

$$d) X \wedge Y \rightarrow Z, X \leftrightarrow Y, Z \rightarrow Y \models X \leftrightarrow Z$$

$$F = \{X \wedge Y \rightarrow Z, X \leftrightarrow Y, Z \rightarrow Y, \neg(X \leftrightarrow Z)\}$$

STEP

FORMULA

RULE

1

$$\{X \wedge Y \rightarrow Z\}$$

ASSUMPTION

2

$$\{X \leftrightarrow Y\}$$

ASSUMPTION

3

$$\{Z \rightarrow Y\}$$

ASSUMPTION

4

$$\{\neg(X \leftrightarrow Z)\}$$

ASSUMPTION

5

$$\{X\}$$

1, d- EXPANSION

6

$$\{Y \rightarrow Z\}$$

1, d- EXPANSION

7

$$\{\neg Y, Z\}$$

6. β - EXPANSION

8

$$\{X \rightarrow Y\}$$

2, BIIMPLICATION

9

$$\{Y \rightarrow X\}$$

2, BIIMPLICATION

10

$$\{\neg X, Y\}$$

8, β - EXPANSION

11

$$\{\neg Y, X\}$$

9, β - EXPANSION

12

$$\{Y, \neg Z\}$$

3, β - EXPANSION

13

$$\{\neg(X \rightarrow Z)\}$$

4, BIIMPLICATION

14

$$\{\neg(Z \rightarrow X)\}$$

4, BIIMPLICATION

15

$$\{X\}$$

13, d- EXPANSION

16

$$\{\neg z\}$$

13, d. FORMULA

17

$$\{z\}$$

14, d. FORMULA

18

$$\{\neg x\}$$

14, d. FORMULA

19

$$\{\neg z, \neg z\}$$

7, 12 RESOLUTION

20

$$\{\neg z\}$$

19, 16 RESOLUTION

21

$$\perp$$

20, 17 RESOLUTION

THE SET HAS A CLOSED EXPANSION \Rightarrow THE STATEMENT IS CORRECT

$$e) X \rightarrow Y \vee Z, Y \leftrightarrow Z \models X \rightarrow Y \wedge Z$$

$$F = \{ X \rightarrow Y \vee Z, Y \leftrightarrow Z, \neg(X \rightarrow Y \wedge Z) \}$$

STEP

FORMULA

RULE

1

$$\{ X \rightarrow Y \vee Z \}$$

ASSUMPTION

2

$$\{ Y \leftrightarrow Z \}$$

ASSUMPTION

3

$$\{ \neg(X \rightarrow Y \wedge Z) \}$$

ASSUMPTION

4

$$\{ \neg X, Y \vee Z \}$$

1, P-EXPANSION

5

$$\{ \neg X, Y, Z \}$$

4, P-EXPANSION

6

$$\{ Y \rightarrow Z \}$$

2, BIIMPLICATION

7

$$\{ Z \rightarrow Y \}$$

2, BIIMPLICATION

8

$$\{ \neg Y \}$$

6, d-EXPANSION

9

$$\{ Z \}$$

6, d-EXPANSION

10

$$\{ \neg Z \}$$

7, d-EXPANSION

11

$$\{ Y \}$$

8, d-EXPANSION

12

$$\{ X \}$$

3, d-EXPANSION

13

$$\{ \neg(Y \wedge Z) \}$$

3, d-EXPANSION

14

$$\{ \neg Y, \neg Z \}$$

13, P-EXPANSION

15

$$\{ Y, Z \}$$

5, 12 RESOLUTION