ALGEBRAIC STRUCTURES

SUPPOSE K IS A FIELD ($K=\mathbb{Q},\mathbb{R},\mathbb{C}$) AND S AN EXISTENT SET.

SET K^S OF ALL FUNCTIONS $f: S\to K$ IS A VECTOR SPACE WITH OPERATIONS:

- $0(x)=0_K \quad (x\in S)$
- $(-f)(x)=-f(x)$
- $(f+g)(x)=f(x)+g(x)$
- $(t f)(x)=t\cdot f(x) \qquad t\in K,\ f,g\in K^S$

WE CHECK THAT AXIOMS ARE SATISFIABLE, SO THAT $K^S\models\varphi$ FOR EVERY φ

IN THE SET OF AXIOMS OF THE THEORY SPACES

1) ASSOCIATIVITY OF ADDITION

2) COMMUTATIVITY OF ADDITION

3) NEUTRAL ELEMENT OF ADDITION

4) OPPOSITE

5) NEUTRAL ELEMENT OF PRODUCT

6) ASSOCIATIVITY OF PRODUCT

7) ADDITIVITY OF PRODUCT

8) DISTRIBUTIVITY

1) ASSOCIATIVITY OF ADDITION

$f, g, h \in K^S \quad (f+g)+h = f+(g+h)?$

$((f+g)+h)(x) = (f+g)(x) + h(x)$ // PROPERTY OF THE GIVEN SET

$= f(x) + g(x) + h(x) =$ // $f(x), g(x) \in K$, WHERE + IS ASSOCIATIVE

$= f(x) + (g(x) + h(x)) = f(x) + (g+h)(x) = (f+(g+h))(x)$

2) COMMUTATIVITY OF ADDITION

$f, g \in K^S \quad f+g = g+f ?$

THE TWO FUNCTIONS ARE EQUAL IF $(f+g)(x) = (g+f)(x)$

$(f+g)(x) = f(x) + g(x) =$ // PROPERTY OF THE GIVEN SET

$= g(x) + f(x)$ // $f(x), g(x) \in K$, WHERE + IS COMMUTATIVE

$= (g+f)(x)$