RESOLUTION

$\{\neg\neg x \vee y, y \to z, \neg((\neg y \to x) \to \neg z)\}$

STEP	FORMULA	RULE
1	$\{\neg\neg x \vee y\}$	ASSUMPTION
2	$\{y \to z\}$	ASSUMPTION
3	$\{\neg((\neg y \to x) \to \neg z)\}$	ASSUMPTION
4	$\{\neg\neg x, y\}$	1, β- EXPANSION
5	$\{\neg y, z\}$	2, β- EXPANSION
6	$\{(\neg y \to x)\}$	3, α- EXPANSION
7	$\{\neg z\}$	3, α- EXPANSION
8	$\{x, y\}$	6, β-EXPANSION

WE CANNOT OPERATE ON $\{z\}$ BECAUSE $\{\neg z\}$ NEVER OCCUR

9	$\{y\}$	4,8 RESOLUTION

WE CANNOT OPERATE ON $\{y\}$ BECAUSE $\{\neg y\}$ NEVER OCCUR

THE GIVEN STATEMENT DOES NOT HAVE A CLOSED EXPANSION

$\Rightarrow F \not\vdash \varphi$

e) $(X \vee Y \vee Z) \wedge (Y \to X) \wedge (Z \to Y) \models X$

F_1 F_2 F_3 F'

SEMANTICAL

X	Y	Z	X∨Y	F_1	F_2	F_3	F'	F	φ
0	0	0	0	0	1	1	0	0	0
0	0	1	0	1	1	0	1	0	0
0	1	0	1	1	0	1	0	0	0
0	1	1	1	1	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1
1	0	1	1	1	1	1	0	0	1
1	1	0	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

VALUATIONS v_5, v_7, v_8 SATISFY BOTH PREMISES AND CONCLUSION $\Rightarrow F \models \varphi$

RESOLUTION

CLOSED EXPANSION

PROVE THAT $\{(X \vee Y \vee Z) \wedge (Y \Rightarrow X) \wedge (Z \Rightarrow Y), \neg X\}$ HAS $\wedge$

STEP	FORMULA	RULE
1	$\{[(X \vee Y \vee Z) \wedge (Y \Rightarrow X)] \wedge (Z \Rightarrow Y)\}$	ASSUMPTION
2	$\{\neg X\}$	ASSUMPTION
3	$\{X \vee Y \vee Z\}$	1, α-EXPANSION
4	$\{Y \Rightarrow X\}$	1, α-EXPANSION
5	$\{Z \Rightarrow Y\}$	1, α-EXPANSION
6	$\{Y, \neg Z\}$	5, α-EXPANSION
7	$\{X, Y, Z\}$	3, β-EXPANSION
8	$\{X, \neg Y\}$	4, β-EXPANSION
9	$\{\neg Y\}$	2,8 RESOLUTION
10	$\{Y, Z\}$	2,7 RESOLUTION
11	$\{Y\}$	6,10 RESOLUTION
12	$\square$	9,11 RESOLUTION

THE SET HAS A CLOSED EXPANSION $\Rightarrow$ THE STATEMENT IS CORRECT

$F \vdash \varphi$