

14

ΣΣΣ

5,9 RESOLUTION

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ΣΤΖΞ

12,13 RESOLUTION

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14,15 RESOLUTION

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THE SET HAS A CLOSED EXPANSION \Rightarrow THE STATEMENT IS CORRECT

$$b) X \wedge Y \rightarrow Z \models (X \rightarrow Z) \wedge (Y \rightarrow Z)$$

WE HAVE TO SEE IF $F = \{X \wedge Y \rightarrow Z, \neg((X \rightarrow Z) \wedge (Y \rightarrow Z))\}$

IS INCONSISTENT

| STEP | FORMULA | RULE |
|------|--|-----------------------|
| 1 | $\{X \wedge Y \rightarrow Z\}$ | ASSUMPTION |
| 2 | $\{\neg((X \rightarrow Z) \wedge (Y \rightarrow Z))\}$ | ASSUMPTION |
| 3 | $\{\neg(X \wedge Y), Z\}$ | 1, β -EXPANSION |
| 4 | $\{\neg X, \neg Y, Z\}$ | 3, β -EXPANSION |
| 5 | $\{\neg(X \rightarrow Z), \neg(Y \rightarrow Z)\}$ | 2, β -EXPANSION |
| 6 | $\{X, \neg(Y \rightarrow Z)\}$ | 5, d-FORMULA |
| 7 | $\{\neg Z, \neg(Y \rightarrow Z)\}$ | 5, d-FORMULA |
| 8 | $\{X, Y\}$ | 6, d-FORMULA |
| 9 | $\{X, \neg Z\}$ | 6, d-FORMULA |
| 10 | $\{\neg Z, Y\}$ | 7, d-EXPANSION |
| 11 | $\{\neg Z\}$ | 7, d-EXPANSION |
| 12 | $\{Y\}$ | 10, 11 RESOLUTION |
| 13 | $\{\neg X, Z\}$ | 4, 12 RESOLUTION |

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TO ELIMINATE IT, WE NEED X AND $\neg Z$ IN \neq STEPS \Rightarrow SET CONSISTENT

\Rightarrow THE STATEMENT IS WRONG

C) $\gamma \rightarrow x \vee z, z \rightarrow \neg y, \neg z \rightarrow \neg x \models x \leftrightarrow y$

F: $\{\gamma \rightarrow x \vee z, z \rightarrow \neg y, \neg z \rightarrow \neg x, \neg(x \leftrightarrow y)\} \models$

STEP

FORMULA

RULE

1

$\{\gamma \rightarrow x \vee z\}$

ASSUMPTION

2

$\{z \rightarrow \neg y\}$

ASSUMPTION

3

$\{\neg z \rightarrow \neg x\}$

ASSUMPTION

4

$\{x \leftrightarrow y\}$

ASSUMPTION

5

$\{\neg y, x \vee z\}$

1, P- EXPANSION

6

$\{x, \neg y, z\}$

5, P- EXPANSION

7

$\{\neg y, \neg z\}$

2, P- EXPANSION

8

$\{\neg x, z\}$

3, P- EXPANSION

9

$\{\neg(x \rightarrow y)\}$

4, BIIMPLICATION

10

$\{\neg(y \rightarrow x)\}$

4, BIIMPLICATION

11

$\{x\}$

9, d- EXPANSION

12

$\{\neg y\}$

9, d- EXPANSION

13

$\{y\}$

10, d- EXPANSION

14

$\{\neg x\}$

10, d- EXPANSION

15

1

4, 12 RESOLUTION

THE SET HAS A CLOSED EXPANSION \Rightarrow THE STATEMENT IS CORRECT