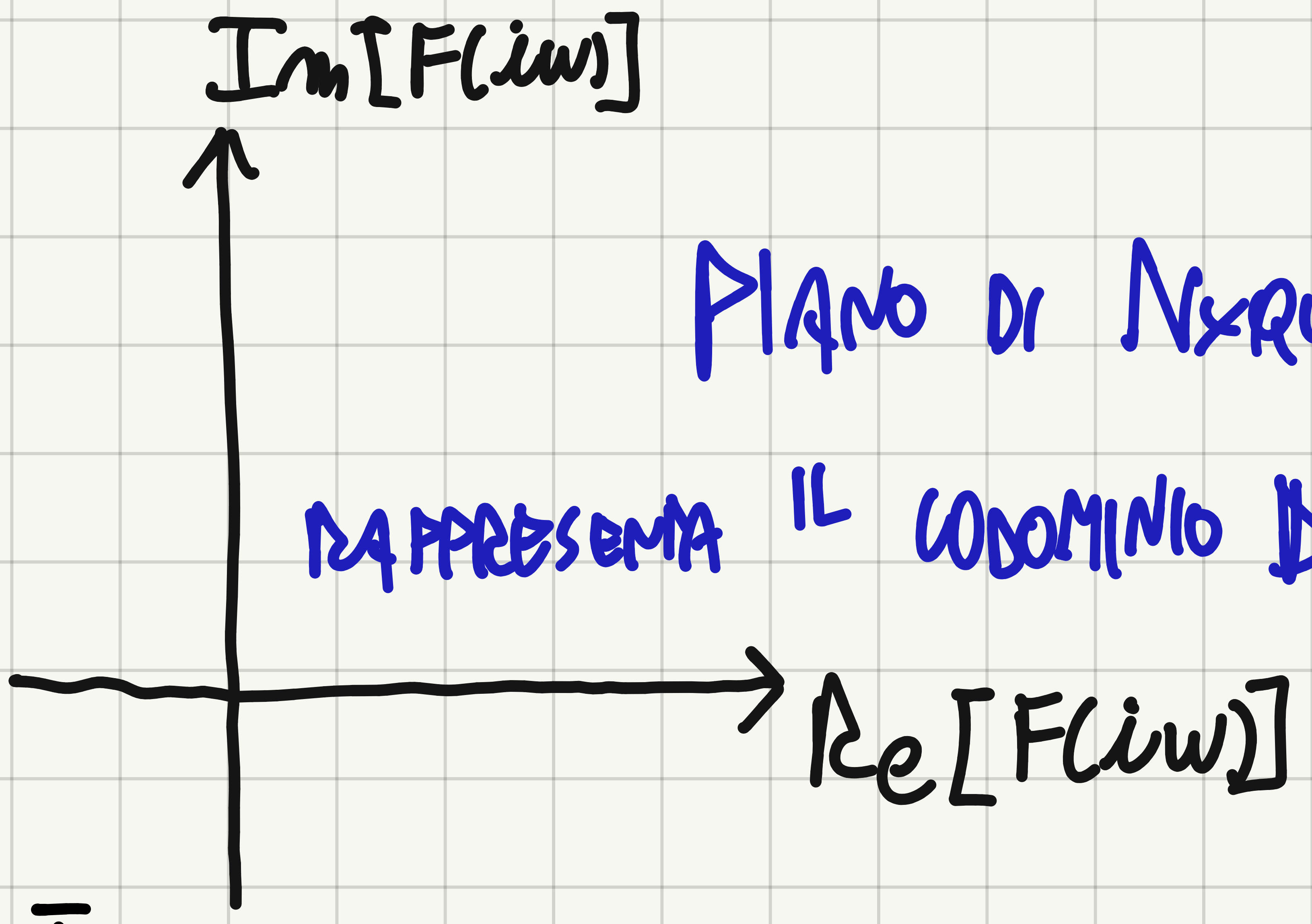
Diagramma Polare
(o diagramma di Nyquist)



Piano di Nyquist, rappresenta il codominio della funzione

Traccia il diagramma qualitativo di Bode della funzione $F(i\omega)$, da cui si estrapolerà un criterio per studiare la stabilità di $W(s)$

Esempio: $F(s)=\frac{8}{s+2}$

1) Forma canonica di Bode $\quad F(s)=\frac{4}{1+\frac{s}{2}}$

2) Scrivere $F(i\omega)$ $\quad F(i\omega)=\frac{4}{1+\frac{i\omega}{2}}$

3) Calcolare modulo e fase di $F(i\omega)$ in $\omega\to0^+$ e $\omega\to+\infty$

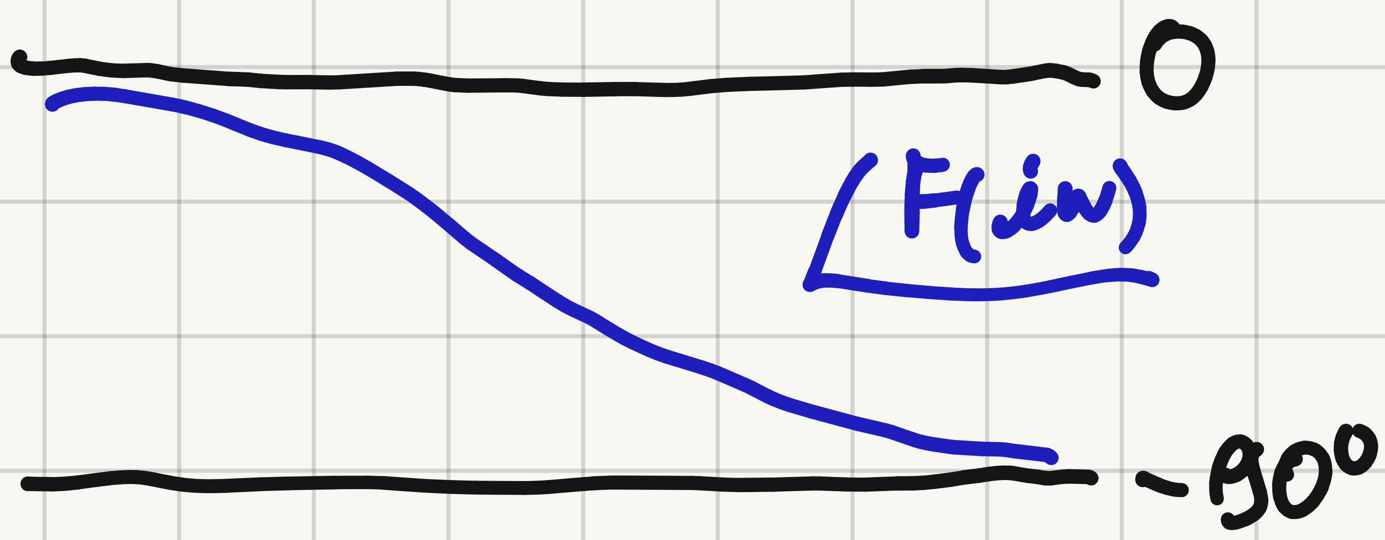
$\lim_{\omega\to0^+}|F(i\omega)|=M(0^+)=4 \qquad \lim_{\omega\to0^+}\angle F(i\omega)=\varphi(0^+)=0°$

$\lim_{\omega\to+\infty}|F(i\omega)|=M(+\infty)=0 \qquad \lim_{\omega\to+\infty}\angle F(i\omega)=\varphi(+\infty)=-90°$

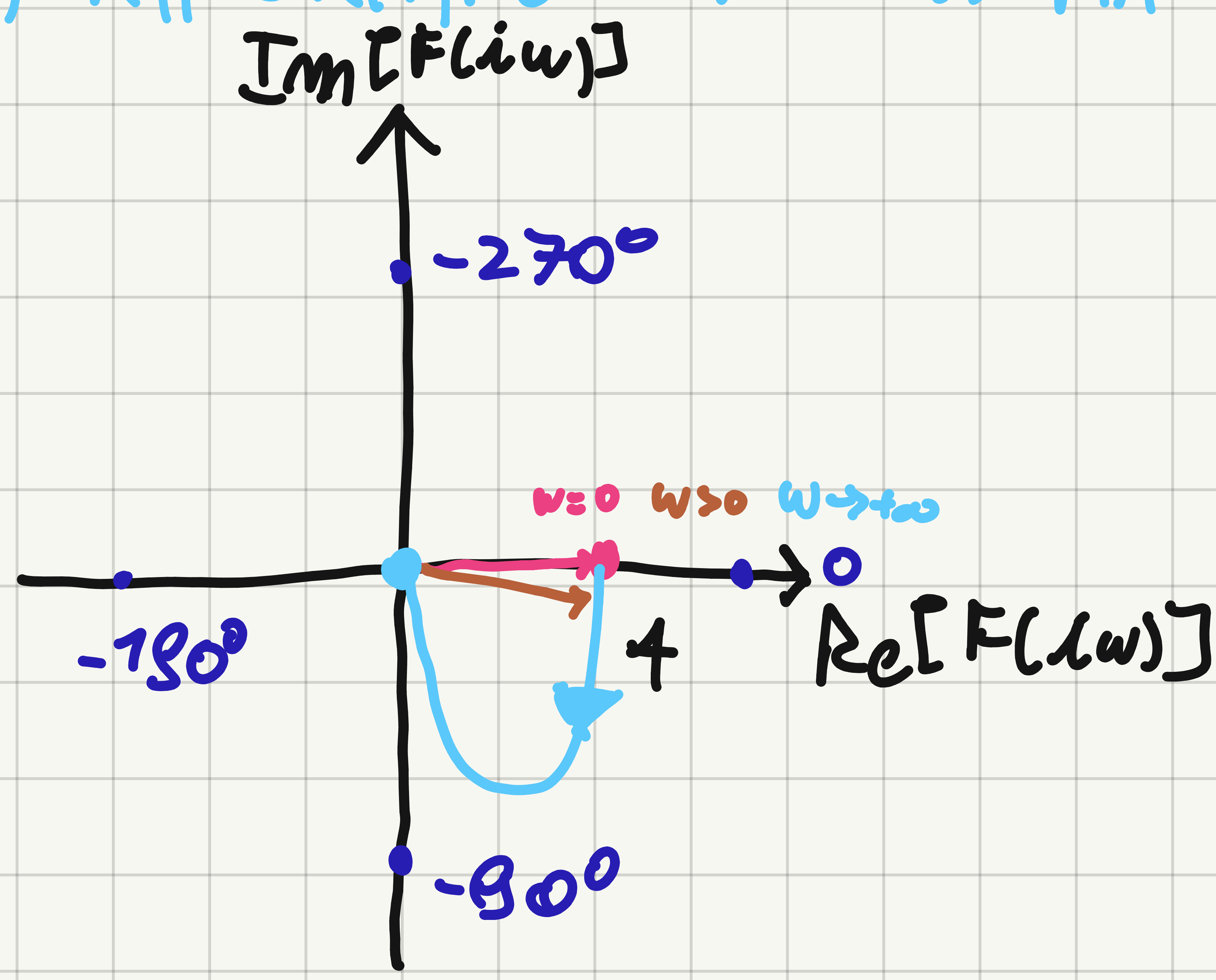
$\angle F(i\omega)=-\arctan\left(\frac{\omega}{2}\right)$

In generale, $\varphi(0^+) = \begin{cases} 0^\circ & K_F > 0 \\ -180^\circ & K_F < 0 \end{cases}$

4) Disegnare il diagramma qualitativo di $\angle F(i\omega)$



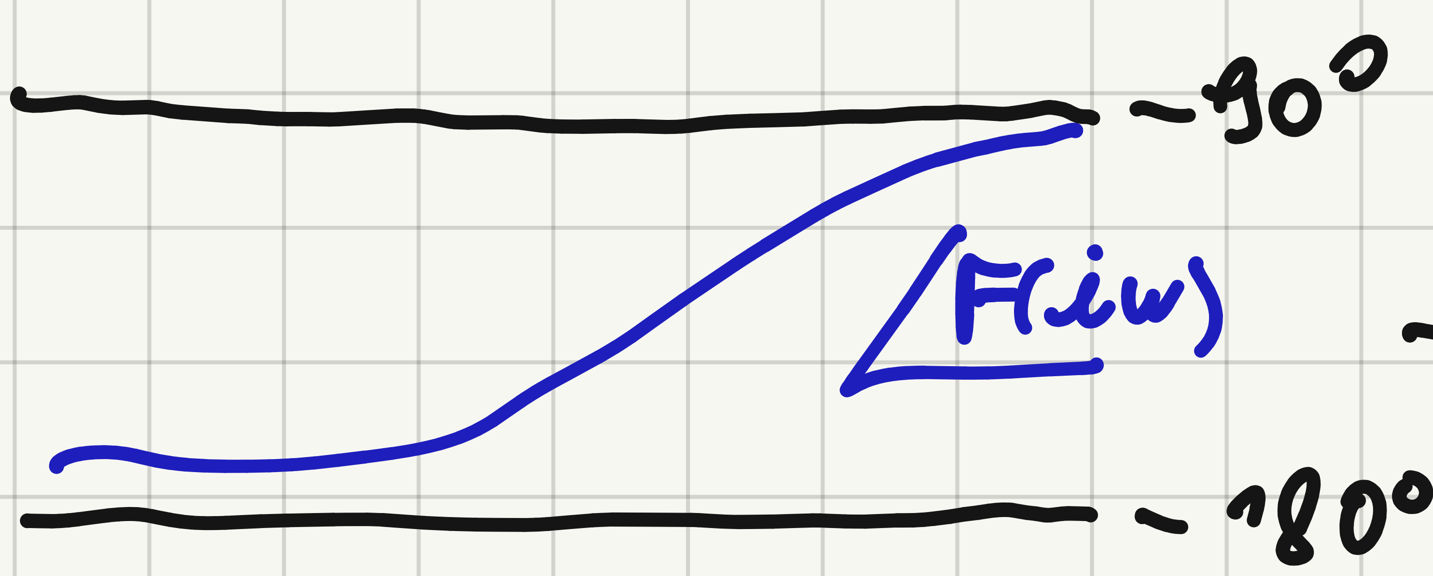
5) Riportare tutto sul piano di Gauss

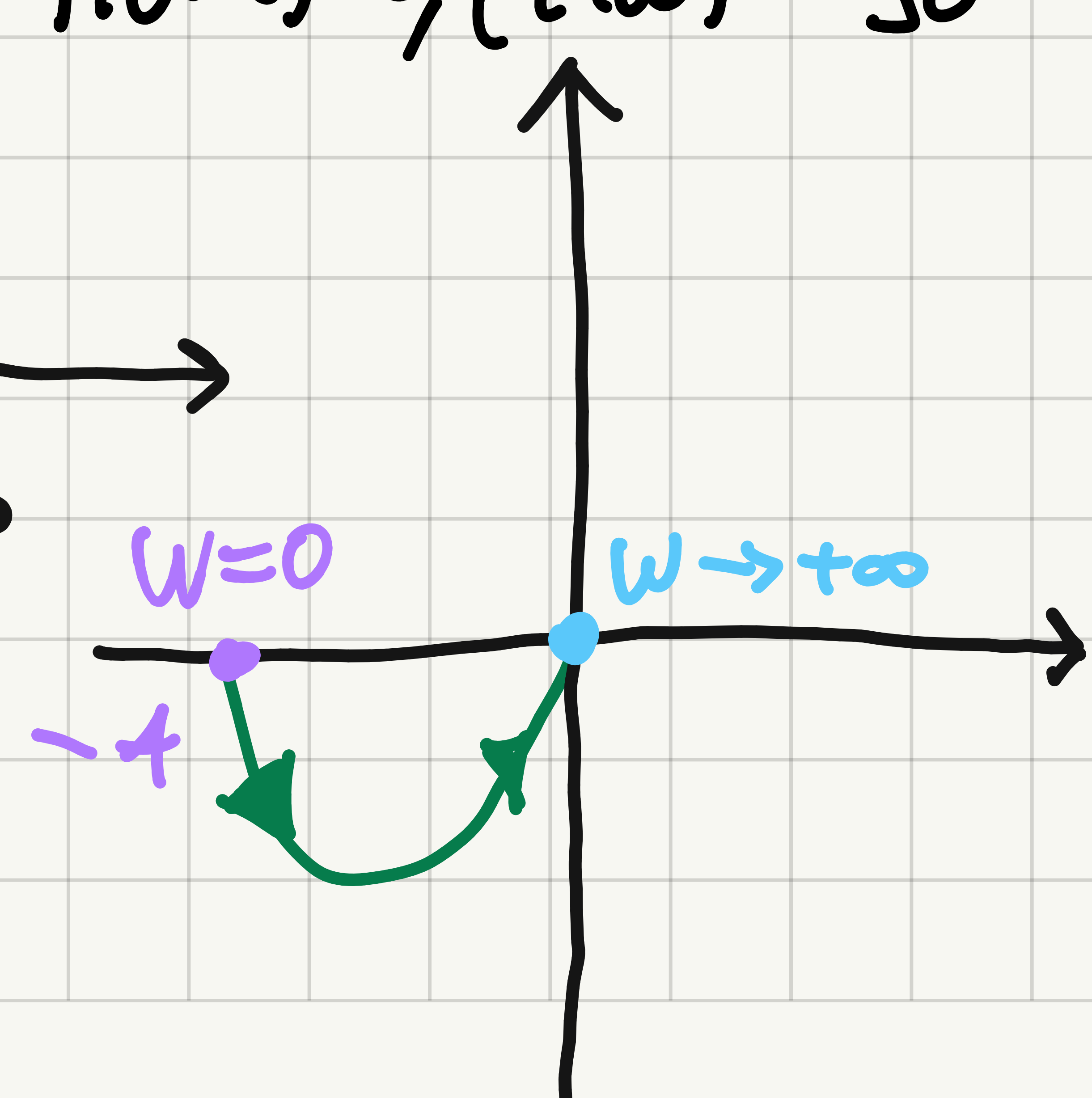


Esempio: $F(s) = \dfrac{8}{s-2}$ $\qquad F(s) = -4 \cdot \dfrac{1}{1-\frac{s}{2}}$

$F(i\omega) = -4 \cdot \dfrac{1}{1-\frac{i\omega}{2}}$ $\qquad \angle F(i\omega) = -180^\circ + \arctan\left(\frac{\omega}{2}\right)$

$M(0^+) = -4, \ \varphi(0^+) = -180^\circ$ $\qquad M(+\infty) = 0, \ \varphi(+\infty) = -90^\circ$



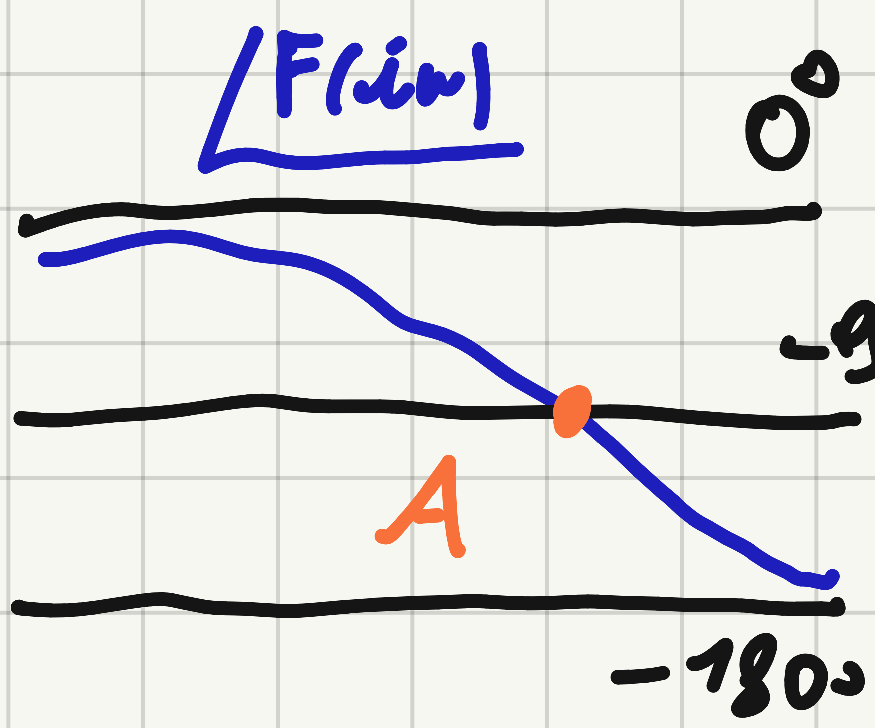


Esempio: $F(s) = \dfrac{20}{(s+2)(s+5)}$

$$F(s) = \frac{2}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \frac{2}{\left(1+\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{5}\right)}$$

$M(0^+) = 2 \quad \varphi(0^+) = 0 \quad M(+\infty) = 0 \quad \varphi(+\infty) = -180^\circ$

$$\angle F(i\omega) = -\arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{5}\right)$$



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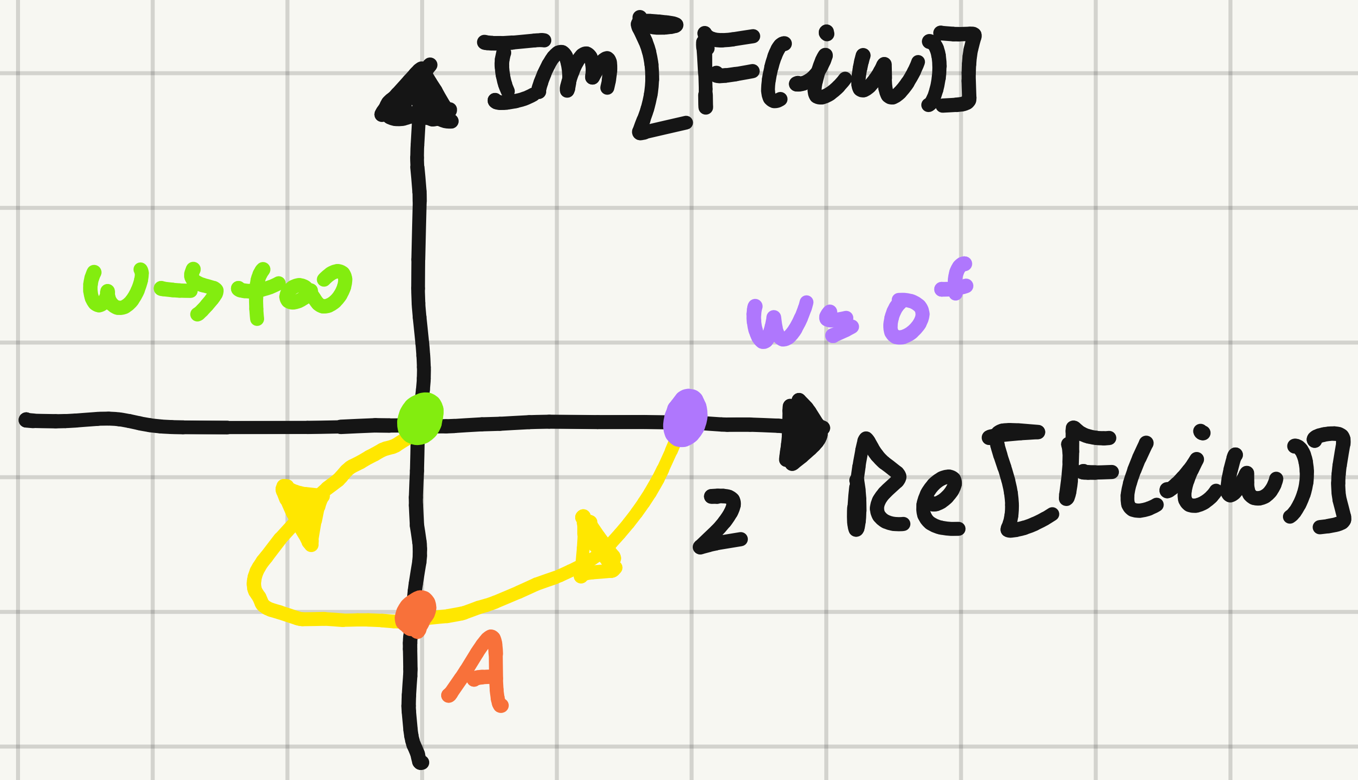


Esempio: $F(s) = \dfrac{20}{(s+2)(s-5)}$

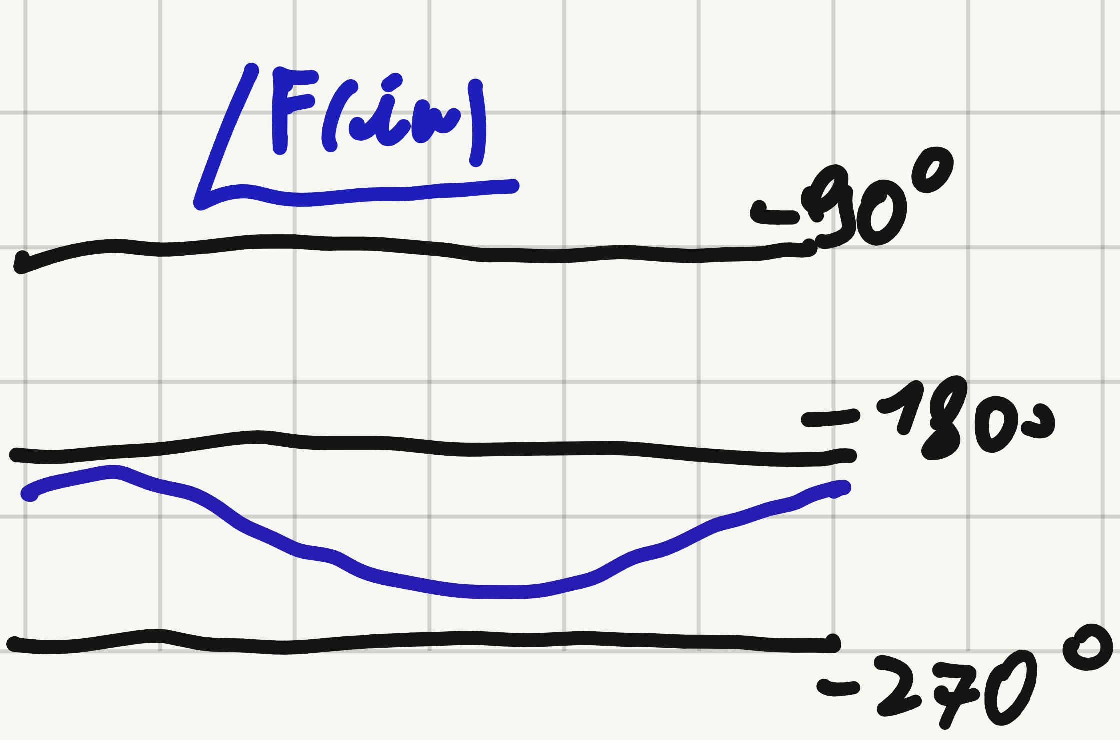
$$F(s) = \frac{-2}{\left(1+\frac{s}{2}\right)\left(1-\frac{s}{5}\right)} \qquad F(i\omega) = \frac{-2}{\left(1+\frac{i\omega}{2}\right)\left(1-\frac{i\omega}{5}\right)}$$

$M(0^+) = -2 \quad \varphi(0^+) = -180^\circ \quad M(+\infty) = 0 \quad \varphi(+\infty) = -180^\circ$

$\omega_1 = 2, \quad \tau_1 > 0$

$\omega_2 = 5, \quad \tau_2 < 0$

$$\angle F(i\omega) = -180^\circ - \arctan\left(\frac{\omega}{2}\right) + \arctan\left(\frac{\omega}{5}\right)$$



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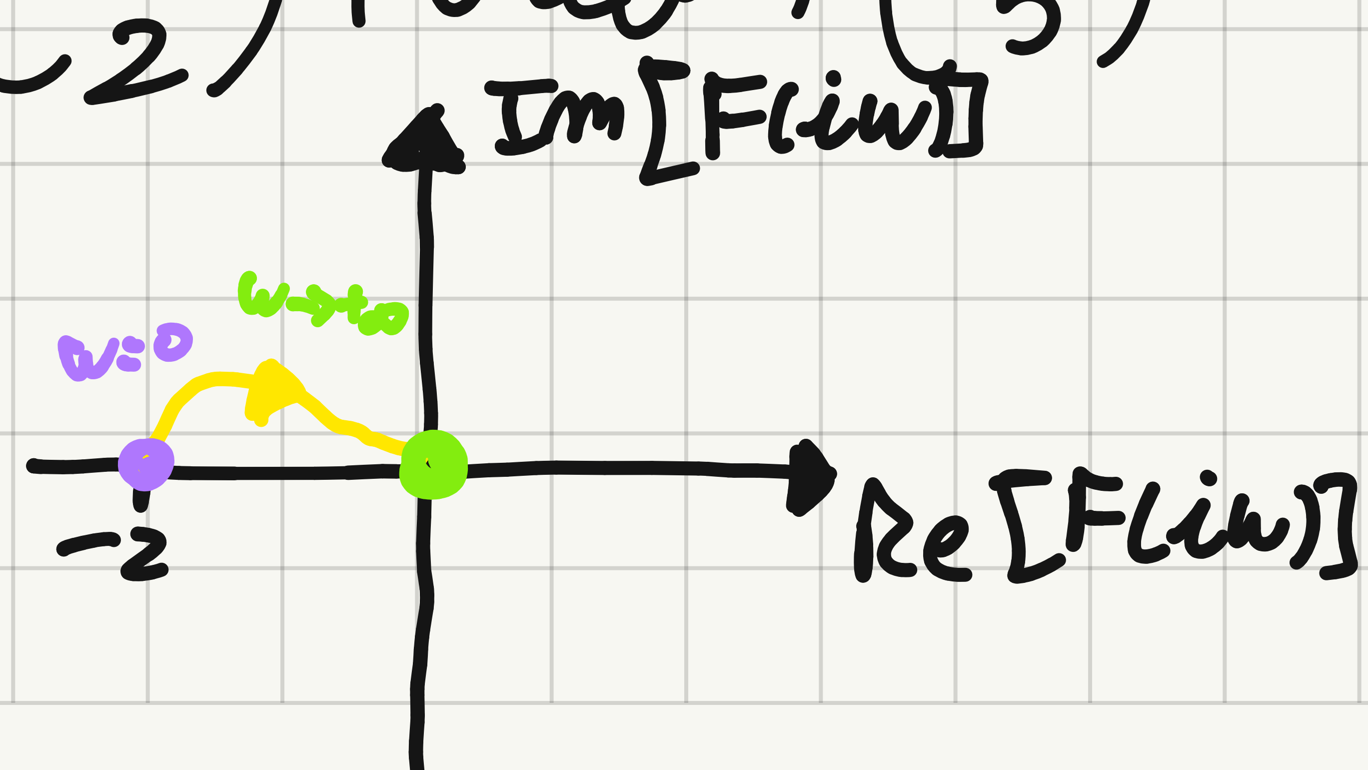
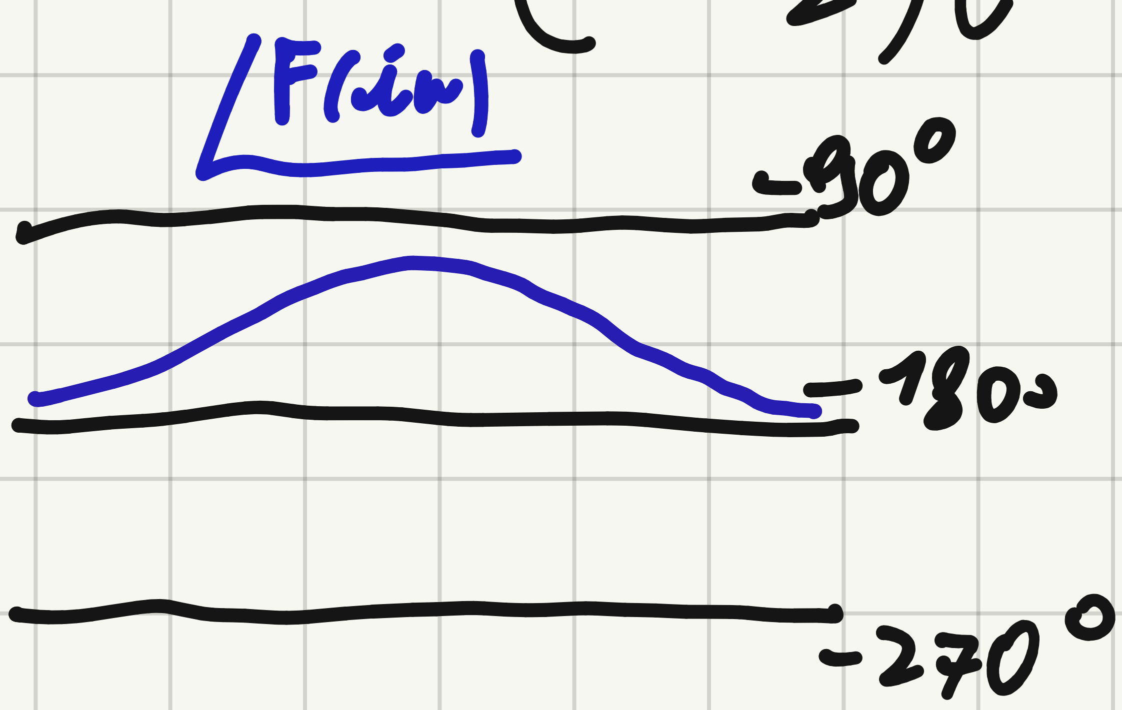
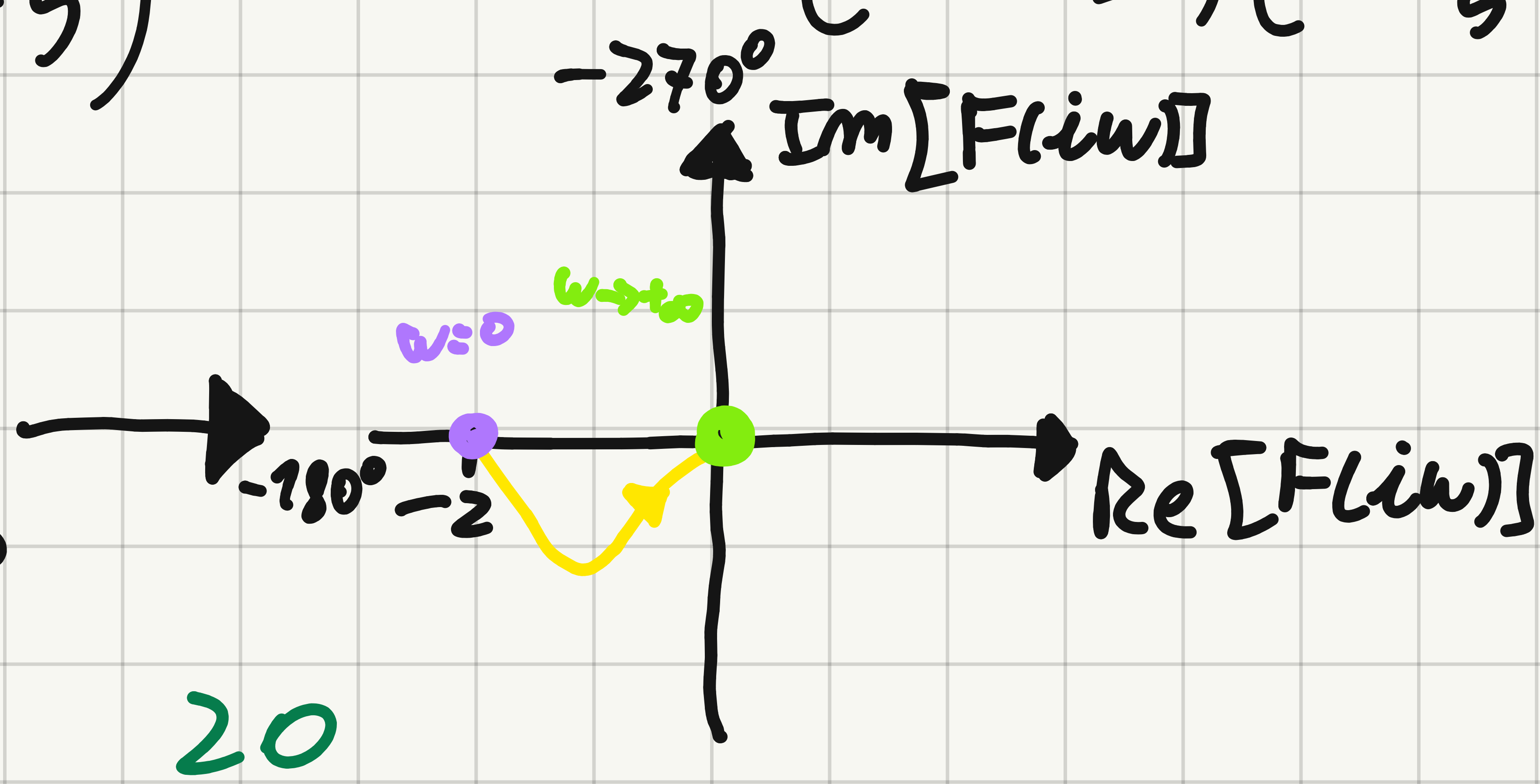


ESEMPIO: $F(s) = \dfrac{20}{(s-2)(s+5)}$

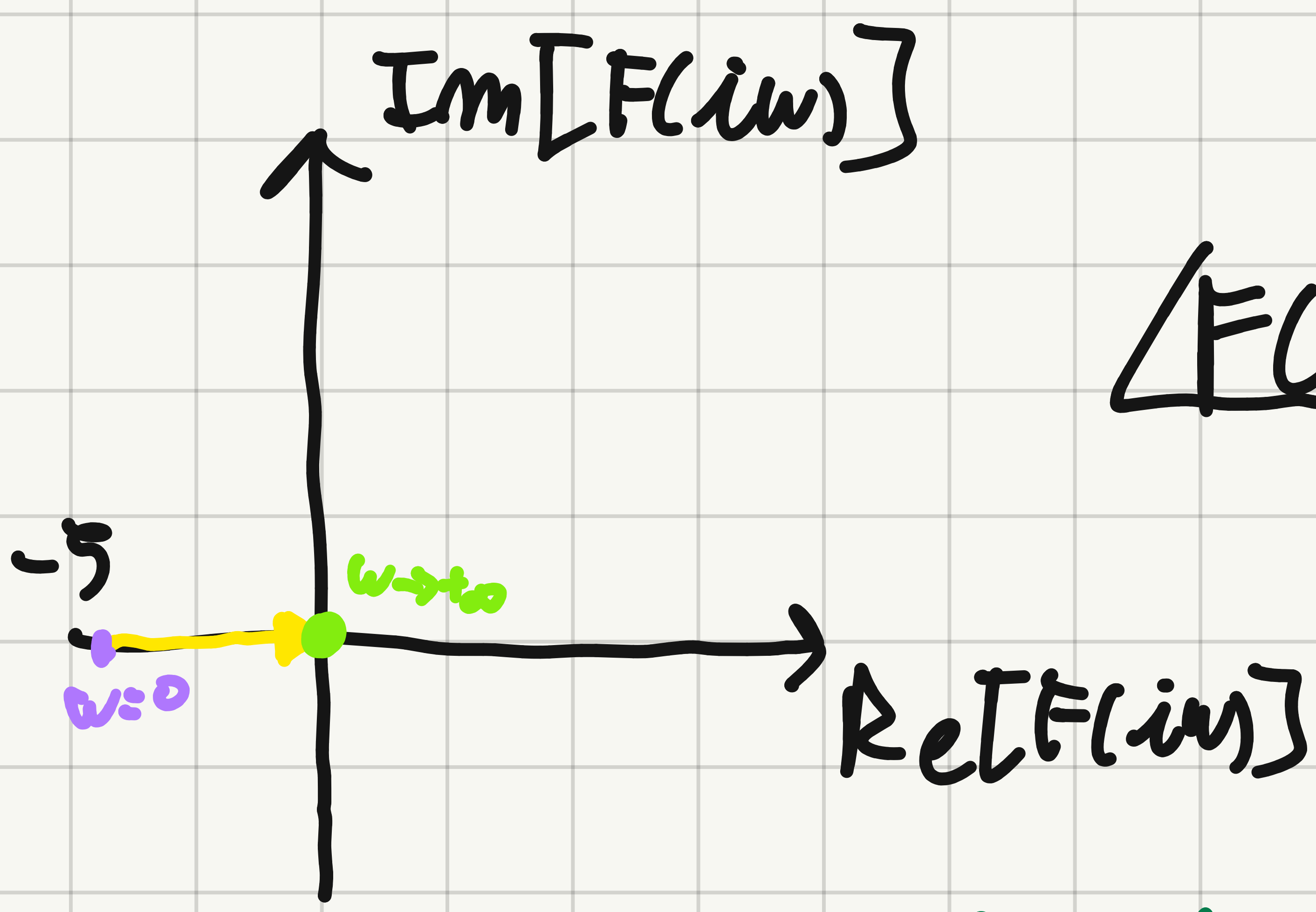
$$F(s) = \frac{-2}{\left(1-\frac{s}{2}\right)\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \frac{-2}{\left(1-\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{5}\right)}$$





ESEMPIO: $F(s) = \dfrac{20}{(s-2)(s+2)}$

$$F(s) = \frac{-5}{\left(1-\frac{s}{2}\right)\left(1+\frac{s}{2}\right)} \qquad F(i\omega) = \frac{-5}{\left(1-\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{2}\right)}$$



$$\angle F(i\omega) = -180^\circ \cancel{+\arctan\frac{\omega}{2}} \cancel{-\arctan\frac{\omega}{2}}$$

ESEMPIO: $F(s) = \dfrac{60(s+0{,}5)}{(s+2)(s+5)}$

$$F(s) = \frac{3\left(1+\frac{s}{0{,}5}\right)}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \frac{3\left(1+\frac{i\omega}{0{,}5}\right)}{\left(1+\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{5}\right)}$$

$\omega_1 = 0{,}5, \quad \tau_1 > 0$

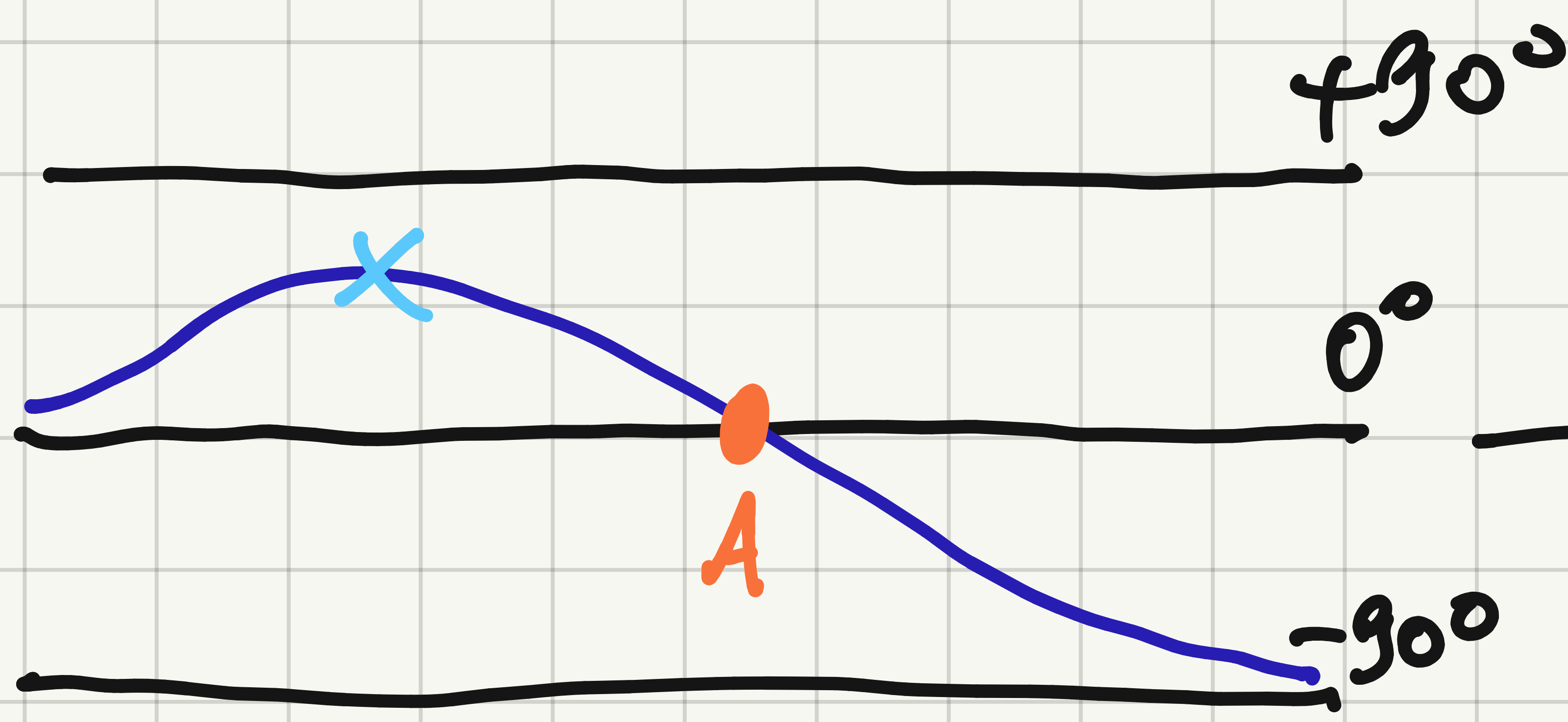
$\omega_2 = 2, \quad \tau_2 > 0$

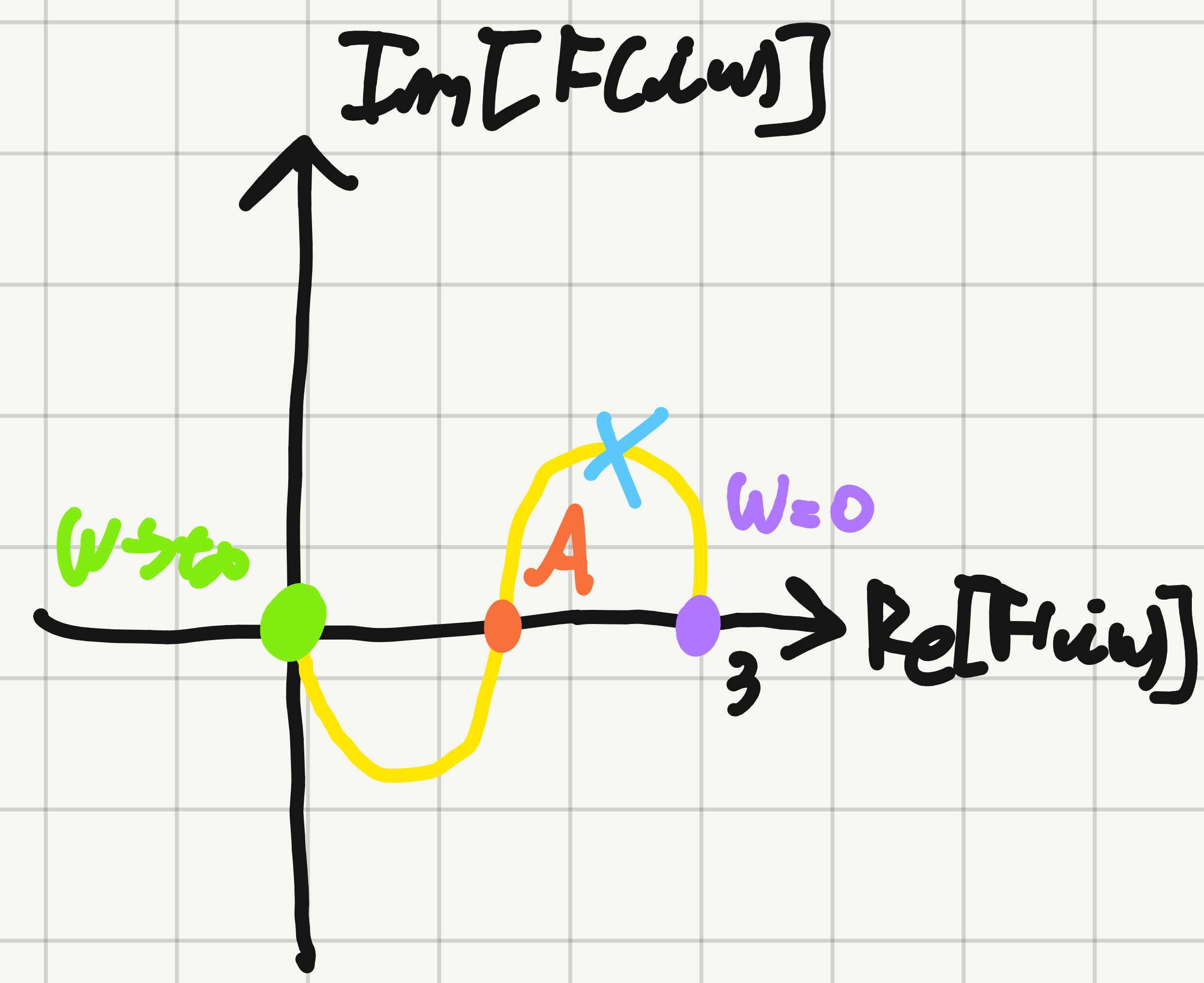
$\omega_3 = 5, \quad \tau_3 > 0$

$M(0^+) = 3 \qquad \varphi(0^+) = 0$

$M(+\infty) = 0 \qquad \varphi(+\infty)^\circ = +90^\circ - 90^\circ - 90^\circ = -90^\circ$

$$\underline{\angle F(i\omega)} = \arctan\left(\frac{\omega}{0{,}5}\right) - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{5}\right)$$



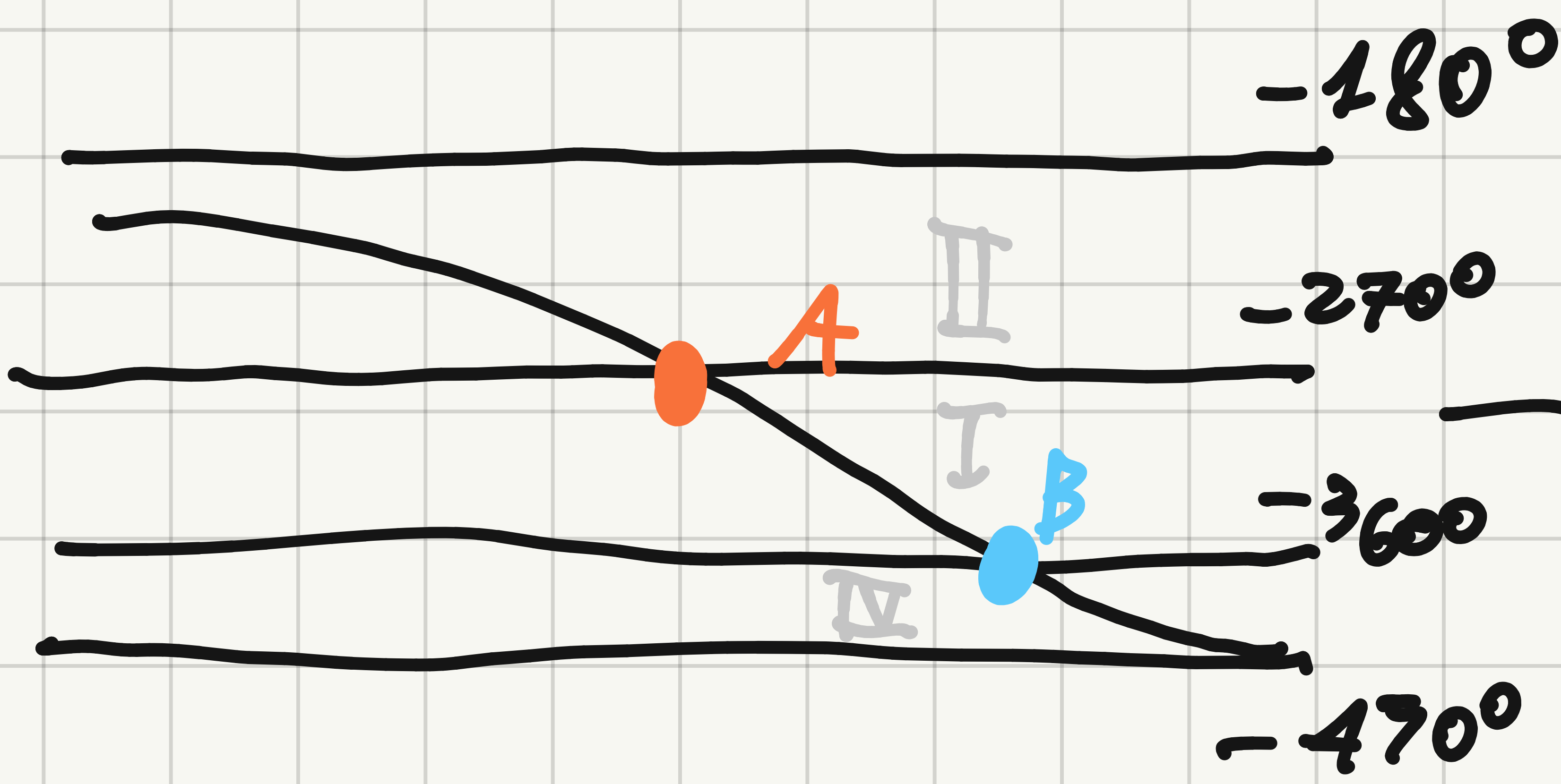


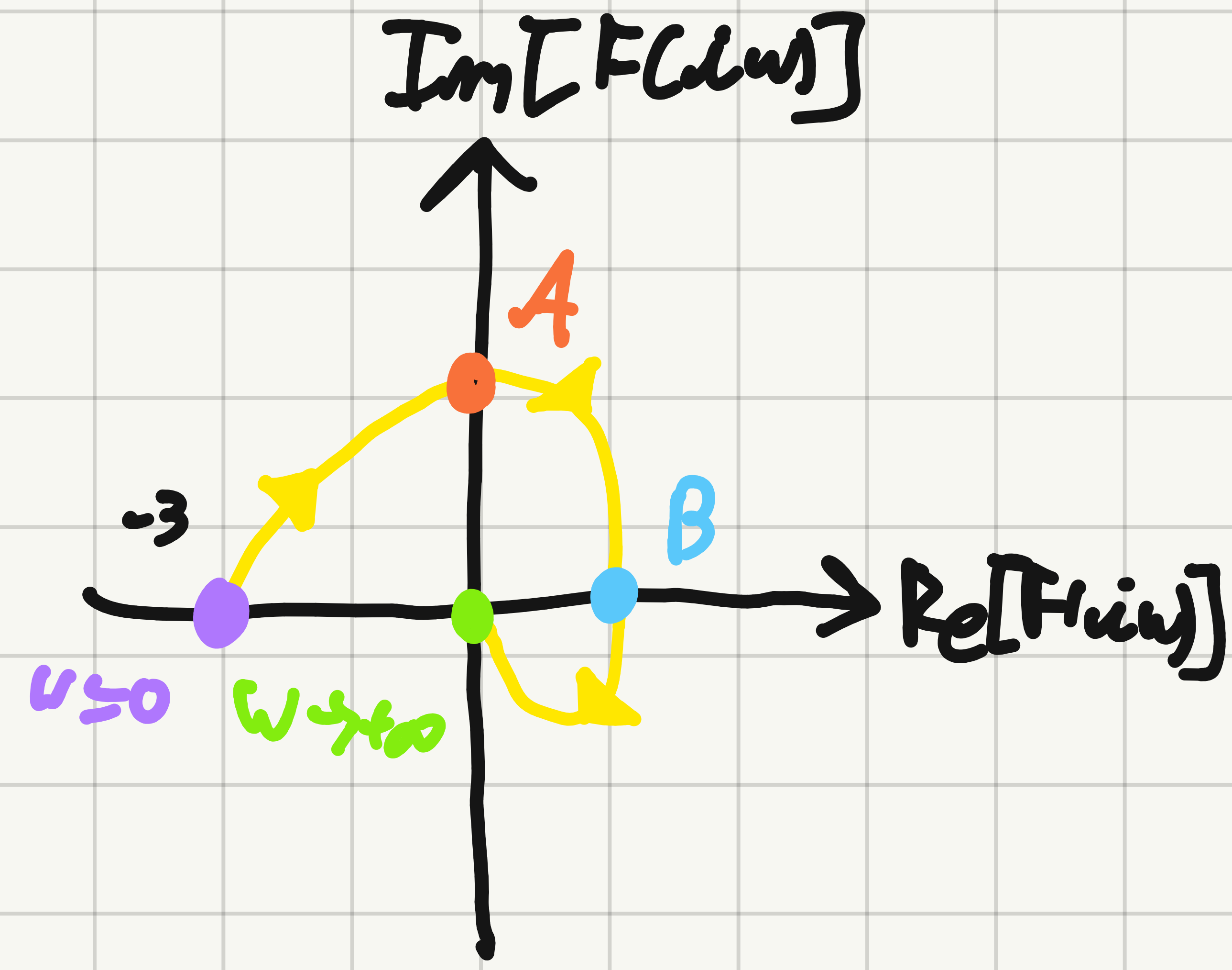
ESEMPIO: $F(s) = \dfrac{60(s-0{,}5)}{(s+2)(s+5)}$

$$F(s) = \frac{-3\left(1 - \frac{s}{0{,}5}\right)}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \frac{3\left(1 + \frac{i\omega}{0{,}5}\right)}{\left(1+\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{5}\right)}$$

$M(0^+) = -3 \qquad \varphi(0^+) = -180^\circ$

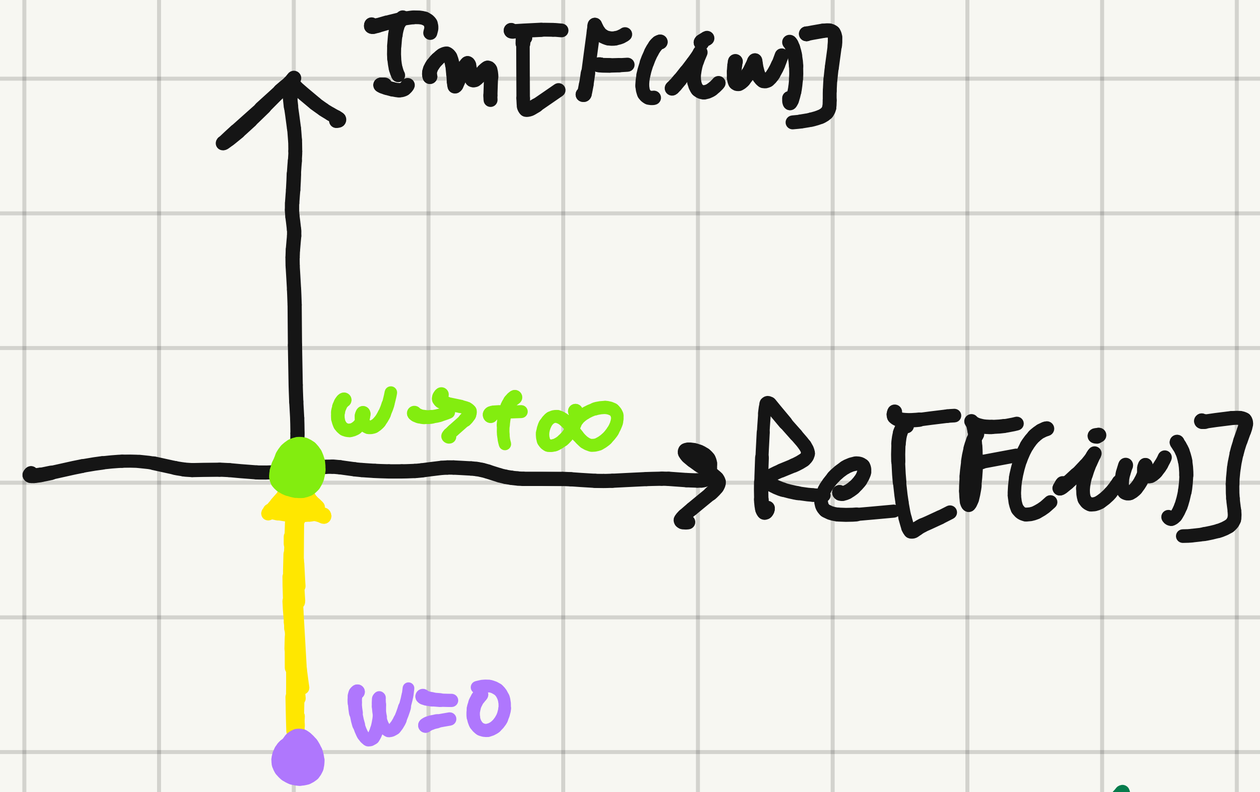
$$\underline{\angle F(i\omega)} = -180^\circ - \arctan\left(\frac{\omega}{0{,}5}\right) - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{5}\right)$$





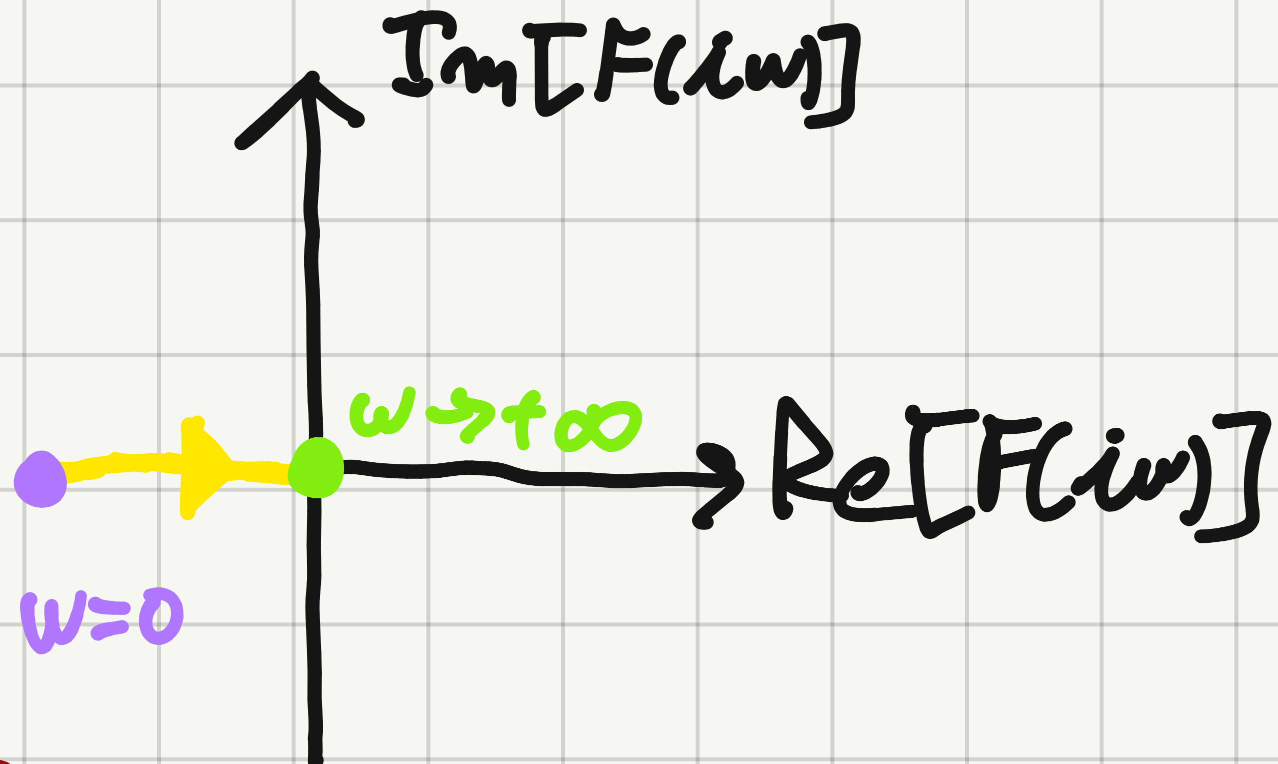
ESEMPIO: $F(s) = \dfrac{1}{s}$ $\quad F(i\omega) = \dfrac{1}{i\omega}$

$M(0^+) \to +\infty \quad M(+\infty) = 0 \quad \angle F(i\omega) = -90^\circ \ \forall\, \omega > 0$



ESEMPIO: $F(s) = \dfrac{1}{s^2}$ $\quad F(i\omega) = \dfrac{1}{(i\omega)^2} = -\dfrac{1}{\omega^2}$

$M(0^+) \to +\infty \quad M(+\infty) = 0 \quad \angle F(i\omega) = 0 \ \forall\, \omega > 0$

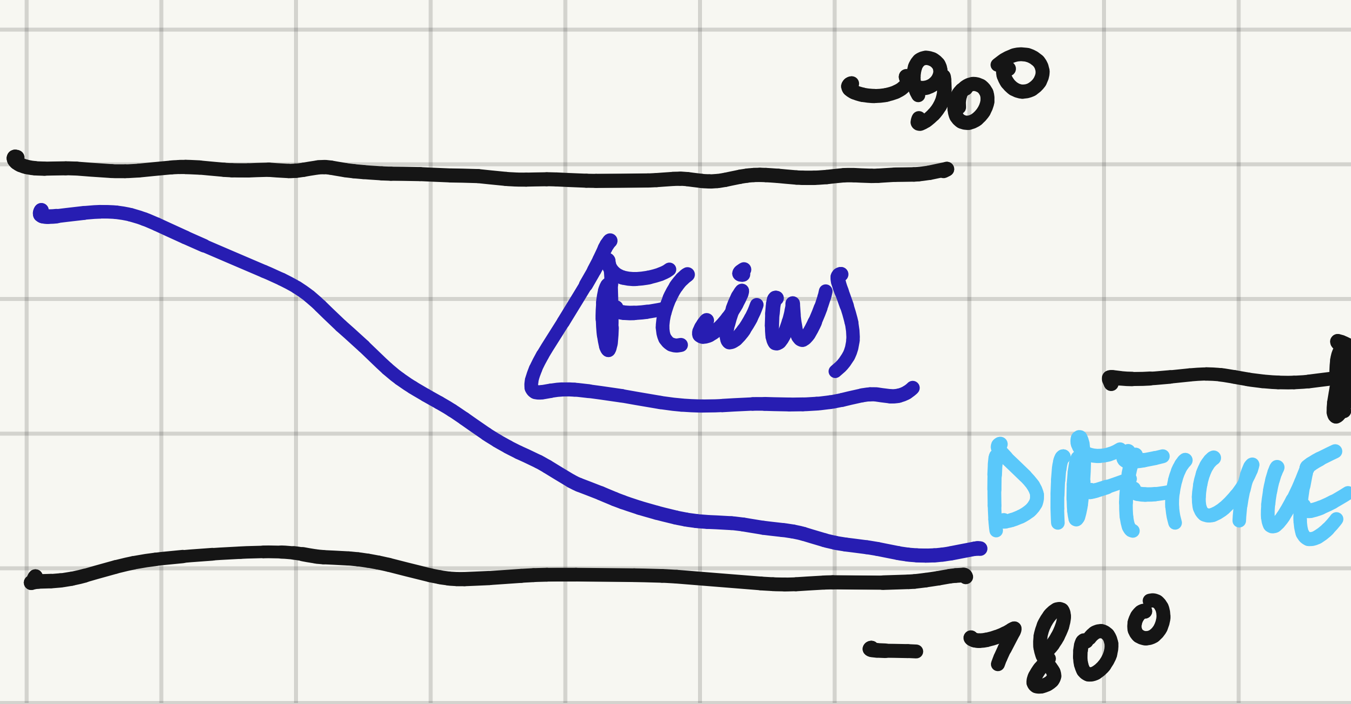


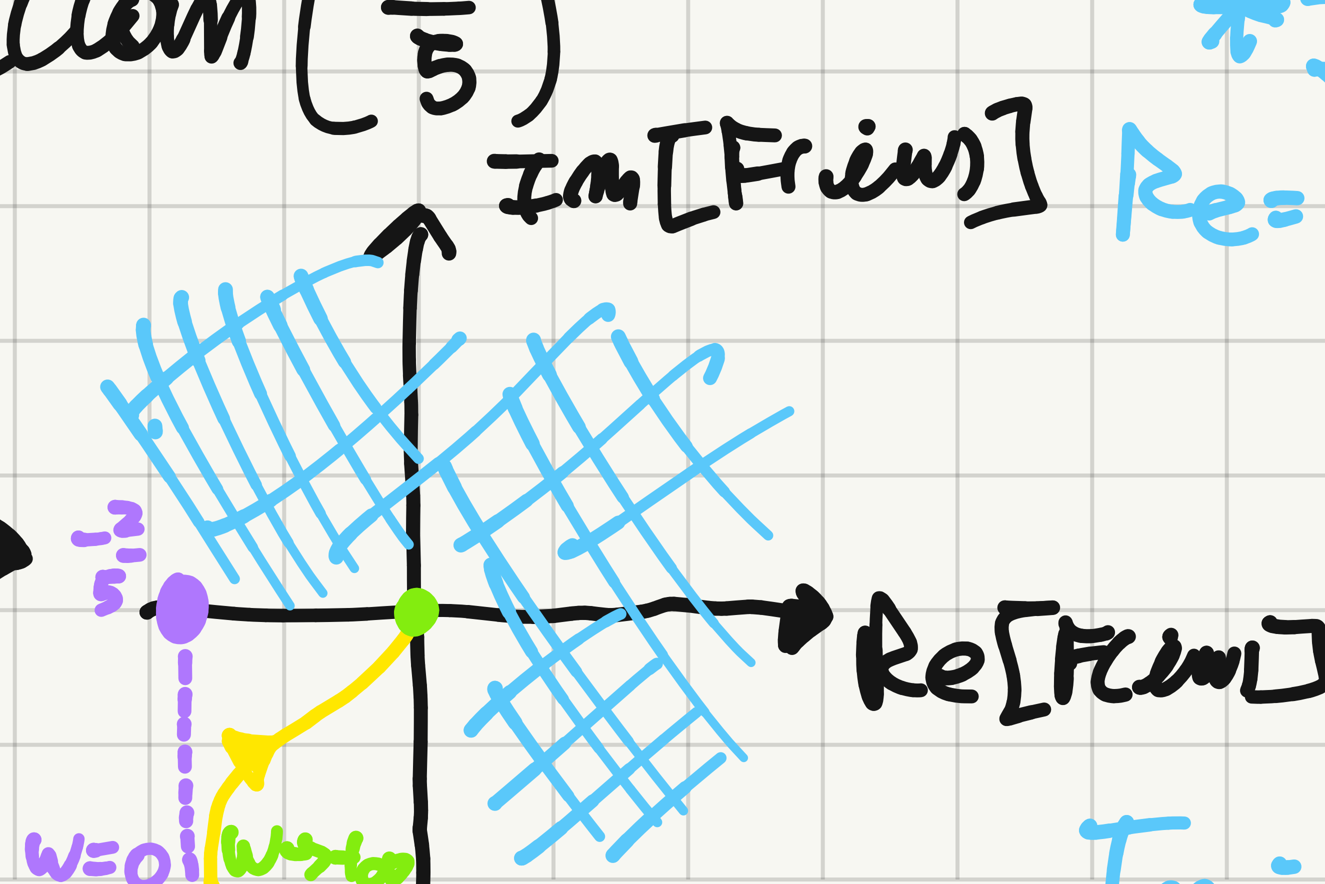
ESEMPIO: $F(s) = \dfrac{10}{s(s+5)}$

$F(s) = \dfrac{2}{s\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \dfrac{2}{i\omega\left(1+\frac{i\omega}{5}\right)} = -\dfrac{2}{\omega} \cdot \dfrac{1}{\frac{\omega}{5} - i} = -\dfrac{2}{\omega} \cdot \dfrac{\frac{\omega}{5} + i}{\frac{\omega^2}{25} + 1}$

$M(0^+) \to +\infty \quad \varphi(0^+) = -90^\circ \quad M(+\infty) = 0 \quad \varphi(+\infty) = -90^\circ - 90^\circ = -180^\circ$

$\angle F(i\omega) = -90^\circ - \arctan\left(\dfrac{\omega}{5}\right)$





* $\mathrm{Re} = \dfrac{-\frac{2}{5}}{1 + \frac{\omega^2}{25}}$

$\mathrm{Im} = \dfrac{-\frac{2}{\omega}}{1 + \frac{\omega^2}{25}}$

$\Rightarrow$ ENTRAMBE < 0

$$\lim_{\omega\to 0^+} Re[F(i\omega)] = -\frac{2}{3} \qquad \lim_{\omega\to 0^+} Im[F(i\omega)] = -\infty$$

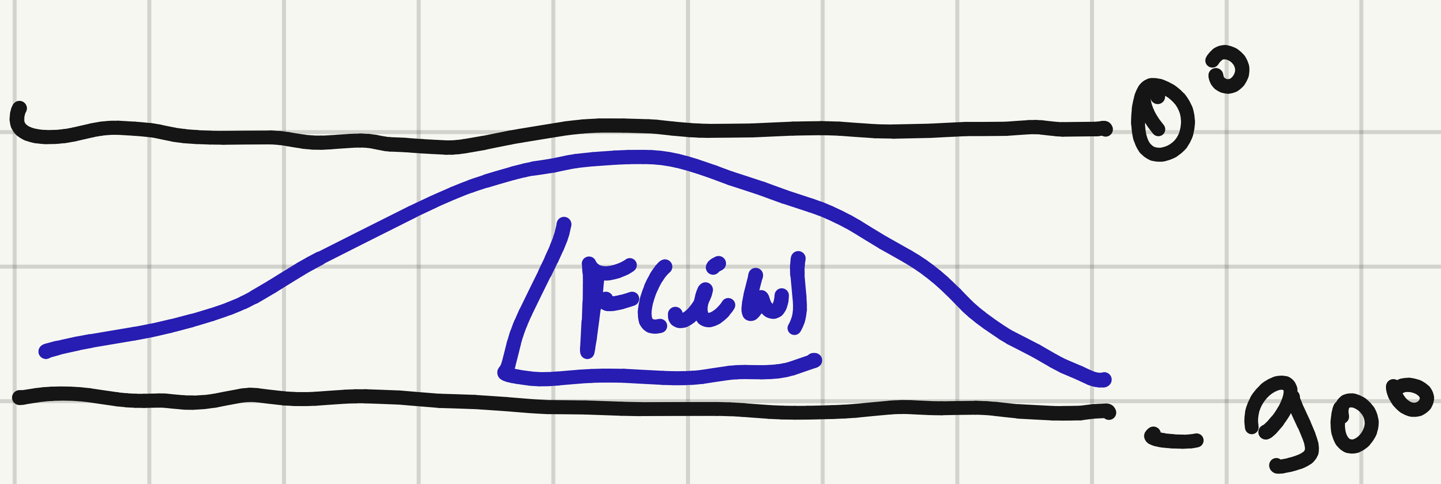
$$\lim_{\omega\to +\infty} Re[F(i\omega)] = 0 \qquad \lim_{\omega\to +\infty} Im[F(i\omega)] = 0$$

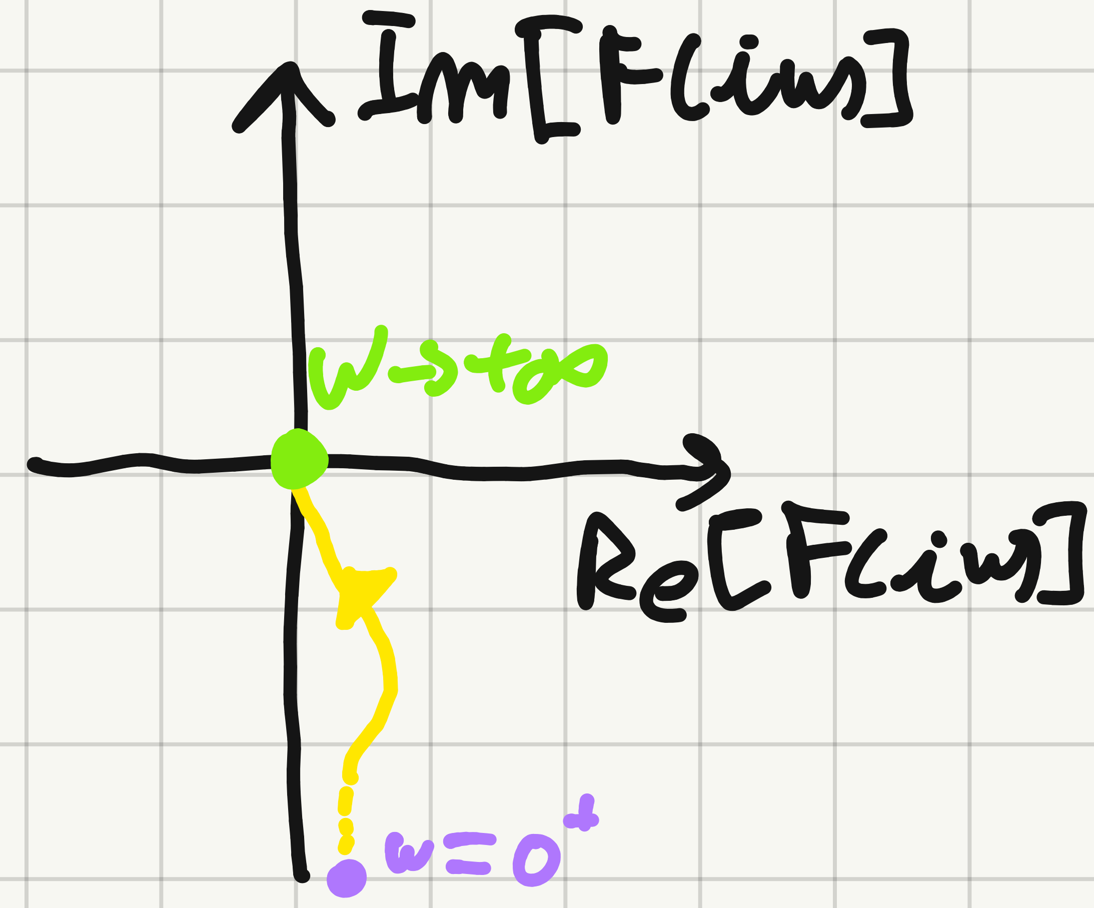
ESEMPIO: $F(s) = \dfrac{10(s+2)}{s(s+5)}$

$$F(s) = \frac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \frac{4\left(1+\frac{i\omega}{2}\right)}{i\omega\left(1+\frac{i\omega}{5}\right)}$$

$M(0^+) = +\infty \qquad \varphi(0^+) = -90^\circ \qquad M(+\infty) = 0 \qquad \varphi(+\infty) = -90^\circ$

$\omega_1 = 2,\ \omega_2 = 5 \quad \angle F(i\omega) = -90^\circ + \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{5}\right)$

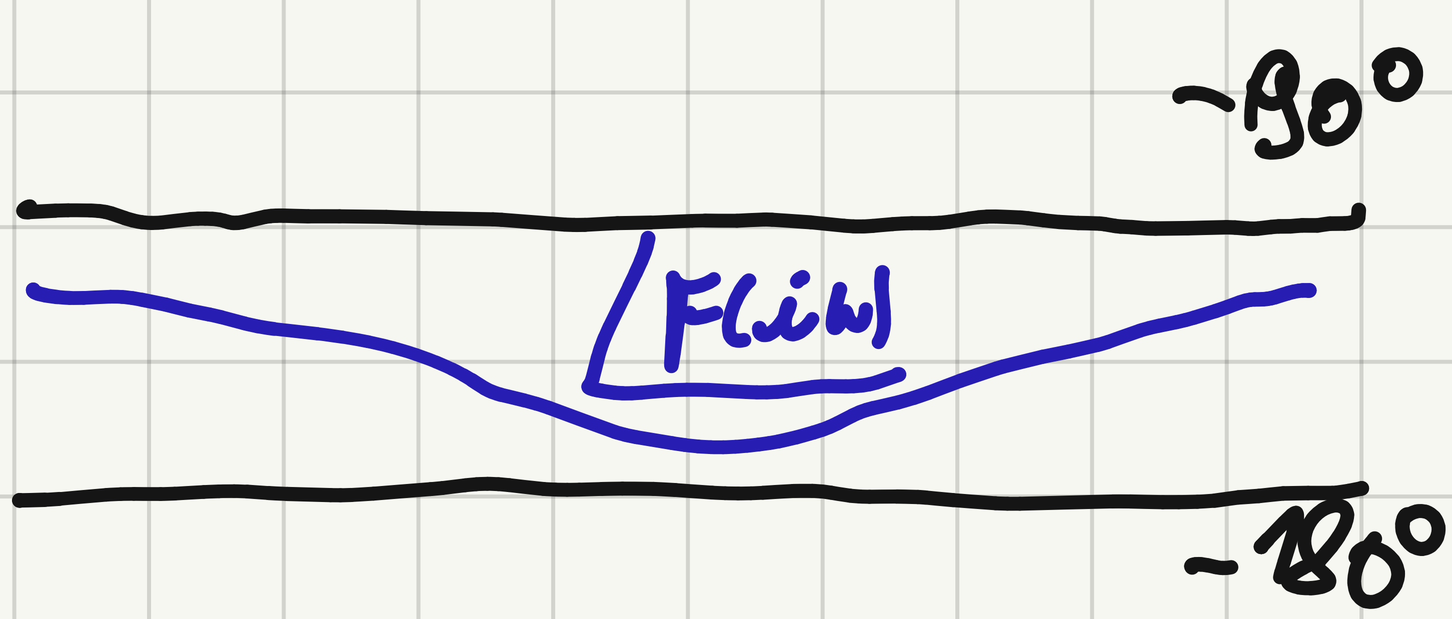


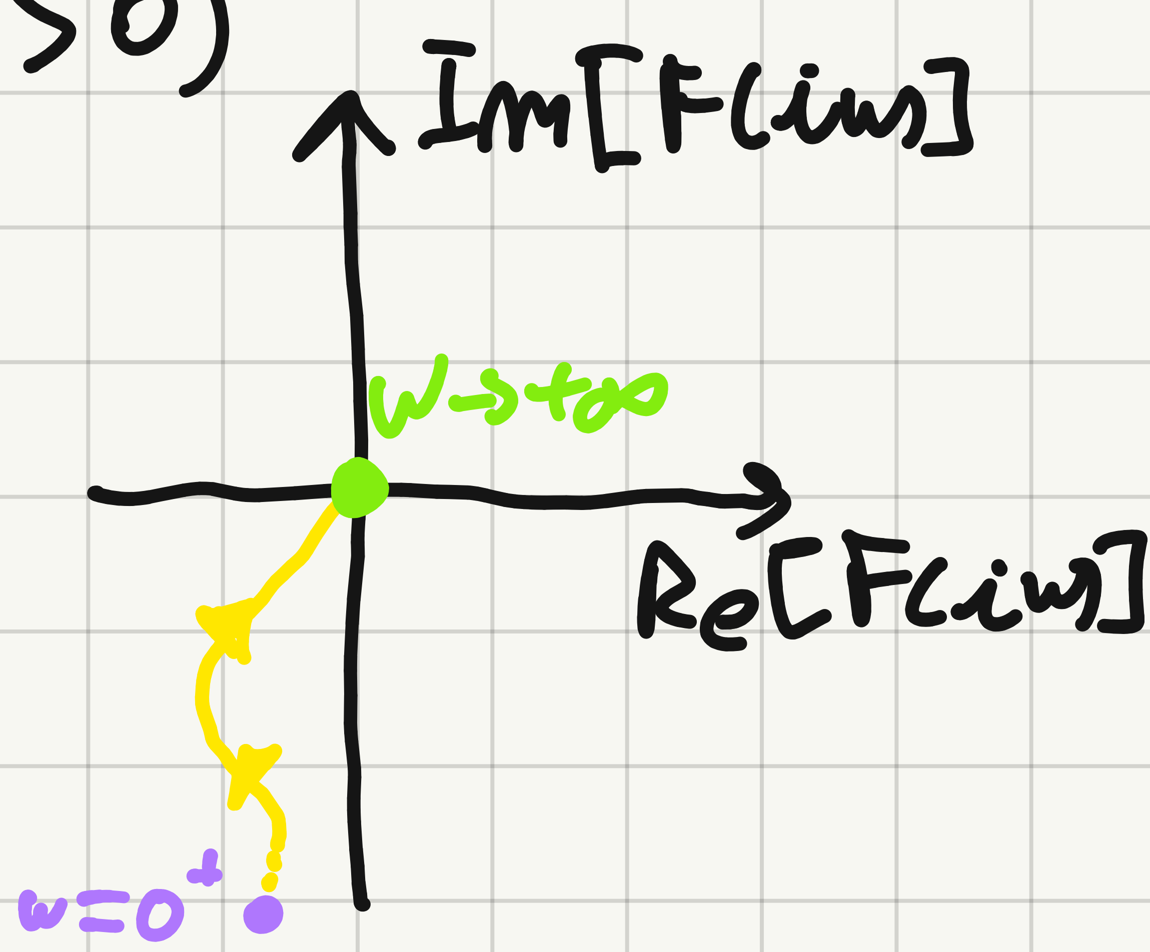


ESEMPIO: $F(i\omega) = \dfrac{10\left(1+\frac{i\omega}{5}\right)}{i\omega\left(1+\frac{i\omega}{2}\right)}$

$\omega_1 = 2 \to$ BINOMIO DENOMINATORE $(\tau_1 > 0)$

$\omega_2 = 5 \to$ BINOMIO NUMERATORE $(\tau_2 > 0)$





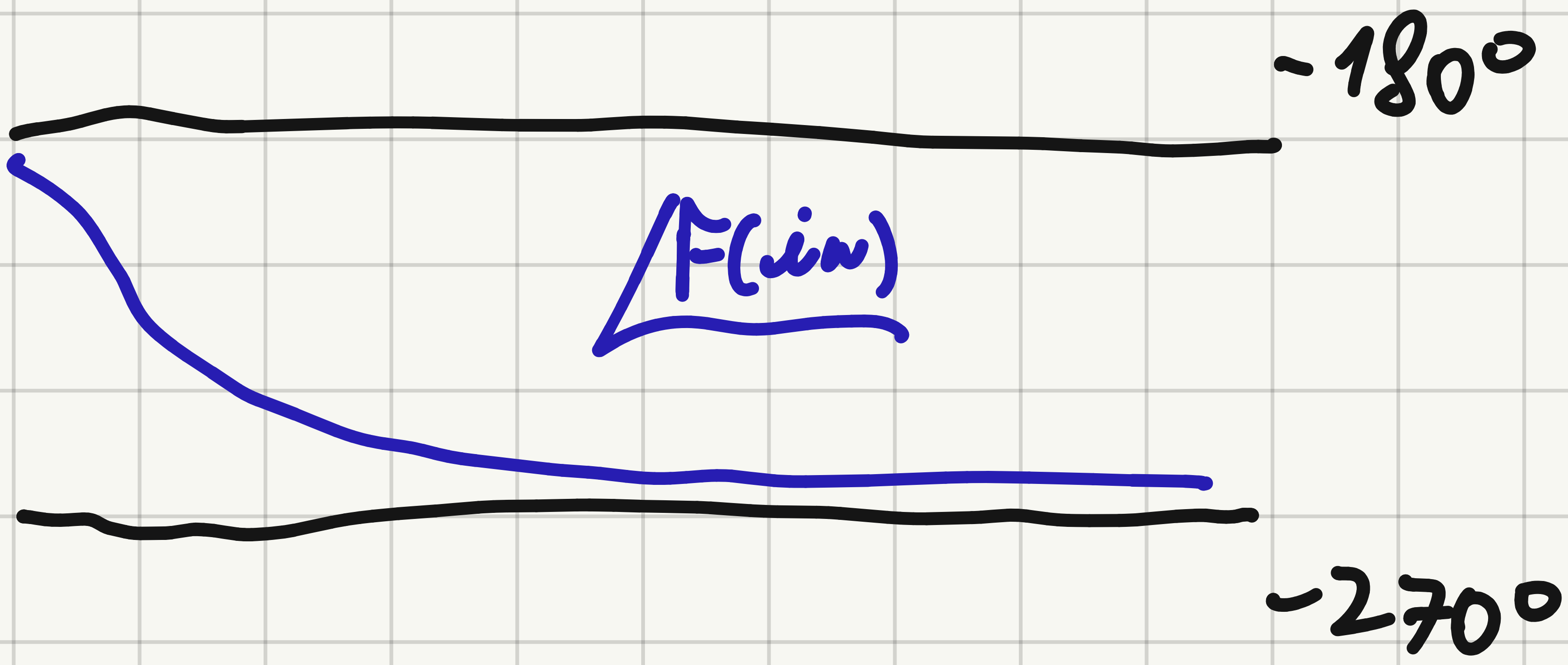
ESEMPIO: $F(s) = \frac{10}{s^2(s+5)}$

$$F(s) = \frac{2}{s^2\left(1+\frac{s}{5}\right)} \qquad F(i\omega) = \frac{2}{(i\omega)^2\left(1+\frac{i\omega}{5}\right)}$$

$M(0^+) = +\infty \quad M(+\infty) = 0 \qquad \omega_1 = 5,\ \tau_1 > 0$

$\varphi(0^+) = -180° \qquad \varphi(+\infty) = -180° - 90° = -270°$

$$\angle F(i\omega) = -180° - \arctan\left(\frac{\omega}{5}\right)$$



$$F(i\omega) = -\frac{2}{\omega^2} \cdot \frac{1}{1+\frac{i\omega}{5}} = -\frac{2}{\omega^2} \cdot \frac{\left(1 - i\frac{\omega}{5}\right)}{1+\frac{\omega^2}{25}}$$

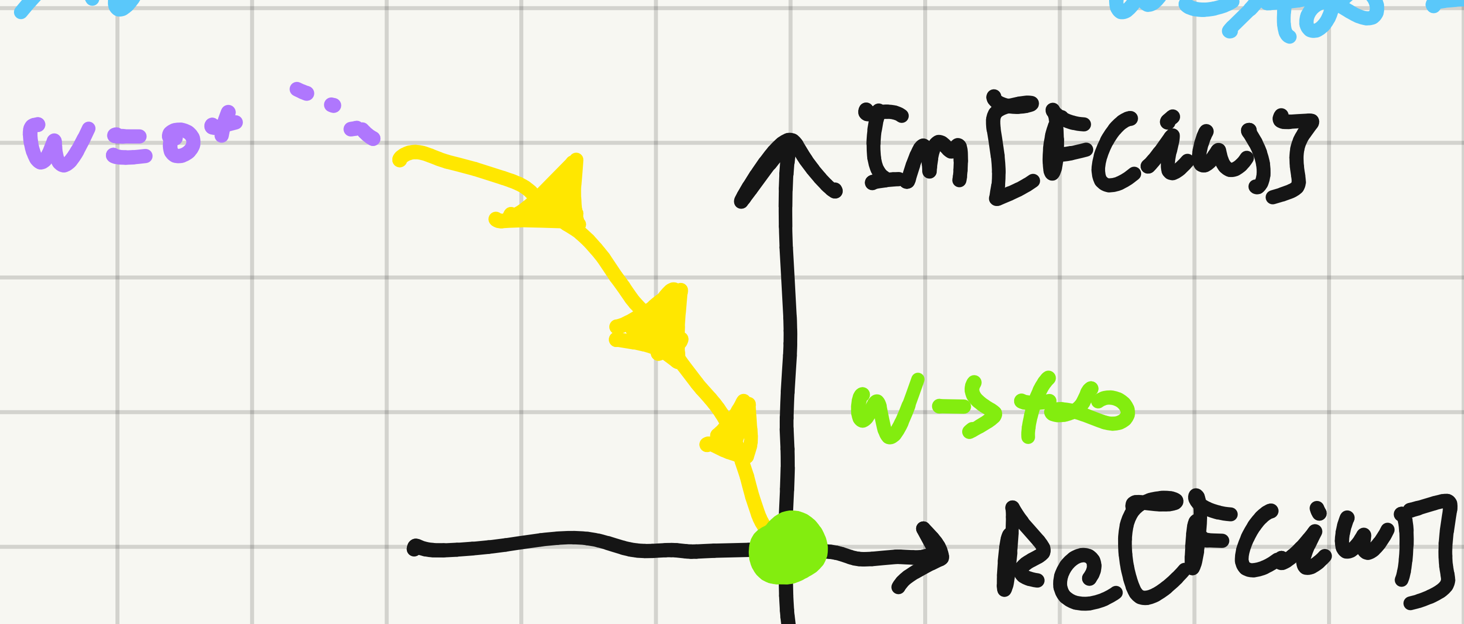
$$Re[F(i\omega)] = -\frac{2}{\omega^2} \cdot \frac{1}{1+\frac{\omega^2}{25}}$$

$$Im[F(i\omega)] = \frac{2}{5\omega} \cdot \frac{1}{1+\frac{\omega^2}{5}}$$

CONFERMA 2° QUADRANTE

$\lim_{\omega \to 0^+} Re[F(i\omega)] = -\infty \qquad \lim_{\omega \to 0^+} Im[F(i\omega)] = +\infty$

$\lim_{\omega \to +\infty} Re[F(i\omega)] = 0 \qquad \lim_{\omega \to +\infty} Im[F(i\omega)] = 0$



ESEMPIO: $F(s) = \dfrac{10(s+2)}{s^2(s+10)}$

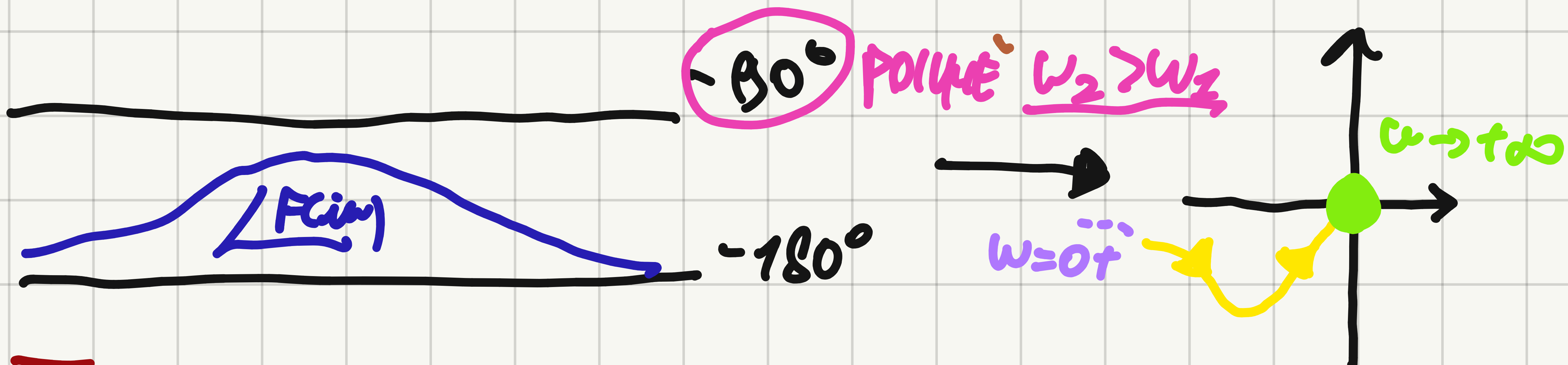
$$F(s) = \frac{2\left(1+\frac{s}{2}\right)}{s^2\left(1+\frac{s}{10}\right)} \qquad F(i\omega) = \frac{2\left(1+\frac{i\omega}{2}\right)}{(i\omega)^2\left(1+\frac{i\omega}{10}\right)}$$

$M(0^+) = +\infty \quad M(+\infty) = 0 \qquad \omega_1 = 2 \quad \tau_1 > 0$

$\omega_2 = 10 \quad \tau_2 > 0$

$$\angle F(i\omega) = -180^\circ + \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{10}\right)$$

$\varphi(0^+) = -180^\circ \qquad \varphi(+\infty) = -180^\circ + 90^\circ - 90^\circ = -180^\circ$



ESEMPIO: $F(s) = \dfrac{10}{(s+2)(s-p)}$, con $p > 0$, $p \neq 2$

ESERCIZIO TIPO DELLA PROVA TEORICA/ORALE

$$F(s) = \frac{10}{2\left(1+\frac{s}{2}\right)(-p)\left(1-\frac{s}{p}\right)} = -\frac{5}{p} \cdot \frac{1}{\left(1+\frac{s}{2}\right)\left(1-\frac{s}{p}\right)}$$

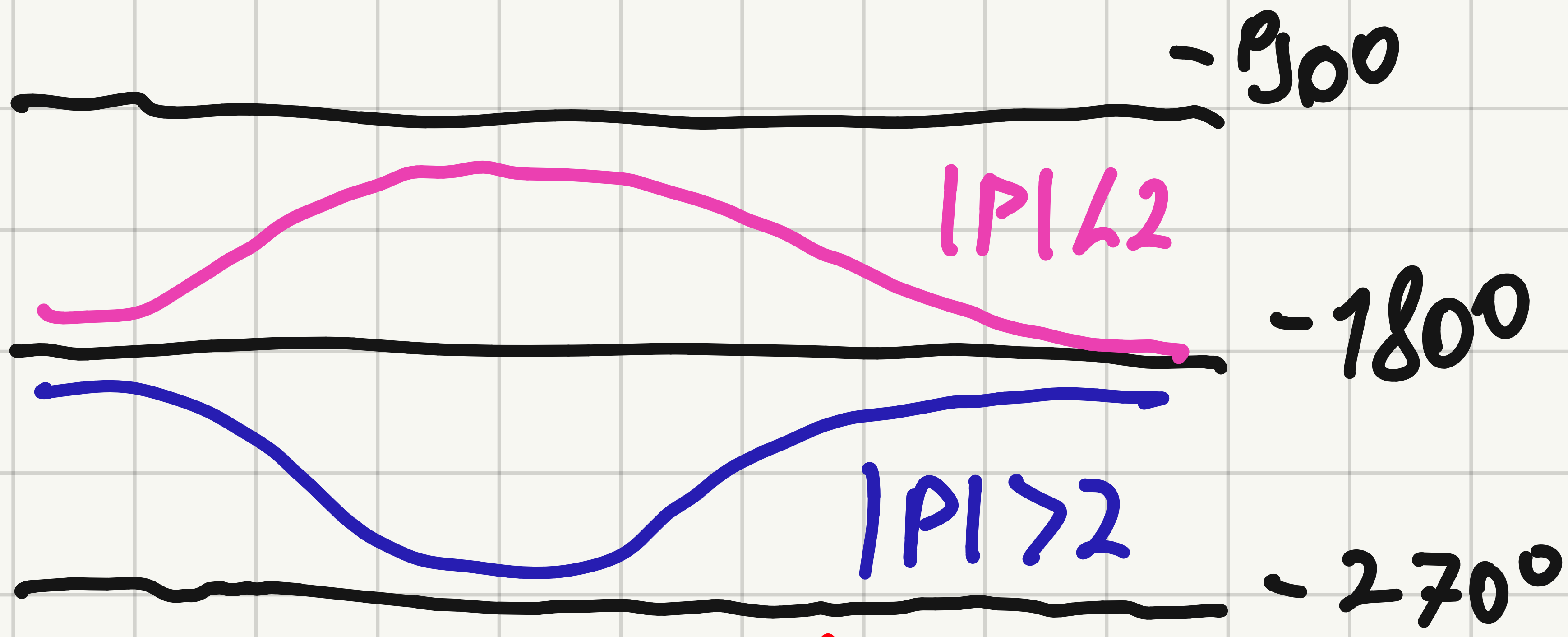
$$F(i\omega) = -\frac{5}{p} \cdot \frac{1}{\left(1+\frac{i\omega}{2}\right)\left(1-\frac{i\omega}{p}\right)}$$

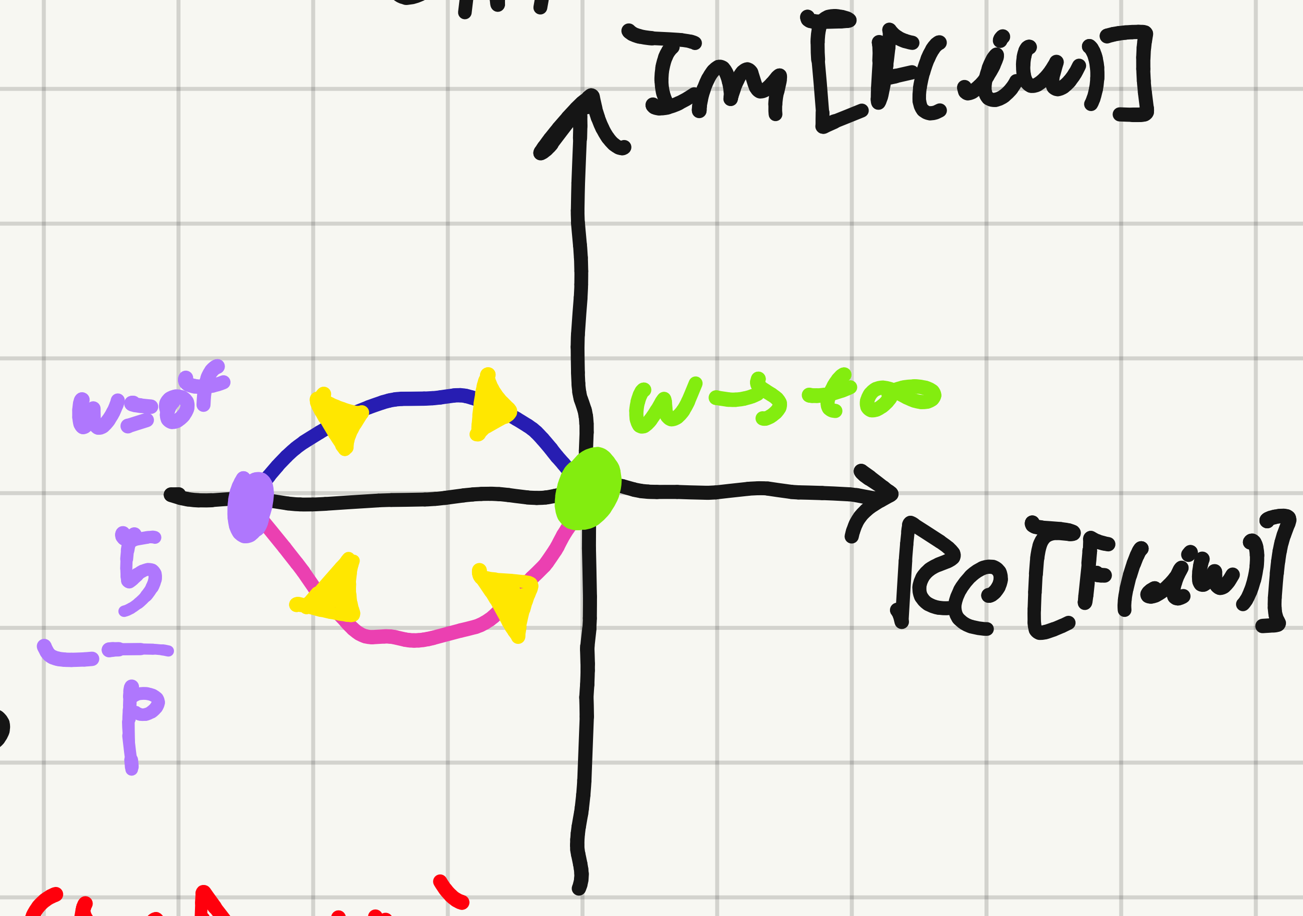
$K_F = -\frac{5}{p} < 0 \qquad M(0^+) = \frac{5}{p}, \quad \varphi(0^+) = -180^\circ$

$M(+\infty) = 0, \quad \varphi(+\infty) = -180^\circ$

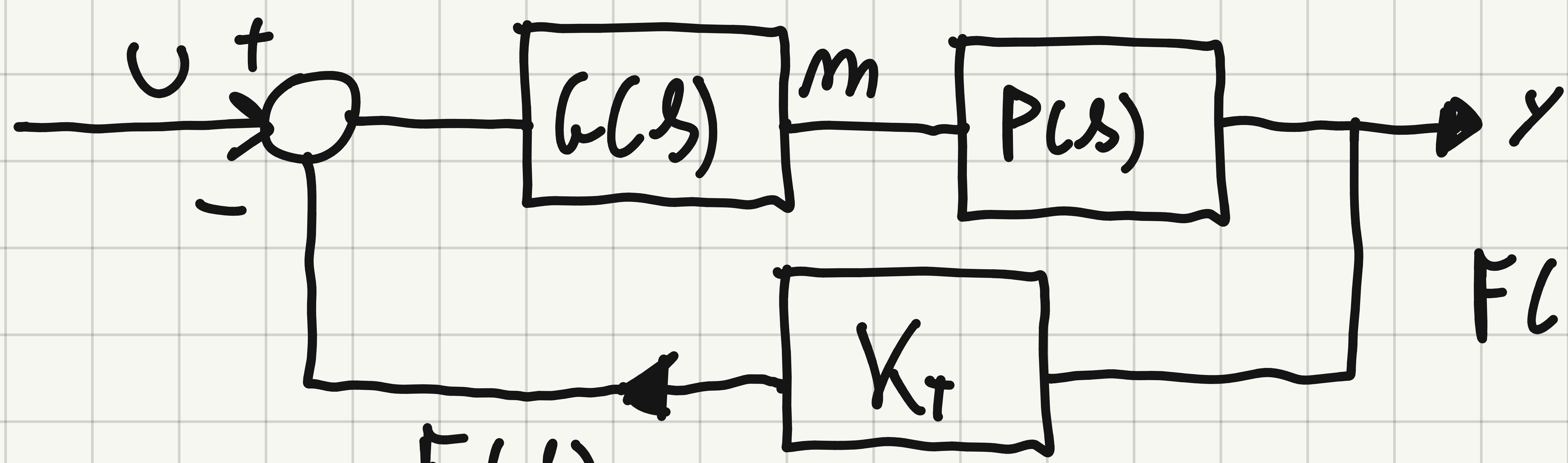
$\omega_1 = 2, \tau_1 > 0 \qquad \omega_2 = p, \tau_2 < 0$

$\angle F(i\omega) = -180° - \arctan\left(\frac{\omega}{2}\right) + \arctan\left(\frac{\omega}{|p|}\right)$





ANALISI DELLA STABILITÀ



$F(s) = G(s) \cdot P(s)$

$W(s) = \dfrac{F(s)}{1 + K_T F(s)}$ DOVRÀ ESSERE ESTERNAMENTE STABILE, CIOÈ TUTTI I POLI DEVONO AVERE PARTE REALE NEGATIVA.

QUESTA CONDIZIONE È RICAVABILE DAL DIAGRAMMA DI NYQUIST, CHE, IN QUESTO CASO, È ANCHE CONDIZIONE NECESSARIA E È SUFFICIENTE. SI RICAVA IL GRAFICO DI $F(i\omega)$, $-\infty < \omega < +\infty$

CRITERIO DI NYQUIST: IL SISTEMA IN CATENA CHIUSA È STABILE ESTERNAMENTE, CIOÈ I POLI DI $W(s)$ SONO TUTTI A PARTE REALE NEGATIVA $\Longleftrightarrow$ IL DIAGRAMMA DI NYQUIST DI $F(i\omega)$, PER $-\infty < \omega < +\infty$, COMPIE UN NUMERO DI GIRI IN SENSO ANTIORARIO INTORNO AL PUNTO $\left(-\frac{1}{K_T} ; i0\right)$ PARI AL

NUMERO DI POLI A PARTE REALE POSITIVA DI $F(s)$, CIOÈ:

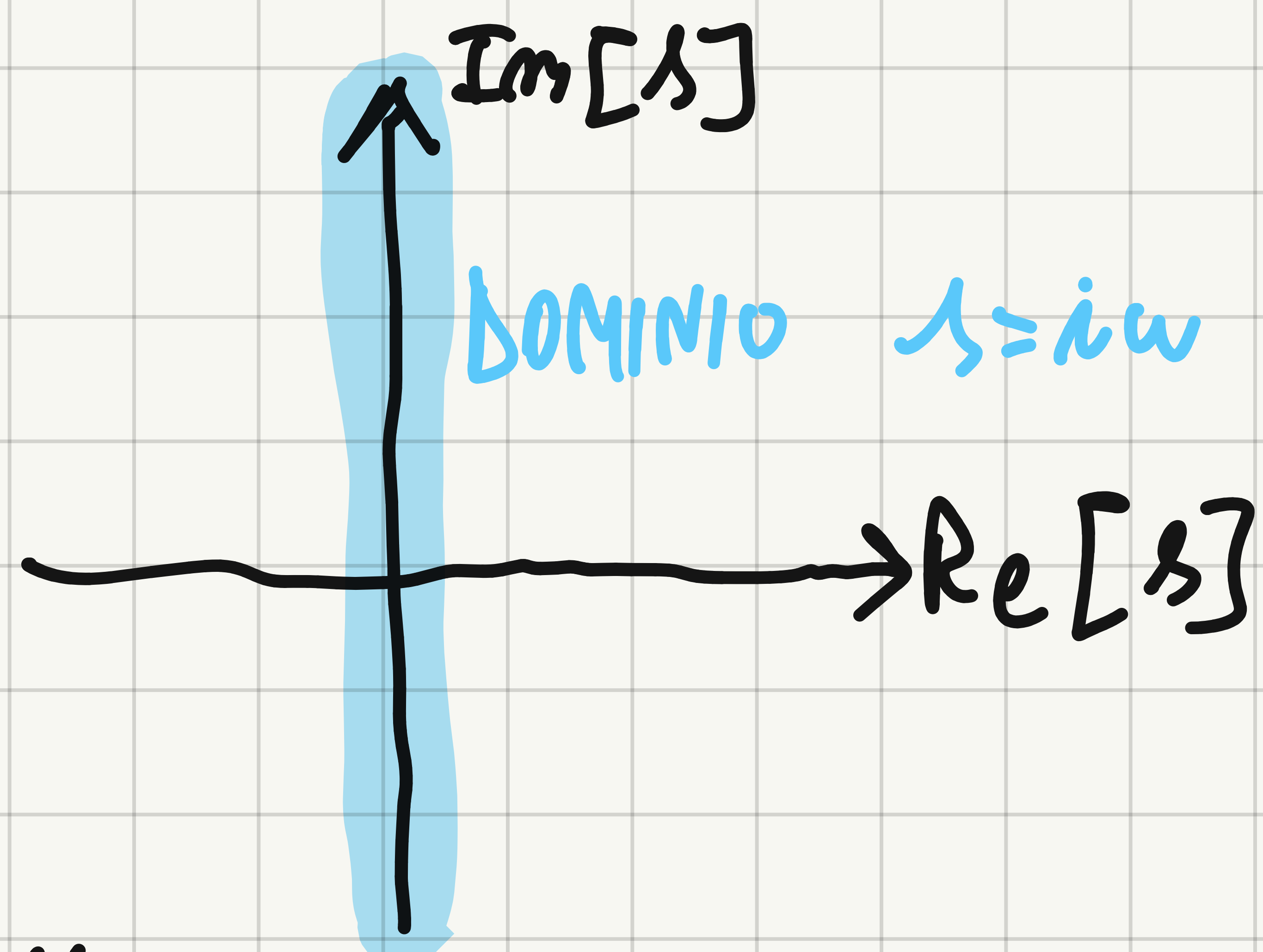
$\overset{\curvearrowright}{N} = -P_+$ 0, IN COERENZA CON L'ENUNCIATO, $\overset{\curvearrowleft}{N} = P_+$

OSSERVAZIONE: SE $P_+ < 0$, IL SISTEMA È INSTABILE

DIM

1) $F(s)$ NON PRESENTA POLI O ZERI SULL'ASSE IMMAGINARIA

$$W = \frac{F(s)}{1 + K_T F(s)}$$



$$F(s) = \frac{N_F(s)}{D_F(s)}$$

2) $F(s)$ È STRETTAMENTE PROPRIA, CIOÈ GRADO DI $D_F(s)$ > DEL GRADO DI $N_F(s)$

$$W(s) = \frac{\frac{N_F(s)}{D_F(s)}}{1 + K_T \frac{N_F(s)}{D_F(s)}} = \frac{N_F(s)}{D_F(s) + K_T \cdot N_F(s)} = \frac{N_W(s)}{D_W(s)}$$

$$\begin{cases} N_W(s) = N_F(s) \\ D_W(s) = D_F(s) + K_T \cdot N_F(s) \end{cases}$$

. DETTO n IL GRADO DI $D_F(s)$, QUAL È IL GRADO DI $D_W(s)$? POICHÉ LA $F(s)$

È STRETTAMENTE PROPRIA, IL GRADO DI $D_W(s) = n$

$\Rightarrow$ IL COEFFICIENTE DEL GRADO MASSIMO DI $D_F(s)$ È UGUALE AL COEFFICIENTE DEL GRADO MASSIMO DI $D_W(s)$.

INFATTI $D_W(s) = \underbrace{D_F(s)}_{\text{GRADO } n} + \underbrace{K_r N_F(s)}_{\text{GRADO } n-1 \text{ O MINORE}}$

DETTA $E(s) = 1 + K_r F(s) = \frac{F(s)}{W(s)}$,

$$E(s) = \frac{F(s)}{\frac{F(s)}{1+K_r F(s)}} = 1 + K_r \cdot \frac{N_F(s)}{D_F(s)} = \frac{D_F(s) + K_r N_F(s)}{D_F(s)}$$

$$\Longrightarrow E(s) = \frac{D_W(s)}{D_F(s)}$$

ESEMPIO: $F(s) = \frac{s+3}{2(s^2+2s+2)}$ $\quad n=2,\ K_r = 1$

$D_F(s) = 2s^2 + 4s + 4$

$D_W(s) = 2s^2 + 4s + 4 + 1 \cdot (s+3) = 2s^2 + 5s + 7$

$$E(s) = \frac{D_W(s)}{D_F(s)} = a_n s^n + \dots + a_1 s + a_0 = a_1 (s - s_1) \cdot \ldots \cdot (s - s_n)$$

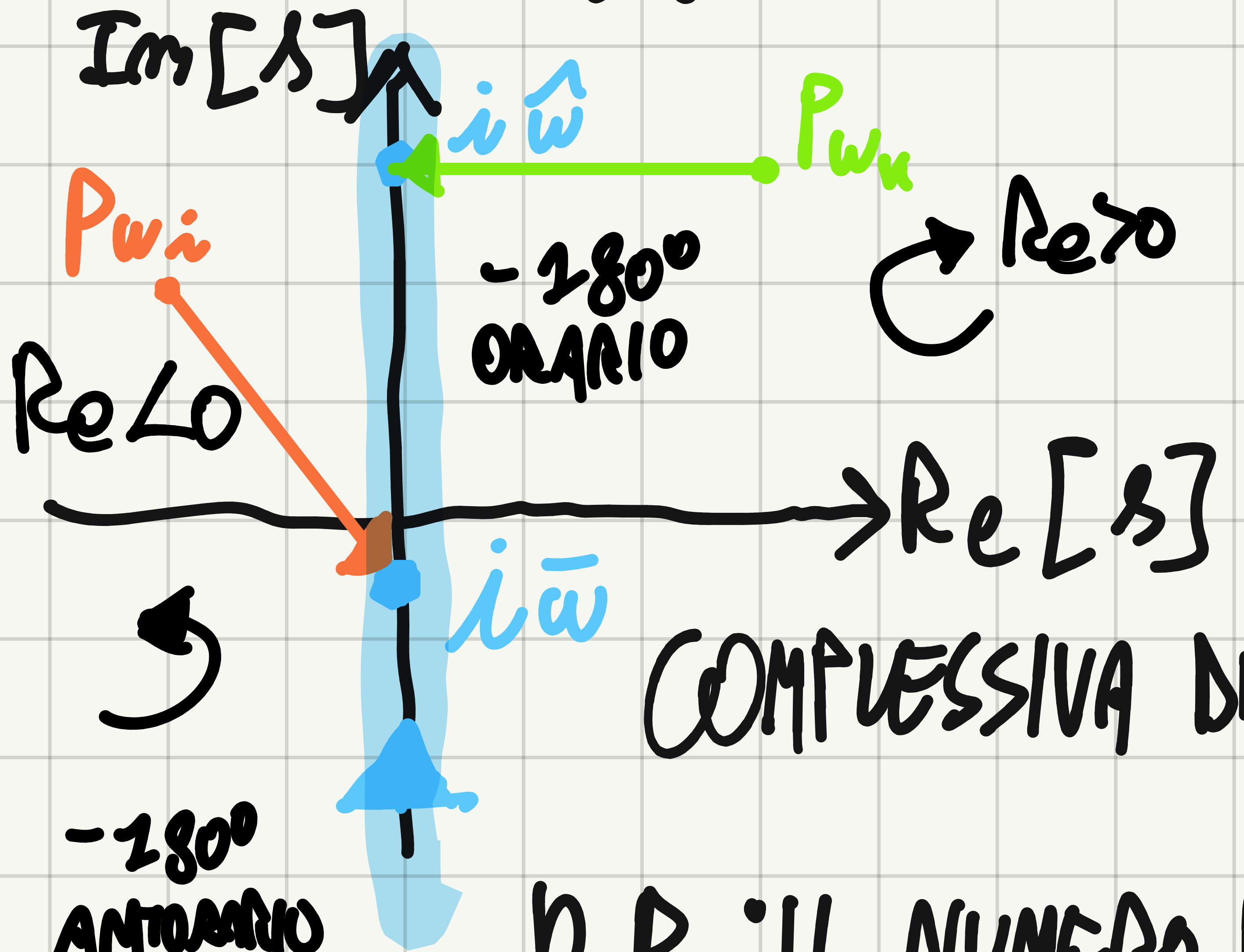
$$= \frac{K_r (s - P_{w_1}) \cdot \ldots \cdot (s - P_{w_n})}{K_r (s - P_{F_1}) \cdot \ldots \cdot (s - P_{F_n})} = \frac{\prod_{i=1}^{n} (i\omega - P_{W_i})}{\prod_{i=1}^{n} (i\omega - P_{F_i})}$$

$E(s) = 1 + K_r F(s) \rightarrow E(i\omega) = 1 + K_r \cdot F(i\omega)$. IL NUMERO DI GIRI DI $F(i\omega)$ INTORNO AL PUNTO $\left(-\frac{1}{K_r}; i0\right)$ COINCIDE CON IL

NUMERO DI GIRI DI $E(i\omega)$ INTORNO ALL'ORIGINE $(0;0)$?

$$E(i\omega)=\frac{\prod_{i=1}^{n}(i\omega-P_{W_i})}{\prod_{i=1}^{n}(i\omega-P_{F_i})}$$

$$\angle E(i\omega)=\sum_{i=1}^{n}\angle i\omega-P_{W_i}-\sum_{i=1}^{n}\angle i\omega-P_{F_i}$$



(ANALOGO IN P_F) $\Rightarrow$ SI SCEGLIE ANTIORARIO

INDICHIAMO CON Φ_E LA VARIAZIONE COMPLESSIVA DI $E(i\omega)$ PER $-\infty<\omega<+\infty$

n, P_+: IL NUMERO DI POLI A PARTE REALE POSITIVA DI $E(s)$ COINCIDE CON IL NUMERO DI POLI A PARTE REALE POSITIVA DI $F(s)$

n, Z_+: IL NUMERO DI ZERI A PARTE REALE POSITIVA DI $E(s)$ COINCIDE CON IL NUMERO DI POLI A PARTE REALE POSITIVA DI $W(s)$.

NUMERATORE DI $E(i\omega)$ DENOMINATORE DI $E(i\omega)$

$$\Phi_E=[\pi\cdot Z_+-(n-Z_+)\pi]-[\pi\cdot P_+-(n-P_+)\pi]$$

$$=\pi Z_+-\cancel{n\pi}+\pi Z_+-\pi P_++\cancel{n\pi}-\pi P_+=2\pi(Z_+-P_+)$$

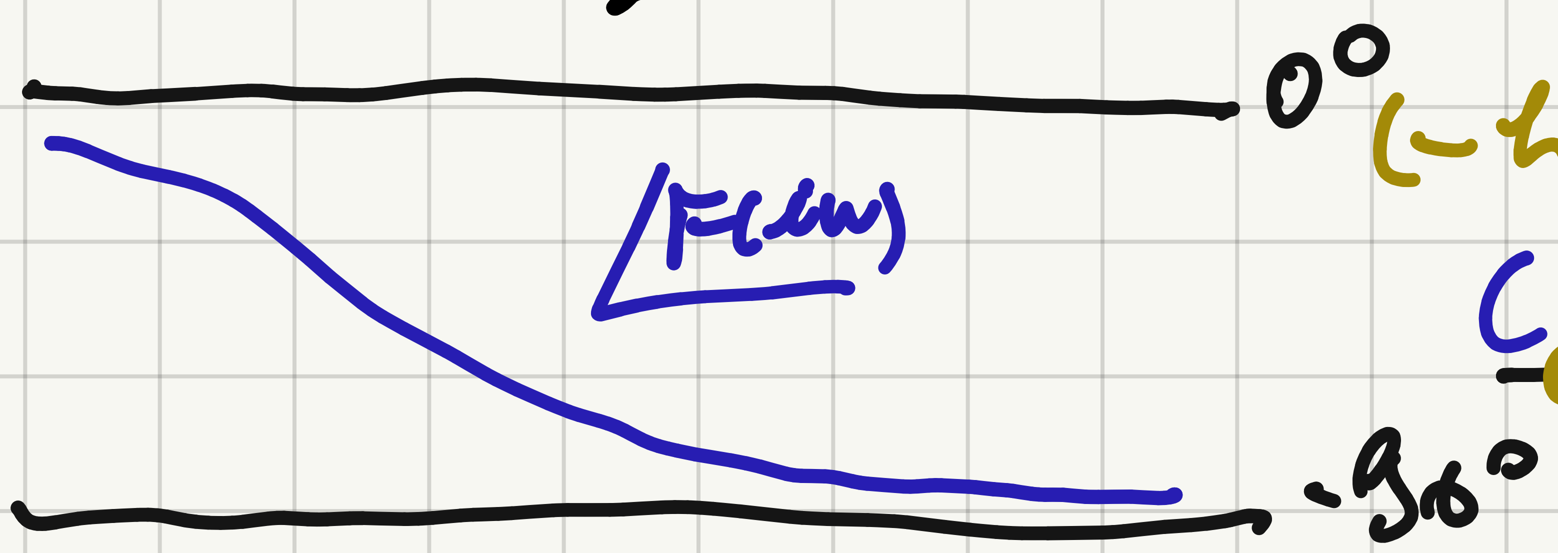
$$\overset{\curvearrowright}{N}=\frac{\Phi_E}{2\pi}=\cancel{Z_+}-P_+$$ $=0$ PER LA CONDIZIONE DI STABILITÀ ESTERNA SU $W(s)$

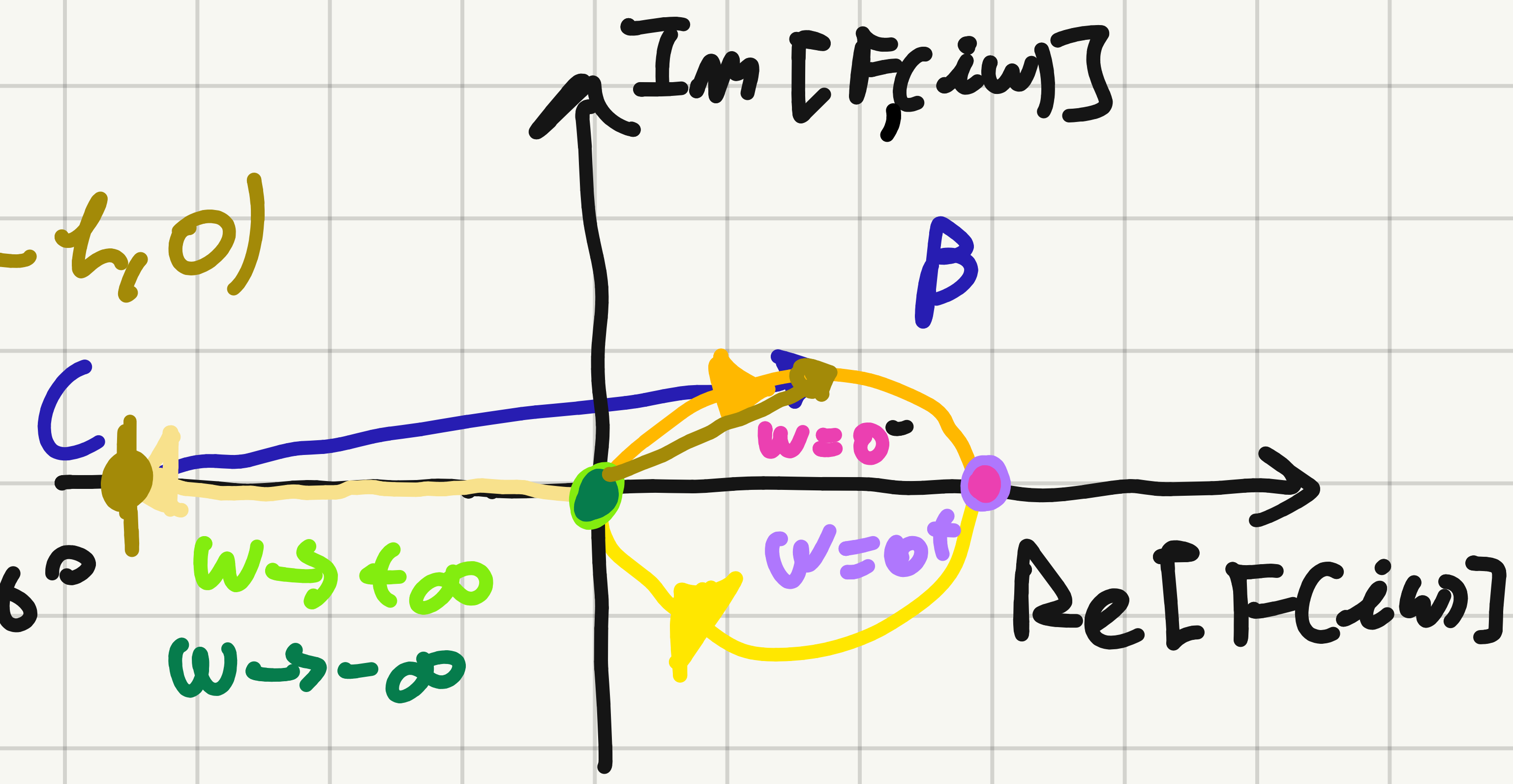
$\Rightarrow \overset{\curvearrowright}{N}=-P_+ \Rightarrow \overset{\curvearrowleft}{N}=P_+$ CVD

→ DOPO DIAGRAMMA, SIMMETRICO RISPETTO $\mathbb{R}$ E CHIUSURA IN VERSO ORARIO IN MODO CHE IL POLO IN 0 SI CONSIDERA A PARTE REALE NEGATIVA

ESEMPIO: $F(s) = \dfrac{10}{s+2}$, $K_r = 1$ $\quad F(s) = \dfrac{5}{1+\frac{s}{2}}$ $\quad P_+ = 0$

$F(i\omega) = \dfrac{5}{1+\frac{i\omega}{2}}$ $\quad M(0^+) = 5,\ \varphi(0^+) = 0°$ $\quad M(+\infty) = 0,\ \varphi(+\infty) = -90°$



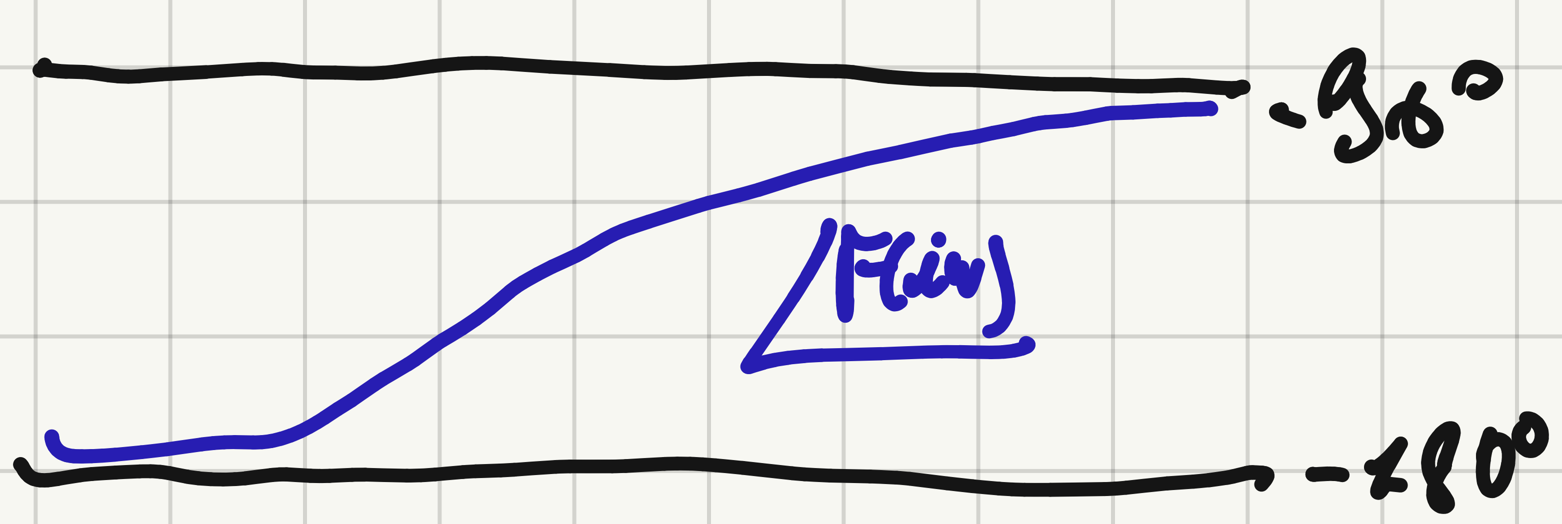


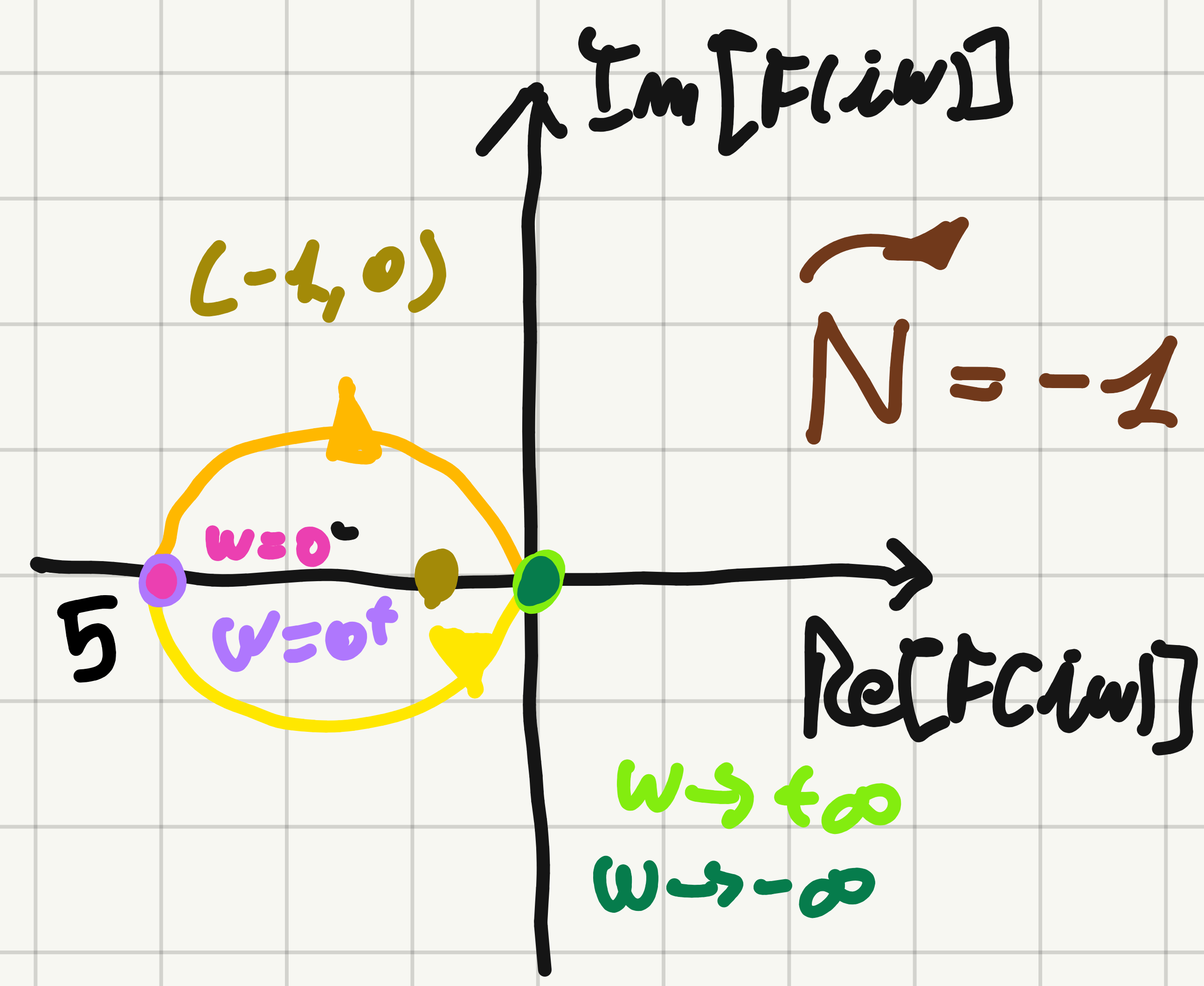
$\overrightarrow{CB} = F(i\omega) + 1 = E(i\omega)$ $\quad \overset{\curvearrowleft}{N} = 0$

$\overset{\curvearrowleft}{N} = -P_+ \Rightarrow$ IL SISTEMA A CICLO CHIUSO È STABILE

ESEMPIO: $F(s) = \dfrac{10}{s-2}$, $K_r = 1$ $\quad F(s) = \dfrac{5}{1-\frac{s}{2}}$ $\quad P_+ = 1$

$F(i\omega) = \dfrac{5}{1-\frac{i\omega}{2}}$ $\quad M(0^+) = 5,\ \varphi(0^+) = 180°$ $\quad M(+\infty) = 0,\ \varphi(+\infty) = -90°$



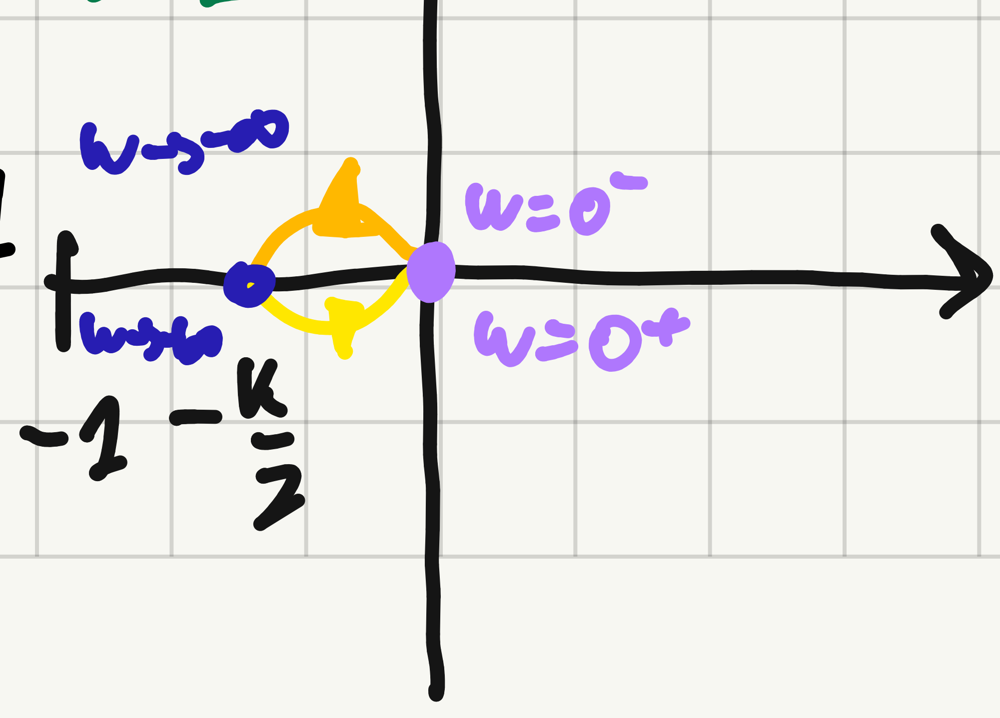


$\overset{\curvearrowleft}{N} = -1$

$\overset{\curvearrowleft}{N} = -P_+ \Rightarrow$ IL SISTEMA A CICLO CHIUSO È STABILE

ESEMPIO: $F(s) = \dfrac{K}{s-2}$ $\quad F(i\omega) = \dfrac{-K/2}{1-\frac{i\omega}{2}}$ $\quad P_+ = 1$

STABILE PER $-\dfrac{K}{2} > -1$

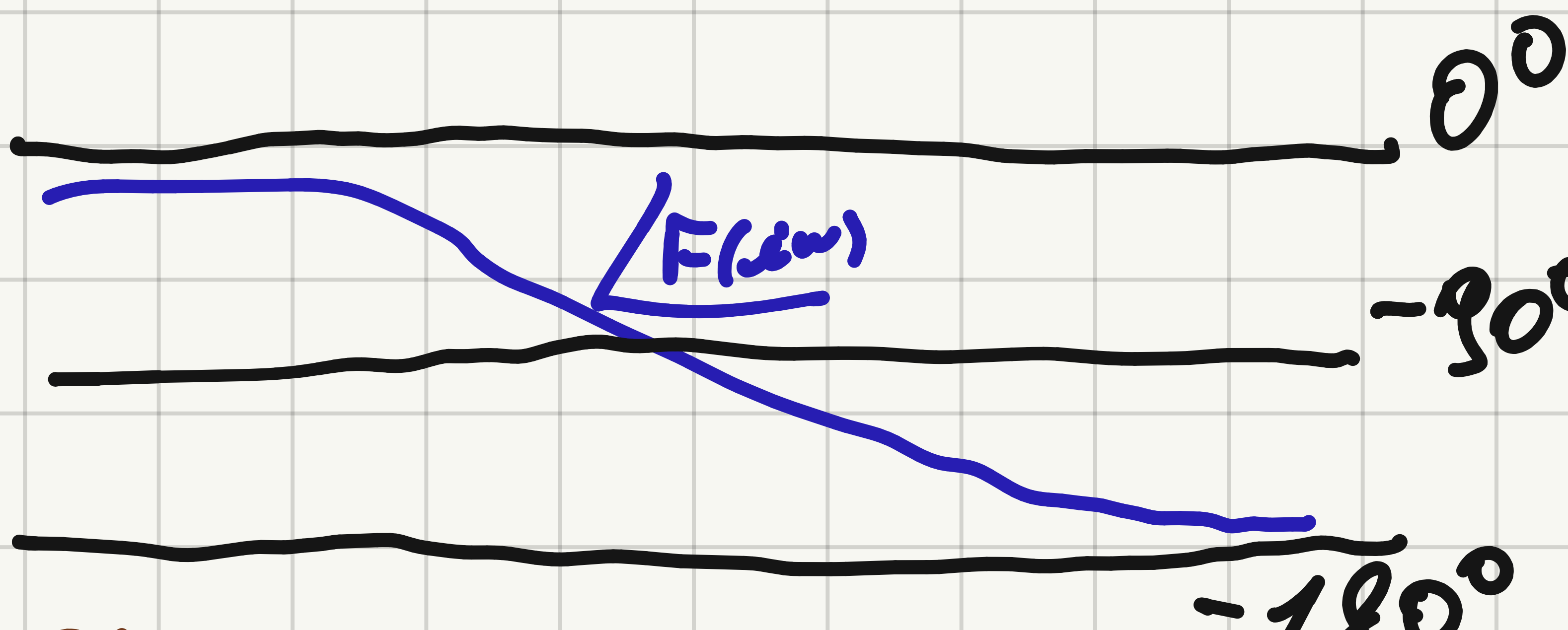


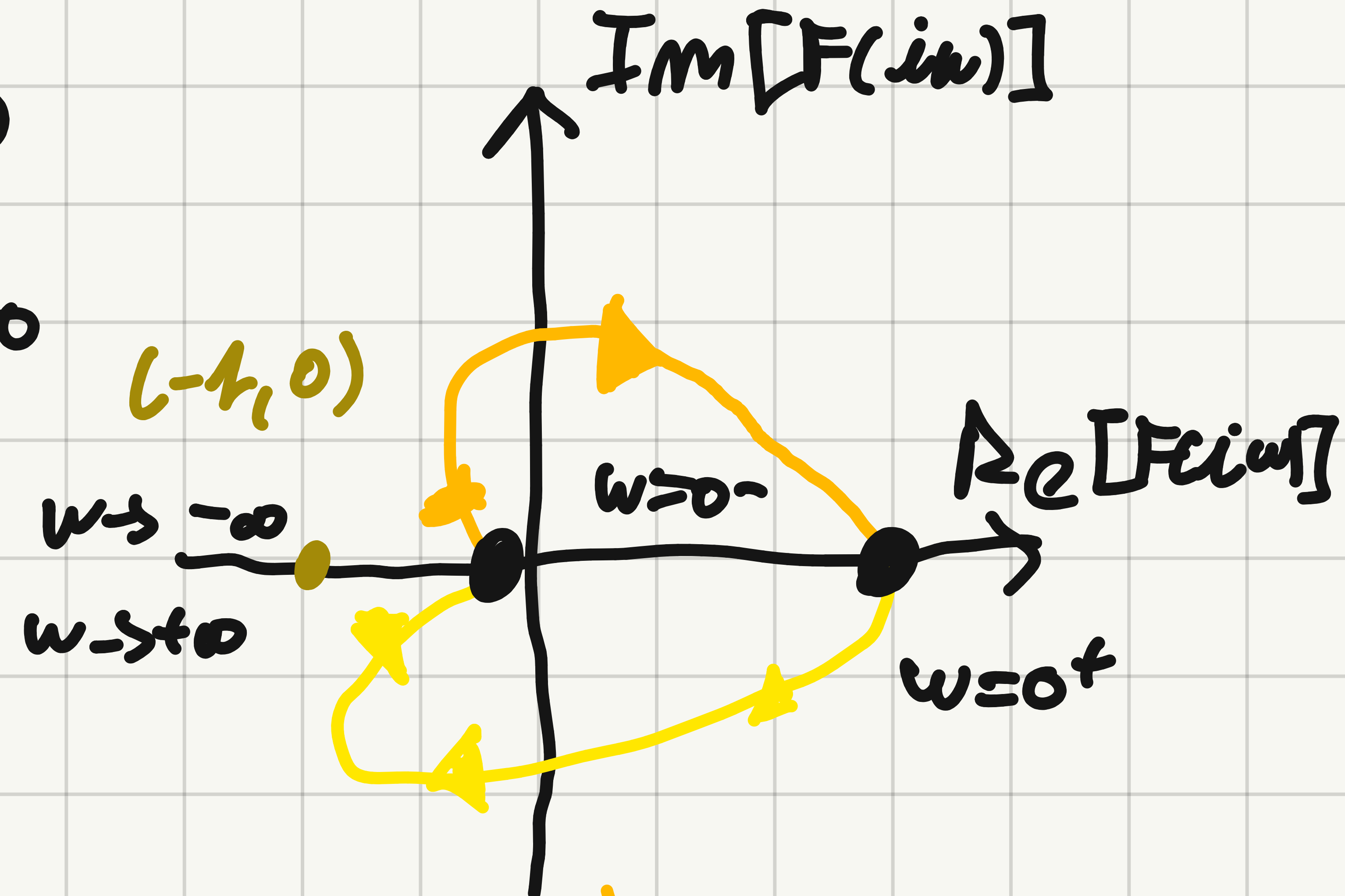
$\Rightarrow K < 2$

ESEMPIO: $F(s) = \dfrac{20}{(s+2)(s+5)}$ $\quad P_+ = 0$

$$F(i\omega) = \frac{5}{\left(1+\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{5}\right)} \qquad \angle F(i\omega) = -\arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{5}\right)$$

$M(0^+) = 2$, $\varphi(0^+) = 0°$, $M(+\infty) = 0$, $\varphi(+\infty) = -180°$





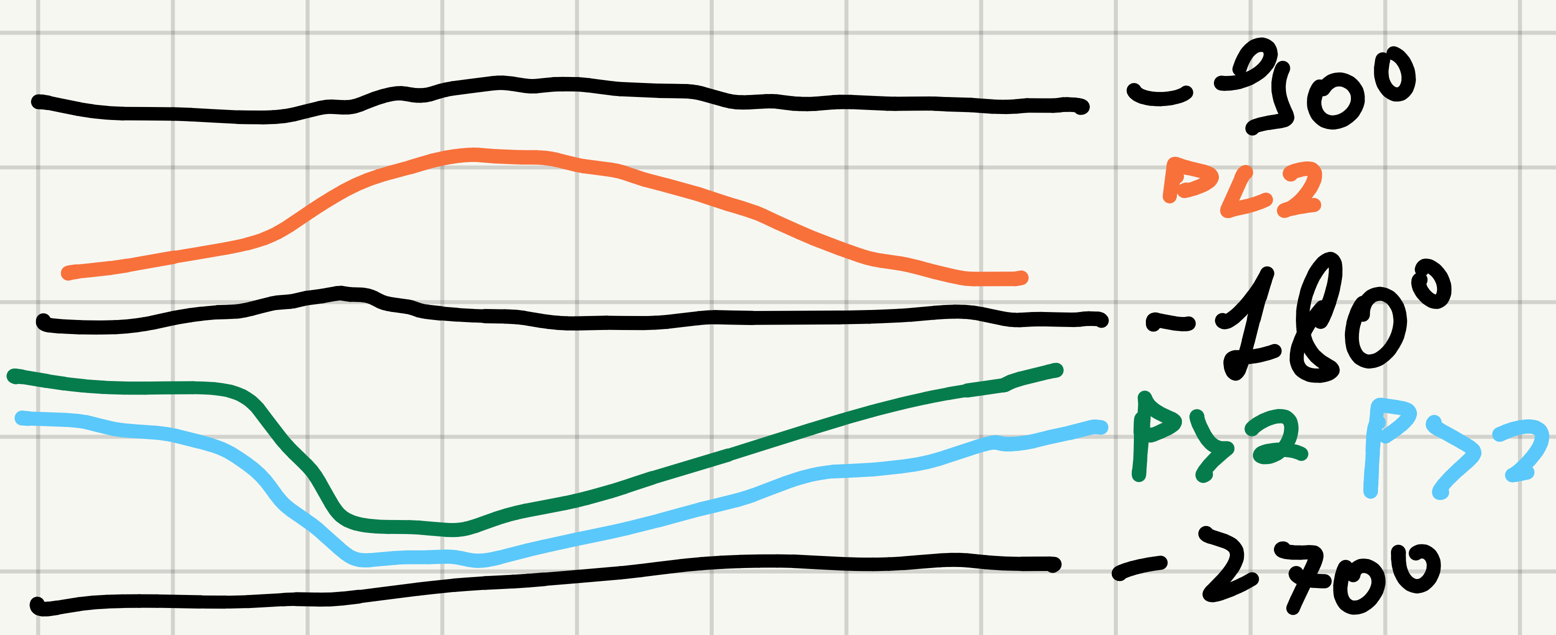
$\overset{\curvearrowright}{N} = 0$

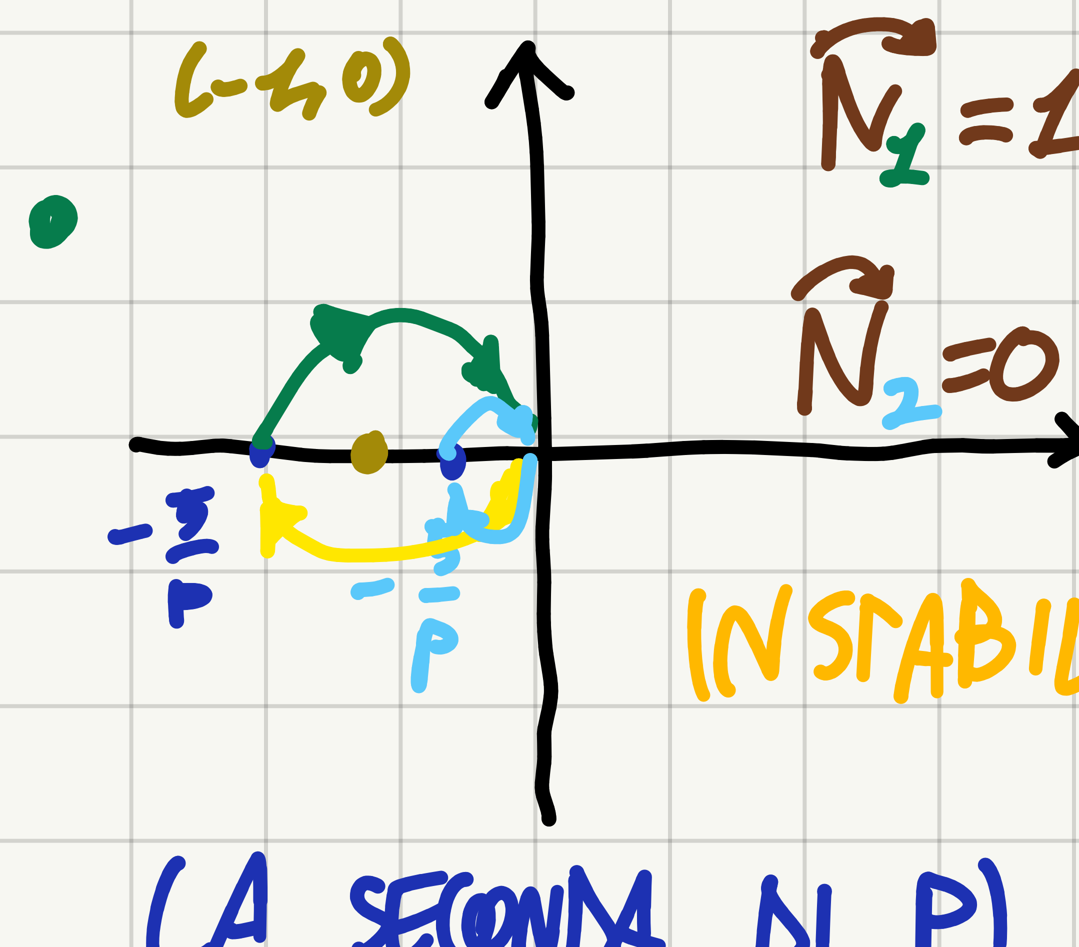
$\overset{\curvearrowright}{N} = -P_+ \Longrightarrow$ IL SISTEMA A CICLO CHIUSO È STABILE

ESEMPIO: $F(s) = \dfrac{10}{(s+2)(s-p)}$, $p > 0$ $\quad P_+ = 1$

$$F(i\omega) = -\frac{5}{p} \cdot \frac{1}{\left(1+\frac{i\omega}{2}\right)\left(1-\frac{i\omega}{p}\right)} \qquad \angle F(i\omega) = -180° - \arctan\left(\frac{\omega}{2}\right) + \arctan\left(\frac{\omega}{p}\right)$$

$M(0^+) = -\frac{5}{p}$, $\varphi(0^+) = -180°$ $\quad M(+\infty) = 0$, $\varphi(+\infty) = -180°$



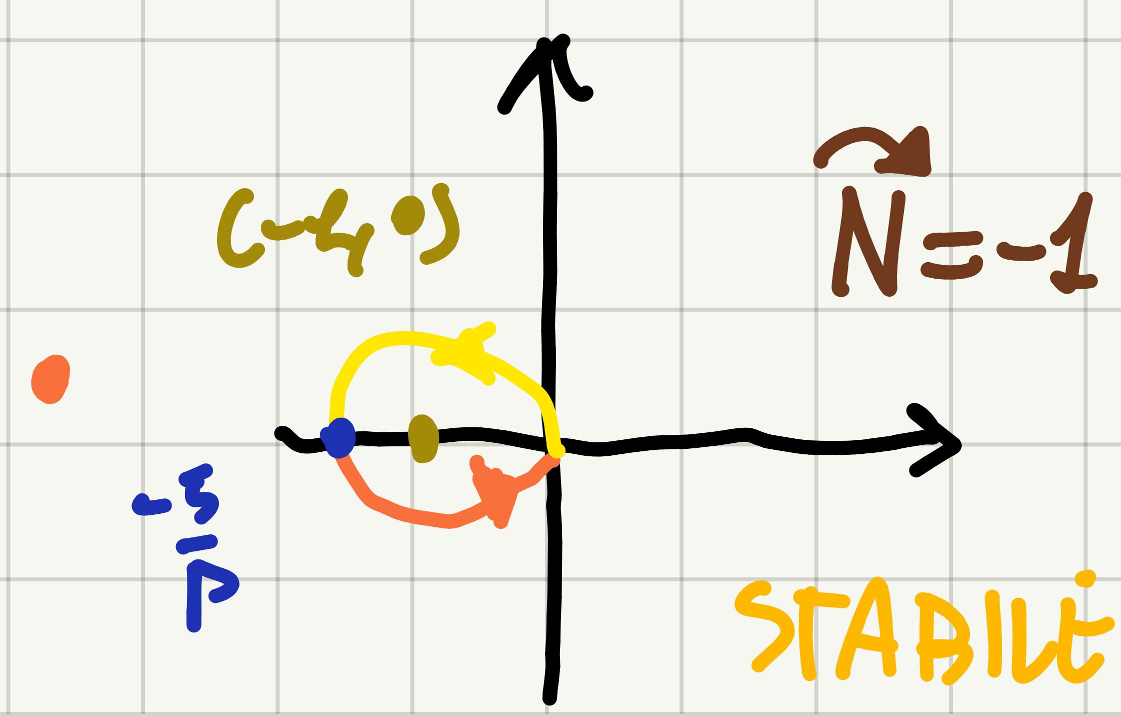


$\overset{\curvearrowright}{N_1} = 1$

$\overset{\curvearrowright}{N_2} = 0$

INSTABILE

(A SECONDA DI P)



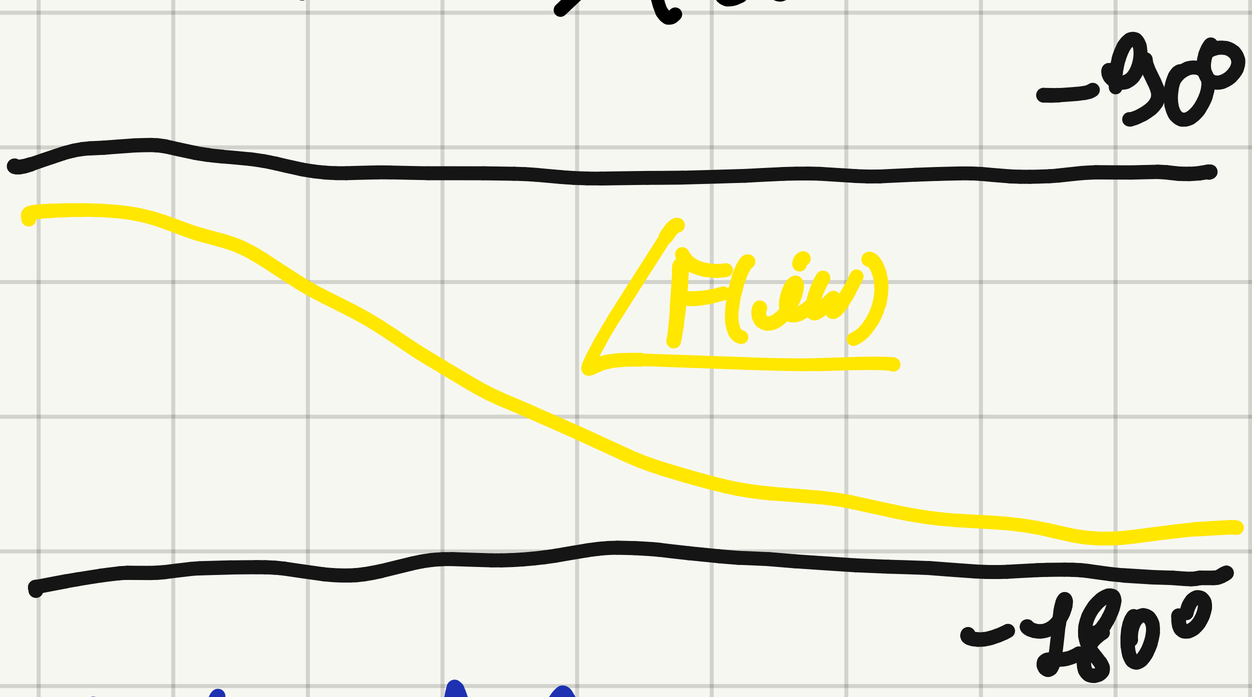
$\overset{\curvearrowright}{N} = -1$

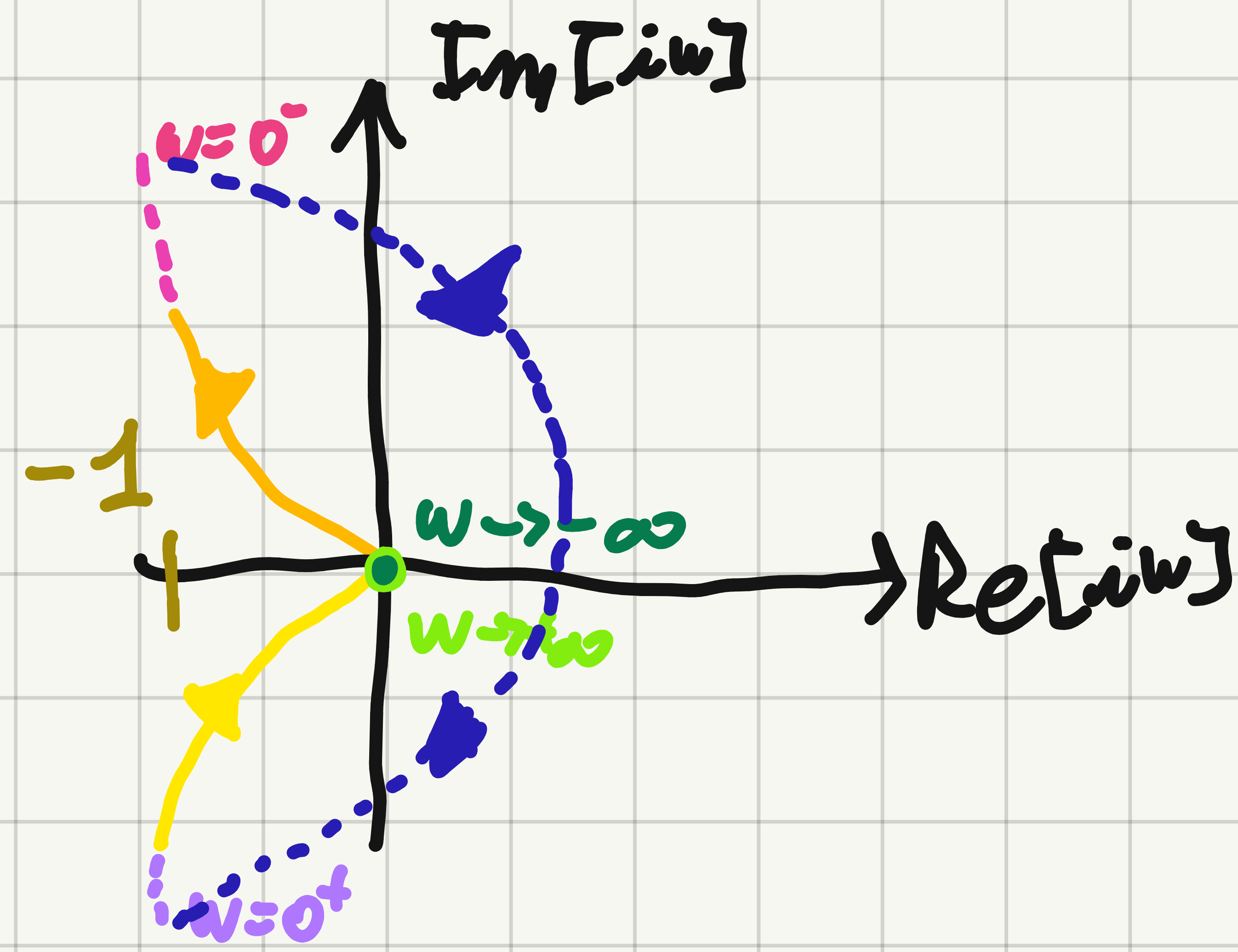
STABILE

Esempio: $F(s) = \frac{K}{s(s+2)}$, $K>0, K_T=1$ $\quad P_+=0$

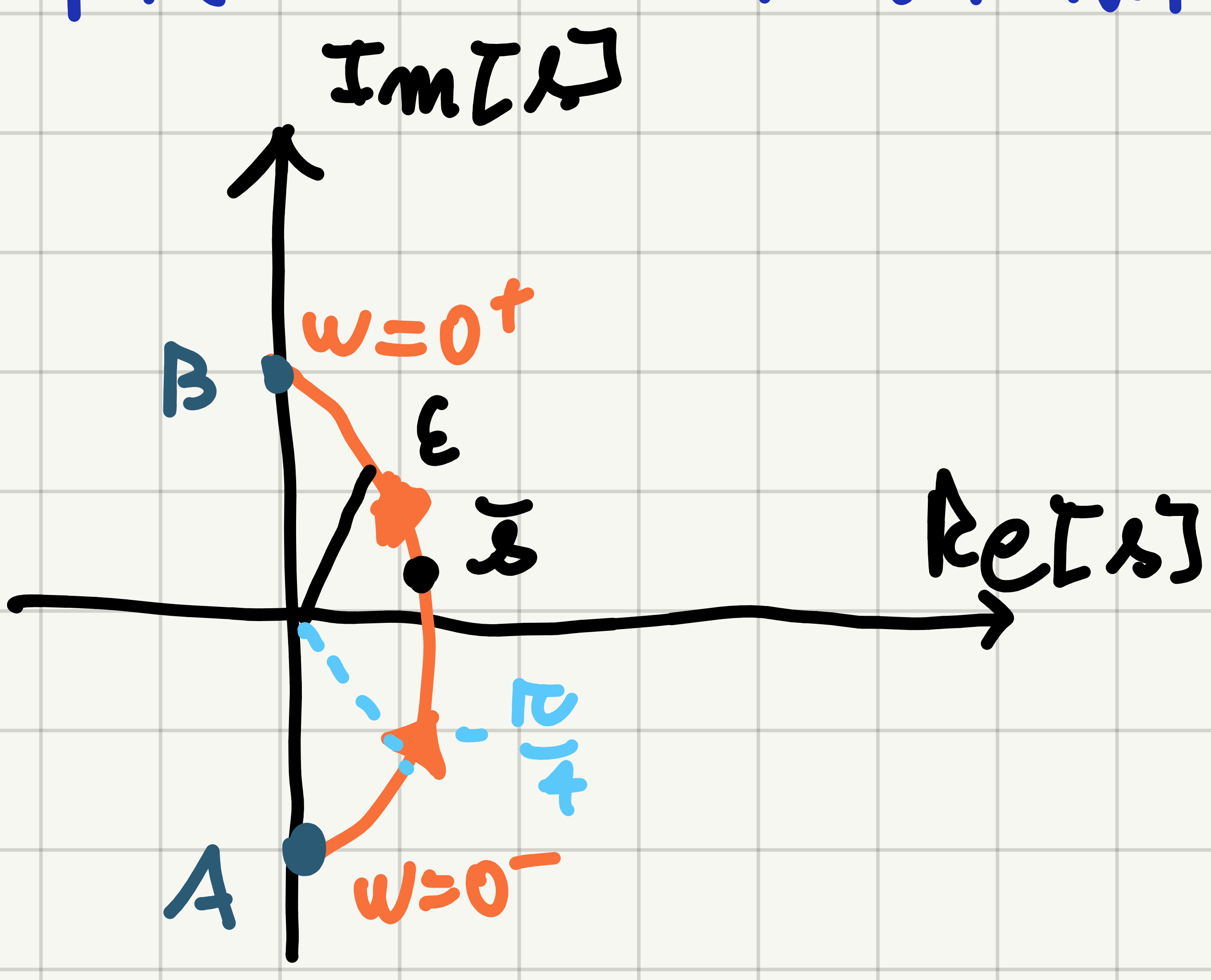
$$F(i\omega) = \frac{K/2}{i\omega\left(1+\frac{i\omega}{2}\right)} \qquad \angle F(i\omega) = -90° - \arctan\left(\frac{\omega}{2}\right)$$

$M(0^+)=+\infty,\ \varphi(0^+)=-90° \qquad M(+\infty)=0,\ \varphi(+\infty)=-180°$





CHIUSURA DA $\omega=0^- \rightarrow \omega=0^+$ IN SENSO ORARIO. CONSENTE LA CONVENZIONE PER CUI $s=0$ SI CONSIDERA COME POLO A PARTE REALE NEGATIVA



$A = \varepsilon e^{-\frac{\pi}{2}i} \qquad B = \varepsilon e^{\frac{\pi}{2}i}$

$\bar{s} = \varepsilon e^{i\varphi},\ \varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$F(s)\Big|_{s=\bar{s}} = \frac{K/2}{\bar{s}\left(1+\frac{\bar{s}}{2}\right)} = \frac{K/2}{\varepsilon e^{i\varphi}\left(1+\frac{\varepsilon}{2}e^{i\varphi}\right)}$$

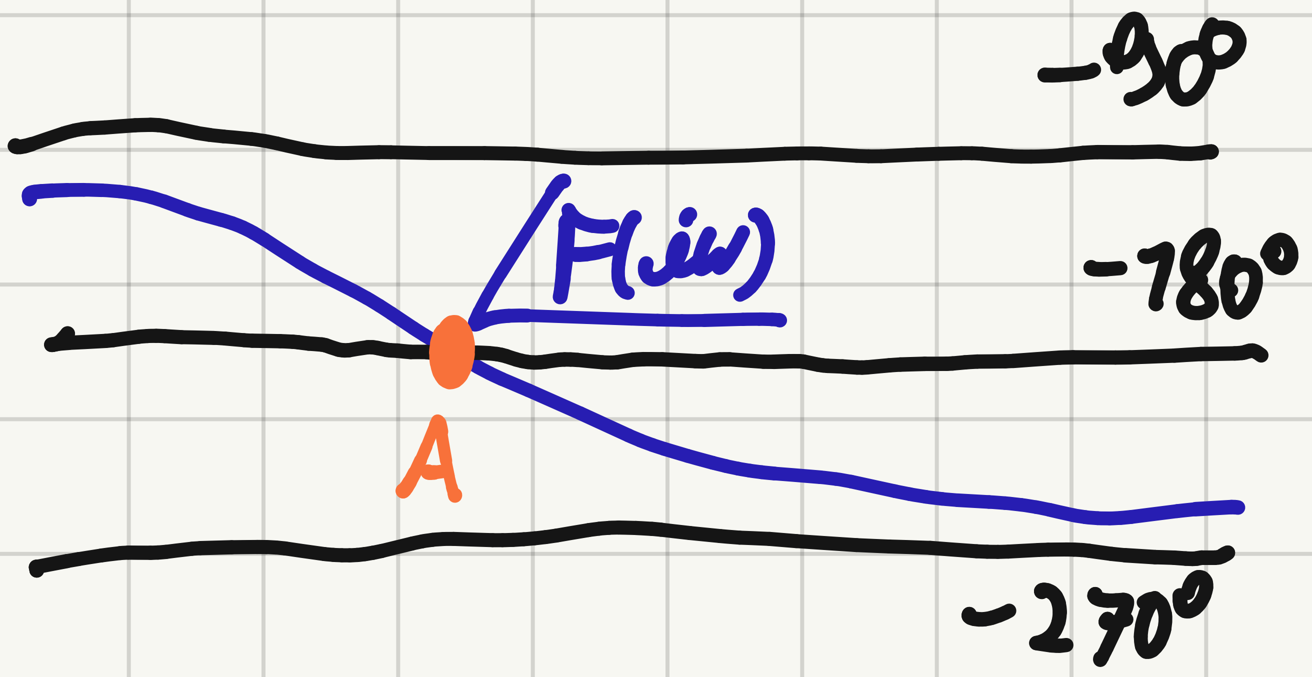
$\tilde{N}=0 \qquad \tilde{N}=-P_+ \Rightarrow$ SISTEMA A CICLO CHIUSO STABILE

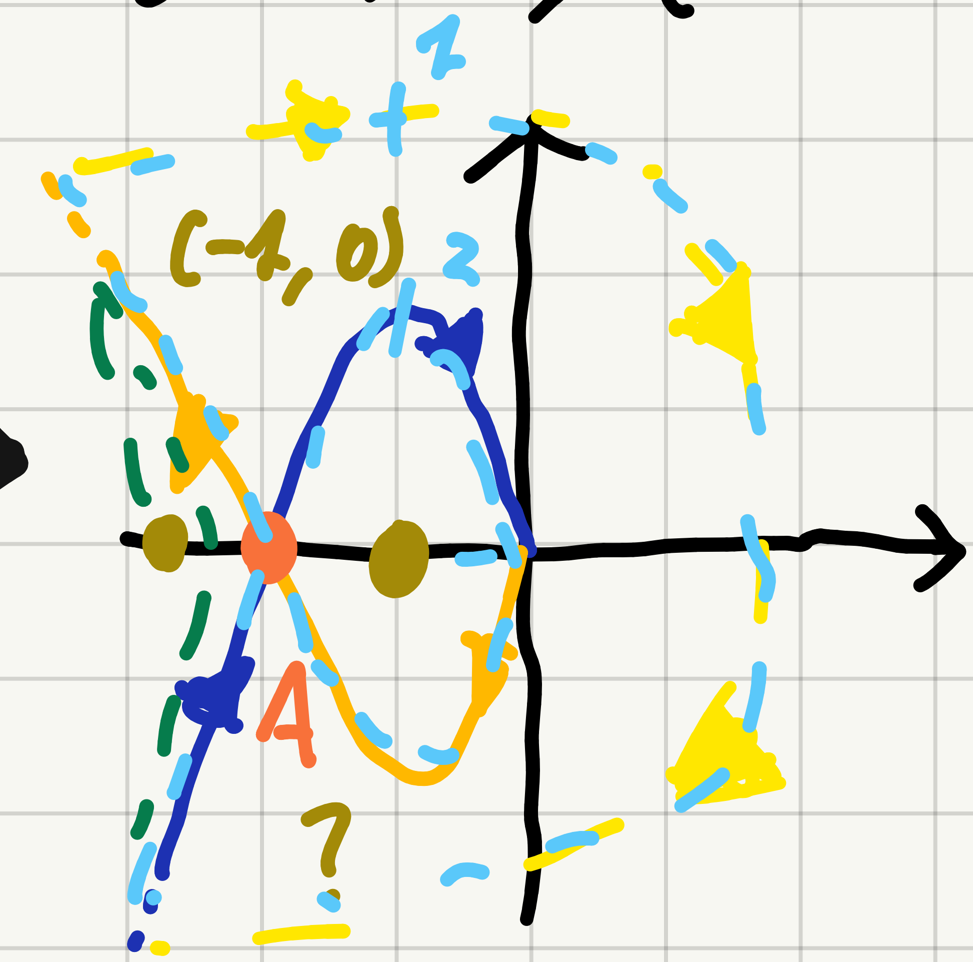
Esempio: $F(s) = \frac{K}{s(s+2)(s+4)}$, $K>0$

$$F(i\omega) = \frac{K}{8} \cdot \frac{1}{i\omega\left(1+\frac{i\omega}{2}\right)\left(1+\frac{i\omega}{4}\right)} \qquad P_+=0$$

$$\angle F(i\omega) = -90^\circ - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{4}\right)$$

$M(0^+) = +\infty,\ \varphi(0^+) = -90^\circ$ $M(+\infty) = 0,\ \varphi(+\infty) = -270^\circ$





- $|A| < 1$ $N = 0$ $N = -P_+$ $\Longrightarrow$ STABILE
- $|A| > 1$ $N = 2$ $N \neq -P_+$ $\Longrightarrow$ INSTABILE

$$F(i\omega) = \frac{K/8}{i\omega(1+\frac{i\omega}{2})(1+\frac{i\omega}{4})} = \frac{K}{8\omega} \cdot \frac{1}{(i-\frac{i\omega}{2})(1+\frac{i\omega}{4})} = -\frac{K}{8} \cdot \frac{1}{(\frac{\omega}{2}-i)(1+\frac{\omega}{4}i)}$$

$$= -\frac{K}{8\omega} \cdot \frac{1}{\frac{3}{4}\omega + i\left(-1+\frac{\omega^2}{8}\right)} = -\frac{K}{8\omega} \cdot \frac{\frac{3}{4}\omega - i\left(\frac{\omega^2}{8}-1\right)}{\frac{9}{16}\omega^2 + \left(\frac{\omega^2}{8}-1\right)^2}$$

$$Re[F(i\omega)] = \frac{-\frac{3K}{32}}{\frac{9\omega^2}{16} + \left(\frac{\omega^2}{8}-1\right)^2} \qquad Im[F(i\omega)] = \frac{K\left(\frac{\omega^2}{8}-1\right)}{8\omega\left(\frac{9}{16}\omega^2 + \left(\frac{\omega^2}{8}-1\right)^2\right)}$$

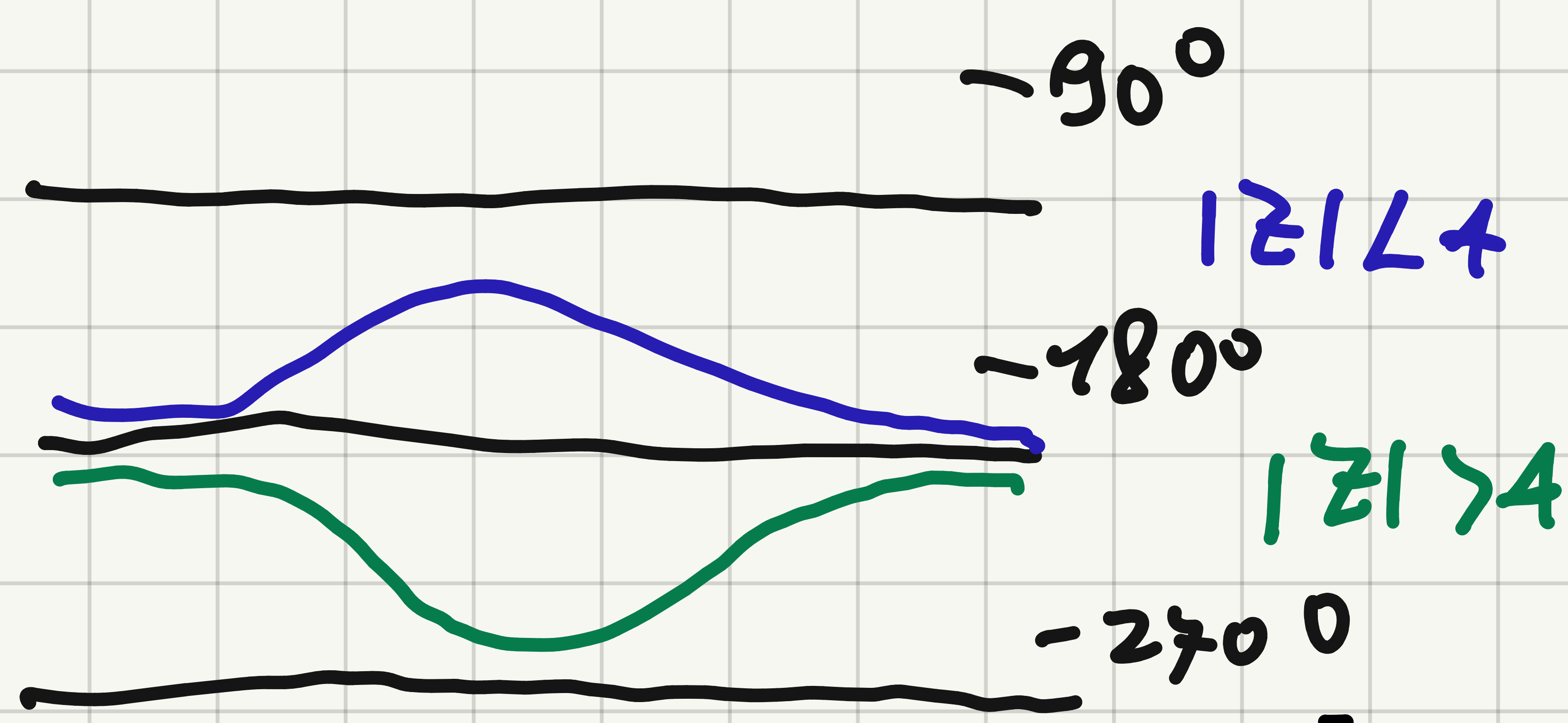
$Im[F(i\omega^*)] = 0 \Longrightarrow \frac{\omega^2}{8} - 1 = 0 \rightarrow \omega = \pm 2\sqrt{2}$

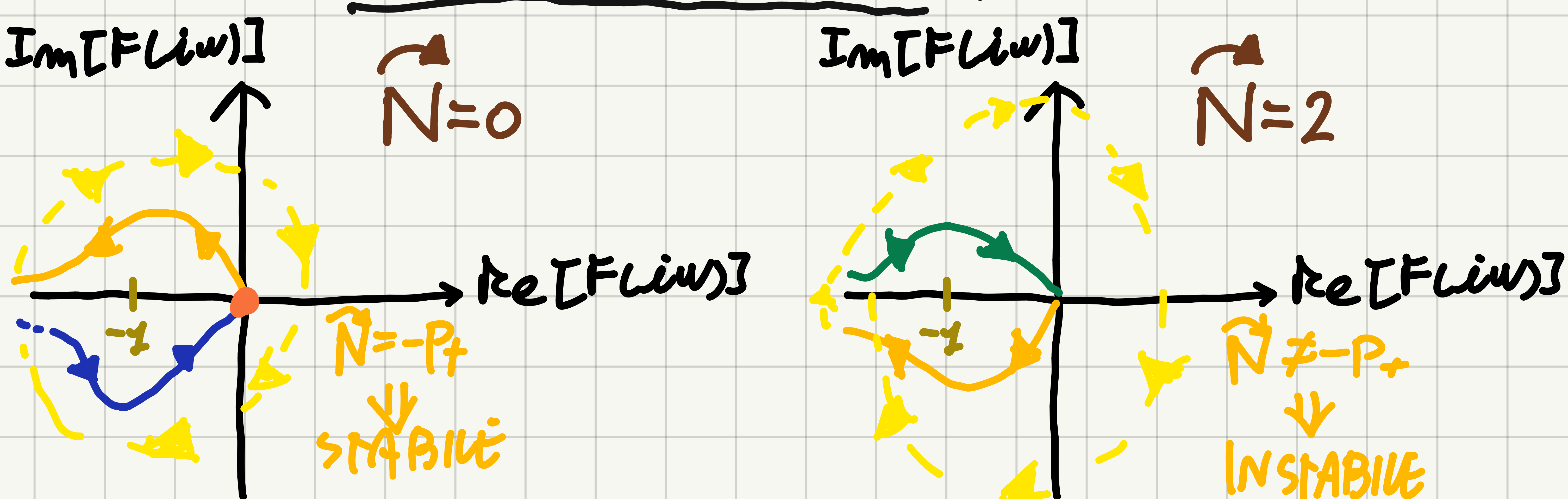
$Re[F(i\omega^*)] = \dfrac{-3K/32}{\frac{9}{16}\cdot(\pm 2\sqrt{2})^2 + 0} = -\dfrac{K}{48} \Longrightarrow A = \left(-\dfrac{K}{48}, 0\right) \Longrightarrow$ STABILITÀ $K < 48$

ESEMPIO: $F(s) = \dfrac{K(s-z)}{s^2(s+4)}$, $K>0$, $z<0$ $\qquad P_+ = 0$

$$F(i\omega) = -\frac{kz}{4}\cdot\frac{\left(1-\frac{i\omega}{z}\right)}{(i\omega)^2\left(1+\frac{i\omega}{4}\right)} \qquad \angle F(i\omega) = -180° + \arctan\left(\frac{\omega}{|z|}\right) - \arctan\left(\frac{\omega}{4}\right)$$

$M(0^+) = +\infty$, $\varphi(0^+) = -180°$ $\qquad M(+\infty) = 0$, $\varphi(+\infty) = -180°$





STABILITÀ CONDIZIONATA

Dati dei valori prefissati, possiamo progettare il sistema in modo che sia stabile a tratti. Ad esempio, dati tre valori $K_1, K_2, K_3 > 0$, il sistema risulta:

- stabile per $0 < K < K_1$
- instabile per $K_1 < K < K_2$
- stabile per $K_2 < K < K_3$
- instabile per $K > K_3$