HERBRAND MODEL

STEP	FORMULA	RULE
1	$\{\neg R(x,y), R(fgx, fgy)\}$	ASSUMPTION
2	$\{\neg R(fx, fy), R(x,y)\}$	ASSUMPTION
3	$\{R(a,b)\}$	ASSUMPTION
4	$\{\neg R(ga,gb)\}$	ASSUMPTION
5	$\{R(fga, fgb)\}$	1,3 RESOLUTION
6	$\{R(ga, gb)\}$	2,5 RESOLUTION
7	$\emptyset$	5,6 RESOLUTION

b) IF f AND g ARE R-SURJECTIVE, g IS R-COMPATIBLE AND R IS TRANSITIVE, THEN fg IS R-SURJECTIVE

$\forall y \exists x R(fx,y) \wedge \forall y \exists x R(gx,y), \forall xy (R(x,y) \rightarrow R(fx,fy)), \forall xyz (R(x,y) \wedge$

$\wedge R(y,z) \rightarrow R(x,z)) \vdash \forall y \exists x R(fgx,y)$

- $\{\forall y \exists x R(fx,y) \wedge \forall y \exists x R(gx,y)\} \mapsto \{\{\forall y \exists x R(fx,y)\}.\{\forall y \exists x R(gx,y)\}\}$

  $\mapsto \{\{R(fh_y,y)\}.\{R(gk_y,y)\}\}$

- $\{\forall xy(R(x,y) \rightarrow R(fx,fy))\} \mapsto \{\neg R(x,y), R(fx,fy)\}$

- $\{\forall xyz(R(x,y) \wedge R(y,z) \rightarrow R(x,z))\} \mapsto \{\neg R(x,y), \neg R(y,z), R(x,z)\}$

- $\{\neg \forall y \exists x R(fgx.y)\} \mapsto \{\exists y \forall x. \neg (R fgx,y)\} \mapsto \{\neg R(fgx,a)\}$

$$C(F) = \{\{R(fhx, y)\}, \{R(gkx, y)\}, \{\neg R(x, y), R(fx, fy)\},$$

$$\{\neg R(x, y), \neg R(y, z), R(x, z)\}, \{\neg R(fgx, a)\}\}$$

$$\frac{\{\neg R(fgx, a)\} \qquad \{\neg R(x, y), \neg R(y, z), R(x, z)\}}{\{\neg R(x, fgx), R(x, a)\}} \left[\frac{fgx}{y}, \frac{a}{z}\right]$$

$$\frac{\{R(gky, y)\} \qquad \{\neg R(x, y), R(fx, fy)\}}{\{R(fgky, fy)\}}$$

$$\left[\frac{ky}{x}\right] \qquad \frac{\{R(fgky, a)\} \qquad \{\neg R(fgx, a)\}}{\emptyset}$$

PROVE USING RESOLUTION THAT EVERY BINARY RELATION R ON A SET WHICH IS SERIAL, SYMMETRIC AND TRANSITIVE IS AN EQUIVALENCE RELATION

SERIAL: $\forall x \exists y. Rxy$

SYMMETRIC: $\forall xy (Rxy \rightarrow Ryx)$

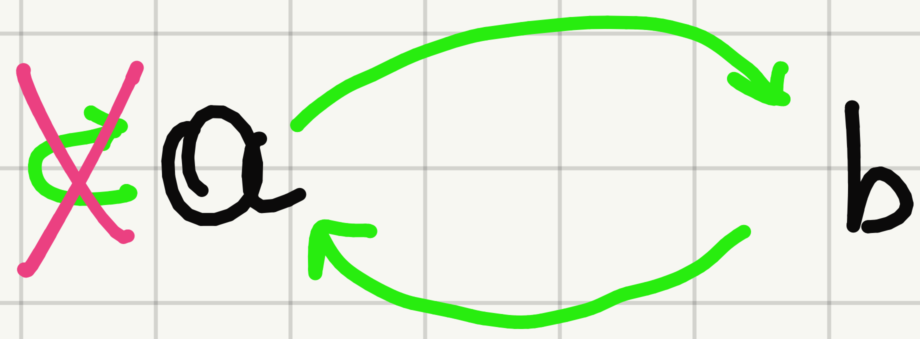
TRANSITIVE: $\forall xyz (Rxy \land Ryz \rightarrow Rxz)$

EQUIVALENCE RELATION: REFLEXIVE, SYMMETRIC AND TRANSITIVE. SINCE SYMMETRIC AND TRANSITIVE CONDITIONS ARE IN THE PREMISES, WE ONLY NEED TO SHOW REFLEXIVITY: $\forall x Rxx$

$\forall x \exists y. Rxy, \forall xy (Rxy \rightarrow Ryx), \forall xyz (Rxy \land Ryz \rightarrow Rxz) \vdash \forall x Rxx$

$F = \{ \forall x \exists y. Rxy, \forall xy (Rxy \rightarrow Ryx), \forall xyz (Rxy \land Ryz \rightarrow Rxz), \neg \forall x Rxx \}$

SUGGESTION: BEFORE TO START, IT IS USEFUL TO HAVE AN IDEA



PREMISES

CONCLUSION

HERBRAND MODEL

STEP	FORMULA	RULE
1	$\{ \forall x \exists y. Rxy \}$	ASSUMPTION
2	$\{ \forall xy (Rxy \rightarrow Ryx) \}$	ASSUMPTION