EXTRA: SKOLEMIZATION EXERCISES. SKOLEMIZE THE FOLLOWING:

a) $\exists x P(x,y) \rightarrow Q(x)$ $\quad \exists \Rightarrow x = f(y)$

$P(f(y), y) \rightarrow Q(f(y))$

b) $\forall x \exists y (P(x,y) \wedge Q(y,z))$ $\quad \exists \Rightarrow y = f(x,z)$ UNCHANGED

$\forall x (P(x, f(x,z)) \wedge Q(f(x,z), z)) \mapsto P(x, f(x,z)) \wedge Q(f(x,z), z))$

c) $\forall x \exists y P(x,y) \wedge \exists x Q(x)$

$\forall \Rightarrow x = a \quad f(a) = b$

$\forall x P(x, f(x)) \wedge \exists Q(x) \mapsto P(a,b) \wedge Q(c)$

d) $\exists x \forall y (P(x,y) \vee Q(y))$ $\quad f(x) = a$ UNCHANGED

$\forall y (P(a,y) \vee Q(y)) \mapsto P(a,y) \vee Q(y)$

e) $\exists x (P(x) \rightarrow \forall y Q(x,y))$ $\quad f(x) = a$ UNCHANGED

$P(a) \rightarrow \forall y Q(a,y)) \mapsto P(a) \rightarrow Q(a,y)$

f) $\exists x \forall y (P(x,y) \wedge \exists z Q(y,z))$ $\quad f(x) = a \quad f(z) = b$

$\exists x \forall y (P(x,y)) \wedge \exists x \forall y \exists z (Q(y,z))$

$\forall y (P(a,y)) \wedge \forall y Q(y,b)$

$P(a,y) \wedge Q(y,b)$

PROVE THAT THE FOLLOWING FORMULAS ARE SATISFIABLE BUT NOT VALID

a) $\forall xy(Rxy \to \exists z(Rxz \land Rzy))$

- **SATISFIABILITY**

$$\forall xy(Rxy \to \exists z(Rxz \land Rzy))$$

$$\forall xy(\neg Rxy) \qquad \forall xy(\exists z(Rxz \land Rzy))$$

$$\neg Raa \implies \text{SATISFIABLE}$$

- **VALIDITY**

$$\neg(\forall xy(Rxy \to \exists z(Rxz \land Rzy)))$$

$$\forall xy(Rxy), \forall xy(\neg \exists z(Rxz \land Rzy))$$

$$\forall xy(Rxy), \forall xy(\exists z(\neg Rxz))$$

$$Raa, \neg Rab \implies \neg\varphi \text{ IS SATISFIABLE}$$

$$\implies \text{NOT VALID}$$

b) $\forall x \exists y . Rxy \to \exists y \forall x . Rxy$

- SATISFIABILITY

$$\forall x \exists y Rxy \to \exists y \forall x Rxy$$

$$\neg(\forall x \exists y Rxy) \qquad \exists y \forall x Rxy$$

$$\neg(\exists y Ray)$$

$$\neg Raa$$

SATISFIABLE

- VALIDITY

$$\neg(\forall x \exists y Rxy \to \exists y \forall x Rxy) \equiv \forall x \exists y Rxy \land \neg \exists y \forall x Rxy$$

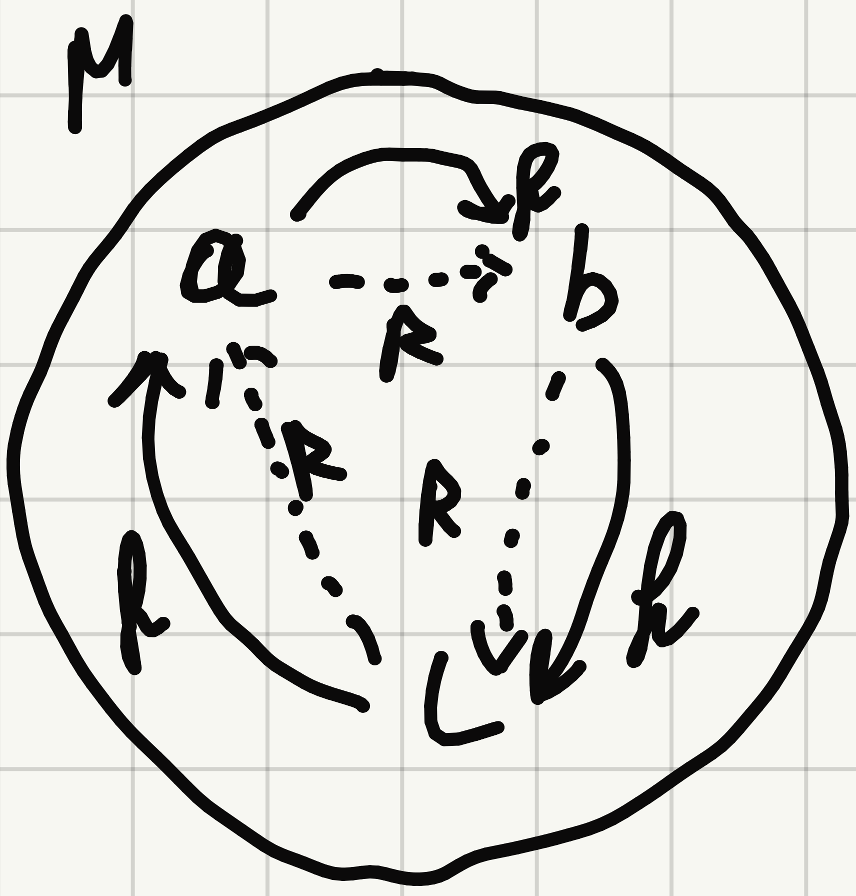
$$\equiv \forall x \exists y Rxy \land \forall x \exists y \neg Ryx \mapsto \forall x (Rxf(x) \land \neg Rg(x)x)$$

FOR SIMPLICITY, SUPPOSE $f(x) = g(x) \mapsto xRf(x) \land \neg f(x)Rx$

$x = a$ $\quad f(a)?$ $\quad \neq a$. OTHERWISE, $aRa \land \neg aRa$

$\to f(a) = b$ $\quad f(b)?$ $\neq a$ $\qquad bRa \land \neg aRb$

$f(b) = c$ $\quad f(c)?$ $= a$ $\qquad bRc \land \neg cRa$ ✓



$\to$ $M : \{a, b, c\}$

$[[R]] : \{(a,b), (b,c), (c,a)\}$

$[[f]] : \{(a,b), (b,c), (c,a)\}$

THAT MAKES $\neg\varphi$ SATISFIABLE $\Rightarrow$ φ IS NOT VALID