UNIFICATION

- $\{\forall x Rx \vee \forall x Sx\} \mapsto \{Rx, Sx\}$
- $\{\neg\forall x (Rx \vee Sx)\} \mapsto \{\exists x. \neg (Rx \vee Sx)\} \mapsto \{\neg Ra\}, \{\neg Sa\}$

$C(F) = \{\{Rx, Sx\}, \{\neg Ra\}, \{\neg Sa\}\}$

$$\text{Ⓐ}\ \frac{\{Rx, Sx\} \quad \{\neg Ra\}}{\{Sa\}}\ [\tfrac{a}{x}] \qquad \text{Ⓑ}\ \frac{\{Sa\} \quad \{\neg Sa\}}{\emptyset}$$

Ⓐ $C_1 = \{Rx, Sx\} \quad E_1 = \{Rx\}$

$C_2 = \{\neg Ra\} = E_2$

$F_1 = E_1 \cup E_2 = \{Rx, Ra\} \qquad S = [\tfrac{a}{x}]$

$F_2 = F_1 S = \{Ra\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)[\tfrac{a}{x}] = \{Sa\}$

Ⓑ $C_1 = E_1 = \{Sa\} \quad C_2 = E_2 = \{\neg Sa\}$

$F = \{Sa\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2) S = \emptyset$

C) $\forall x(Rx \to Sx) \vdash \exists x Rx \to \exists x Sx$

$\{\neg(\forall x(Rx \to Sx), \neg(\exists x Rx \to \exists x Sx)\}$

HERBRAND MODEL

STEP	FORMULA	RULE
1	$\{\forall x(Rx \to Sx)\}$	ASSUMPTION
2	$\{\neg(\exists x Rx \to \exists x Sx)\}$	ASSUMPTION
3	$\{\neg\forall x Rx\}$	1, β-EXPANSION
4	$\{\forall x Sx\}$	1, β-EXPANSION
5	$\{\neg Ra\}$	3, γ-EXPANSION
6	$\{Sa\}$	4, δ-EXPANSION
7	$\{\exists x Rx, \neg\exists x Sx\}$	2, α-EXPANSION
8	$\{Ra, \neg Sa\}$	7, δ-EXPANSION
9	$\{\neg Sa\}$	5,8 RESOLUTION
10	$\emptyset$	6,9 RESOLUTION

UNIFICATION

- $\{\forall x(Rx \to Sx)\} \mapsto \{\forall x(\neg Rx, Sx)\} \mapsto \{\neg Rx, Sx\}$
- $\{\neg(\exists x Rx \to \exists x Sx)\} \mapsto \{\exists x Rx\}, \{\neg \exists x Sx\} \mapsto \{Ra\}, \{\neg Sx\}$

$C(F) = \{\{\neg Rx, Sx\}, \{Ra\}, \{\neg Sx\}\}$

$$\frac{\{\neg Rx, Sx\} \quad \{Ra\}}{\emptyset}\,[\tfrac{a}{x}]$$

Ⓐ $C_1 = \{\neg Rx, Sx\}$ $E_1 = \{\neg Rx\}$

$C_2 = \{Ra\} = E_2$ $F_1 = E_1 \cup E_2 = \{Rx, Ra\}$ $S = [\tfrac{a}{x}]$

$F_2 = F_1 S = \{Ra\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = (\{Sx\} \cup \emptyset)[\tfrac{a}{x}] = \{Sa\}$

Ⓑ $C(F) = \{\{Sa\}, \{\neg Sx\}\}$

$C_1 = \{Sa\} = E_1$ $C_2 = \{\neg Sx\} = E_2$

$F_1 = E_1 \cup E_2 = \{Sa, Sx\}$ $S = [\tfrac{a}{x}]$

$F_2 = \{Sa\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = \emptyset$