

# TUTORATO G61 22-23

## 1 - EQUAZIONI FRAZIONE E POLINOMI DI 2<sup>o</sup> GRADO

1a)  $\frac{1 + \frac{1}{x}}{\frac{1}{x} - 1} = \frac{x + \frac{1}{2}}{x - \frac{1}{2}}$

C.E.:  $\begin{cases} x \neq 0 \\ x \neq 1 \\ x \neq \frac{1}{2} \end{cases}$

$$(1 + \frac{1}{x})(x - \frac{1}{2}) = (x + \frac{1}{2})(\frac{1}{x} - 1)$$

$$x - \frac{1}{2} + 1 - \frac{1}{2x} = 1 - x + \frac{1}{2x} - \frac{1}{2}$$

$$2x - \frac{1}{x} = 0$$

$$2x^2 - 1 = 0$$

$$x = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$$

1c)  $\frac{2 - \frac{5}{x+3}}{2 + \frac{4}{x-3}} + \frac{x+2}{x-1} - \frac{15}{x+3} = 0$

C.E.:  $\begin{cases} x \neq \pm 3 \\ x \neq 1 \end{cases}$

$$\frac{\frac{2x+4}{x+3}}{\frac{2x-2}{x-3}} + \frac{x+2}{x-1} - \frac{15}{x+3} = 0$$

$$\frac{(2x+1)(x-3)}{2(x+3)(x-1)} + \frac{x+2}{x-1} - \frac{15}{x+3} = 0$$

$$\frac{(2x+1)(x-3) + 2(x+2)(x+3) - 30(x-1)}{2(x+3)(x-1)} = 0$$

$$2x^2 - 5x - 3 + 2x^2 + 10x + 12 - 30x + 30 = 0$$

$$4x^2 - 25x + 29 = 0$$

$$x = \frac{25 \pm \sqrt{625 - 464}}{8} = \frac{25 \pm \sqrt{13}}{8} = \frac{25}{8} \pm \frac{\sqrt{13}}{8}$$

2a)  $P(x) = x^2 + ax + b$   $a, b \in \mathbb{R}$   $P(1) = 1, P(-1) = -1$   $x?$

$$\begin{cases} 1 + a + b = 1 \\ 1 - a + b = -1 \end{cases} \rightarrow \begin{cases} a + b = 0 \\ -a + b = -2 \end{cases} \begin{cases} b = -a \\ 2b = -2 \end{cases} \begin{cases} a > 1 \\ b = -1 \end{cases}$$

$$-1 \pm \sqrt{5}$$

$$P(x) = x^2 + x - 1 = 0 \rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

2c)  $a+b=q$ ;  $P=a \cdot b$   $\triangleright$  MAX PBR  $a, b$ ?

$$b = q - a \Rightarrow P = a(q - a)^2 \rightarrow P' = q - 2a = 0 \quad q - 2a > 0 \Rightarrow a < \frac{q}{2}$$

$$a = \frac{q}{2} \Rightarrow b = \frac{q}{2}$$

METODO SENZA DERIVATE

DAUE EQUAZIONI PARAMETRICHE,  $x^2 - qx + p = 0$

$$\Rightarrow x = \frac{q \pm \sqrt{q^2 - 4p}}{2}$$

MASSIMO PER  $q^2 - 4p \geq 0$

$$\Rightarrow P_{\text{MAX}} = \frac{q^2}{4} \Rightarrow a \cdot b = q(a - a^2) \Rightarrow a = \frac{q}{2}, b = \frac{q}{2}$$

## 2 - EQUAZIONI DI GRADO SUPERIORE AL 2°

1a)  $x^6 - 5x^5 + 6x^4 + 4x^3 - 24x^2 + 16x + 32 = 0$

$$\begin{array}{r|cccccc|c} 1 & 5 & 6 & 4 & -24 & 16 & 32 \\ \hline -1 & & -1 & 6 & -22 & 8 & 16 & -32 \\ \hline 1 & -6 & 12 & -8 & -16 & 32 & 0 \end{array}$$

$$(x+1)(x^5 - 6x^4 + 12x^3 - 8x^2 - 16x + 32) = 0$$

$$\begin{array}{r|ccccc|c} 1 & -6 & 12 & -8 & -16 & 32 \\ \hline 2 & 2 & -8 & 8 & 0 & -32 \\ \hline 1 & -4 & 4 & 0 & -16 & 0 \end{array}$$

$$(x+1)(x-2)(x^4 - 4x^3 + 4x^2 - 16) = 0$$

$$x^4 - 4x^3 + 4x^2 - 16 = x^2(x^2 - 4x + 4) - 16$$

$$= x^2(x-2)^2 - 16 = [x(x-2) - 4][x(x-2) + 4]$$

$$= (x^2 - 2x - 4)(x^2 - 2x + 4)$$

$$\rightarrow (x+1)(x-2)(x^2 - 2x - 4)(x^2 - 2x + 4) = 0$$

- $x = -1$

- $x = 2$

- $x = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$

$$1b) 2x^8 + 3x^4 - 2 = 0 \quad x^4 = t$$

$$2t^2 + 3t - 2 = 0$$
$$t = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} = \begin{cases} \frac{1}{2} \\ -2 \end{cases}$$

$$\left\{ \begin{array}{l} \frac{1}{2} = x^4 \rightarrow x = \pm \sqrt[4]{\frac{1}{2}} \\ -2 = x^4 \rightarrow x = \sqrt[4]{-2} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{1}{2} = x^4 \rightarrow x = \pm \sqrt[4]{\frac{1}{2}} \\ -2 = x^4 \rightarrow x = \sqrt[4]{-2} \text{ IMP.} \end{array} \right.$$

### 3-DISIEQUAZIONI FRAZIONE

Ia)  $\frac{x-1}{x} + \frac{x+1}{x-1} < 3 - \frac{2}{x(x-1)}$

$$\frac{x^2 - 2x + 1 + x^2 + x + 2 - 3x^2 + 3x}{x(x-1)} < 0$$

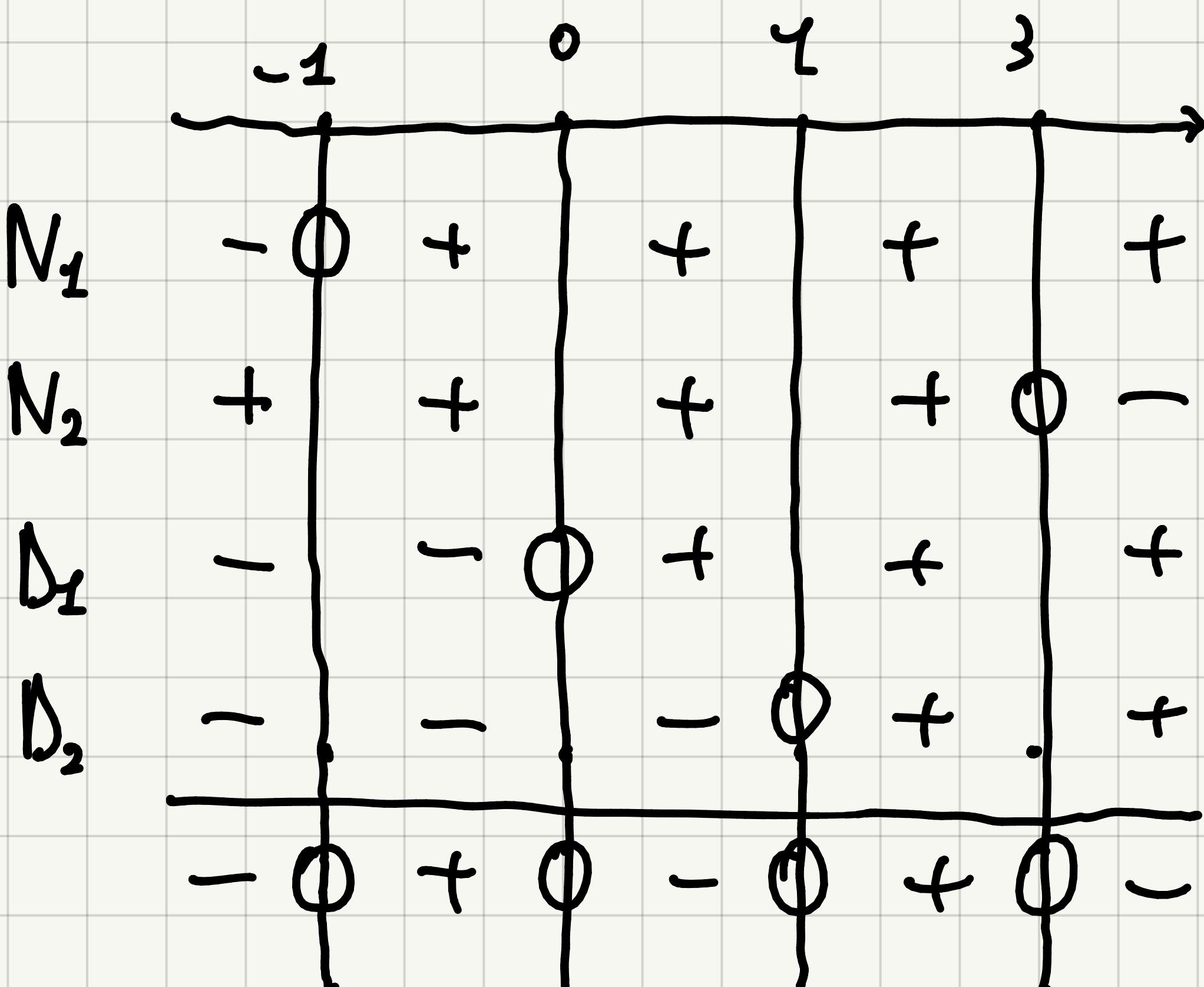
$$\frac{-x^2 + 2x + 3}{x(x-1)} < 0$$



$$\frac{(x+1)(-x+3)}{x(x-1)} < 0$$

$$N_1 \quad x > -1 \quad N_2 \quad x < 3$$

$$D_1 \quad x > 0 \quad D_2 \quad x > 1$$



$x < -1, 0 < x < 1, x > 3$

$$1b) (1+x^2-2x)(1-6x)(6x-x^2) < 0$$

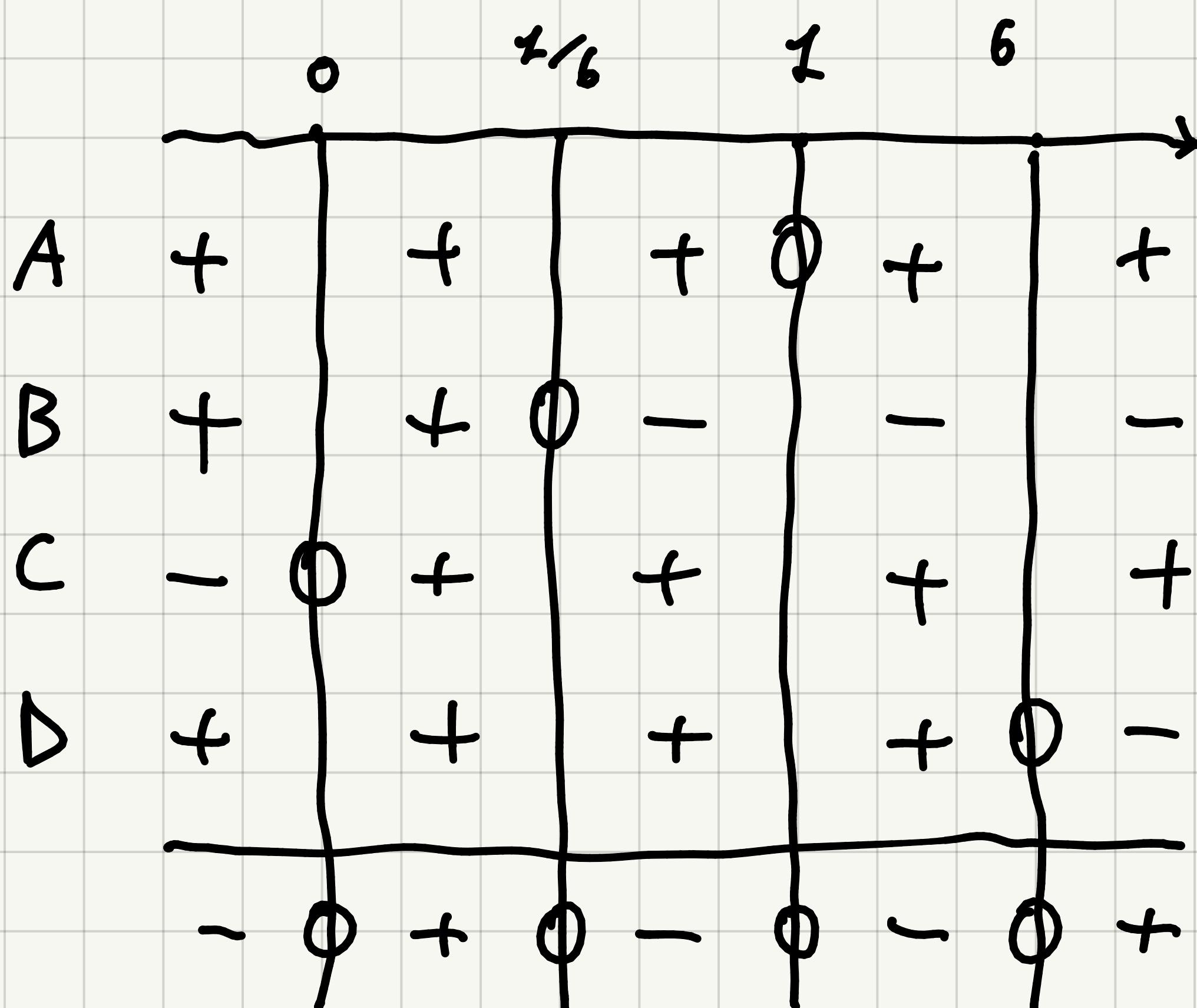
$$(x-1)^2(1-6x)(6-x) < 0$$

A •  $x \neq 1$

B •  $x < \frac{1}{6}$

C •  $x \geq 0$

D •  $x < 6$



$$x < 0, \frac{1}{6} < x < 1, 1 < x < 6$$

1c)

$$\left\{ \begin{array}{l} x(x-4) < 12 \\ x(2x-4) + 3 > 4x \end{array} \right. \quad A$$

$$\left\{ \begin{array}{l} x^2 + x > 1 \\ x(2x-4) + 3 > 4x \end{array} \right. \quad B$$

$$\left\{ \begin{array}{l} x^2 + x > 1 \\ x^2 - 5x + 3 > 0 \end{array} \right. \quad C$$

$$A \quad x = \frac{4 \pm \sqrt{16+48}}{2} = 2 \pm 4 = \begin{cases} 6 \\ -2 \end{cases}$$



$$B \quad 2x^2 - x + 3 - 4x > 0 \quad 2x^2 - 5x + 3 > 0$$

$$x = \frac{5 \pm \sqrt{25-24}}{4} = \begin{cases} 3/2 \\ 1 \end{cases}$$



$$C \quad x^2 + x + 1 > 0 \quad x = \frac{-1 \pm \sqrt{1-3}}{2}$$

$$\Delta < 0 \Rightarrow C > 0 \quad \forall x \in \mathbb{R}$$



A



$$-2 < x < 1, \quad 3/2 < x < 6$$

## 4 - EQUAZIONI E DISEQUAZIONI IRRAZIONALI

1a)  $\sqrt{x} + x = 6$  C.E. :  $x \geq 0$

$$\sqrt{x} = 6 - x \quad 6 - x \geq 0 \rightarrow x \leq 6$$

$$x = x^2 - 12x + 36 \quad x^2 - 13x + 36 = 0$$

$$x = \frac{13 \pm \sqrt{25}}{2} = \frac{13 \pm 5}{2} = \begin{cases} 9 & \text{NO} \\ 4 & \text{SI} \end{cases}$$

1b)  $\sqrt{x-1} - 2\sqrt{x+6} = -5$   $\begin{cases} x-1 \geq 0 \\ x+6 \geq 0 \end{cases}$

$$\sqrt{x-1} = 2\sqrt{x+6} - 5 \Rightarrow x \geq 1$$

$$x-1 = 4(x+6) + 25 - 20\sqrt{x+6}$$

$$x-1 - 4x - 24 - 25 = -20\sqrt{x+6}$$

$$3x + 50 = 20\sqrt{x+6}$$

$$2\sqrt{x+6} - 5 \geq 0 \Rightarrow x+6 \geq \frac{25}{4}$$

$$9x^2 + 300x + 2500 = 400x + 2400$$

$$x \geq \frac{1}{4}$$

$$9x^2 - 100x + 100 = 0$$

$$\begin{cases} x \geq 1 \\ x \leq \frac{1}{4} \end{cases}$$

$$x = \frac{100 \pm \sqrt{6400}}{18} = \begin{cases} 10/9 \\ 10 \end{cases}$$

$$\Rightarrow x \geq 1$$

$$Ic) \sqrt[3]{2+x^3} - x = 1$$

$$\sqrt[3]{2+x^3} = x + 1$$

$$2 + \cancel{x^3} = \cancel{x^3} + 3x^2 + 3x + 1$$
$$3x^2 + 3x - 1 = 0 \quad x = \frac{-3 \pm \sqrt{21}}{6}$$

$$II) \sqrt{x-1} + \frac{1}{\sqrt{x-1}} = \underline{\hspace{2cm}}$$
$$x-1 > 0 \quad \downarrow \quad x > 1$$

$$x-1 + 1 = x \quad 0=0 \quad \text{VERIFICATO } \forall x > 1$$

$$2\alpha) X-1 < \sqrt{x^3-4x+3}$$

$$S_1 \left\{ \begin{array}{l} X-1 < 0 \\ x^3-4x+3 \geq 0 \end{array} \right.$$

$$S_2 \left\{ \begin{array}{l} X-1 \geq 0 \\ (X-1)^2 < x^3-4x+3 \end{array} \right.$$

$$\begin{array}{c|ccc|c} & 1 & 0 & -4 & 3 \\ \hline 1 & & 1 & -4 & -3 \\ \hline & 1 & 1 & -3 & 0 \end{array}$$

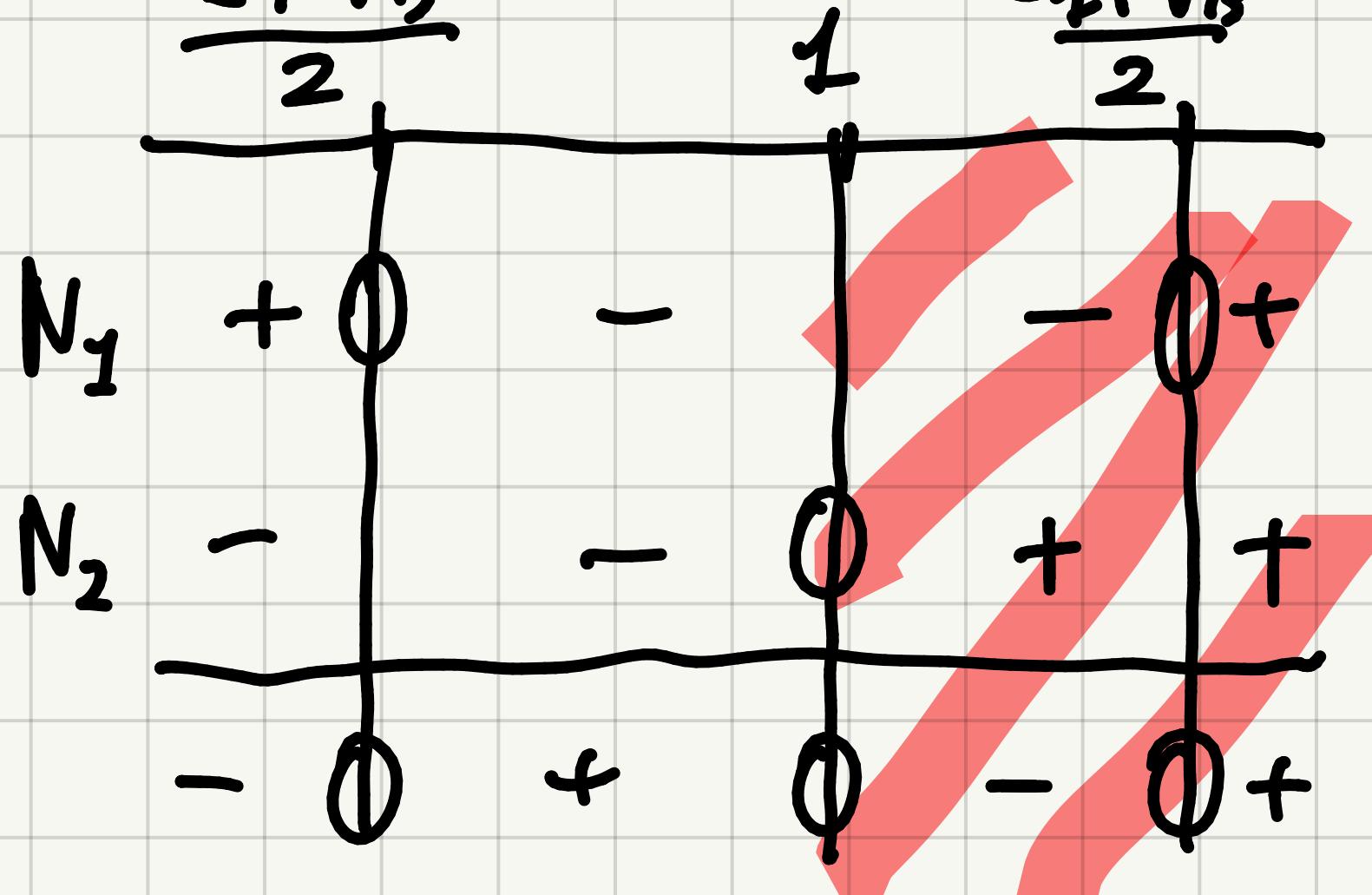
$$\rightarrow (X-1)(X^2+X-3)=0$$

$$\left\{ \begin{array}{l} X < 1 \\ (X-1)(X^2+X-3) > 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} X \geq 1 \\ (X-1)^2 < (X-1)(X^2+X-3) \end{array} \right.$$

$$X = \frac{-1 \pm \sqrt{13}}{2}$$

$$\Rightarrow X < \frac{-1-\sqrt{13}}{2}, X > \frac{-1+\sqrt{13}}{2}$$



$$\rightarrow \frac{-1-\sqrt{13}}{2} < X < 1$$

$S_2$

$$\left\{ \begin{array}{l} X \geq 1 \\ X-1 < X^2+X-3 \end{array} \right.$$

$$\left\{ \begin{array}{l} X \geq 1 \\ X^2-2 > 0 \end{array} \right. \rightarrow X > \sqrt{2}$$

$$\Rightarrow -\frac{1-\sqrt{13}}{2} < X < 1, X > \sqrt{2}$$

$$2b) \sqrt{2x+3} > \sqrt{4x^2-2x-6}$$

$$\left\{ \begin{array}{l} 2x+3 \geq 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} 4x^2-2x-6 \geq 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} 2x+3 \geq 0 \end{array} \right.$$

$$x \leq -1, x \geq \frac{3}{2}$$

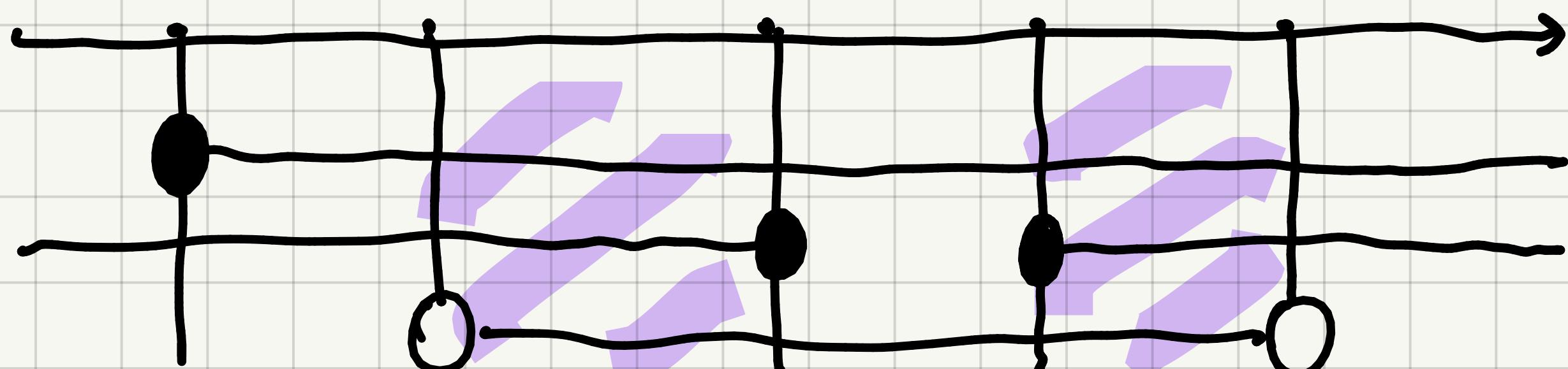
$$2x+3 > 4x^2-2x-6$$

$$4x^2-4x-9 < 0$$

$$x = \frac{4 \pm \sqrt{16+144}}{8} = \frac{4 \pm 4\sqrt{10}}{8} = \frac{4 \pm \sqrt{10}}{2}$$

$$\left\{ \begin{array}{l} x \geq -\frac{3}{2} \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} x \leq -1, x \geq \frac{3}{2} \\ \frac{1-\sqrt{10}}{2} < x < \frac{1+\sqrt{10}}{2} \end{array} \right.$$



$$\frac{1-\sqrt{10}}{2} < x \leq -1, \frac{3}{2} \leq x < \frac{1+\sqrt{10}}{2}$$

$$2c) \sqrt{x+2} < \sqrt{6-x} - \sqrt{5-x}$$

$$\left\{ \begin{array}{l} x+2 \geq 0 \\ 6-x \geq 0 \\ 5-x \leq 0 \end{array} \right.$$

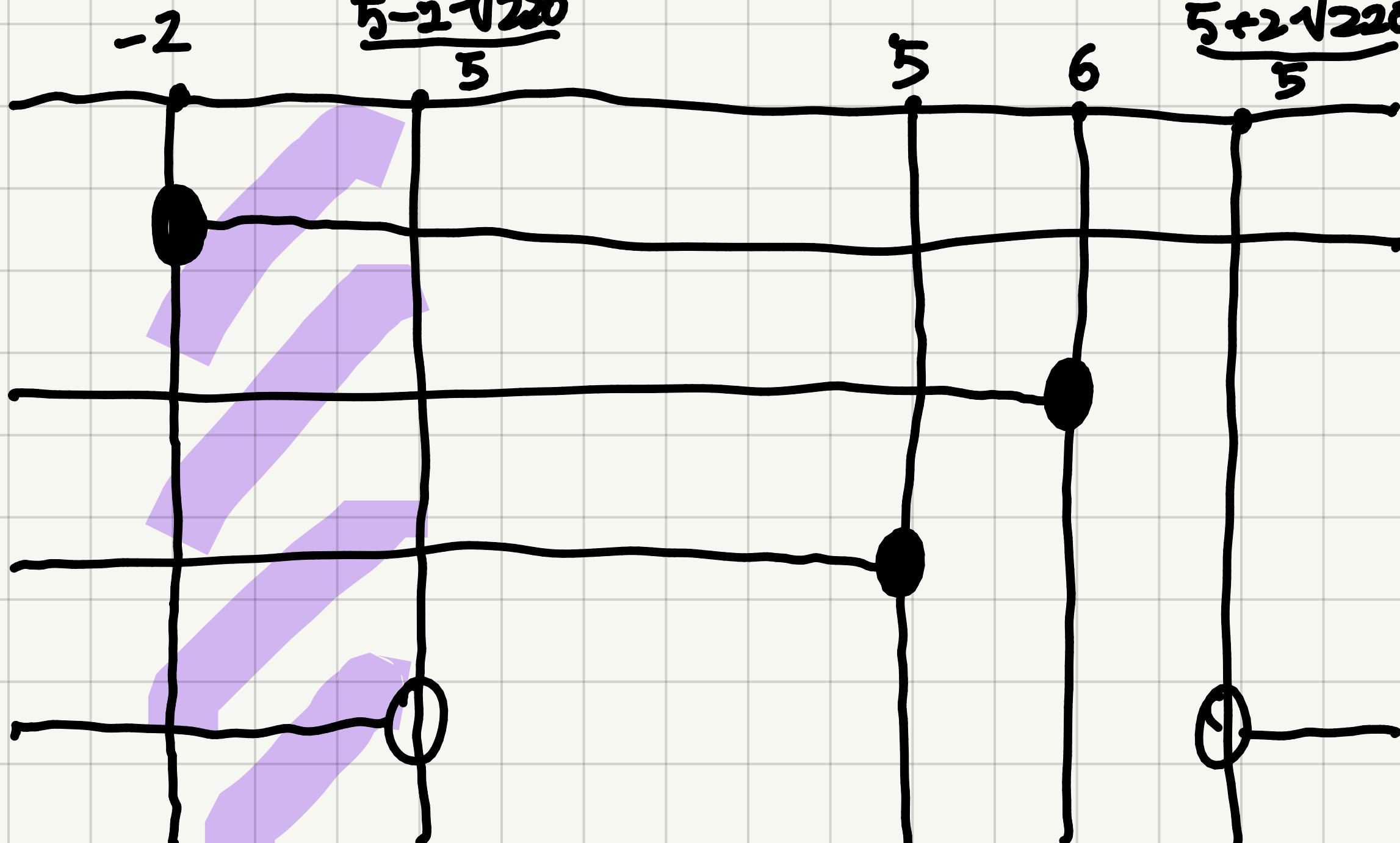
$$x+2 < 6-x + 5-x - 2\sqrt{(6-x)(5-x)}$$

$$\left\{ \begin{array}{l} x \geq -2 \\ x \leq 6 \\ x \leq 5 \end{array} \right.$$

$$*4(x^2 + 11x + 30) < 9x^2 - 54x + 81$$

$$5x^2 - 10x - 39 > 0 *$$

$$2\sqrt{(6-x)(5-x)} < 9-3x * \quad *x < \frac{5-2\sqrt{220}}{5}; x > \frac{5+2\sqrt{220}}{5}$$



$$-2 \leq x < \frac{5-2\sqrt{220}}{5}$$

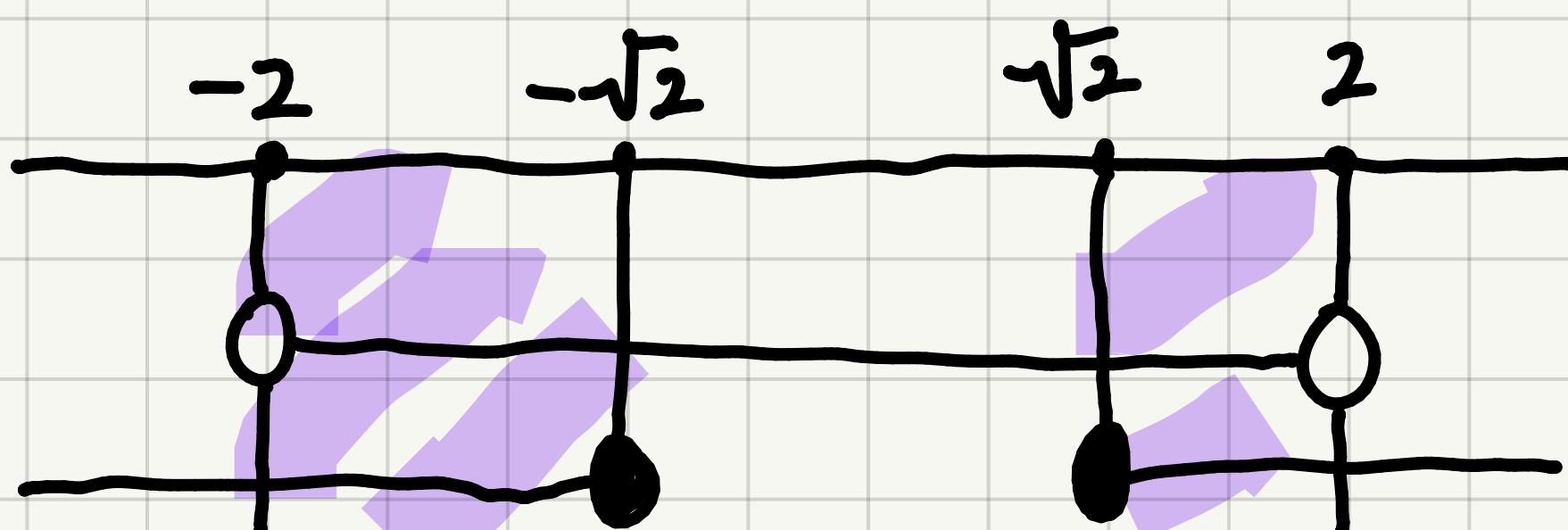
$$2d) \frac{1}{\sqrt{4-x^2}} - \frac{1}{x} \geq 0 \quad 4-x^2 > 0 \rightarrow -2 < x < 2, x \neq 0$$

$$\frac{1}{\sqrt{4-x^2}} \geq \frac{1}{x}$$

$$\sqrt{4-x^2} \leq x$$

$$4-x^2 \leq x^2$$

$$x^2 \geq 2 \rightarrow x \leq -\sqrt{2}, x \geq \sqrt{2}$$



$$-\sqrt{2} \leq x \leq \sqrt{2}$$

## 5 - EQUAZIONI E DISEQUAZIONI CON IL MODULO



1a)  $|x-1| = 1 - |x|$

$$\begin{cases} x < 0 \\ 1-x > 1+x \end{cases} !$$

$$\begin{cases} 0 \leq x \leq 1 \\ 1-x = 1-x \end{cases} \quad \cancel{x}$$

$$\begin{cases} x > 1 \\ x-1 = 1-x \end{cases} ! \Rightarrow 0 \leq x \leq 1$$

$$\frac{1}{x+1} = \frac{4}{|x|-1}$$

C.E. :  $x \neq \pm 1$

$$\begin{cases} x \geq 0 \\ \frac{1}{x+1} = \frac{4}{x-1} \end{cases}$$

$$\begin{cases} x < 0 \\ \frac{1}{x+1} = \frac{4}{-x-1} \end{cases}$$

$$\begin{cases} x \geq 0 \\ x-1 = 4x+4 \end{cases}$$

$$\begin{cases} x < 0 \\ -x-1 = 4x+4 \end{cases}$$

$$\begin{cases} x \geq 0 \\ 3x = -5 \end{cases}$$

$$\begin{cases} x < 0 \\ 3x = -5 \end{cases}$$

$$\begin{cases} x \geq 0 \\ x = -\frac{5}{3} \end{cases} \quad \text{NO}$$

$$\begin{cases} x < 0 \\ x = -1 \end{cases} \quad \text{NO}$$

$\Rightarrow \emptyset \in \mathbb{R}$

$$1c) \frac{3x+1}{\sqrt{x^2+1}} - 3 \cdot \frac{x}{|x|} \quad C.E.: x \neq 0$$

•  $x > 0$

$$\frac{3x+1}{\sqrt{x^2+1}} - 3 \cdot \frac{x}{x} = 0$$

$$\frac{3x+1}{\sqrt{x^2+1}} = 3$$

$$\frac{9x^2+6x+1}{x^2+1} = 9$$

$$\cancel{9x^2+6x+1} = \cancel{9x^2+9}$$

$$x = \frac{4}{3} \checkmark$$

•  $x < 0$

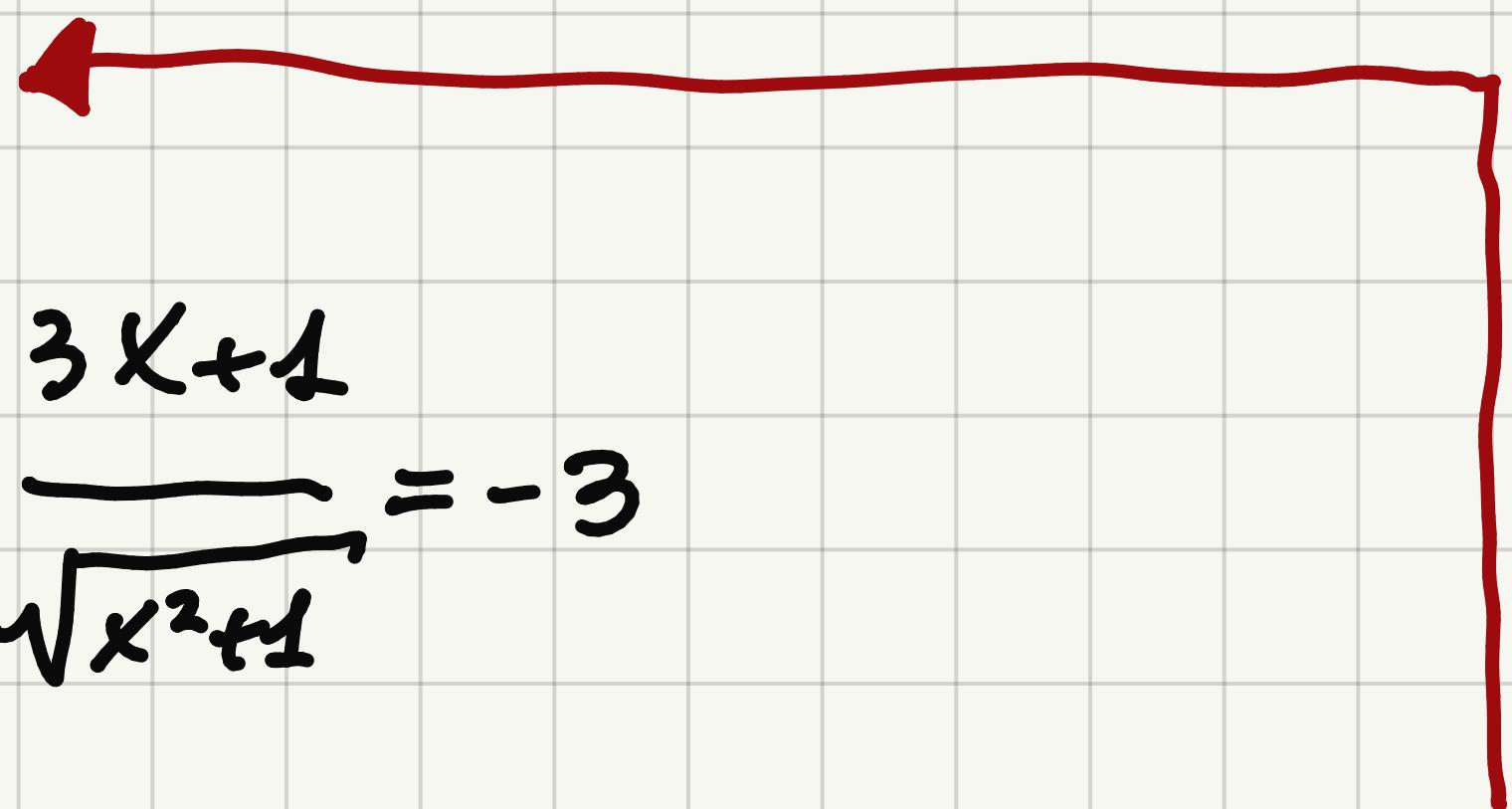
$$\frac{3x+1}{\sqrt{x^2+1}} - 3 \cdot \frac{x}{-x} = 0$$

$$\frac{3x+1}{\sqrt{x^2+1}} = -3$$

$$\frac{9x^2+6x+1}{x^2+1} = 9$$

$$\cancel{9x^2+6x+1} = \cancel{9x^2+9}$$

$$x = -\frac{4}{3} \times$$



$$2a) |1 - |1 - x^2|| > 0$$

$$|1 - x^2| < 1$$

$$\begin{cases} 1 - x^2 < 1 \\ 1 - x^2 > -1 \end{cases} \quad \begin{cases} x^2 > 0 \\ x^2 < 2 \end{cases} \quad \begin{cases} x \neq 0 \\ -\sqrt{2} < x < \sqrt{2} \end{cases}$$

$$2b) | |x| - 1 | > 0$$

$$| |x| - 1 | < 1$$

$$\begin{cases} |x| - 1 < 1 \\ |x| - 1 > -1 \end{cases} \quad \begin{cases} |x| < 2 \\ |x| > 0 \end{cases} \quad \begin{cases} -2 < x < 2 \\ x \neq 0 \end{cases}$$

$$2c) |1 + |x - 1|| \leq |1 - |x + 1||$$

$$|1 - |x - 1|| \geq 1 + |x - 1|$$

$$S_1 \begin{cases} 1 - |x + 1| \leq 0 \\ 1 - |x + 1| \leq -1 - |x - 1| \end{cases}$$

$$S_2 \begin{cases} 1 - |x + 1| \geq 0 \\ 1 - |x + 1| \geq 1 + |x - 1| \end{cases}$$

$$S_1 \begin{cases} |x + 1| \geq 1 \\ |x - 1| - |x + 1| \leq -2 \end{cases}$$

$$S_2 \begin{cases} |x + 1| \leq 1 \\ |x - 1| + |x + 1| \leq 0 \end{cases}$$

$S_1$

$$\begin{cases} |x + 1| > 1 \\ |x + 1| > |x - 1| + 2 \end{cases}$$

$$\begin{cases} x < -2, x > 0 \\ |x + 1| > |x - 1| + 2 \end{cases}$$

$$\bullet x < -2 \rightarrow -x - 1 = -x + 1 + 2 \quad \forall x \in \mathbb{R}$$

$$\bullet x > 0 \rightarrow x + 1 \geq |x - 1| + 2 \quad |x - 1| \leq x - 1 \rightarrow x \geq 1$$

$$\begin{cases} |x+1| \leq 1 \\ |x+1| \leq -(x-1) \end{cases} \rightarrow \exists x \in \mathbb{R}$$

$s_2$

## 6 - EQUAZIONI E DISEQUAZIONI LOGARITMICHE

CE

1a)  $\log x = \log(x^2 - 2)$

$$x = x^2 - 2$$

$$\begin{cases} x > 0 \\ x^2 - 2 \geq 0 \Rightarrow x \geq \sqrt{2}, \end{cases} \Rightarrow x > \sqrt{2}$$

$$x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{3}}{2} \quad \begin{matrix} 2 \\ -1 \end{matrix} \Rightarrow x = 2$$

CE

1b)  $\log(x-1) + \log(x^2+3) = \log(x^2-1)$

$$\begin{cases} x-1 > 0 \\ x^2+3 > 0 \\ x^2-1 \leq 0 \end{cases}$$

$$\log((x-1)(x^2+3)) = \log(x^2-1)$$

$$x^2 - x - 2 + 3x - 1 = x^2 - 1$$

$$x^2 - 2x + 3 = 0$$

$$\begin{cases} x > 1 \\ \forall x \in \mathbb{R} \\ x \neq 1, x \neq -1 \end{cases} \Rightarrow x > 1$$

$$\Delta = 4 - 12 < 0 \Rightarrow \exists x \in \mathbb{R}$$

1c)  $3\log(x-1) - (\log x - 1)^{\frac{1}{3}} - 2 = 0$

$$\begin{cases} x-1 > 0 \Rightarrow x > 1 \\ x > 0 \end{cases}$$

$$\log x - 1 = t$$

$$3t - \sqrt[3]{t} - 2 = 0$$

$$t = (3t - 2)^3$$

$$t = 27t^3 - 54t^2 + 36t - 8 \quad 27t^3 - 54t^2 + 35t - 8 = 0$$

$$\begin{array}{c|ccc|c} & 27 & -54 & 36 & -8 \\ \hline 1 & & 27 & -27 & 8 \\ \hline & 27 & -27 & 8 & 0 \end{array}$$

$\Delta < 0$

$$(t-1)(27t^2 - 27t + 8) = 0$$

$$t-1=0 \rightarrow t=1 \Rightarrow \log x - 1 = 1 \quad \log x = 2 \quad x = 100$$

$$2a) \log_{\frac{3}{4}}(6+5x) \geq 0 \quad \bullet \quad \alpha = \frac{3}{4} < 1$$

$$\text{C.E. } \begin{cases} 6+5x > 0 \\ 6+5x \leq 1 \end{cases} \quad \begin{cases} x > -\frac{6}{5} \\ x \leq -1 \end{cases} \rightarrow -\frac{6}{5} \leq x \leq -1$$

$$2b) 2\log(x-3) - \log(3x-1) > \log(1+x^2) \quad \exists x \in \mathbb{R}$$

$$\left. \begin{array}{l} \{ x-3 > 0 \\ \{ 3x-1 > 0 \\ 1+x^2 > 0 \end{array} \right. \quad \bullet \quad \forall x \in \mathbb{R}$$

$$\log\left(\frac{(x-3)^2}{(3x-1)(1+x^2)}\right) > 0 \quad \rightarrow \frac{(x-3)^2}{(3x-1)(1+x^2)} > 1$$

$$\left. \begin{array}{l} \{ x > 3 \\ \{ x > 1/3 \\ \frac{(x-3)^2 - (3x-1)(1+x^2)}{(3x-1)(1+x^2)} > 0 \end{array} \right.$$

$$\left. \begin{array}{l} \{ x > 3 \\ \{ x > 1/3 \\ \frac{x^2 - 6x + 9 - 3x^3 + x^2 - 3x + 1}{3x-1} > 0 \end{array} \right.$$

$$\left. \begin{array}{l} \{ x > 3 \\ \{ \frac{-3x^3 + 2x^2 - 9x + 10}{3x-1} > 0 \end{array} \right.$$

	-3	2	-9	10
1	-3	-1	10	
	-3	-1	-10	0

$$\left. \begin{array}{l} \{ x > 3 \\ \{ \frac{(x-3)(-3x^2 - 1 - 10)}{3x-1} > 0 \end{array} \right. \quad \Delta L0 \quad \left. \begin{array}{l} \{ x > 3 \\ \{ x-1 > 0 \\ x < 1 \end{array} \right. \Rightarrow \exists x \in \mathbb{R}$$

$$2c) \log(\sqrt{36-x^2}-x) > \log(\sqrt{1+x^2}-x)$$

$$\left. \begin{array}{l} C \\ E \end{array} \right\} \sqrt{36-x^2}-x > 0$$

$$\sqrt{1+x^2}-x > 0$$

$$\sqrt{36-x^2}-x > \sqrt{1+x^2}-x$$

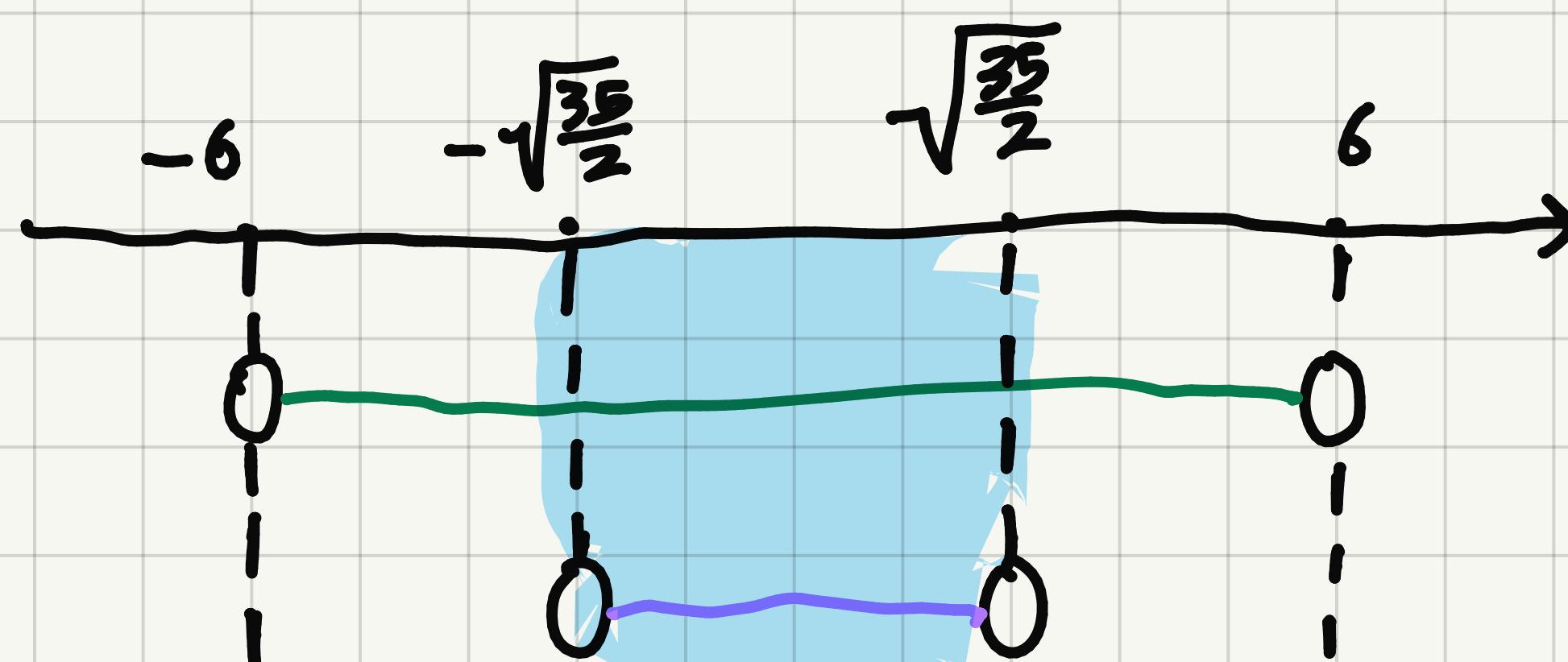
$$\left. \begin{array}{l} 36-x^2 > x^2 \\ 1+x^2 > x^2 \end{array} \right\}$$

$$36-x^2 > 1+x^2$$

$$\bullet -6 < x < 6$$

$$1 > 0 \quad \forall x \in \mathbb{R}$$

$$\bullet -\sqrt{\frac{35}{2}} < x < \sqrt{\frac{35}{2}}$$

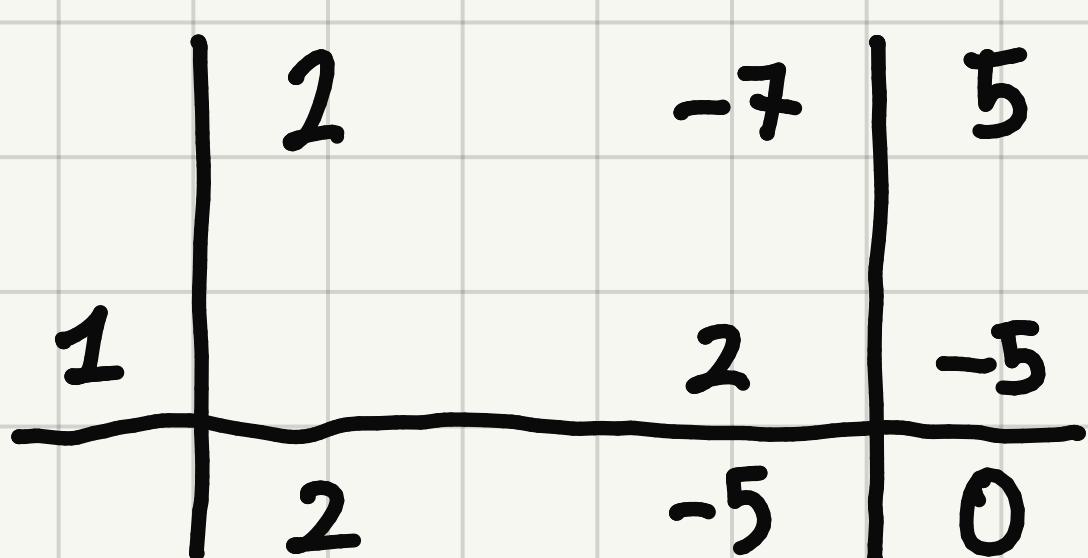
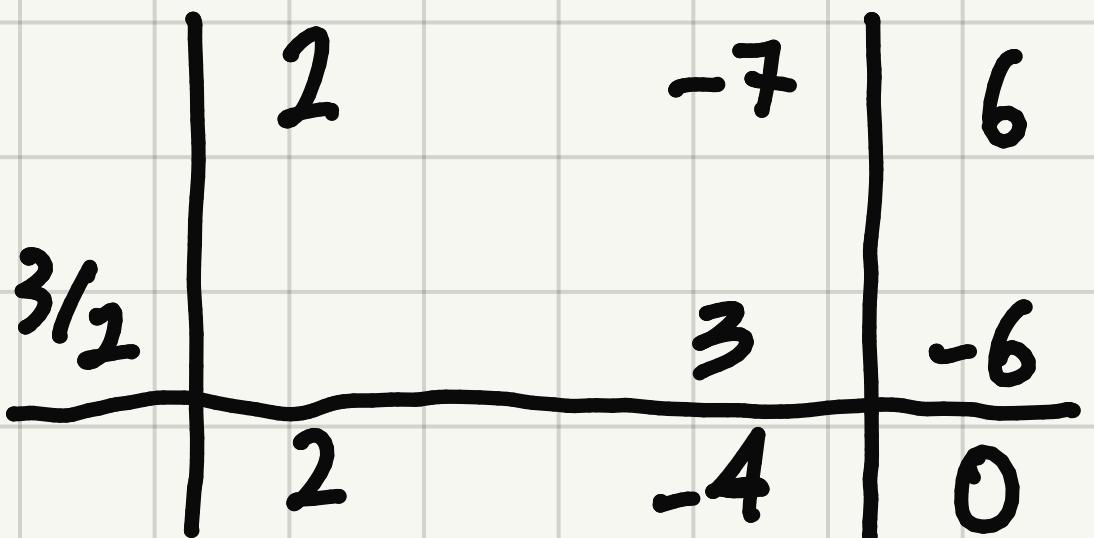


$$-\sqrt{\frac{35}{2}} < x < \sqrt{\frac{35}{2}}$$

$$21) \log_{\sqrt{2x^2-7x+6}}\left(\frac{x}{3}\right) > 0$$

$$\left\{ \begin{array}{l} 2x^2 - 7x + 6 > 0 \\ \sqrt{2x^2 - 7x + 6} > 1 \end{array} \right.$$

$$\left\{ \begin{array}{l} \sqrt{2x^2 - 7x + 6} > 1 \text{ BASE} > 1 \\ \frac{x}{3} > 1 \end{array} \right. \cup \left\{ \begin{array}{l} \sqrt{2x^2 - 7x + 6} < 1 \text{ BASE} < 1 \\ \frac{x}{3} < 1 \end{array} \right.$$



$$\left\{ \begin{array}{l} (x - \frac{3}{2})(2x - 4) > 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} (x - 1)(2x - 5) > 0 \end{array} \right.$$

$$x > 3$$

$$\left\{ \begin{array}{l} (x - \frac{3}{2})(2x - 4) > 0 \end{array} \right.$$

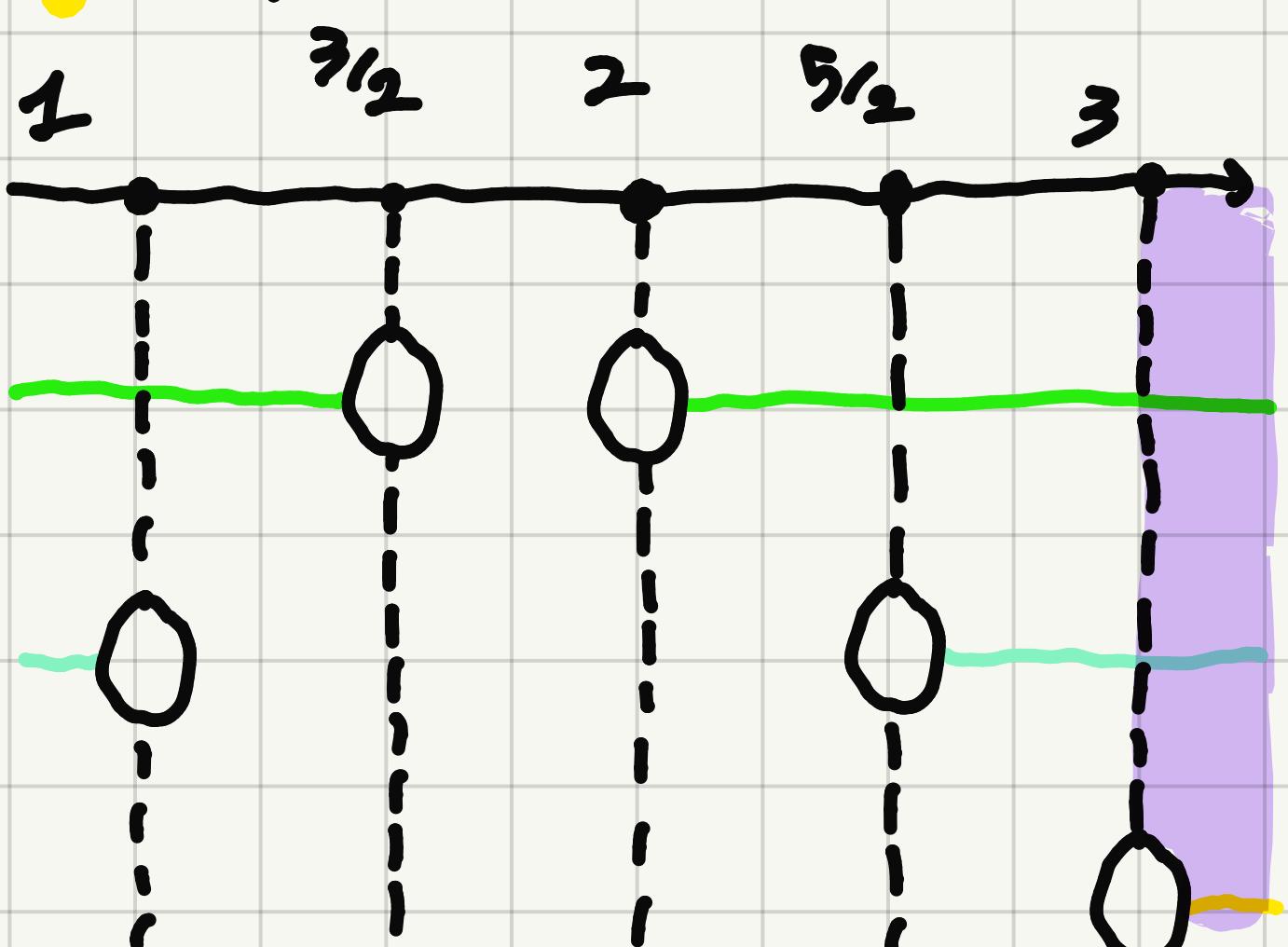
$$\left\{ \begin{array}{l} (x - 1)(2x - 5) < 0 \end{array} \right.$$

$$x < 3$$

$$\bullet \left\{ \begin{array}{l} x < \frac{3}{2} \vee x > 2 \end{array} \right.$$

$$\bullet \left\{ \begin{array}{l} x < 1 \wedge x > \frac{5}{2} \end{array} \right.$$

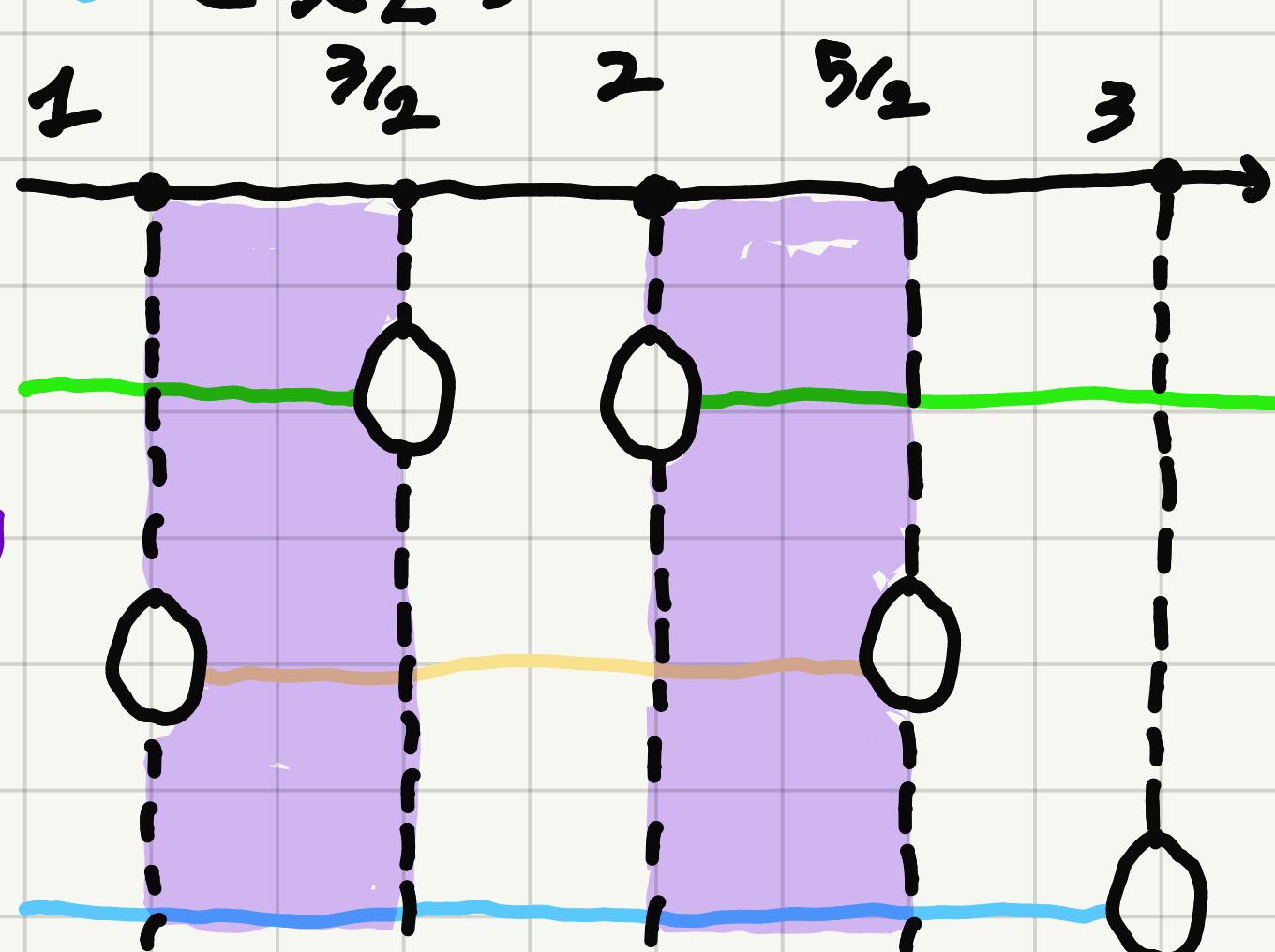
$$\bullet \left\{ \begin{array}{l} x > 3 \end{array} \right.$$



$$\bullet \left\{ \begin{array}{l} x < \frac{3}{2} \wedge x > 2 \end{array} \right.$$

$$\bullet \left\{ \begin{array}{l} 1 < x < \frac{5}{2} \end{array} \right.$$

$$\bullet \left\{ \begin{array}{l} x < 3 \end{array} \right.$$



$$1 < x < \frac{3}{2} \vee 2 < x < \frac{5}{2} \vee x > 3$$

## 7 - EQUAZIONI E DISEQUAZIONI ESPONENZIALI

1a)  $2e^{2x} - 6e^x + 3 = 0 \quad e^x = t$

$$2t^2 - 6t + 3 = 0 \quad t = \frac{6 \pm \sqrt{32}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

$$\Rightarrow e^x = \frac{3 \pm \sqrt{3}}{2} \Rightarrow x = \ln\left(\frac{3 \pm \sqrt{3}}{2}\right)$$

1b)  $5^{2-x} = 25 \cdot 5^{x^2-1}$

$$5^{2-x} = 5^2 \cdot 5^{x^2-1} \quad 5^{2-x} = 5^{x^2+1}$$

$$1-x = x^2+1 \quad x^2+x=0$$

$$x = \begin{cases} 0 \\ -1 \end{cases}$$

1c)  $2-e^x + 2\sqrt{|e^x-1|} = 0$

$$2\sqrt{|e^x-1|} = e^x - 2$$

$$!e^x - 2 > 0 \rightarrow x > \ln 2$$

$$4|e^x-1| = e^{2x} - 4e^x + 4$$

$$4e^x - 4 = e^{2x} - 4e^x + 4$$

$$e^{2x} - 8e^x + 8 = 0$$

$$e^x = t$$

$$t^2 - 8t + 8 = 0$$

$$t = \frac{8 \pm \sqrt{32}}{2} = 4 \pm 2\sqrt{2}$$

$$\Rightarrow e^x = 4 + 2\sqrt{2} \Rightarrow x = \ln(4 + 2\sqrt{2})$$

$$2a) \frac{e^x + e^{-x}}{2} < 3$$

$$e^x \cdot (e^x + e^{-x}) < 6 \cdot e^x$$

$$e^{2x} - 6e^x + 1 < 0$$

$$t^2 - 6t + 1 < 0$$

$$t_{1,2} = \frac{6 \pm \sqrt{32}}{2} = 3 \pm \sqrt{8} = 3 \pm 2\sqrt{2}$$

$$\Rightarrow -2\sqrt{2} + 3 < t < 2\sqrt{2} + 3$$

$$\Rightarrow -2\sqrt{2} + 3 < e^x < 2\sqrt{2} + 3$$

$$\Rightarrow \ln(-2\sqrt{2} + 3) < x < \ln(2\sqrt{2} + 3)$$

$$2b) 3^{2x-1} < 3^4x^2-x-1$$

$$\text{BASE} = 3 > 1 \Rightarrow 2x-1 < 4x^2-x-1$$

$$4x^2 - 3x > 0 \quad x(4x-3) > 0$$

$$x < 0 \vee x > \frac{3}{4}$$

$$2c) x^{\sqrt{x}} \geq (\sqrt{x})^x$$

$$(x^{\sqrt{x}})^2 \geq (x^{x/2})^2 \quad x > 0 \quad x > 0, \text{ PER EVITARE } 0^0$$

$$x^{2\sqrt{x}} \geq x^x$$

$$\ln(x^{2\sqrt{x}}) \geq \ln(x^x)$$

$$2\sqrt{x} \ln(x) \geq x \ln(x)$$

$$-\sqrt{x} \ln(x) (2-\sqrt{x}) \geq 0$$

NON HO PROBLEMI DI SEGNO POICHÉ,  
PER PROPRIETÀ DEGLI ESPONENZIALI,  
 $e^x > 0 \quad \forall x \in \mathbb{R}$   
 $e^x = t$

$$\left\{ \begin{array}{l} x > 0 \\ x \geq 0 \\ \sqrt{x} \geq 0 \\ h(x) \geq 0 \\ 2 - \sqrt{x} \geq 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x > 0 \\ x \geq 1 \\ x \leq 4 \end{array} \right. \Rightarrow 1 \leq x \leq 4$$