Computing Infrastructures
Exercises on performance I

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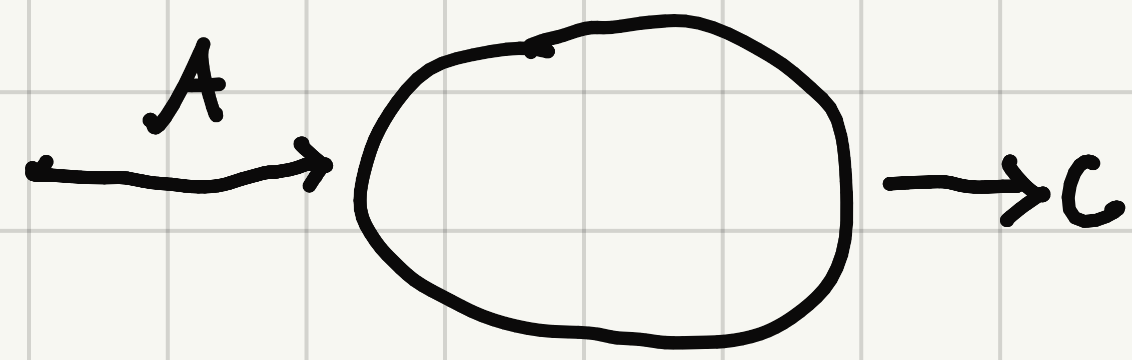
Credits to S. Cereda, E. Barbierato, R. Pinciroli

QUICK REVIEW

ASSUMPTION: WE CONSIDER ONE SYSTEM ONLY

DEFINITIONS:



- T → LENGHT OF TIME THE SYSTEM IS OBSERVED
- A_K → NUMBER OF REQUEST ARRIVALS FOR RESOURCE K
- C_K → NUMBER OF REQUEST COMPLETIONS FOR RESOURCE K
- B_K → AMOUNT OF TIME RESOURCE K IS BUSY ($B_K \leq T$)
- N_K → AVERAGE NUMBER OF JOBS IN THE RESOURCE K
- R_K → AVERAGE RESPONSE TIME FOR RESOURCE K
- Z_K → AVERAGE THINK TIME FOR RESOURCE K

- $\lambda_K = A_K/T$ ARRIVAL RATE
- $X_K = C_K/T$ THROUGHPUT/COMPLETION RATE
- $U_K = B_K/T$ UTILISATION $0 \leq U_K \leq 1$
- $S_K = B_K/C_K$ MEAN/AVERAGE SERVICE TIME PER COMPLETED JOB

UTILIZATION LAW $U_K = X_K S_K$

LITTLE'S LAW $N = X \cdot R$

RESPONSE TIME LAW $R = N/X - Z$

Exercise 1

- A server has completed 80 jobs in 10 minutes. The utilization that has been measured has been 75%

Determine:

1. The throughput of the system
2. The average service time
3. The busy time of the system
4. The average response time

Exercise 2

- A processing system, is working on the average on 35 requests at the same time. It can complete 28 jobs in one hour.

Determine:

1. The average response time
2. The utilization of the system

Exercise 3

- The busy time of a virtual machine is 3 hour every 4 hours. Its utilization is 80%, and on the average it is serving 5 jobs at the same time.

Determine:

1. The system throughput
2. The average response time

Exercise 3b

- The busy time of a virtual machine is 3 hour every 4 hours. In that time, it has completed 15 jobs, and on the average it is serving 5 jobs at the same time.

Determine:

1. The system throughput
2. The average response time

Exercise 4

In a course there 100 students. Each student studies for a week, then sends an email to the professor, waits for an answer, studies one more week and then sends another email and so on. The professor replies to 5 emails every day. How much time does every student wait, on average, for an answer?

Exercise 5

Software monitor data for an interactive system shows a CPU utilization of 75%, a 3 second CPU service demand, a response time of 15 seconds, and 10 active users. What is the average think time of these users?

Exercise 6

Suppose an interactive system is supporting 100 users with 15 second think times and a system throughput of 5 interactions/second.

1. What is the response time of the system?
2. Suppose that the service demands of the workload evolve over time so that system throughput drops to 50% of its former value (i.e., to 2.5 interactions/second). Assuming that there still are 100 users with 15 second think times, what would their response time be?

Exercise 7

A user request submitted to the system must queue for memory, and may begin processing (in the central subsystem) only when it has obtained a memory partition.

1. If there are 100 active users with 20 second think times, and system response time (the sum of memory queueing and central sub- system residence times) is 10 seconds, how many customers are competing for memory on average?
2. If memory queueing time is 8 seconds, what is the average number of customers loaded in memory?

Exercise 8

Consider the following measurement data for an interactive system with a memory constraint:

T	1 hour
N	80
R	1 second
N in memory	6
C	36000
U_{cpu}	75%
U_{D1}	50%
U_{D2}	50%
U_{D3}	25%

1. What was throughput (in requests / second)?
2. What was the average "think time"?
3. On the average, how many users were attempting to obtain service (i.e., not "thinking")?
4. On the average, how much time does a user spend waiting for memory (i.e., not "thinking" but not memory-resident) ?

Exercise 9

By monitoring a single class interactive system, we are able to measure the following data:

- Disk demand: 4 seconds/transaction
- CPU demand: 1 seconds/transaction
- CPU utilization: 80%
- Response time: 20 seconds/transaction
- Monitoring period: 6 minutes
- Number of users: 16

Which is the average think time of these users?

Exercise 10

Consider the following measurement data for an interactive system:

- Measurement interval: 2 minutes
- number of users: 15
- average response time per transaction: 10 seconds
- Disk demand: 0.5 seconds/transaction
- CPU demand: 0.2 seconds/transaction
- Number of servers: 20
- Number of completed transactions: 60

On average, how many users are thinking?

1

$C=80$ $T=10$ MINUTES $U=75\%$ a) X? b) S? c) B? d) R?

a) $X = C/T = 8$/MINUTE

b) $U = XS \Rightarrow S = \frac{U}{X} = 0{,}0938$ MINUTE $= 5{,}625$ SECOND

c) $U = B/T \Rightarrow B = U \cdot T = 7{,}5$ MINUTE

d) $R = N/X - Z$ INSUFFICIENT INFORMATIONS

2

$N=35$ $X = 28$/HOUR a) R? b) U?

a) $R = N/X = 1{,}25$ HOUR

b) $U = B/T = XS = X\,B/C$ INSUFFICIENT INFORMATIONS

3

$B = 3$ HOUR $T=4$ HOUR $U=80\%$ $N=5$ X? R?

! $U \neq B/T$. INCORRECT MEASUREMENTS (NEVER HAPPENS IN THE EXAM)

THE EXERCISE CANNOT BE SOLVED

3b $B = 3$ HOUR $T=4$ HOUR $C=15$ $N=5$ X? R?

a) $X = C/T = 3{,}75$/HOUR

b) $R = N/X = 1{,}33$ HOUR

④

$N=100$ $Z=1$ WEEK $X=5/\text{DAY}$ $R?$

$R=\frac{N}{X}-Z=\frac{100}{5/\text{DAY}}-1\cdot 7\,\text{DAY}=13$ DAYS

⑤

$U=75\%$ $S=3$ SECOND $R=15$ SECOND $N=10$ $Z?$

$R=\frac{N}{X}-Z \Rightarrow Z=\frac{N}{X}-R=\frac{N}{U/S}-R=25$ SECOND

⑥

$N=100$ $Z=15$ SECOND $X=5/$SECOND a) $R?$ b) $X'=2{,}5/$SECOND $R'?$

a) $R=\frac{N}{X}-Z=5$ SECOND b) $R'=\frac{N}{X'}-Z=25$ SECOND

⑦

a) $N=100$ $Z=20$ SECOND $R=10$ SECOND $N'=XR?$ b) $T=8$ SECOND $N_{TOT}?$

a) $R=\frac{N}{X}-Z \Rightarrow X=\frac{N}{R+Z}=3{,}33/$SECOND $N'=X\cdot R=33{,}3$

b) $X=\frac{C}{T} \Rightarrow C=X\cdot T=26{,}6$ $N_{TOT}=N'-C=6{,}7$

⑧ $T=1$ HOUR $N=80$ $R=1$ SECOND N(IN MEMORY)$=6$ $C=36000$ $U_{CPU}=75\%$

$U_{D1}=50\%$ $U_{D2}=50\%$ $U_{D3}=25\%$ a) $X?$ b) $Z?$ c) $Z=0\rightarrow N?$ d) $R_{OUT}?$

a) $X=\frac{C}{T}=36000/\text{HOUR}=10/$SECOND b) $R=\frac{N}{X}-Z \Rightarrow Z=\frac{N}{X}-R=7$ SECOND

c) $R=\frac{N}{X} \Rightarrow N=10$ d) $N_{OUT}=N-N$(IN MEMORY)$=4$ $R=\frac{N_{OUT}}{X}=0{,}4$ SECOND

9 TO VERIFY

$S = 1$ SECOND $U = 80\%$ $R = 20$ SECOND $T = 6$ MINUTE $N = 16$ Z?

$$R = \frac{N}{X} - Z = \frac{N}{U/S} - Z \Rightarrow Z = \frac{N}{U/S} - R = (20 - 20)\text{ SECOND} = 0$$

10 TO VERIFY

$T = 2$ MINUTE $N = 15$ $R = 10$ SECOND $S = 0{,}2$ SECOND $C = 60$ N_{THINK}?

$$N_{ACTIVE} = X \cdot R = \frac{C}{T} \cdot R = \frac{60}{2 \cdot 60\text{ SECOND}} \cdot 10\text{ SECOND} = 5$$

$$N_{THINK} = N - N_{ACTIVE} = 10$$

Computing Infrastructures

Exercises on performance II

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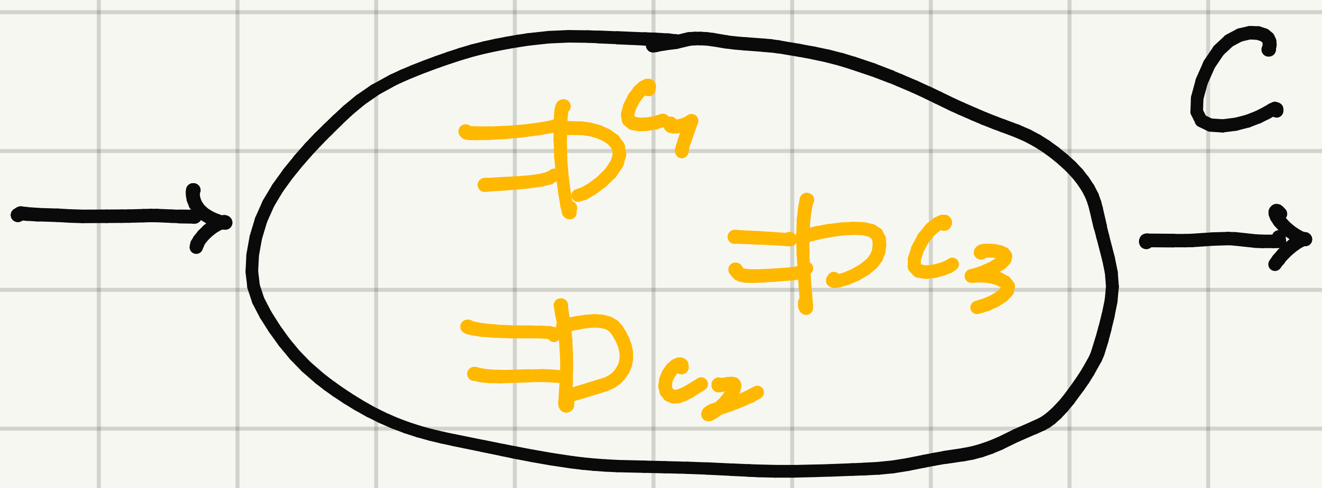
QUICK REVIEW

SYSTEMS WITH MULTIPLE COMPONENTS AND WE DON'T KNOW WHICH OF THEM WILL BE USED EXACTLY ONCE



DEFINITIONS:

- $V_K = \frac{C_K}{C}$ → VISIT RATIO FOR COMPONENT K
- $X_K = V_K \cdot X$ (FORCED FLOW LAW) → THROUGHPUT FOR COMPONENT K

  $X_K = \frac{C_K}{T}$
- $D_K = \frac{B_K}{C} = V_K \cdot S_K$ → DEMAND
- $\overset{\text{RESIDENCE}}{R_K} = \overset{\text{RESPONCE TIME}}{R_K} \cdot V_K$ → RESIDENCE TIME
- $N_K = X \cdot \overset{\text{RESIDENCE TIME}}{R_K} = X_K \cdot R_K$ → RESPONSE TIME
- $U_K = X_K S_K = X \cdot D_K$

Exercise 11

A ski resort accommodates 1000 visitors per day. The resort has a nice ski slope which every skiers visits, on average, 10 times.

How many visits does the slope see in an entire day?

Exercise 12

An interactive system with 80 active terminals shows an average think time of 12 seconds. On average, each interaction causes 15 paging disk accesses. If the service time per paging disk access is 30 ms and this disk is 60% busy, what is the average system response time?

Exercise 13

In a 30 minute observation interval, a particular disk was found to be busy for 12 minutes. If it is known that jobs require 320 accesses to that disk on average, and that the average service time per access is 25 milliseconds, what is the system throughput (in jobs/second)?

Exercise 14

In a batch system, a specific disk is performing, on average, 12 operation per second. We know that each batch transaction requires, on average, 6 accesses to this disk. Another disk in the system is handling 18 operations per second. Which is the average number of accesses to this second disk require by every batch transaction?

Exercise 15

The storage server of an intranet consists of two groups of disks, A and B, each having service times with means S_A = 5ms and S_B = 3ms. The mean number of visits for the two components are V_A = 20 and V_B = 30. The throughput of A is 150 op./sec. The above data were collected when the system is processing a workload generated by 300 users with think time Z = 15 sec.

1. Compute the System Throughput X and the Utilization of B.
2. Compute the system response time.
3. If the number of users increases to 400, which will be the new response time?

Exercise 16

The throughput of a disk is 100 I/O operations per second. To complete a given request 20 visits to the disk are required. The number of users is 100 and the response time is 15 seconds.

Compute the users think time.

Exercise 17

A web server of a company is connected to an intranet and is accessed by the employees that work internally in the company resulting in a population of fixed size: N=21 users. The average think time of the users is Z=20 sec. A complete execution of a request generates a load of V_S = 20 operations to a specific storage device whose utilization is U_S = 0.30. The service time of the storage device per each visit is S_S = 0.025 sec.

1. Determine the average system response time R
2. Compute the average throughput and system response time with N=40 users.

11

$X = 1000/\text{DAY}$ $V_{SLOPE} = 10$ X_{SLOPE}?

$X_{SLOPE} = V_{SLOPE} \cdot X = 10000$

12

$N = 80$ $Z = 12$ SECOND $V_{DISK} = 15$ $S_{DISK} = 30$ MILLISECOND $U_{DISK} = 60\%$ R?

$R = \frac{N}{X} - Z$ $U_{DISK} = X \cdot D_{DISK} \Rightarrow X = \frac{U_{DISK}}{D_{DISK}} = \frac{U_{DISK}}{V_{DISK} \cdot S_{DISK}} =$

$= \frac{0{,}6}{15 \cdot 0{,}03} = 1{,}333/\text{SECOND}$ $R = \frac{80}{1{,}333} - 12 = 48$ SECOND

13

$T = 30$ MINUTE $B_{DISK} = 12$ MINUTE $V_{DISK} = 320$ $S_{DISK} = 15$ MILLISECOND X?

$T = 1800$ SECOND $B_{DISK} = 720$ SECOND $S_{DISK} = 0{,}015$ SECOND

$U_{DISK} = X \cdot D_{DISK} \Rightarrow X = \frac{U_{DISK}}{D_{DISK}} = \frac{B_{DISK}/T}{V_{DISK} \cdot S_{DISK}} = 0{,}05/\text{SECOND}$

14

$X_{DISK_1} = 12/\text{SECOND}$ $V_{DISK_1} = 6$ $X_{DISK_2} = 18/\text{SECOND}$ V_{DISK_2}?

$V_{DISK_1} = \frac{X_{DISK_1}}{X} \Rightarrow X = \frac{X_{DISK_1}}{V_{DISK_1}} = 2/\text{SECOND}$ $V_{DISK_2} = \frac{X_{DISK_2}}{X} = 9$

15

$S_A = 0{,}005$ SECOND $S_B = 0{,}003$ SECOND $V_A = 20$ $V_B = 30$ $N = 300$

$X_A = 150/$SECOND $Z = 15$ SECOND a) X? U_B? b) R? c) $N' = 400$ R'?

a) $X_A = V_A X \rightarrow X = \frac{X_A}{V_A} = 7{,}5/\text{SECOND}$ $U_B = X_B S_B = (X V_B) S_B = 67{,}5\%$

b) $R = \frac{N}{X} - Z = 25$ SECOND

c) $R' = \frac{N'}{X} - Z = 38{,}33$ SECOND INCORRECT SINCE WE DON'T KNOW

IF X AND Z CHANGES. IT HAS TO BE EXPLICITLY ASSUMED THAT X

AND Z REMAINS THE SAME

16

$X_{DISK} = 100/$SECOND $V_{DISK} = 20$ $N = 100$ $R = 15$ SECOND Z?

$R = \frac{N}{X} - Z = \frac{N}{X_{DISK}/V_{DISK}} - Z \Rightarrow Z = \frac{N}{X_{DISK}/V_{DISK}} - R = 5$ SECOND

17

$N = 21$ $Z = 20$ SECOND $V_S = 20$ $U_S = 0{,}30$ $S_S = 0{,}025$ SECOND a) R? b) $N' = 40$ X'? R'?

a) $R = \frac{N}{X} - Z = \frac{N}{U_S/D_S} - Z = \frac{N}{U_S/(V_S \cdot S_S)} - Z = 15$ SECOND

b) INSUFFICIENT DATA

Computing Infrastructures

Exercises on performance III

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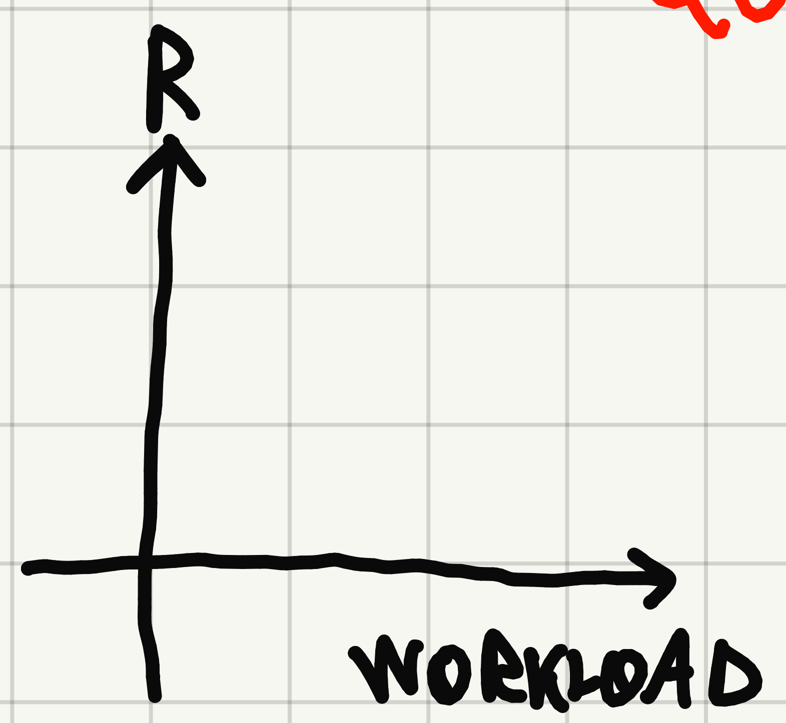
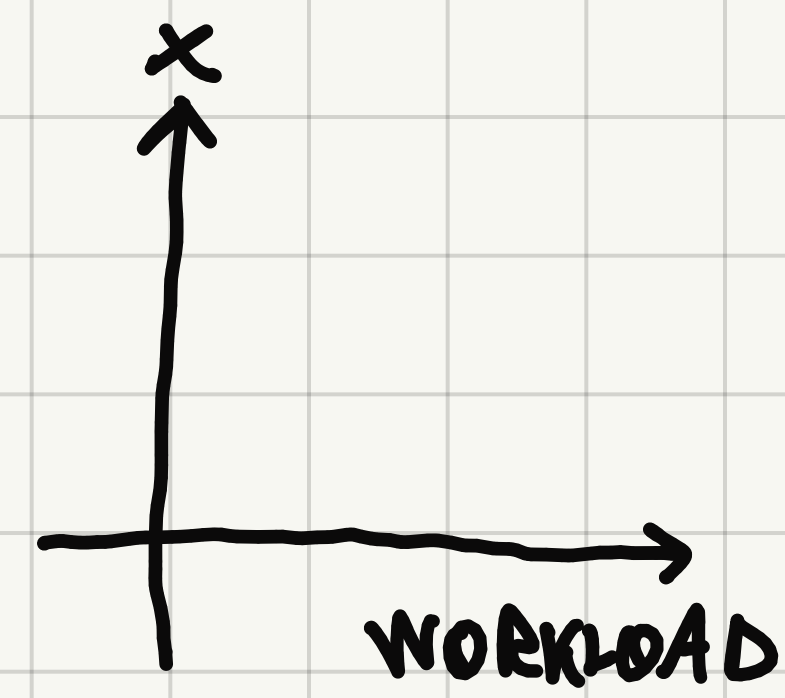
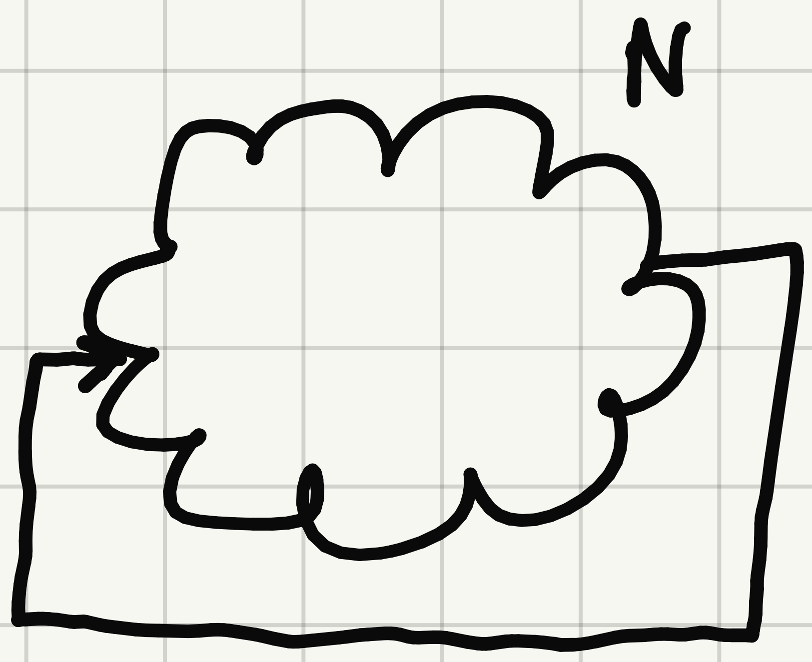
QUICK REVIEW

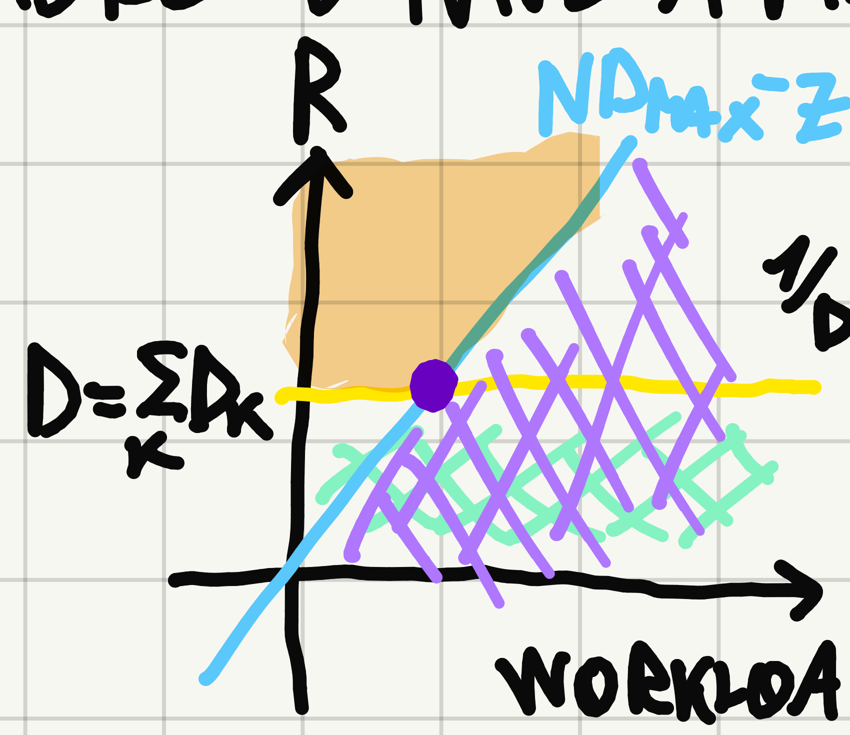


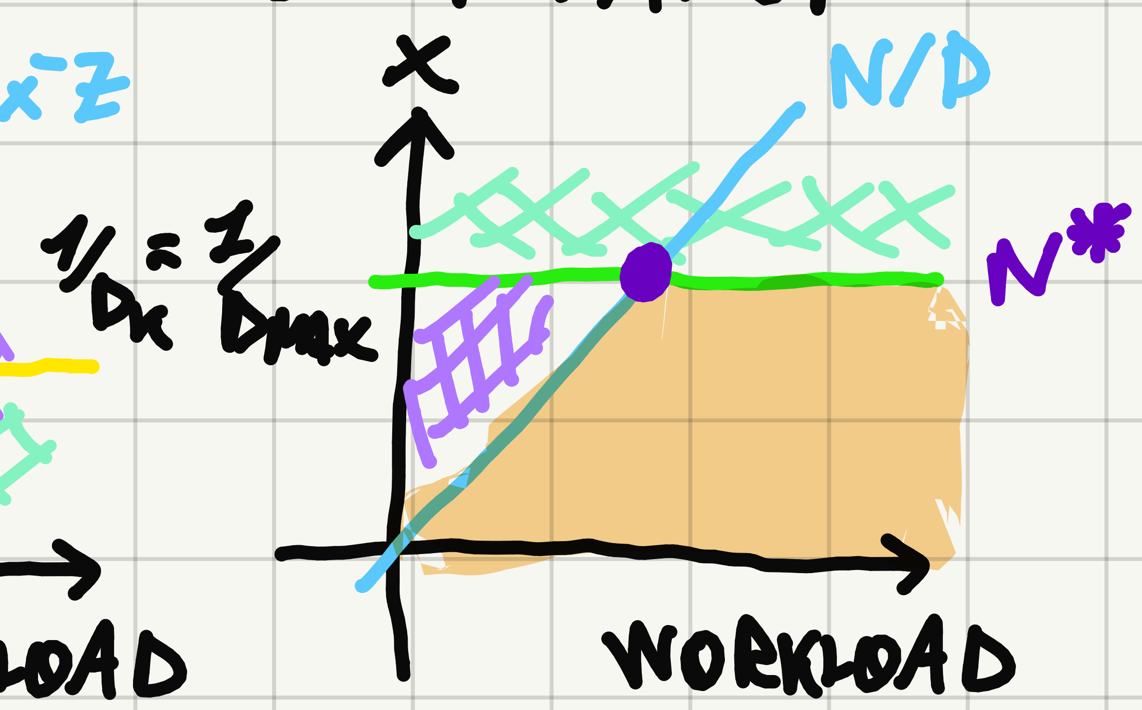


BATCH SYSTEM

CLOSED SYSTEMS WHERE WE HAVE A FIXED POPULATION

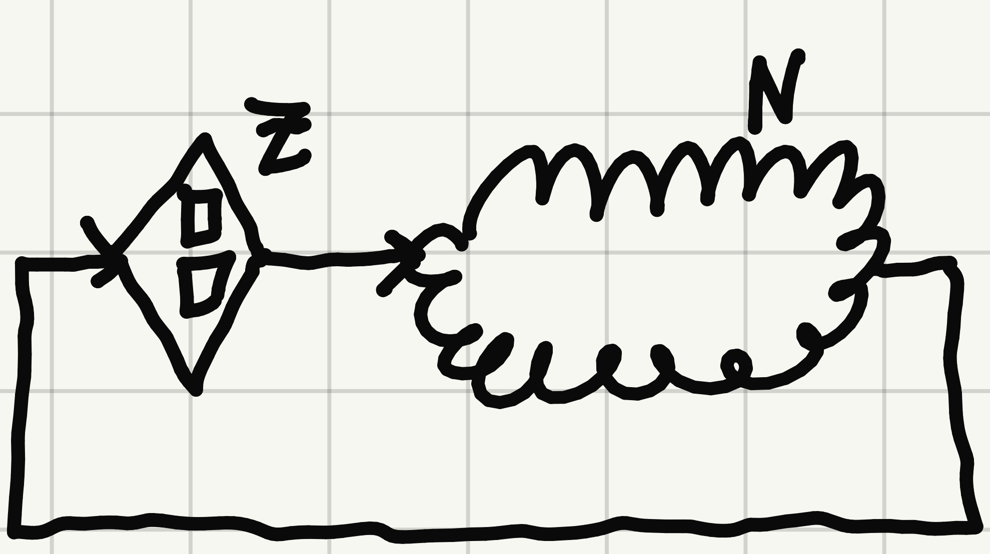






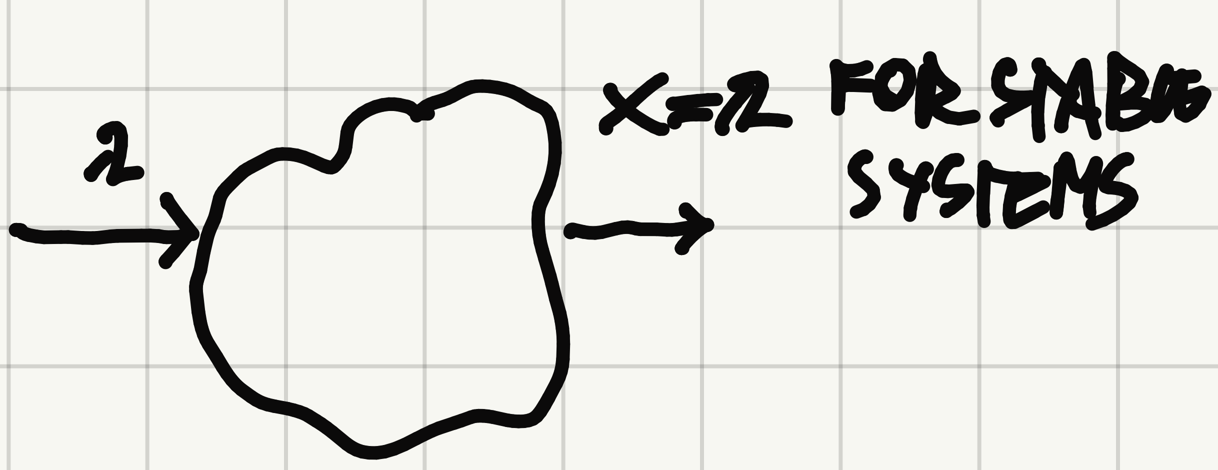
TERMINAL SYSTEM

CLOSED SYSTEMS WHERE WE HAVE A FIXED POPULATION AND EACH USER, BEFORE ENTERING IN THE SYSTEM FOR ANOTHER REQUEST THEY WAIT A CERTAIN TIME Z



TRANSACTION SYSTEM

OPEN SYSTEMS WHERE WE MEASURE THE TRANSACTIONS PER SECOND λ



Fundamentals laws:

$D = \sum_{k=1}^{K} D_k$ and $D_{max} = \max_k D_k$ $\quad N^* = \frac{D+Z}{D_{max}}$

batch	$\frac{1}{D} \leq X(N) \leq \min\left(\frac{N}{D}, \frac{1}{D_{max}}\right)$
terminal	$\frac{N}{ND+Z} \leq X(N) \leq \min\left(\frac{N}{D+Z}, \frac{1}{D_{max}}\right)$
transaction	$X(\lambda) \leq \frac{1}{D_{max}}$

batch	$\max(D, ND_{max}) \leq R(N) \leq ND$
terminal	$\max(D, ND_{max} - Z) \leq R(N) \leq ND$
transaction	$D \leq R(\lambda)$

$R \geq D \rightarrow \frac{N}{X} - Z \geq D$

$\rightarrow X \leq \frac{N}{D+Z}$

$\rightarrow X \geq \frac{N}{ND+Z}$

$R = \frac{N}{X} - Z \quad R+Z = \frac{N}{X}$

$\rightarrow N \leq \frac{R+Z}{D_{MAX}} \rightarrow R \geq ND_{MAX} - Z$

Tricks:

- Always compute D_k
- Then compute D and D_{max}
- N^* is necessary only if explicitly requested
- If you are lazy, just remember the terminal formula:
  - The batch can be obtained with Z=0
- If you are lazier, just remember the formula for the response time (which is simpler), and obtain the other using the Response Time law. Only pay attention that the min on the left, will become a max on the right.

- Two of the three bounds are always valid:
  - $R >= D$ [from the definition of D]
  - $X < 1/D_{max}$ [form the utilization law, and U<1]
- In closed models, using the Response time law, they generate two other bounds.
- In closed model we also have $R(N) < ND$

$$\frac{N}{ND+Z} \leq X(N) \leq \min\left(\frac{N}{D+Z}, \frac{1}{D_{max}}\right)$$

$$\max\left(D, ND_{max} - Z\right) \leq R(N) \leq ND$$

Exercise 18

- A Web Application, is created over three servers (a web + application server, and two storage servers), characterized by the following average service times and visits:

$$S_{WAS} = 200ms, \quad v_{WAS} = 2$$

$$S_{SS1} = 20ms, \quad v_{SS1} = 15$$

$$S_{SS2} = 50ms, \quad v_{SS2} = 8$$

Determine:

1. The maximum throughput of the system
2. The minimum response time

Exercise 19

- A Batch processing system of a bank, is composed by an application server, a database server and a storage server.

$$S_{APP} = 20ms, \;\; v_{APP} = 25$$

$$S_{DB} = 15ms, \;\; v_{DB} = 20$$

$$S_{SS} = 50ms, \;\; v_{SS} = 10$$

Determine:

1. The maximum throughput of the system for N = 5
2. The minimum response time for N = 10

Exercise 20

- A Interactive system, where users have a think time *$Z = 200s$*, is composed by an application server, a database server and a web server.

$$S_{APP} = 20ms,\ \ v_{APP} = 5$$

$$S_{DB} = 10ms,\ \ v_{DB} = 20$$

$$S_{WEB} = 40ms,\ \ v_{WEB} = 2$$

Determine:

1. The number of users for which the system switches from low to high load
2. The minimum and maximum response time for N = 50
3. The minimum and maximum throughput for N = 2000

Exercise 21

Let's consider an IT infrastructure consisting of a Web Server (WS), an Application Server (AS) and a Storage Server (SS).

After 1 hour measurement, during which N = 50 users were working continuously, the following data have been collected:

- C total number of jobs executed by the system: 5400 j
- C_{WS} Number of WS completed operations: 54000 op
- C_{AS} Number of AS completed operations: 32400 op
- C_{SS} Number of SS completed operations: 10800 op
- B_{WS} WS total activity time: 1800 sec
- B_{AS} AS total activity time: 720 sec
- B_{SS} SS total activity time: 900 sec
- Z Mean think time 5 sec

Using Operational Analysis equations:

1. Compute the visits V_i to the three servers during a complete job execution, their global service requests D_i and determine the bottleneck resource of the IT infrastructure
2. Compute response time when $N = 50$ users are connected, as well as the maximum throughput when the number of users tends to infinity (asymptotic value)
3. Let's substitute the bottleneck resource determined at point 1 with another, two times (2x) more powerful. Does the bottleneck migrate to another resource? If so, which one? Compute the new value of the asymptotic throughput.

Exercise 22

Let's consider an intranet that can be accessed by a large number of users. The execution of a single request pass through an application server (AS), which has a service time $S = 300ms$, then through a database server (DS), which has a service time $S = 250ms$, and then back through the application server. A request must pass through the system firewall before entering the intranet and before exiting from it. The firewall service time per visit is $S = 10ms$.

1. Compute the maximum throughput of the system.
2. Is it possible to have a Response Time $R < 9s$? At which conditions?

Exercise 23

A session of a graphical multi-user workstation, using a disk with an average service time $S_{disk} = 25ms$, yields the following measurements:

- average think-time $z = 10s$
- average CPU service demand, $D_{cpu} = 4s$
- average disk service demand, $D_{disk} = 5s$
- fraction of the busy time in which the CPU performs floating point operations 75%

Evaluate, using asymptotic bounds, which of the following modifications is more advantageous:

1. adding a FPU, which is 10 times as fast as the CPU, to offload floating point operations
2. replacing the disk with a new one with $S'_{disk} = 15ms$

Exercise 24

Consider a web application deployment structured as follows:

- one web server (WS) that renders dynamic pages,
- one application server (AS) that constructs such dynamic pages,
- one database server (DB) that holds the data needed by the application server.

The web server is the most demanded center, it has indeed $D_{WS} = 5ms$. The application server and the database server have a service demand of, respectively, $D_{AS} = 4ms$ and $D_{DB} = 3ms$.

The we application is tested in a closed deployment with a terminal workload of 20 customers with a think time of 7 milliseconds.

1. What is the primary bottleneck center?
2. What is the secondary bottleneck center?
3. What is the minimum response time that can be achieved?
4. What is the effect of replacing the application server with a new one that is twice as fast?

Exercise 25

- Consider a batch system with one CPU and two disks, for which the following measurements have been obtained:
- Monitoring period: 500 seconds
- CPU busy time: 144 seconds, • CPU completed operations: 200
- Slow disk busy time: 86 seconds, • Slow disk completed operations: 100
- Fast disk busy time: 226 seconds, • Fast disk completed operations: 500
- Completed transactions: 200
- Number of concurrent jobs: 2

Using only the information available, shift files between disks (= change the relative visits) in order to balance load between the two disks and increase the expected maximum throughput.

Using asymptotic bounds, which is the maximum throughput for the new, improved system after you have moved the files? *Visits are not required to be an integer number.*

18

$S_{WAS} = 0{,}2$ SECOND $V_{WAS} = 2$ $S_{SS1} = 0{,}02$ SECOND $V_{SS1} = 15$

$S_{SS2} = 0{,}05$ SECOND $V_{SS2} = 8$ X_{MAX}? R_{MIN}?

$$D = \sum_{i=1}^{3} D_i = \sum_{i=1}^{3} V_i S_i = V_{WAS} \cdot S_{WAS} + V_{SS1} \cdot S_{SS1} + V_{SS2} \cdot S_{SS2} =$$

$= (0{,}4 + 0{,}3 + 0{,}4)$ SECOND $= 1{,}1$ SECOND $D_{MAX} = 0{,}4$ SECOND

$X \leq 1/D_{MAX} \Rightarrow X_{MAX} = 1/D_{MAX} = 2{,}5/\text{SECOND}$

$R \geq D \Rightarrow R = D = 1{,}1$ SECOND

19

$S_{APP} = 0{,}02$ SECOND $V_{APP} = 25$ $S_{DB} = 0{,}015$ SECOND $V_{DB} = 20$

$S_{SS} = 0{,}05$ SECOND $V_{SS} = 10$ a) $N = 5$ X_{MAX}? b) $N = 10$ R_{MIN}?

$D_{APP} = V_{APP} \cdot S_{APP} = 0{,}5$ SECOND $D_{DB} = V_{DB} \cdot S_{DB} = 0{,}3$ SECOND

$D_{SS} = V_{SS} \cdot S_{SS} = 0{,}5$ SECOND $D = 1{,}3$ SECOND $D_{MAX} = 0{,}5$ SECOND

a) $X \leq \min\left(\frac{N}{D}, \frac{1}{D_{MAX}}\right) = \min(3.846, 2) \Rightarrow X_{MAX} = 2/\text{SECOND}$

b) $R \geq \max(D, N \cdot D_{MAX}) = \max(1.3, 5) \Rightarrow R_{MIN} = 5$ SECOND

20 ! INTERACTIVE = TERMINAL

$Z = 200$ SECOND $D_{APP} = 0{,}1$ SECOND $D_{DB} = 0{,}2$ SECOND $D_{WEB} = 0{,}08$ SECOND

a) N^*? b) $N = 50$ R_{MIN}, R_{MAX}? c) $N = 2000$ X_{MIN}, X_{MAX}?

$D = 0{,}38$ SECOND $D_{MAX} = 0{,}2$ SECOND

a) $N^* = \frac{D+Z}{D_{MAX}} = 1001{,}9$! SINCE N^* IS A THRESHOLD, WE DON'T WORRY ABOUT IT BEING AN INTEGER OR NOT

b) $\max(D, ND_{MAX} - Z) = \max(0{,}38, -181) \leq R \leq ND = 19 \Rightarrow$

$R_{MIN} = 0{,}38$ SECOND $R_{MAX} = 19$ SECOND

c) $\frac{N}{ND+Z} \leq X \leq \min\left(\frac{N}{D+Z}, \frac{1}{D_{MAX}}\right) \Rightarrow 2{,}08 \leq X \leq \min(9.981, 5)$

$X_{MIN} = 2{,}08$ $X_{MAX} = 5$

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N, C, C_i, B_i, Z, T a) V_i? D_i? BOTTLENECK = SERVER WITH D_{MAX}?

b) R? X_{MAX} FOR $N \to +\infty$? c) $D'_{B.N.} = \frac{D_{B.N.}}{2}$ BOTTLENECK? X_{MAX} FOR $N \to +\infty$

a) $D_i = \frac{B_i}{C} \Rightarrow D_{WS} = 0{,}333$ SECOND $D_{AS} = 0{,}133$ SECOND $D_{SS} = 0{,}167$ SECOND

$V_i = \frac{C_i}{C} \Rightarrow C_{WS} = 10$ $C_{AS} = 6$ $C_{SS} = 2$

$D = 0{,}633$ SECOND BOTTLENECK = WEBSERVER

b) $R = \frac{N}{X} - Z = \frac{N}{C/T} - Z = 28{,}33$ SECOND $X_{MAX} = \min\left(\infty, \frac{1}{D_{MAX}}\right) = 3$/SECOND

c) $D'_{WS} = 0{,}167$ SECOND, SAME AS SS $\Rightarrow$ TWO POSSIBLE BOTTLENECKS: WEBSERVER AND DISK $X_{MAX} = 6$/SECOND

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$S_{AS} = 0{,}3$ SECOND $S_{DS} = 0{,}25$ SECOND $S_{FW} = 0{,}01$ SECOND $V_{AS} = 2$ $V_{DS} = 1$

$V_{FW} = 2$ a) X_{MAX}? b) $R < 9$ SECOND?

$D_{AS} = S_{AS} \cdot V_{AS} = 0{,}6$ SECOND $D_{DS} = 0{,}25$ SECOND $D_{FW} = 0{,}02$ MS

a) $X_{MAX} = \frac{1}{D_{MAX}} = \frac{1}{D_{AS}} = 1{,}67$/SECOND

b) $R = \frac{N}{X_{MAX}} - Z < 9$ SECOND $\Rightarrow$ $N < 60 + Z$

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$S_{DISK} = 0{,}025$ SECOND $Z = 10$ SECOND $D_{CPU} = 4$ SECOND $D_{DISK} = 5$ SECOND

$B_{CPU} = 75\%$ ① $FPU = \frac{CPU}{10}$ ② $S'_{DISK} = 0{,}015$ SECOND ① OR ②?

$D_{FPU} = (1 - B_{CPU}) \cdot D_{CPU} + \frac{B_{CPU} \cdot D_{CPU}}{10} = 1{,}3$ SECOND $D'_{DISK} = \frac{S'_{DISK}}{S_{DISK}} D_{DISK} = 3$ SECOND

① $D_{MAX} = D_{DISK}$ 5 SECOND ② $D_{MAX} = D_{CPU} = 4$ SECOND

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$D_{WS} = 0{,}005$ SECOND $D_{AS} = 0{,}004$ SECOND $D_{DB} = 0{,}003$ SECOND $N = 20$ $Z = 0{,}007$ SECOND

a) PRIMARY BOTTLENECK? b) SECONDARY BOTTLENECK? c) R_{MIN}? d) $D_{AS} = \frac{D_{AS}}{2}$ => ?

a) $D_{MAX} = D_{WS} = 0{,}005$ SECOND b) $D_{CMAX-1} = D_{AS} = 0{,}004$ SECOND

c) $R_{MIN} = \max(D, N \cdot D_{MAX} - Z)$ $D = 0{,}012$ SECOND $R_{MIN} = 0{,}1$ SECOND

d) $D' = 0{,}01$ SECOND R_{MIN} REMAINS THE SAME

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$T = 500$ SECOND $B_{CPU} = 144$ SECOND $C_{CPU} = 200$ $B_{SD} = 86$ SECOND

$C_{SD} = 100$ $B_{FD} = 226$ SECOND $C_{FD} = 500$ $C = 200$ $N = 2$

a) $V_{SD} = \frac{C_{SD}}{C} = 0{,}5$ $V_{FD} = \frac{C_{FD}}{C} = 2{,}5$

$S_{SD} = \frac{B_{SD}}{C_{SD}} = 0{,}86$ SECOND $S_{FD} = \frac{B_{FD}}{C_{FD}} = 0{,}452$ SECOND

$$\begin{cases} C'_{SD} \cdot S_{SD} = C'_{FD} \cdot S_{FD} \\ C'_{SD} + C'_{FD} = C_{SD} + C_{FD} \end{cases} \Rightarrow C'_{SD} = 206{,}707 \quad C'_{FD} = 393{,}293$$

$\rightarrow V'_{SD} = \frac{C'_{SD}}{C} = 1{,}0335$ $V'_{FD} = \frac{C'_{FD}}{C} = 1{,}9665$

CHECK

$D'_{SD} = V'_{SD} \cdot S_{SD} = 0{,}889$ SECOND $D'_{FD} = V'_{FD} \cdot S_{FD} = 0{,}889$ SECOND ✓

b) $X_{MAX} = \min\left(\frac{N}{D}, \frac{1}{D_{MAX}}\right)$ $D_{CPU} = \frac{B_{CPU}}{C} = 0{,}72$ SECOND $\Rightarrow D_{MAX} = D_{DISK}$

$D = D'_{SD} + D'_{FD} + D_{CPU} = 2{,}496$ SECOND

$X_{MAX} = \min\left(\frac{2}{2{,}496}, \frac{1}{0{,}889}\right) = 0{,}8013$ /SECOND