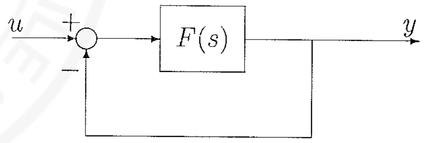
Domanda Scritta di Controlli Automatici (9CFU) - 13/01/2014

Esercizio 1

È dato il sistema di controllo:



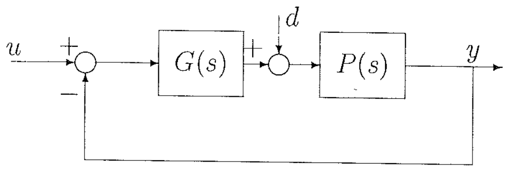
in cui:

$$F(s) = \frac{K(s+4)^2(s+8)}{(s+1)^2(s+6)(s+7)}$$

- Tracciare il luogo positivo delle radici.
- Tracciare il luogo negativo delle radici.
- Determinare, se esiste, l'intervallo di valori di K che garantisce la stabilità a ciclo chiuso.
- Esiste un valore di $K > 0$ per cui una radice a ciclo chiuso sia pari a -4.1? Motivare la risposta.

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{2}{s(s+10)^2}; \quad d(t) = \delta_{-1}(t);$$

Progettare $G(s)$ con la sintesi per tentativi in ω in modo che:

- $|\tilde{y}_d(t)| \leq 0.005$, in cui $\tilde{y}_d(t)$ è la risposta a regime permanente al disturbo $d(t)$;
- $B_3 \simeq 2.5\ Hz$;
- $M_r \leq 2\ dB$.

Calcolare infine la risposta a regime permanente all'ingresso: $u(t) = (2t+3) \cdot \delta_{-1}(t)$.

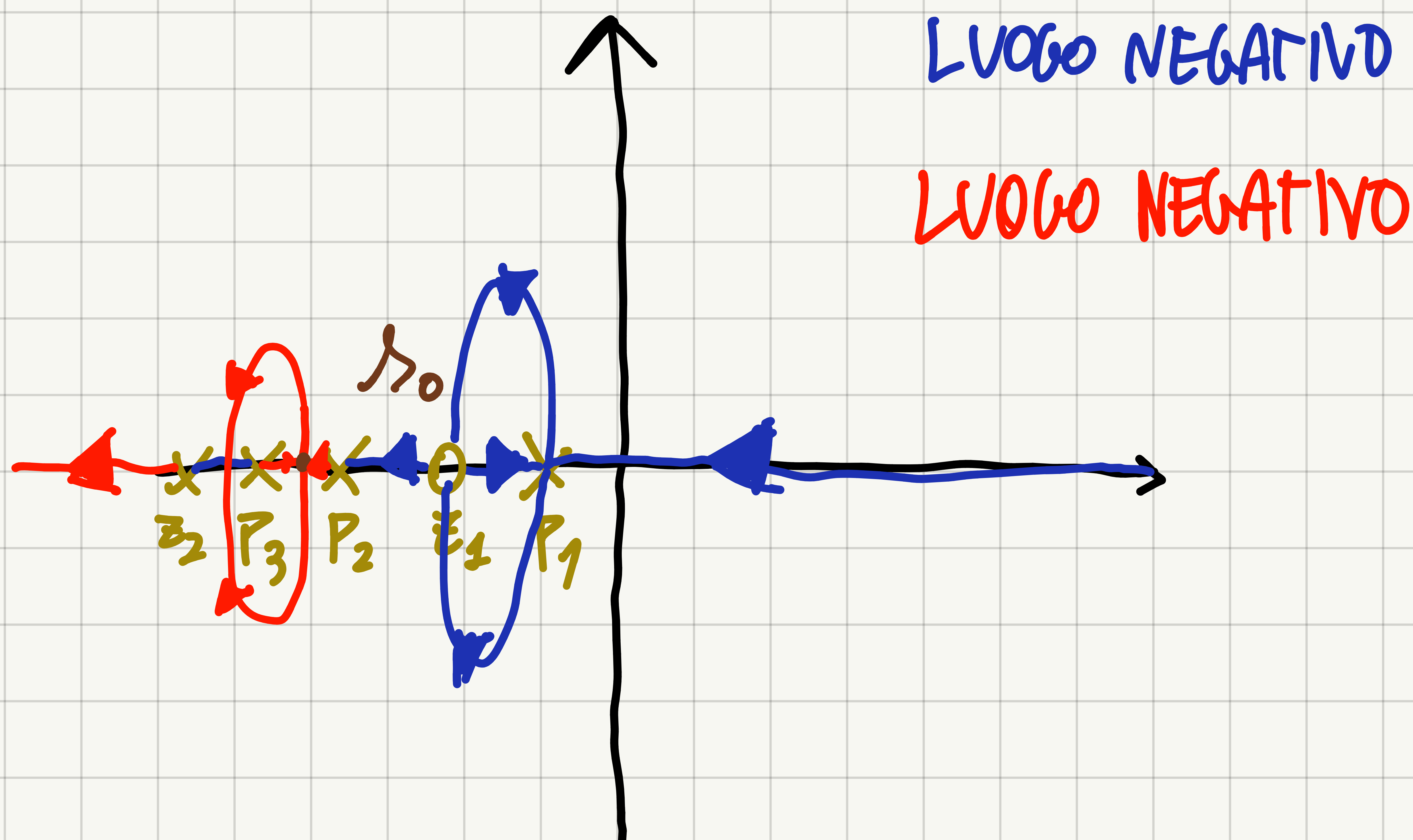
1

$$F(s) = K \cdot \frac{(s+4)^2 (s+8)}{(s+1)^2 (s+6)(s+7)}$$

$n = 4,\ m = 3 \qquad n - m = 1$

$p_1 = -1,\ \mu_{p_1} = 2 \qquad p_2 = -6,\ \mu_{p_2} = 1 \qquad p_3 = -7,\ \mu_{p_3} = 1$

$z_1 = -4,\ \mu_{z_1} = 2 \qquad z_2 = -8,\ \mu_{z_2} = 1$



$$\varphi(s, K)\big|_{K=0} = 0$$

$$\varphi(s, K) = (s+1)^2 (s+6)(s+7) + K (s+4)^2 (s+8)$$

$42 + 128K = 0 \rightarrow$ STABILITÀ PER $K > -\frac{21}{64}$

$4,1$ È NEL LUOGO NEGATIVO $\Rightarrow \nexists\ K > 0$

②

$|Y_d(\infty)| = \left|\frac{K_d}{K_G}\right| \leq 0{,}005 \qquad K_d = 1 \Rightarrow K_G \geq 200$

$G(s) = K$ (SISTEMA ASTATICO RISPETTO DISTURBO DI TIPO 1, POLO IN $s=0$ GIÀ PRESENTE)

$$F(s) = 400 \cdot \frac{1}{s(s+10)^2}$$

$M_r \leq 2dB \Rightarrow M_\varphi \geq 47°$ $\qquad B_3 \simeq 2{,}5\, Hz \Rightarrow \omega_t = 4B_3 = 10 \frac{rad}{s}$

$$F(i\omega) = 4 \cdot \frac{1}{i\omega\left(1+\frac{i\omega}{10}\right)^2} \qquad 4 \to 20\log_{10} 4 = 12dB$$

PUNTI DI ROTTURA

- $\omega = 0$ $\quad -20\,dB \quad -90° \quad -20\,dB \quad -90°$
- $\omega = 10$ $\quad -40\,dB \quad -180° \quad -60\,dB \quad -270°$

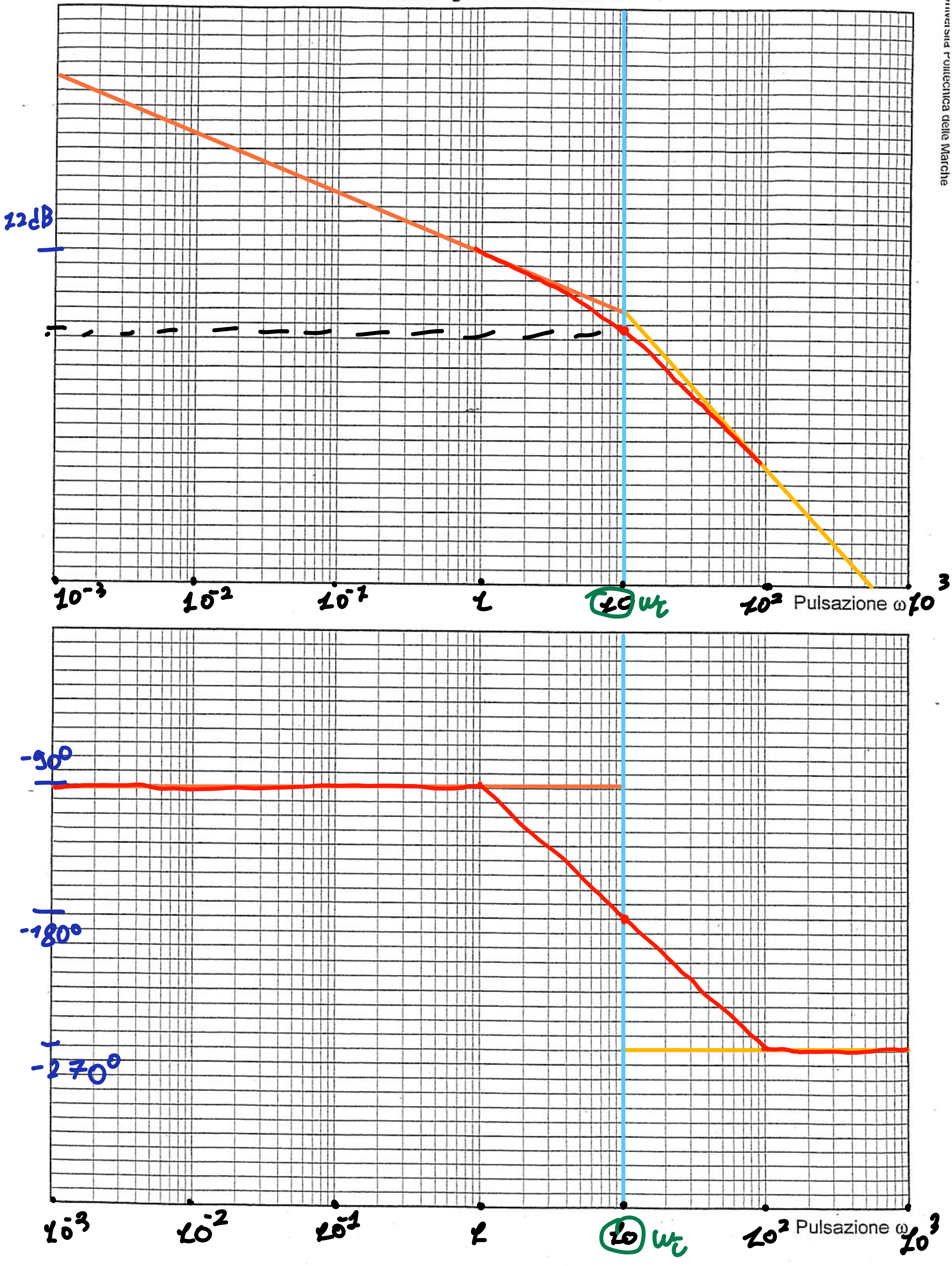
CORREZIONE MODULO

ω	$\left(1+\frac{i\omega}{10}\right)^2$
10	$-6\,dB$

CORREZIONE FASE

ω	$i\omega$	$\left(1+\frac{i\omega}{10}\right)^2$	TOT
10	$-90°$	$-90°$	$-180°$

Carta semilogaritmica a 6 decadi



$|F(i\omega_t)| = -12\,dB$

$\angle F(i\omega_t) = -180^\circ \Rightarrow M_\varphi = 0$

OBIETTIVO:

$|F(i\omega_t)| = 0 \qquad M_\varphi \geq 47^\circ$

$\Downarrow$

FUNZIONE ANTICIPATRICE

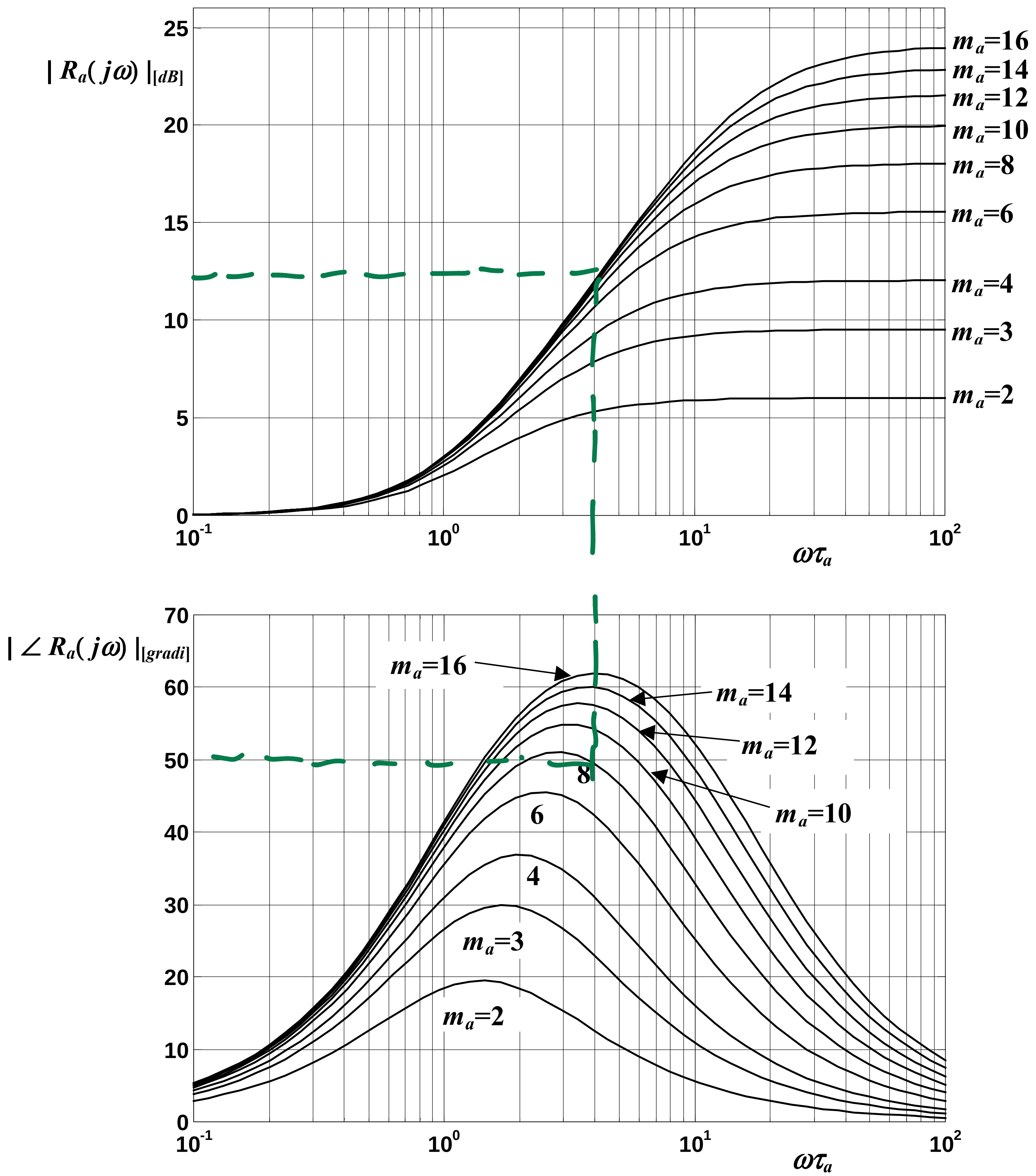
$m_a = 8$

$$\omega_t \tau_a = 4 \Rightarrow \omega_a = \frac{\omega_t}{4} = 2{,}5 \ \frac{rad}{s}$$

$$R_a(s) = \frac{1 + \frac{s}{\omega_a}}{1 + \frac{s}{m_a \omega_a}} = \frac{1 + \frac{s}{2{,}5}}{1 + \frac{s}{20}}$$

$$\Rightarrow G(s) = 200 \cdot \frac{1 + \frac{s}{2{,}5}}{1 + \frac{s}{20}}$$

$$F(s) = 4 \cdot \frac{1}{s\left(1 + \frac{s}{20}\right)^2} \cdot \frac{1 + \frac{s}{2{,}5}}{1 + \frac{s}{20}}$$



| $R_a(j\omega)$ |[dB]
25
20
15
10
5
0
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
m_a=16
m_a=14
m_a=12
m_a=10
m_a=8
m_a=6
m_a=4
m_a=3
m_a=2
| ∠ $R_a(j\omega)$ |[gradi]
70
60
50
40
30
20
10
0
m_a=16
m_a=14
m_a=12
m_a=10
8
6
4
m_a=3
m_a=2
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$

$$F(i\omega) = 4 \cdot \frac{1}{i\omega\left(1+\frac{i\omega}{10}\right)^2} \cdot \frac{1+\frac{i\omega}{2,5}}{1+\frac{i\omega}{20}}$$

$4 \rightarrow 20\log_{10} 4 = 12\,dB$

PUNTI DI ROTTURA

- $\omega = 0$ $-20\,dB$ -90° $-20\,dB$ -90°
- $\omega = 2,5$ $+20\,dB$ $+90^\circ$ 0 0
- $\omega = 10$ $-40\,dB$ -180° $-40\,dB$ -180°
- $\omega = 20$ $-20\,dB$ -90° $-60\,dB$ -270°

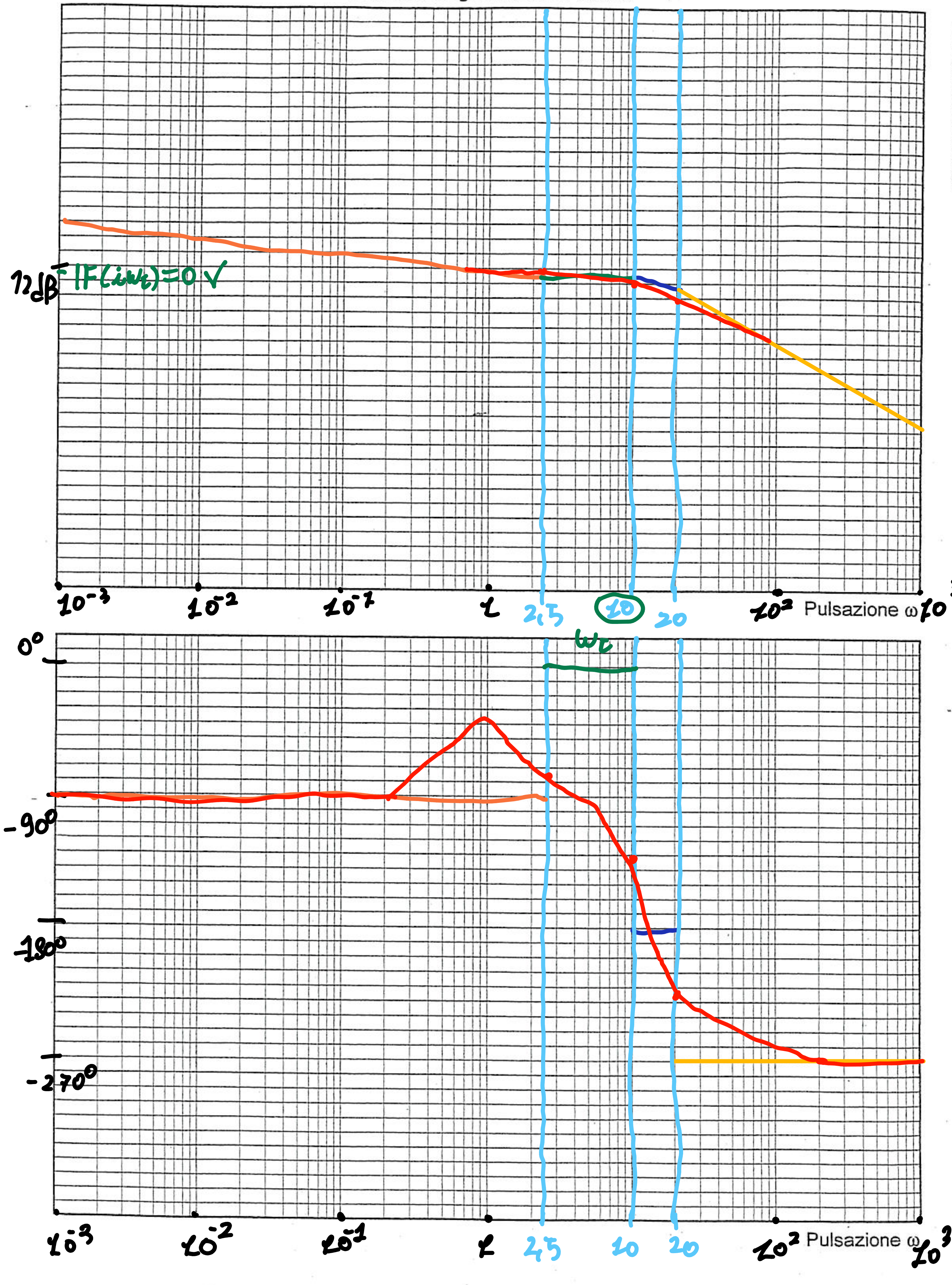
CORREZIONE MODULO

ω	$1+\frac{i\omega}{2,5}$	$\left(1+\frac{i\omega}{10}\right)^2$	$1+\frac{i\omega}{20}$	TOT
2,5	$+3\,dB$	$-1\,dB$	0	$+2\,dB$
10	0	$-6\,dB$	$-1\,dB$	$-7\,dB$
20	0	$-2\,dB$	$-3\,dB$	$-5\,dB$

CORREZIONE FASE

ω	$i\omega$	$1+\frac{i\omega}{2,5}$	$\left(1+\frac{i\omega}{10}\right)^2$	$1+\frac{i\omega}{20}$	TOT
2,5	-90°	$+45^\circ$	-30°	0	-75°
10	-90°	$+75^\circ$	-90°	-25°	-130°
20	-90°	0	-130°	-45°	-175°

Carta semilogaritmica a 6 decadi



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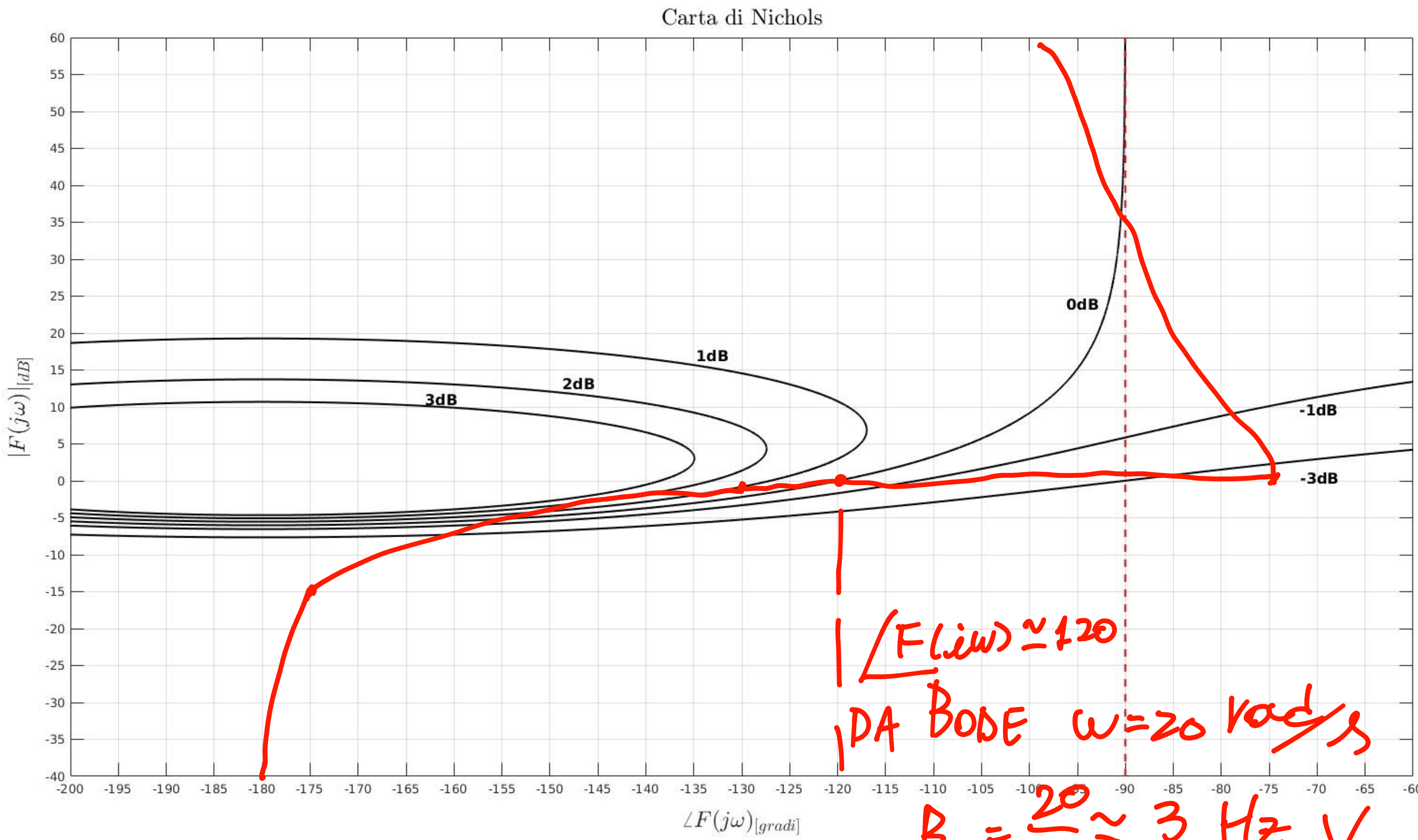


$B_3 = \frac{20}{2\pi} \simeq 3$ Hz V

$$u(t) = (2t+3)\delta_{-1}(t) = 2(t)\delta_{-1}(t) + (3)\delta_{-1}(t)$$

$$= 2u_1(t) + 3u_2(t)$$

- $u_1(t)$

$$\tilde{e}_{u_1}(t) = K_d u_1(t) - \tilde{y}_{u_1}(t)$$

$$\tilde{e}_{u_1}(t) = \frac{1}{K_d \cdot K_p} = \frac{1}{K_F} = \frac{1}{4}$$

$$\tilde{y}_{u_1}(t) = K_d u_1(t) - \tilde{e}_{u_1}(t) = \left(t - \frac{1}{4}\right)\delta_{-1}(t)$$

- $u_2(t)$

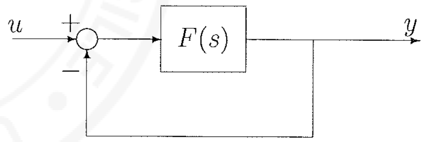
GRADO DI $u_2(t) <$ TIPO DI $F(s) \Rightarrow \tilde{y}_{u_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 2\left(t - \frac{1}{4}\right)\delta_{-1}(t) + 3\delta_{-1}(t)$$

Domanda Scritta di Controlli Automatici (9 CFU) - 17/02/2014

Esercizio 1

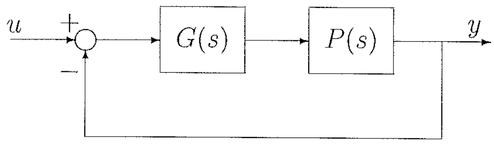
È dato il sistema di controllo:



in cui: $F(s) = \dfrac{K(s+p)}{s(s-1)^2}$. Utilizzando il criterio di Nyquist, studiare la stabilità del sistema a ciclo chiuso, per $K > 0$, $p < 0$ ($p \neq 0$, $p \neq -1$).

Esercizio 2

È dato il sistema di controllo:



in cui: $P(s) = \dfrac{s+4}{s(s+3)^2}$. Utilizzando il luogo delle radici, progettare $G(s)$ in modo che:

- $|\tilde{e}_1| \leq 0.05$;
- tutti i poli a ciclo chiuso abbiano parte reale minore od uguale a -2.

Calcolare infine la risposta a regime permanente all'ingresso $u(t) = (3t+2)\delta_{-1}(t)$.

1

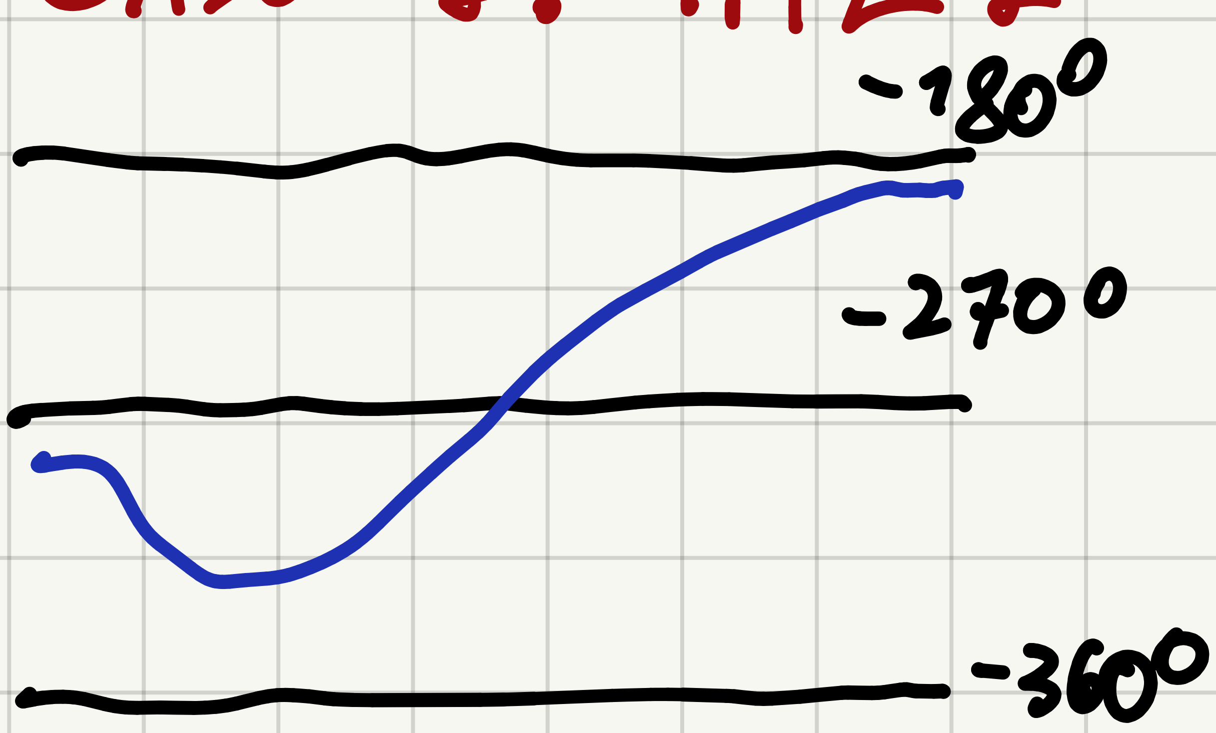
$$F(s) = K \cdot \frac{(s+p)}{s(s-1)^2} \qquad K>0,\ p<0 \qquad p\neq 0,\ p\neq -1$$

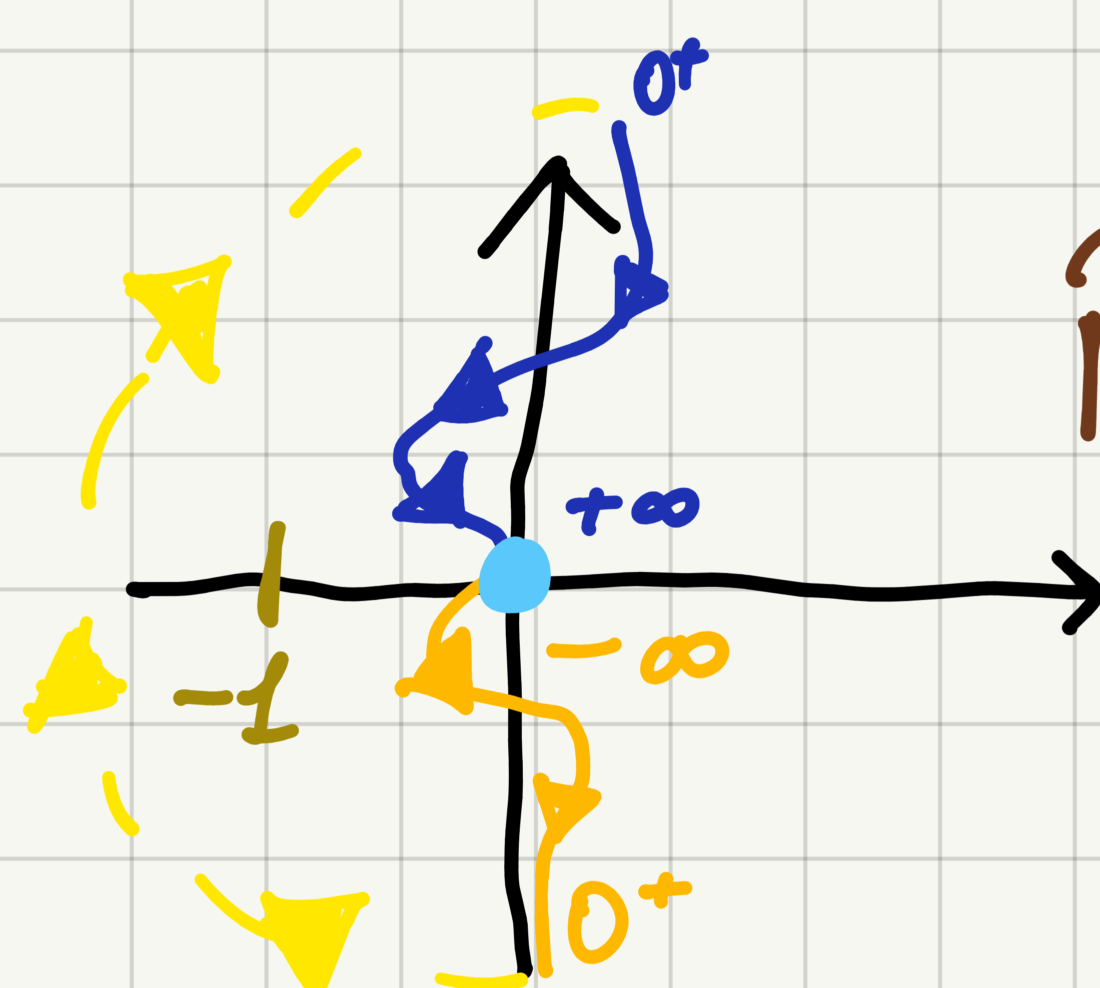
$$F(s) = Kp \cdot \frac{\left(1+\frac{s}{p}\right)}{s(1-s)^2} \Rightarrow F(i\omega) = Kp \cdot \frac{\left(1+\frac{i\omega}{p}\right)}{i\omega(1-i\omega)^2}$$

$$\angle F(i\omega) = -270^\circ - \arctan\left(\frac{\omega}{|p|}\right) + 2\arctan(\omega) \qquad P_+ = 2$$

$M(0^+) = \infty$, $\varphi(0^+) = -270^\circ$ $\quad M(+\infty) = 0$, $\varphi(+\infty) = -180^\circ$

CASO 1: $|p| < 1$

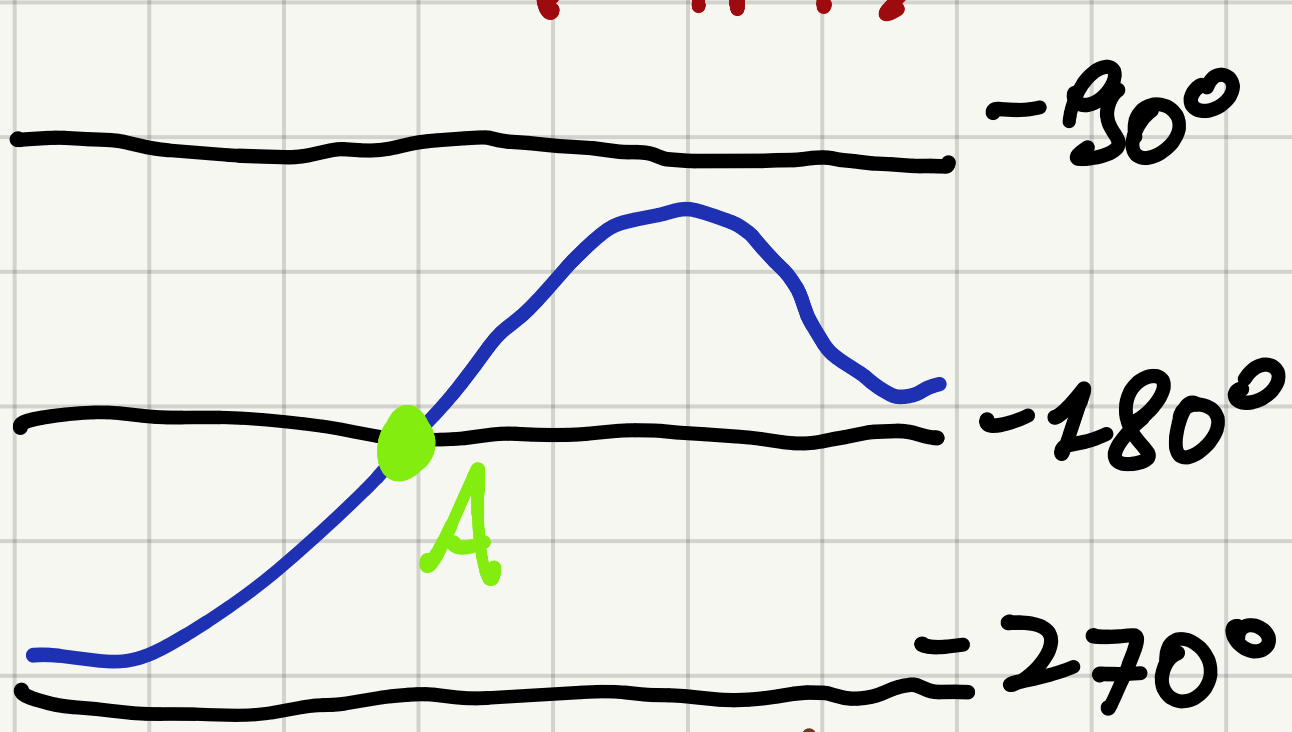


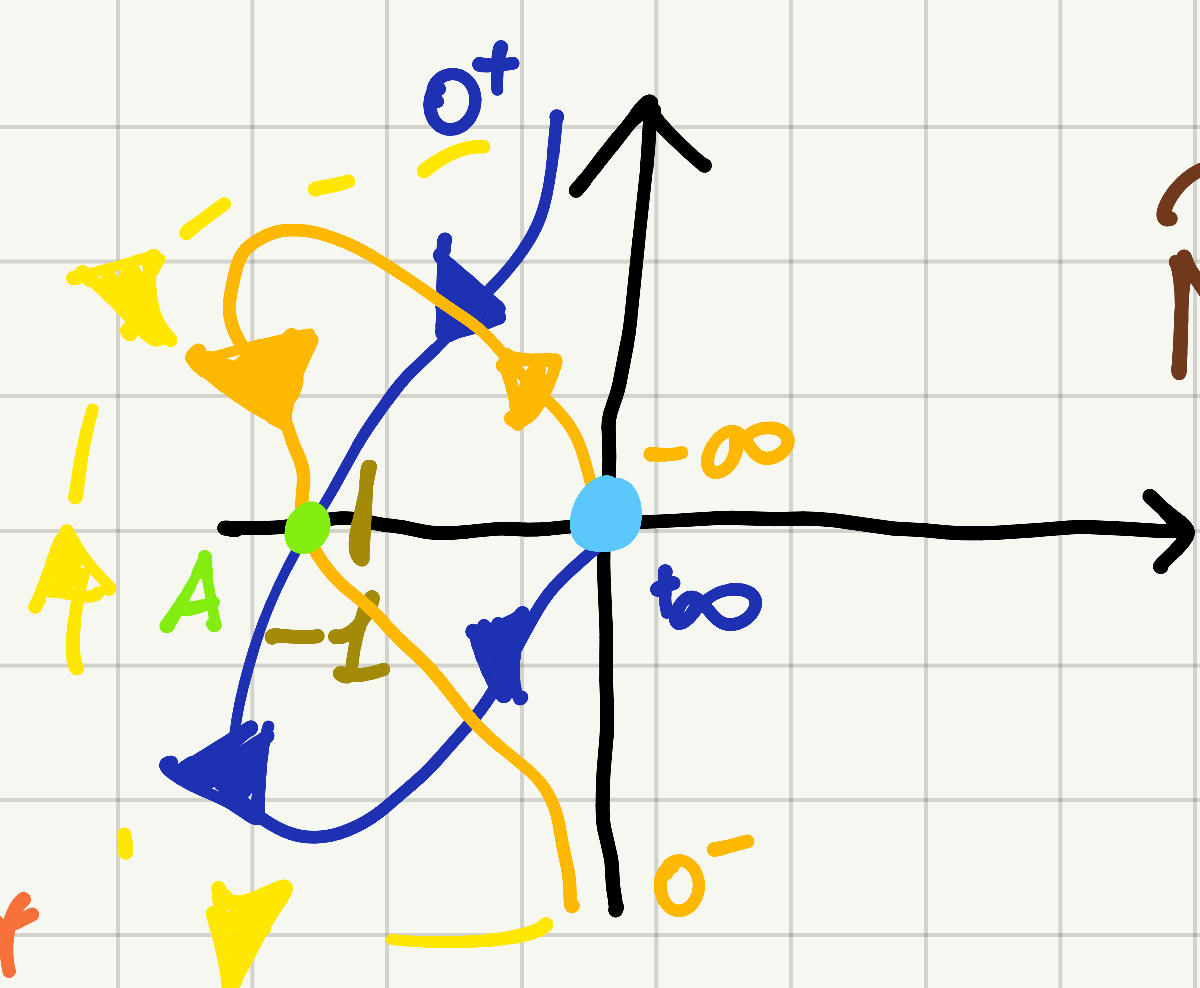


$\overset{\curvearrowright}{N} = 1$

$\overset{\curvearrowright}{N} \neq -P_+ \Rightarrow$ SISTEMA INSTABILE

CASO 2: $|p| > 1$





$\overset{\curvearrowright}{N} = 1$

$|A| < 1$ $\quad \overset{\curvearrowright}{N} = -1 \neq -P_+$

$|A| > 1$ $\quad \overset{\curvearrowright}{N} = -1 \neq -P_+$

SISTEMA INSTABILE $\forall\, p, A$

2

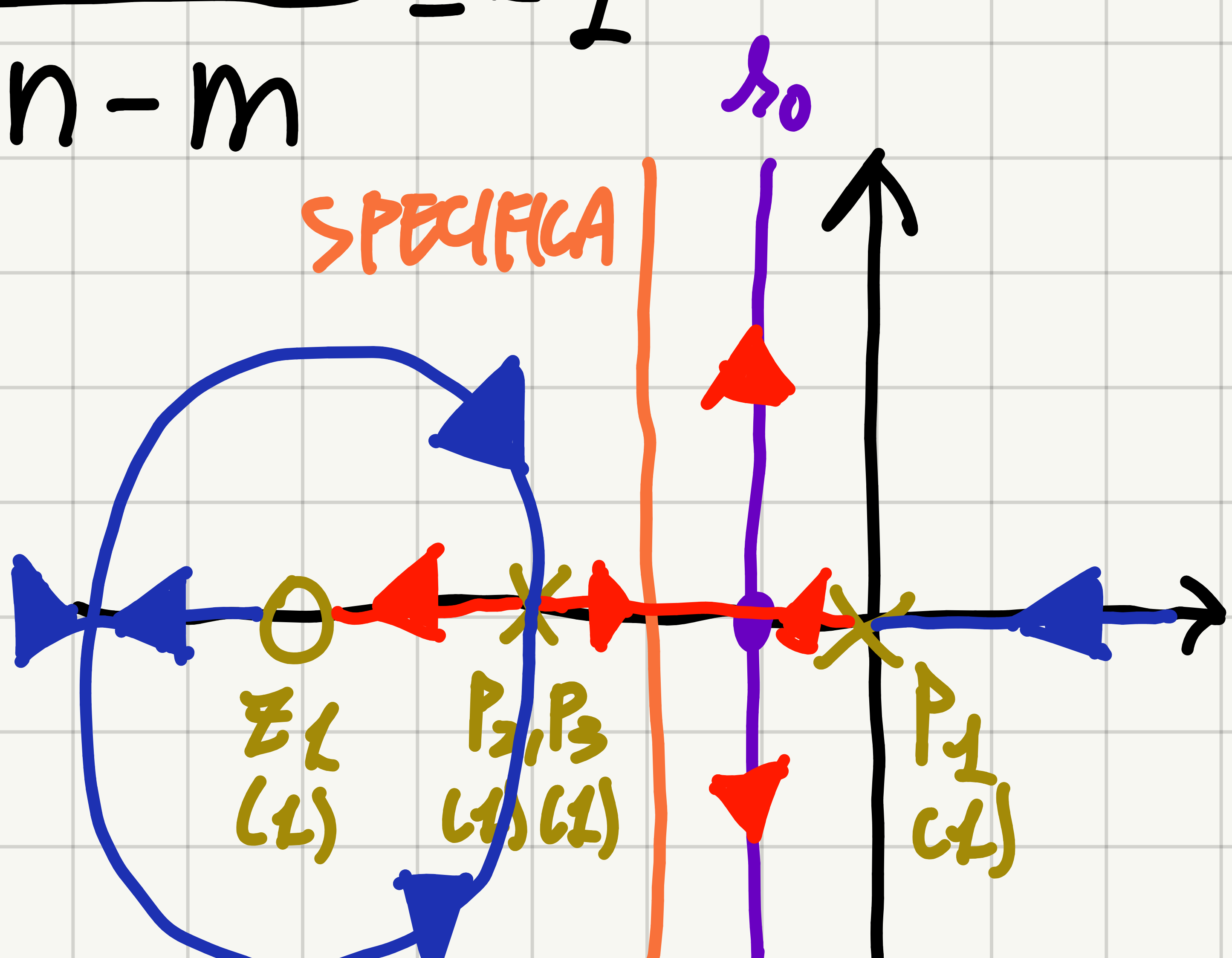
$$|\tilde{e}_1| = \left| \frac{1}{K_G K_P} \right| \leq 0{,}05 \qquad K_P = \frac{4}{9} \Rightarrow K_G \geq 45$$

$$G(s) = K \qquad F(s) = G(s) \cdot P(s) = K \cdot \frac{s+4}{s(s+3)^2}$$

$$n = 3, m = 1 \Rightarrow n - m = 2$$

$$Z_1 = -4 ; \; P_1 = 0, \, P_2 = -3, \, P_3 = -3$$

$$s_0 = \frac{\Sigma P - \Sigma Z}{n - m} = -1$$



SPECIFICA NON SODDISFATTA

INTRODUCO COPPIA POLO-ZERO PER SPOSTARE $s_0 < 2$ (POICHÈ $G(s)$ HA ZERO POLI, NON POSSO INTRODURRE NUOVI ZERI DA SOLI) PONGO $s_0 = -4$

$$G(s) = K \cdot \frac{(s-z)}{(s-p)} \Rightarrow F(s) = K \cdot \frac{(s+4)(s-z)}{s(s+3)^2 (s-p)}$$

$$s_0 = -4 \qquad -4 = \frac{p - 3 - 3 - (-4 + z)}{2}$$

SCELGO $z = -3$
$\Rightarrow p = -9$

$$\Rightarrow G(s) = K \cdot \frac{(s+3)}{(s+9)} \Rightarrow F(s) = K \cdot \frac{(s+4)}{s(s+3)(s+9)}$$

$$K_G \geq 45 \Rightarrow K \cdot \frac{3}{9} \geq 45 \Rightarrow K \geq 135$$

$Z_1 = -4$; $P_1 = 0$, $P_2 = -3$; $P_3 = -9$

$$f(s, K) = s(s+3)(s+9) + K(s+4) = 0$$

$$s = \bar{s} - 2$$

$$\Rightarrow (\bar{s}-2)(\bar{s}+1)(\bar{s}+7) + K(\bar{s}+2) = 0$$

$$(\bar{s}^2 - \bar{s} - 2)(\bar{s}+7) + K(\bar{s}+2) = 0$$

$$\bar{s}^3 + 6\bar{s}^2 + (K-9)\bar{s} + (2K-14) = 0$$

3	1	$K-9$
2	6	$2K-14$
1	$\frac{2K-20}{3}$	
0	$2K-14$	

$$\begin{cases} \frac{2K-20}{3} > 0 \\ 2K-14 > 0 \end{cases} \Rightarrow K > 10$$

$$\Rightarrow \begin{cases} K \geq 135 \\ K > 10 \end{cases} \Rightarrow G(s) = 140 \cdot \frac{s+3}{s+9}$$

$$U(t) = (3t+2)\delta_{-1}(t) = 3(t)\delta_{-1}(t) + (2)\delta_{-1}(t)$$

$$= 3U_1(t) + 2U_2(t)$$

- $U_1(t)$

$$\tilde{e}_{U_1}(t) = K_d U_1(t) - \tilde{y}_{U_1}(t)$$

$$\tilde{e}_{U_1}(t) = \frac{1}{K_G \cdot K_P} = \frac{1}{K_F} = 945$$

$$\tilde{y}_{U_1}(t) = K_d U_1(t) - \tilde{e}_{U_1}(t) = (t-945)\delta_{-1}(t)$$

- $U_2(t)$

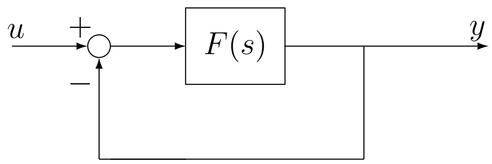
GRADO DI $U_2(t) <$ TIPO DI $F(s) \Rightarrow \tilde{y}_{U_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 3(t-945)\delta_{-1}(t) + 2\delta_{-1}(t)$$

Domanda Scritta di Controlli Automatici - 15/06/2015

Esercizio 1

È dato il sistema in controreazione:



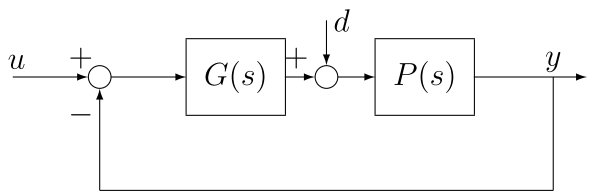
in cui:

$$F(s) = \frac{K(s-z)}{s^2(s+6)}, \quad K \in \mathbb{R}, \quad z \in \mathbb{R}, \quad K \neq 0, \quad z \neq 0 \quad z \neq -6.$$

Utilizzando il criterio di Nyquist, studiare la stabilità del sistema in catena chiusa al variare di K e z.

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{s+6}{s^2+8s+25}; \qquad d(t) = \delta_{-1}(t).$$

Utilizzando il luogo delle radici, progettare $G(s)$ in modo che:

- il sistema sia astatico rispetto a $d(t)$;
- tutti i poli a ciclo chiuso abbiano parte reale minore od uguale a -3.

Calcolare infine la risposta a regime permanente all'ingresso $u(t) = (5t-3)\delta_{-1}(t)$.

1

$$F(s) = -KZ \cdot \frac{(1-\frac{s}{Z})}{s^2(1+\frac{s}{6})} \qquad F(i\omega) = -KZ \cdot \frac{(1-\frac{i\omega}{Z})}{(i\omega)^2(1+\frac{i\omega}{6})}$$

CASO 1: $K>0, Z>0$ $\quad P_+ = 0$

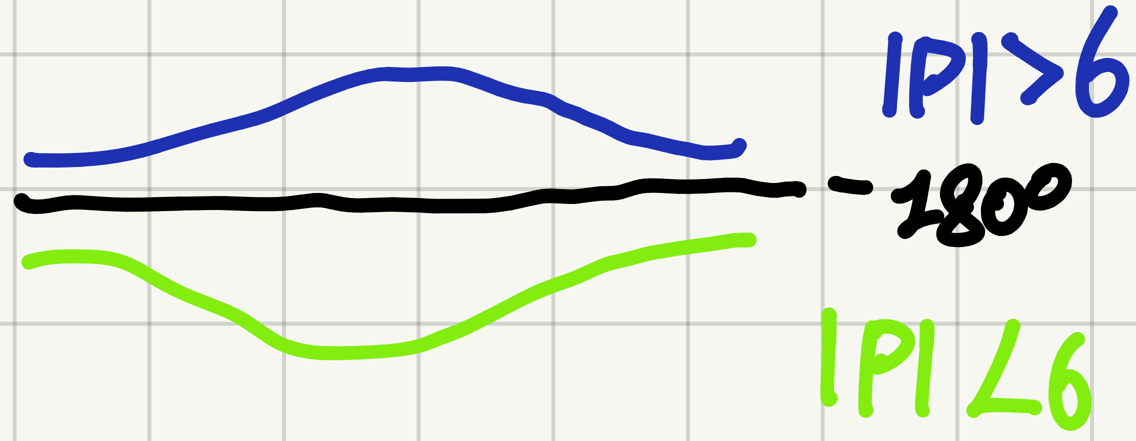
$M(0^+) = \infty, \varphi(0^+) = -360°$ $\quad M(+\infty) = 0, \varphi(+\infty) = -540°$

$\tilde{N} = 1$

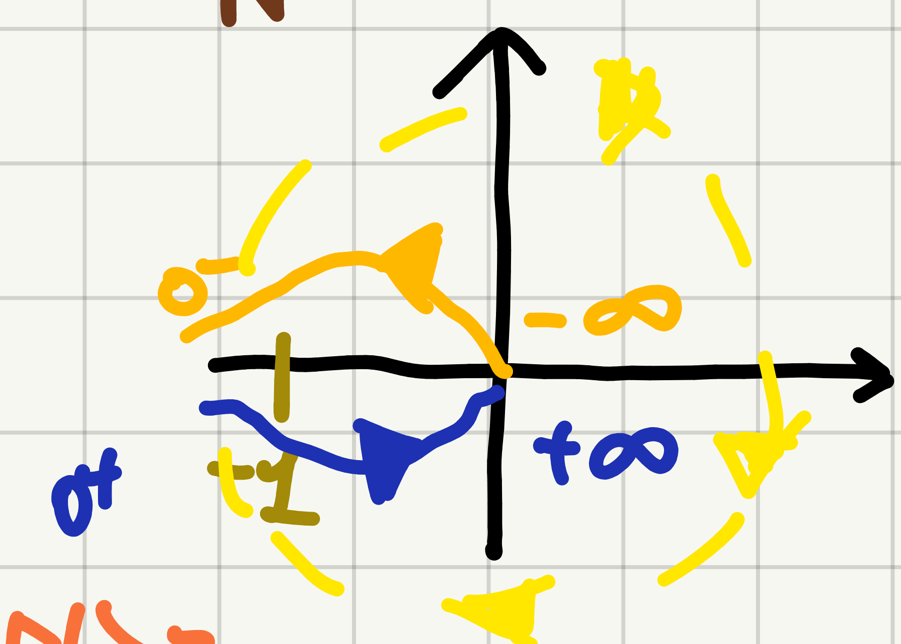
$\tilde{N} \neq -P_+ \Rightarrow$ SISTEMA INSTABILE

CASO 2: $K>0, Z<0$ $\quad P_+ = 0$

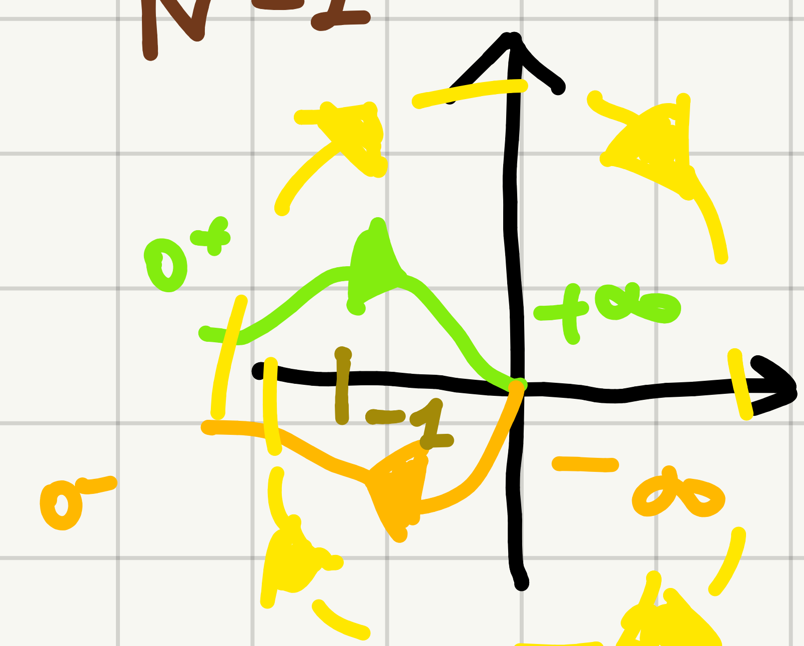
$M(0^+) = \infty, \varphi(0^+) = -180°$ $\quad M(+\infty) = 0, \varphi(+\infty) = -180°$



$\tilde{N} = 0$



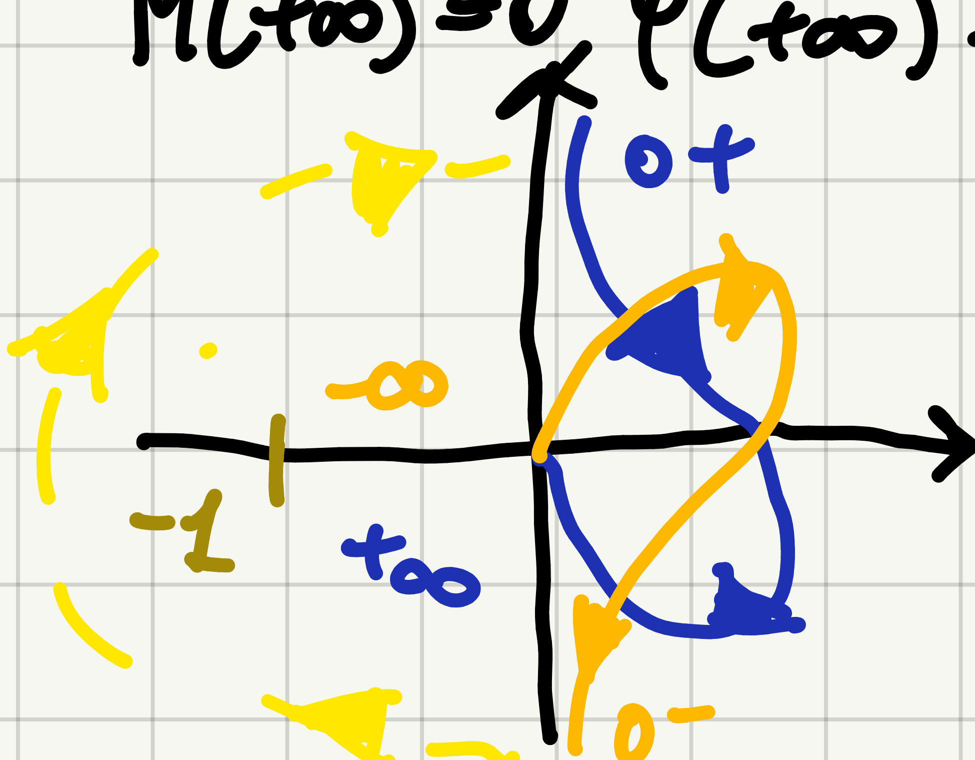
$\tilde{N} = 1$



SISTEMA STABILE PER $|P|>3$

CASO 3: $K<0, Z>0$ $\quad P_+ = 0$

$M(0^+) = \infty, \varphi(0^+) = -180°$ $\quad M(+\infty) = 0, \varphi(+\infty) = -360°$

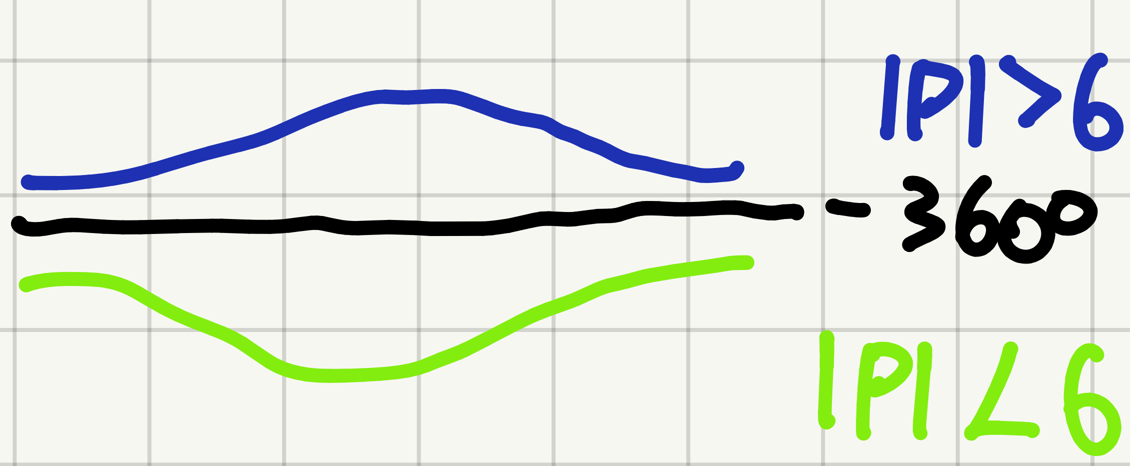


$\tilde{N} = 1$

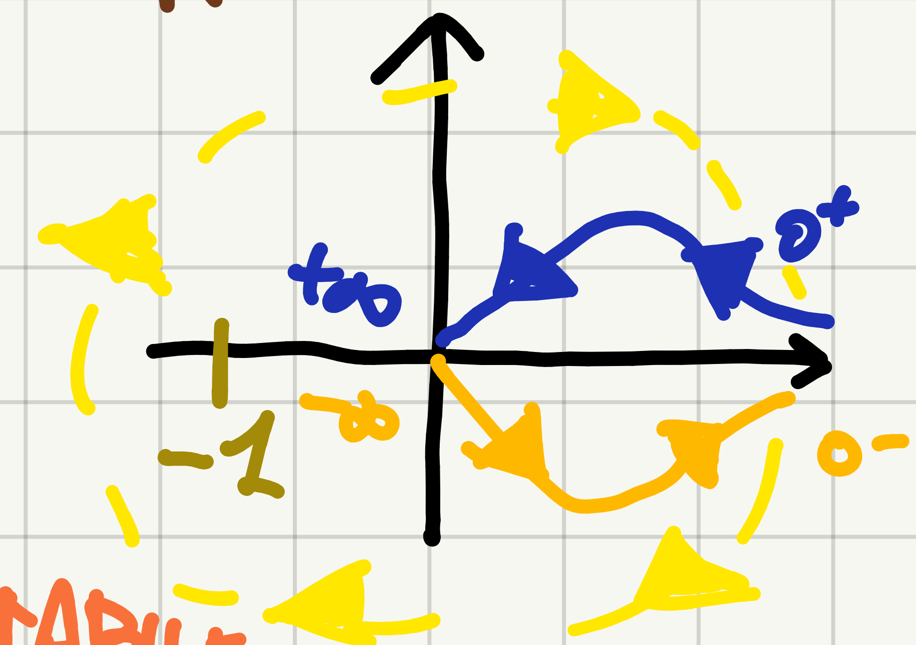
$\tilde{N} \neq -P_+ \Rightarrow$ SISTEMA INSTABILE

CASO 4: $K<0, Z<0$ $\quad P_+ = 0$

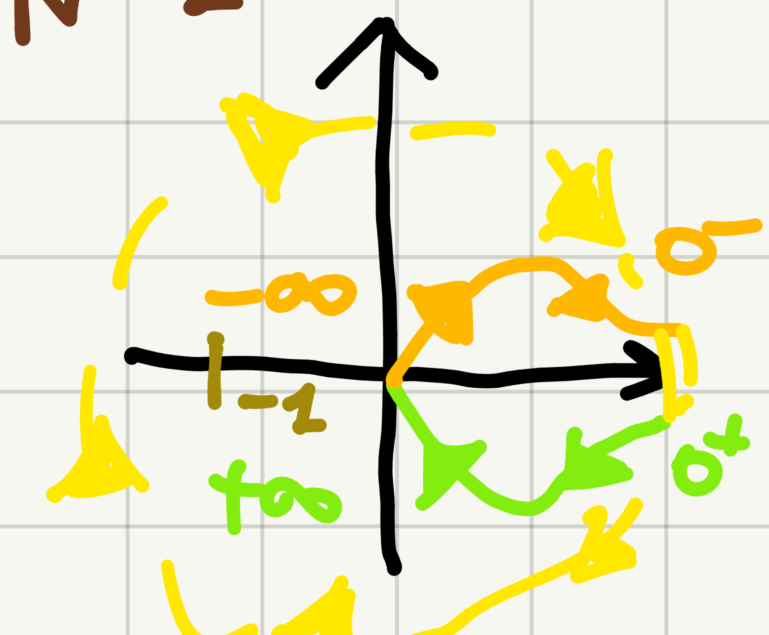
$M(0^+) = \infty, \varphi(0^+) = -360°$ $\quad M(+\infty) = 0, \varphi(+\infty) = -360°$



$\tilde{N} = 1$



$\tilde{N} = 1$



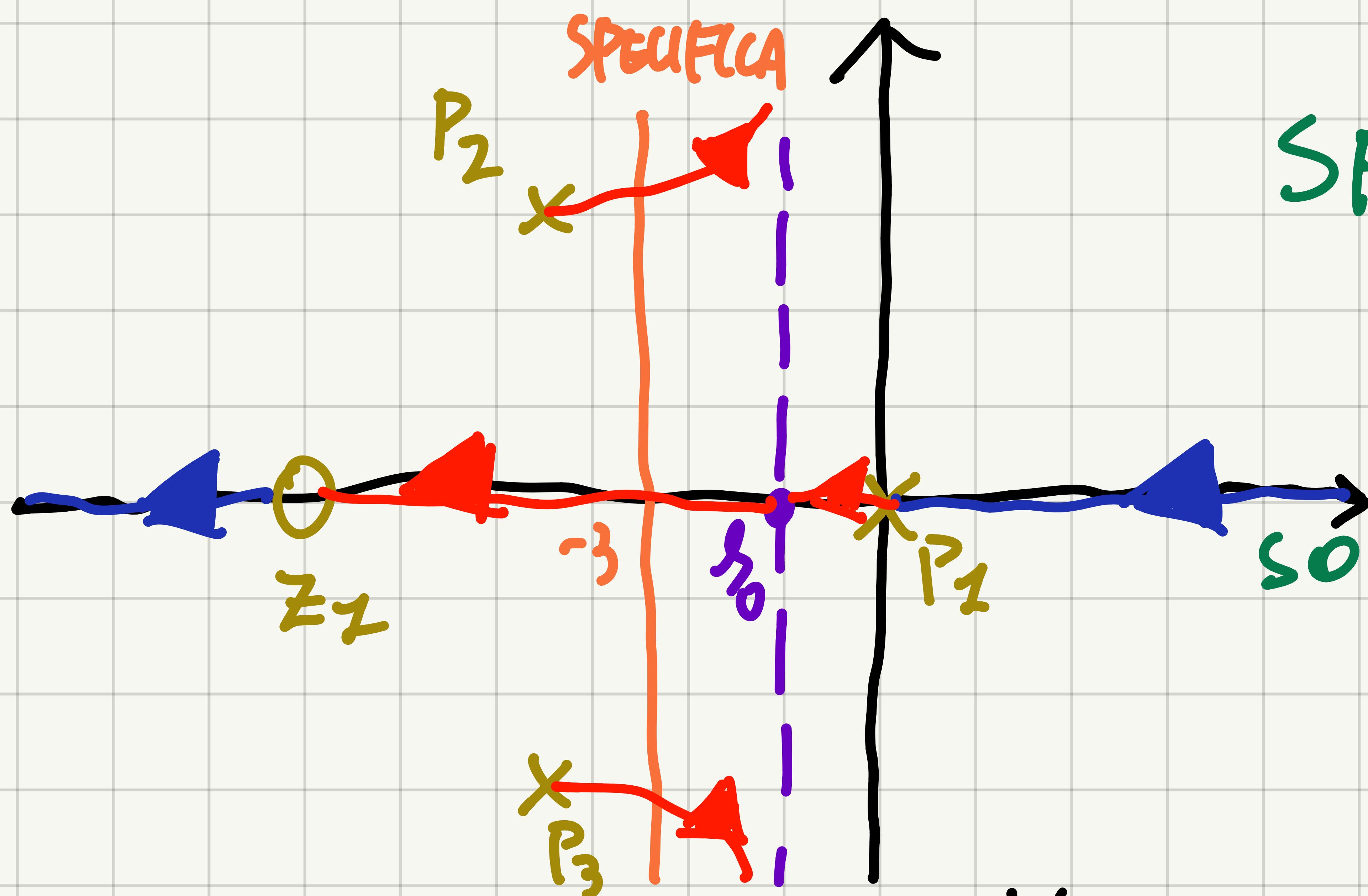
$\tilde{N} \neq -P_+ \Rightarrow$ SISTEMA INSTABILE

2

$$P(s) = \frac{s+6}{s^2+8s+25} \qquad G(s) = \frac{K}{s} \quad \text{CASTATISMO } \delta_{-1}(t)$$

$$F(s) = G(s) \cdot P(s) = K \cdot \frac{s+6}{s(s^2+8s+25)} \qquad n=3, m=1 \Rightarrow n-m=2$$

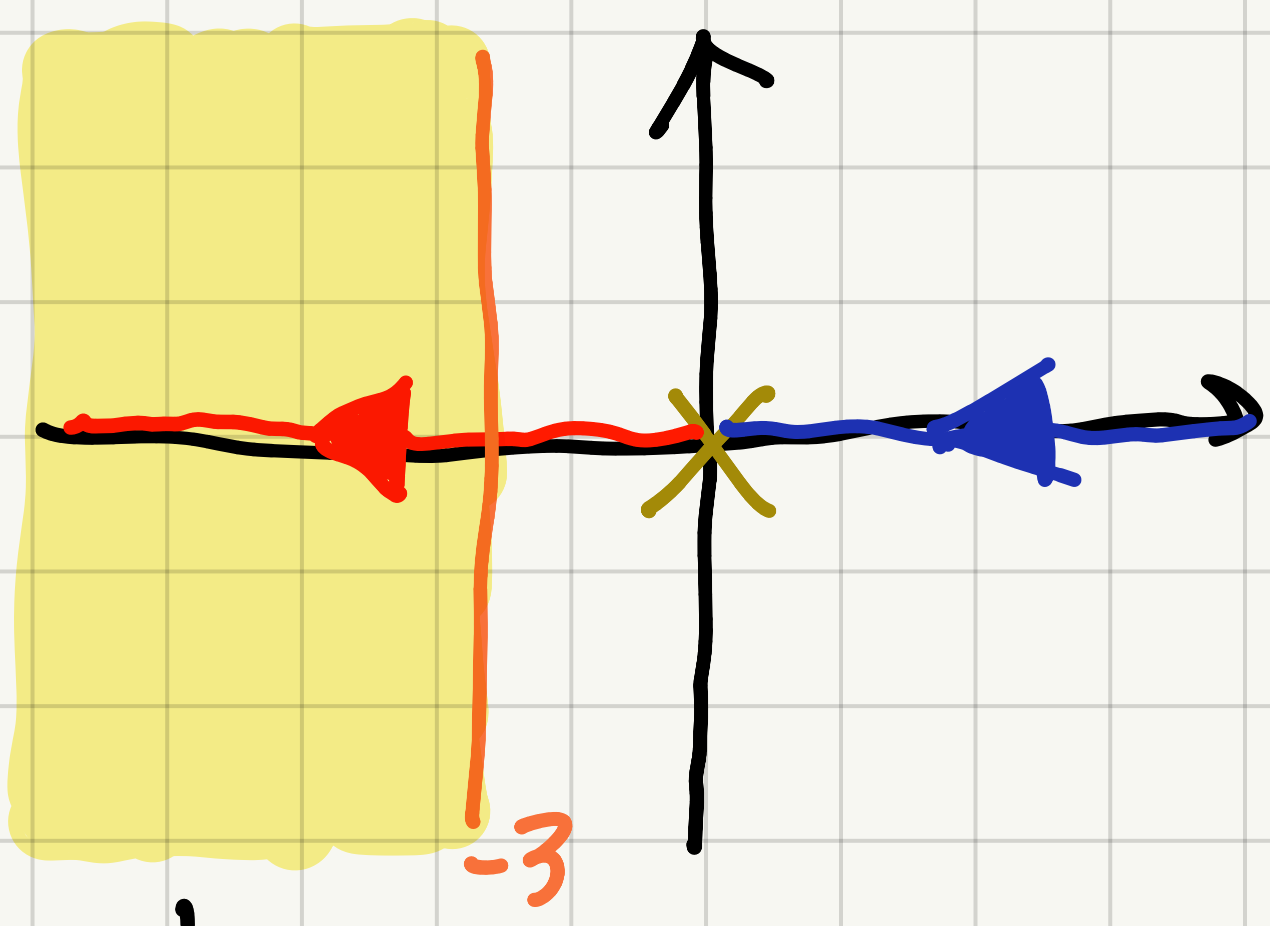
$$Z_1 = -6;\ P_1 = 0,\ P_2 = -4+3i,\ P_3 = -4-3i \qquad s_0 = \frac{\Sigma P - \Sigma Z}{n-m} = -1$$



SPECIFICA NON SODDISFATTA

$$n-m=1 \longrightarrow G(s) = \frac{K}{s} \cdot (s-Z_1) \cdot \frac{(s-z_2)}{(s-p)}$$

$$\longrightarrow G(s) = \frac{K}{s} \cdot \frac{s^2+8s+25}{s+6} \Rightarrow F(s) = \frac{K}{s}$$



$$p(s,K) = s+K \Big|_{s=-3} = 0 \Rightarrow \text{STABILITÀ } \forall\ K > 3$$

$$\Rightarrow F(s) = \frac{4}{s}$$

$$u(t) = (5t-3)\delta_{-1}(t) = 5(t)\delta_{-1}(t) + (-3)\delta_{-1}(t)$$

$$= 5u_1(t) - 3u_2(t)$$

- $u_1(t)$

$$\tilde{e}_{u_1}(t) = K_d u_1(t) - \tilde{y}_{u_1}(t)$$

$$\tilde{e}_{u_1}(t) = \frac{1}{K_d \cdot K_p} = \frac{1}{K_F} = \frac{1}{4}$$

$$\tilde{y}_{u_1}(t) = K_d u_1(t) - \tilde{e}_{u_1}(t) = \left(t - \frac{1}{4}\right)\delta_{-1}(t)$$

- $u_2(t)$

GRADO DI $u_2(t)$ $<$ TIPO DI $F(s)$ $\Rightarrow \tilde{y}_{u_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 5\left(t - \frac{1}{4}\right)\delta_{-1}(t) - 3\delta_{-1}(t)$$