USING SEMANTIC TREE

① CHECK FOR SATISFIABILITY. WE JUST NEED ONE NODE TO BE SATISFIABLE

$(X \to (Y \to Z)) \to ((X \to Y) \to (X \to Z))$

β-FORMULA

$\neg(X \to (Y \to Z))$ $\quad (X \to Y) \to (X \to Z)$

$\neg(X \to Y)$ $\quad X \to Z$

β-FORMULA

$\neg X$ $\quad Z$

THE FORMULA IS SATISFIABLE

② CHECK FOR VALIDITY. THE EASIEST WAY IS TO CHECK IF $\neg\varphi$ IS SATISFIABLE OR UNSATISFIABLE

$\neg((X \to (Y \to Z)) \to ((X \to Y) \to (X \to Z)))$

| α-EXPANSION

$X \to (Y \to Z), \neg((X \to Y) \to (X \to Z))$

| α-EXPANSION

$X \to (Y \to Z), X \to Y, \neg(X \to Z)$

| α-EXPANSION

$X \to (Y \to Z), X \to Y, X, \neg Z$

β-EXPANSION

UNSATISFIABLE

$X \to (Y \to Z), \neg X, X, \neg Z$ $\quad X \to (Y \to Z), Y, X, \neg Z$

β-EXPANSION

UNSATISFIABLE

$\neg X, Y, X, \neg Z$ $\quad (Y \to Z), Y, X, \neg Z$

β-EXPANSION

UNSATISFIABLE

UNSATISFIABLE

$\neg Z, Y, X, \neg Z$ $\quad Z, Y, X, \neg Z$

$\Rightarrow \neg\varphi$ IS UNSATISFIABLE $\Rightarrow \neg(\neg\varphi) = \varphi$ IS VALID

CONTRADICTION

0	0	1	0	1	1	1	1
0	0	1	0	1	1	0	1
0	0	1	0	1	1	1	1
0	0	1	0	1	1	0	1
0	0	1	0	1	1	1	0
0	1	0	1	1	0	1	0
0	0	1	0	1	1	0	0
0	1	0	1	1	0	0	0
⊥	$Z \vdash V(A \leftarrow X)$	$Z \leftarrow (A \leftarrow X)$	ZL	$A \leftarrow X$	Z	A	X

$(Z \vdash V(A \leftarrow X)) \leftrightarrow (Z \leftarrow (A \leftarrow X))$ (9

c) $(X \wedge Y \to Z) \to ((X \to Z) \wedge (Y \to Z))$

X	Y	Z	$X \wedge Y$	$X \to Z$	$Y \to Z$	$X \wedge Y \to Z$	$(X \to Z) \wedge (Y \to Z)$	φ
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	1	0	1	0	0
0	1	1	0	1	1	1	1	1
1	0	0	0	0	1	1	0	0
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

NEITHER OF THEM