EXTRA: TRANSLATE THE FOLLOWING ASSERTION IN PROPOSITIONAL LOGIC. THEN VERIFY IT USING BOTH SEMANTIC AND RESOLUTION CALCULUS

SE TEMPERATURA E PRESSIONE RESTANO COSTANTI ALLORA NON PUÒ PIOVERE. LA TEMPERATURA È RIMASTA COSTANTE. PIOVE, DUNQUE LA PRESSIONE NON È RIMASTA COSTANTE

X = PIOVE Y = TEMPERATURA COSTANTE Z = PRESSIONE COSTANTE

$Y \wedge Z \Rightarrow \neg X,\ Y,\ X \models \neg Z$

SEMANTIC

X	Y	Z	$X \wedge Z$	$\neg X$	F_1	F_2	F_3	φ
0	0	0	0	1	1	0	0	1
0	0	1	0	1	1	0	0	0
0	1	0	0	1	1	1	0	1
0	1	1	0	1	1	1	0	0
1	0	0	0	0	1	0	1	1
1	0	1	1	0	0	0	1	0
1	1	0	0	0	1	1	1	1
1	1	1	1	0	0	1	1	0

VALUATION v_7 SATISFIES BOTH PREMISES AND CONCLUSION

$\Rightarrow F \models \varphi$ ✓

RESOLUTION

$\{Y \wedge Z \Rightarrow \neg X, Y, X, \neg(\neg Z)\}$

STEP	FORMULA	RULE
1	$\{Y \wedge Z \rightarrow \neg X\}$	ASSUMPTION
2	$\{Y\}$	ASSUMPTION
3	$\{X\}$	ASSUMPTION
4	$\{\neg(\neg Z)\}$	ASSUMPTION
5	$\{\neg(Y \wedge Z), \neg X\}$	1, β-EXPANSION
6	$\{\neg X, \neg Y, \neg Z\}$	1, β-EXPANSION
7	$\{Z\}$	4, DOUBLE NEGATION
8	$\{\neg Y, \neg Z\}$	6,3 RESOLUTION
9	$\{\neg Z\}$	8,2 RESOLUTION
10	$\perp$	9,7 RESOLUTION

$F \cup \{\neg\varphi\}$ HAS A CLOSED EXPANSION $\Rightarrow$ THE STATEMENT IS CORRECT

$F \vdash \varphi$

FORMULA:

DNF $\varphi = X \wedge Y \wedge \neg Z$

RELATIONS AND FUNCTIONS

CONSIDER THE SETS $A=\{1,2,3\}$; $B=\{1,2,3,4\}$; $C=\{1,2\}$

AND LET $R: A \to B$ AND $S: B \to C$ BE THE RELATIONS DEFINED BY THE MATRICES $R=\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$ AND $S=\begin{vmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{vmatrix}$.

COMPUTE THE RELATION RS

$R: A \to B$ $\quad R=\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$

$S: B \to C$ $\quad S=\begin{vmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{vmatrix}$ $\qquad R \cdot S?$

$$R \cdot S = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{vmatrix}$$

$RS: A \to C$ ✓