LET R BE A BINARY RELATION SYMBOL. PROVE THAT $\forall xy \exists z (Rzx \land Rzy)$, $\forall xyz (Rxy \land Ryz \to Rxz) \vdash \forall xyz \exists w (Rwx \land Rwy \land Rwz)$

$$\Gamma = \{\forall xy \exists z (Rzx \land Rzy), \forall xyz (Rxy \land Ryz \to Rxz), \neg(\forall xyz \exists w (Rwx \land Rwy \land Rwz))\}$$

- $\{\forall xy \exists z (Rzx \land Rzy)\} \mapsto \{R(f(x,y),x)\}, \{R(f(x,y),y)\}$
- $\{\forall xyz (Rxy \land Ryz \to Rxz)\} \mapsto \{\neg Rxy, \neg Ryz, Rxz\}$
- $\{\neg(\forall xyz \exists w (Rwx \land Rwy \land Rwz))\} \mapsto \{\exists xyz \forall w. \neg(Rwx \land Rwy \land Rwz)\} \mapsto$

  $\{\neg Rwa, \neg Rwb, \neg Rwc\}$

$$C(F) = \{\{R(f(x,y),x)\}, \{R(f(x,y),y)\}, \{\neg Rxy, \neg Ryz, Rxz\}, \{\neg Rwa, \neg Rwb, \neg Rwc\}\}$$

$$\dfrac{\{R(f(x,y),x)\} \quad \{\neg Rwa, \neg Rwb, \neg Rwc\}}{\{\neg R(f(a,y),b), \neg R(f(a,y),c)\}} \qquad \dfrac{\{R(f(x,y),y)\} \quad \{\neg Rxy, \neg Ryz, Rxz\}}{\{\neg Ryz, R(f(x,y),z)\}}$$

$$\dfrac{\{\neg R(f(a,y),b), \neg R(f(a,y),c)\} \quad \{R(f(x,y),y)\}}{\{\neg R(f(a,y),c)\}} \qquad \dfrac{\{\neg Ryz, R(f(x,y),z)\} \quad \{R(f(x,y),x)\}}{\{R(f(x,y),z)\}}$$

$$\dfrac{\{\neg R(f(a,y),c)\} \quad \{R(f(x,y),z)\}}{\emptyset}$$

Assume R and S are binary relations on a set X. Use resolution to prove that if both are symmetric and $SR \subseteq RS$, then RS is symmetric

$$\forall xy(Rxy \to Ryx), \forall xy(Sxy \to Syx); \forall xy(\exists z(Sxz \land Rzy) \to$$
$$\to \exists w(Rxw \land Swy)) \vdash \forall xy(\exists z(Rxz \land Szy) \to \exists w(Ryw \land Swx)$$

$$\Gamma: \{\forall xy(Rxy \to Ryx), \forall xy(Sxy \to Syx); \forall xy(\exists z(Sxz \land Rzy) \to$$
$$\to \exists w(Rxw \land Swy)), \neg \forall xy(\exists z(Rxz \land Szy) \to \exists w(Ryw \land Swx)\}$$

- $\{\forall xy(Rxy \to Ryx)\} \mapsto \{\neg Rxy, Ryx\}$
- $\{\forall xy(Sxy \to Syx)\} \mapsto \{\neg Sxy, Syx\}$
- $\{\forall xy(\exists z(Sxz \land Rzy) \to \exists w(Rxw \land Swy))\} \mapsto$

  $\{\neg Sxz, \neg Rzy, R_{xfx}\}, \{\neg Sxz, \neg Rzy, S_{fxy,y}\}$
- $\{\neg \forall xy(\exists z(Rxz \land Szy) \to \exists w(Ryw \land Swx)\} \mapsto$

  $\{\exists xy.\neg(\exists z(Rxz \land Szy) \to \exists w(Ryw \land Swx))\} \mapsto \{\neg \exists z(Raz \land Szb) \to$

  $\to \exists w(Rbw \land Swa))\} \mapsto \{Rac\}, \{Scb\}, \{\neg Rbw, \neg Swa\}$

$$C(\Gamma) = \{\{\neg Rxy, Ryx\}, \{\neg Sxy, Syx\}, \{\neg Sxz, \neg Rzy, S_{fxy,y}\}$$
$$\{\neg Sxz, \neg Rzy, R_{xfx}\}, \{Rac\}, \{Scb\}, \{\neg Rbw, \neg Swa\}\}$$

$\{Rac\}$ $\{\neg Rxy, Ryx\}$ $\{Scb\}$ $\{\neg Sxy, Syx\}$

$\{Rya\}$ $\{Sbc\}$

$\{\neg Sxz, \neg Rzy, Sfxy, y\}$ $\{Scb\}$ $\{\neg Syz, \neg Rzy, Rxfx\}$

$\{\neg Rby, Sfby, y\}$ $\{\neg Rcy, R(b, fby)\}$

$\{Sfba, a\}$ $\{R(b, fba)\}$ $\{\neg Rbx, \neg Sxa\}$

$\{\neg Sfba, a\}$

$\varnothing$