$$\begin{vmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$T = mRm^{-1} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \cdot$$

$$\begin{vmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \Rightarrow O = V$$

EXTRA: IN THE DIAGRAM BELOW, ASSUME f IS A FUNCTION, E AN EQUIVALENCE RELATION ON A, F THE INVERSE IMAGE OF E ALONG f AND p, q THE CORRESPONDING QUOTIENTS. PROVE THAT f INDUCES A FUNCTION g BETWEEN THE QUOTIENTS, I.E. THERE EXISTS A UNIQUE FUNCTION g SUCH THAT $fp = qg$. PROVE THAT g IS INJECTIVE AND f IS SURJECTIVE. THEN g IS AN ISOMORPHISM AND DETERMINE g WHEN

$A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$, $M(E) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$, $M(f) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$$\begin{tikzcd}
B \arrow[r, "f"] \arrow[d, "F"'] & A \arrow[d, "E"] \\
B \arrow[r, "f"] \arrow[d, "q"'] & A \arrow[d, "p"] \\
B/F \arrow[r, dashed, "g"] & A/E
\end{tikzcd}$$

$g = q^{op} f p$

BEING QUOTIENT SET, q IS SURJECTIVE $\Rightarrow \mathrm{Ker}(q) = q q^{op} = F =$

$= f E f^{op} = f p p^{op} f^{op} = (fp)(fp)^{op} = \mathrm{Ker}(fp)$ *

$\Rightarrow \mathrm{Ker}(g) = q_*(\mathrm{Ker}(q)) = q_*(q q^{op}) = q^{op}(q q^{op}) = 1$

$\Rightarrow$ g IS INJECTIVE

*

BY EPI-MONO FACTORIZATION, $fp = qg$, g UNIQUE

$\Rightarrow$ g IS SURJECTIVE $\Rightarrow$ g IS AN ISOMORPHISM

CALCULUS

$g = q^{op} f p$. FROM M(E), WE HAVE $p = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{vmatrix}$.

$f = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$. $q : B \to A/F$ $\quad F = f E f^{op}$

$$F = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \bullet \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}$$

* 0 (PROF. NOTES...)

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{vmatrix} =$$

$$= \begin{array}{c} \begin{array}{ccccc} [1] & [2] & [3] & [4] & [5] \end{array} \\ \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{vmatrix} \end{array} \Rightarrow q = \begin{array}{c} \begin{array}{cc} [1] & [3] \end{array} \\ \begin{vmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{vmatrix} \end{array}$$

$$g = \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \bullet \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \bullet \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$