USE RESOLUTION TO PROVE THAT THE FOLLOWING FORMULAS ARE VALID:

$\exists x \forall y (Rx \to Sy) \leftrightarrow \forall y \exists x (Rx \to Sy)$

$\{\exists x \forall y (Rx \to Sy) \leftrightarrow \forall y \exists x (Rx \to Sy)\} =$ //DEDUCTION THEOREM

$= \underbrace{\exists x \forall y (Rx \to Sy) \vdash \forall y \exists x (Rx \to Sy)}_{A} ; \underbrace{\forall y \exists x (Rx \to Sy) \vdash \exists x \forall y (Rx \to Sy)}_{B}$

A $\{\exists x \forall y (Rx \to Sy), \neg \forall y \exists x (Rx \to Sy)\}$

- $\{\exists x \forall y (Rx \to Sy)\} \mapsto \{\forall y (Ra \to Sy)\} \mapsto \{Ra \to Sy\}$

  $\mapsto \{\neg Ra, Sy\}$

- $\{\neg \forall y \exists x (Rx \to Sy)\} \mapsto \{\exists y \forall x. \neg (Rx \to Sy)\} \mapsto \{\neg (Rx \to Sb)\}$

  $\mapsto \{Rx\}, \{\neg Sb\}$

$C(F) = \{\{\neg Ra, Sy\}, \{Rx\}, \{\neg Sb\}\}$

$$\dfrac{\{\neg Ra, Sy\} \quad \{Rx\}}{\{Sy\}}\left[\tfrac{a}{x}\right] \quad (1)$$

$$\dfrac{\{Sy\} \quad \{\neg Sb\}}{\emptyset}\left[\tfrac{y}{b}\right] \quad (2)$$

(1) $C_1 = \{\neg Ra, Sy\}$ $E_1 = \{\neg Rx\}$

$C_2 = \{Rx\} = E_1$

$F_1 = E_1 \cup \overline{E_2} = \{\neg Ra, \neg Rx\}$ $S = [a/x]$

$F_2 = F_1 S = \{\neg Ra\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2) S = \{Sy \cup \emptyset\}[a/x] = \{Sy\}$

(E) $C_1 = \{Sy\} = E_1 \qquad C_2 = \{\neg Sb\} = E_2$

$F_1 = E_1 \cup \bar{E}_2 = \{Sy, Sb\}$ $\qquad S = [b/y]$

$F_2 = \{Sb\}$

$R(C_1, C_2) = \{(C_1 \setminus E_1 \cup C_2 \setminus E_2)S = \emptyset$

(B) $\{\forall y \exists x (Rx \to Sy), \neg \exists x \forall y (Rx \to Sy)\}$

- $\{\forall y \exists x (Rx \to Sy)\} \mapsto \{R(fy) \to Sy\} \mapsto \{\neg R(fy) \vee Sy\}$

  $\mapsto \{\neg R(fy), Sy\}$

- $\{\neg \exists x \forall y (Rx \to Sy)\} = \{\forall x \exists y . \neg (Rx \to Sy)\} = \{\neg (Rx \to S(gx))\}$

  $\mapsto \{Rx \wedge \neg S(gx)\} \mapsto \{Rx\}, \{\neg S(gx)\}$

$C(F) = \{\{\neg R(fy), Sy\}, \{Rx\}, \{\neg S(gx)\}$

$$(\underset{\bullet}{\sqcup})\ \frac{\{\neg R(fy), Sy\} \quad \{Rx\}}{\{Sy\}} \qquad (E)\ \frac{\{Sy\} \qquad \{\neg S(gx)\}}{\emptyset}$$

($\underset{\bullet}{\sqcup}$) $C_1 = \{\neg R(fy), Sy\} \quad E_1 = \{\neg R(fy)\} \quad C_2 = \{Rx\} = E_2$

$F_1 = E_1 \cup \bar{E}_2 = \{\neg R(fy), Rx\}$ $\quad S = [fy/x] \quad F_2 = \{Rx\}$

$R(C_1, C_2) = (C_1 \setminus E_1 \cup C_2 \setminus E_2)S = \{Sy\}$

(E) $C_1 = \{Sy\} = E_1 \quad C_2 = \{\neg S(gx)\} = E_2 \quad F_1 = \{Sy, S(gx)\} \quad S = [y/gx]$

$F_2 = \{Sy\} \qquad R(C_1 . C_2) = \emptyset$

USE RESOLUTION TO PROVE THAT:

a) $\forall xy (x \le y) \to (fx \le fy) \wedge (gx \le gy) \vdash \forall xy ((x \le y) \to (gfx \le gfy))$

$\{\forall xy (x \le y) \to (fx \le fy) \wedge (gx \le gy), \neg \forall xy ((x \le y) \to (gfx \le gfy))\}$

UNIFICATION

- $\{\forall xy ((x \le y) \to (fx \le fy) \wedge (gx \le gy))\}$

  $\mapsto \{(x \le y) \to (fx \le fy) \wedge (gx \le gy)\}$

  $\mapsto \{\neg (x \le y) \vee ((fx \le fy) \wedge (gx \le gy))\}$

  $\mapsto \{\neg (x \le y), fx \le fy\}, \{\neg (x \le y), gx \le gy\}$

- $\{\neg \forall xy ((x \le y) \to (gfx \le gfy))\}$

  $\mapsto \{\exists xy. \neg ((x \le y) \to (gfx \le gfy))\}$

  $\mapsto \{\exists xy. ((x \le y) \wedge \neg (gfx \le gfy))\}$

  $\mapsto \{a \le b\}, \{\neg (gfa \le gfb)\}$

$C(CF) = \{\{\neg (x \le y), fx \le fy\}, \{\neg (x \le y), gx \le gy\}, \{a \le b\}, \{\neg (gfa \le gfb)\}\}$

A: $\dfrac{\{\neg (x \le y), fx \le fy\} \qquad \{a \le b\}}{\{fa \le fb\}}$

B: $\dfrac{\{fa \le fb\} \qquad \{\neg (x \le y), gx \le gy\}}{\{gfa \le gfb\}}$

C: $\dfrac{\{gfa \le gfb\} \qquad \{\neg (gfa \le gfb)\}}{\varnothing}$

A $\quad C_1 = \{\neg (x \le y), fx \le fy\} \quad E_1 = \{\neg (x \le y)\} \quad C_2 = \{a \le b\} = E_2$

$F_1 = \{E_1 \cup E_2\} = \{x \le y, a \le b\} \quad S_1 = [\frac{a}{x}]$