$F_2 = F_1 S_1 = \{a \leq y, a \leq b\}$ $S_2 = [\frac{b}{y}]$

$F_3 = F_2 S_2 = \{a \leq b\}$ $S = S_1 S_2$

$R(C_1, C_2) = \{C_1 \setminus E_1 \cup C_2 \setminus E_2\} S = \{fx \leq fy\}[\frac{a}{x}, \frac{b}{y}] = \{fa \leq fy\}$

B $C_1 = \{fa \leq fb\} = E_1$ $C_2 = \{\neg(x \leq y), gx \leq gy\}$ $E_2 = \{\neg(x \leq y)\}$

$F_1 = \{fa \leq fb, x \leq y\}$ $S_1 = [\frac{fa}{x}]$

$F_2 = \{fa \leq fb, fa \leq y\}$ $S_2 = [\frac{fb}{y}]$

$F_3 = \{fa \leq fb\}$ $S = S_1 S_2$

$R(C_1, C_2) = \{gx \leq gy\}[\frac{fa}{x}, \frac{fb}{y}] = \{gfa \leq gfb\}$

C $C_1 = E_1 = \{gfa \leq gfb\}$ $C_2 = E_2 = \{\neg(gfa \leq gfb)\}$

$F_1 = E_1 \cup \bar{E}_2 = \{gfa \leq gfb, gfa \leq gfb\} = \{gfa \leq gfb\}$

$R(C_1, C_2) = \emptyset$

HERBRAND MODEL

STEP		FORMULA	RULE
1		$\{\forall xy((x \le y) \to (fx \le fy) \wedge (gx \le gy))\}$	ASSUMPTION
2		$\{\neg\forall xy((x \le y) \to (gfx \le gfy))\}$	ASSUMPTION
3		$\{\forall y((a \le y) \to (fa \le fy) \wedge (ga \le gy))\}$	1, γ-EXPANSION
4		$\{(a \le b) \to (fa \le fb) \wedge (ga \le gb)\}$	3, γ-EXPANSION
5		$\{\neg(a \le b), (fa \le fb) \wedge (ga \le gb)\}$	4, β-EXPANSION
6		$\{\neg(a \le b), (fa \le fb)\}$	5, α-EXPANSION
7		$\{\neg(a \le b), (ga \le gb)\}$	5, α-EXPANSION
8		$\{\exists y.\neg((a < y) \to (gfa \le gfy)\}$	2, δ-EXPANSION
9		$\{\neg(a < b \to gfa \le gfb\}$	8, δ-EXPANSION
10		$\{a \le b\}$	9, α-EXPANSION
11		$\{\neg(gfa \le gfb)\}$	9, α-EXPANSION
12		$\{fa \le fb\}$	6,10 RESOLUTION
13		$\{ga \le gb\}$	7,10 RESOLUTION
14	?	$\{g(fa) \le g(fb)\}$	13,12 COMPOSITION
15		$\emptyset$	11,14 RESOLUTION

(I AM NOT 100% SURE)

b) $\forall x(x \doteq x), \forall xy(fx \doteq y \leftrightarrow x \doteq gy), \forall xy(fx \doteq y \leftrightarrow x \doteq hy) \vdash \forall x(gx \doteq hx) \wedge \forall x(hx \doteq gx)$

$\{\forall x(x \doteq x), \forall xy(fx \doteq y \leftrightarrow x \doteq gy), \forall xy(fx \doteq y \leftrightarrow x \doteq hy), \neg(\forall x(gx \doteq hx) \wedge \forall x(hx \doteq gx)\}$

UNIFICATION

- $\{\forall x(x \doteq x)\} \mapsto \{x \doteq x\}$
- $\{\forall xy(fx \doteq y \leftrightarrow x \doteq gy)\} \mapsto \{\forall xy(fx \doteq y \rightarrow x \doteq gy) \wedge \forall xy(x \doteq gy \rightarrow fx \doteq y)\}$

  $\mapsto \{\neg fx \doteq y, x \doteq gy\}, \{\neg x \doteq gy, fx \doteq y\}$
- $\{\forall xy(fx \doteq y \leftrightarrow x \doteq hy)\} \mapsto \{fx \doteq y, \neg x \doteq hy\}, \{\neg fx \doteq y, x \doteq hy\}$
- $\{\neg(\forall x(gx \doteq hx) \wedge \forall x(hx \doteq gx)\} \mapsto \{\neg(ga \doteq ha)\}, \{\neg(ha \doteq ga)\}$

$C(F) = \{\{x \doteq x\}, \{\neg fx \doteq y, x \doteq gy\}, \{\neg x \doteq gy, fx \doteq y\},$
$\{fx \doteq y, \neg x \doteq hy\}, \{\neg fx \doteq y, x \doteq hy\}, \{\neg(ga \doteq ha)\}, \{\neg(ha \doteq ga)\}\}$

A $$\frac{\{x \doteq x\} \qquad \{\neg x \doteq gy, fx \doteq y\}}{\{fgy \doteq y\}}\left[\frac{x}{gy}\right]$$

B $$\frac{\{fgy \doteq y\} \qquad \{\neg fx \doteq y, x \doteq hy\}}{\{gy \doteq hy\}}\left[\frac{x}{gy}\right]$$

C $$\frac{\{gy \doteq hy\} \qquad \{\neg(ga \doteq ha)\}}{\emptyset}\left[\frac{a}{y}\right]$$