) =$$

$$= \left(-\frac12\hat i - \frac12\hat j\right) = a_G^{(t)} \qquad |\vec a_G^{(t)}| = 0{,}707\ m/s^2$$

$\Rightarrow \frac{d}{dt}K = 15{,}31\ W$ $\qquad |\vec v_d| = |\dot\alpha| = 1{,}58\ m/s$

$$\sum P = \left(-P_{||}v_G + C_m\dot\beta\right) + \left(-P_d v_d + C_r\,\omega_{DISCO} - \mu_v N R\,\omega_{DISCO}\right)$$



$$\begin{array}{l} \\ \\ B \end{array}\begin{cases} -N - R_B\sin30 = 0 \\ T - P - m_d a_B - R_B\cos30 = 0 \\ TR - C_r - J_d\dot\omega_{DISCO} - \underbrace{\mu_v N R}_{\text{ATTRITO VOLVENTE}} = 0 \end{cases}$$

$$\begin{cases} N = -R_B/2 \\ T = M_d(g + a_B) + \sqrt{3}/2\, R_B \\ (M_d g + \sqrt{3}/2\, R_B)R - C_r - J_d \dot{\omega}_{DISCO} - \mu_v N R = 0 \end{cases} \Rightarrow R_B = -2N$$

$$(M_d(g + a_D) - \sqrt{3}N)R - C_r - J_d \dot{\omega}_{DISCO} - \mu_v N R = 0$$

$$(-\mu_v R - \sqrt{3}R)N = -M_d(g + a_D)R + C_r + J_d \dot{\omega}_{DISCO}$$

$$N = \frac{M_d(g + a_D)R - C_r - J_d \dot{\omega}_{DISCO}}{(\mu_v + \sqrt{3})R} = 20{,}7\,N$$

$$-mg v_G \cos(135) + C_m \dot{\beta} - M_d g v_d + C_r \omega_{DISCO} - \mu_v N R \omega_{DISCO} = 15{,}31\,W$$

$$\Rightarrow C_m = 23{,}7\ N \cdot m$$