

11	$\{\forall r(R_{ar} \rightarrow R_{ra})\}$	3, γ -EXPANSION
12	$\{R_{ab} \rightarrow R_{ba}\}$	11, γ -EXPANSION
13	$\{\neg R_{ab}, R_{ba}\}$	12, β -EXPANSION
14	$\{\exists x \forall z. \neg(R_{xz} \wedge R_{yz} \rightarrow R_{xz})\}$	4, δ -EXPANSION
15	$\{\exists y z. \neg(R_{ay} \wedge R_{yz} \rightarrow R_{az})\}$	14, δ -EXPANSION
16	$\{\exists z. \neg(R_{ab} \wedge R_{bz} \rightarrow R_{az})\}$	15, δ -EXPANSION
17	$\{\neg(R_{ab} \wedge R_{ba} \rightarrow R_{aa})\}$	16, δ -EXPANSION
18	$\{R_{ab} \wedge R_{ba}\}$	17, d-EXPANSION
19	$\{\neg R_{aa}\}$	17, d-EXPANSION
20	$\{R_{ab}\}$	19, d-EXPANSION
21	$\{R_{ba}\}$	19, d-EXPANSION
22	\emptyset	5, 21 RESOLUTION

LET R BE A BINARY RELATION WHICH IS SYMMETRIC AND

TRANSITIVE. USE RESOLUTION TO PROVE THAT R SATISFIES THE FORMULA

$$\exists x \forall y. Rxy \rightarrow \forall x y. Rxy$$

$$\forall x y (xRy \rightarrow yRx), \forall x y z (xRy \wedge yRz \rightarrow xRz) \vdash \exists x \forall y. Rxy \rightarrow Ryx$$

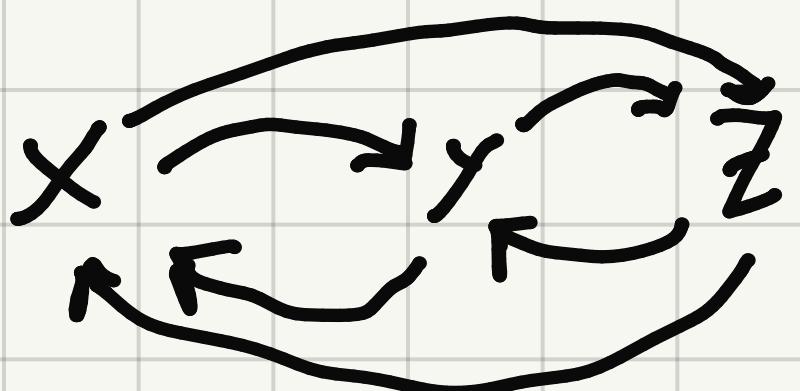
$$\{ \forall x y (Rxy \rightarrow Ryx), \forall x y z (Rxy \wedge Ryz \rightarrow \neg Rxz), \neg (\exists x \forall y. Rxy \rightarrow \forall xy. Rxy) \}$$

• $\{ \forall x y (Rxy \rightarrow Ryx) \} \vdash \{ Rxy \rightarrow Ryx \} \vdash \{ \neg Rxy, Ryx \}$

• $\{ \forall x y z (Rxy \wedge Ryz \rightarrow \neg Rxz) \} \vdash \{ Rxy \wedge Ryz \rightarrow \neg Rxz \}$

$$\vdash \{ \neg (Rxy \wedge Ryz) \vee Rxz \} \vdash \{ \neg Rxy \vee \neg Ryx \vee Rxz \}$$

$$\vdash \{ \neg Rxy, \neg Ryx, Rxz \}$$



• $\{ \neg (\exists x \forall y (Rxy \rightarrow (\forall xy. Rxy))) \}$

$$\vdash \{ \forall x \exists y. \neg (Rxy \rightarrow (\forall zw. Rzw)) \}$$

$$\vdash \{ \neg (Rx\alpha \rightarrow (\forall zw. Rzw)) \} \vdash \{ Rx\alpha \wedge \neg (\forall zw. Rzw) \}$$

$$\vdash \{ Rx\alpha \wedge \exists z w. \neg (Rzw) \} \vdash \{ Rx\alpha \}, \neg Rb\alpha$$

$$CC(F) = \{ \{ \neg Rxy, Ryx \}, \{ \neg Rx\alpha \}, \{ \neg Rx\alpha \}, \{ Rx\alpha \}, \{ \neg Rb\alpha \} \}$$

(A) $\{ \neg Rxy, Ryx \}, \{ Rx\alpha \}$

(B) $\{ Rx\alpha \}, \{ \neg Rxy, \neg Ryx, Rxz \}$

(C) $\{ Rx\alpha \}$

$\{ \neg Rb\alpha \}$

\emptyset

$$\textcircled{A} \quad C_1 = \{\neg R_{xy}, R_{yx}\} \quad E_1 = \{\neg R_{xy}\} \quad C_2 = \{R_{xa}\} = E_2$$

$$F_1 = \bar{E}_1 \cup E_2 = \{R_{xy}, R_{xa}\} \quad S = [a_{xy}]$$

$$F_2 = F_1 S = \{R_{xa}\}$$

$$RC(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = (\{R_{yx}\} \cup \emptyset)[a_{xy}] = \{R_{ax}\}$$

$$\textcircled{B} \quad C_1 = \{R_{ax}\} = E_1 \quad C_2 = \{\neg R_{ky}, \neg R_{yz}, R_{xz}\} \quad E_2 = \{\neg R_{xy}, \neg R_{yz}\}$$

$$F_1 = E_1 \cup \bar{E}_2 = \{R_{ax}, R_{ky}, R_{yz}\} \quad S_1 = [a_{ky}]$$

$$F_2 = \{R_{ax}, R_{ay}, R_{yz}\} \quad S_2 = [x_{yz}]$$

$$F_3 = \{R_{ax}, R_{xz}\} \quad S_3 = [a_{xz}]$$

$$F_4 = \{R_{aa}, R_{az}\} \quad S_4 = [a_{az}]$$

$$F_5 = \{R_{aa}\} \quad S = S_1 S_2 S_3 S_4$$

$$RC(C_1, C_2) = \{R_{xz}\} \left[\frac{a}{k}, \frac{y}{z}, \frac{a}{x}, \frac{a}{z} \right] = \{R_{aa}\}$$

$$\textcircled{C} \quad C_1 = \{R_{aa}\} = E_1 \quad C_2 = \{\neg R_{bc}\} = E_2$$

$$F_1 = \{R_{aa}, R_{bc}\} \quad S_1 = [a_b]$$

$$F_2 = \{R_{aa}, R_{ac}\} \quad S_2 = [a_c]$$

$$F_3 = \{R_{aa}\} \quad S = S_1 S_2$$

$$RC(C_1, C_2) = \emptyset$$