DETERMINE WHETHER THE FOLLOWING RELATIONS $R \to R$ ARE FUNCTIONS. WHEN THEY ARE, DETERMINE WHETHER THEY ARE INJECTIVE, SURJECTIVE, BIJECTIVE

a) $xRy \Leftrightarrow x^3 - x^2 - y^2 = 0$

$y^2 = x^2(x-1) \to y = \sqrt{x^2(x-1)}$ $|f^*(y)| > 1$

$x=0$ AND $x=1$ HAS THE SAME $y \Rightarrow$ IT IS NOT A FUNCTION

b) $xRy \Leftrightarrow x^2y - 2x + y = 0$

$y(x^2+1) = 2x \to y = \dfrac{2x}{x^2+1}$

$x^2+1 \neq 0 \ \forall x \in A$ ✓; THE SAME x DON'T HAVE $\neq y \Rightarrow$ IT IS A FUNCTION

$x = \dfrac{2 \pm \sqrt{4 - 4y^2}}{2y}$

- INJECTIVITY: THE SAME y HAVE $\neq x \Rightarrow$ IT IS NOT INJECTIVE
- SURJECTIVITY: THERE ARE NO SOLUTIONS FOR $y < -1, y > 1$ $\Rightarrow$ IT IS NOT SURJECTIVE

c) $xRy \Leftrightarrow x^3y + xy^3 - x^2 - y^2 = 0$

$x^2(xy-1) + y^2(x-1) = 0$ $(x^2+y^2)(xy-1) = 0$

CONSIDERING BOTH $x \neq 0 \wedge y \neq 0$ $xy - 1 = 0$

$\to y = \dfrac{1}{x}$ SAME $x \to$ SAME y $\Rightarrow$ IT IS A FUNCTION

$x = \dfrac{1}{y}$

- INJECTIVITY: THE SAME y HAVE $= x \Rightarrow$ IT IS INJECTIVE
- SURJECTIVITY: THERE ARE SOLUTIONS $\forall y \neq 0 \Rightarrow$ IT IS SURJECTIVE

$\Rightarrow$ IT IS BIJECTIVE

FOR EACH OF THE FOLLOWING RELATION, DETERMINE IF THEY ARE FUNCTION (OR IF THEY CONTAIN/ARE CONTAINED IN A FUNCTION) AND, IF THEY ARE, VERIFY IF THEY ARE INJECTIVE, SURJECTIVE, BIJECTIVE

A) $A=\{1,2,3,4\}$ $B=\{1,2,3\}$ $R: A \to B$

$$M(R) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

- R IS NOT A FUNCTION. IN FACT, THE 2^{ND} ROW HAS MORE "1"
- IS THERE ANY $f: A \to B \mid R \subseteq f$?

$R \subseteq f \Leftrightarrow M(R) \leq M(f) \Leftrightarrow \forall_{ij} (M(R)_{ij} \leq M(f)_{ij})$. SO,

$$M(f) = \begin{vmatrix} 1 & * & * \\ 1 & 1 & * \\ * & 1 & * \\ * & 1 & 1 \end{vmatrix}$$

2^{ND} AND 4^{TH} ROW DO NOT PERMIT IT TO BE A FUNCTION

- IS THERE ANY $f: A \to B \mid f \subseteq R$? SIMILARLY, $M(f) \leq M(R)$

$$M(f) = \begin{vmatrix} * & 0 & 0 \\ * & * & 0 \\ 0 & * & 0 \\ 0 & * & * \end{vmatrix} \to M(f) = \begin{vmatrix} 1 & 0 & 0 \\ * & * & 0 \\ 0 & 1 & 0 \\ 0 & * & * \end{vmatrix}$$

THERE ARE 4 DIFFERENT $f \subseteq R$:

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

WE CAN CLASSIFY THEM:

1) NEITHER INJECTIVE NOR SURJECTIVE
2) NEITHER INJECTIVE NOR SURJECTIVE
3) NOT INJECTIVE, IT IS SURJECTIVE
4) NOT INJECTIVE, IT IS SURJECTIVE

B) $A = \{1,2,3,4\}$ $B = \{1,2,3,4\}$ $R: A \to B$

$$M(R) = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

- R IS NOT A FUNCTION. 2^{ND} AND 4^{TH} ROW HAVE NO "1"
- IS THERE ANY $f: A \to B \mid R \subseteq f$? $M(R) \leq M(f)$

$$M(f) = \begin{vmatrix} * & * & * & * \\ * & 1 & * & * \\ * & * & * & * \\ 1 & * & * & * \end{vmatrix} =$$

$$= \begin{vmatrix} * & * & * & * \\ 0 & 1 & 0 & 0 \\ * & * & * & * \\ 1 & 0 & 0 & 0 \end{vmatrix}.$$ THERE ARE 16 DIFFERENT $R \subseteq f$:

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix},$$