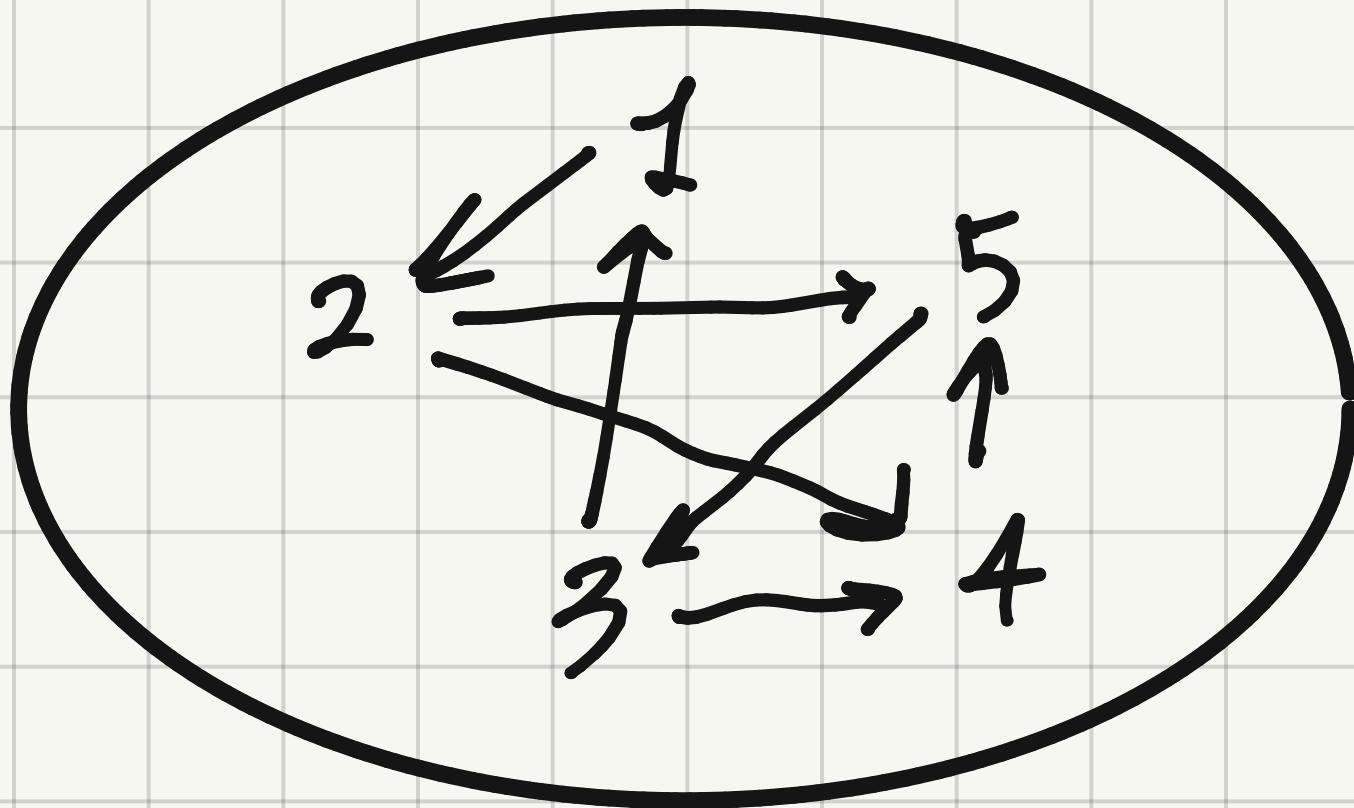


b)

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 |



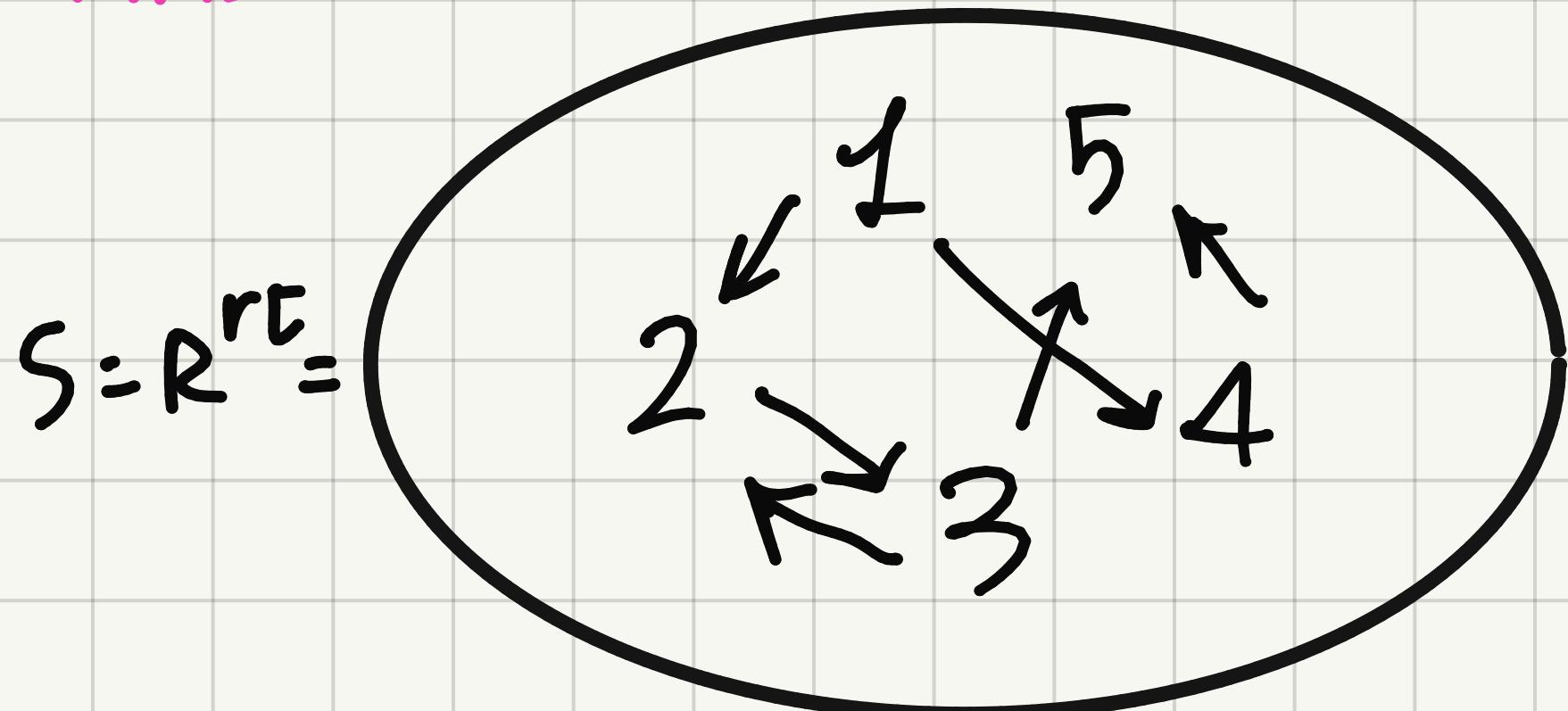
| | | | | | | |
|------------|---|---|---|---|---|--------------|
| $R^{rc} =$ | 1 | 1 | 1 | 1 | 1 | REFLEXIVITY |
| | 1 | 1 | 1 | 1 | 1 | TRANSITIVITY |
| | 1 | 1 | 1 | 1 | 1 | NO PATHS |
| | 0 | 0 | 0 | 1 | 0 | |

CLEARLY, IT IS NOT ANTI-SYMMETRIC $\Rightarrow R$ IS NOT CONTAINED IN ANY ORDER RELATION

$$A = \{1, \dots, 5\}$$

$$R: A \rightarrow A \quad M(R) = \begin{vmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} \quad \text{ORDER INDUCED BY } R?$$

① FIND TRANSITIVE CLOSURE



$$M(S) = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

② FIND EQUIVALENCE RELATION

$$E = S \cap S^{op} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cap \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$[2] = [3]$$

③ QUOTIENT SET $A/E = \{[1], [2], [4], [5]\}$

④ THE PROJECTION $P: A \rightarrow A/E$ IS

$$P = \begin{vmatrix} [1] & [2] = [3] & [4] & [5] \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

⑤ CALCULATE THE INDUCED ORDER $\bar{s} = P \circ P^T$

$$P^T = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\bar{s} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}.$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$