

NON VALIDITY: FIND AT LEAST A STRUCTURE THAT DOES NOT SATISFY THE FORMULA

$$\neg((\forall x.R_x \rightarrow \forall x.S_x) \rightarrow \forall x(\forall x(R_x \rightarrow S_x)))$$

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$$\forall x R_x \rightarrow \forall x S_x, \neg(\forall x(\forall x(R_x \rightarrow S_x)))$$

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$$\neg \forall x R_x, \neg(\forall x(\forall x(R_x \rightarrow S_x)))$$

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$$\forall x S_x, \neg(\forall x(\forall x(R_x \rightarrow S_x)))$$

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$$\neg R_a, \neg(R_b \rightarrow S_b)$$

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$$\neg(R_a \rightarrow S_a), S_a$$

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$$\neg R_a, R_b, \neg S_b$$

SATISFIABLE

$$R_a, \neg S_a, S_a^x$$

$$M = \{a, b\} \quad [I[R]] = \{b\}, [I[S]] = \emptyset$$

$\rightarrow \neg \varphi$ IS SATISFIABLE $\Rightarrow \varphi$ IS NOT VALID

3.12

PROVE THAT THE FOLLOWING FORMULA IS SATISFIABLE BUT NOT VALID:

$$\forall x \exists y (R_x \rightarrow S_y) \rightarrow \forall x (R_x \rightarrow S_x)$$

SATISFIABILITY

$$\forall x \exists y (R_x \rightarrow S_y) \rightarrow (\neg S_x \wedge \forall x (R_x \rightarrow S_x))$$

$$\neg (\forall x \exists y (R_x \rightarrow S_y)) \quad \neg (\neg S_x \wedge \forall x (R_x \rightarrow S_x))$$

$$\neg (\neg S_x \wedge \forall x (R_x \rightarrow S_x))$$

SKOLEMIZATION

$$\neg (\exists y (R_a \rightarrow S_y))$$

$$\neg (\forall x R_x) \quad \forall x S_x$$

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$$M = \{a\}$$

$$\neg (R_a \rightarrow S_a)$$

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$$\neg R_a$$

$$S_a$$

$$[R] = \emptyset$$

$$R_a, \neg S_a$$

VALIDITY

$$\neg(\forall x \exists y (R_x \rightarrow S_y) \rightarrow \forall x (R_x \rightarrow S_x))$$

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$$\forall x \exists y (R_x \rightarrow S_y), \neg(\forall x (R_x \rightarrow S_x))$$

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$$\forall x \exists y (R_x \rightarrow S_y), \forall x R_x, \neg \forall x S_x$$

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$$\forall x \exists y (R_x \rightarrow S_y), R_a, \neg S_a$$

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$$\exists y (R_b \rightarrow S_y), R_a, \neg S_a$$

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$$\neg R_b, R_a, \neg S_a$$

SATISFIABLE $\rightarrow \varphi$ IS NOT VALID