USE RESOLUTION TO PROVE THE FOLLOWING ASSERTIONS

a) $\forall x(Rx \to Sx) \vdash \forall x Rx \to \forall x Sx$

$F \cup \{\neg\varphi\} = \{\forall x(Rx \to Sx), \neg(\forall x Rx \to \forall x Sx)\}$

HERBRAND MODEL

STEP	FORMULA	RULE
1	$\{\forall x(Rx \to Sx)\}$	ASSUMPTION
2	$\{\neg(\forall x Rx \to \forall x Sx)\}$	ASSUMPTION
3	$\{\forall x Rx\}$	2, α-EXPANSION
4	$\{\neg\forall x Sx\}$	2, α-EXPANSION
5	$\{\neg Sa\}$	4, δ-EXPANSION
6	$\{Ra\}$	3, γ-EXPANSION
7	$\{Ra \to Sa\}$	1, γ-EXPANSION
8	$\{\neg Ra, Sa\}$	7, β-EXPANSION
9	$\{Sa\}$	6,8 RESOLUTION
10	$\emptyset$	5,9 RESOLUTION

$F \cup \{\neg\varphi\}$ HAS A CLOSED EXPANSION $\Rightarrow F \vdash \varphi$ IS SATISFIABLE

UNIFICATION

- $\{\forall x(Rx \to Sx)\} \mapsto \{\forall x(\neg Rx \vee Sx)\} \mapsto \{\neg Rx, Sx\}$
- $\{\neg(\forall x Rx \to \forall x Sx)\} \mapsto \{\neg(\forall x Rx \to \forall y Sy)\} \mapsto \{\neg \forall y \exists x (Rx \to Sy)\}$

  $\mapsto \{\exists y \forall x. \neg(Rx \to Sy)\} \mapsto \{\forall x. \neg(Rx \to Sa)\} \mapsto \{Rx\}, \{\neg Sa\}$

$C(F) = \{\neg Rx, Sx\}, \{Rx\}, \{\neg Sa\}$

$$\text{A}\quad \dfrac{\{\neg Rx, Sx\} \qquad \{Rx\}}{\{Sx\}}$$

$$\text{B}\quad \dfrac{\{Sx\} \qquad \{\neg Sa\}}{\emptyset}$$

A $C_1 = \{\neg Rx, Sx\} \qquad E_1 = \{\neg Rx\}$

$C_2 = \{Rx\} \qquad E_2 = \{Rz\}$

$F_1 = \bar{E}_1 \cup E_2 = \{Rx, Rz\}$ $\qquad S = [\frac{x}{z}]$

$F_2 = \{Rx\}$ $\quad R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = (\{Sx\} \cup \emptyset)[\frac{x}{z}] = \{Sx\}$

B $C(F) = \{\{Sx\}, \{\neg Sa\}\}$

$C_1 = \{Sx\} = E_1$

$C_2 = \{\neg Sa\} = E_2$

$F_1 = E_1 \cup \bar{E}_2 = \{Sx, Sa\}$ $\qquad S = [\frac{a}{x}]$

$F_2 = F_1 S = \{Sa\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = \emptyset$

b) $\forall x Rx \vee \forall x Sx \vdash \forall x (Rx \vee Sx)$

$F \cup \{\neg\varphi\} = \{\forall x Rx \vee \forall x Sx, \neg \forall x (Rx \vee Sx)\}$

HERBRAND MODEL

STEP	FORMULA	RULE
1	$\{\forall x Rx \vee \forall x Sx\}$	ASSUMPTION
2	$\{\neg \forall x (Rx \vee Sx)\}$	ASSUMPTION
3	$\{\forall x Rx\}$	1, β-EXPANSION
4	$\{\forall x Sx\}$	1, β-EXPANSION
5	$\{Ra\}$	3, γ-EXPANSION
6	$\{Sa\}$	4, γ-EXPANSION
7	$\{\exists x. \neg(Rx \vee Sx)\}$	2, δ-EXPANSION
8	$\{\exists x. \neg Rx, \exists x. \neg Sx\}$	7, α-EXPANSION
9	$\{\neg Ra, \neg Sa\}$	8, δ-EXPANSION
10	$\{\neg Sa\}$	9, 5 RESOLUTION
11	$\varnothing$	6, 10 RESOLUTION