

16

 $\{\neg x \vee y\}$ 3, α -EXPANSION

17

 $\{\neg z\}$ 3, α -EXPANSION

18

 $\{\neg x, y\}$ 16, β -EXPANSION

19

 $\{\neg x, \neg y\}$

5,17 RESOLUTION

20

 $\{\neg x, y\}$

10,17 RESOLUTION

21

 $\{\neg x\}$

19,20 RESOLUTION

22

 $\{y\}$

18,21 RESOLUTION

23

 $\{\neg y, z\}$

14,21 RESOLUTION

24

 $\{z\}$

22,23 RESOLUTION

25

1

17,24 RESOLUTION

THE SET HAS A CLOSED EXPANSION \Rightarrow THE STATEMENT IS CORRECT

b) $Z \rightarrow (X \leftrightarrow Y), \neg Z \rightarrow (\neg X \wedge \neg Y) \models X \vee Y \rightarrow X \wedge Y$

SEMANTICAL

X	Y	Z	$X \leftrightarrow Y$	F_1	$\neg Z$	$\neg X \wedge \neg Y$	$(\neg X \wedge \neg Y)$	F_2	$X \vee Y$	$X \wedge Y$	φ
0	0	0	1	1	1	1	1	1	0	0	1
0	0	1	1	1	0	1	1	1	0	0	1
0	1	0	0	1	1	1	0	0	1	0	0
0	1	1	0	0	0	1	0	0	1	1	0
1	0	0	0	1	1	0	1	0	0	1	0
1	0	1	0	0	0	0	1	0	1	1	0
1	1	0	1	1	1	0	0	0	1	1	1
1	1	1	1	1	0	0	0	0	1	1	1

VALUATIONS v_1, v_2, v_3 SATISFY BOTH THE PREMISES AND THE

CONCLUSION $\Rightarrow F \models \varphi \checkmark$

RESOLUTION

$$\{ \exists \rightarrow (x \leftrightarrow y), \neg z \rightarrow (\neg x \wedge \neg y), \neg(x \vee y \rightarrow x \wedge y) \}$$

STEP	FORMULA	RULE
1	$\exists \rightarrow (x \leftrightarrow y)$	ASSUMPTION
2	$\neg z \rightarrow (\neg x \wedge \neg y)$	ASSUMPTION
3	$\neg(\neg(x \vee y \rightarrow x \wedge y))$	ASSUMPTION
4	$\neg \neg z, x \leftrightarrow y$	1, P-EXPANSION
5	$\neg \neg z, x \leftrightarrow y$	4, BIIMPLICATION
6	$\neg \neg z, y \rightarrow x$	4, BIIMPLICATION
7	$\neg \neg x, y, \neg z$	5, P-EXPANSION
8	$x, \neg y, \neg z$	6, P-EXPANSION
9	$\neg z, \neg x \wedge \neg y$	2, P-EXPANSION
10	$\neg \neg x, \neg z$	9, D-EXPANSION
11	$\neg \neg y, \neg z$	9, D-EXPANSION
12	$x \vee y$	3, D-EXPANSION
13	$\neg(\neg x \wedge \neg y)$	3, D-EXPANSION
14	$\neg x, \neg y$	12, P-EXPANSION
15	$\neg \neg x, \neg y$	13, P-EXPANSION