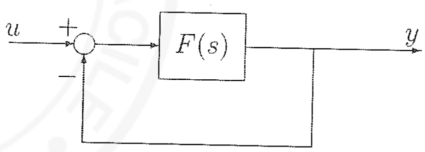
Domanda Scritta di Controlli Automatici (9CFU) - 19/11/2012

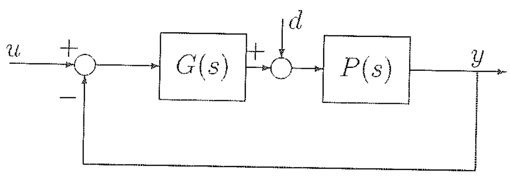
Esercizio 1 È dato il sistema di controllo:



in cui: $F(s) = \dfrac{K}{s(s-p)}$. Utilizzando il criterio di Nyquist, studiare la stabilità del sistema a ciclo chiuso, per $K \in \mathbb{R}$, $K \neq 0$, $p \in \mathbb{R}$, $p \neq 0$.

Esercizio 2

È dato il sistema di controllo:



in cui $P(s) = \dfrac{(s+3)}{s(s+4)}$; $\quad d(t) = \delta_{-1}(t)$.

Utilizzando la sintesi con il luogo delle radici, progettare $G(s)$ in modo che:

- il sistema sia astatico rispetto al disturbo $d(t)$.
- tutti i poli della funzione di trasferimento in catena chiusa abbiano parte reale minore di -3.

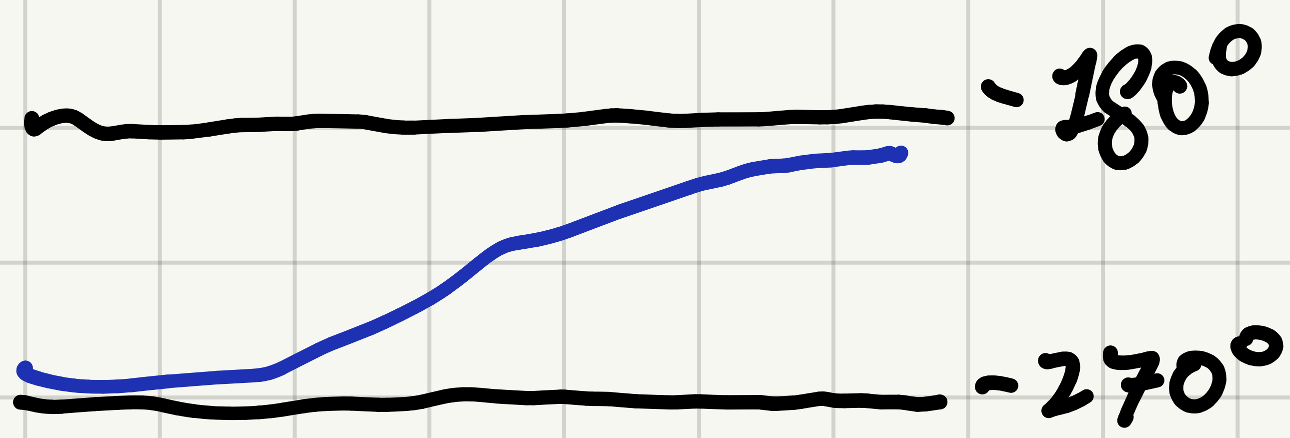
Calcolare infine la risposta a regime permanente all'ingresso $u(t) = (3t+2)\delta_{-1}(t)$.

1

$$F(s) = -\frac{K}{P} \cdot \frac{1}{s\left(1-\frac{s}{P}\right)} \qquad F(i\omega) = -\frac{K}{P} \cdot \frac{1}{i\omega\left(1-\frac{i\omega}{P}\right)}$$

CASO 1: $K>0, P>0$ $\quad P_t = 1$

$M(0^+)=\infty, \varphi(0^+) = -270°$ $\quad M(+\infty)=0, \varphi(+\infty) = -180°$

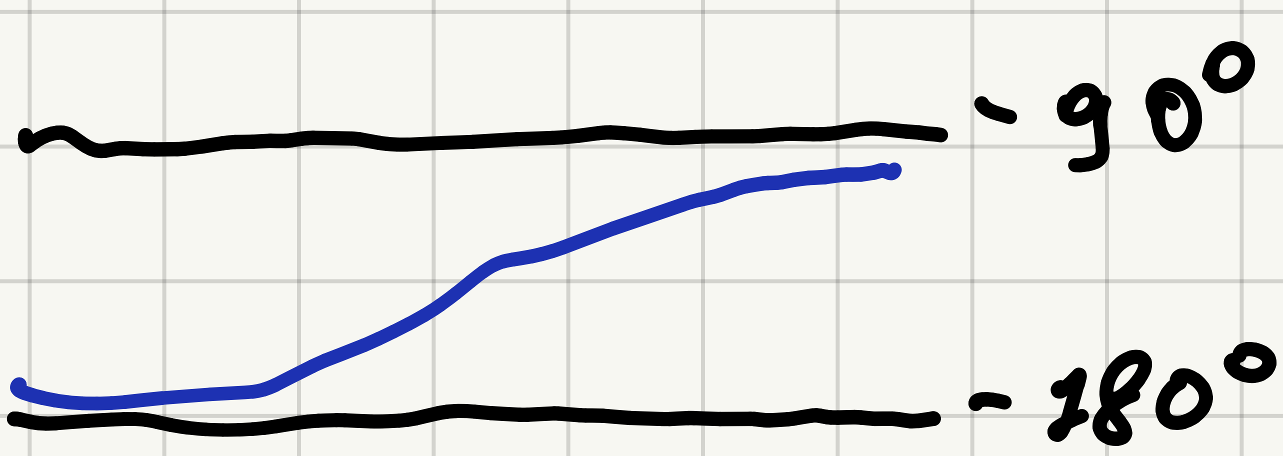


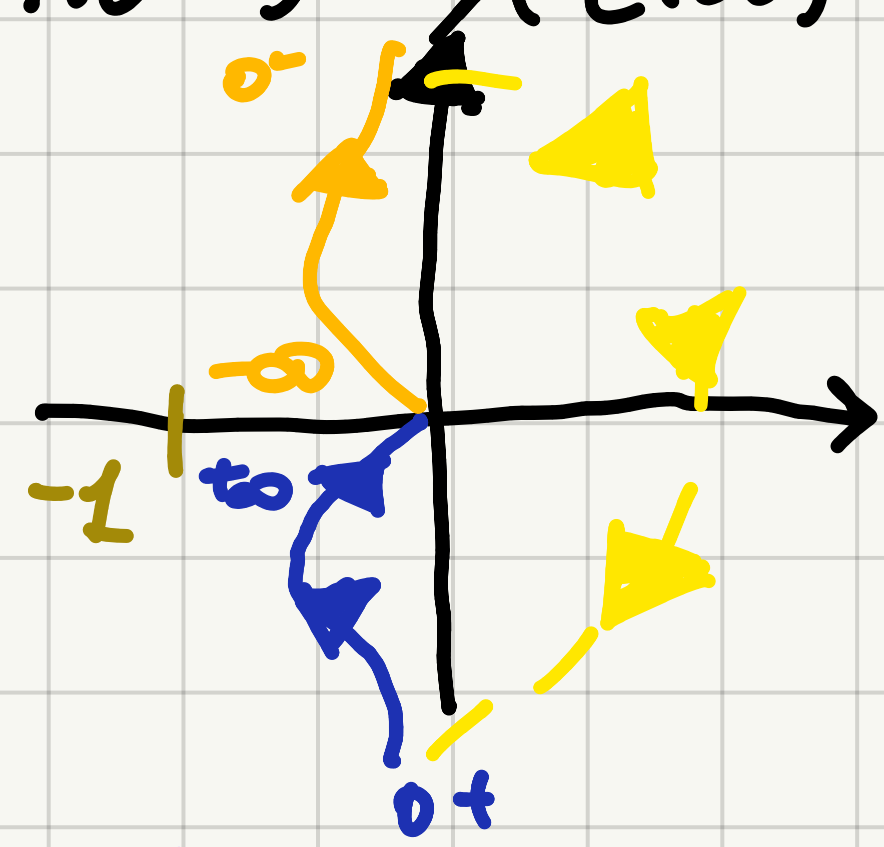
$\overset{\curvearrowleft}{N} = 2$

$\overset{\curvearrowleft}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 1: $K>0, P<0$ $\quad P_t = 0$

$M(0^+)=\infty, \varphi(0^+) = -180°$ $\quad M(+\infty)=0, \varphi(+\infty) = -90°$



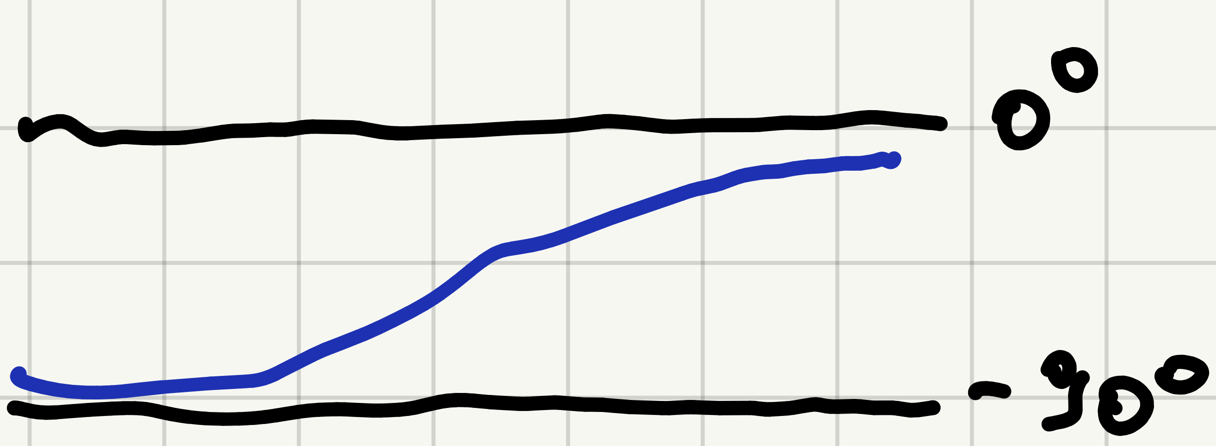


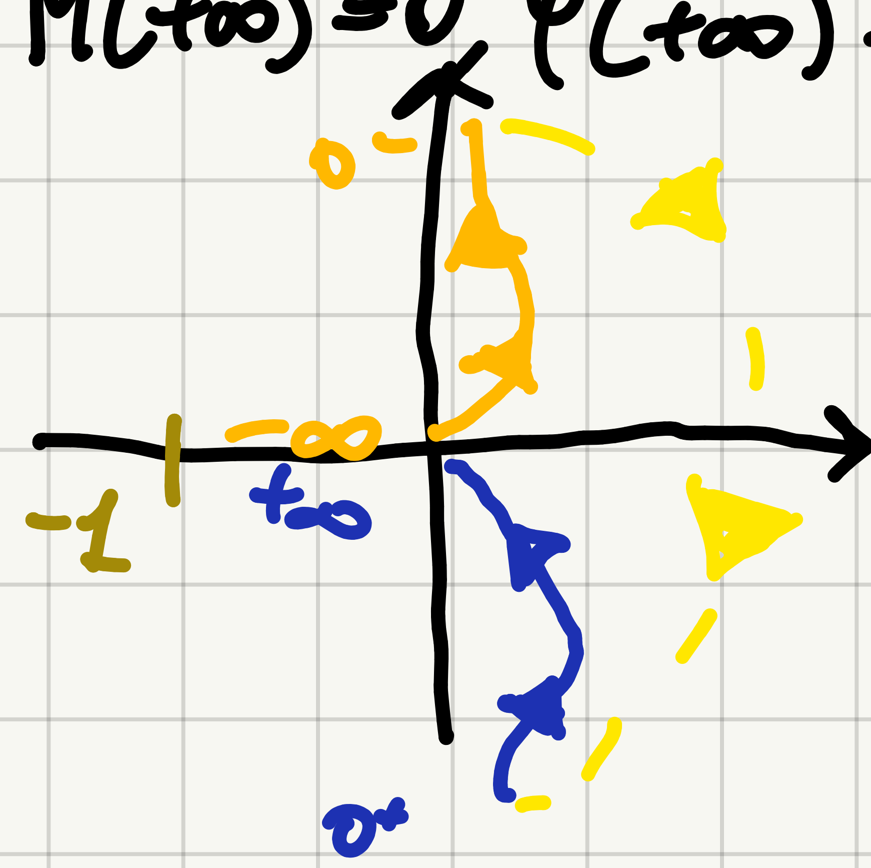
$\overset{\curvearrowleft}{N} = 0$

$\overset{\curvearrowleft}{N} = -P_t \Rightarrow$ SISTEMA STABILE

CASO 1: $K<0, P>0$ $\quad P_t = 1$

$M(0^+)=\infty, \varphi(0^+) = -90°$ $\quad M(+\infty)=0, \varphi(+\infty) = 0$



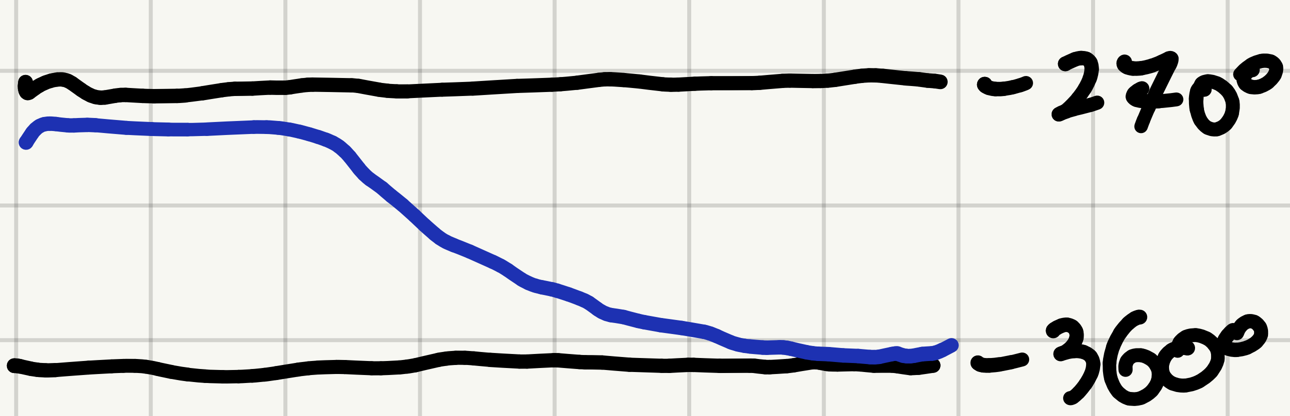


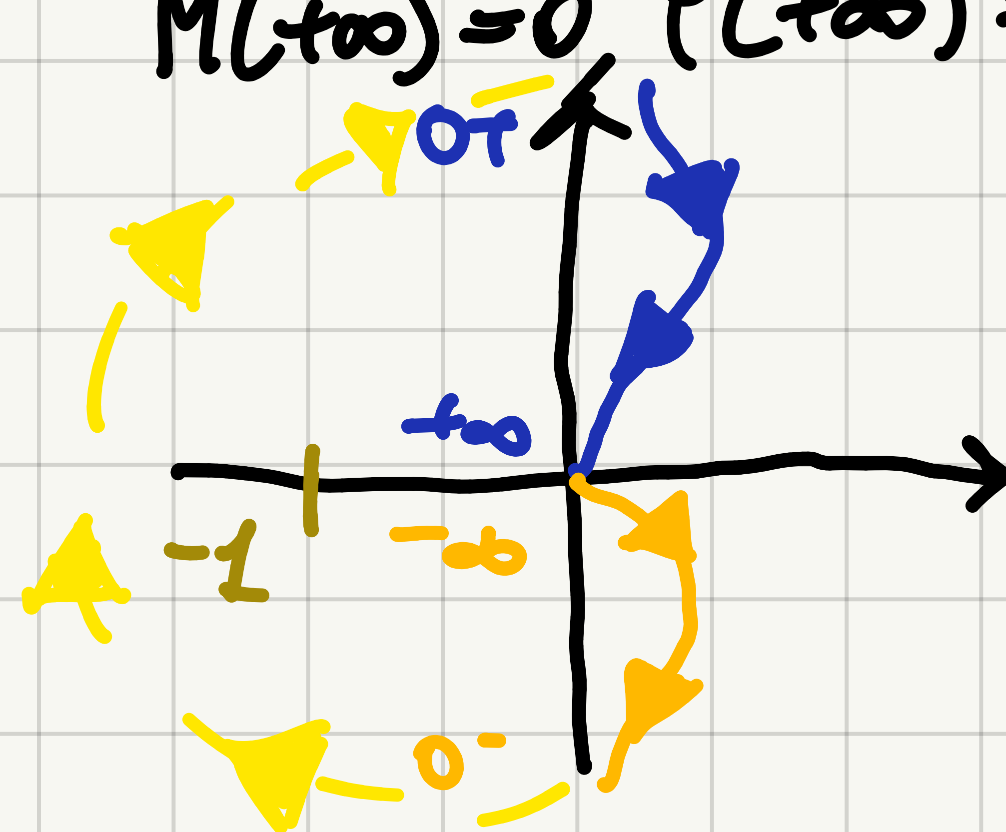
$\overset{\curvearrowleft}{N} = 0$

$\overset{\curvearrowleft}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

CASO 1: $K<0, P<0$ $\quad P_t = 0$

$M(0^+)=\infty, \varphi(0^+) = -270°$ $\quad M(+\infty)=0, \varphi(+\infty) = -360°$





$\overset{\curvearrowleft}{N} = 1$

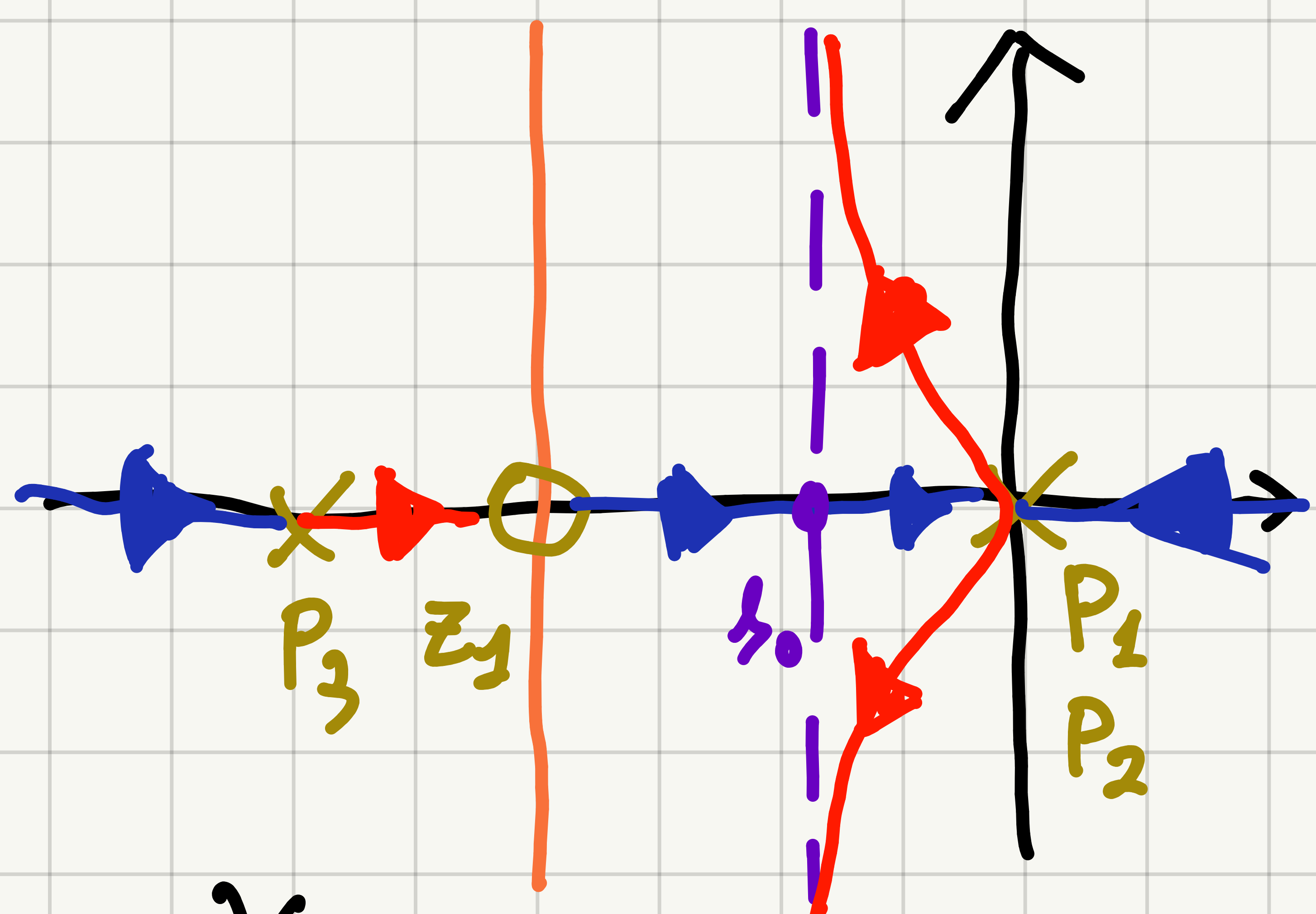
$\overset{\curvearrowleft}{N} \neq -P_t \Rightarrow$ SISTEMA INSTABILE

2

$G(s) = \frac{K_G}{s}$ $\quad F(s) = G(s)\cdot P(s) = K_G \cdot \frac{(s+3)}{s^2(s+4)}$

$n=3,\ m=1 \Rightarrow n-m=2$

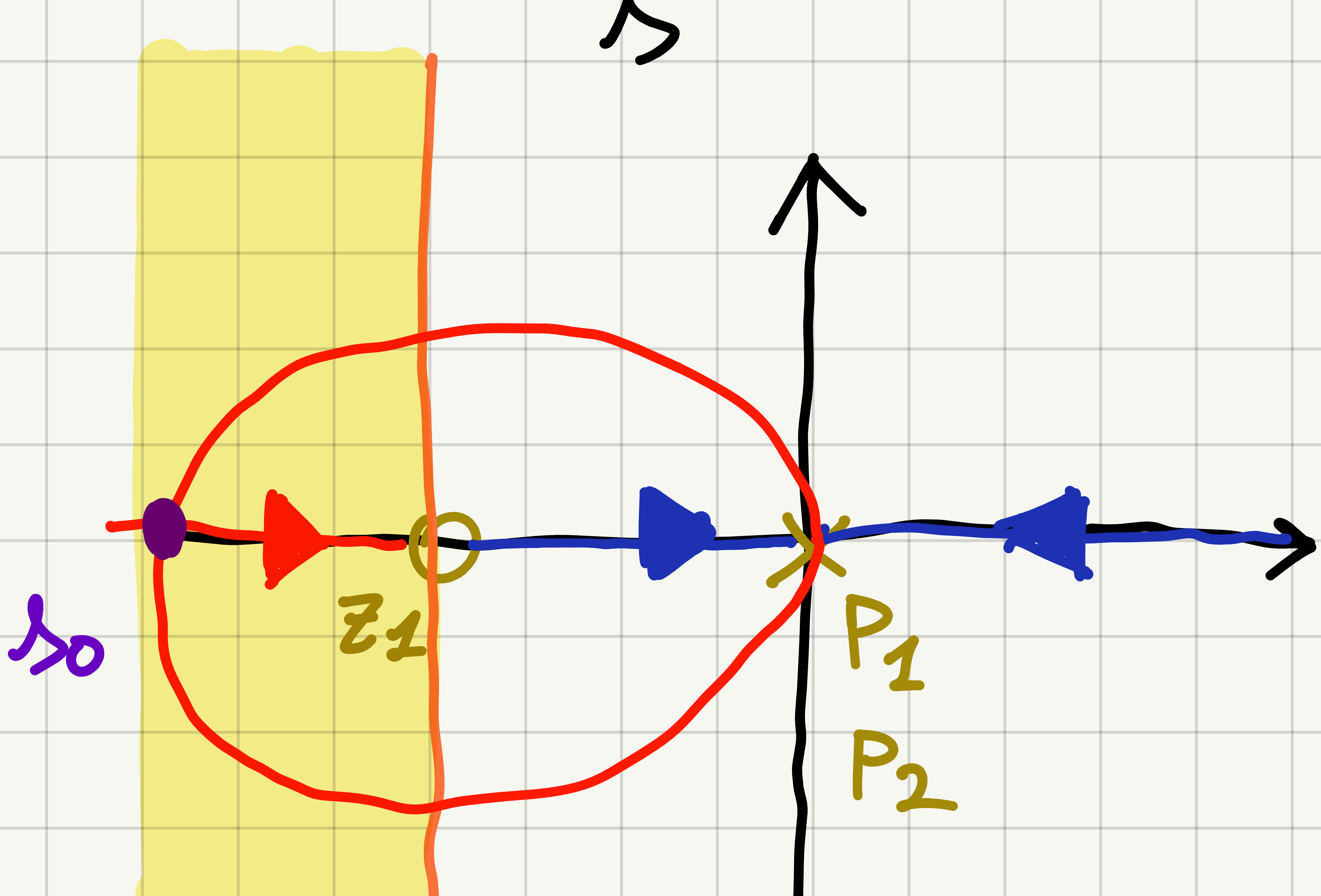
$P_1 = P_2 = 0,\ P_3 = -4;\ z_1 = -3$ $\quad s_0 = \frac{\sum P - \sum z}{n-m} = -\frac{1}{2}$



SPECIFICA NON SODDISFATTA

$G(s) = \frac{K_G}{s}\cdot(s - z_1) \Rightarrow n-m=1$ $\quad z_1 = -4$

$\Rightarrow G(s) = \frac{K_G\cdot(s+4)}{s} \Rightarrow F(s) = K_G\cdot\frac{(s+3)}{s^2}$



$f(s,k) = s^2 + K_G(s+3) \to (\bar{s}-3)^2 + K_G\,\bar{s} = 0$

$\bar{s}^2 + (K_G-6)\bar{s} + 9 = 0$

$$
\begin{array}{c|cc}
 & 3 & \\
2 & 1 & 9 \\
1 & K_f - 6 & \\
0 & 9 &
\end{array}
\quad \Rightarrow K_f > 6
$$

$$\Rightarrow G(s) = 8 \cdot \frac{(s+4)}{s} \qquad F(s) = 8 \cdot \frac{(s+3)}{s^2}$$

$$u(t) = (3t+2)\delta_{-1}(t) = 3(t)\delta_{-1}(t) + (2)\delta_{-1}(t)$$

$$= 3u_1(t) + 2u_2(t)$$

- $u_1(t)$

$$\tilde{e}_{u_1}(t) = K_d u_1(t) - \tilde{y}_{u_1}(t)$$

$$\tilde{e}_{u_1}(t) = \frac{1}{K_f \cdot K_p} = \frac{1}{K_F} = \frac{1}{24}$$

$$\tilde{y}_{u_1}(t) = K_d u_1(t) - \tilde{e}_{u_1}(t) = \left(t - \frac{1}{24}\right)\delta_{-1}(t)$$

- $u_2(t)$

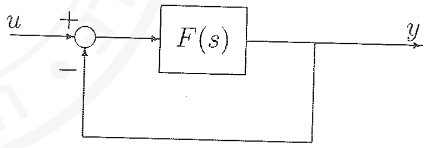
GRADO DI $u_2(t)$ $<$ TIPO DI $F(s)$ $\Rightarrow \tilde{y}_{u_2}(t) = \delta_{-1}(t)$

$$\Rightarrow \tilde{y}(t) = 3\left(t - \frac{1}{24}\right)\delta_{-1}(t) + 2\delta_{-1}(t)$$

Domanda Scritta di Controlli Automatici (9CFU) - 17/12/2012

Esercizio 1

È dato il sistema in controreazione:

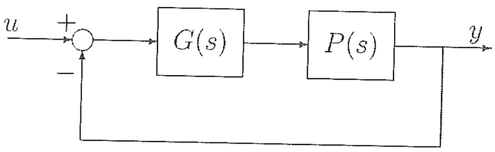


in cui $F(s) = \dfrac{K(s+1)^2}{(s-1)(s+3)(s+5)(s+8)}$, $K \in \mathbb{R}$.

- Tracciare il luogo positivo delle radici;
- tracciare il luogo negativo delle radici;
- determinare per quali valori di K il sistema a ciclo chiuso è asintoticamente stabile;
- se $K = 10$, esiste la risposta a regime permanente a ciclo chiuso per un ingresso a gradino? Motivare la risposta.

Esercizio 2

È dato il sistema di controllo:



in cui:

$$P(s) = \frac{50(s+1)}{(s+4)^2}.$$

Progettare $G(s)$ con la sintesi per tentativi in ω in modo che:

- $|\tilde{e}_1(t)| \leq 0.04$, essendo $\tilde{e}_1(t)$ l'errore a regime permanente per un ingresso di riferimento a rampa unitaria;
- $M_r \leq 2\ dB$;
- $B_3 \simeq 1.5\ Hz$.

Calcolare infine la risposta a regime permanente all'ingresso: $u(t) = (3t+5) \cdot \delta_{-1}(t)$.

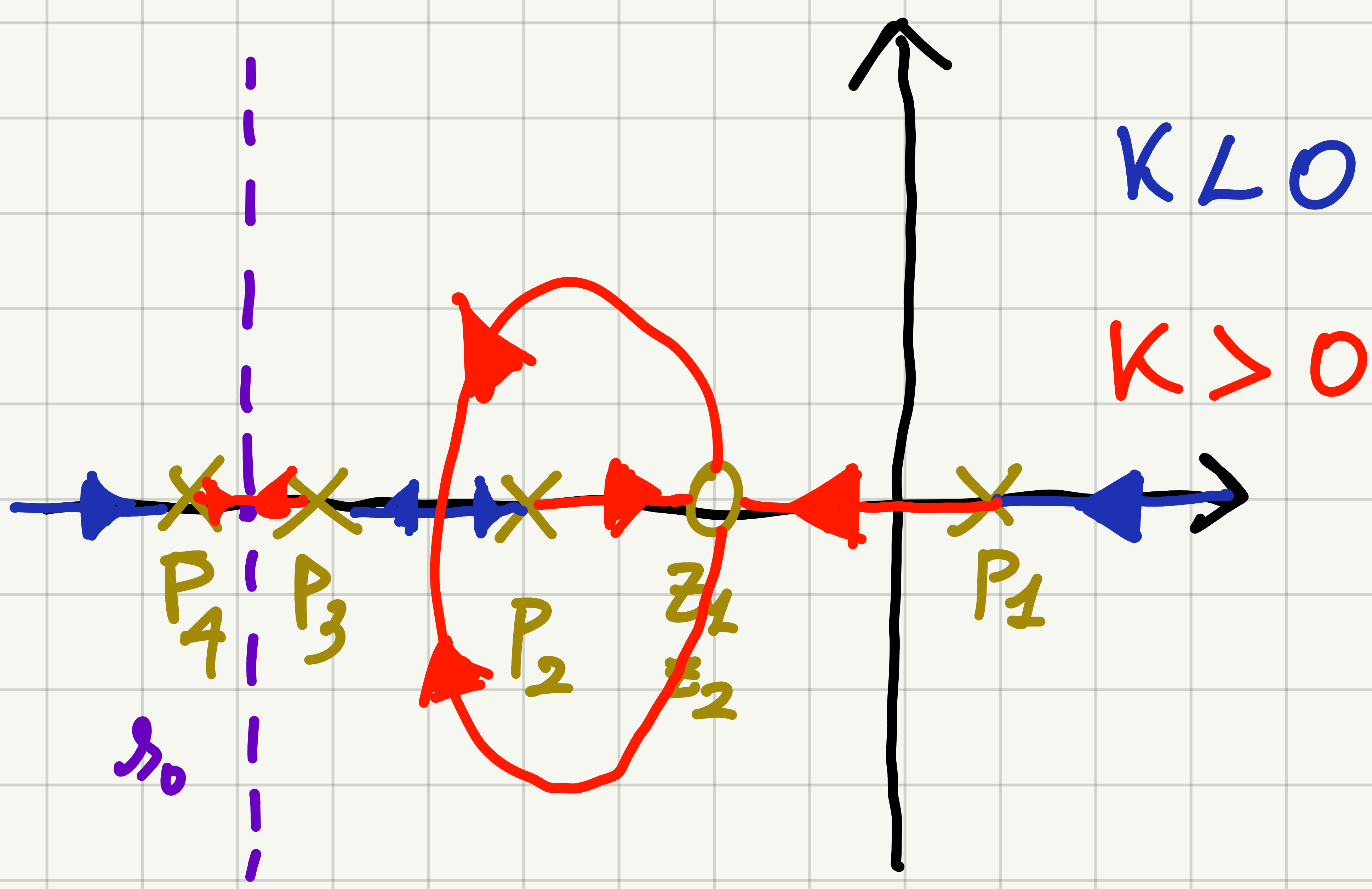
1

$$F(s) = K \cdot \frac{(s+1)^2}{(s-1)(s+3)(s+5)(s+8)} \qquad K \in \mathbb{R}$$

$n=4,\ m=2 \implies n-m=2$

$P_1 = 1 \qquad P_2 = -3 \qquad P_3 = -5 \qquad P_4 = -8$

$$Z_1 = Z_2 = -1 \qquad s_0 = \frac{\Sigma p - \Sigma z}{n-m} = \frac{-13}{2} = -6{,}5$$



$$\varphi(s,K) = (s-1)(s+3)(s+5)(s+8) + K(s+1)^2 \Big|_{s=0} = 0$$

$-120 + K = 0 \qquad K = 120$

SISTEMA STABILE $\forall K > 120$

$\implies \nexists$ RISPOSTA A REGIME PERMANENTE PER $K = 10$

2

$G(s) = \frac{K}{s}$ $\quad |\tilde{e}_r| = \left|\frac{K_d^2}{K_G \cdot K_p}\right| \le 0{,}04$ $\quad K_p = \frac{50}{16} \Rightarrow K_G \ge 8$

$M_r \le 2\,dB \Rightarrow M_\varphi \ge 47^\circ$

$B_3 \simeq 1{,}5\,Hz \Rightarrow \omega_t = 3 \div 5\, B_3 = 4 B_3 \simeq 6\,\frac{rad}{s}$

$$F(s) = G(s) \cdot P(s) = 25 \cdot \frac{(1+s)}{s\left(1+\frac{s}{4}\right)^2}$$

$$F(i\omega) = 25 \cdot \frac{(1+i\omega)}{i\omega\left(1+\frac{i\omega}{4}\right)^2}$$ $\quad 25 \to 20\log_{10}(25) = 28\,dB$

PUNTI DI ROTTURA

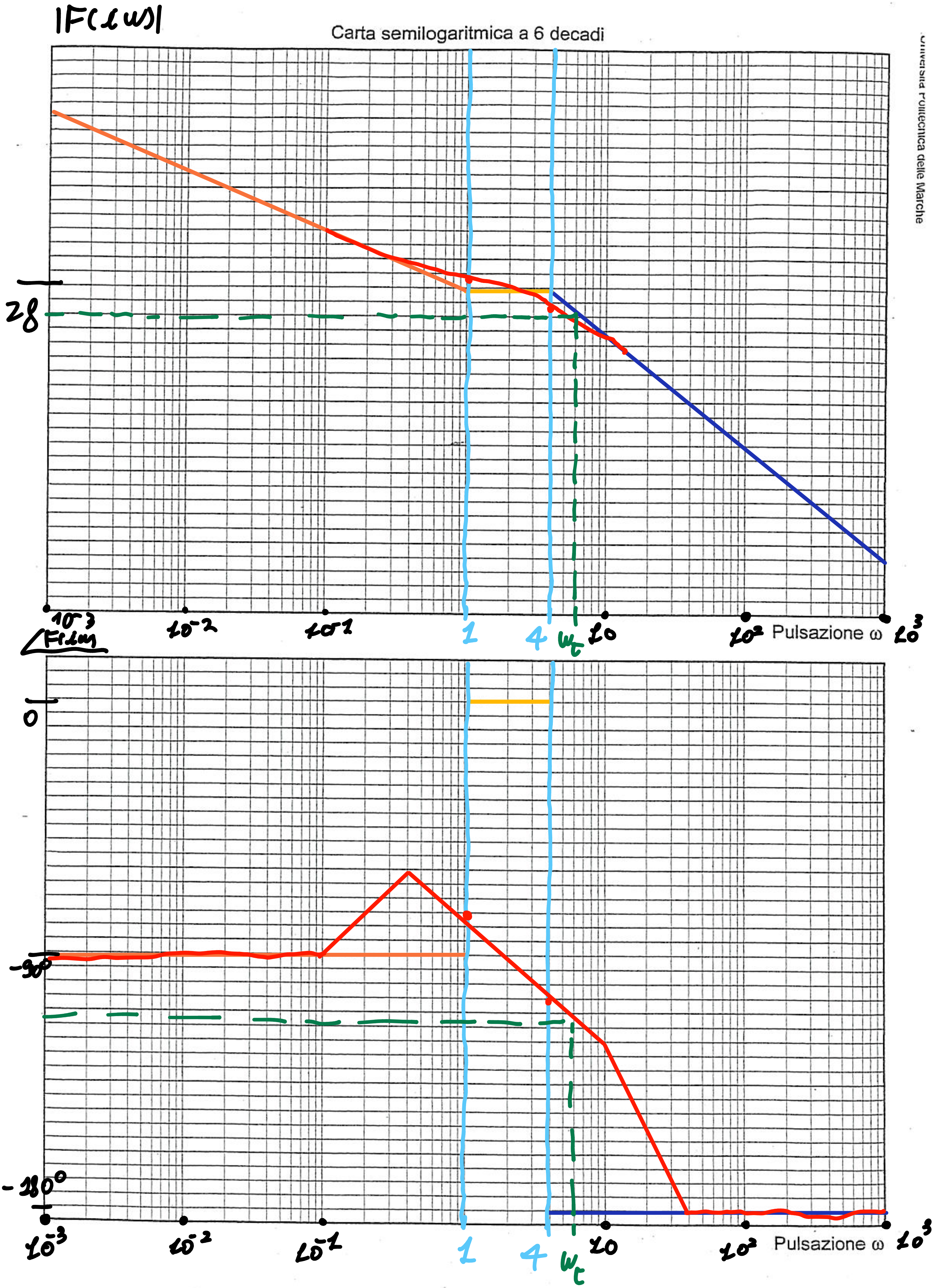
- $\omega = 0$ ● $-20\,dB$ $\quad -90^\circ$ $\qquad -20\,dB$ $\quad -90^\circ$
- $\omega = 1$ ● $+20\,dB$ $\quad +90^\circ$ $\qquad 0$ $\quad 0$
- $\omega = 4$ ● $-40\,dB$ $\quad -180^\circ$ $\qquad -40\,dB$ $\quad -180^\circ$

CORREZIONE MODULO

ω	$1+i\omega$	$\left(1+\frac{i\omega}{4}\right)^2$	TOT
1	$+3\,dB$	0	$+3\,dB$
4	0	$-6\,dB$	$-6\,dB$

CORREZIONE FASE

ω	$i\omega$	$1+i\omega$	$\left(1+\frac{i\omega}{4}\right)^2$	TOT
1	-90°	$+45^\circ$	-30°	-75°
4	-90°	$+75^\circ$	-90°	-105°



|F(jω)|
Carta semilogaritmica a 6 decadi
Università Politecnica delle Marche
28
10⁻³
10⁻²
10⁻¹
1
4
ωt
10
10²
Pulsazione ω
10³
∠F(jω)
0
-90°
-180°
10⁻³
10⁻²
10⁻¹
1
4
ωt
10
10²
Pulsazione ω
10³

$|F(i\omega_c)| = 18\,dB$

$\angle F(i\omega_c) = -115^\circ \Rightarrow M_\varphi = 65^\circ$

OBIETTIVO:

$|F(i\omega_c)| = 0$ ✗

$M_\varphi \geq 47^\circ$ ✓ $\Rightarrow$ FUNZIONE ATTENUATRICE

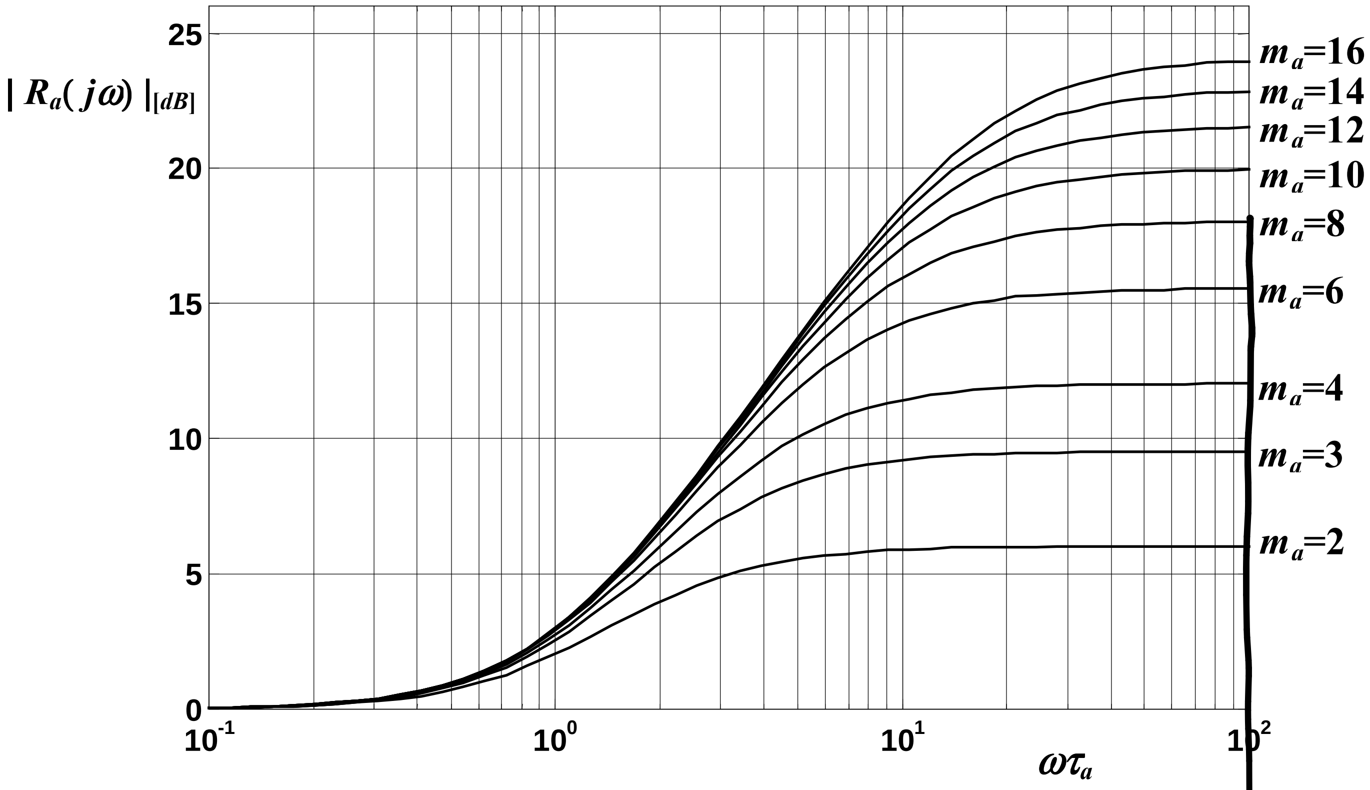
$\omega_c \tau_i = 100,\ m_i = 8 \quad \omega_i = 0{,}06$

$$\Rightarrow R_i(i\omega) = \frac{1 + \frac{i\omega}{m_i \omega_i}}{1 + \frac{i\omega}{\omega_i}} = \frac{1 + \frac{i\omega}{0{,}48}}{1 + \frac{i\omega}{0{,}06}}$$

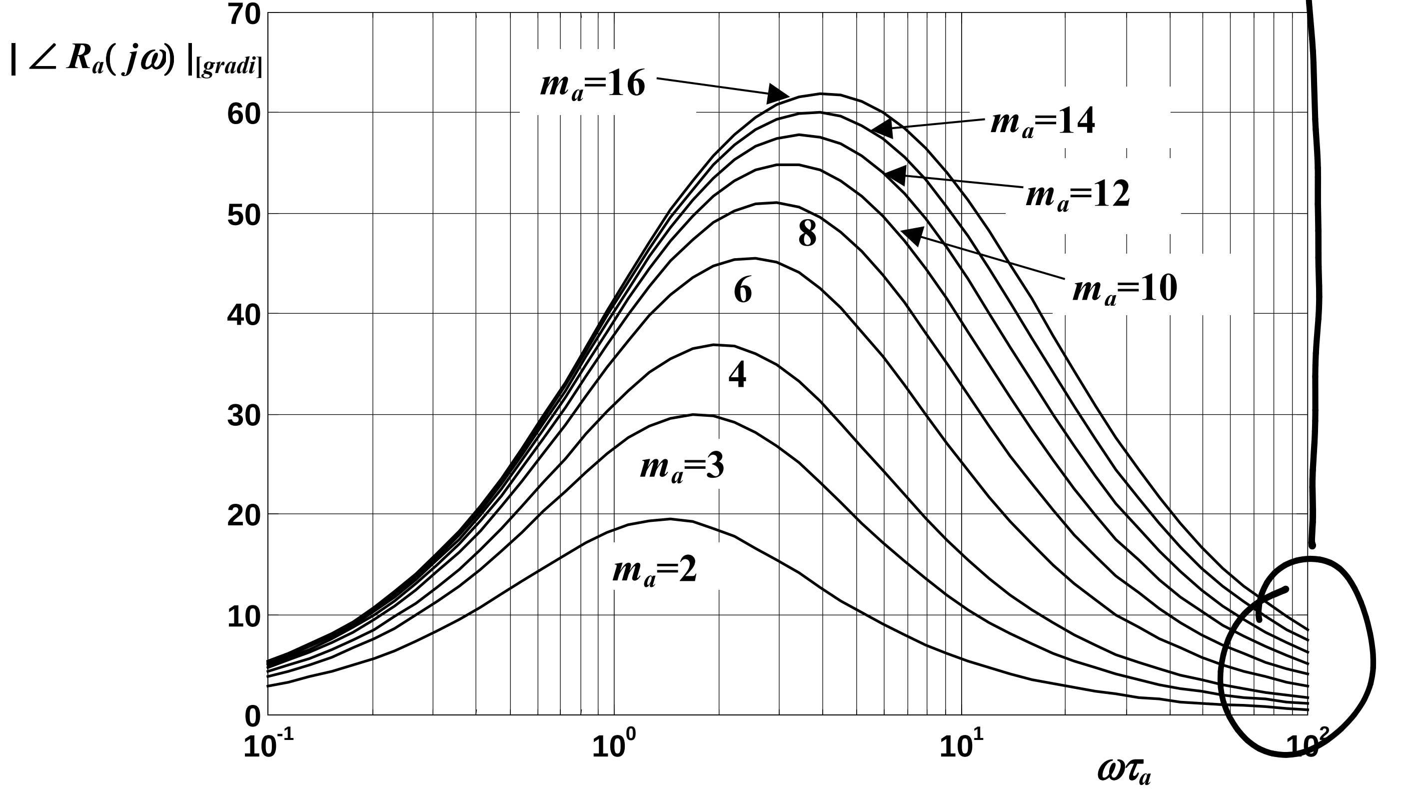
$$\Rightarrow G(s) = K \cdot \frac{1 + \frac{s}{0{,}48}}{s\left(1 + \frac{s}{0{,}06}\right)}$$

$$F(i\omega) = 25 \cdot \frac{(1 + i\omega)}{i\omega\left(1 + \frac{i\omega}{4}\right)^2} \cdot \frac{1 + \frac{i\omega}{0{,}48}}{1 + \frac{i\omega}{0{,}06}}$$

• $\omega = 0$	$-20\,dB$	-90°	$-20\,dB$	-90°
• $\omega = 0{,}06$	$-20\,dB$	-90°	$-40\,dB$	-180°
• $\omega = 0{,}48$	$+20\,dB$	$+90^\circ$	$-20\,dB$	-90°
• $\omega = 1$	$+20\,dB$	$+90^\circ$	0	0
• $\omega = 4$	$-40\,dB$	-180°	$-40\,dB$	-180°



25
20
15
10
5
0
$|R_a(j\omega)|_{[dB]}$
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
m_a=16
m_a=14
m_a=12
m_a=10
m_a=8
m_a=6
m_a=4
m_a=3
m_a=2



70
60
50
40
30
20
10
0
$|\angle R_a(j\omega)|_{[gradi]}$
10^{-1}
10^{0}
10^{1}
10^{2}
$\omega\tau_a$
m_a=16
m_a=14
m_a=12
m_a=10
8
6
4
m_a=3
m_a=2

CORREZIONE MODULO

ω	$1+\frac{i\omega}{0{,}06}$	$1+\frac{i\omega}{0{,}48}$	$1+i\omega$	$\left(1+\frac{i\omega}{4}\right)^2$	TOT
0,06	-3dB	0	0	0	-3dB
0,48	0	+3dB	+1dB	0	+4dB
1	0	+1dB	+3dB	0	+4dB
4	0	0	0	-6dB	-6dB

CORREZIONE FASE

ω	$i\omega$	$1+\frac{i\omega}{0{,}06}$	$1+\frac{i\omega}{0{,}48}$	$1+i\omega$	$\left(1+\frac{i\omega}{4}\right)^2$	TOT
0,06	-90°	-45°	+5°	0	0	-130°
0,48	-90°	-80°	+45°	+25°	0	-100°
1	-90°	-90°	+65°	+45°	-20°	-90°
4	-90°	-90°	+80°	+75°	-90°	-115°

$$F(i\omega)=25\cdot\frac{(1+i\omega)}{i\omega\left(1+\frac{i\omega}{4}\right)^2}\,\frac{1+\frac{i\omega}{0{,}48}}{1+\frac{i\omega}{0{,}06}}$$