PROVE THAT $\forall x(Rx \to Sx), \exists x(Sx \to Tx) \not\models \forall x(Rx \to Tx)$

$F \models \varphi \Leftrightarrow F \cup \{\neg\varphi\}$ IS UNSATISFIABLE. So,

$F \not\models \varphi \Leftrightarrow F \cup \{\varphi\}$ IS SATISFIABLE

$$\forall x(Rx \to Sx), \exists x(Sx \to Tx), \neg(\forall x(Rx \to Tx))$$

$$Sa \to Ta, Ra \to Sa, \neg\forall x(Rx \to Tx), \forall x(Rx \to Sx)$$

$$Sa \to Ta, Ra \to Sa, \neg(Rb \to Tb), Rb \to Sb, \forall x(Rx \to Sx)$$

α- FORMULA

$$Sa \to Ta, Ra \to Sa, Rb, \neg Tb, Rb \to Sb, \forall x(Rx \to Sx)$$

β - FORMULA

$$Sa \to Ta, Ra \to Sa, Rb, \neg Tb, Sb$$

β-FORMULA

$$Ta, Ra \to Sa, Rb, \neg Tb, Sb$$

β-FORMULA

SATISFIABLE LEAF $\longleftarrow$ $Ta, Sa, Rb, \neg Tb, Sb$

$M = \{a, b\}$ $[[R]] = \{b\}$ $[[S]] = \{a, b\}$ $[[T]] = \{a\}$

PROVE THE FOLLOWING ASSERTIONS

a) $\forall x(Rx \to Sx) \models \forall x Rx \to \forall x Sx$

BY DEDUCTION THEOREM $\forall x(Rx \to Sx), \forall x Rx \models \forall x Sx$

WE PROVE $F \cup \{\neg\varphi\} = \{\forall x(Rx \to Sx), \forall x Rx, \neg\forall x Sx\}$ IS UNSATISFIABLE

$$\forall x(Rx \to Sx), \forall x Rx, \neg\forall x Sx$$

$$|$$

$$Ra \to Sa, Ra, \neg Sa, \ldots$$

β- EXPANSION

$$\neg Ra, Ra, \neg Sa \qquad\qquad Sa, Ra, \neg Sa$$

BOTH UNSATISFIABLE LEAF $\to F \cup \{\neg\varphi\}$ UNSATISFIABLE

$\to F \models \varphi$

b) $\forall x Rx \to \exists x Sx \models \exists x (Rx \to Sx)$

$F \models \varphi \Leftrightarrow F \cup \{\neg \varphi\}$ IS UNSATISFIABLE

$F \cup \{\neg\varphi\} = \{\forall x Rx \to \exists x Sx, \neg(\exists x (Rx \to Sx))\}$

$\forall x Rx \to \exists x Sx, \neg(\exists x (Rx \to Sx))$

| α-EXPANSION

$\forall x Rx \to \exists x Sx, \exists x Rx, \neg \exists x Sx$

$\neg(\forall x Rx), \exists x Rx, \neg\exists x Sx, \ldots$

$\exists x Sx, \exists x Rx, \neg \exists x Sx, \ldots$

$\exists x \neg Rx, \exists x Rx, \neg E x Sx$

$Ra, Sa, \neg Sa$

$\neg Ra, Ra, \neg Sa$

BOTH UNSATISFIABLE LEAFS $\Rightarrow F \cup \{\neg\varphi\}$ UNSATISFIABLE

$\Rightarrow F \models \varphi$