EXTRA: LET $A = \{1,2,3,4,5,6\}$ AND $R \subseteq A \times A$

$R = \{(1,1),(1,2),(1,3),(2,4),(3,4),(4,5),(5,6)\}$

FIND TRANSITIVE CLOSURE

$$M_R = \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$T = \bigcup_{n>0}^{k} M_R^{n} \;\Big|\; M_R^{k} = M_R^{k-1}$$

$$M_R^2 = \begin{vmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$M_R^3 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$M_R^4 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$M_R^5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = M_R^4$$

$$T = \bigcup_{n=1}^{4} M_R^n = M_R \cup M_R^2 \cup M_R^3 \cup M_R^4 =$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

EXTRA: LET $M_R = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}$

a) WHICH PROPERTIES DOES $M_R$ HAVE?

b) FIND $M_s$

c) FIND $M_{Re}$

a) • REFLEXIVITY $\Leftrightarrow I \subseteq R \Leftrightarrow M(I) \leq M(R)$

$(M_R)_{2,2} \neq 0 \Rightarrow$ IT IS NOT REFLEXIVE

• SIMMETRY $\Leftrightarrow R^{op} \subseteq R^{op} \Leftrightarrow M_R^T \leq M_R$

$$M_{R^T} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \Rightarrow \text{IT IS NOT SYMMETRIC}$$

• TRANSITIVITY $\Leftrightarrow R^2 \subseteq R \Leftrightarrow M_{R^2} \leq M_R$

$$M_{R^2} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

IT IS TRANSITIVE

b) $M_s = M_R + I + M_{R^{op}}$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$