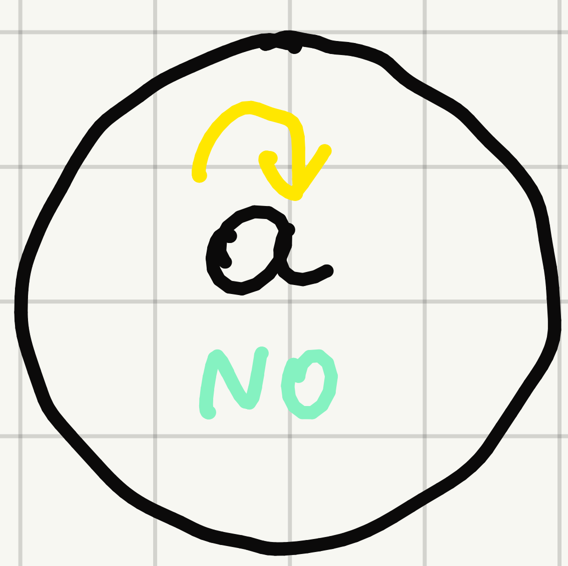
c) $\{\exists xy \forall z(xRz \wedge zRy), \exists xy(\neg xRy \wedge \neg yRx), \forall xyz(xRy \wedge yRz \rightarrow xRz)\}$

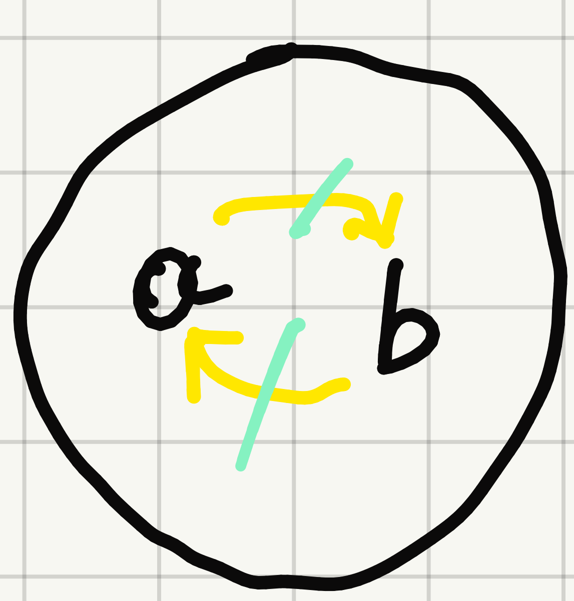
- $\exists xy(\forall z(xRz \wedge zRy))$
- $\exists xy(\neg xRy \wedge \neg yRx)$
- $\forall xyz(xRy \wedge yRz \rightarrow xRz)$

$M=\{a\}$



✓
✗

$M=\{a,b\}$



$z=a$ $z=b$

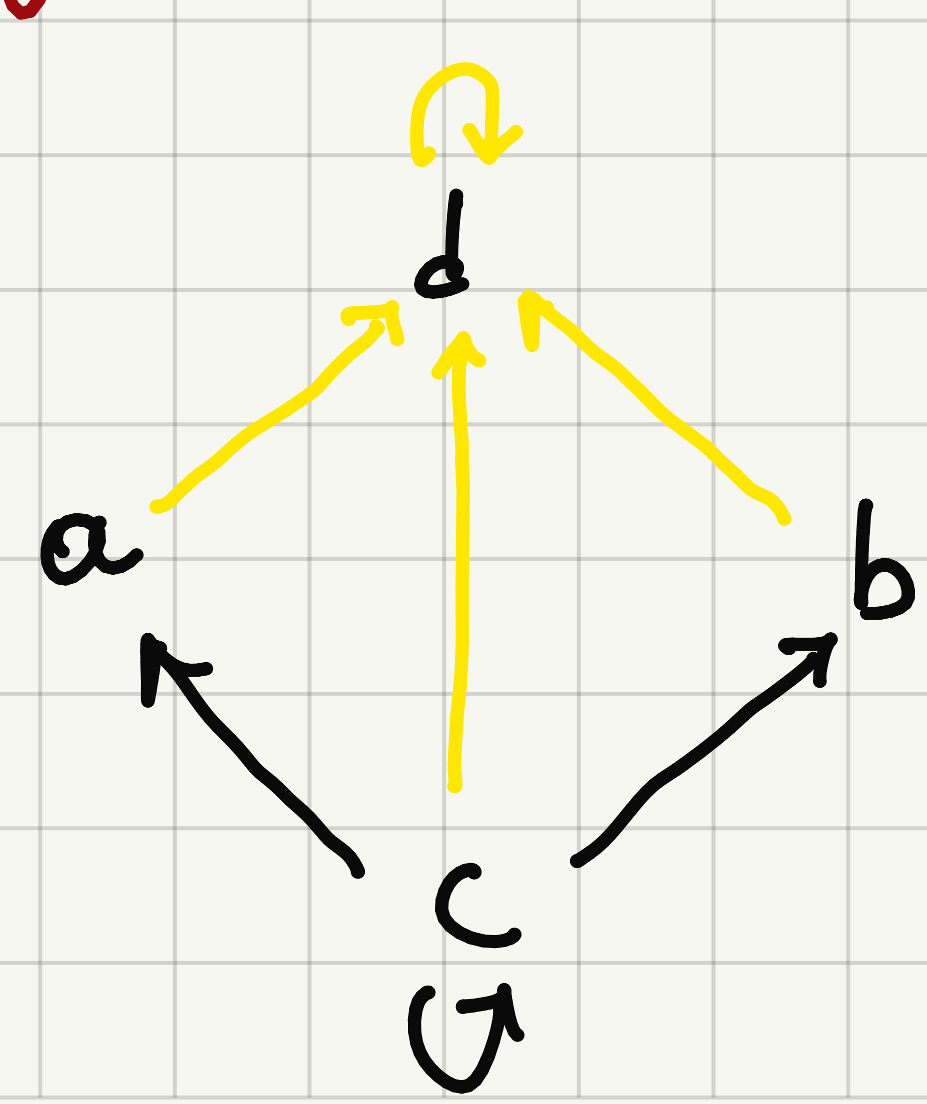
$\exists xy$, IN GENERAL $x \neq y$ ✗

$M=\{a,b,c\}$

$z=a$ $z=b$ $z=c$

O! NO

$M=\{a,b,c,d\}$



PROVE THAT THE FOLLOWING FORMULAS ARE VALID

a) $\forall x(Rx \to Sx) \to (\forall x Rx \to \forall x Sx)$

IN ORDER TO HAVE A VALID FORMULA φ, WE VERIFY UNSATISFIABILITY OF $\neg\varphi$

$$\neg(\forall x(Rx \to Sx) \to (\forall x Rx \to \forall x Sx))$$

| α- FORMULA

$$\forall x(Rx \to Sx), \neg(\forall x Rx \to \forall x Sx)$$

| α-FORMULA

$$\forall x(Rx \to Sx), \forall x Rx, \neg(\forall x Sx)$$

|

$$Ra \to Sa, Ra, \neg Sa, \ldots$$

/ \ β-FORMULA

$$\neg Ra, Ra, \neg Sa \qquad Sa, Ra, \neg Sa$$

$\Rightarrow$ $\neg\varphi$ NOT SATISFIABLE $\to$ φ IS VALID

b) $\exists x(Rx \to Sx) \leftrightarrow (\forall x Rx \to \exists x Sx)$

$$\neg(\exists x(Rx \to Sx) \leftrightarrow (\forall x Rx \to \exists x Sx))$$

$\neg(\exists x(Rx \to Sx) \to (\forall x Rx \to \exists x Sx))$ | $\neg((\forall x Rx \to \exists x Sx) \to \exists x(Rx \to Sx))$

$\exists x(Rx \to Sx), \neg(\forall x Rx \to \exists x Sx)$ | $\forall x Rx \to \exists x Sx, \neg\exists x(Rx \to Sx)$

$\exists x(Rx \to Sx), \forall x Rx, \neg\exists x Sx$ | $\forall x Rx \to \exists x Sx, \exists x Rx, \neg\exists x Sx$

$\neg\exists x Rx, \forall x Rx, \neg\exists x Sx$ | $\exists x Sx, \forall x Rx, \neg\exists x Sx$ | $\neg\forall x Rx, \exists x Rx, \neg\exists x Sx$ | $\exists x Sx, \exists x Rx, \neg\exists x Sx$

$\neg Ra, Ra, \neg Sa$ | $Sa, Ra, \neg Sa$ | $\neg Ra, Ra, \neg Sa$ | $Sa, Ra, \neg Sa$

$\Rightarrow$ $\neg\varphi$ IS NOT SATISFIABLE $\Rightarrow$ φ IS VALID