b) $\{\exists x \forall y Rxy, \forall xy(Rxy \to \neg Ryx)\}$

HERBRAND MODEL

NOT NECESSARY TO SPECIFY

STEP	FORMULA	RULE
1	$\{\exists x \forall y Rxy\}$	ASSUMPTION
2	$\{\forall xy(Rxy \to \neg Ryx)\}$	ASSUMPTION

USUALLY, IT IS BETTER TO START WITH δ- FORMULAS

STEP	FORMULA	RULE
3	$\{\forall y Ray\}$	1, δ-EXPANSION
4	$\{Raa\}$	3, γ-EXPANSION $[a/y]$
5	$\{\forall y(Ray \to \neg Rya)\}$	2, γ-EXPANSION $[a/x]$
6	$\{Raa \to \neg Raa\}$	5, γ-EXPANSION
7	$\{\neg Raa, \neg Raa\}$	6, β-EXPANSION
8	$\{\neg Raa\}$	6, β-EXPANSION
9	$\emptyset$	4,7 RESOLUTION

UNIFICATION

- $\{\exists x \forall y Rxy\} \mapsto \{\forall y Ray\} \mapsto \{Ray\}$
- $\{\forall xy (Rxy \to \neg Ryx)\} \mapsto \{\forall xy (\neg Rxy, \neg Ryx)\} \mapsto \{\neg Rxy, \neg Ryx\}$

$C_1 = \{Raz\} = E_1 \qquad C_2 = \{\neg Rxy, \neg Ryx\} = E_2$

$F_1 = E_1 \cup \overline{E_2} = \{Raz, Rxy, Ryz\} \qquad S_1 = [\frac{a}{x}]$

$F_2 = F_1 S_1 = \{Raz, Ray, Ryz\} \qquad S_2 = [\frac{y}{z}]$

$F_3 = F_2 S_2 = \{Ray, Ryy\} \qquad S_3 = [\frac{a}{y}]$

$F_4 = \{Raa\} \qquad S = S_1 S_2 S_3 = [\frac{a}{x}, \frac{a}{y}, \frac{a}{z}]$

$R(C_1 \setminus E_1 \cup C_2 \setminus E_2) = (\emptyset \cup \emptyset)[\frac{a}{x}, \frac{a}{y}, \frac{a}{z}] = \emptyset$

USE RESOLUTION TO PROVE THAT $\forall xy\, R(x,y) \vdash \exists xy\, R(x, f(y))$

WE PROVE THAT $F = \{\forall xy.\, Rxy, \neg\exists xy.\, R(x, f(y))\}$ HAS A CLOSED EXPANSION

- CONSTRUCT SKOLEM NORMAL FORM
  - $\forall xy.\, Rxy \mapsto \{Rxy\}$
  - $\neg\exists xy.\, R(x, f(y)) \mapsto \forall xy.\, \neg R(x, f(y)) \mapsto \{\neg R(x, f(y))\}$
- PROCEED AS ALWAYS

$F = \{\{Rxy\}, \{\neg R(x, f(y))\}\}$

$C_1 = \{R(x,y)\}$ $\quad\quad E_1 = \{R(x,y)\}$

$C_2 = \{\neg R(x, f(y))\}$ $\quad\quad E_2 = \{\neg R(x, f(y))\}$

$F_1 = E_1 \cup \overline{E_2} = \{R(x,y), R(x, f(y))\}$

IN THIS CASE, WE CAN'T DO MORE $\quad\quad$ IL MIO PANE

WE CAN PROCEED BY SUBSTITUTING THE VARIABLES IN C_1

$C_1 = \{R(u,v)\}$ $\quad\quad E_1 = \{R(u,v)\}$

$C_2 = \{\neg R(x, f(y))\}$ $\quad\quad E_2 = \{\neg R(x, f(y))\}$

$F_1 = E_1 \cup \overline{E_2} = \{R(u,v), R(x, f(y))\}$ $\quad\quad S_1 = [\frac{x}{u}]$

$F_2 = \{R(x,v), R(x, f(y))\}$ $\quad\quad S_2 = [\frac{f(y)}{v}]$

$F_3 = \{R(x, f(y))\}$ $\quad\quad S = S_1 S_2 = [\frac{x}{u}, \frac{f(y)}{v}]$

$R(C_1 \setminus E_1 \cup C_2 \setminus E_2) = (\emptyset \cup \emptyset) S = \emptyset$